Special Session 8: Propagation Phenomena Appearing in Reaction-Diffusion Systems

Hirokazu Ninomiya, Meiji University, Japan
Masaharu Taniguchi, Tokyo Institute of Technology, Japan

This special session is concerned with mathematical analysis on propagation phenomena or pattern formation appearing in reaction-diffusion systems. Related topics are traveling waves, equilibrium states and asymptotic behavior of solutions.

Existence and uniqueness of spiral waves of a wave front interaction model in a plane

Yan-Yu Chen
Meiji University, Taiwan
sumocat@meiji.ac.jp
Jonq-Shenq Guo, Hirokazu Ninomiya

To study the spiral wave in an unbounded excitable medium, we consider the wave front interaction model which derived by Zykov in 2007. This model consists of two systems of ordinary differential equations which describe the wave front and wave back, respectively. First, we derive some properties of the back by shooting argument and comparison principle. Next we show the global existence of the solution of the back. Then, we study its asymptotic behavior at infinity. Finally, we prove the uniqueness of the solution.

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Monotone traveling waves of the nonlocal Fisher-KPP equation

Jian Fang
York University, Canada
jfang@mun.ca
Xiaoqiang Zhao

We consider traveling waves of the nonlocal Fisher-KPP equation. It is known that narrow nonlocal interactions do not change the monotonicity of traveling waves but some wide interactions do. We establish the critical value of the rate of nonlocal interactions for the existence of monotone traveling waves and further prove that such traveling wave is unique up to translation.

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Dynamics of traveling fronts in some heterogeneous diffusive media

Hideo Ikeda
University of Toyama, Japan
ikeda@sci.u-toyama.ac.jp

We consider two component reaction-diffusion systems with a specific bistable and odd symmetric nonlinearity, which have the bifurcation structure of pitchfork-type traveling front solutions with opposite velocities. We introduce a spatial heterogeneity, for example, a Heaviside-like abrupt change at the origin in the space, into diffusion coefficients. Numerically, the responses of traveling fronts via the heterogeneity can be classified into four types of behaviors depending on the strength of the heterogeneity: passage, stoppage, and two types of reflection. The goal is to reduce the PDE dynamics to finite-dimensional ODE systems on a center manifold and show the mathematical mechanism to produce the four types of responses in the PDE systems using finite-dimensional ODE systems. The reduced ODE systems include the terms (referred to as heterogeneous perturbations) originating from the interaction between traveling front solutions and the heterogeneity, which is very important in order to determine the dynamics of the ODE systems. Using these results, we discuss what play the role of separators of the four different behaviors.

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Stability analysis for a planar traveling wave solution in an excitable system

Kota Ikeda
Meiji University, Japan
ikeda@isc.meiji.ac.jp

Excitable systems appear in various phenomena in nature. The first example is the propagation of impulses along a nerve axon. The second one is the simplified chemical model, called Oregonator, of BZ reaction. The last one is the combustion in micrograbity which has various spatial patterns depending on a typical parameter. These phonomena were modeled by reaction-diffusion equations which commonly exhibit excitability and generate a traveling wave solution with a pulse shape. On the other hand, a diffusive coefficient in the three equations changes widely in these three phonomena, which is crucial in the stability of planar traveling waves. In this talk, we show the relation between the diffusive coefficient and the stability of a traveling wave solution with a pulse shape.

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Bifurcation structure of radially symmetric positive stationary solutions for a competition-diffusion system

Yukio Kan-on
Ehime University, Japan
kanon@ed.ehime-u.ac.jp

In this talk, we consider a reaction-diffusion system that describes the dynamics of population for two competing species community, and investigate the global bifurcation structure of radially symmetric positive stationary solutions for the system by assuming the habitat of the community to be a ball. To do this, we shall treat the dimension of the habitat and the diffusion rates of the system as bifurcation parameters, and employ the comparison principle and the bifurcation theory.

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Semilinear solutions in a sector for a curvature flow equation

Bendong Lou
Tongji University, Peoples Rep of China
blou@tongji.edu.cn

We study a two-point free boundary problem in a sector for a curvature flow equation. The inhomogeneous boundary conditions are assumed to be spatially and temporally "similar" in a special way. We prove the existence and uniqueness of an expanding solution, as well as a shrinking solution, which is selfsimilar at discrete times.

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Planar standing front waves of the FitzHugh-Nagumo system

Yoshihisa Morita
Ryukoku University, Japan
morita@rins.ryukoku.ac.jp
Chao-Nien Chen, Shih-Yin Kung

We are dealing with the FitzHugh-Nagmumo system, $u_t = d\Delta u + f(u) - v,\ v_t = \Delta v - \gamma v + u$, where $f(u) = u(u-\beta)(1-u), \beta \in (0, 1/2)$, in the whole space.We assume a balanced condition, $\gamma = 9(2\beta^2 - 5\beta + 2)^{-1}$.Then for $d > \gamma^{-2}$ the existence of a planar standing front wave is shown by using variational method. We also show the uniqueness of the solution with a symmetry by the comparison method under an additional restriction for β.Moreover, we can discuss the dynamical stability of the solution in one-space dimensional setting if β is sufficiently close to 1/2.

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Existence of recurrent traveling waves in a two-dimensional undulating cylinder: the virtual pinning case

Ken-ichi Nakamura
Kanazawa University, Japan
k-nakamura@se.kanazawa-u.ac.jp

In this talk we study traveling wave solutions for a curvature-driven motion of plane curves in a two-dimensional infinite cylinder with undulating boundary. Here a traveling wave in non-periodic inhomogeneous media is defined as a time-global solution whose shape is "a continuous function of the current environment". Under suitable conditions on the boundary undulation we show the existence of traveling waves which propagates over the entire cylinder with zero lower average speed. Such a peculiar situation called "virtual pinning" never occurs if the boundary undulation is periodic.

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Singular limit of a damped wave equation with bistable nonlinearity

Mitsunori Nara
Iwate University, Japan
nara@iwate-u.ac.jp
Danielle Hilhorst

We consider the interfacial phenomena and the singular limit of a damped wave equation with bistable type nonlinearity. Under the assumption that the damping effect is very strong, the solution will behave like that of parabolic equations. We establish the comparison principle for the damped wave equation and construct suitable supersolutions and subsolutions.

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Stability and bifurcation of periodic traveling waves in a dispersive system

Kunimochi Sakamoto
Hiroshima University, Japan
kuni@math.sci.hiroshima-u.ac.jp

Stable periodic traveling waves bifurcate from a stable branch of trivial solutions. The trivial solutions do not change its stability (i.e., remain stable) before and after the periodic traveling waves bifurcate. Such bifurcation naturally seems contradictory in dissipative systems and stability is meant asymptotic stability. However, situation is different in dispersive systems and the stability is meant orbital stabiity.In this talk, an example of such bifurcations is presented in a derivative nonlinear Schrödinger equation with the periodic boundary condition.

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Front speeds in reaction-diffusion systems: slow pushed and accelerated pulled fronts

Arnd Scheel
University of Minnesota, USA
scheel@umn.edu
Matt Holzer

We show two examples where front speeds in coupled reaction-diffusion systems deviate from the typical distinction between pulled (slow linear) and pushed (fast nonlinear) predictions. In the first example, a Lotka-Volterra system, we show that the selected front speed is much slower than the linear prediction and give an asymptotic expansion using geometric desingularization of the traveling-wave problem. In the second example, we study multi-stage invasion problems, arising in pattern-forming systems (roll-hexagon competition), phase-separation problems (spinodal decomposition and coarsening fronts), and population models. We show that a primary front can accelerate the speed of the secondary invasion process, leading to locked modes of propagation and, more surprisingly, constant acceleration at arbitrary large distances. We give expansions for locked plateaus and determine the speed of the secondary front in the unlocked case.

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Convergence and blow-up of solutions for a complex-valued heat equation with a quadratic nonlinearity

Masahiko Shimojo
Department of Mathematics, Hokkaido University, Japan
Shimojo@math.sci.hokudai.ac.jp
Jong-Shenq Guo,Hirokazu Ninomiya, Eiji Yanagid

We study the Cauchy problem for a system of parabolic equations, which is derived from a complex-valued equation with a quadratic nonlinearity. Our equation has a strong relation with the viscous Constantin-Lax-Majda equation, which is a one dimensional model for the vorticity equation. We first study the asymptotic behavior for global solutions. We also construct solutions that both components of this system blow up simultaneously, although only the real part of the corresponding ODE blows up.

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Precise asymptotic formulas of critical eigenfunctions for 1D bistable reaction diffusion equations

Tohru Wakasa
Kyushu Institute of Technology, Japan
wakasa@mns.kyutech.ac.jp

In 1D bistable reaction diffusion equations with a small diffusion paramter, it is known that a transient dynamics of solutions are characteriaced by a super slow motion of thin interfaces connecting two stable states. In order to understand this pattern dynamics, the existence of critical eigenvalues of the corresponding linearized operators for nontrivial steady-states plays a crucial role.In this talk, we will introduce a precise asymptotic formulas for 1 dimensional linearized operators for n mode stationary solutions.

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Nonplanar traveling wave solutions in Lotka-Volterra competition-diffusion system

Zhi-Cheng Wang
Lanzhou University, Peoples Rep of China
wangzhch@lzu.edu.cn
Wan-Tong Li, Shigui Ruan

We study the nonplanar traveling fronts of the Lotka-Volterra competition-diffusion system

$$\begin{cases} \frac{\partial}{\partial t}u_1(\mathbf{x},t) & = & \Delta u_1(\mathbf{x},t) \\ & + & u_1(\mathbf{x},t)\left[1- \ u_1(\mathbf{x},t) - k_1 u_2(\mathbf{x},t)\right] \\ \frac{\partial}{\partial t}u_2(\mathbf{x},t) & = & d\Delta u_2(\mathbf{x},t) \\ & + & r u_2(\mathbf{x},t)\left[1 - u_2(\mathbf{x},t) - k_2 u_1(\mathbf{x},t)\right], \end{cases}$$

with $\mathbf{x} \in \mathbb{R}^m, t > 0$.

For the bistable case, namely $k_1, k_2 > 1$, it is known that the system admits an one-dimensional traveling front $\mathbf{\Phi}(x+ct) = (\Phi_1(x+ct), \Phi_2(x+ct))$ connecting two stable equilibria $\mathbf{E}_u = (1,0)$ and $\mathbf{E}_v = (0,1)$, where $c \in \mathbb{R}$ is the unique wave speed. Assume $c > 0$. For any $s > c > 0$, we establish the V-shaped fronts in $\mathbb{R}^2$, pyramidal traveling fronts and conical traveling fronts in $\mathbb{R}^3$. For the V-shaped fronts and pyramidal traveling fronts, we also prove their uniqueness and stability. For conical traveling fronts, we show that they locally uniformly converge to the planar traveling front when s tends to c. We also show the nonexistence of conical traveling fronts.

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Asymptotic behavior in a 2-allele genetic model with population control

Hans Weinberger
University of Minnesota, USA
hfw@math.umn.edu

The standard model for the evolution of the densities of the three genotypes aa, aA, and AA is

$$\partial\rho_{aa}/\partial t - \Delta\rho_{aa} = \frac{r[2\rho_{aa}+\rho_{aA}]^2}{2[\rho_{aa}+\rho_{aA}+\rho_{AA}]} - \tau_{aa}\rho_{aa}$$
$$\partial\rho_{aA}/\partial t - \Delta\rho_{aA} = \frac{2r[2\rho_{aa}+\rho_{aA}][2\rho_{AA}+\rho_{aA}]}{2[\rho_{aa}+\rho_{aA}+\rho_{AA}]} - \tau_{aa}\rho_{aa}$$
$$\partial\rho_{AA}/\partial t - \Delta\rho_{AA} = \frac{r[2\rho_{AA}+\rho_{aA}]^2}{2[\rho_{aa}+\rho_{aA}+\rho_{AA}]} - \tau_{AA}\rho_{aa},$$

where the parameters are constants. It has recently been shown by Souplet and Winkler that in the heterozygote intermediate case $\tau_{aa} \geq \tau_{aA} \geq \tau_{AA}$, $\tau_{aa} > \tau_{AA}$ the gene fraction $u := [2\rho_{aa}+\rho_{aA}]/\{2[\rho_{aa}+\rho_{aA}+\rho_{AA}]\}$ converges to 0. However, the proof in the biologically interesting case depends on the fact that ρ_{AA} converges to infinity, which, as the authors pointed out, makes the model unrealistic. When $[\tau_{aa}+\tau_{AA}]/2 \leq \tau_{aA} < \tau_{aa}$, we produce a large class of models of the above form with the birth rate r replaced by a suitable density-dependent function. These models have the property that, for a reasonable set of conditions, the density $\rho_{aa}+\rho_{aA}$ of those individuals which have at least one a-allele converges to zero uniformly in every bounded set as t approaches infinity, while the total population density remains bounded and uniformly positive.

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Symmetric and asymmetric spikes for the two-dimensional Schnakenberg model

Matthias Winter
Brunel University, England
matthias.winter@brunel.ac.uk
Juncheng Wei

We consider the existence and stability of symmetric and asymmetric spikes for the two-dimensional Schnakenberg model in two space dimensions. For existence we will derive and solve an algebraic system for the amplitudes using a nondegeneracy condition. The positions of the spikes are then given by the nondegenerate critical points of some Green's function. For stability we will investigate large eigenvalues of order O(1) and small eigenvalues of order o(1) separately, which are connected with the amplitudes and positions of the spikes, respectively.

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The existence and stability of fraveling front solutions for some autocatalytic systems

Yaping Wu
Capital Normal University, Peoples Rep of China
yaping_wu@hotmail.com
Yi Li

In this talk we shall talk about the existence and stability of traveling fronts for the following typical autocatalytic chemical reaction system

$$\begin{cases} u_t = u_{xx} - u^q v^p, \\ v_t = dv_{xx} + u^q v^p. \end{cases}$$

For $p \geq 1$, $q \geq 1$ and $d > 0$, it is known that there exists a critical speed $c^*(p,q,d)$ such that for any $c \geq c^*(p,q)$ there exist travelling front solutions $(u(x-ct), v(x-ct))$ connecting $(0,1)$ and $(1,0)$. For the cases $p > 1$ or $q > 1$, the travelling waves with noncritical speed decay algebraically in space at $+\infty$ or $-\infty$.

In this talk we shall be more interested in the linear and nonlinear asymptotic stability of the waves with noncritical speeds and with algebraic spatial decay when d is near 1. We shall first talk about our recent work on the asymptotic stability of the waves with algebraic spatial decay in some polynomially weighted spaces for the system when $d = 1$. Further we shall introduce our recently obtained abstract results on the existence and analyticity of Evans function for the more general ODE systems with slow algebraic decaying coefficients, and our recent work on the linear exponential stability of waves with algebraic decay in some exponentially weighted spaces for the case $p > 1$ and $q \geq 1$ when d is near 1. Finally for the case $p > 1$ and $q = 1$ we shall prove the nonlinear asymptotic stability of the waves with noncritical speeds when d is near 1.

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