Special Session 55: Nonlinear Elliptic and Parabolic Problems

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Nonlinear parabolic problems model a great variety of phenomena in physics, chemistry, biochemistry, engineering, ecology and economics. Quite strikingly, they can model very different natural systems with the same set of mathematical models, establishing very abstract rule bodies for describing and understanding the evolution of nature. Moreover, they provide with excellent mathematical toys for testing and developing new mathematical techniques which later apply to wide areas of mathematics. Consequently, their study is fundamental from a series of different perspectives. Nonlinear elliptic problems provide us with the steady-state solutions of parabolic problems. This session gathers to some of the very best world experts in this field in an attempt for lightening the most relevant advances of the last decade.

Positive solutions of semilinear boundary value problems of logistic type with nonlinear mixed boundary conditions

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The main goal of this talk is analyzing the existence, uniqueness or multiciplicity and stability of the positive solutions of a very general class of semilinear boundary value problems of logistic type with nonlinear mixed boundary conditions of sub and superlinear type. We will analyze the structure of the set of positive solutions of our problem, depending of the sign of the potential which appears on the boundary condition in front of the nonlinearity, and depending of the relative position with respect to the boundary, of the vanishing set of the potential in front of the nonlinearity in the partial differential equation. Local and global bifurcation, monotonicity and continuation methods and rescaling arguments, are the main technical tools used to obtain the main results.

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Exporing the evolutionary advantages of quasi-linear dispersal

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In this talk we will discuss how parabolic partial differential equations may be used to study the evolution of dispersal strategies. We will begin with a brief historical overview to set our context. We will then focus on our primary interest, namely exploring the relative advantages of fitness-dependent and random dispersal in a two species competition model. Both species have the same population dynamics, but one species adopts a combination of random and fitness-dependent dispersal and the other adopts random dispersal. In so doing we regard the species as ecologically identical, differing only in their dispersal strategies. The model is realized as a quasi-linear parabolic system that can be regarded as a triangular cross-diffusion system. Global existence of smooth solutions to the system is established under some conditions. When the single species which combines random and fitness-dependent is considered in the absence of its competitor, we show that a strong tendency of the species to move up its fitness gradient leads to a stable equilibrium that can approximate the ideal free distribution. For the two-species competition model, if one species has a strong tendency to move up its fitness gradient, such approximately ideal free dispersal is evolutionarily advantageous relative to random dispersal. Further, bifurcation analysis shows that the two competing species can coexist when one species has only an intermediate tendency to move up its fitness gradient and the other species has a smaller random dispersal rate. We will conclude our discussion by highlighting a number of open mathematical problems that are prompted by our analysis

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Global bifurcation of solutions for crime modeling equations

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This talk will present results on pattern formation in a quasilinear system of two elliptic equations that has been developed by Short et al. [2] as a model for residential burglary. That model is based on the observation that the rate of burglaries of houses that have been burglarized recently and their close neighbors is typically higher than the average rate in the larger community, which creates hotspots for burglary. The patterns generated by the model describe the location of those hotspots. We prove that the system supports global bifurcation of spatially varying solutions from the spatially constant equilibrium, leading to the formation of spatial patterns. The analysis is based on recent results on global bifurcation in quasilinear elliptic systems derived by Shi and Wang [1]. We show in some cases that near the bifurcation point the bifurcating spatial patterns are stable.

[1]. J. Shi and X. Wang. On global bifurcation for quasilinear elliptic systems on bounded domains, Journal of Differential Equations, v.7 (2009), 2788-

2812.

[2]. M.B. Short, M.R D'Orsogna, V.B. Pasour, G.E. Tita, P.J. Brantingham, A.L. Bertozzi and L.B. Chayes. A Statistical model of criminal behavior, Math Models and Methods in Applied Sciences, v.18 (2008), 1249-1267.

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On homocliniic solutions for singular Hamiltonian systems

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We show existence of infinitely many homoclinic orbits at the origin for a class of singular second-order Hamitonian systems. Variational methods are used under the assumption that the potential satisfies the so-called "Strong-Force" condition.

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A forward-backward regularization of the Perona-Malik equation

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Since its introduction in 1990 as a tool of image processing, the Perona-Malik equation has attracted a great deal of interest in the mathematical community. Its numerical implementations are characterized by the onset of jump discontinuities, a phenomenon aptly called staircasing. A novel regularization will be presented in this talk which offers a suggestive mathematical explanation of this phenomenon.

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Diffusion-driven instability for non-autonomous problems

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Reaction-diffusion systems on domains evolving in time can be studied by transforming the original system into a reaction-diffusion system on a fixed domain with time-dependent diffusion coefficients and non-autonomous reaction terms. If the original system satisfies no flux boundary conditions, a spatially homogeneous solution can be stable in the absence of diffusion (as a solution of the resulting system of ordinary differential equations), but unstable in the presence of diffusion. I will discuss conditions for this diffusion-driven instability.

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Single phytoplankton growth on light and nutrient in a water column

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In this talk we shall present a mathematical model of single phytoplankton growth yon light and nutrient in a water column. The model takes form of nonlocal system of Parabolic PDEs. We shall do the steady states analysis in terms of two bifurcation parameters, the light incidence rate I_0 from the top of the water column and nutrient input rate, S_0 from the bottom. In the $I_0 - S_0$ bifurcation plane, we have three regions, namely I-limited region, S-limited region on the entire water column and partial S-limited, partial I-limited in the water column.

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An optimal algebraic invariant to detect any changes of the topological degree

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Through the modern theory of algebraic multiplicities developed by J. Esquinas and C. Mora-Corral under the supervision of the author, one can axiomatize the theory of algebraic multiplicities of eigenvalues of linear operators and construct an optimal invariant detecting any change of the topological degree for general operator pencils. This algebraic-analytic invariant provides us with optimal results in local and global bifurcation theory.

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On the dependence of the population size on the dispersal rate,

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Song Liang

This talk concerns the dependence of the population size for a single species on its random dispersal rate and its applications on the invasion of species. The population size of a single species often depends on its random dispersal rate in non-trivial manners. Previous results show that the population size is usually not a monotone function of the dispersal rate. We construct some examples to illustrate that the population size, as a function of the dispersal rate, can have at least two local maxima. As an application we illustrate that the invasion of species depends upon the dispersal rate of the resident species in complicated manners. Previous results show that the total population is maximized at some intermediate dispersal rate for several classes of local intrinsic growth rates. We find one family of local intrinsic growth rates such that the total population is maximized exactly at zero dispersal rate. We show that the population distribution becomes flatter in average if we increase the dispersal rate, and the environmental gradient is always steeper than the population distribution, at least in some average sense.

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Steady state analysis for a relaxed cross diffusion model

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The mechanism of cross diffusion has been introduced by Shigesada Kawasaki and Terramoto to model the trend of a species to avoid another one and thereby, possibly segregate. In this pioneer paper, cross diffusion pressure takes the form of a linear dependency to population density in the diffusion coefficient in a reaction diffusion system. In this talk, we will study the following system with cross diffusion and relaxation, in the absence of reaction

$$\begin{cases} \partial_t u - \Delta[a(\tilde{v})u] = 0, \text{ in } \Omega, \\ \partial_t v - \Delta[b(\tilde{u})v] = 0, \text{ in } \Omega, \\ -\delta\Delta\tilde{u} + \tilde{u} = u, \text{ in } \Omega, \\ -\delta\Delta\tilde{v} + \tilde{v} = v, \text{ in } \Omega, \\ \partial_n u = \partial_n v = \partial\tilde{u} = \partial_n\tilde{u} = 0, \text{ on } \partial\Omega, \end{cases}$$

which was proposed in a recent paper by Bendahmane et al. In this situation, it is necessary to incorporate nonlinear cross diffusion to generate segregating behaviour. We will address questions such as the global existence of solutions, existence of nonconstant solutions and its linear stability. This is joint work with Thomas Lepoutre (INRIA).

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Pointwise estimates for solutions of singular parabolic problems in $\mathbb{R}^N \times [0, +\infty)$

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Let us consider the following homogeneous quasilinear parabolic equation

$$u_t = divA(x, t, u, Du), \quad (x, t) \in \mathbb{R}^N \times [0, +\infty), \tag{1.1}$$

where the functions $A := (A_1, ..., A_N)$ are assumed to be only measurable in $(x, t) \in \mathbb{R}^N \times [0, +\infty)$, continuous w.r.t. u and Du for almost all (x, t). For p-laplacian-type equation we ask A to satisfy the following structure conditions:

$$A(x,t,u,\eta)\cdot\eta \ \geq c_0|\eta|^p, \quad |A(x,t,u,\eta)| \leq c_1|\eta|^{p-1}, c_0, c_1 > 0 \tag{1.2}$$

for almost all $(x, t) \in \mathbb{R}^N \times [0, +\infty)$ and $(u, \eta) \in \mathbb{R} \times \mathbb{R}^N$ with $\frac{2N}{N+1} < p < 2$ (supercritical range). Moreover we assume that there exists $L > 0$ such that

$$\begin{cases} A(x,t,\xi,z_1) - A(x,t,\xi,z_2)\cdot(z_1 - z_2) \ \geq 0, \\ |A(x,t,\xi_1,z) - A(x,t,\xi_2,z)| \leq L\ |\xi_1 \\ \qquad -\xi_2|(1 + |z|^{p-1}), \end{cases} \tag{1.4}$$

for almost all $(x, t) \in \mathbb{R}^N \times [0, +\infty)$ and all $\xi, \xi_i \in \mathbb{R}$ and $z, z_i \in \mathbb{R}^N, i = 1, 2$.
We prove that, if u is a non negative, locally bounded weak solution of

$$\begin{cases} u_t = divA(x, t, u, Du), \quad (x, t) \in \mathbb{R}^N \times [0, +\infty), \\ u(x, 0) = \delta(0), \quad x \in \mathbb{R}^N, \end{cases} \tag{1.5}$$

with A satisfying (1.2), (1.4), p in the supercritical range, $\delta(0)$ the Dirac mass on $\mathbb{R}^N$, then there exist positive constants $\underline{\gamma}, \overline{\gamma}$ depending only upon N, p, c_0, c_1 such that $\forall x \in \mathbb{R}^N$ and $\forall t > 0$, $\underline{\gamma}B_p(x,t) \leq u(x,t) \leq \overline{\gamma}B_p(x,t)$, with $B_p = t^{-\frac{N}{\lambda}}\Big[1 + \gamma_p\Big(\frac{|x|}{t^{1/\lambda}}\Big)^{\frac{1}{p-1}}\Big]^{\frac{p-1}{p-2}}$ and $\lambda = N(p-2) + p > 0$. These results can be extended to the case of porous medium type equation and to general nonnegative L^1 initial data.

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Optimization of the first eigenvalue of equations with indefinite weights

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Reaction-diffusion models for the dynamics and genetics of spatially distributed populations can lead to eigenvalue problems where the weight function may change sign. Typically, a prediction of persistence or extinction for a population in a reaction-diffusion model, depends on the size of certain parameters relative to the principal eigenvalue of the Laplacian with a spatially varying weight function. If an environment contains some regions that are favorable for a given population but others that are unfavorable then the weight function will change sign. If the overall amounts of favorable and unfavorable habitat are fixed in some sense it is natural to ask which arrangements of those habitats are most likely to lead to persistence and which arrangements are most likely to lead to extinction. That question is of practical importance in the contexts of reserve design and pest control. Mathematically, the question can be formulated as asking which weight functions in a suitable class minimize or maximize the principal eigenvalue. This talk addresses that question in case where the class of weight functions is restricted to be the set of rearrangements of a given function. Suppose that $\Omega \subset \mathbb{R}^2$ is a smooth bounded domain representing a region occupied by a population that diffuses at rate D and grows or declines locally at a rate $g(x)$, so that $g(x) > 0$ corresponds to local growth and $g(x) < 0$ to local decline. Suppose that the exterior of Ω is hostile to the population, and that the population is scaled so that the carrying capacity is equal to 1. If $u(x,t)$ is the population density, the global behavior of the population is described by the equation

$$\frac{\partial u}{\partial t} = D\Delta u + (g(x) - u)u \quad \text{in } \Omega \times (0,T),$$
$$u = 0 \quad \text{on } \partial\Omega \times (0,T),$$

where Δu denotes the spatial Laplacian of $u(x,t)$. This equation predicts persistence if and only if $D < 1/\lambda_g$ where λ_g is the positive principal eigenvalue in

$$\Delta\phi + \lambda g(x)\phi = 0 \quad \text{in } \Omega, \quad \phi = 0 \quad \text{on } \partial\Omega.$$

In the present talk we will consider the question: for weights $g(x)$ within the set of rearrangements of a given weight function $g_0(x)$, which, if any, maximize or minimize λ_g ?

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Qualitative behavior of a diffusive predator-prey model

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This talk describes the study of the qualitative behavior of non-constant positive solutions of a diffusive predator-prey model with a functional response describing behavioral mechanism of certain predator's spatial foraging in linear schools versus school under homogeneous Neumann boundary condition. We investigate the local and globalattractor for non-negative time dependent solutions and provide some sufficient conditions for local stability of the positive constant solutions. Nevertheless, we prove the non-existence and existence of non-constant positive steady-state solutions in the sequel. Ecological implications of the analytical and numerical results are described.

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An elliptic system with chemotaxis term and nonlinear boundary conditions

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M. Delgado, C. Morales-Rodrigo

The main goal of this talk is to show some theoretical results concering to an elliptic system containing a chemotaxis term and nonlinear boundary condition. This model arises from the angiogenesis process, crucial step in the growth tumour. In this process new blood vessels grow from existing ones following the gradient of chemical substances, angiogenic factors. These substances are segregated by the tumour when it attains an specific size. Firstly, we use bifurcation methods to prove the existence of positive solutions and then we give some biological interpretations of the main results. These results are obtained joint Profs. M. Delgado and C. Morales-Rodrigo, Univ. of Sevilla, SPAIN. Supported by MICINN and FEDER under grant MTM2009-12367.

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Measure valued solutions of the 2D Keller-Segel system

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We deal with the two-dimensional Keller-Segel system describing chemotaxis in a bounded domain with smooth boundary under the nonnegative initial data. As for the Keller-Segel system, the L1 norm is the scaling invariant one for the initial data, and so if the initial data is sufficiently small in L1, then the solution exists globally in time. On the other hand, if its L1 norm is large, then the solution blows up in a finite time. The first purpose of my talk is to construct a time global solution as a measure valued function beyond the blow-up time even though the initial data is large in L1. The second purpose is to show the existence of two measure valued solutions of the different type depending on the approximation, while the classical solution is unique before the blow-up time. For this purpose, we also discuss on the possibility of the aggregation mass greater than 8pi.

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Theoretical and numerical analysis of complex bifurcation diagrams related to a class of superlinear indefinite problems

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We will present a multiplicity result for large solutions of a class of second order boundary value problems of superlinear indefinite type. Considering one of the parameters involved in the setting of the problem as a bifurcation parameter, we obtain some complex bifurcation diagrams. We will also illustrate the technical problems arising in their numerical computation.

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Entire solutions for competition-diffusion systems and a priori estimates

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Henri Berestycki, Kelei Wang, Juncheng Wei

We study the qualitative properties of a limiting elliptic system arising in phase separation for multiple states Bose-Einstein condensates:

$$\begin{cases} \Delta u = uv^2, \\ \Delta v = vu^2, \\ u, v > 0 \quad \text{in } \mathbf{R}^N. \end{cases}$$

We first prove that stable solutions in $\mathbf{R}^2$ with linear growth must be one-dimensional. Then we construct entire solutions with polynomial growth $|x|^d$ for any positive integer $d \geq 1$. The construction is also extended to multi-component elliptic systems. Finally, we show the connection between the existence/nonexistence of entire solutions and the qualitative properties of the optimal partitions.

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On the effect of spatial heterogeneity in logistic type elliptic equations with nonlinear boundary conditions

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This talk is devoted to the study of global bifurcation of positive solutions for some semilinear elliptic equation of logistic type with nonlinear boundary conditions. The purpose of this talk is to understand the role that an indefinite coefficient of the equation plays in determining the global bifurcation structure. For this we use the implicit function theorem and blow-up arguments to discuss the existence of turning points of the bifurcation component. We argue the nonexistence of positive solutions by stability arguments. By use of the local bifurcation analysis of the reduced equation we obtain the global nature of the bifurcation component.

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Positive solutions of elliptic equation with Hardy potential

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In this paper, we consider the existence of maximal positive solution and minimal positive solution of

$$-\Delta = \lambda \frac{u}{d^2(x)} - b(x)u^p, \quad x \in \Omega,$$

where $p > 1, d(x) = dist(x, \partial\Omega), b$ is a nonnegative continuous function over $\bar{\Omega}$. We obtain the existence of maximal positive solution and minimal positive solution depending on parameter λ for two different type of b. For the degenerate case, we infer that the maximum point x_λ of the minimal positive solution u_λ converges to $\partial\Omega_0$ as $\lambda \to \lambda^*$ under suitable conditions.

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Blow-up rate and uniqueness of singular radial solutions for a class of quasi-linear elliptic equations

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We establish the uniqueness and the blow-up rate of the large positive solution of the quasi-linear elliptic problem $-\triangle_p u = \lambda u^{p-1} - b(x)h(u)$ in $B_R(x_0)$ with boundary condition $u = +\infty$ on $\partial B_R(x_0)$, where $B_R(x_0)$ is a ball centered at $x_0 \in \mathbb{R}^N$ with radius R, $N \geq 3$, $2 \leq p0$ are constants and the weight function b is a positive radially symmetrical function. We only require $h(u)$ to be a locally Lipschitz function with $h(u)/u^{p-1}$ increasing on $(0, \infty)$ and $h(u) \sim u^{q-1}$ for large u with $q > p - 1$.

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Pairs of nodal solutions for a class of super-sublinear problems

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We consider various boundary value problems associated to the second-order nonlinear ODE

$$-u'' = \lambda f(x, u),$$

where $f(x, u)/u \to 0$ as $u \to 0$ and $u \to \infty$. Under mild sign conditions on $f(x, u)$ we obtain multiplicity results of solutions with prescribed nodal properties for certain values of the parameter $\lambda > 0$. Our work is related to some classical results by P.H. Rabinowitz (P.H. Rabinowitz: A note on pairs of solutions of a nonlinear Sturm-Liouville problem, Manuscripta Math. 11 (1974), 273-282) and some recent contributions (A. Boscaggin and F. Zanolin: Positive periodic solutions of second order nonlinear equations with indefinite weight: Multiplicity results and complex dynamics, J. Differential Equation 252 (2012), 2922-2950).

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