Special Session 75: Heteroclinic Cycles: Theory and Applications

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Heteroclinic cycles were first studied in the 1980's to explain aperiodic switching between different steady-states observed in certain hydrodynamical experiments. These flow invariant objects are non generic, however they can be structurally as well as dynamically stable when symmetries or other structures are imposed on the system. Symmetry-breaking bifurcation theory is typically used to study them. Since their discovery, further applications have been found as well as new mechanisms to create such cycles. Typical questions relate to notions of stability, understanding of nearby dynamics and the effects of forced symmetry breaking.

The aim of the workshop is to collect together various approaches and up-to-date results. We aim to bring together mathematicians and other scientists to increase interactions between the more theoretical aspects and applications. Recent progress in experimental and computational neuroscience and other life sciences lead us to expect that new applications of heteroclinic cycles will show up in these domains.

We also aim to bring together established researchers and younger scientists, in order to generate new ideas and to provide input for new developments.

Bifurcation of robust heteroclinic cycles in spherically invariant systems with $\ell = 3, 4$ mode interaction

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Bifurcation with spherical symmetry and mode interaction is known to produce robust heteroclinic cycles between axisymmetric steady-states as it occurs for the mode interaction with representation of SO(3) of degrees l=1 and 2 in the context of the onset of Rayleigh-Bénard convection. The existence of such heteroclinic cycles for the mode interaction between two consecutive modes (l,l+1) was proved generically. However the (3,4) mode interaction is an exception of this last analyze. Moreover it may correspond to the onset of convection of an experiment performed recently in the International Space Station designed in order to provide a system with (nearly) spherical symmetry (GeoFlow project). Motivated initially by this experiment, we have analyzed the occurrence of robust heteroclinic cycle for this (3,4) mode interaction. This case is highly complex but, applying the methods of equivariant bifurcation theory, we have shown the existence of (generalized) robust heteroclinic cycles involving, not only axisymmetric states, but also and principally states with cubic symmetry. These objects are observable in the numerical simulations of the dynamics on the center manifold for parameter values relevant as well for geophysics and as for the GeoFlow project.

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Robust heteroclinic cycles in delay-differential equations

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Robust heteroclinic cycles in delay-differential equations can arise naturally on finite-dimensional centre manifolds; in this case, the theory is straightforward. I will discuss an example of a heteroclinic cycle in a symmetric delay-differential equation which does not lie in a finite-dimensional submanifold and robust with respect to a specific class of perturbations. These are called Inherently Infinite Dimensional Cycles. The example is a generalization of the Guckenheimer-Holmes cycle seen as a delay-coupled system of three cells. I will also present some numerical simulations and some open questions.

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Stability and dynamics along a heteroclinic network

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The literature on heteroclinic networks has introduced several notions of stability that relate to attraction properties of the network. At the same time, different types of complex behaviour near the network have been studied. These are often associated to the existence of some type of switching at nodes or along connections of the network. The object of this talk is the discussion of the relation between different types of stability and admissible types of dynamics that may be observed. This will be illustrated by means of a Rock-Scissors-Paper game with two players and a problem in convection.

Some of the results presented in this talk were obtained jointly with M. Aguiar, I. Labouriau and O. Podvigina.

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Characteristic features of the heteroclinic networks with a child-cycle

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Heteroclinic networks with various types of structure can be found in dynamical systems with a certain class of constraints, including heteroclinic networks with depth larger than one, i.e. network with connections from fixed point saddle to saddle-like heteroclinic cycle/network (child-cycle/network). In this talk, we will mainly concentrate on the origin of effective nonlinearity that induces complex phenomena like the coexistence of infinitely many sequence of attractors in the neighborhood of such heteroclinic networks.

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Bifurcation of a heteroclinic network in a problem of pattern formation in the Poincaré disk

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We consider the spontaneous formation of steady solution of a PDE or integro-differential equation set on the Poincaré disk $\mathbb{D}$, which is invariant under the action of the octagonal lattice group Γ. This problem was introduced as an example of spontaneous pattern formation in a model of image feature detection by the visual cortex where the features are assumed to be represented in the space of structure tensors. Under "generic" assumptions the bifurcation problem reduces to an ODE which is invariant by an irreducible representation of the group of automorphisms $\mathcal{G}$ of the compact Riemann surface $\mathbb{D}/\Gamma$. The irreducible representations of $\mathcal{G}$ have dimension one, two, three and four. In this presentation, we show that for one of the four-dimensional representation there is a generic bifurcation of a heteroclinic network connecting equilibria with two different orbit types. We also present some results about the stability of this heteroclinic network together with some numerical simulations.

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Heteroclinic cycles in complex systems

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We describe recent work on robust heteroclinic cycles in large asymmetric adaptive networks of approximately identical cells and as well as in hybrid networks mixing deterministic and random dynamics. We also describe some examples using a SLOGALS architecture (SLOppy Globally Asynchronous Locally Synchronous).

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Dynamics of codimension one homoclinic cycles

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We discuss the dynamics near codimension one homoclinic cycles in flows that are equivariant under actions of the group D_m. We show how suspensions of subshifts of finite type appear in the unfolding. The descriptions are provided in terms of the geometry of the homoclinic cycle. The analysis is not complete for a certain type of D_m cycles. We show how those cycles can be constructed, and we present a more complete picture of the dynamics based on numerical simulations.

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Asymptotic stability of robust heteroclinic cycles

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Ian Melbourne

In this talk I present the results on asymptotic stability of heteroclinic cycles obtained in joint work with Ian Melbourne. In our first article we identified a class of cycles for which we could find an optimal condition for stability. However our condition was not optimal for another class of cycles studied by Hofbauer and Sigmund and by Field and Swift. These authors obtained a different optimal condition using a transition matrix technique. In a later work we combined the two approaches, generalizing both of them. Finally in a recent work, we obtained a further generalization of the transition matrix method

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On relations between the stability index and attraction properties of heteroclinic cycles

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In this talk I will present recent results on the relationship between the stability index for heteroclinic cycles that Peter Ashwin and Olga Podvigina defined in their paper "On local attraction properties and a stability index for heteroclinic connections" and different stability properties of the cycle. The index quantifies the extent of the basin of attraction of the cycle in a small ε-ball around it. It is related to stability properties of the cycle, but not always in an obvious way. For example, if there is a point on the cycle where the index exists and is greater than $-\infty$, then the cycle is essentially asymptotically stable (e.a.s.). A similar result can be shown for predominant asymptotic stability (p.a.s.) of the cycle in the case that the index exists everywhere and is greater than some constant $c > 0$. On the way to proving these results we establish a more general statement, namely that any cycle with a set of positive measure in its basin of attraction is e.a.s. One crucial idea for the proof of this was developed by Olga Podvigina.

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Synaptic cellular automaton for description the sequential dynamics of excitatory neural networks

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Neurophysiological experiments have indicated that some neural processes are accompanied by transient short-time activity of individual neurons or small groups of neurons. In such a process, a sequence of activity phases of different neurons emerges successively in time. It was shown that this behavior can be related to the existence of a collection of metastable invariant sets joined by heteroclinic trajectories in the phase space (heteroclinic network) and thus can be thought of as a process of successive switching among these metastable sets. There are some models in which the heteroclinic network can be rigorously established. However, it is not always a simple problem. We have proposed an approach of the reduction of continuous sequential dynamics of excitatory neural networks to a discrete one, in the form of a cellular automata (CA) on the graph of connections. In our approach the main role is played by the dynamics of synapses but not by the specific features of neurons. The CA represents a network of synapses with a finite number of states which alternate with each other, according to some fixed rules. We illustrate our approach on an example of network of Morris-Lecar neurons coupled by chemical synapses with short-term plasticity.

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Stability and bifurcations of heteroclinic cycles of type Z

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Dynamical systems, equivariant under the action of a non-trivial symmetry group, can possess structurally stable heteroclinic cycles. We consider stability properties of a class of structurally stable heteroclinic cycles in $\mathbb{R}^n$, which we call heteroclinic cycles of type Z. It is well-known that a heteroclinic cycle, that is not asymptotically stable, can attract nevertheless a positive measure set from its neighbourhood. We call such cycles fragmentarily asymptotically stable. Necessary and sufficient conditions for fragmentary asymptotic stability are expressed in terms of eigenvalues and eigenvectors of transition matrices. Finally, we discuss bifurcations occurring when the conditions for asymptotic stability or for fragmentary asymptotic stability are broken.

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Resonance of robust heteroclinic networks

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It is well known that heteroclinic cycles and networks can exist robustly in systems with symmetry. Resonance bifurcations are one way in which heteroclinic cycles can change stability. Such bifurcations occur when an algebraic condition on the eigenvalues of the equilibria in the cycle is satisfied, and generically are accompanied by the birth or death of a long-period periodic orbit. Although resonance bifurcations of heteroclinic cycles have been extensively studied, very little is known about resonances of heteroclinic networks. In this talk, I will describe new work on understanding resonance bifurcations of heteroclinic networks. In a network, at least one unstable manifold is two-dimensional; I will describe a technique to account for all the trajectories on these manifolds. We find that the sub-cycles of the network undergo resonance bifurcations as might be expected if they were isolated from the network. There is an additional resonance point due to the structure of the network at which the periodic orbits bifurcating from the different sub-cycles of the network interact.

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Heteroclinic bifurcations near non-reversible homoclinic snaking

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Non-reversible homoclinic snaking is a scenario where the bifurcation curve of a codimension-one homoclinic orbit to a hyperbolic equilibrium exists on a snaking curve in parameter space. We consider systems without any particular structure and give analytical and geometric statements when such a scenario is to be expected. The numerical analysis of heteroclinic bifurcations near the non-reversible homoclinic snaking is presented, in particular equilibrium-to-periodic heteroclinic connections.

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Heteroclinic phenomena

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For systems with symmetry, it is possible to find robust heteroclinic cycles and networks, which may be seen as the skeleton for the understanding of complicated dynamics. In this talk, we discuss the geometric behaviour of trajectories in a neighbourhood of a special class of heteroclinic networks, near which we observe chaotic dynamics. More precisely, we present two persistent phenomena which we call switching and double cycling, and some dynamical consequences.

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Timing control of networks with switching dynamics

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Timing control is a fundamental principle of cognition and behavior. The idea that intrinsic dynamics of the neuronal networks allows them to generate the different time intervals for the spatiotemporal encoding is supported by theoretical and experimental studies. Still the mechanisms by which this temporal control is achieved remain unclear. We have focused on the properties of inhibitory neuronal network sequential dynamics since they are a substantial factor on the generation of a rhythm in central pattern generators. Here we consider the new mechanism that is related to the multi-neuronal network dynamics - winnerless competition (WLC) between three unidirectionally inhibitory clusters of Hodgkin-Huxley modeled neurons inducing closed limit cycles that remind the heteroclinic cycles included the saddle cycle. Findings indicate a sensitive dependence of the sequential timing on the effective synaptic strength of the connections between clusters. For our particular purpose, neurons within the clusters were not connected supporting the idea that such connectivity is not necessary for timing control of the network. Role of the cluster size on the control cycling activity timing is discussed.

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