Special Session 62: PDEs and Dynamical Systems, and Their Applications

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Many real life problems have been understood by theories of partial differential equations and dynamical systems. On the other hand, modeling of physical problems give rise to many challenging mathematical problems. In this session, we will learn many recent developments of static and dynamic problems arising from mechanics, physics, and materials science from the view points of both modeling and mathematical theory.

Global stability of the normal state of superconductors under the effect of strong electric current

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Consider a superconducting wire whose temperature is lower than the critical one. When one flows a sufficiently strong current through the wire, it is well known from experimental observation that the wire becomes resistive, behaving like a normal metal. We prove that the time-dependent Ginzburg-Landau model anticipates this behaviour. We first prove that, for sufficiently strong currents, the semi-group associated with the model, becomes a contraction semi-group. Then, we obtain an upper bound for the critical current where the semi-group becomes stable. We relate this current to the resolvent of the linearized elliptic operator.

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Analysis of liquid crystals with defects of degree one-half

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Jinhae Park, Daniel Phillips

We investigate the structure of nematic liquid crystal thin films described by the Landau–de Gennes tensor-valued order parameter with Dirichlet boundary conditions of nonzero degree. We prove that as the elasticity constant goes to zero a limiting uniaxial texture forms with disclination lines corresponding to a finite number of defects, all of degree $\frac{1}{2}$ or all of degree $-\frac{1}{2}$

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The local structure of the set of steady-states to the 2D incompressible Euler equations of hydrodynamics

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Steady-states are of great interest in the study of the dynamics of 2D incompressible Euler flows. In this talk I will consider the local structure of the *set* of steady-states under certain non-degeneracy assumptions. The space of vorticity fields is formally foliated into the coadjoint orbits of the group $\mathcal{D}_{\rm area}(\Omega)$ of area-preserving diffeomorphisms of the region Ω filled with the ideal fluid. The Euler evolution is a Hamiltonian system on each orbit, with the kinetic energy as Hamiltonian H. In particular, critical points of H restricted to the orbits correspond to steady-states. Hence, these should be, locally and under suitable assumptions, in one-to-one correspondence with the coadjoint orbits. In finite dimensions, that is, when one considers a"genuine" Lie group instead of $\mathcal{D}_{\rm area}(\Omega)$, this can be established by a routine application of the classical Implicit Function Theorem. In the case of $\mathcal{D}_{\rm area}(\Omega)$, it seems difficult to give the orbits a satisfactory structure as submanifolds in the space of vorticity fields. Nevertheless, the problem can be approached in an indirect way and an analogue of the finite-dimensional result can be established via the Nash-Moser Inverse Function Theorem. This is joint work with V. Šverák.

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Recurrences, limit sets and shadowing property in dynamical systems

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Jaeyoo Choy, Se-Hyun Ku, Jong-Suh Park

We focus on certain dynamic phenomena as recurrences, limit sets, shadowing property and their applications. In this talk, firstly, we discuss the envelope of flows on compact spaces which is a gereralized notion of the functional envelope of maps in discrete case. Next, we deal with the concepts of chain recurrences, attractors and shadowing property in several categories.

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Asymptotics of Maxwell-Chern-Simons vortices on the unit disc

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In this talk, we are concerned with the asymptotic behavior of the relativistic Maxwell-Chern-Simons(MCS) vortices on the unit disc. We establish the existence of a radial solution of the static MCS equations which is a minimizer of the MCS functional in the class of radial functions. We also derive the asymptotic limit of the radial solution. As a consequence, we obtain a set of elliptic linear equations which is a generalization of the London equation for the Ginzburg-Landau model.

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A Poincaré–Dulac normal form for Navier-Stokes equations

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We study the incompressible Navier-Stokes equations with potential body forces on the three-dimensional torus. We show that the normalization introduced in the paper [Ann. Inst. H. Poincaré Anal. Non Linéaire, 4(1):1–47, 1987] produces a Poincaré-Dulac normal form which is obtained by an explicit change of variable. This change is the formal power series expansion of the inverse of the normalization map. Each homogeneous term of a finite degree in the series is proved to be well-defined in appropriate Sobolev spaces and is estimated recursively by using a family of homogeneous gauges which is suitable for estimating homogeneous polynomials in infinite dimensional spaces.

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On the singular limit of thim film equations with small Born repulsion force

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We consider dewetting thin film model with both van der Waals and Born repulsion force. We show that as the Born repulsion force tends to zero, the energy minimizers, passing to a subsequence if necessary, converge to a Dirac mass located on the boundary. The blow up profile of the energy minimizers is identified and the location of spike will be discussed.

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Local estimates of weak solutions for steady-state non-Newtonian fluid flows

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We consider steady-state non-Newtonian Stokes and Navier-Stokes equations confined in bounded domains in dimension three. We obtain various estimates of higher derivatives of weak solutions near boundary as well as in the interior, when no-slip boundary conditions are given.

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Lyapunov functions, attractors and shadowing property in set-valued dynamics

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In this talk, we are concerned with set-valued dynamical systems. In the systems, we obtain some results about Lyapunov functions and attractors, and then we also treat the shadowing property. Moreover, we deal with the Conley's theorem, which is about chain recurrences, on non-compact spaces.

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On some coupled system with the Navier-Stokes equations

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In this talk, we consider some mathematical models coupled with the Navier-Stokes equations. At first, we consider the magnetohydrodynamics equations in 2 and 3 dimensions. We consider the existence of classical solutions and the regularity criterion. Secondly, we consider the Navier-Stokes-Vlasov-Fokker-Planck equations derived from the combustion theory. Thirdly, we consider the Navier-Stokes-Keller-Segel equations which governs the motion of the swimming bacteria. Lastly, we consider some system which is a simplified model of the Ericksen-Leslie system.

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Global existence and nonrelativistic limit for the Vlasov-Maxwell-Chern-Simons system

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A system of partial differential equations is studied. Considering interaction between the Maxwell-Chern-Simons electrodynamics and the Vlasov matter, the Vlasov-Maxwell-Chern-Simons system is introduced. Global existence of classical solutions is proved, and then its nonrelativistic limit is investigated. When the speed of light tends to infinity, it is proved that solutions of the Vlasov-Maxwell-Chern-Simons system converge to solutions of the Vlasov-Yukawa system.

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Planar bistable liquid crystal device and dynamics of switching

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A planar bistable liquid crystal device, reported in Tsakonas et al., is modeled within the Landau-de Gennes theory for nematic liquid crystals. This planar device consists of an array of square micron-sized wells. We obtain six different classes of equilibrium profiles and these profiles are classified as *diagonal* or *rotated* solutions. In the strong anchoring case, we propose a Dirichlet boundary condition that mimics the experimentally imposed *tangent boundary conditions*. In the weak anchoring case, we present a suitable surface energy and study the multiplicity of solutions as a function of the anchoring strength. We find that diagonal solutions exist for all values of the anchoring strength $W \geq 0$ while rotated solutions only exist for $W \geq W_c > 0$, where W_c is a critical anchoring strength that has been computed numerically. We propose a dynamic model for the switching mechanisms based on only dielectric effects. For sufficiently strong external electric fields, we numerically demonstrate diagonal to rotated and rotated to diagonal switching by allowing for variable anchoring strength across the domain boundary.

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$W^{1,p}$ bound and compactness at $p* = \frac{np}{n-p}, 1 \leq p < n$

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Abstract. In respect of the Rellich-Kondrachov compact embedding theorem, a mathematically simple reason is given as to why the well known counter examples in existence for many years and showing non-compactness at the critical value $p^* = \frac{np}{n-p}, 1 \leq p < n$, do not actually survive (become vacuous) for computations in the $L^{p^*}-$ norm. The Gagliardo-Nirenberg-Sobolev condition (GNS) $1 - \frac{n}{p} + \frac{n}{p^*} = 0$ is clearly seen to impose a restriction on the sequence index in the definition of the sequence for such counter examples, in so far as the sequence index disappears if the $L^{p^*}-$ norm is computed without using the gradient, where as, the sequence index need not disappear while using GNS if the p^*-norm estimation makes use of the gradient of the sequence with the corresponding 'implied value'. We thus obtain a different and natural value for the $L^{p^*}-$ norm estimate, showing ambiguity in $L^{p^*}-$ norm computation if the two 'restricted ' and 'unrestricted' computations are compared; note that it is indeed compulsory to use the gradient of the sequence and its 'implied value' (in integration) from the pointwise definition of the sequence to check for the $W^{1,p}$ bound, so computation using the gradient has to be considered 'canonical'. To achieve this, we make use of the strong norm bounds for the Hardy-Littlewood maximal function along with the well known 'potential formula' which expresses a C^1 function of compact support in terms of its gradient. Thus, and very surprisingly, this leaves open the question of compactness at p^*.

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Regularity of solutions to the linearized Monge-Ampère equation

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The linearized Monge-Ampère equation is a second-order degenerate elliptic equation which appears in several applications such as affine differential geometry, complex geometry and fluid mechanics. We study smoothness properties of solutions to the equation. By employing a perturbation argument, we establish interior estimates for the first and second derivatives of its solutions.

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An analysis of textures in smectic-C films with multiple defects

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We analyze a model for the elastic energy of planer c-director patterns in a smectic film. Because of boundary conditions and polar fields topological defects form in these patterns. We use a Ginzburg Landau model that allows the director field to have variable length and to vanish at the defect cores. We prove that if the model's G-L parameter is small then low energy states develop degree +(-) onedefects that tend to a minimal energy configuration with a limiting far field texture. Our main contribution is that we are able to treat the case of unequal splay and bend elasticity constants. Earlier analytic work for the G-L functional had been limited to the equal constant case.

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Mathematical analysis and diffusive interface modeling of membrane movements

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The role of a biological membrane is to act as a barrier between ionic solutions. One way life controls ionic solution is through ion channels. A second more drastic way is by introducing a hole in the membrane itself. For example, in hemolysis, the osmotic swelling and rupture of a red-blood cell, a single hole forms in the membrane leading to the leak out of the contents of the cell. Similarly, in exocytosis a hole is formed by joining to membrane bilayers. These processes are mathematically challenging to study because they involve physical forces on multiple scales and predicting the time course is more consequential than the equilibrium end states. This talk will show how such complicated fluid mechanical problems yield to quantitative modeling and simulation when using the diffusive interface and energetic approach.

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Propagation of the advantageous genes in a population with multiple alleles at a locus

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We extended the classical result of Fisher (1937) from the case of two alleles to multiple alleles. Consider a population living in a homogeneous one-dimensional infinite habitat. Individuals in this population carry a gene that occurs in k forms, called alleles. Under the joint action of selection and migration, the allele frequencies, $p_i, i = 1, \ldots, k$, satisfy a system of differential equations. We first showed that under the conditions A_1A_1 is the most fit among the homozygotes, the system is cooperative, the state that only allele A_1 is present in the population is stable, and the state that only allele A_1 is absent in the population is unstable, there exists a positive constant, c^*, such that allele A_1 propagates asymptotically with speed c^* as $t \to \infty$. We then showed that traveling wave solutions connecting these two states exist for $|c| \geq c^*$. Finally, we showed that under certain additional conditions, c^* has an explicit formula. These results allow us to estimate how fast an advantageous gene propagates in a population under selection and migration forces as $t \to \infty$ and help to predict the genetic makeup of a population in the long run. This is a joint work with Roger Lui.

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Dynamic transitions and pattern formations for Cahn-Hilliard model with long-range repulsive interactions

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In this talk, I shall address the order-disorder phase transitionand pattern formation for systems with long-range repulsive interactions. The main focus is on the Cahn-Hilliard model with a nonlocal term in the corresponding energy functional, representing the long-range repulsive interaction. First, we show that as soon as the linear problem loses stability, the system always undergoes a dynamic transition to one of the three types, forming different patterns/structures. The types of transition are then dictated by a nondimensional parameter, measuring the interactions between the long-range repulsive term andthe quadratic and cubic nonlinearities in the model. The derived explicit form of this parameter offers precise information for the phase diagrams. Second, we obtain a novel and explicit pattern selection mechanism associated with the competition between the long-range repulsive interaction and the short-range attractive interactions. In particular, the hexagonal pattern is unique to the long-range interaction, and is associated with novel two-dimensional reduced transition equations on the center manifold generated by the unstable modes, consisting of (degenerate) quadratic terms and non-degenerate cubic terms. Finally, explicit information on the metastability and basin of attraction of different disordered/ordered states and patterns are derived as well.

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Liouville Theorem for higher order elliptic systems

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We study the following higher order elliptic system in R^n

$$\Delta^m u = v^p$$
$$\Delta^m v = u^q$$

We prove that for $1/(p+1)+1/(q+1) > 1-2m/n$, there are no positive radial solutions to the system. If $p > 1, q > 1$ and $\max(2m(p+1)/(pq-1), 2m(q+1)/(pq-1)) >= n-2m$, there are no positive solutions to the system. The proof of the radial case uses Rellich identity and the proof in the general case uses estimates on spherical average of solutions.

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Crystalline surface diffusion

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We study the surface diffusion of the boundary of a planar polygonal domain in the setting of a crystalline surface energy. It is a motion law mimicking the evolution of a surface in which the surface normal velocity equals the Laplacian of mean curvature. We characterize the velocity in the framework of canonical restriction and study its properties. In particular, the phenomena of facet splitting is analyzed.

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Kinetic theory and simulations of active nematic suspensions

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A kinetic model is proposed for dilute or semidilute active nematic suspensions, which generalizes the Smoluchowski equation for rod-like or plate-like nematic polymers. Numerical simulations show the longwave instability of the local concentration field and the long time phenomena of the band structure that forms and breaks periodically.

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