Special Session 69: Dissipative Systems and Applications

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Dissipative systems arise in many applications. The special session will feature a broad spectrum of research from infinite dynamical systems theory and random dynamical systems to evolutionary partial differential equations and numerical simulation. The scope of applications reaches from reaction-diffusion systems with local and nonlocal dispersal from ecology to climate modeling.

Attractors of impulsive dissipative semidynamical systems

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We consider a class of dissipative semidynamical systems with impulses and we define some types of attractors for these systems. We present results which relate attractors and dissipative systems (point, bounded and compact). Also, we apply our results for a nonlinear reaction diffusion equation of type $u' - \Delta u + g(u) = f$ with impulse condition.

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Some results on Hopf type bifurcation in delayed complex Ginzburg-Landau equations

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We consider the complex Ginzburg-Landau equation with feedback control given by some delayed linear terms (possibly dependent of the past spatial average of the solution). We prove several bifurcation results by using the delay as parameter. We start by considering the case of the whole space and, later, that of a bounded domain with periodicity conditions. A linear stability analysis is made with the help of computational arguments (showing evidence of the fulfilment of the delicate transversality condition). In the last section the bifurcation takes place starting from a uniform oscillation and originates a path over a torus. This is obtained by the application of an abstract result over suitable functional spaces.

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Regularity for strongly coupled parabolic systems via homotopy

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We introduce a new technique, the so-called nonlinear heat approximation and BMO preserving homotopy, to investigate regularity properties of BMO weak solutions of strongly coupled nonlinear parabolic systems consisting of more than one equations defined on domain of any dimension.

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On a cross-diffusion PDE system arising as limit of repelling particle models

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Starting from a stochastic ODE model for two finite populations of particles attracted by the same environmental potential but repelled by intra and inter-population overcrowding, we heuristically identify the limit PDE system of equations satisfied by the population densities when the number of particles tend to infinity. The resulting system if of nonlinear cross-diffusion type, for which we present a proof of existence of solutions based on a finite element approximation. Some numerical demonstrations showing the phenomenum of segregation of populations are also provided.

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On the local behavior of solutions to thin-film Ginzburg-Landau equations near vortices

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We study the properties of solutions to the thin-film Ginzburg-Landau equations near their zeros and establish that they cannot vanish of infinite order unless identically zero. The method is based on an

extension of a classical result by Hartman and Wintner which allows the introduction of a non-analytic variable thickness function and a non-analytic applied magnetic field.

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A finite volume scheme for the numerical approximation of a 2D climatological model.

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In this work we compute the numerical solution of a global climate model including deep ocean effect. The model is based on that proposed by Watts and Morantine but including a coalbedo temperature dependent and nonlinear diffusion at the boundary. One of the main features of the model, which makes the problem of particular interest, is the dynamic and diffusive boundary condition that represents the coupling between ocean and atmosphere. The numerical method used is a finite volume scheme with high order WENO reconstruction in space and third order Runge-Kutta TVD for time discretization.

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Positive stationary solutions and spreading speeds of KPP equations in locally spatially inhomogeneous media

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The current paper is concerned with positive stationary solutions and spatial spreading speeds of KPP type evolution equations with random or nonlocal or discrete dispersal in locally spatially inhomogeneous media. It is shown that such an equation has a unique globally stable positive stationary solution and has a spreading speed in every direction. Moreover, it is shown that the localized spatial inhomogeneity of the medium neither slows down nor speeds up the spatial spreading in all the directions.

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Faster vs slower diffusers

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We study the dynamics of a reaction-diffusion-advection model for two competing species in a spatially heterogeneous environment. We consider the outcome of the competition between two species which are assumed to have the same population dynamics but different dispersal strategies: both species diffuses by random diffusion and advection along the environmental gradient, but with different diffusion and/or advection rates. We show that when the advection rates are large, then the faster diffuser wins the competition, this is in contrast to the previously known result that the slower diffuser prevails when there is small or no advection (Dockery et. al).

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On a nonlocal Bernoulli-type problem with unknown measure data

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Our aim is to study the existence of solutions for a nonlinear nonlocal Bernoulli-type free boundary problem with a unknown measure data. The problem arises in several nonlinear flow laws and physical situation. The elliptic problem was studied by J.I. Díaz, J.F. Padial and J.M. Rakotoson in *On some Bernoulli free boundary type problems for general elliptic operators*, Proceedings of the Royal Society of Edinburgh, 137A (2007), 895-911. Here, one will give an approach for the evolution case. We introduce a semi-implicit time differencing in order to obtain a family of elliptic problems. For each one of this problems, we find weak solution by applying a general mountain pass principle due to Ghoussoub-Preiss for a sequence of approximate nonsingular problems. Finally, apriori estimates allow us to obtain the solution by passing to the limit.

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On a free boundary problem for a cross-diffusion system

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We study a free boundary problem for a cross-diffusion system describing the tumor growth. The free boundary is the surface of segregation for the cells of different kind. The method is based on the introduction of a local system of Lagrangian coordinates which renders the free boundary stationary. We prove that the problem admits a weak solution, derive the equation of motion of the free boundary and study its regularity.

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On a climate energy balance model with continents distribution.

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We study a global climate model with continents distribution. The model is based on a long time average energy balance and leads to a nonlinear parabolic equation for the mean surface temperature. This energy balance model is coupled with a deep ocean model in the oceanic areas. We extend some results on the mathematical treatment of the model without land-sea distribution.

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Inverse problems in lubrication theory

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We study a system of equations concerning equilibrium positions of journal bearings. The problem consists of two surfaces in relative motion separated by a small distance filled with a lubricant. The shape of the inlet surface is circular, while the other surface has a more general shape. An exterior force F is applied on the inner cylinder (shaft) which turns with a given velocity. The force applied produces an unknown displacement of the inner cylinder and therefore a new unknown (the displacement) appears in the problem. We will see results concerning the existence of at least one equilibrium of the problem, when F is constant in time.

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Approximations of random dispersal operators by nonlocal dispersal operators

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This talk is concerned with the approximations of random dispersal operators/equations by nonlocal dispersal operators/equations. It first proves that the solutions of properly rescaled nonlocal dispersal initial-boundary value problems converge to the solutions of the corresponding random dispersal initial-boundary value problems. Next, it proves that the principal spectrum points of nonlocal dispersal operators with properly rescaled kernels converge to the principal eigenvalues of the corresponding random dispersal operators. Finally, it proves that the unique positive stationary solutions of nonlocal dispersal KPP equations with properly rescaled kernels converge to the unique positive stationary solutions of the corresponding random dispersal KPP equations.

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Spatial spread and front propagation dynamics of nonlocal monostable equations in periodic habitats

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This talk is concerned with the spatial spread and front propagation dynamics of monostable equations with nonlocal dispersal in spatially periodic habitats. Such equations arise in modeling the population dynamics of many species which exhibit nonlocal internal interactions and live in spatially periodic habitats. Firstly, we establish a general principal eigenvalue theory for spatially periodic nonlocal dispersal operators. Secondly, applying such theory and comparison principle for sub- and super-solutions, we obtain the existence, uniqueness, and global stability of spatially periodic positive stationary solutions and the existence of a spatial spreading speed in any given direction of a general spatially periodic nonlocal equation. Such features are generic for nonlocal monostable equations in the sense that they are independent of the assumption of the existence of the principal eigenvalue of the linearized nonlocal dispersal operator at 0. Finally, under the above assumption we also investigate the front propagation feature for monostable equations with non-local dispersal in spatially periodic habitats.

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