Special Session 63: Infinite Dimensional Dynamics and Applications

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Many partial differential equations of mathematical physics, particular those of fluid dynamics, can be recast as infinite-dimensional dynamical systems. This allows one to use the qualitative ideas from dynamical systems to try to understand the behaviour of these systems. This may be the time-asymptotic behaviour, studied using the theory of attractors or inertial manifolds, the existence of invariant measures, etc. There is also considerable overlap with the more classical approach to qualitative properties of PDEs - at its most basic, the generation of any dynamical system relies on existence and uniqueness results for the original model. This session aims to cover all these aspects, from the classical PDE theory to applications.

Markus-Sell's theorem for infinite dimensional asymptotically almost periodic systems.

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This talk is dedicated to the study of asymptotic stability of asymptotically almost periodic systems. We formulate and prove the analog of Markus-Sell's theorem for asymptotically almost periodic systems (both finite and infinite dimensional cases). We study this problem in the framework of general non-autonomous dynamical systems.The obtained general results we apply to different classes of non-autonomous evolution equations: Ordinary Differential Equations, Difference Equations, Functional Differential Equations (both with finite retard and neutral type) and Semi-linear Parabolic Equations.

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Uniform global attractor of the 3D Navier-Stokes equations

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We investigate the long time behavior of the 3D Navier-Stokes equations (NSE) with a time-dependent force and obtain the existence and the structure of the weak uniform global attractor. This is done by studying an autonomous evolutionary system without uniqueness, whose trajectories are solutions to the nonautonomous 3D NSE.The established method can also be applied to other nonautonomous partial differential equations.

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Norm inflation for incompressible magneto-hydrodynamic system in $\dot{B}_{\infty}^{-1,\infty}$

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We demonstrate that the solutions to the Cauchy problem for the three dimensional incompressible magneto-hydrodynamics (MHD) system can develop different types of norm inflation in $\dot{B}_{\infty}^{-1,\infty}$. Particularly the magnetic field can develop norm inflation in short time even when the velocity remains small and vice versa. Another interesting case is that, even with zero initial velocity, the velocity field can develop norm inflation in short time. We construct different initial data to obtain these results using plane waves. The intuition is from the construction method introduced by Bourgain and Pavlović for Navier-Stokes equations.

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Time-dependent attractors for the oscillon equation

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The oscillon equation

$$\begin{cases} \partial_{tt}u(x,t) + H\partial_t u(x,t) - \mathrm{e}^{-2Ht}\partial_{xx}u(x,t) \\ \qquad\qquad +V'(u(x,t)) = 0,\ x \in (0,1), t \in \mathbb{R}, \\ u(0,t) = u(1,t),\ t \in \mathbb{R}, \end{cases}$$

where V is a nonlinear potential, arises from relativistic mechanics as a one-dimensional model for wave propagation in an expanding universe.

We study the asymptotic behavior of the process generated by the oscillon equation, as well as its three-dimensional generalizations, in the ad-hoc framework of pullback attractors in time-dependent spaces. The explicit time-dependence of the phase space allows us to restrict the basin of attraction to the physically meaningful initial data, recover a

suitable notion of dissipation, and construct a finite-dimensional attractor of optimal regularity for the process.

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Computer assisted rigorous proof of chaos in some infinite dimensional dynamical systems

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The Conley index, introduced by Charles C. Conley in 1971, has become a powerful tool to study dynamical systems. This is a topological invariant for investigating the structure of isolated invariant sets of continuous or discrete dynamical systems. And recently, the Conley index was generalized to a class of discontinuous systems by Xinchu Fu and Kaihua Wang. However, in most cases, calculating the index, or even finding an index pair, for systems with complicated dynamics, is impossible without exploiting computer assisted methods. Hence many researchers developed the Conley index for multi-valued systems, which are computer representations of the single-valued one. The homotopy invariant property of Conley index ensures that with an appropriate error bound, we can numerically compute the index correctly. This allows us to use the computer to give mathematically rigorous arguments for the existence of chaotic sets. The idea of a computer assisted proof of chaotic dynamics based on topological invariants appeared first in the work of Mischaikow and Mrozek. They used the discrete Conley index to prove the existence of chaotic dynamics for the Henon map and the Lorenz equations. With the development of the algorithms, the Conley index has been successfully applied to detect all kinds of chaotic attractors. Recently, automatic method for the efficient computation of a database of global dynamics of a multi-parameter dynamical system is introduced by Z. Arai et al., which uses Conley index as a main tool. The aim of this talk is to use the numerical method mentioned above to rigorously prove the chaos in some infinite dimensional dynamical systems, e.g., the Ikeda-like map coupled by a network with infinite size. It is shown that the Ikeda-like system with particular parameters has an isolated invariant set containing two periodic points, and a heteroclinic orbit connecting them, then a semiconjugacy between the invariant set and a nontrivial subshift system is established, which implies the system is chaotic. By the homotopy invariant property of the Conley index, all the proof are rigorous.

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Discrete data assimilation for the 2D Navier-Stokes equations

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We present an algorithm for dynamically constructing an approximate solution v(t) to the 2D Navier-Stokes equations from observational data of the form Ih(u(tn)) where Ih is a suitable interpolation operator and tn is a series of discrete points in time. We then show that the difference between the approximate solution and the reference solution tends to zero as t tends to infinity.

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Global existence and finite time blow up in a class of stochastic non linear wave equations.

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We consider a stochastic extension of a class of wave equations with nonlinear damping of viscoelastic type and nonlinear forcing. We show global existence for the stochastic equation, for certain range of parameters. Additionally, when the source term dominates the damping term, and when the initial data are large enough, we show that solutions of the stochastic system blow up in finite time, with positive probability. In the presence of noise, we have also extended the previously known range of initial data corresponding to blow-up. We also comment on asymptotic behavior of solution, in terms of attractors, for the good range of parameters.

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Embedding of compact invariant sets of dynamical systems on infinite-dimensional manifolds into finite-dimensional spaces

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Takens embedding method and some its generalizations for dynamical systems on Banach spaces use an embedding theorem which is due to B. R. Hunt and V. Yu. Kaloshin. In contrast to this we consider dynamical systems on Banach manifolds for the embedding of compact invariant sets with finite fractal dimension and use an embedding theorem due to T. Okon. As an example we consider a dynamical system generated by the coupled Maxwell-Dirac equations.

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3D Navier-Stokes equations: numerical verification of regularity for bounded sets of initial data

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James C. Robinson, Pedro Marin-Rubio

The paper concerns three-dimensional Navier-Stokes equations with periodic boundary conditions and zero forcing. As it is very well known global regular solutions to these equations exist for small initial data in H^1. The problem of regularity of solutions arising from arbitrarily large sets of initial data in H^1 remains open. In the paper we consider the following statement: all initial conditions in a given ball (of arbitrary radius R) in H^1 give rise to global regular solutions of the Navier-Stokes equations. We prove that this statement can be verified computationally in a finite time (that is assuming that the statement holds, there exists a numerical algorithm that will verify veracity of the statement in a finite time). This is a joint paper with James C. Robinson and Pedro Marin-Rubio.

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Separation and bifurcation phenomena for flows interacting with a boundary

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In this talk we shall review some rigorous and numerical results on the evolution of boundary layers for incompressible Navier-Stokes equation. The possibility of interpreting the phenomena leading to separation of the boundary layer in terms of bifurcation of equilibria will be explored.

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On representatives of solutions and those properties independent of the chosen representative.

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James C. Robinson

Once a solution $u \in L^p(0, T; L^q(\mathbb{R}^n; \mathbb{R}^n))$ of a PDE has been found, for example a weak solution of the Navier-Stokes equations, it is often of interest to switch from the Eulerian to the Lagrangian viewpoint and describe trajectories of this solution by solving $\dot{X} = u(X, t)$. We discuss how the trajectories can change when u is altered on a set of null measure. In particular, we demonstrate that if we alter u on a set that is 'small' in a fractal sense, u is sufficiently integrable and there exists a generalised flow solution of the ODE then almost every trajectory is unchanged.

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Global existence for the critical semilinear heat equation

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We will present a method of proving global existence of classical solutions for a critical semilinear heat equation. The result is an extension of the well-known technique based on the construction of supersolutions used in the theory of supercritical nonlinearities. A nonstandard smallness assumption on the initial data will be derived.

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On the structure of the global attractor for reaction-diffusion equations

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In this work we study a scalar reaction-diffusion equation for which the non-linear term satisfy some growth and dissipativity assumptions ensuring global existencce of solutions for the Cauchy problem, but not uniqueness. We define several multivalued semiflows generated by weak, regular and strong solutions of the equation and study the structure of the global attractor. More, precisely, we prove that the attractor can be characterized as the union of all unstable manifolds of the set of stationary points and of the stable ones as well.

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Moments and the Navier-Stokes equations

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We will discuss some results related to the moment problem and statistical solutions of the 3D Navier-Stokes equations

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Quasi-periodic solutions for 1 dimensional generalized Boussinesq equation

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In this paper, one-dimensional(1-D)generalized Boussinesq equation:

$$u_{tt} - u_{xx} + (f(u) + u_{xx})_{xx} = 0$$

with hinged boundary conditions is considered, where $f(u) = u^3$. It is proved that the above equation admits small-amplitude quasi-periodic solutions corresponding to finite dimensional invariant tori of an associated infinite dimensional dynamical system. The proof is based on an infinite dimensional KAM theorem and partial birkhoff normal form.

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