Special Session 68: Analysis and Simulations of Nonlinear Systems

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The aim of the special session is to address analytic and computational aspects of nonlinear systems and differential equations arising particularly in mechanics and mathematical biology. The session will focus on recent advances in mathematical analysis and simulations of various systems of mixed types (including PDE models, delay equations, and impulsive differential equations), equally with applications to mechanics, population dynamics, wave phenomena, formation of patterns and other physical contexts.

Pullback random attractor for the FitzHugh-Nagumo system on unbounded domains

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We study the asymptotic behavior of solutions of the FitzHugh-Nagumo system defined on unbounded domains with non-autonomous deterministic as well as stochastic terms. We first prove the pullback asymptotic compactness of solutions and then establish the existence of a unique pullback random attractor for the system. We further show that the random attractor is periodic when the non-autonomous deterministic terms are periodic.

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Some computational challenges in analyzing global dynamics of certain nonlinear discrete dynamical systems

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We present some computational challenges involved in analyzing global behavior of solutions to certain classes of nonlinear discrete dynamical systems which have applications in mathematical biology and ecology. We also suggest some innovative geometric and computer-based approaches to overcome these challenges.

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Mathematical modeling with reaction-diffusion-advection systems and its application in biology

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In this talk, I will discuss about the computational challenges to solve reaction diffusion advection equations and how to overcome these difficulties by designing effective and robust numerical techniques. Moreover, I will show some interesting applications of reaction diffusion models arising from complex biological systems.

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A fast finite difference method for fractional diffusion equations

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Fractional diffusion equations model phenomena exhibiting anomalous diffusion that can not be modeled accurately by the second-order diffusion equations. Because of the nonlocal property of fractional differential operators, the numerical methods have full coefficient matrices which require storage of $O(N^2)$ and computational cost of$O(N^3)$ where N is the number of grid points. In this talk, I will discuss a fast finite difference method for fractional diffusion equations, which only requires storage of$O(N)$ and computational cost of $O(N \log^2 N)$ for a one dimensional fractional diffusion equation or $O(N \log N)$ for a two dimensional fractional diffusion equation while retaining the same accuracy and approximation property as the regular finite difference method. Numerical experiments are presented to show the utility of the method.

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Local and global well-posedness for KdV-systems

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Systems of Korteweg-de Vries type arise in various applications. We study here a class of such systems and obtain local and global well-posedness results together with some associated examples of blow-up in finite time.

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Hopf bifurcation analysis of a predator-prey dystem with discrete and distributed delays

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In this paper, a ratio dependent predator-prey system with both discrete and distributed delays is investigated. First we consider the stability of the positive equilibrium and the existence of local Hopf bifurcations. Then, by choosing the delay time τ as a bifurcation parameter, we show that Hopf bifurcation can occur as the delay time τ passes some critical values. Using normal form theory and central manifold argument, we also establish the direction and the stability of Hopf bifurcation. Finally, we perform some numerical simulations to support theoretical results.

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Initial-boundary-value problem of systems of nonlinear dispersive equations

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Considered here is the system

$$\begin{cases} u_t + u_x - u_{xxt} + P(u,v)_x = 0, \quad x \in (0,L),\, t > 0, \\ v_t + v_x - v_{xxt} + Q(u,v)_x = 0, \quad x \in (0,L),\, t > 0, \\ u(0,t) = f_1(t), v(0,t) = g_1(t), \quad t > 0, \\ u(L,t) = f_2(t), v(L,t) = g_2(t), \quad t > 0, \\ u(x,0) = u_0(x), v(x,0) = v_0(x) \quad x \in (0,L) \end{cases}$$

of coupled BBM-equations posed on bounded domain for x in the bounded domain $[0,L]$, where $u = u(x,t), v = v(x,t)$ are functions defined on $(0,L)\times\mathbb{R}^+$, $P(u,v) = Au^2+Buv+Cv^2$ and $Q(u,v) = Du^2+Euv+Fv^2$ in which $A, B, \cdots, F$ are fixed real numbers. We investigate conditions on both boundary and initial data to guarantee the system to be well-posed.

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Lyapunov stability of elliptic periodic solutions of nonlinear damped equations

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Based on the third order approximation developed by Ortega, we present the formula of the first twist coefficient for a nonlinear damped equation. As examples, we consider the Lyapunov stability of a superlinear damped differential equation and a singular damped differential equations.

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Existence and global attractivity of positive periodic solution to a Lotka-Volterra model

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In this talk, a Volterra model with mutual interference and Holling III type functional response is investigated, some sufficient conditions which guarantee the existence and global attractivity of positive periodic solution for the system are obtained, two examples are given to verify the results by using MatLab. We improve the main results of the references.

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First integral of Duffing-van der Pol oscillator system

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In this talk, we restrict our attention to a nonlinear Duffing–van der Pol–type oscillator system by means of the first-integral method. This system has physical relevance as a model in certain flow-induced structural vibration problems, which includes the van der Pol oscillator and the damped Duffing oscillator etc as particular cases. Through applying the first-integral method and the Lie symmetry method, we present a general first integral formula of the Duffing–van der Pol–type oscillator system.

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Exact solutions of the Burgers-Huxley equation

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In this talk, we apply the Lie symmetry reduction method to solve exact solutions of partial differential equations such as the Burgers-Huxley equation and the generalized Fisher equation. Through analyzing the symmetry condition and the resultant determining system, we construct classical and non-classical infinitesimal generators, and obtain exact solutions accordingly.Joint with Zhaosheng Feng

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The nonlinear Schrödinger equation created by the vibrations of an elastic plate and its dimensional expansion

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In the course of studying the theory of plates the classical, Kirchhoff plate theory due to Timoshenko (1970) and Ugural (1981) in which transverse normal and shear stresses are neglected to study bending, buckling, and natural vibrations of rectangular plates, was first established. The first order shear deformation plate theory extends the kinematics of the classical, Kirchhoff plate theory by relaxing the normality restriction and allowing for arbitrary but constant rotation of transverse normals and finite element models are developed for the precise analysis of the plate characteristics in real problem. In general, the Schrödinger equation governs the spatial and temporal evolution of the amplitude of a wavepacket propagating transversely in any dispersive, lossless medium. The Schrödinger equation governs an envelope created by a wavepacket by Nohara(2003). The nonlinear Schrödinger equation governs the non-linearity of the envelope. Many studies of a wavepacket has been carried out in water waves, fiber-optic communication systems, and some other area as well.

In this paper, we, first, survey the two-dimensional governing equation that describes the propagation of a wavepacket on an elastic plate using the method of multiple scales. Then we expand the governing equation to the multi-dimensional case not only in the sense of mathematical science but also engineering. For example the governing equation for 3-dimensional coordinate (x, y, z) shows the vibrational dynamics inside the plate. On the other hand 2-dimensionl wavenumber (k_1, k_2) means the vertical wavenumber k_1 and horizontal wavenumber k_2 on the plate, respectively.

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On the paradoxes of enrichment and pesticides

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Frank Feng, Yipeng Yang

For the paradox of enrichment (which arose from models of population dynamics), we present a resolution (joint with Frank Feng). For the paradox of pesticides (which arose from experiments), we study mathematical models and present a theory for the resolution (joint with Yipeng Yang).

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Intravenous glucose tolerance test model and its global stability

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Diabetes mellitus has become a prevalent disease in the world. Diagnostic protocol for the onset of diabetes mellitus is the initial step in the treatments. The intravenous glucose tolerance test (IVGTT) has been considered as the most accurate method to determine the insulin sensitivity and glucose effectiveness. It is well known that there exists a time delay in insulin secretion stimulated by the elevated glucose concentration level. However, the range of the length of the delay in the existing IVGTT models are not fully discussed and thus in many cases the time delay may be assigned to a value out of its reasonable range. In addition, several attempts had been made to determine when the unique equilibrium point is globally asymptotically stable. However, all these conditions are delay-independent. In this talk, we review the existing IVGTT models, discuss the range of the time delay and provide easy-to-check delay-dependent conditions for the global asymptotic stability of the equilibrium point for a simplified but practical IVGTT model through Liapunov function approach. Estimates of the upper bound of the delay for global stability are given in corollaries. In addition, the numerical simulation in this work is fully incorporated with functional initial conditions, which is natural and more appropriate in delay differential equation systems.

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Burst synchronization and rhythm dynamics in neuronal networks

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Physiological experiments have confirmed the existence of synchronous oscillations of neurons in different areas of the brain. Synchronization of neurons plays a significant role in neural information processing. Clinical evidences also point out that the synchronization of individual neurons plays a key role in some pathological conditions like Parkinson's disease, essential tremor, and epilepsies. According to experimental results, we constructed a small world neuronal network to study the effects of the coupling scheme, the intrinsic property of individual neurons and the network topology on burst synchronization and the rhythm dynamics of the network. The results are of fundamental significance to understand information activities in nervous systems.

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Analytical solutions for the BoitiLeon-Pempinelli equation

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We apply the classical Lie symmetry reduction method and an constructive algorithm to solve exact solutions of the BoitiLeonPempinelli equation. Infinitesimal generators, Backlund transformation and various types of analytical solutions of the BoitiLeon-Pempinelli equation are obtained.

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Hyper-spherical harmonics and jumps in financial markets

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The technique of expanding functions in terms of spherical harmonics turns out to be very useful in solving integral equations. This motivates us to apply the same technique in mathematical finance. The governing equation in mathematical finance is mostly a parabolic partial differential equation. We present a technique of solving Heat equation and Black-Scholes equation using the method of spherical harmonics. We also discuss the method related to the Black-Scholes equation in annular domain and some generalized Black-Scholes equations. Finally we solve some integro-differential equation arising in financial models with jumps by using the method of spherical and hyper-spherical harmonics.

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A mathematical model with time-varying antiretroviral therapy for the treatment of HIV

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We present a mathematical model to investigate theoretically and numerically the effect of immune effectors in modeling HIV pathogenesis. Additionally, by introducing drug therapy, we assess the effect of treatments consisting of a combination of several antiretroviral drugs. A periodic model of bang-bang type and a pharmacokinetic model are employed to estimate the drug efficiencies. Nevertheless, even in the presence of drug therapy, ongoing viral replication can lead to the emergence of drug-resistant virus variances. Thus, by including two viral strains, wild-type and drug-resistant, we show that the inclusion of the CTL compartment produces a higher rebound for an individuals healthy helper T-cell compartment than does drug therapy alone. We investigate numerically how time-varying drug efficacy due to drug dosing regimen and/or suboptimal adherence affects the antiviral response and the emergence of drug resistance.

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Investigation of the long-time evolution of localized solutions of a dispersive wave system

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We consider the long-time evolution of the solution of a system with dispersion and nonlinearity which is the progenitor of the different Boussinesq equations. As initial condition we use a wave system comprised by the superposition of two analytical soliton solutions. We use a strongly implicit difference scheme with internal iterations which allows us to follow the evolution of the solution at very long times. We focus on the dynamical behavior of traveling localized solutions developing from critical initial data. The system is rendered into a seven-diagonal band-matrix form and solved effectively by specialized solver with pivoting which is stable to round-off errors even for 2.500.000 points of spatial resolution. The main solitary waves appear virtually non-deformed from the interaction, but additional oscillations are excited at the trailing edge of each one of them. We extract the perturbations and track their evolution for very long times when they tend to adopt a self-similar shape: their amplitudes decrease with the time while the length scales increase. We test a hypothesis about the dependence on time of the amplitude and the support of Airy-function shaped coherent structures which gives a very good quantitative agreement with the numerically obtained solutions. This investigation is supported by the Scientific Foundation of the Bulgarian Ministry of Education, Youth, and Science under Grant DDVU02/71.

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Entire and blow-up solutions: from semilinear to fully nonlinear elliptic equations

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Semilinear elliptic equations arise in many fields of the applied sciences, in particular in mechanics, engineering and life sciences. We discuss existence, uniqueness and qualitative properties of solutions in the whole space as well as in a bounded domain, possibly explosive on the boundary, for second order elliptic operators with a superlinear growth in the

solution, both in the case of a linear main term and of a fully nonlinear one.

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Models and applications for minority health Studies

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Minority health status is of big concern to our society. In many cases, researchers seek to understand the effects of predictors (such as demographic measures, social-economic measures, insurance etc.) on health or intervention outcomes. Generalized linear models are used in our studies to explore such relationships. Application examples will be illustrated.

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Qualitative properties of positive solutions of a class of boundary blow-up problems

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We will consider boundary blow-up problems on an unbounded domain. For the problems, we will study the existence, uniqueness, blow-up rate of positive solutions. Our results extend some results of Professor Julian Lopez Gomez's papers.

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The effects of singular lines in nonlinear wave equations

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In this talk, the dynamical system approach to study traveling wave solutions of nonlinear wave equations is introduced. We pay more careful attentions to the travelling wave solutions of the nonlinear wave equations with nonlinear dispersive terms which corresponding traveling wave system may have singular lines and thus will result in a variety of singular waves. The singular traveling waves of the generalized nonlinear Klein-Gordon model equation are studied as an example to demonstrate the effects of singular lines in nonlinear wave equations.

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Energy identity for a class of approximate biharmonic maps into sphere in dimension four

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We considered in dimension four weakly convergent sequences of approximate biharmonic maps into sphere with bi-tension fields bounded in L^p for some $p > 1$. We prove an energy identity that accounts for the loss of Hessian energies by the sum of Hessian energies over finitely many nontrivial biharmonic maps on $\mathbb{R}^4$.

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