Special Session 53: Greedy Algorithms and Tensor Product Representations for High-Dimensional Problems

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The curse of dimensionality remains a major obstacle to numerical simulations in various fields such as quantum chemistry, molecular dynamics or uncertainty quantification for instance. Tensor product representations of multivariate functions are a promising way to avoid this difficulty. Besides, greedy algorithms have been used in many contexts in order to provide satisfactory, but not usually optimal, tensor product representations. The aim of this symposium is to bring together scientists from the greedy algorithms and tensor product representation communities, in order to interact on these subjects.

Sparse adaptive Taylor approximation algorithms for parametric and stochastic elliptic PDEs

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The numerical approximation of parametric partial differential equations is a computational challenge, in particular when the number of involved parameter is large. We consider a model class of second order, linear, parametric, elliptic PDEs on a bounded domain D with diffusion coefficients depending on the parameters in an ane manner. For such models, it was shown that under very weak assumptions on the diusion coecients, the entire family of solutions to such equations can be simultaneously approximated by multivariate sparse polynomials in the parameter vector y with a controlled number N of terms. The convergence rate in terms of N does not depend on the number of parameters, which may be arbitrarily large or countably infinite, thereby breaking the curse of dimensionality. However, these approximation results do not describe the concrete construction of these polynomial expansions, and should therefore rather be viewed as benchmark for theconvergence analysis of numerical methods. We present an adaptive numerical algorithmfor constructing a sequence of sparse polynomials that is proved to converge toward the solution with the optimal benchmark rate. Numerical experiments are presented in large parameter dimension, which confirm the effectiveness of the adaptive approach.

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Greedy algorithms for non symmetric linear problems with uncertainty

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Greedy algorithms, also called Progressive Generalized Decomposition algorithms, are known to give very satisfactory results for the approximation of the solution of minimization problems, typically symmetric hermitian uncertain problems with a large number of random parameters. Indeed, their formulation leads to discretized problems whose complexity evolves linearly with respect to the number of parameters, while avoiding the curse of dimensionality. However, naive adaptations of these methods to non-symmetric problems may not converge towards the desired solution. In this talk, different versions of these algorithms will be presented, along with convergence results.

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Geometric structures in tensor representations

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In this talk we discuss about the geometric structures associated with tensor representations based in subspaces. In particular, we use the Grassmann Banach manifold to characterize the manifold of tensors in Tucker format with fixed rank. It allows to extend the dynamical tensor approximation framework to tensor Banach spaces.

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The POD-Greedy method: convergence rates and applications

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Iterative approximation algorithms are successfully applied in parametric approximation tasks. In particular, reduced basis (RB) methods make use of a greedy algorithm for approximating solution sets of parametrized partial differential equations. Recently, a-priori convergence rate statements for this algorithm have been given (Buffa et al. 2009, Binev et al. 2010). When addressing time-dependent parametric problems with RB-methods, the POD-Greedy algorithm (Haasdonk and Ohlberger 2008) is typically applied. In this algorithm, each greedy step is invoking a temporal compression by performing a proper orthogonal decomposition (POD). Using a suitable coefficient representation of the POD-Greedy algorithm, we recently have shown that the existing convergence rate results of the Greedy algorithm can be extended. In particular, exponential or algebraic convergence rates of the Kolmogorov n-widths are maintained by the POD-Greedy algorithm.

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Applications in mathematical finance of a greedy algorithm for solving high-dimensional partial differential equations.

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We study an algorithm which has been proposed in Ammar et al. (2007) and analyzed in Cancès et al. (2010) and Le-Bris et al. (2009) to solve high dimensional partial differential equations. The idea is to represent the solution as a sum of tensor products and to compute iteratively the terms of this sum. We will present the application of this non linear approximation method to the option pricing problem. This leads us to consider two extensions of the standard algorithm, which applies to *symmetric linear*partial differential equations: (i) *nonsymmetric linear* problems to value European options, (ii) *nonlinear variational* problems to price American options. For European options, to overcome the fact that we have to solve a non-symmetrical problem we use an implicit-explicit scheme. For American options, we consider the idea proposed in Cancès et al. (2010) of penalizing the constraints of the problem in order to obtain a symmetrical problem. Finally, we study the interest of this method as a variance reduction method. The idea is to use the method to solve the backward Kolmogorov equation associated to a pricing problem in relatively high dimension to solve related pricing problems in very high dimension.

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Regularized reconstruction with series kernels

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The main focus of this talk lies on scattered data approximation using series kernels. The latter are positive definite kernel functions that possess expansions in terms of simple basis functions. In many examples these simple basis functions are elementary tensors products, i.e., tensor products of univariate basis functions. We focus on two aspects: The first aspect is the subtle interplay between the intrinsic (multi-scale, tensor product) structure of the kernel and the choice of data locations in scattered data approximation problems. The second aspect concerns the choice of coefficients with respect to the basis of the trial space. We propose a discrete optimization problem to find the coefficients, based on the well-established compressed sensing techniques. These techniques are known to be greedy in the sense that, under certain conditions, they minimize the number of non-zero coeficients. Convergence results for the function reconstruction are obtained by using sampling inequalities. This is partly based on joint works with M. Griebel and B. Zwicknagl.

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Optimization in hierarchical tensor formats

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We will discuss recently introduced hierarchical Tucker representation (Hackbusch, et al.), HT format or tree tensor networks. The particular case of TT-tensors can be written by a matrix product representation (matrix product states (MPS) in quantum information theory). We consider numerical methods solving optimizations problem within a prescribed format, focusing on i) L_2-approximation, ii) linear equations and iii) eigenvalue problems. We will develop differntial equations of gradient flow, and projected gradient methods and an alternating linear scheme, or alternating direction scheme, which is a generalization ot an alternating least square (ALs) together with a modification (MALS) which resembles the density matrix renormalization group algorithm (DMRG). Additionally we will discuss a Newton type method. All these methods are local opitmzation schemes and may suffer from the existence of local minima. Greedy type approaches may be applied for these formats as well.

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