Special Session 82: Multi-component Integrable Systems, Solitons, and Nonlinear Waves

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The aim of this session broadly is to discuss recent developments in integrable systems and evolution equations related to nonlinear waves. Topics of special focus will be generalizations of Camassa-Holm and Hunter-Saxton equations, multi-component NLS and KdV-type equations, analysis of breaking waves, soliton solutions, bi-Hamiltonian structures, and connections with geometry.

Multi-component soliton equations

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In this talk we derive new multi-component soliton equations of mKdv-type, NLS-type and Sine-Gordon type. The method uses a general construction of bi-Hamiltonian integrable systems from inelastic curve flows in symmetric spaces applied to 3 examples of hermitian spaces: SU(n+1)/U(n) , SO(n+2)/SO(n)xSO(2) , SO(2n)/U(n). The resulting integrable systems in these spaces exhibit unitary invariance.

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Inelastic curve flows in 2-3 dimensional Minkowskian space

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The purpose of this talk to derive integrable systems from inelastic curve flows in 2 and 3 dimensional Minkowskian space by using Hasimoto variables. We introduce a Lorentzian version of a moving parallel frame and show that its structure equations encode the Hasimoto variables in natural way. For timelike/spacelike curves in the Minkowskian plane, we obtain the defocusing mKdV equation and its bi-Hamiltonian structure. For null curves, we find Burgers equation. For timelike curves in 3 dimensional Minkowskian space, we have derive the complex defocusing mKdV and the NLS equations whereas for spacelike curves, we find similar equations with complex numbers replaced by hyperbolic numbers.

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Multi-component soliton equations from geometric curve flows

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I will survey some recent work on deriving multi-component soliton equations (and their integrability structure) from geometric curve flows.

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C^3 ill-posedness of the gravity-capillary problem

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We prove via explicitly constructed initial data that solutions to the gravity-capillary wave system in $\mathbb{R}^3$ representing a $2d$ air-water interface immediately fail to be C^3 with respect to the initial data if the initial data $(h_0, \psi_0) \in H^{s+\frac{1}{2}} \otimes H^s$ for $s < 3$.

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The Cauchy problem for the Novikov equation

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We shall discuss well-posedness of the initial value problem for a class of weakly dispersive nonlinear evolution equations,including the Camassa-Holm, the Degasperis-Procesi, and the Novikov equation. The focus will be continuity properties of the data-to-solution map in Sobolev spaces. This talk is based on work in collaboration with Carlos Kenig, Gerard Misiolek and Curtis Holliman and Katelyn Grayshan.

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On small quasi-periodic perturbation of two-dimensional hyperbolic-type degenerate nonlinear systems

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This paper considers a class of two-dimensional nonlinearquasi-periodic systems with small perturbations. The unperturbed system has zero as a hyperbolic-type degenerate equilibrium point. If the quasi-periodic frequency satisfies the Diophantine conditions, by KAM iteration we prove that it can be reduced to asuitable normal form with zero as an equilibrium point by a nonlinear quasi-periodic transformation. Thus, we obtain a small quasi-periodic solution for the perturbed system.

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Wave breaking and global existence for the generalized periodic two-component Hunter-Saxton system

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In this paper, we study the wave-breaking phenomena and global existence for the generalized two-component Hunter-Saxton system in the periodic setting. We first establish local well-posedness for the generalized two component Hunter-Saxton system. We obtain a wave-breaking criterion for solutions and results of wave-breaking solutions with certain initial profiles. We also determine the exact blow-up rate of strong solutions. Finally, we give a sufficient condition for global solutions.

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Integrability, wave breaking and peakons for a modified μ-Camassa-Holm equation

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In this talk, a modified μ-Camassa-Holm equation with cubic nonlinearity is introduced. It is a integrable equation in the sense that it admits Lax-pair and bi-Hamiltonian structure. The formation of singularities and the existence of peaked travelingwave solutions for the equation are investigated. It is shown to admit a single peaked soliton and multi-peak on solutions, of a similar character of the μ-Camassa-Holm equation. Singularities of the solutions can occur only in the form of wave-breaking, and a wave-breaking mechanism for solutions with certain initial profiles is described in detail. Sufficient conditions for blow up of solutions to initial value problem are also given.

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Generalized EulerPoincare equations on Lie groups and Homogeneous spaces, orbit invariants and applications

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Cornelia Vizman

We develop the necessary tools, including a notion of logarithmic derivative for curves in homogeneous spaces, for deriving a general class of equations including EulerPoincare equations on Lie groups and homogeneous spaces. Orbit invariants play an important role in this context and we use these invariants to prove global existence and uniqueness results for a class of PDE. This class includes EulerPoincare equations that have not yet been considered in the literature as well as integrable equations like Camassa Holm, DegasperisProcesi, CH and DP equations, and the geodesic equations with respect to right-invariant Sobolev metrics on the group of diffeomorphisms of the circle.

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Painleve and Lax-pair tests for the integrability of a two-component nonlinear Schrödinger equations with variable coefficients and its nonautonomous solitons

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In the literature, we have found that the Painleve and the Lax-pair conditions of integrability are the same for some models and that those conditions are different for some other models. Having this in mind, we apply the Painleve test and the Lax-pair test on the generalized two-component Nonlinear Schrodinger (NLS) equations with variable coefficients and the external potentials with the goal to see in which category we can classify it. The results obtained so far show that this system fails the Painleve test but it passes the Lax-pair test. By employing the homogeneous balance principle and the F-expansion technique, we construct abundant exact traveling wave and abundant solitary wave-like solutions including bright-dark-,bright-bright-, and dark-dark-like solitary pairs. The solutions are obtained by choosing special forms of the gain/loss and dispersion terms. More precisely, we allow those coefficients to timely modulate by Hermite-Gaussian functions of different orders. Our analytical results suggest a way of controlling the dynamics of solitary

wave-like solutions by an appropriate time modulation of those coefficients.

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The existence of ground states for quasilinear asymptotically periodic Schrodinger equations

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In this talk we will establish the existence of ground states for quasilinear asymptotically periodic Schrodinger equations in R^n. The proof is based on the methods of Nehari manifold and concentration compactness principle.

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Semi-discrete AKNS system: Hamiltonian structures and applications

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Yufeng Cao, Wei Fu, Zhijun Qiao

The Ablowitz-Ladik (AL) spectral problem is known as a discrete version of the Ablowitz-Kaup-Newell-Segur (AKNS) spectral problem, but it can not directly lead to semi-discrete AKNS hierarchy. In the talk I will start with integrable aspects of the AL hierarchy. By suitable combination we get the semi-discrete AKNS system and its symmetries, conservation laws and Hamiltonian structures. These results will go to the correspondence of the continuous AKNS system in continuous limit. Like in the continuous case, the semi-discrete AKNS system admits reductions and we can get the KdV, modified KdV and nonlinear Schrödinger equations in semi-discrete version. We will mainly focus the semi-discrete KdV equation and investigate its tri-Hamiltonian structures. Then using Hamiltonian operators we may reach to a semi-discrete model which is related to the negative order KdV equation and Camassa-Holm equation, but still many questions are open...

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On continuous limits theory and integrability for a semidiscrete system

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In this talk, we discuss continuous limits theory and integrabilityfor a semidiscrete system including the Lax pairs, conservationlaws, the Darboux transformation and soliton solutions.

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