Special Session 58: Variational Analysis and Equilibrium Problems

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The aim of the session is to present state-of-the-art and current research directions in variational analysis and in applications to equilibrium problems. Variational analysis encompasses a large area of modern mathematics, including the classical calculus of variations, the theories of perturbation, approximation, subgradient, set convergence, variational inequalities and partial differential equations. Such theories have been applied to various models such as spatial markets, auctions, traffic networks, oligopolistic markets, supernetworks, financial frameworks, electric power supply chains, the Internet, ... This special session will provide a forum for researchers to present current results.

A variational formulation for dynamic market equilibrium problems with excesses

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The aim of the talk is to present the evolutionary variational formulation for market equilibrium problems with production and demand excesses. The equilibrium conditions are expressed according to Counot-Nash principle and in terms of Lagrange multipliers, and, by means of a recent infinite-dimensional duality theory, the equivalence of both the conditions with an evolutionary variational inequality is proved. The variational formulation allows us to obtain existence and regularity results for the equilibrium solution. Moreover, we introduce a discretization procedure for computing dynamic equilibrium solutions and we provide a numerical example.

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Generalized Nash Games and Evolutionary Variational Inequalities

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In this work we show how generalized Nash (GN) games with shared constraints can be related to evolutionary variational and quasivariational inequalities. This association can be used for two things: the first is to compute solutions of such class of GN games, the second is to study long term behaviour of a GN game solution set from a dynamical systems perspective, such as stability and reachability. We outline our approach and possible future research avenues.

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Supply chain networks with corporate financial risks and trade credits under economic uncertainty

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The focus of this paper is to provide an analytical framework which can be used to investigate how financial risks affect the values of interconnected supply chain firms from a network perspective, and how financial risks affect the supply chain firms' profitability as well as the cash and credit transactions. In particular, we develop a variational inequality equilibrium model in conjunction with capital asset pricing model (CAPM) and the net present value (NPV) to determine the optimal supply chain prices, profits, and implicit equity values of supply chain firms under financial risks and economic uncertainty. We illustrate the analytical framework with numerical examples which yield interesting managerial implications to the following questions: 1) How do financial risks and economic uncertainty affect the values of interconnected supply chain firms from a network perspective? 2) How do financial risks and economic uncertainty affect the supply chain firms' profitability as well as the cash and credit transactions? 3) How does the effect of financial risks change under different competition scenarios?

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Electric and economic supply chains: a variational formulation

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First, we present the static electric power supply chain network model with known demands consisting of power generators, power suppliers, power transmitters, and the ultimate consumers. The behavior of the various economic decision-makers associated with the nodes of the network is made explicit. We also derive the governing finite-dimensional variational inequality formulation. Then, we present the time-dependent economic supply chain network

model with three tiers of decision-makers (manufacturers, retailers, and consumers) in the case when excesses of production and excesses of demand of the commodity are present. For such a framework we furnish, using the infinite dimensional duality theory, the equilibrium conditions for the representatives of each tier of the supernetwork, and the time-dependent variational formulation governing the complete supply chain supernetwork.

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The infinite dimensional Lagrange multiplier rule for convex optimization problems

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In this talk an infinite dimensional generalized Lagrange multipliers rule for convex optimization problems is presented and necessary and sufficient optimality conditions are given in order to guarantee the strong duality. Furthermore, an application is presented, in particular the existence of Lagrange multipliers associated to the bi-obstacle problem is obtained.

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A survey on duality theory in elastic-plastic torsion problem

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The talk deals with the problem of existence of Lagrange multipliers associated to the elastic-plastic torsion problem. According to R.Von Mises, the elastic-plastic torsion problem of a cylindrical bar with cross section Ω is to find a function $\psi(x)$ which vanishes on the boundary $\partial\Omega$ and, together with its first derivatives, is continuous on Ω; nowhere on Ω the gradient of ψ must have an absolute value less than or equal to a given positive constant τ; whenever in Ω the strict inequality holds, the function ψ must satisfy the differential equation $\Delta\psi = -2\mu\theta$, where the positive constants μ and θ denote the shearing modulus and the angle of twist per unit length respectively. Aim of the talk is to provide a survey on very recent results on the existence of a Lagrange multiplier associated to the elastic-plastic torsion problem as a Radon measure and as a L^∞ function. These results are obtained by means of the duality theory.

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Evolution solutions of network problems: a hybrid dynamical system approach

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In this talk a new approach for modeling the evolution of networks in states other than equilibrium is presented. Currently, network problems which can be formulated as an evolutionary variational inequality can be modeled in non-equilibrium states, called evolution solutions via a parameterized projected dynamical system. Through the combination of discrete and continuous dynamical systems, namely hybrid dynamical systems, an alternative approach for the modeling of such evolution solutions of networks is constructed. We illustrate examples depicting the advantages/differences of the two approaches and outline stability criteria.

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A generalized quasi-variational inequality for an economic equilibrium problem

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In this talk a competitive economic equilibrium model integrated with exchange, consumption and production is presented. In this model the utility functions are assumed concave, proper and uppersemi-continuous. The problem is characterized by means of a suitable generalized quasi-variational inequality. The aim is to give an existence result of the equilibrium by using the variational approach.

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Two-dimensional approximation of three dimensional piezoelectric membrane shells using gamma convergence

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In this paper we consider piezoelectric membrane shells of very small thickness subjected only to mechanical forces and we show that functions which minimize the energy associated with the three dimensional models converge to the function which minimizes the energy of the two dimensional model of elastic membrane shells.

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Local structure in a class of variational problems

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There is a considerable amount of local structure in variational problems that can be posed as finding zeros of C^1 functions restricted to the graphs of maximal monotone operators. Some of this structure exists when the operators act on Hilbert spaces, but considerably more is known when the underlying spaces are finite-dimensional and the operators are the normal-cone operators of polyhedral convex sets. This latter class includes a vast array of useful models including complementarity problems, more general variational inequalities, and reformulations of variational conditions on sets defined by nonlinear constraints. This local structure is useful, for example, in constructing fast solution algorithms or in transforming the problems into versions more suitable for solution. We will give a general explanation of the structural analysis and will relate it to topics such as parametric analysis and computational solution.

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A quasi-variational approach to the joint implementation of environmental projects

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The 1997 Kyoto Protocol prescribes that some industrialized, labeled as Annex I Par-ties, must reduce their greenhouse gas emissions at least 5 per cent below the 1990 levels for the 2008-2012 period. In this paper we focus on the so-called joint implementation (JI),a mechanism that allows Parties, with emission reduction or limitation commitments, to collect rewards in the form of emission reduction units (ERUs) from an emission-reductionor emission removal project in another Annex I Party, where the abatement costs are lower. We develop a time-dependent pollution control model in which different countries aim to determine the optimal investment allocation in environmental projects and the tolerable pollutant emissions, so as to maximize their welfare. We provide the equilibrium conditions governing the model and derive the characterization in terms of an infinite dimensional quasi-variational inequality problem. The existence of solutions is then investigated.

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A variational inequality formulation of economic network equilibrium models with nonlinear constraints

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Variational inequality theory facilitates the formulation of equilibrium problems in economic networks. Examples of successful applications include models of supply chains, financial networks, transportation networks, and electricity networks. Previous economic network equilibrium models that were formulated as variational inequalities only included linear constraints. In this paper, we first highlight with an application from the context of reverse logistics why the introduction of nonlinear constraints is beneficial. We then show mathematical conditions that ensure that the models have unique solutions and we suggest algorithms that can be applied to solve the models.

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A brief overview on a variational approach for the study of an economic equilibrium problem

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Competitive economic equilibrium problems are presented. These problems are reformulated by means of suitable quasi-variational inequalities. Thanks this characterization, a variational method is presented in order to derive existence and regularity results of equilibria.

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The influence of technical, market and legislative factors on e-waste flows

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The establishment of efficient reverse logistics for end-of-life products, especially, electrical and electronic equipment waste (WEEE), has been recognized as a crucial building block for sustainable economies. One of the major concerns of many WEEE take-back schemes is whether adequate amounts of WEEE flow into the designed recycling systems. We formulate the e-waste network flow model as a variational inequality problem and analyze how technical, market, and legislative factors influence the total amount of e-waste that is collected, recycled and (legally and illegally) disposed of, the prices that sources of waste, processors and demand markets face, and the profits of collectors and processors.

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