Special Session 52: Fractional Differential and Integral Equations, Theory and Applications

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Fractional Differential Equations are met in modeling a great variety of phenomena in Visco-elasticity/damping, Chaos, biology, electronics, chemistry, signal processing, diffusion and wave propagation, percolation, as well as in mathematical economy. This session will focus on the recent developments in the theory of fractional differential equations (such as global existence/blow-up, regularity, long-time behavior, oscillation), their numerical analysis, and recent developments in practical applications.

On the applications of Volterra equations in image processing and restoration

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A generalized understanding of PDEs involving fractional time derivatives/integrals leads to Volterra equations which somehow interpolate parabolic and hyperbolic models, therefore enjoying intermediates properties, in fact properties related to diffusion/smoothing of the image. In this talk we provide precise details of this approach and practical results supporting the goodness of this approach.

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Global existence of the solutions of a class fractional - differential equations with a Legendre derivative

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In this talk we consider the problem

$$D^\alpha\Big(t^k D^\beta\Big)u(x,t) = t^\gamma \Delta u(x,t)$$
$$+f(t,x,u,u_{x_1},u_{x_2},\dots,u_{x_n}), \quad t>0, x\in\mathbb{R}^n,$$
$$\lim_{t\longrightarrow 0} u(x,t) = u(x,0), \quad x\in\mathbb{R}^n,$$

where D^α and D^β are fractional derivatives in t, 0

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Asymptotic stability of abstract dissipative systems with infinite memory

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We consider in this work the problem of asymptotic behavior of solutions to an abstract linear dissipative integrodifferential equation with infinite memory (past history) modeling linear viscoelasticity. Under a boundedness condition on the history data, we show that the stability of the system holds for a much larger class of the convolution kernels than the one considered in the literature, and we provide a relation between the decay rate of the solutions and the growth of the kernel at infinity. Additionally, our decay estimates improve, in some particular cases, the decay rates known in the literature. Several particular applications are also given.

These results have been recentely published in J. Math. Anal. Appl., 382 (2011), 748-760.

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Solvability of some partial integral equations in Hilbert space

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An integral equation of contact problem of the theory of visco elasticity of mixed Fredholm and Volterra type with spectral parameter depending on time is considered. In the case where the final value of parameter coincides with some isolated point of the spectrum of Fredholm operator the additional conditions of solvability are established.

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Solving fractional Riccati differential equations using modified variational iteration method

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In this paper, a modified variational iteration method (MVIM) for solving Fractional Riccati differential equations will be introduced. Also we solve the fractional Riccati differential equation using variational iteration method by considering Adomians polynomials for nonlinear terms. The main advantage of the MVIM is that it can enlarge the convergence region of iterative approximate solutions. Hence, the solutions obtained using the MVIM give good approximations for a larger interval. Numerical results show that the method is simple and effective.

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Finite-dimensional behavior in a thermosyphon with a viscoelastic fluid

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We analyze the motion of a Maxwellian viscoelastic fluid [Faith Morrison,2001] in the interior of a closed loop thermosyphon under the effects of natural convection and a given external heat flux in the diffusion case. This nonlinear model for viscoelastic fluids is, in some sense, a generalization of the previous models [A. Rodriguez-Bernal and E.S. Van Vleck, 1998 and Angela Jimenez-Casas and Alfonso Matias Lozano Ovejero, 2001 between others]. We study the asymptotic properties of the fluid inside the thermosyphon and we use the result and techniques from [J. Yasappan, A. Jimenez-Casas and M. Castro, 2012] to obtain an exact finite dimensional system representing the dynamics on the inertial manifold for this diffusion case.

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Differences between fractional- and integer-order dynamics

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The qualitative theory of fractional-order dynamical systems and it applications to the sciences and engineering is a recent focus of interest of many researchers. In addition to natural similarities that can be drawn between fractional- and integer-order derivatives and fractional- and integer-order dynamical systems, very important differences arise as well. Since in many cases, qualitative properties of integer-order dynamical systems cannot be extended by generalization to fractional-order dynamical systems, the analysis of fractional-order dynamical systems is a very important field of research.This talk is devoted to presenting qualitative contrasts between fractional- and integer-order dynamical systems. For example, even though the integer-order derivative of a periodic function is obviously a periodic function of the same period, the fractional-order derivative of a non-constant periodic function cannot be a periodic function of the same period. As a consequence, periodic solutions do not exist in a wide class of fractional-order dynamical systems. Moreover, important differences will be highlighted concerning the asymptotic stability analysis of fractional- and integer-order dynamical systems, as well. Numerical simulations will also be presented to substantiate the theoretical results.

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Lyapunov stability for differential systems of fractional order

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In this talk we introduce the fractional comparison principle. Then we discuss some results about boundedness and Mittag-Leffler stability of solutions of fractional differential systems using Lyapunov-type functions. Also, we give the converse Lyapunov theorem for the Mittag-Leffler stability.

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A theory of non-local linear drift wave transport

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Transport events in tokamak turbulent plasmas often exhibit non-local or non-diffusive action at a distance features that so far have eluded a conclusive theoretical description. A prominent candidate for explaining the suggestive non-local features of plasma turbulence is the inclusion of a fractional velocity derivative in the Fokker-Planck (FP) equation leading to an inherently non-local description as well as giving rise to non-Gaussian Probability Distribution Functions (PDFs) of e.g. densities and heat flux. The non-locality is introduced through the integral description of the fractional derivative and the non-Maxwellian distribution function drives the observed PDFs of densities and heat flux far from Gaussian. The aim of this study is to elucidate the effects of a non-Maxwellian distribution function induced by the fractional velocity derivative in the Fokker-Planck equation. Using fractional generalizations of the Liouville equation, kinetic descriptions have been developed previously. It has been shown that the chaotic dynamics can be described by using the FP equation with coordinate fractional derivatives as a possible tool for the description of anomalous diffusion. Much work has been devoted on investigation of the Langevin equation with Levy white noise, or related fractional FP equation. Furthermore, fractional derivatives have been introduced into the FP framework in a similar manner as the present work but a study including drift waves is still called for.

To this end we quantify the effects of the fractional derivative in the FP equation in terms of a modified dispersion relation for density gradient driven linear plasma drift waves where we have considered a case with constant external magnetic field and a shear-less slab geometry. In order to calculate an equilibrium PDF we use a model based on the motion of a charged Levy particle in a constant external magnetic field obeying non-Gaussian, Levy statistics. This assumption is the natural generalization of the classical example of the motion of a charged Brownian particle with the usual Gaussian statistics. The fractional derivative is represented with the Fourier transform containing a fractional exponent. We find a relation for the deviation from Maxwellian distribution described by ϵ through the quasi-neutrality condition and the characteristics of the plasma drift wave are fundamentally changed, i.e. the values of the growth-rate γ and real frequency ω are significantly altered. A deviation from the Maxwellian distribution function alters the dispersion relation for the density gradient drift waves such that the growth rates are substantially increased and thereby may cause enhanced levels of transport.

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Exponential decay in thermoelastic systems with boundary delay

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In this paper we consider a thermoelastic system with boundary delay. Under suitable assumption on the weight of the delay, we prove the well posedness of the system and use the energy method to show that the damping effect through heat conduction is still strong enough to uniformly stabilize the system even in the presence of boundary time delay. Our result extends the stability region of the system and improves earlier results existing in the literature.

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Almost periodic mild solutions to evolutions equations with stepanov almost periodic coefficients

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In this work, we study sufficient conditions for the existence of almost periodic mild solutions to autonomous fractional differential equation, where the fractional derivative is understood in the Riemann-Liouville sense:

$$D_t^{\alpha} u(t) = Au(t) + D_t^{\alpha-1} f(t, u(t)), \quad t \in \mathbb{X}.$$

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Some results to evolutions equations with stepanov-like pseudo-almost periodic coefficients

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In this work we make use of the properties of Stepanov-like pseudo-almost periodic functions to study the existence of pseudo-almost periodic solutions to fractional differential equation and integro-differential equation with Stepanov pseudo-almost periodic coefficients.

We study the existence of pseudo-almost periodic mild solutions to fractional differential equation:

$$D_t^{\alpha} u(t) = Au(t) + D_t^{\alpha-1} f(t, u(t)),\ t \in \mathbb{R}.$$

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