Special Session 77: The Navier-Stokes Equations and Related Problems

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The Navier-Stokes equations represent the fundamental equations in mathematical fluid dynamics. Recently researchers from many countries around the world have developed important new results to the theory of these important equations and their applications: Existence proofs for weak or even strong solutions for time-dependent as well as for stationary boundary value problems of the fully nonlinear Navier-Stokes equations or for the linearized Stokes equations, considered with a wide variety of boundary conditions, uniqueness classes in the frame of suitably adapted abstract spaces, questions of asymptotic behavior and stability of solutions, and of maximum regularity, and convergence results with vanishing viscosity. A further important part of research is the interaction of fluid flow with moving bodies or particles or with additional heat flow, leading to enlarged dynamical systems which combine the Navier-Stokes equations with other equations of evolution. The strong progress in theory opens the way to efficient numerical schemes for problems in technology and medicine. The aim of our special session will be to bring together leading researchers from all parts of the world and from different working directions as mentioned above, and to initiate exchange of ideas as well as future cooperations.

The L^∞-Stokes semigroup in exterior domains

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Analyticity of semigroups is one of fundamental properties for semigroups associated with parabolic equations. In this talk we study analyticity of the Stokes semigroup in spaces of bounded functions both in bounded and unbounded domains. In particular, we consider the Stokes system in an exterior domain as a typical example of an unbounded domain. Even the existence of solution is nontrivial in that case. For the proof of analyticity of the Stokes semigroup, we appeal to a priori L^∞-estimates which is derived from a blow-up argument.

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L^p-theory for Stokes and Navier-Stokes equations with non-standard boundary conditions

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We consider here elliptical systems as Stokes and Navier-Stokes problems in a bounded domain, eventually multiply connected, whose boundary consists of multi-connected components. We investigate the solvability in L^p theory, with $1 < p < \infty$, under the non standard boundary conditions

$$\boldsymbol{u}\cdot\boldsymbol{n} = g, \quad \mathbf{curl}\,\boldsymbol{u}\times\boldsymbol{n} = \boldsymbol{h} \quad \text{or}$$
$$\boldsymbol{u}\times\boldsymbol{n} = \boldsymbol{g}, \quad \pi = \pi_* \quad \text{on}\,\Gamma.$$

The main ingredients for this solvability are given by the Inf-Sup conditions, some Sobolev's inequalities for vector fields and the theory of vector potentials satisfying

$$\boldsymbol{\psi}\cdot\boldsymbol{n} = 0, \quad \text{or} \quad \boldsymbol{\psi}\times\boldsymbol{n} = \mathbf{0} \quad \text{on}\,\Gamma.$$

Those inequalities play a fundamental key and are obtained thanks to Calderon-Zygmund inequalities and integral representations. In the study of ellpitical problems, we consider both generalized solutions and strong solutions that very weak solutions.

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Boundary regularity for the steady stokes type flow with shear thickening viscosity

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We work on the boundary regularity for weak solutions to the stationary Stokes type equations with shear dependent viscosity. Using a weighted estimate near the boundary, we obtain the Holder continuity of the solution for the shear thickening fluid without the convection term.

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An obstacle problem for capillary surfaces

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Let $G := \Omega\times\mathbb{R}^+$, $\Omega\subseteq\mathbb{R}^2$ a bounded domain, be a cylinder that is partly filled with liquid; $\mathcal{B}$ is a rigid body that is floating on it, and the interface between the fluid and the air above is described as a graph of a real function u.

If we assume that the shape of the interface is governed by surface tension then the unknowns of the problems, which are the function u and the position of $\mathcal{B}$, are determined by a variational problem for the energy $\mathcal{E}$ of the configuration.

$\mathcal{E}$ consists of the interfacial energy which is proportional to the area of the graph of u, the adhesion energy, which is proportional to the wetted part of the boundary of $\mathcal{B}$ and of the cylindrical boundary $\partial\Omega \times \mathbb{R}^+$ as well as the gravitational energies of the fluid and of the floating body.

Because of the presence of $\mathcal{B}$ the capillary surface Σ is bounded by some curve on $\partial\Omega \times \mathbb{R}^+$ as well as a contact line Γ on $\partial\mathcal{B}$; therefore in our formulation of the variational problem the body $\mathcal{B}$ acts as an obstacle for u.

We show the existence of a minimizer and investigate some of its properties, in particular the regularity. This is based on the first variation of the energy which also gives a variant of Archimedes' principle that includes the forces exerted on $\mathcal{B}$ by Σ.

References

[1] J. Bemelmans, G. P. Galdi, M. Kyed: Fluid Flows around Floating Bodies, I: The Hydrostatic Case, preprint, RWTH Aachen University, to appear in Journal of Mathematical Fluid Mechanics, 2012

[2] J. Bemelmans, G. P. Galdi, M. Kyed: Capillary Surfaces and Floating Bodies, preprint, RWTH Aachen University, 2012

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Maximum modulus estimate in nonstationary Stokes system

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A maximum modulus estimate for the nonstationary Stokes system in smooth domain is found. The singular part and regular part of Poisson kernel are analyzed. The singular part is a gradient potential related to only normal component of the boundary data. Furthermore, the normal velocity near the boundary is bounded if the boundary data is bounded. If the normal component of the boundary data is Dini-continuous and the tangential component of the boundary data is bounded, then the maximum modulus of velocity is bounded in whole domain

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Some properties of solutions to liquid crystal systems

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In this talk, I will present some work on Liquid Crystal systems. The existence of global weak solutions and regularity of solutions to the nematic liquid crystals with non-constant fluid density were established. We also established a long time behavior result for the regular solutions to the liquid crystal system with constant density, providing small initial condition.

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Pointwise decay of incompressible flows around rigid bodies

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We consider pointwise spatial decay of flows around rigid bodies moving steadily in an incompressible viscous fluid.

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Well-posedness of the Euler equations in planar convex domains

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Let $\Omega \subset \mathbb{R}^2$ be a bounded convex domain. We consider the well-posedness of the Euler equations

$$\begin{cases} \partial_t \mathbf{u}(x,t) + (\mathbf{u}\cdot\nabla)\mathbf{u}(x,t) + \nabla p(x,t) = \mathbf{f}(x,t), \\ \qquad\qquad\qquad\qquad\qquad x \in \Omega, t \in (0,T) \\ \nabla \cdot \mathbf{u}(x,t) = 0, \quad x \in \Omega, t \in (0,T) \end{cases}$$

with impermeability boundary conditions. For divergence-free initial data $\mathbf{u}_0 = \mathbf{u}(t=0) \in H^1(\Omega)$, we show the existence of a weak solution

$$\mathbf{u} = \mathbf{u}(x,t) \in L^\infty((0,T), H^1(\Omega)^2) \\ \cap W^{1,2}((0,T); L^p(\Omega)^2)$$

to the Euler equations. Under the additional assumptions that Ω is a polygonal-type domain, and that $\operatorname{curl} \mathbf{u}_0 \in L^\infty(\Omega)$, the obtained solution $\mathbf{u}$ is unique, and possesses the additional regularity

$$D\mathbf{u} \in L^\infty((0,T), \mathrm{Exp}L^1(\Omega)).$$

Even for smooth domains, these results improve on the classical theorem of Kato; they also extend to polygonal-type domains the uniqueness theorem of Yudovich, thus partly improving results by Taylor, and Gèrard-Varet and Lacave.

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On the steady equations for compressible radiative gas

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We study the equations describing the steady flow of a compressible radiative gas with newtonian rheology. Under suitable assumptions on the data which include the physically relevant situations (i.e. the pressure law for monoatomic gas, the heat conductivity growing with square root of the temperature) we show the existence of a variational entropy solution to the corresponding system of partial differential equations. Under additional restrictions we also show the existence of a weak solution to this problem.

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Weak solutions for the motion of a self-propelled deformable structure in a viscous incompressible fluid

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We deal with a model of swimming where we consider that the muscles of the creature are enough to perform a given deformation. Using this model we investigate the self-propelled motion in a viscous fluid. We study the existence of weak solution. The proof is based on a penalization method.

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On a stationary compressible flow with slip-inflow boundary conditions

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In my talk I am going to discuss the issue of existence of stationary solutions to the Navier-Stokes system describing the flow of a compressible fluid in a cylindrical domain. On the boundary we prescribe inhomogeneous slip conditions on the velocity. I will consider both the barotropic flow and the complete system with thermal effects. In both cases we show the existence of a solution in a vicinity of a given special solution such as a constant flow with nonzero velocity or a Poiseuille-like profile. The main problem to face in the proof is the lack of compactess in the continuity equation, I will discuss different ways to overcome this problem, such as elliptic regularization, successive approximations or reformulation of the problem in a kind of Lagrangian coordinates. The results have been obtained together with Piotr B. Mucha and Milan Pokorný.

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Hopf-Galerkin approach to vorticity transport & diffusion.

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To the initial boundary value problem of vorticity transport & diffusion in a smoothly bounded 3-dimensional domain we present a convergent Hopf-Galerkin scheme including error estimates locally in time.

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Maximal regularity on cross-sections implies maximal regularity on a cylinder

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The proof of maximal regularity for general boundary value problems on domains with non-compact boundaries relies typically on intricate localization procedures. In the talk presented, I will show that for the L^p-approach such procedures can be avoided, if the domain (and the corresponding differential operator) is cylinder like, i.e., of the form $\Omega = \Omega_1 \times \Omega_2$ and maximal regularity is known on $\Omega_1 \subset \mathbb{R}^k$ and $\Omega_2 \subset \mathbb{R}^m$. For domains of this type there is a much more elegant approach based on operator-valued functional-calculus techniques. The approach has also some advantages in comparison to localization procedures, as e.g. it also applies to Lipschitz cross-sections. The methods apply to Stokes and Navier-Stokes equations as well, as will be demonstrated in the talk, too. This is a joint project with Tobias Nau at the University of Konstanz.

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On questions of decay for solutions to liquid crystals

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I will discuss the asymptotic behavior for solutions to a nematic liquid crystals system in the whole space of three dimensions. The fluid under consideration has constant density and small initial data.

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The fractional Laplacian in an exterior domain

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We study the fractional Laplacian in the complement of an obstacle and use our result to obtain results about solutions of some fluid dynamics equations in exterior domains. The work relies heavily on a generalization of the Fourier transform due to Ikebe and Ramm, and the characterization of the fractional Laplacian as a local operator due to Caffarelli and Silvestre.

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Fluid-structure interaction problems in hemodynamics

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Blood flow interacts mechanically with the vessel wall, giving rise to pressure waves propagating in arteries, which deform under the action of blood pressure. In order to capture these phenomena, complex fluid-structure interaction (FSI) problems must be considered, coupling physiologically meaningful models for both the blood and the vessel wall. From the theoretical point of view, this is extremely difficult because of the high non-linearity of the problem and the low regularity of the displacement of the fluid-structure interface. So far, mathematical results have been obtained only in simplified cases. In this talk, simulations of the mechanical interaction between blood flow and vessel walls will be shown, based on a partitioned approach. A 3D FSI model in a compliant vessel is used to describe the pressure wave propagation. The 3D fluid is described through the Navier-Stokes equations (or a shear-thinning generalized Newtonian model) and the structure by a 3D hyperelastic model. In order to cope with the spurious reflections due to the truncation of the computational domain, several absorbing boundary conditions are analyzed. This work has been done in collaboration with A. Moura, J. Janela, and A. Gambaruto.

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On the motion of a fluid-rigid ball system at the zero limit of the rigid ball radius

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We investigate the limit of a coupled system "rigid ball + Navier-Stokes liquid" when the radius of the ball goes to 0. Dashti and Robinson [Arch. Ration. Mech. Anal., 2011] solved this problem in dimension two, in the absence of rotation. Our aim is to improve their result, considering the two-dimensional case when the disk is also spinning and the general three-dimensional case.

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(Almost) everything you always wanted to know about the Helmholtz decomposition but were afraid to ask.

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The Helmholtz decomposition of L^p-spaces (and in its wake the definition of the Stokes operator) is a crucial tool in the theory of Navier-Stokes equations. This contribution collects results on this topic with a particular emphasis on why certain arguments do not work in particular situations.

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Compressible Navier-Stokes equations with slip boundary conditions in time dependent domains

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We consider the compressible (barotropic) Navier-Stokes system on time-dependent domains, supplemented with slip boundary conditions. Our approach is based on penalization of the boundary behaviour, viscosity, and the pressure in the weak formulation. Global-in-time weak solutions are obtained.

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Extensions of Serrin's condition for weak solutions of the Navier-Stokes equations

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We consider weak solutions of the non-stationary Navier-Stokes equations in a smoothly bounded three-dimensional domain and develop some extensions of Serrin's uniqueness and regularity condition for this case.

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On a new 3D model for incompressible Euler and Navier-Stokes equations

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In this talk, we investigate some new properties of the incompressible Euler and Navier-Stokes equations by studying a 3D model for axisymmetric 3D incompressible Euler and Navier-Stokes equations with swirl. The 3D model is derived by reformulating the axisymmetric 3D incompressible Euler and Navier-Stokes equations and then neglecting the convection term of the resulting equations. Some properties of this 3D model are reviewed. Finally, some potential features of the incompressible Euler and Navier-Stokes equations such as the stabilizing effect of the convection are presented. (The research was supported by National Basic Research Program of China (973 Program, 2011CB808002), the NSFC (11071009) and PHR-IHLB (200906103))

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Chemically reacting mixtures

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Development of rigorous mathematical theory for reactive flows is of fundamental need for purposes of many practical applications. In most of them, one has to deal with multicomponent mixtures undergoing reactions that are, in general, completly reversible. In this talk I will summarize the recent results on the existence of solutions to the full Navier-Stokes system coupled with the species mass balance equations and present the open problems.

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