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ABSTRACTS

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Invited Keynote Lectures

Nalini Anantharaman
Université Paris-Sud, France
Nalini.Anantharaman@math.u-psud.fr
http://www.math.u-psud.fr/∼anantharaman/

Nalini Anantharaman is a Professor at the Department of Mathematics (Université Paris-Sud, Orsay, France). She obtained her PhD in Paris 6, then she moved to the École Normale Supérieure de Lyon as Maître de Conferences, to Ecole Polytechnique as a CNRS researcher and Hadamard Professor, and finally to Paris-Sud as a Full Professor. She was awarded the Prix Gabrielle Sand et Marie Guido Triossi de l'Académie des Sciences, 2007, the 2010 Salem Prize and the 2012 Henri Poincaré Prize. She has been invited speaker at the ICMP 2006, the ECM 2008 and the ICM 2010. Prof. N. Anantharaman is a member of the Editorial Board of Annales Sci. ENS. She has made original contributions to the area of quantum chaos, dynamical systems and Schrödinger equations, including a remarkable advance in the problem of quantum unique ergodicity.

Dispersion and Controllability for Linear Schrödinger Equations on Compact Riemannian Manifolds

Abstract
I will review recent results related to the controllability and dispersive properties of the linear Schrödinger equation on a compact Riemannian manifolds, putting the emphasis on the role of the geometry. I will discuss, in particular, the case of negatively curved manifolds, where the classical propagation of rays is chaotic, and of flat tori and of the 2-dimensional disk, where the classical propagation is completely integrable. Joint work with Gabriel Rivière, Fabricio Macià, Matthieu Léautaud, Clotilde Fermanian.

Diego Córdoba
Consejo Superior de Investigaciones Científicas, Spain
dcg@icmat.es

Diego Córdoba is a Research Professor at the Instituto de Ciencias Matemáticas (ICMT, CSIC). He obtained his PhD at Princeton, and was a Member of the Institute for Advanced Study Princeton, L. E. Dickson Instructor at the University of Chicago, and later Assistant Professor at Princeton University. In 2002 he moved to the CSIC. He was been awarded an Alfred P. Sloan Doctoral Dissertation Fellowship (1997-1998), the SEMA Prize (Sociedad Española de Matemática Aplicada) for young researchers (2005), and the Miguel Catalán Young Award 2011 from the Comunidad Autónoma de Madrid. In 2008 he obtained a Starting Independent Research Grant from the European Research Council (2008-2013). His research interest is focused on Partial Differential Equations, Analysis and Fluid Mechanics.

Analysis of the Muskat Problem

Abstract
The Muskat equation governs the motion of an interface separation of two incompressible fluids in a porous media. In this talk I will discuss the existence of weak solutions, well-posedness, ill-posedness, global existence and finite time singularities for the Muskat problem.

Ingrid Daubechies
Duke Universite, USA
ingrid@math.duke.edu
http://fds.duke.edu/db/aas/math/ingrid

Ingrid Daubechies studied theoretical physics at the Free University of Brussels, where she earned her PhD in 1980. The following year she moved to the United States to pursue a two-year postdoctoral fellowship, and in 1987 she settled definitively in the United States. Her first job was as a member of the technical staff at AT&T Bell Laboratories, where she stayed until 1994. That same year she took up a professorship in the Department of Mathematics at Princeton University. Daubechies spent the next 16 years at Princeton, where she was active especially in the Program in Applied and Computational Mathematics. In 2010, she was elected President of the International Mathematical Union, and the following year obtained the Benjamin Franklin Medal for Electrical Engineering and left Princeton to take up her current appointment at Duke University. In 2012 she was awarded the BBVA Foundation Frontiers of Knowledge Prize. Ingrid Daubechies is very well-known for her work on wavelets.

Applied Mathematics Helping Art Historians and Conservators: Digital Cradle Removal

Abstract
Between the 12th to the 17th century, European artists typically painted on wooden boards. To remediate or prevent structural or insect damage, conservators in the 19th and first half of the 20th century first thinned the panels to a few mm, and then strengthened the much thinner wood structures by (permanently) attaching to their backs hardwood lattices called cradles. These cradles are highly visible in X-ray images of the paintings. X-rays of paintings are a useful tool for art conservators and art historians to study the condition of a painting, as well as the techniques used by the artist and subsequent restorers. The cradling artifacts obstruct a clear "reading" of the X-rays by these experts. We introduce an algorithm that removes these artifacts, using a variety of mathematical tools, including Bayesian algorithms. Joint work with Rachel Yin, Bruno Cornelis and David Dunson.

Weinan E
Princteon University, USA
weinan@math.princeton.edu

Weinan E received his Ph.D. from UCLA in 1989. After being a visiting member at the Courant Institute of NYU and the Institute for Advanced Study at Princeton, he joined the faculty at NYU in 1994. He is now a professor of mathematics at Princeton University, a position he has held since 1999. He is also a Changjiang professor at the Peking University. Weinan E's research interest is in multiscale and stochastic modeling. His work covers issues that include mathematical foundation of stochastic and multiscale modeling, design and analysis of of algorithms and applications to problems in various disciplines of science and engineering, with particular emphasis on the modeling of rare events, material sciences and fluid dynamics. Weinan E is the recipient of the Presidential Early Career Awards for Scientists and Engineers (PECASE), the Feng Kang Prize, the SIAM R. E. Kleinman Prize, and the ICIAM Collatz Prize. He is a member of the Chinese Academy of Sciences, a fellow of the American Mathematical Society, a SIAM fellow and a fellow of the Institute of Physics.

Mathematical Theory of Solids: from Quantum Mechanics to Continuum Models

Abstract
The problem of trying to understand solids from microscopic and macroscopic viewpoints goes back to Cauchy and was taken up again by Born. In this talk, I will present the progress that has been made on this Cauchy-Born program. In particular, I will discuss how macroscopic continuum models of solids can be derived from models of quantum mechanics and molecular dynamics. I will also discuss stability of solids at the macroscopic, atomic and electronic scales.

Charles L. Fefferman
Princeton University, USA
cf@math.princeton.edu

Charles Louis Fefferman is a Full Professor at Princeton University. In 1966, he received his Bachelor's degrees in physics and mathematics from the University of Maryland, and three years later a PhD in mathematics from Princeton University under the supervision of Elias Stein. In 1971, Fefferman received a full professorship at the University of Chicago. He returned three years later to Princeton. Awards: Fields Medal, Salem Prize, Bôcher Memorial Prize, and Alan T. Waterman Award.

Formation of Singularities in Fluid Interfaces

Abstract
The talk describes several problems for which which initially smooth fluid interfaces form singularities in finite time.

Bernold Fiedler
Freie Universität Berlin, Germany
fiedler@math.fu-berlin.de
http://dynamics.mi.fu-berlin.de/persons/fiedler.php

Bernold Fiedler obtained his Doctorate and Habilitation in Heidelberg, and is currently Professor at the Institute of Mathematics (Free University Berlin). His research interests are: Differential Equations, Dynamical Systems, global attractors of certain parabolic PDEs, pattern formation, bifurcation theory, and a broad range of applications.

Reaction, Diffusion, and Advection in One Space Dimension – an Invitation

Abstract
We give an introduction to the rich theory and phenomenology of scalar reaction-advection-diffusion PDEs

$$u_t = u_{xx} + f(x, u, u_x),$$

specifically, on the unit interval under Neumann boundary conditions. Besides growth conditions on $f \in C^2$, to guarantee existence of the (Sturm) global attractor, we only assume hyperbolicity of all equilibrium solutions $u_t = 0$. Rather than addressing superficial issues like existence and finite dimension, however, we ask: what do those Sturm global attractors actually look like?

Easy as this may sound, please recall that already the complications of the chaotic equilibrium equation $u_t = 0$, a forced pendulum, are legendary. The ODE boundary value problem exhibits intriguing links to topics like braids, billiards, and Temperley-Lieb algebras.

We give a combinatorial characterization of the PDE heteroclinic orbits between the ODE equilibria. On the more geometric and topological side, we observe how the unstable manifolds form a regular CW-complex, due to a Schoenflies Theorem on their boundaries. For illustration we show how any single closed 3-ball arises as a Sturm global attractor, but not any pair of 3-balls.

The results are joint work with Carlos Rocha (IST Lisboa) and others. They are all based on one-dimensional nodal properties, which amount to a nonlinear version of Sturm oscillation theory. See also http://dynamics.mi.fu-berlin.de/

Zhi-Ming Ma
Chinese Academy of Sciences, Peoples Republic of China
mazm@amt.ac.cn
http://www.amt.ac.cn/member/mazhiming/eindex.html

Zhi-Ming Ma is a Professor at the Institute of Applied Mathematics (AMSS, CAS) and fellow of the Chinese Academy of Sciences. He has been President of Chinese Mathematical Society, Chairman of the Organizing Committee of ICM2002 and a member of the Executive committee of IMU. He is currently President of the Chinese Society of Probability and Statistics. He has obtained a number of awards and honors, among them, the Max-Plank Research Award,1992; the Chinese National Natural Sciences Prize, 1993; Invited Speaker at the International Congress of Mathematicians, 1994; the S.S. Chern Mathematics Prize, 1995, and Honorary Degree of Doctor of Science, Loughborough University, U.K., 2004. His research is focused on the study of Random Complex Networks and Information Retrieval.

Lampability of Continuous States Markov Jump Processes

Abstract
In the settings that the state space is finite or discrete, whether a transformation of a Markov chain enjoys still Markov property is known as "lampability of Markov chains". In this talk I shall describe a verifiable condition for a transformation of a continuous states valued Markov jump process enjoys still Markov property, and discuss some aspects related to this topic. Our results have applications in the study of modeling genetic coalescent processes with recombination. The talk is based on my recent joint work with Xian Chen.

Philip K. Maini
University of Oxford, England
maini@maths.ox.ac.uk
http://people.maths.ox.ac.uk/maini/

Philip K. Maini is a Professor at the Centre for Mathematical Biology, Mathematical Institute at the University of Oxford. He co-authored a Bellman Prize winning paper (1997), was awarded a Royal Society Leverhulme Trust Senior Research Fellowship for 2001-2 and a Royal Society-Wolfson Research Merit Award (2006-11). In 2009 he was awarded the LMS Naylor Prize and Lectureship. His present research projects include the modelling of avascular and vascular tumours, normal and abnormal wound healing, and a number of applications of mathematical modelling in pattern formation in early development, as well as the theoretical analysis of the mathematical models that arise in all these applications.

Modelling Collective Cell Movement in Biology

Abstract
Collective cell movement occurs in many areas of biology, both in normal circumstances and in disease. Here, we review some recent work in three different areas - acid-mediated cancer cell migration, cranial neural crest cell migration, and epithelial cell movement. These applications lead to three different mathematical frameworks, namely, a coupled system of nonlinear partial differential equations, a hybrid model combining a partial differential equation with an off-lattice individual-based model, and an individual-based model for epithelial sheets. We show that the results are consistent with experimental observations as well as predicting new, and subsequently validated, biological phenomena and that mathematically, these models reduce to nonlinear transport equations with the macroscopic tissue level diffusion coefficient incorporating the microscopic cell level behaviour.

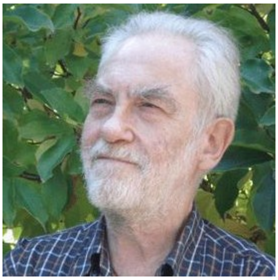
Sylvia Serfaty
Université Pierre et Marie Curie Paris 6, France
serfaty@ann.jussieu.fr
http://www.ann.jussieu.fr/∼serfaty/

Sylvia Serfaty is a Professor at the Laboratoire Jacques-Louis Lions, Université Pierre et Marie Curie Paris 6. She has been awarded a Sloan Foundation Research Fellowship and a NSF CAREER award (2003), the 2004 European Mathematical Society Prize, 2007 EURYI (European Young Investigator) award, and has been invited speaker at the International Congress of Mathematicians (2006), Plenary speaker at the European Congress of Mathematics (2012) and has recently received the IAMP Henri Poincaré prize in 2012. Her research is focused on the study of Nonlinear Partial Differential Equations, calculus of variations and mathematical physics, in particular the Ginzburg-Landau superconductivity model.

Questions of Crystallization in Coulomb Systems

Abstract
I will discuss systems of points with Coulomb interaction. An instance is the classical Coulomb gas, another is vortices in the Ginzburg-Landau model of superconductivity, where one observes in certain regimes the emergence of densely packed point vortices forming perfect triangular lattice patterns, named Abrikosov lattices in physics. In joint works with Etienne Sandier and with Nicolas Rougerie, we studied both systems and derived a "Coulombian renormalized energy". I will present it, examine the question of its minimization and its link with the Abrikosov lattice and weighted Fekete points. I will show how it leads to expecting crystallization, and discuss this sort of problems.

Carles Simó
Universitat de Barcelona, Spain
carles@maia.ub.es



Carles Simó is a Professor at the Universitat de Barcelona. He is the founder of the Spanish group on Dynamical Systems. Simó is a fellow of the Academies of Sciences of Barcelona and Madrid, and has recently be awarded the Premi Nacional de Recerca 2012 (Fundació Catalana per a la Recercai la Innovació). His current research topics are Hamiltonian systems, Celestial mechanics, Numerical indicators of dynamics, and Computer assisted studies in dynamical systems.

Measuring the Total Amount of Chaos in some Hamiltonian Systems

Abstract
We consider some simple Hamiltonian systems, variants or generalizations of the Hénon-Heiles system, in two and three degrees of freedom, around a positive definite elliptic point, in resonant and non-resonant cases. After reviewing some theoretical background, we determine a measure of the domain of chaoticity by looking at the frequency of positive Lyapunov exponents in a sample of initial conditions. The question we study is how this measure depends on the energy and parameters and which are the dynamical objects responsible for the observed behaviour.

Cédric Villani
Institut Henri Poincaré, France
villani@ihp.fr
http://cedricvillani.org/

CédricVillani is a Professor at the l'Université de Lyon and Director of the l'Institut Henri Poincaré. From 2000 to 2010 he was professor at École Normale Supérieure de Lyon, and occupied visiting professor positions in Atlanta, Berkeley and Princeton. He has been awarded the Jacques Herbrand Prize (French Academy of Sciences, 2007); Prize of the European Mathematical Society (2008); Fermat Prize (2009); Henri Poincaré Prize (2009) and the Fields Medal (2010). His main research interests are in kinetic theory (Boltzmann and Vlasov equations and their variants), and optimal transport and its applications.

Synthetic Theory of Ricci Curvature, when Monge Meets Riemann

Abstract
This is the story of the encounter between three fields of mathematics: non-Euclidean geometry and more precisely the study of Ricci curvature, minimal transport, and statistical physics; this encounter comes with mutual enrichment.

Amie Wilkinson
University of Chicago, USA
wilkinso@math.uchicago.edu
http://math.uchicago.edu/~wilkinso/

Amie Wilkinson is currently a Professor at the University of Chicago. Wilkinson received her PhD from the University of California, Berkeley, in 1995. She was the recipient of a National Science Foundation Postdoctoral Fellowship and has given American Mathematical Society invited addresses at Salt Lake City (2002), Rio de Janeiro (2007) and San Francisco (2010). She was also an invited speaker at the 2010 International Congress of Mathematics in Hyderabad, India. Wilkinson was the 2011 recipient of the American Mathematical Society Ruth Lyttle Satter Prize in Mathematics. Her research is focused on ergodic theory and smooth dynamical systems.

Robust Mechanisms for Chaotic Dynamics

Abstract
What are the underlying mechanisms for robustly chaotic behavior in smooth dynamics? In addressing this question, I'll focus on the study of diffeomorphisms of a compact manifold, where "chaotic" means "mixing" and and "robustly" means "stable under smooth perturbations." I'll describe recent advances in constructing and using tools called "blenders" to produce stably chaotic behavior with arbitrarily little effort.

Special Session 1: Mathematical Aspects of Fluid Dynamics

Angel Castro, ICMAT-CSIC-UAM, Spain
Diego Cordoba, ICMAT-CSIC, Spain
Charles Fefferman, Princeton University, USA
Francisco Gancedo, Universidad de Sevilla, Spain

This session will be focus in recent developments in the mathematical theory of incompressible fluids. Topics will include well-posedness and regularity of the Navier-Stokes, Euler equations and other related fluid models, as well as numerical aspects.

Global solutions for gravity water waves

Thomas Alazard
Ecole normale superieure, Paris, France
alazard@dma.ens.fr
Jean-Marc Delort

The aim of this talk is to present a well-posedness result for the incompressible Euler equation with free surface. We prove a global existence theorem for the gravity water waves equation with smooth, small, and decaying at infinity Cauchy data. We obtain moreover an asymptotic description of the solution which shows that modified scattering holds.

On the construction of suitable Weak Solutions

Luigi Berselli
Università di Pisa, Italy
berselli@dma.unipi.it

In this talk I will discuss the problem of approximations to the Navier-Stokes equations producing solutions, which are *suitable* in the sense of Scheffer and Caffarelli-Kohn-Nirenberg. This notion of solution is very relevant for partial regularity results, but also the local behavior of energy seems a natural request for numerical methods. I will present a recent result obtained with S. Spirito, showing that solutions obtained by means of the Navier-Voigt model are suitable, even when studied in a bounded domain, with Dirichlet boundary conditions.

Singularity formation for the Hall-MHD equations

Dongho Chae
Chung-Ang University, Korea
dchae@cau.ac.kr
Shangkun Weng

In this talk we present that the incompressible Hall-MHD system without resistivity is not globally in time well-posed in any Sobolev space $H^m(\mathbb{R}^3)$ for any $m > \frac{7}{2}$. Namely, either the system is locally ill-posed in $H^m(\mathbb{R}^3)$, or it is locally well-posed, but there exists an initial data in $H^m(\mathbb{R}^3)$, for which the $H^m(\mathbb{R}^3)$ norm of solution blows-up in finite time if $m > 7/2$.

Boundary layer problem: Navier-Stokes equations and

Nikolai Chemetov
CMAF / University of Lisbon, Portugal
chemetov@ptmat.fc.ul.pt
F. Cipriano

We consider an incompressible viscous fluid in domains with permeable walls. The permeability is described by the Navier slip boundary conditions. The goal is to study the fluid behavior at vanishing viscosity. We show that the vanishing viscous limit is a solution of the Euler equations with the Navier slip boundary condition on the inflow region of the boundary. This is a joint work with Fernanda Cipriano (GFM / New University of Lisbon).

On the inviscid limit

Peter Constantin
Princeton University, USA
const@math.princeton.edu
Igor Kukavica, Vlad Vicol

We give one-sided conditions for the convergence of solutions of the incompressible Navier-Stokes equations to solutions of the incompressible Euler equations. These generalize conditions of Oleinik and Kato.

The stability of laminar shear flows

Weinan E
Princeton University, USA
weinan@math.princeton.edu

In 1883, Reynolds published his classical work on the experimental study of the stability of shear show and transition to turbulence. Since then the issue of the critical Reynolds number at which laminar flows become unstable has been studied by numerous people, includig Sommerfeld, Heisenberg, C. C. Lin, Orazag, and more recently, Trefethen, Hof, Barkley, Eckhardt, etc. Despite this great deal of effort, the theoretical question as to how the critical Reynolds number should be determined still remains open. In this talk, we present an approach using ideas drawn from statistical physics and large deviation theory. We also discuss applications of this theoretical framework to a few examples of other non-equilibrium systems.

Thin knotted vortex tubes in steady Euler flows

Alberto Enciso
ICMAT, Spain
aenciso@icmat.es

I will outline some recent results concerning the existence of steady solutions to the Euler equation in $\mathbf{R}^3$ with a prescribed set of (possibly knotted and linked) thin vortex tubes.

Shell models for turbulent flows

Susan Friedlander
University of Southern California, USA
susanfri@usc.edu
Nathan Glatt-Holtz and Vlad Vicol

We discuss a shell model first introduced by Desnianskii and Novikov to simulate the cascade process of energy transmission in turbulent flows. We review results that reproduce Onsager's conjecture and Kolmogorov's laws for the dyadic model with constant forcing. We then discuss recent results for the stochastically driven model. Here localized, gaussian and white in time forcing serves as a proxy for generic large scale prosesses driving turbulent cascades.

On well-posedness and small data global existence for a damped free boundary fluid-structure model

Mihaela Ignatova
Stanford University, USA
mihaelai@stanford.edu
I. Kukavica, I. Lasiecka, A. Tuffaha

We address a fluid-structure system which consists of the incompressible Navier-Stokes equations and a damped linear wave equation defined on two dynamic domains. The equations are coupled through transmission boundary conditions on the free moving interface separating the two domains. Given sufficiently small initial data, we prove the global in time existence of solutions by establishing a key energy inequality which in addition provides the exponential decay of solutions.

Holder continuous Euler flows

Philip Isett
MIT, USA
isett@math.mit.edu

The study of incompressible Euler flows with Holder regularity is motivated by the theory of hydrodynamic turbulence. In connection with this theory, L. Onsager conjectured that solutions to incompressible Euler with Holder regularity less than 1/3 may fail to satisfy the conservation of energy. C. DeLellis and L. Székelyhidi, Jr. have pioneered an approach to constructing such irregular flows based on an iteration scheme known as convex integration. This approach involves correcting "approximate solutions" by adding rapid oscillations which are designed to reduce the error term in solving the equation. In this talk, I will discuss an improved framework which leads to solutions with Holder regularity as much as $1/5-$.

Mixing of passive scalars by incompressible enstrophy-constrained flows.

Gautam Iyer
Carnegie Mellon University, USA
gautam@math.cmu.edu
A. Kiselev, G. Iyer, X. Xu

Consider a diffusion-free passive scalar θ being mixed by an incompressible flow u on the torus $\mathbb{T}^d$. We study how well this scalar can be mixed under an enstrophy constraint on the advecting velocity field. Our main result shows that the mix-norm ($\|\theta(t)\|_{H^{-1}}$) is bounded below by an exponential function of time. The exponential decay rate is morally the measure of the support of the initial data, and agrees with both physical intuition and numerical simulations. The main idea behind our proof is to use the notion of "mixed to scale δ" and recent work of Crippa and DeLellis towards the proof of Bressan's rearrangement cost conjecture.

Regularity, blow up, and small scale creation in fluids

Alexander Kiselev
Rice University, USA
sasha243022@gmail.com
Vladimir Sverak

The Euler equation of fluid mechanics describes flow of inviscid and incompressible fluid, and has been first written in 1755. The equation is both nonlinear and nonlocal, and its solutions often create small scales easily and tend to be unstable. I will review some of the background, and then discuss a recent sharp result on small scale creation in solutions of 2D Euler equation. I will also indicate links to the long open question of finite time blow up for solutions of 3D Euler equation.

On the absence of "splash" singularities in the case of two-fluid interfaces

Victor Lie
Purdue University, USA
vlie@math.purdue.edu
Charles Fefferman and Alexandru Ionescu

We show that the water-wave splash scenario discovered by Castro-Cordoba-Fefferman-Gancedo-Gomez-Serrano cannot develop in the case of locally smooth solutions of the two-fluid interfaces in two dimensions.

The exponential-like moments of the Boltzmann equation without cutoff

Natasa Pavlovic
University of Texas at Austin, USA
natasa@math.utexas.edu
Irene Gamba and Maja Taskovic

We consider the spatially homogeneous Boltzmann equation without the angular cutoff in the case of variable hard potentials and provide a new proof of the generation of exponential moments of order up to the rate of potentials. We also investigate a behavior of exponential moments of order beyond the rate of potentials and for that purpose we introduce Mittag-Leffler moments (which can be understood as a generalization of the exponential moments) and prove their propagation. The talk is based on a recent joint work with Irene Gamba and Maja Taskovic.

On a coupled parabolic-elliptic system arising from MHD

Jose Rodrigo
University of Warwick, England
J.Rodrigo@warwick.ac.uk
C. Fefferman

In this talk I will present several results connected with the idea of magnetic relaxation for MHD, including some new commutator estimates (and a counterexample to the estimate in the critical case). (Joint work with various subsets of D. McCormick, J. Robinson, C. Fefferman and J-Y. Chemin.)

Interface singularities for the Euler equations

Steve Shkoller
University of Oxford, USA
shkoller@gmail.com
D. Coutand

I will discuss the problem of interface singularities for the 3-D Euler equations. In the case of one-fluid interfaces, I will discuss the so-called "splash" and "splat" singularities, wherein the crest of a breaking wave crashes onto the trough, and hypersurfaces collide. In the case of two-fluid interfaces, I will explain why such singularities cannot form. This is joint work with D. Coutand.

Self-similar blow-up for the Euler equation

Roman Shvydkoy
University of Illinois at Chicago, USA
shvydkoy@uic.edu
Anne Bronzi

TBA

Uniform boundedness and long-time asymptotics for the two-dimensional Navier-Stokes equations in an infinite cylinder

Sinisa Slijepcevic
University of Zagreb, Croatia
slijepce@math.hr
Thierry Gallay

We study the incompressible Navier-Stokes equations in the two-dimensional strip $R \times [0, L]$, with periodic boundary conditions and no exterior forcing. If the initial velocity is bounded, we prove that the solution remains uniformly bounded for all times, and that the vorticity distribution converges to zero as $t \to \infty$. We deduce that, after a transient period, a laminar regime emerges in which the solution rapidly converges to a shear flow governed by the one-dimensional heat equation. Our approach is constructive and gives explicit estimates on the size of the solution and the lifetime of the turbulent period in terms of the initial Reynolds number.

Absence of splash singularities for surface quasi-geostrophic sharp fronts and the Muskat problem

Robert Strain
University of Pennsylvania, USA
strain@math.upenn.edu
Francisco Gancedo

The formation of singularities for the evolution of the interphase between fluids with different characteristics is a fundamental problem in mathematical fluid mechanics. These contour dynamics problems are given by fundamental fluid laws such as Euler's equation, Darcy's law and Surface Quasi-geostrophic equations (SQG). In this talk, we present a proof that contours can not intersect at a single point while the free boundary remains smooth, a splash singularity, for either the sharp front SQG equation or the Muskat problem. Alternatively splash singularities have been shown for water waves. The SQG equation has seen numerical evidence of single pointwise collapse with curvature blow up. We prove that maintaining control of the curvature will remove the possibility of pointwise interphase collapse, confirming the numerical experiments. This is a joint work with Francisco Gancedo.

Recent Advances Concerning the 3D Primitive Equations of Oceanic and Atmospheric Dynamics

Edriss Titi
Weizmann Institute & UC - Irvine, USA
etiti@math.uci.edu
Chongsheng Cao, Slim Ibrahim, Jinkai Li and Kenji Nakanishi

In this talk, we consider the initial-boundary value problem of the viscous 3D primitive equations for oceanic and atmospheric dynamics with only vertical diffusion in the temperature equation. Local and global well-posedness of strong solutions are established for this system with H^2 initial data. Furthermore, we will show that for certain class of initial data the 2D and 3D primitive equations are either ill-posed or develop finite-time shock-like singularities.

On the local well-posedness of the Prandtl and the hydrostatic Euler equations with multiple monotonicity regions

Vlad Vicol
Princeton University, USA
vvicol@math.princeton.edu
I. Kukavica, N. Masmoudi, TK. Wong

We find a new class of data for which the Prandtl boundary layer equations and the hydrostatic Euler equations are locally in time well-posed.

Special Session 2: Nonlinear Evolution PDEs and Interfaces in Applied Sciences

Gunduz Caginalp, University of Pittsburgh, USA
Maurizio Grasselli, Politecnico di Milano, Italy
Alain Miranville, University of Poitiers, France

Many issues in applied science can be formulated as interface problems which can be regarded as limiting cases of evolution equations exhibiting transition layers. The study of phase field or diffusive interface problems, Allen-Cahn and Cahn-Hilliard equations have been an active area for the past few decades. This has also been an additional motivation for studying general nonlinear evolution equations. This session will focus on the mathematical properties of these equations including well-posedness, regularity, stability and asymptotic behavior of solutions, as well as their implications for applications.

Cahn-Hilliard Equation with Nonlocal Singular Free Energies

Helmut Abels
University of Regensburg, Germany
helmut.abels@mathematik.uni-regensburg.de
Stefano Bosia, Maurizio Grasselli

We consider the Cahn-Hilliard equation with a nonlocal free energy. In contrast to previous works the nonlocal part of the free energy is given by a strongly singular integral kernel, which gives rise to an integro-differential operator similar to a fractional power of the Laplacian. Moreover, the homogenuous free energy density is singular as well. We prove existence of a unique solution for all times and the existence of a global attractor. Finally, we discuss the boundary regularity of the solutions.

Non-smooth degenerating elliptic equations for damage models

Elena Bonetti
University of Pavia, Italy
elena.bonetti@unipv.it
Francesco Freddi, Antonio Segatti

The thermo-mechanical behaviour of (visco)elastic and special materials is determined by the coexistence of different configurations at the micro-scale, changing due to thermo-mechanical loads (as in the case of degrading mechanical properties in quasi-brittle materials). The continuum damage theory, introduced in the framework of solid-solid phase transitions, turns out to perform a good description of the evolving damage phenomenon in materials losing their stiffness. In particular, an order parameter delineates the state of the cohesion in the body. The resulting PDE system we investigate couples thermo-mechanical actions and phase dynamics and it is strongly nonlinear and possibly degenerating in the elliptic equations for deformations. We discuss existence of a solution in the case of complete damage. This problem is still open in its original formulation, but some answer has been given for suitable weak notion of solutions. We introduce a new notion of solution, accounting for a new "interior" stress which implicitly accounts for the state of damage of the material and write an appropriate free energy. We recover a new PDE system describing complete damage and we show the existence of a weak solution.

Outflow boundary conditions for non-homogeneous flows

Franck Boyer
Aix-Marseille Universite, France
franck.boyer@univ-amu.fr

In this talk, I will describe some theoretical results concerning (nonlinear) outflow boundary conditions for the incompressible non-homogeneous Navier-Stokes equations. I will also present a few numerical results showing the influence of the outflow boundary conditions modeling for instance in the framework of diffuse interface models for multiphase flows.

Global weak solution and blow-up criterion of the general Ericksen-Leslie system for nematic liquid crystal flows.

Cecilia Cavaterra
Università degli Studi di Milano, Italy
cecilia.cavaterra@unimi.it
Elisabetta Rocca, Hao Wu

We investigate the three dimensional general Ericksen-Leslie (EL) system with Ginzburg-Landau type approximation modeling nematic liquid crystal flows. First, we prove existence of global-in-time weak solutions under physically meaningful boundary conditions and suitable assumptions on the Leslie coefficients, which ensures that the total energy of the EL system is dissipated. Moreover, for the EL system with periodic boundary conditions, we prove the local well-posedness of classical solutions under the so-called Parodi's relation and establish a blow-up criterion in terms of the temporal integral of both the maximum norm of the curl of the velocity field and the maximum norm of the gradient of the liquid crystal director field.

On a generalized Cahn-Hilliard equation with biological applications

Laurence Cherfils
La Rochelle University, France
laurence.cherfils@univ-lr.fr
A. Miranville, S. Zelik

We will discuss the asymptotic behavior of a generalization of the Cahn-Hilliard equation with a proliferation term and endowed with Neumann boundary conditions. Such a model has, in particular, applications in biology. We show that either the average of the local density of cells is bounded, in which case we have a global in time

solution, or the solution blows up in finite time. We will also prove that the relevant, from a biological point of view, solutions converge to 1 as time goes to infinity. We will end with some numerical simulations which confirm the theoretical results.

On a variation of the Cahn-Hilliard approach

Pierluigi Colli
University of Pavia, Italy
pierluigi.colli@unipv.it

A toy problem which results from a variation of the Cahn-Hilliard model will be introduced and some results will be discussed. This results fro a research project with E. Bonetti and G. Tomassetti.

Asymptotic structure of the attractor for processes on time-dependent spaces

Monica Conti
Politecnico di Milano, Italy
monica.conti@polimi.it
Vittorino Pata

We compare the asymptotic structure of the time-dependent attractor A_t generated by the partial differential equation

$$\epsilon u_{tt} + \alpha u_t - \Delta u + f(u) = g,$$

where the positive function $\epsilon = \epsilon(t)$ tends to zero as $t \to \infty$, with the global attractor A_∞ of its formal limit

$$\alpha u_t - \Delta u + f(u) = g.$$

We establish an abstract result and we apply it to the proof of the convergence $A_t \to A_\infty$.

On the stability of the weak attractor of the 3D Navier-Stokes equations

Michele Coti Zelati
Indiana University, USA
micotize@indiana.edu

We consider the three-dimensional Navier-Stokes-Voigt (NSV) equations and we analyze, from the asymptotic behavior viewpoint, its Navier-Stokes (NS) limit as the relaxation parameter vanishes. We show that the NSV-attractors converge to the weak NS-attractor in the Hausdorff semidistance induced by the weak L^2-metric on the absorbing set of the Navier-Stokes equations. Some results related to the strong topology of L^2 are also proved.

On optimal mixing schemes

Julien Dambrine
Universite de Poitiers, France
julien.dambrine@math.univ-poitiers.fr

Optimal stirring is an important issue in chemical engineering. The underlying optimization problem is the following: given a color function c transported with a solenoidal velocity, what is the velocity $(t, X) \to V(t, X)$ that ensures the quickest mixing? Of course, the answer depends on : - The definition of the mixing criterion - The energy constraint on the velocity Recent works of Mathew et. al. have shown that a good criterion for measuring the mixing of two fluids in the periodic case is the $H^{-\frac{1}{2}}$ norm. An explicit locally-in-time optimal mixing scheme has been suggested in subsequent works from Lin et. al. In this talk we will investigate this mixing scheme both numerically and through a linear stability analysis, in a framework that is as general as possible. In particular we will show the ill-posedness of the linearized model when the energy constraint on the velocity is taken as the kinetic energy, and the well-posedness when this energy constraint is the viscous dissipation energy.

Asymptotic behavior of a generalization of the Caginalp phase-field system

Filippo Dellóro
Universita di Brescia, Italy
filippo.delloro@polimi.it
Monica Conti, Alain Miranville

We discuss the asymptotic behavior of a generalized nonlinear Caginalp phase-field system, based on the theory of heat conduction of type III devised by Green and Naghdi. In the case of two nonlinearities of polynomial critical growth, we prove the existence of global attractors of optimal regularity.

Grisvard's shift theorem near L^∞ and Yudovich theory in domains with corners

Francesco Di Plinio
Università di Roma “Tor Vergata”, Italy
diplinio@mat.uniroma2.it
Roger Temam

The planar Euler equations describe the motion of a 2-D inviscid incompressible fluid, and also arise as a model problem for the study of the barotropic mode (to put it simply, the vertical average) of the Primitive equations of the ocean. It is a result by Yudovich that, in the space-periodic case, there exist a unique weak solution to the Euler system whenever the initial data has bounded vorticity. Relying on a refinement of the sharp A_p weighted bounds for singular integrals by Buckley, we prove an L^∞ version of Grisvard's shift theorem on domains with corners, and extend the Yudovich theory of weak solutions for the Euler equations to this class of domains. We also discuss analogous results for the barotropic mode of the Primitive equations. This is partly joint work with Roger Temam and Claude Bardos.

Regularized families of simplified Ericksen-Leslie (RSEL) models

Ciprian Gal
Florida International University, USA
cgal@fiu.edu
Theodore Tachim Medjo

We consider a general family of regularized flows for the simplified Ericksen-Leslie (RSEL) model for the hydrodynamics of liquid crystals in 2 and 3-dimensional compact Riemannian manifolds. The system contains the Navier-Stokes equations, the Navier-Stokes-Voight and the Navier-Stokes alpha-model equations as special cases, and many others. We establish existence, stability, reg-

ularity results and singular perturbation results, and we also show the existence of a global attractor and exponential attractor for the general family. Then we establish precise conditions under which each trajectory converges to a single equilibrium by means of a Lojasiewicz-Simon inequality.

Evolution equations of fast phase transitions in solutions and applications

Peter Galenko
University of Jena, Germany
peter.galenko@uni-jena.de

A model of fast phase transitions is analyzed. Examples of transitions described by non-linear evolution equations are given for a number of tasks from material physics. In particular, analytical and numerical solutions of problems on spinodal decomposition, eutectic transformation, rapid solidification, and order-disorder transition are discussed.

Dynamics of fluid interfaces with surface viscosity and Helfrich forcing

Harald Garcke
University Regensburg, Germany
harald.garcke@ur.de

In two-phase flow the interface often has important additional aspects. First we discuss the Boussinesq-Scriven surface fluid model and a model for biomembranes due to Arroyo and DeSimone taking curvature elasticity forcing into account. We then present analytical properties of the model and introduce a parametric finite element method for the evolution problem. For the discrete equations we show stability estimates and finally we present numerical computations in two and three space dimensions which show different phenomena stemming from surface viscosity and from forces originating from the Helfrich energy.

Pullback exponential attractor for a Cahn-Hilliard-Navier-Stokes system in 2D

Stefania Gatti
University of Modena and Reggio Emilia, Italy
stefania.gatti@unimore.it
Stefano Bosia

We address the long term behavior of a $2D-$Cahn-Hilliard-Navier-Stokes system with polynomial double-well potential, proving that it possesses a pullback exponential attractor. In particular the regularity estimates we obtain depend on the initial data only through fixed powers of their norms and these powers are independent of the growth of the polynomial potential considered in the Cahn-Hilliard equation.

Well-posedness for the Navier-slip thin-film equation in complete wetting

Lorenzo Giacomelli
Sapienza University of Rome, Italy
lorenzo.giacomelli@sbai.uniroma1.it

We are interested in the thin-film equation with quadratic mobility, modeling the spreading of a thin liquid film with a Navier-slip condition at the solid substrate. This degenerate fourth-order parabolic equation has the contact line (where liquid, solid, and vapor meet) as a free boundary. There, a zero-contact angle condition is imposed, modeling the so-called "complete wetting" regime. We first argue that the self-similar source-type solution, once its leading order profile is factored-off, is analytic as a function of two variables (x, x^β) with β irrational, where x denotes the distance from the contact line. Motivated by this preliminary, we then argue that the full free-boundary problem is well-posed in weighted L^2-spaces which capture the leading order terms of such (x, x^β)-expansion. This is part of a joint project with Manuel V. Gnann, Hans Knüpfer, and Felix Otto.

On the Cahn-Hilliard equation with dynamic boundary conditions and a dominating boundary potential

Gianni Gilardi
University of Pavia, Italy
gianni.gilardi@unipv.it

Several results for the Cahn-Hilliard equation with dynamic boundary conditions are known. In particular, well poseness holds if the potential entering the equations in the domain dominates the potential involved in the boundary condition. In this talk, the opposite case is considerered, i.e., the dominating potential is the boundary potential. Well posedness and regularity results obained in a joint paper with P. Colli and J. Sprekels are presented both for the Cahn-Hilliard equation and for the viscous Cahn-Hilliard equation. Moreover, for the latter, an optimal control problem is discussed.

The Gel'fand problem for the biharmonic operator

Olivier Goubet
Universite de Picardie Jules Verne, France
olivier.goubet@u-picardie.fr

We study stable solutions of the equation $\Delta^2 u = e^u$ in bounded domains $\Omega \subset \mathbb{R}^N$. We prove that the extremal solutions are smooth if $N \leq 12$.

On convergent numerical schemes for two-phase flow of incompressible fluids with different mass densities

Günther Grün
Erlangen University, Germany
gruen@am.uni-erlangen.de

In this talk, we will be concerned with convergence results of numerical schemes for diffuse interface models for two-phase flow of immiscible, incompressible viscous fluids with different mass densities. In contrast to the case of identical mass densities, for general mass densities only recently diffuse interface models have been suggested which are consistent with thermodynamics and which allow for a solenoidal velocity field (see Abels, Garcke, Grün M3AS 2012). These models consist of a new momentum equation for the velocity field coupled to a Cahn-Hilliard equation for the evolution of the order parameter. A subtle discretization of the convective coupling between the flux of the phase-field and the momen- tum equation allows to formulate a numerical scheme which satisfies a discrete counterpart of the energy estimate. By higher regularity results for discrete solutions of convective Cahn-Hilliard equations, we prove its convergence in two and in three space dimensions. Finally, we shall present numerical simulations to underline the full practicality of our approach and to identify physical settings for which the new coupling term suggested in (Abels, Garcke, Grün, M3AS 2012) seems to be indispensable for numerical stability.

Convergence to equilibrium for smectic-A liquid crystals in 3D domains

Francisco Guillen-Gonzalez
Universidad de Sevilla, Spain
guillen@us.es
Blanca Climent-Ezqierra

In this talk, we focus on a smectic-A liquid crystal model in 3D domains, obtaining three main results: the proof of an adequate Lojasiewicz-Simon inequality in a strong framework, the rigorous proof (via a Galerkin approach) of the existence of global in time weak solutions which are strong (and unique) for large times, and the convergence to equilibrium of the whole trajectory as time goes to infinity. Given any regular initial data, the existence of a unique global in time regular solution (bounded up to infinite time) and the convergence to an equilibrium have been previously proved, but under the constraint of large enough viscosity. Now, all results are obtained without imposing large viscosity.

The singular limit of an Allen-Cahn equation with a random source term

Danielle Hilhorst
University of Paris-Sud, Orsay, France
Danielle.Hilhorst@gmail.com
Thanh Nam Nguyen, Matthias Roeger

We revisit the singular limit of an Allen-Cahn equation in the case of a random source term which only depends on time; the scaling which we consider is such that the limiting motion equation does not involve the mean curvature of the moving interface. Our main results involve generation and propagation of interface properties.

A well posedness result for nonlinear viscoelastic equations with memory

Elsa Maria Marchini
Politecnico di Milano, Italy
elsa.marchini@polimi.it
Monica Conti, Vittorino Pata

In this talk we discuss existence, uniqueness and continuous dependence results for the weak solutions to a nonlinear viscoelastic equation with hereditary memory on a bounded three-dimensional domain. This equation arises in mechanics, in the description of the vibrations of thin rods whose material density is not constant.

Asymptotic behaviour for a double time-delayed 2D-Navier-Stokes model

Pedro Marin-Rubio
Universidad de Sevilla, Spain
pmr@us.es
Julia Garcia-Luengo, Gabriela Planas

In this contribution we analyze the well-posedness and asymptotic behaviour of a double time-delayed 2D-Navier-Stokes model with delay effects in the nonlinear term. Some surprising facts arise in this simple perturbation, for instance about uniqueness of solution, and therefore a suitable approach must be used in order to ensure the existence of attractor. For a suitable choice of the phase-space we prove that the problem is certainly well-posed and have an attractor.
This is a joint work with Julia García-Luengo, from Universidad de Sevilla (Spain), and Gabriela Planas, from Universidade Estadual de Campinas (Brazil).

Analysis of a fully nonlinear reaction-diffusion system describing multicomponent reactive flows

Martine Marion
Ecole Centrale de Lyon, France
Martine.Marion@ec-lyon.fr
Roger Temam

We consider combustion problems in the presence of complex chemistry and nonlinear diffusion laws leading to fully nonlinear multispecies reaction-diffusion equations. We establish results of existence of solution and maximum principle, i.e. positivity of the mass fractions, which rely on specific properties of the models. The nonlinear diffusion coefficients are obtained by resolution of the so-called Stefan-Maxwell equations.

The phase-field transition system endowed with a general regular potential and nonlinear dynamic boundary conditions of non-homogeneous type and non-constant thermal conductivity

Costica Morosanu
Alexandru Ioan Cuza University, Romania
costica.morosanu@uaic.ro

The work is devoted to the study of a Caginalp phase-field transition system, endowed with a general regular potential and a general class of nonlinear and non-homogeneous dynamic boundary conditions (in both unknown functions), as well as non-constant thermal conductivity. The existence, uniqueness and regularity of solutions is established. This extends previous works, including the already studied boundary conditions, which makes the preset mathematical model capable to reveal the complexity of the real physical phenomena, including the phase change.

Coupled motion by mean curvature with surface diffusion

Amy Novick-Cohen
Technion IIT, Israel
amync@tx.technion.ac.il

In the context of thin polycrystalline films, evolutionary problems arise very naturally where motion by mean curvature is coupled with motion by surface diffusion. We formulate and study such problems in a number of specific geometries. The implications of our results for the stabiity of thin films and measurement of kinetic coefficients are discussed.

A numerical analysis of the Cahn-Hilliard equation with non-permeable walls

Madalina Petcu
University of Poitiers, France
Madalina.Petcu@math.univ-poitiers.fr
Laurence Cherfils

In this talk we consider the numerical analysis of the Cahn-Hilliard equation in a bounded domain with non-permeable walls, endowed with dynamic-type of boundary conditions. The dynamic-type boundary conditions that we consider were proposed by Goldstein, Miranville and Schimperna in order to describe the interactions with the wall of a binary material. The equation is semi-discretized using a finite element method for the space and error estimates between the exact and the approximation solution are obtained. We also prove the stability of a fully discrete problem based on the backward Euler scheme for the time discretization.

Regularity results for a Cahn-Hilliard-Navier-Stokes system with shear dependent viscosity

Dalibor Prazak
Charles University, Prague, Czech Rep
prazak@karlin.mff.cuni.cz
M. Grasselli

We consider a diffuse interface problem, where the fluid is described by a non-Newtonian type model with shear-dependent viscosity of polynomial growth. We will discuss the qualitative behavior of solutions, with particular emphasis on the regularity results.

Entropic solutions to a PDE system for phase transitions and damage in thermoviscoelastic materials

Riccarda Rossi
Università degli studi di Brescia, Italy
riccarda.rossi@unibs.it
Elisabetta Rocca

We focus on the analysis of a PDE system modelling (non-isothermal) phase transitions and damage in thermoviscoelastic materials. The model is thermodynamically consistent: in particular, no small perturbation assumption is adopted, which results in the presence of quadratic terms on the right-hand side of the temperature equation. The whole system has a highly nonlinear character. We address the existence for a weak notion of solution, referred to as "entropic", where the temperature equation is formulated with the aid of an entropy inequality, and of a total energy inequality. This solvability concept reflects the basic principles of thermomechanics and the thermodynamical consistency of the model, and allows us to obtain *global-in-time* existence theorems without imposing any restriction on the size of the initial data. We prove our results by passing to the limit in a time-discrete scheme carefully tailored to the nonlinear features of the PDE system and of the a priori estimates performed on it.

Strichartz estimates and smooth attractors for wave equations with fractional damping in bounded domains

Anton Savostianov
University of Surrey, England
a.savostianov@surrey.ac.uk
Sergey Zelik

We consider Dirichlet problem for a semi-linear wave equation with damping term $(-\Delta)^\alpha\partial_t u$, where $\alpha\in[0,\frac{1}{2}]$, in a bounded smooth domain $\Omega\subset\mathbb{R}^3$, assuming initial data from usual energy space $H^1_0(\Omega)\times L^2(\Omega)$. First, we establish control of $L^5([0,T];L^{10}(\Omega))$ norm of solutions for corresponding linear non-autonomous problem in terms of energy norm, which does not follow from energy estimate as well as Strichartz estimates for pure wave equation. Then treating semi-linear equation as perturbation of the linear problem we establish its well-posedness in the class of energy solutions with finite $L^5_{loc}(\mathbb{R}_+,L^{10}(\Omega))$

norm. Moreover, we show that solutions from the mentioned class possess smoothing property analogous to solutions of parabolic equation. Finally we show that dynamical system generated by these solutions possesses a smooth global attractor.

From fractional Cahn Hilliard to fractional Porous Medium

Antonio Segatti
University of Pavia, Italy
antonio.segatti@unipv.it
Goro Akagi, Giulio Schimperna

In this talk I will report on a joint work with Goro Akagi (Kobe) and Giulio Schimperna (Pavia) in which we highlight some relations between a fractional Cahn Hilliard type equation and a fractional Porous Medium equation. In particular, we will start with a fractional Cahn Hilliard equation (for which we show existence and uniqueness of solutions) and we will rigorously prove that when the fractional order of differentiation tends to zero the solutions will converge to the solution of a fractional Porous Medium equation.

Nonlinear evolution as convex minimization

Ulisse Stefanelli
University of Vienna, Austria
ulisse.stefanelli@univie.ac.at

I intend to overview some global-in-time variational techniques for evolution equations. In particular, we shall be interested in the possible reformulation of evolution systems, either of dissipative or dispersive type, in terms of minimization problems. The idea is that of moving the successful machinery of the Calculus of Variations (direct method, gamma-convergence, relaxation..) to evolutionary situations. I will in particular present results for gradient and doubly nonlinear flows as well as for nonlinear waves. This is joint work with Goro Akagi.

Finite Number of Determining Parameters for the Navier-Stokes Equations with Applications into Feedback Control and Data Assimilations

Edriss Titi
Weizmann Institute & UC - Irvine, USA
etiti@math.uci.edu
A. Azouani, H. Bessaih, C. Foias, M. Jolly, R. Kravchenko and E. Olson

In this talk we will implement the notion of finite number of determining parameters for the long-time dynamics of the Navier-Stokes equations (NSE), such as determining modes, nodes, volume elements, and other determining interpolants, to design finite-dimensional feedback control for stabilizing their solutions. The same approach is found to be applicable for data assimilations. In addition, we will show that the long-time dynamics of the NSE can be imbedded in an infinite-dimensional dynamical system that is induced by an ordinary differential equations, named *determining form*, which is governed by a globally Lipschitz vector field. The NSE are used as an illustrative example, and all the above mentioned results hold also to other dissipative evolution PDEs.

Flows in karst geometry

Xiaoming Wang
Florida State University, USA
xwang@fsu.edu

Karst type geometry is a particular type of configuration that consists of both conduit/channel (or vug/chamber) together with porous media. Many important applications involve fluid flows in karstic geometry. Well-known examples include contaminant transport in karst aquifer, oil recovery in karst oil reservoir, proton exchange membrane fuel cell technology, cardiovascular modeling, and carbon-dioxide sequestration among others. The mathematical study of flows in karst geometry is a challenge due to the coupling of the flows in the conduits and flows in the surrounding matrix which are governed by different physical processes, the possibly complex geometry of the network of conduits, the vastly disparate spatial and temporal scales, the strong heterogeneity and the enormous associated uncertainty with natural karst aquifer, and the multi-phase nature of many important applications. In this talk, we will present recent results on the modeling, analysis and simulation of single phase as well as two-phase flows in karstic geometry.

Rayleigh-Taylor instability for the two-phase Navier-Stokes equations with surface tension

Mathias Wilke
Martin-Luther University Halle-Wittenberg, Germany
mathias.wilke@mathematik.uni-halle.de

This talk is concerned with the dynamic behaviour of two immiscible and incompressible fluids in a cylindrical container, e.g. a capillary, which are separated by a sharp interface. In case that the heavier fluid overlies the lighter fluid one expects that the heavier fluid sinks down into the lighter one. This effect is known as Rayleigh-Taylor instability. The main result yields the existence of a critical surface tension with the following property. In case that the surface tension of the interface between the two fluids is smaller than the critical surface tension, one has Rayleigh-Taylor instability. On the contrary, if the interface has a greater surface tension than the critical value, the instability effect does not occur and one has exponential stability of the interface. The last part of the talk is concerned with the bifurcation of nontrivial equilibria in multiple eigenvalues. The invariance of the bifurcation equation with respect to rotations and reflections yields the existence of bifurcating subcritical equilibria. Finally it is proven that the bifurcating equilibria are unstable.

Special Session 3: Mathematical Models in the Systems Biology of Cancer

Philip K. Maini, University of Oxford, England
Gabriel F. Calvo, University of Castilla-La Mancha, Spain
Juan Belmonte-Beitia, UCLM, Spain

The aim of this Special Session is to bring together researchers from all over the world to present their new results on mathematical/computational models in the field of tumour biology. These will include new advances in mathematical analysis, as well as novel diagnostic, prognostic and therapeutic ideas.

Predicting the regrowth of diffuse low-grade gliomas under radiotherapy with an edema-based model

Mathilde Badoual
Paris Diderot University, France
badoual@imnc.in2p3.fr
C. Gerin, C. Deroulers, B. Grammaticos, J.-F. Llitjos, C. Oppenheim, P. Varlet, J. Pallud

Diffuse low-grade gliomas are rare and slowly growing tumours that transform inexorably into more aggressive forms. Optimizing treatments, for example with modeling, could help to delay the tumor regrowth and the anaplastic transformation. We present here a model for the effect of radiotherapy on diffuse low-grade gliomas. We complement a migration-proliferation equation by an equation describing the appearance and draining of edema, and we argue that the latter effect accounts for the long decrease of the tumour's radius (sometimes even years) after the end of the radiotherapy. Using our four-parameter model, we are able to fit the data of the evolution of the tumor radius along time, of 28 patients [1]. The model predicts a strong correlation between a high proliferation coefficient and a low progression-free gain of lifetime among the patients, in agreement with clinical studies [1]. Moreover, by measuring or fixing the values of the parameters, we show that it is possible to predict, at the time of the radiotherapy, the duration of the tumour's radius decrease.

References

[1] Pallud J. et al. Dynamic imaging response following radiation therapy predicts long-term outcomes for diffuse low-grade gliomas Neuro Oncol. 2012;14:1-10.

From simplicity to complexity in modelling cancer as an ecosystem

David Basanta
H. Lee Moffitt Cancer Center, USA
david@CancerEvo.org
Arturo Araujo, Leah Cook, Conor Lynch

Cancer is a disease driven by Darwinian evolution which means that understanding how tumour cells interact with each other, with normal cells and with the physical microenvironment is key if we want to understand disease progression. These cell-cell and cell-microenvironment interactions constitute the natural selection of the somatic evolution characterising carcinogenesis. In this talk I will introduce simple but also more complex approaches to model this where we consider prostate cancer metastases to the bone as new species invading an existing ecosystem. Tools like Evolutionary Game Theory allow us to quickly explore interaction-based cancer evolutionary dynamics whereas agent-based models allow us to obtain more detailed and quantiative predictions that would allow us to have a better insight about the best therapeutic options for a given patient. While mathematical models should strive to be as simple as possible, modelling metastases requires us to embrace the complexity of all the interactions that characterise homeostasis and homeostasis disruption in the bone.

Anomalous transport and migration-proliferation dichotomy of cancer cells.

Sergei Fedotov
The University of Manchester, England
sergei.fedotov@manchester.ac.uk

We examine proliferation and migration dichotomy of the tumor cells in terms of non-Markovian models involving anomalous transport. We consider the tumor spheroid consisting of the tumor core with a high density of cells and the outer invasive zone. We consider two different regions of the outer invasive zone and develop models for both zones. In the first model we analyze the near-core-outer region, where biased migration away from the tumor spheroid core takes place. We suggest non-Markovian switching between the migrating and proliferating phenotypes of tumor cells. Nonlinear master equations for mean densities of cancer cells of both phenotypes are derived. In anomalous switching case we find a sublinear growth of the average size of the near-core-outer region. In the second model we consider the outer zone, where the density of cancer cells is very low. We obtain an integrodifferential equation for the total density of cells which gives the overall spreading rate of cancer.

Mathematical modeling of tumor growth and metasttatic spreading: Validation in tumor-bearing mice

Florence Hubert
Université d'Aix-Marseille, France
florence.hubert@univ-amu.fr
D. Barbolosi, A. Benabdallah, J. Ciccolini, N. Hartung, S. Mollard

Initial staging of cancer disease is critical in clinical oncology since it impacts on the subsequent treatment strategies. In this respect, early detection of occult metastasis remaining invisible upon imaging is an unmet medical need. Mathematical models describing metastatic spreading could be attractive tools to better estimate the risk for a given tumor to spread when clinical evidence is not available. In this work, we have adapted a top-down model, constituted by a transport equation describing metastatic growth and endowed with a boundary condition for metastatic emission. Model predictions have been confronted with nonclinical experimental data. Nod

Scid mice were orthotopically xenografted with MDA-231 Luc+ breast cancer cells. Main tumor growth and metastatic spreading and growth were monitored over up to six weeks by bioluminescence tomography. A total of 166 observations on primary tumor size and 63 observations on metastatic burden were obtained. For model building, a tailored computational approach permitted to use the Monolix software, classically employed for Ordinary Differential Equations (ODE), for a Partial Differential Equation (PDE) model.

From MTD towards Metronomic Chemotherapy: The Role of the Tumor Microenvironment

Urszula Ledzewicz
Southern Illinois University, USA
uledzew@siue.edu
Heinz Schaettler

In mathematical models for homogeneous tumor populations of chemotherapeutically sensitive cells, optimal controls are bang-bang reaffirming the medical paradigm of maximum tolerated doses (MTD). However, as more aspects of the tumor microenvironment are taken into account—such as tumor heterogeneity, the tumor vasculature or tumor-immune system interactions— alternatives to this paradigm become a viable option. There is mounting medical evidence that "more is not necessarily better", but that a so-called biologically optimal dose (BOD) should be sought. This, for example, might be realized by specific time-varying, reduced dose rate regimes known under the name of metronomic dosing. It has been shown that chemotherapy administered in such a manner, although having obvious lower cytotoxic effects, exhibits antiangiogenic and immune-stimulatory effects and contributes to resensitizations. In this talk, we shall address modeling aspects of this problem using an optimal control approach. Interestingly, protocols that provide an alternative to MTD strategies appeared as optimal solutions in the analysis of various models for tumor growth in connection with antiangiogenic treatments and tumor immune system interactions as well as in models for heterogeneous tumor populations. We shall discuss a model that encompasses all these features and provides a framework for the analysis of metronomic chemotherapy.

Mathematical modeling of Interleukin-27 induction of anti-tumor T cells response

Kang-Ling Liao
Mathematical Biosciences Institute, The Ohio State Univeristy, USA
liao.92@mbi.osu.edu
Xue-Feng Bai, Avner Friedman

Interleukin-27 is an immunoregulatory cytokine from the Interleukin-12 family. In recent years, Interleukin-27 has been considered as a potential anti-tumor agent. Recent experiments *in vitro* and *in vivo* have shown that cancer cells transfected with IL-27 activate $CD8^+$ T cells to promote the secretion of anti-tumor cytokines Interleukin-10, although, at the same time, IL-27 inhibits the secretion of Interferon-γ by $CD8^+$ T cells. We develop a mathematical model based on these experimental results. The model involves a dynamic network which includes tumor cells, $CD8^+$ T cells and cytokines Interleukin-27, Interleukin-10 and Interferon-γ. Simulations of the model show how Interleukin-27 promotes $CD8^+$ T cells to secrete Interleukin-10 to inhibit tumor growth. On the other hand Interleukin-27 inhibits the secretion of Interferon-γ by $CD8^+$ T cells which somewhat diminishes the inhibition of tumor growth. Our numerical results are in qualitative agreement with experimental data. We use the model to design protocols of IL-27 injections for the treatment of cancer and find that, for some special types of cancer, with a fixed total amount of drug, within a certain range, continuous injection has better efficacy than intermittent injections in reducing the tumor load while the treatment is ongoing.

Clonal selection and therapy resistance in acute leukemias: Mathematical modelling explains different proliferation patterns at diagnosis and relapse

Anna Marciniak-Czochra
University of Heidelberg, Germany
anna.marciniak@iwr.uni-heidelberg.de
Natalia Baran, Anthony D. Ho and Thomas Stiehl

Recent experimental evidence suggests that acute myeloid leukemias may originate from multiple clones of malignant cells. However, it is not known how the observed clones may differ with respect to cell properties such as proliferation and self-renewal. There are scarcely any data on how these cell properties change due to chemotherapy and relapse. We propose a new mathematical model to investigate the impact of cell properties on multi-clonal composition of leukemias. Model results imply that enhanced self-renewal may be a key mechanism in the clonal selection process. Simulations suggest that fast proliferating and highly self-renewing cells dominate at primary diagnosis while relapse following therapy-induced remission is triggered mostly by highly self-renewing but slowly proliferating cells. Comparison of simulation results to patient data demonstrates that the proposed model is consistent with clinically observed dynamics. To explain the underlying phenomenon, we investigate a structured population model with a continuum of different cell clones. We show mass concentration of the model solutions, which reflects the process of selection of clones with highest self-renewal potential.

Extreme protraction as a novel therapeutical strategy for low grade gliomas: Theory and ongoing work in animal models

Victor Perez-Garcia
Universidad de Castilla-La Mancha, Spain
victor.perezgarcia@uclm.es

Low-grade gliomas are primary brain tumors with slow evolution and a mean patients survival of five years. Currently, patients receive surgery to remove as much tumor as posible and proceed with chemotherapy and radiotherapy only when there are indications of risk of transformation to a faster growing higher-grade tumor (e.g. due to growth to a too large tumor size and/or tumor growth acceleration and/or large initial size and patient age beyond 40 years). When applying cytotoxic therapies such as radiotherapy and/or chemotherapy (temozolomide), the pattern of administration replicates the one used for high grade brain tumors. In those cases a very intensive therapy delays the tumor growth a few months after which it invariably recurs. In this communication I will present mathematical models able to describe the response of low-grade gliomas to the therapy. Using optimization techniques we have found novel protracted therapeutical schedules that may be preferable for the treatment of low grade gliomas achieving a better tumor control with reduced side effects. I will also describe the difficultities for the experimental implementation and how these have been overcome in a study with animal models. If successful the study may lead to a future clinical trial.

Molecular Programming of Cell and Vessel Phenotypes in Cancer

Amina Qutub
Rice University, USA
aminaq@rice.edu

Oxygen is fundamental to life on Earth. In diseases affecting the vasculature including cancer, abberrant hypoxic response is a critical part of the disease. Limited oxygen can lead to more aggressive tumors; on the other hand, appropriate manipulation of proteins involved in cellular hypoxic response can help halt cancer's progression. A challenge lies in understanding the complex cellular response to hypoxia both across different cancers and across patients with the detail needed to develop effective therapies. In this presentation, I will share how we are developing and integrating methods in mathematical modeling, machine learning, molecular biology, and microscopy image analysis to tackle the challenge of interpreting how changes at the molecular level affect cellular response and cancer progression. Results of our analyses have identified (1) molecular signatures of leukemia that predict patient outcome and (2) a set of unique vascular cell phenotypes. My lab's goal is to use this computational systems biology approach to understand cellular response to oxygen, and apply this knowledge to drive the development of new therapeutic strategies.

Multi-scale modelling of palisade formation in glioblastomas

Ignacio Ramis Conde
University of Castilla la Mancha, Spain
ignacio.ramis@uclm.es
Alfonso Caiazzo

Palisades are multi-cellular circular structures that can be localized around occluded vascular vessels in glioblastomas. This geometry suggests the following formation process. Initially the tumour grows around the vasculature in a cylindrical shape. This structure facilitates the tumour supply of oxygen and nutrients. The cells that are closest positioned to the vessel obtain have a better access to nutrients and oxygen than those positioned at the outer parts. At a certain point, due to tumor growth, the pressure exerted by the tumor cells on the vessel is such that to cause vessel collapse. This moment represents a dramatical change for those cells that were positioned nearby to the vessel. After collapse they loose oxygen and nutrient supply as they are at the centre of tumour. As a consequence they become hypoxic and, if the tumour geometry or the irrigating vasculature do not change, they die. At the outer rim of the tumour, the effects of vessel collapse are different. In this region, the cells are not immersed in the tumour mass and when lack of oxygen and nutrients occurs they can activate migration mechanisms that allow them to invade the nearby tissue in the search for areas where oxygen and nutrients are available. The collective migration results in the formation of a palisade. Observation of palisades under microscopy is constrained to two dimensional layers as a consequence of the experimental setups. Advanced microscopy has been able to reproduce three dimensional images of particular tissue structures, however, up to our knowledge, a three dimensional shape of palisades has not been reproduced yet. This is due to the difficult constraints that glioblastomas represent to be observed in-vivo. In this talk we present three dimensional realistic simulations, based on a multiscale coupling of the evolution of tumor cells and the oxygen diffusion in tissue (treating blood vessels as immersed boundaries), that depict the shape of palisades during its formation. These images can provide to clinicians and biologists for a better understanding of the palisades 3-D structure and how glioblastomas grow.

Data assimilation in tumor growth modeling: towards patient calibrated models using imaging devices

Olivier Saut
Institut Mathematiques Bordeaux, CNRS, France
Olivier.Saut@inria.fr
T. Colin, F. Cornelis, J. Jouganous, M. Martin

Clinicians are routinely following the evolution of the tumor of a patient thanks to imaging devices. They are evaluating the growth or a response to a treatment using quantitative criteria (like RECIST) computed from these images. Our goal is to improve these criteria by using mathematical models for tumor growth and to be able to quantify the agressivness of a tumor or even obtain a prognosis. For this matter, two main challenges have to be overcome. First, one has to design an accurate deterministic mathematical model of the disease that is able to reproduce the observed behaviors on patients. These models typically involve many parameters (which are e.g tuning the interplays between the various phenomena influencing the tumor growth) that are practically difficult or even impossible to recover them from experimental data or clinical routine. Hence, the second challenge is to find a way to recover reasonable values of these parameters, that allow the computed evolution to match the observed ones, exclusively with the information available to clinicians. I will describe these challenges and the methods we have developed to overcome them in the clinical context of evaluating the agressivness of metastases (from bladder, thyroid) to the lung.

Stability in the fluctuations of the cell membrane potential and how they change with cancer

Aneta Stefanovska
Lancaster University, England
aneta@lancaster.ac.uk
Yevhen Suprunenko, Kirsten Jenkins

Ever since the experimental work of Hodgkin and Huxley, where the transmembrane voltage was clamped to a constant value, the dynamics of the cell membrane potential has been largely neglected. We now consider the origin of its fluctuations and investigate their time-dependent deterministic characteristics. In particular, we describe its time-varying dynamics with a time-dependent point attractor (driven steady state). In this way the system is able to resist external perturbations. In this talk we will present a mathematical model of the cell membrane potential and of how this could change with cancer. In particular, the interplay between the production of ATP in mitochondria and due to glycolysis is considered and the role of the oxygen supply is investigated. Cancer is associated with a state for which the oxygen supply is at a low level. The proposed model results in regions when the system can or cannot maintain a stable frequency of ATP production, in accordance with the recently-proposed theory of chronotaxic systems (from *chronos* – time and *taxis* – order). A comparison between theoretical and experimental results will be discussed. We conclude that the dynamics of the cell membrane potential, which is often considered to be fully stochastic, can contain a strong deterministic component due to its stability properties. In this way, the dynamics of cell membrane potential may be used to distinguish a cancer cell from normally functioning cell.

Special Session 4: Delay Equations Applied to Population Dynamics

Philipp Getto, TU Dresden, Germany
Gergely Rost, University of Szeged, Hungary
Julia Sanchez, BCAM-Basque Center for Applied Mathematics, Spain

If the present changes of a system depend on the state of the system in the past, its dynamics can be modelled as a differential equation with time delay, also known as a functional differential equation. Examples are the development of a population as a function of birth rates or the spread of an infectious disease as a function of infectivity. As indicated by the references below, both, theory and applications of delay equations have a great tradition but are also a subject of modern research. In this session we propose to focus on applicability to problems from biology, in particular the dynamics of populations of humans, animals, cells and, related to this, the spread of infectious diseases.

On waning immunity and immune system boosting

Maria Vittoria Barbarossa
University of Szeged, Hungary
barbarossamv@gmail.com
Gergely Röst

When the body gets infected by a pathogen, the immune system develops pathogen-specific immunity. Disease-induced immunity decays in time and years after recovery an individual might become susceptible again. Exposure to the pathogen boosts the immune system thus prolonging the time in which a recovered (or vaccinated) individual is immune. We propose a new approach to model SIR dynamics monitoring immune status of individuals, including waning immunity and immune system boosting. Under particular assumptions on the general PDE model, we can recover known examples such as SIS models and SIRS with constant delay. A new class of SIRS models with state-dependent delays can be also obtained in this framework.

General model of a cascade of reactions with time delays: global stability analysis

Marek Bodnar
University of Warsaw, Poland
mbodnar@mimuw.edu.pl

We considered a model of a cascade of reactions that can be interpreted as a generalisation of the Hes1 gene expression model. It is a system of ordinary differential equations with discrete as well as distributed delays. Using techniques proposed in the paper by Liz and Ruiz-Herrera (J. Diff. Eqs., 2013) we prove sufficient conditions for global stability of the steady state for the abstract general model. Then this result is applied to the Hes1 gene expression model.

Numerical analysis for eigenvalues of structured population dynamics: the Daphnia model

Dimitri Breda
University of Udine, Italy
dimitri.breda@uniud.it
Philipp Getto, Julia Sanchez Sanz, Rossana Vermiglio

Consumer-resource dynamics are recently modeled by a Retarded Functional Equation (RFE) for a structured consumer population coupled with a Retarded Functional Differential Equation (RFDE) for an unstructured resource. In addition, size and survival probability of the structured consumer are determined as solutions of external Ordinary Differential Equations (ODEs). If, on the one side, the proposed models can furnish a more realistic description of the dynamics, on the other side the intrinsic complexity limits the use of theoretical tools and thus asks for advanced numerical schemes. In this talk, the analysis of the asymptotic stability of the equilibria is tackled by adapting a recently proposed pseudospectral method to compute the eigenvalues of the infinitesimal generator of linearized systems of coupled RFEs/RFDEs to the peculiarities of a model of Daphnia Magna consuming algae where external ODEs are solved by Runge-Kutta techniques with dense output.

Stability and Hopf Bifurcation for a State-Dependent Delay Differential Equation describing Hematopoietic Stem Cell Dynamics

Fabien Crauste
University Lyon 1, France
crauste@math.univ-lyon1.fr
M. Adimy, M.L. Hbid, R. Qesmi

Hematopoietic stem cells are pluripotent cells at the root of the blood production system. They have unique abilities to self-renew and differentiate in all blood cell types. Hematopoietic stem cell dynamics have been modeled for a long time, back to the end of the seventies and the work of Mackey (1978) based on a delay differential equation describing the dynamics of the cell population, where the delay accounts for an average cell cycle duration. We consider a modification of Mackey's model, in which cell cycle durations are assumed to depend on the cell number. Mackey's equation writes then as a nonlinear differential equation with a state-dependent delay. We first show existence and uniqueness of solutions, as well as boundedness and positivity results. We then focus on the stability analysis. A sufficient condition for the global asymptotic stability of the trivial steady state is obtained using a Lyapunov-Razumikhin function. A unique positive steady state is shown to appear through a transcritical bifurcation of the trivial steady state. The analysis of the positive steady state behavior, through the study of a first order exponential polynomial characteristic equation, concludes to the existence of a local Hopf bifurcation and gives criteria for stability switches.

Asymptotic behaviour for a Nicholson system with multiple delays

Teresa Faria
University of Lisbon, Portugal
tfaria@ptmat.fc.ul.pt
Gergely Rost

We consider an n-dimensional Nicholson's blowflies model with multiple discrete delays,

$$x_i'(t) = -d_i x_i(t) + \sum_{j=1, j\neq i}^{n} a_{ij} x_j(t) + \sum_{k=1}^{m} \beta_{ik} x_i(t-\tau_{ik}) e^{-x_i(t-\tau_{ik})}, \quad i = 1, \ldots, n,$$

where $d_i > 0$, $a_{ij} \geq 0$ for $j \neq i$, $\beta_{ik} \geq 0$, $\sum_{k=1}^{m} \beta_{ik} > 0$, $\tau_{ik} > 0$, and study several features of the asymptotic behaviour of its solutions. This system fits as a population model for the growth of simple or multiple species divided into n patches or classes, with migration of the populations among them. It is shown that if the spectral bound of the community matrix is non-positive, then the population becomes extinct on each patch, whereas the total population uniformly persists if the spectral bound is positive. In the latter case, the existence of a unique positive equilibrium is established, as well as a sharp criterion for its absolute global asymptotic stability, improving results in the recent literature. Several sharp threshold-type results about its dynamics are proven, even when the community matrix is reducible, a case usually not treated in the literature.

A model of humoural mediated immune response with delays

Urszula Forys
University of Warsaw, Poland
urszula@mimuw.edu.pl
Marek Bodnar

Recently, Feyissa and Banerjee proposed a model of interactions between immune system and cancerous with two delays included. Analysis presented in is made under the assumption that solutions are positive. However, this statement is not true. We propose a wide class of initial functions for which the solutions are negative. Moreover, the model is incorrectly constructed in our opinion, and therefore we propose corrected version, where delays cannot lead to destabilisation, while the main result for is appearance of delayed-induced oscillations. This is not the first time when oscillations due to delays are impossible in the corrected model and also not the first model proposed in the literature, for which solutions are negative, why the authors of the models do not pay attention to that problem.

References

[1] S. Feyissa, S. Banerjee, Delay-induced oscillatory dynamics in humoral mediated immune response with two time delays, Nonlinear Analysis: RWA 14(1) (2013) 35-52.

[2] J. Miekisz, J. Poleszczuk, M. Bodnar, U. Forys, Stochastic models of gene expression with delayed degradation, Bull. Math. Biol. 73(9) (2011) 2231-2247.

[3] U. Forys, M. Bodnar, J. Poleszczuk, Negativity of delayed induced oscillations in a simple linear DDE, Appl. Math. Lett. 24(6) (2011) 982-986.

Delay equations as models of physiologically structured populations

Mats Gyllenberg
University of Helsinki, Finland
mats.gyllenberg@helsinki.fi

In this talk I will explain how a large class of models for physiologically structured populations can be formulated in terms of delay equations (nonlinear renewal equations) coupled with delay-differential equations. I will show how these models can be analysed using perturbation theory of adjoint semigroups (sun-star calculus). Linear (in)stability, Hopf bifucation and centre manifold theorems are conveniently obtained within this framework. This is joint work with Odo Diekmann.

Long time behavior of the solutions of population models with delays

István Győri
University of Pannonia, Hungary
gyori@almos.uni-pannon.hu

In this paper we investigate the growth/decay rate of solutions of ordinary and abstract differential and difference equations with delays. Our results can be applied for the case when the characteristic equation of an associated linear equation has complex dominant eigenvalue with higher than one multiplicity. Examples are given for describing the asymptotic behavior of solutions in a class of quasi linear differential and difference equations arising in nonlinear population models. The sharpness of the results and their applicability to some abstract equations which appear in the theory of age dependent population models are also discussed.

State-dependent neutral delay equations from population dynamics

Karl Hadeler
University of Tuebingen, Germany
hadeler@uni-tuebingen.de
Maria Vittoria Barbarossa, Christina Kuttler

The Gurtin-MacCamy system is the standard model for a population structured by age when birth and death rates depend on total (adult) population size. It is well known that (neutral) delay equations can be derived from this system when the population is divided into a juvenile and an adult class and rates within these classes do not depend on age. The delay corresponds to the length of the juvenile phase. The blowfly equation is one example. Here it is assumed that the length of the juvenile phase is a function of the total adult population size. Then a novel class of state-dependent delay equations is obtained which includes neutral equations. These equations are very different from the standard delay equations with state-dependent delay since preservation of mass requires non-linear correction factors. These equations can be written as systems for two variables consisting of an ordinary differential equation and a generalized shift, a form suitable for mathematical analysis and numerical calculations.

Delayed population models with Allee effects and exploitation

Eduardo Liz
Universidad de Vigo, Spain
eliz@dma.uvigo.es
Alfonso Ruiz-Herrera

Models of single-species populations with strong Allee effects typically posses the trivial equilibrium and two positive equilibria. The smallest one is the so-called Allee threshold, and the biggest one usually stands for the carrying capacity. If the population is subject to exploitation, increases in mortality interact with the Allee threshold in different ways. Moreover, while in simple ODEs models stability properties of the carrying capacity are not affected by the interplay between exploitation and the Allee effects, a rich behavior has been reported in discrete models with overcompensating density dependence, where a chaotic attractor may coexist with the trivial attractor, and boundary collisions may result in essential extinction. Having in mind the dynamics of an associated discrete model, we investigate a model governed by a delay-differential equation with a recruitment function exhibiting a strong Allee effect, and subject to exploitation or predation at a constant effort rate. Based on theoretical results and numerical simulations, we show that the introduction of a delay gives rise to some interesting phenomena; we also improve some stability results that have recently appeared in the literature.

Global exponential stability of nonautonomous neural network models with delays

Jose J. Oliveira
University of Minho, Portugal
jjoliveira@math.uminho.pt
Salete Esteves and Elcin Gokmen

For the following system of nonautonomous differential equations with distributed delays,

$$x_i'(t) = -b_i(t, x_i(t)) + f_i(t, x_t), t \geq 0, i = 1, \dots, n,$$

with b_i positive and f_i Lipschitz on the second variable, we give sufficient conditions for the global exponential stability of an equilibrium point. This system includes most of the delayed models of neural networks of Hopfield type with time-varying coefficients and distributed delays. For these models, we establish sufficient conditions for their global exponential stability. The existence and global exponential stability of a periodic solution is also addressed.

Analysis of the family of angiogenesis models with distributed time delays

Monika Piotrowska
University of Warsaw, Poland
monika@mimuw.edu.pl
Marek Bodnar

We study a family of angiogenesis models with distributed delays. Two types of kernels are considered: the Erlang (shifted and non-shifted) distributions and the piecewise linear distributions. The basic mathematical properties of the considered family of models such as the global existence and the uniqueness of the solutions are proven. We analytically derive conditions, involving the parameters defining the distribution kernels, guaranteeing the stability or instability of the steady state. Theorems guaranteeing the existence of stability switches due to the Hopf bifurcation are proven. Theoretical results are illustrated by numerical analysis for parameters estimated by Hahnfeldt *et al.* (*Cancer Res., 1999*). Moreover, for this particular set of parameters we investigate the stability regions of steady state. We compare the results for both the distributed Hahnfeldt et al. and the distributed d'Onofrio-Gandolfi models in the case of all considered distributions. In general case, it is hard to say for which model the stability region is larger since it strongly depends on the considered distribution kernels and theirs shapes. However, we observe a certain similarities.

A Model of Platelet Production: Stability Analysis and Oscillations

Laurent Pujo-Menjouet
University of Lyon, France
pujo@math.univ-lyon1.fr
Mostafa Adimy and Fabien Crauste

We propose here a new model of platelet production and regulation taking into account the recent biological discoveries related to this topic, including the role played by thrombopoietin (TPO), a platelet regulation cytokine. We consider four different cell compartments corresponding to different cell maturity levels: the stem cell, megakaryocytic progenitors, megakaryocytes and platelets compartements. To the best of our knowledge, the progenitor compartment has never been taken into account in previous platelet production models. We consider also the quantity of circulating TPO that influences the dynamics of each cell populations.
Our model consists in a non linear age structured partial differential equation system, where each equation corresponds to a compartment. This system can be reduced to a single non linear delay differential equation describing the dynamics the platelet population.
After a brief introduction of the model, we prove the existence of an unique steady state for the delay differential equation. We set up then conditions to get local and global asymptotic stability of this steady state. We determine then necessary and sufficient conditions for the existence of oscillating solutions.

Global attraction and bifurcation in systems of delay differential equations with non-monotonic feedbacks and Hopfield's model

Alfonso Ruiz-Herrera
University of Szeged, Hungary
alfonsoruiz@ugr.es
Eduardo Liz

The aim of this talk is to give criteria of attraction and bifurcation in general systems of delay differential equations. Our approach relies on a link between a system of functional differential equations and a finite-dimensional discrete dynamical system. As we will see, for a system of delayed neural networks of Hopfield type, we can give sharp criteria of global attractivity, multistability, and bifurcations without assuming monotonicity conditions in the activation functions. Our abstract setting is not limited to applications in systems of neural networks and we illustrate its use in equations with distributed delay motivated by biological models.

Global dynamics of delay differential equations with mixed feedback

Gergely Röst
University of Szeged, Hungary
rost@math.u-szeged.hu
Alfonso Ruiz-Herrera, Eduardo Liz

The method of associating a discrete dynamical system to a scalar delay differential equation has been very useful in the study of the global dynamics of several classical model equations (including Nicholson-blowflies and Mackey-Glass equation). We extend this method to equations which involve mixed terms, in the sense that some terms may include both delayed and instantenous feedback, or the equation has both positive and negative feedback terms. Applications are given to a blood placelet model of Belair and Mackey with two delays, and to an epidemic model that includes delayed adaptation of the population. For the latter, we find a novel bifurcation diagram for epidemic models, what we call endemic bubble.

Dynamics of a cholera model with hyperinfectivity and temporary immunity

Zhisheng Shuai
University of Central Florida, USA
shuai@ucf.edu
Joseph Tien, Pauline van den Driessche

In this talk we present a new mathematical model for cholera that incorporates hyperinfectivity and temporary immunity using distributed delays. The basic reproduction number is defined and proved to be a sharp threshold determining whether cholera dies out. Numerical simulations show that disease oscillations occur for different infectivity kernels.

Virus Competition for a Bacterial Host in a Petri Dish

Hal Smith
Arizona State University, USA
halsmith@asu.edu

If a single virus (bacteriophage) particle is dropped onto a Petri dish in which a homogeneous "lawn" of susceptible bacteria reside, a circular plaque forms with radius increasing linearly with time. This is the result of a traveling wave of infection spreading across the dish. It can happen that the virus mutates along the way so that several strains compete for the bacterial host. Now, the spatial spread is much more complicated. We examine a delayed partial differential equation model of this competition where the delay represents the latent period of infection.

Delay differential systems for tick population dynamics

Horst Thieme
Arizona State University, USA
hthieme@asu.edu
Guihong Fan, Huaiping Zhu

Ticks play a critical role as vectors in the transmission and spread of Lyme disease, an emerging infectious disease. To understand the population dynamics of ticks, we formulate a system of delay differential equations which models the stage structure of the tick population. Temperature can alter the length of time delays in each developmental stage, and so the time delays can vary geographically (and seasonally which we do not consider). We define the basic reproduction number which separates uniform population persistence from local stability of the extinction equilibrium. We present sufficient conditions under which the unique positive equilibrium point is globally asymptotically stable. In general, the positive equilibrium can be unstable and the system show oscillatory behavior. These oscillations are primarily due to negative feedback within the tick system, but can be enhanced by the time delays of the different developmental stages.

Abstract algebraic-delay differential systems and age structured population dynamics

Jianhong Wu
York University, Canada
wujh@yorku.ca
N. Kosovalic, F.M.G. Magpantay, Y. Chen

We present some recent results on the fundamental theory and numerical analysis of algebraic-delay differential systems, and discuss its motivation from and applications to structured population dynamics.

On the basins of attraction for a class of delay differential equations with nonmonotone bistable nonlinearities

Xingfu Zou
University of Western Ontario, Canada
xzou@uwo.ca

Consider delay differential equation $x'(t) = -g(x(t)) + f(x(t-r))$ with bistable equilibrium structure: there are three equilibria $x_0 = 0 < x_1 < x_2$ with x_0 and x_2 being stable and x_1 being stable for the ODE: $x'(t) = -g(x(t)) + f(x(t))$. I will present some results characterizing subsets of basins of attraction of the equilibria. The results will be applied to a particular model equation describing the matured population of some species demonstrating the Alee effect.

Special Session 5: Differential Delay Equations

Fatihcan M. Atay, Max Planck Institute for Mathematics in the Sciences (Leipzig), Germany
Bernhard Lani-Wayda, Justus-Liebig-University Giessen, Germany
Hans-Otto Walther, Justus-Liebig-University Giessen, Germany

The session aims to cover recent developments in the qualitative theory of differential delay equations, for example structure of attractors, bifurcation, global dynamical behavior, equations with state-dependent delay.

Probabilistic methods for a class of equations with rescaling

Gregory Derfel
Ben Gurion University, Israel
derfel@cs.bgu.ac.il

The question about the existence and characterization of bounded solutions to linear functional-differential equations with both advanced and delayed arguments was posed in early 1970s by T. Kato in connection with the analysis of the pantograph equation, $y'(x) = ay(qx) + by(x)$. In our talk, we answer this question for the *balanced* generalized pantograph equation of the form $y'(x) + y(x) = \sum_i p_i\, y(\alpha_i x)$, under the balance condition $\sum_i p_i = 1$ $(p_i \geq 0)$. Namely, setting $K := \sum_1^l p_i \ln \alpha_i$, we prove that if $K \leq 0$ then the equation does not have nontrivial (i.e., nonconstant) bounded solutions, while if $K > 0$ then such a solution exists. The result in the critical case, $K = 0$, settles a long-standing problem. The proofs are based on the link with the theory of Markov processes and employ martingale technique. Same approach may be applied also for other types of equations with rescaling (i.e. functional, integral and integro-differential ones). The talk is based on joint work with Leonid Bogachev (Leeds, UK), Stanislav Molchanov (North Carolina at Charlotte, USA) and John Ockendon (Oxford, UK).

Global dynamics for some classes of Lotka-Volterra systems with infinite delay

Teresa Faria
University of Lisbon, Portugal
tfaria@ptmat.fc.ul.pt

We analyse some aspects of the global dynamics for autonomous n-dimensional Lotka-Volterra systems with infinite delay and patch structure, such as extinction, persistence, global attractivity of a positive equilibrium x^* (when it exists), existence of a positive heteroclinic solution connecting zero to x^*. Our approach exploits the theory of monotone dynamical systems, which is applied even for non-cooperative systems via comparison results with an auxiliary cooperative system.

Quiescent phases and delay equations

Karl Hadeler
University of Tuebingen, Germany
hadeler@uni-tuebingen.de

We consider systems where a vector field is diffusively coupled to the zero field. In a particle interpretation such systems describe particles switching between an active and a quiescent phase. We apply this concept to delay equations and derive three distinct classes of vector valued delay equations with quiescent phases showing different qualitative behavior. Quiescent phases leave stationary points unchanged but affect stability, oscillatory behavior, Hopf bifurcations, and periodic orbits. When all components of the solution vector go quiescent with the same rates, then quiescent phases act like damping. If different components go quiescent with different rates then there may be excitation phenomena: stable stationary points undergo Hopf bifurcations. In the case of two dependent variables exact conditions for these phenomena can be found.

On neutral differential equations with state-dependent delays

Ferenc Hartung
University of Pannonia, Hungary
hartung.ferenc@uni-pannon.hu

In this talk we consider a class of neutral differential equation with state-dependent delays in both in the neutral and the retarded terms. We discuss the well-posedness and continuous dependence issues and differentiability of the solutions with respect to parameters.

Global Hopf bifurcation of differential equations with threshold type state-dependent delay

Qingwen Hu
University of Texas at Dallas, USA
qingwen@utdallas.edu
Zalman Balanov, Wieslaw Krawcewicz

We develop global Hopf bifurcation theory of differential equations with state-dependent delay using the S^1-equivariant degree and investigate a two-degree-of-freedom mechanical model of turning processes. For the model of turning processes we show that the extreme points of each vibration component of the non-constant periodic solutions can be embedded into a manifold with explicit algebraic expression. This observation enables us to establish analytical upper and lower bounds of the amplitudes of the periodic solutions in terms of the system parameters and to exclude certain periods. Using the achieved global bifurcation theory we reveal that if the relative frequency between the natural frequency and the turning frequency varies in a certain interval, then generically every bifurcated continuum of periodic solutions must terminate at a bifurcation point. This termination means that the underlying system with parameters in the stability region near the vertical asymptotes of the stability lobes is less subject to chatter. In the process, several sufficient conditions for the non-existence of nonconstant periodic solutions are also obtained.

Travelling Waves for Fully Discretized Bistable Reaction-Diffusion Problems

Hermen Jan Hupkes
Leiden University, Netherlands
hjhupkes@hotmail.com
E. S. Van Vleck

We study various temporal and spatial discretization methods for bistable reaction-diffusion problems. The main focus is on the functional differential operators that arise after linearizing around travelling waves in the spatially discrete problem and studying how the subsequent discretization of time affects the spectral properties of these operators. This represents a highly singular perturbation that we attempt to understand via a weak-limit method based on the pioneering work of Bates, Chen and Chmaj (2003). Once this perturbation is understood, one can study the existence and (non)-uniqueness of waves in the fully discretized reaction-diffusion system.

Periodic solutions of a singular delay equation with a Farey nonlinearity

Anatoli Ivanov
Pennsylvania State University, USA
afi1@psu.edu

A scalar differential delay equation of the form

$$\varepsilon x'(t) + x(t) = f(x(t-1))$$

is considered where function f is a Farey-type nonlinearity defined by

$$G(x) = \begin{cases} mx + A & \text{if} \quad x \leq 0 \\ mx + B & \text{if} \quad x > 0 \end{cases}$$

for some $0B$. The map f as a dynamical system always has a unique globally attracting cycle. The problem of existence of periodic solutions of the differential delay equation and their properties in relation to the one-dimensional map f are studied for small $\varepsilon > 0$. Theoretical results are demonstrated and supported by numerical calculations.

Oscillation speed and periodic solutions for a class of integro-differential equations with negative feedback

Benjamin Kennedy
Gettysburg College, USA
bkennedy@gettysburg.edu

We consider a class of scalar autonomous nonlinear integro-differential equations of the form

$$(E) \quad x'(t) = \int_0^1 v(s, x(t-s))\, ds,$$

where v satisfies the negative feedback condition $v(s,u)u \leq 0$. Our chief motivation is heuristic modeling of systems with state-dependent delay. We discuss some connections between (E) and some previously studied differential delay equations. For a particular subclass of equations (E), we formulate a non-increasing "oscillation speed" for solutions, and show the existence of nontrivial slowly oscillating periodic solutions.

The structure of unstable sets for delayed monotone feedback

Tibor Krisztin
University of Szeged, Hungary
krisztin@math.u-szeged.hu
Gabriella Vas

We consider the delay differential equation $\dot{x}(t) = f(x(t), x(t-1))$ with a monotone feedback condition on the smooth $f : \mathbb{R}^2 \to \mathbb{R}$. It is shown that certain stable and unstable manifolds of periodic orbits and stationary points intersect transversally. This plays a key role in the description of the geometric structure of the global attractor.

Flow invariance for state-dependent delay differential equations

Wolfgang Ruess
Universitaet Duisburg-Essen, Germany
wolfgang.ruess@uni-due.de

For a state-dependent differential delay equation $\dot{u}(t) = F(u_t)$, $u_0 = \varphi \in C$, in a general Banach state space X and initial-history space $C = C([-r, 0]; X)$, and closed subsets $\hat{X}$ of X, and $\hat{C}$ of C, we discuss a subtangential condition in terms of F, $\hat{X}$ and $\hat{C}$ ensuring flow invariance of $\hat{X}$ for solutions to the equation, and of $\hat{C}$ for their history-segments, with F almost locally Lipschitzian, and a differentiability assumption on F common in this context.

Attraction property of local center-unstable manifolds for differential equations with state-dependent delay

Eugen Stumpf
University of Hamburg, Germany
eugen.stumpf@math.uni-hamburg.de

We consider a class of functional differential equations of the form

$$\dot{x}(t) = f(x_t) \tag{1}$$

with f defined on an open subset of $C^1([-h, 0], \mathbb{R}^n)$, $h > 0$. Under certain conditions, which are typically satisfied in cases where Eq. (1) represents an autonomous differential equation with state-dependent delay, the associated Cauchy problems define a smooth semiflow on a submanifold of $C^1([-h, 0], \mathbb{R}^n)$. In particular, it is known that at a stationary point of the semiflow there exist so-called local center-unstable manifolds. Here, we discuss an attraction property of these manifolds. More precisely, we prove that, after fixing some local center-unstable manifold W_{cu} of Eq. (1) at a stationary point ϕ, each solutions which exists and remains close enough to ϕ for all $t \geq 0$ converges exponentially for $t \to \infty$ to a solution on the local center-unstable manifold W_{cu}.

Rigorous integration of Delay Differential Equations

Robert Szczelina
Jagiellonian University, Poland
robert.szczelina@uj.edu.pl

We will present a rigorous integrator for Delay Differential Equations of the form $\dot{x} = f(x(t-1), x(t))$ and we will show how to implement rigorous Poincare Map for such systems. The integrator, together with topological arguments, can be used in rigorous computer assisted proofs of existence of various dynamical phenomena, such as stable/unstable stationary points, periodic solutions, connecting orbits, etc. We will show some preliminary results which can be obtained for scalar equations when $f(x(t-1), x(t)) = -K \cdot sin(x(t-1))$ and $f(x(t-1), x(t)) = -x(t) + g(x(t-1))$, where g is polynomial of degree 5. Our method can be used for any r.h.s. sufficiently smooth and may be simply extended to any dimension and any number of discrete delays.

The unstable set of a periodic orbit for delayed positive feedback

Gabriella Vas
Hungarian Academy of Sciences, University of Szeged, Hungary
vasg@math.u-szeged.hu
Tibor Krisztin

In the paper [Large-amplitude periodic solutions for differential equations with delayed monotone positive feedback, JDDE 23 (2011), no. 4, 727-790], we have constructed large-amplitude periodic orbits for the scalar delay equation

$$\dot{x}(t) = -\mu x(t) + f(x(t-1)),$$

where $\mu > 0$ and f is a strictly increasing, continuously differentiable nonlinearity (positive feedback case). We have shown that the unstable sets of the large-amplitude periodic orbits constitute the global attractor besides the so called spindle-like structures. In this talk we focus on a large-amplitude periodic orbit $\mathcal{O}_p$ with two Floquet multipliers outside the unit circle, and we intend to characterize the geometric structure of its unstable set $\mathcal{W}^u(\mathcal{O}_p)$. We show that $\mathcal{W}^u(\mathcal{O}_p)$ is a three-dimensional C^1-submanifold of the phase space and admits a smooth global graph representation. Within $\mathcal{W}^u(\mathcal{O}_p)$, there exist heteroclinic connections from $\mathcal{O}_p$ to three different periodic orbits. These connecting sets are two-dimensional C^1-submanifolds of $\mathcal{W}^u(\mathcal{O}_p)$ and homeomorphic to the two-dimensional open annulus. They form C^1-smooth separatrices in the sense that they divide the points of $\mathcal{W}^u(\mathcal{O}_p)$ into three subsets according to their ω-limit sets. The research of Gabriella Vas was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TAMOP-4.2.4.A/2-11/1-2012-0001 'National Excellence Program'.

Amplitude equations for DDEs with large delay

Matthias Wolfrum
WIAS Berlin, Germany
wolfrum@wias-berlin.de
S. Yanchuk, L. Lücken, A. Mielke

The singular limit of large delay in a DDE is relevant for a variety of applied problems. In several examples of such equations it has been demonstrated by formal asymptotics that the dynamics close to the destabilization of an equilibrium can be described by an amplitude equation of Ginzburg-Landau type. We present here some recent results that provide a more general and rigorous framework for this phenomenon. Based on a general theory for the spectrum of linearized DDEs with large dealy, we classify possible types of instabilities and provide for certain cases of this general framework a rigorous result about the validity of the amplitude equation. In particular, we discuss similarities and differences to analogous results about the validity of amplitude equations for spatially extended systems.

Pattern formation in systems with multiple delayed feedbacks

Serhiy Yanchuk
Humboldt University, Germany
yanchuk@math.hu-berlin.de
Giovanni Giacomelli

Dynamical systems with complex delayed interactions arise commonly when propagation times are significant, yielding complicated oscillatory instabilities. In this talk, we consider a class of systems with multiple, hierarchically long time delays. Using a suitable space-time representation, we uncover features otherwise hidden in their temporal dynamics. Erratic behavior in the case of two delays is shown to "encode" two-dimensional spiral defects and defects turbulence. A multiple scale analysis sets the equivalence to a complex Ginzburg-Landau equation. We also demonstrate this phenomenon for a semiconductor laser with two delayed optical feedbacks.

Special Session 6: Random Dynamical Systems in the Life Sciences

Xiaoying Han, Auburn University, United States
Peter Kloeden, J.W. Goethe-Universitat, Germany

Bio-mathematics has made extensive use of concepts and methods from the theory of autonomous dynamical systems. However, many systems in the life sciences are intrinsically non-autonomous due to time seasonally changes and the presence of noise and this classical theory is inadequate. The purpose of this workshop is to highlight and make better known of new concepts and results from the recently developed theory of random dynamical systems such as random attractors and to show their relevance and applicability in the life sciences.

Doubly stochastic processes in neurobiology

Janet Best
The Ohio State University, USA
best.82@osu.edu
Michael Reed

Individual neurons are often modeled as Poisson processes firing with intensity λ. However, the ultimate interest in neuroscience is not in the behavior of individual neurons but in the behavior of networks of neurons that influence each other. That is, the intensity λ of any individual neuron will change stochastically as a function of time as it receives input from other neurons. Thus consideration of network properties automatically leads to considering doubly stochastic processes. We will illustrate this by considering nuclei that are involved in the regulation of sleep/wake cycles. Understanding how networks of doubly stochastic neurons behave poses new challenges for neurobiologists and mathematicians.

Chemostats with random inputs

Tomas Caraballo
Universidad de Sevilla, Spain
caraball@us.es
Xiaoying Han, Peter E. Kloeden

Chemostat refers to a laboratory device used for growing microorganisms in a cultured environment, and has been regarded as an idealization of nature to study competition modeling of mathematical biology. The simple form of chemostat model assumes that the availability of nutrient and its supply rate are both fixed. However, these assumptions largely limit the applicability of chemostat models to realistic competition systems. In this work, we relax these assumptions and study the chemostat models with random nutrient supplying rate or random input nutrient concentration. This leads the models to random dynamical systems and requires the concept of random attractors developed in the theory of random dynamical systems. We will report on the existence of uniformly bounded non-negative solutions, existence of random attractors and geometric details of random attractors for different value of parameters.

A stochastic model for the dynamics of two interacting neurons periodically driven

Maria Francesca Carfora
Università degli Studi di Napoli Federico II, Italy
f.carfora@iac.cnr.it
Enrica Pirozzi

A stochastic model for describing the firing activity of a couple of interacting neurons subject to time-dependent stimuli is proposed. Two stochastic differential equations suitably coupled and including periodic terms to represent stimuli imposed to one or both neurons are considered . We investigate the first passage time densities through specified firing thresholds for the involved time non-homogeneous Gauss-Markov processes. For different values of amplitude, frequency and phase, simulation results and numerical approximations of the firing densities, that we are able to provide, are compared and discussed. Asymptotic behaviors of the firing densities are also given and validated for suitable ranges of parameters values

The stochastic resonance as a case study for bifurcations in random dynamical systems

Anna Maria Cherubini
Università del Salento, Italy
amcherubini@gmail.com
Jeroen S.W. Lamb, Martin Rasmussen and Yuzuru Sato

Stochastic resonance has been the object of extensive research in the last decades. The term describes a phenomenon in nonlinear systems where a weak signal can be be amplified and optimized by the presence of noise. This rather counter-intuitive effect was first presented in works by R. Benzi and collaborators to address the problem of periodically recurrent ice ages. A standard model for SR is given by a damped particle in a periodically oscillating double-well potential with white noise: if the periodic forcing is too weak for the particle to move periodically between wells, noise-induced hopping synchronized with the forcing can nevertheless be observed for optimal values of the parameters. The phenomenon is relevant in a number of fields such as signal analysis, biology, neurosciences and climate studies. Though widely studied in statistical physics and the different fields of application, and despite showing the characters of a "bifurcation", there are few theoretical results on the dynamical systems underlying the phenomenon, in particular in term of bifurcation theory. Starting from results by H. Crauel and F. Flandoli and by J. Lamb, M. Rasmussen and collaborators, we studied the SR in the framework of bifurcation theory for random dynamical systems.

Implicit Simulation Methods for Stochastic Chemical Kinetics

Xiaoying Han
Auburn University, USA
xzh0003@auburn.edu
Tae-Hyuk Ahn, Adrian Sandu

In biochemical systems some of the biological/chemical species are present with only small numbers of molecules. In this situation discrete and stochastic simulation approaches are more relevant than continuous and deterministic ones. The fundamental Gillespie's stochastic simulation algorithm (SSA) accounts for every reaction event, which occurs with a probability determined by the configuration of the system. This approach requires a considerable computational effort for models with many reaction channels and biological/chemical species. In order to improve efficiency, tau-leaping methods represent multiple firings of each reaction during a simulation step by Poisson random variables. For stiff systems the mean of this variable is treated implicitly in order to ensure numerical stability. We developed fully implicit tau-leaping-like algorithms that treat implicitly both the mean and the variance of the Poisson variables. The construction is based on adapting weakly convergent discretizations of stochastic differential equations to stochastic chemical kinetic systems. Theoretical analyses of accuracy and stability of the new methods are performed on a standard test problem. Numerical results demonstrate the performance of the proposed tau-leaping methods.

Viral kinetic modeling approaches in chronic hepatitis

Eva Herrmann
Goethe University Frankfurt, Germany
herrmann@med.uni-frankfurt.de
Yusuke Asai, Dimitra Bon

In this talk, we will give an overview on new approaches for viral kinetic modeling in chronic hepatitis. There exist some well known approaches for modeling the decline of viremia in patients treated with appropriate antiviral drug regimes. Mathematical modeling approaches can help to reveal treatment mechanismen, quantify treatment efficacy and assess potential synergies or antagonism between different drugs. Typically, such models base on nonlinear and non-autonomous ordinary differential equation systems. New multistage models have been proposed to analyse new direct acting antiviral drugs in hepatitis C. Nevertheless, in such models it is more difficult to assess theoretical and practical identifiability. We will show some model examples and data analyses applied to data from recent clinical trials. Furthermore, we will compare results from such ordinary differential equation systems with random ordinary differential equation systems allowing random pertubationsof the model parameters.

Pulse bifurcations in stochastic neural fields

Zachary Kilpatrick
University of Houston, USA
zpkilpat@math.uh.edu
Gregory Faye

We study the effects of additive noise on traveling pulse solutions in spatially extended neural fields with linear adaptation. Neural fields are evolution equations with an integral term characterizing synaptic interactions between neurons at different spatial locations of the network. We introduce an auxiliary variable to model the effects of local negative feedback and consider random fluctuations by modeling the system as a set of spatially extended Langevin equations whose noise term is a Q-Wiener process. Due to the translation invariance of the network, neural fields can support a continuum of spatially localized bump solutions that can be destabilized by increasing the strength of the adaptation, giving rise to traveling pulse solutions. Near this criticality, we derive a stochastic amplitude equation describing the dynamics of these bifurcating pulses when the noise and the deterministic instability are of comparable magnitude. Away from this bifurcation, we investigate the effects of additive noise on the propagation of traveling pulses and demonstrate that noise induces wandering of traveling pulses. Our results are complemented with numerical simulations.

Nonautonomous and random dynamical systems in the life sciences

Peter Kloeden
Goethe University, Germany
kloeden@math.uni-frankfurt.de

Biomathematical models that include seasonal effects or noise are intrinsically nonautonomous and classical concepts and methods from the theory of autonomous dynamical systems are inadequate or not applicable. New concepts of nonautonomous and random attractors will briefly reviewed and illustrated here in terms of SIR and chemostat models as well as Markov chains with randomly varying transition matrx entries that model the effects of a random environment.

Fluctuation Models for Suspensions of Swimming Microorganisms

Peter Kramer
Rensselaer Polytechnic Institute, USA
kramep@rpi.edu
Patrick Underhill, Kajetan Sikorski, Yuzhou Qian

The collective dynamics of swimming microorganisms ("microswimmers") such as bacteria and algal cells have been of considerable recent interest, both as paradigms of collective patterns arising from individual autonomous agents and for their relevance to technological issues such as biofilm formation and power sources for microdevices. The dynamics of microswimmers have been examined through experiment, direct numerical simulations, and mean field theories. The mean field theories have shown a good degree of explanatory power, primarily through stability analysis of the isotropic state, but the mean field assumptions also restrict the theory's ability to describe the actual non-equilibrium steady states of the microswimmers. We will present some recent efforts to incorporate

stochastic fluctuations and correlations into a continuum partial differential equation framework for the effective microswimmer dynamics in a suspension. To develop insight in this endeavor, we have thus far been developing these richer statistical descriptions for simpler microswimmer models than the usual models of force dipoles interacting through Stokes flow hydrodynamics.

Dynamics of Large Networks: Taking It to the Limit

Georgi Medvedev
Drexel University, USA
medvedev@drexel.edu

The continuum limit is an approximate procedure, by which coupled dynamical systems on large graphs are replaced by an evolution integral equation on a continuous spatial domain. This approach has been instrumental for studying dynamics of diverse networks throughout physics and biology. We use the combination of ideas and results from the theories of graph limits and nonlinear evolution equations to develop a rigorous justification for using the continuum limit in a variety of dynamical models on deterministic, random, and quasirandom graphs. As a specific application, we discuss synchronization in small-world networks of Kuramoto oscillators.

References

[1] Georgi S. Medvedev, The nonlinear heat equation on dense graphs and graph limits, SIAM J. Math. Analysis, submitted

[2] Georgi S. Medvedev, The nonlinear heat equation on W-random graphs, Archive for Rational Mechanics and Analysis (2013)

[3] Georgi S. Medvedev, Small-world networks of Kuramoto oscillators, Physica D 266, 13-22 (2014).

Stochastic Dynamics in Signal Transduction, Stem Cells, and Development

Qing Nie
University of California - Irvine, USA
qnieact@gmail.com

Noise and stochastic effects exist in most biological systems due to many intrinsic and extrinsic factors. In this talk, I will discuss strategies and principles for noise attenuation and robustness to genetic and environmental perturbations in signal transduction, embryonic patterning, and regeneration driven by stem cells. In one case, I will introduce a critical quantity that succinctly captures the capability of attenuating temporal noise in feedback systems. In another case, I will show that noise actually can be beneficial through an example in which noise in gene regulation enables reduction of the overall stochastic effects during embryonic development. In addition to presenting experimental data that supports our predictions, I will introduce several multi-scale and stochastic models and new computational tools for simulating such complex biological systems. Lastly, the main mathematical challenges will be discussed for analysis of several modeling frameworks shown in this talk.

Prey-Predator systems with unbounded time-dependent coefficients

Felipe Rivero
UNAM, Mexico
lfeliperiverog@gmail.com
Juan C. Jara

In this talk we are going to see the asymptotic behaviour of the following prey-predator system

$$\begin{cases} A' = \alpha f(t)A - \beta g(t)A^2 - \gamma AP \\ P' = \delta h(t)P - \lambda m(t)P^2 + \mu AP, \end{cases}$$

where functions $f, g : \mathbb{R} \to \mathbb{R}$ are not necessarily bounded above and f, g, h, m can be seen as non-autonomous or random coefficients. I also show the existence of the pullback attractor and the permanence of solutions for any positive initial data and initial time, making a previous study of a logistic equation with unbounded terms, where one of them can be negative for a bounded interval of time. The analysis of a non-autonomous logistic equation with unbounded coefficients is also needed to ensure the permanence of the model.

Mathematical concepts in pharmacokinetics and pharmacodynamics

Johannes Schropp
University of Konstanz, Germany
johannes.schropp@uni-konstanz.de
Gilbert Koch

The objective of this talk is to present and to analyze fundamental concepts of pharmacokinetic (PK) and pharmacodynamic (PD) modeling from a mathematical point of view. It turns out that any PKPD model is a nonautonomous dynamical system driven by the drug concentration. Moreover, we state convergence results between two widely used models, namely, transit compartments and lifespan models.

Systems that can stabilise their rates: lessons from the circulatory system

Aneta Stefanovska
Lancaster University, England
aneta@lancaster.ac.uk
Philip Clemson and Yevhen Suprunenko

The blood circulation is characterised by oscillatory processes. Two of them are well known. They are associated with the rhythmic pumping of the heart at a frequency of 1 Hz in a healthy relaxed human, and with the cyclic breathing at a frequency around 0.25 Hz. Neither of these two frequencies is constant – but continuously varies around these average values. The question is then: What is the most optimal model to account for the randomness in the frequency variation? In this talk we will describe a subset of non-autonomous systems, named chronotaxic (from *chronos* – time and *taxis* – order), that are capable of stabilising their phases. They can thus be characterised by amplitudes and phases that are time-varying but stable and by a time-dependent point attractor moving on a limit cycle. Applying this model we will illustrate that much of the complexity of the cardiac and respiratory dynamics, conventionally treated as stochastic, results from underlying deterministic contributions.

A stochastic multiscale model for acid mediated tumor invasion

Christina Surulescu
University of Kaiserslautern, Germany
surulescu@mathematik.uni-kl.de
Sandesh Hiremath

A recent approach in cancer therapy is based on the role of tumor microenvironment in determining cancer malignancy. Hypoxia and acidity, for instance, are factors that can trigger the progression from benign to malignant growth. The low pH in the peritumoral region directly influences the metastatic potential of tumor cells by inducing apoptosis of normal cells. The macroscopic (population level) behavior of cancer cells is conditioned by and influences biochemical events on the intracellular level. Experiments suggest stochasticity in pH dynamics (production of protons and their in- and outward transport through the cell membrane); this refers to variations and uncertainties in the behaviour of each cell even though they all follow the same biochemical mechanisms. We propose a multiscale model for acid-mediated tumor invasion, which connects the subcellular microscale with the processes on the macroscopic level and accounts for stochasticity in the proton dynamics on the microlevel.

Emergence of synchrony in randomly coupled networks

Jonathan Touboul
College de France, France
jonathan.touboul@college-de-france.fr

We will investigate the role of connectivity disorder in the emergence of synchronized activity in large neuronal networks, possibly subject to noise. We will start by presenting the limiting behavior of these system as their size tend to infinity. With random coupling, the resulting equation is a non-Markovian stochastic equation, an implicit equation in the space of stochastic processes. Thorough analysis of the solutions, and a fruitful analogy with deterministically coupled networks, allow to infer the qualitative dynamics of such systems. We will focus on a surprising transition from stationary states to synchronized activity as disorder is increased. This theory breaks down for balanced networks (in which the net input received by a neuron vanishes). We will discuss how these networks with such balanced connectivity may be analyzed, and will show that random transitions occur in these systems, in relationship with the type of eigenvalue with largest real part of the random coupling matrix.

Stability Criteria for Switched Epidemiological Models

Fabian Wirth
IBM Research Ireland, Ireland
fabwirth@ie.ibm.com
Mustapha Ait Rami, Vahid S. Bokharaie, Oliver Mason

We study the spread of disease in an SIS model of the form

$$\dot{x} = (-D_{\sigma(t)} + B_{\sigma(t)} - \mathrm{diag}(x)B_{\sigma(t)})x\,.$$

Here D_i is a diagonal matrix, representing the recovery coefficients of the different (groups of) individuals and B_i is a weighted adjecency matrix representing the infection graph. In order to model changing circumstances for the spread of the disease we have a set of scenarios $\{D_1, \ldots, D_m\}$, $\{B_1, \ldots, B_m\}$. The time-dependent switching signal σ represents the time-varying change of scenario. The model considered is a time-varying, switched model, in which the parameters of the SIS model are subject to abrupt change. We show that the joint spectral radius can be used as a threshold parameter for this model in the spirit of the basic reproduction number for time-invariant models. We also present conditions for persistence and the existence of periodic orbits for the switched model and results for a stochastic switched model. The results extend in a fairly straightforward manner to switched SIR or SIRS models and we briefly comment on this.

Special Session 7: Topological and Combinatorial Dynamics

Lluis Alseda, Universitat Autonoma de Barcelona, Spain,
Francisco Balibrea, Universidad de Murcia, Spain
Piotr Oprocha, AGH University, Poland

The session will be focused on the topological and combinatorial aspects of low dimensional discrete dynamical systems including entropy, chaos, limit sets, sets of periods, rotation theory and the like. Applications of this theory to Economics, Physics, Engineering, Biology and others are highly welcome. Related areas of dynamical systems, combinatorial aspects of dynamical systems and ergodic theory are not excluded.

Entropy and switching systems

Jose Amigo
Miguel Hernandez University, Spain
jm.amigo@umh.es
Peter Kloeden, Angel Gimenez

Switching systems are an example of non-autonomous dynamical systems, which are well-known in control theory as instances of programmed (or closed loop) control systems. We will discuss the relation between the complexity of the control sequences (as measured by the topological entropy of the control sequence generator) and the complexity of the resulting dynamics (as measured by its topological entropy). We will bring our result in connection with the so-called Parrondo's paradox in other areas of science.

Two folk theorems in topological dynamics

Joe Auslander
University of Maryland, USA
jna@math.umd.edu

Let (X,T) be a flow, where X is a compact Hausdorff space. Recall that by definition (X,T) is distal if the proximal relation P is trivial ($P=\Delta$) and an elementary argument shows that (X,T) is equicontinuous if the regionally proximal relation RP is trivial. One would expect the following to be the case: THEOREM. Let $\pi : X \to Y$ be a homomorphism (continuous surjective equivariant map). If $\pi(P)=\Delta$ then (Y,T) is distal, and if $\pi(RP)=\Delta$ then (Y,T) is equicontinuous. An immediate consequence is that for any flow the distal structure relation is the closed invariant equivalence relation generated by P, and similarly for the equicontinuous structure relation with RP. The theorem is well known (and true) but I have not seen it in print. Moreover, surprisingly, the proof is not immediate.

Strange attractors and rotational chaos on the 2-torus

Jan Boronski
Institute of Research and Applications of Fuzzy Modeling, Czech Rep
jan.boronski@osu.cz
Piotr Oprocha

In my talk I shall present a construction of a 2-torus homeomorphism h homotopic to the identity with an attracting R.H. Bing's pseudocircle C such that the rotation set of $h|C$ is not a unique vector. The inverse limit construction provides new examples of strange attractors with rotational chaos, that were first studied by G.D. Birkhoff in 1932. In this context, time permitting, I shall also discuss a result concerning topological entropy of graph maps, that give hereditarily indecomposable inverse limits.

Matching interval maps

Henk Bruin
University of Vienna, Austria
henk.bruin@univie.ac.at
Carlo Carminati

A somewhat curious phenomenon, observed in e.g. the family of alpha-continued fraction maps, is that certain orbits will synchronize (called matching), and lead to a better form of invariant density and monotonicity properties of entropy than might a priori be expected. The same phenomenon occurs for families of piecewise linear interval maps, but especially the non-matching set becomes very interesting, and I will present results on this.

The forbidden set of some rational difference equations

Antonio Cascales-Vicente
Universidad de Murcia, Spain
antoniocascales@yahoo.es
Francisco Balibrea Gallego

Let $f : \mathbb{A}^n \to \mathbb{A}$ be a rational function. We state the general problem of describing the set of initial conditions $(x_{-n+1},\dots,x_0) \in \mathbb{A}^n$ such that the iteration $x_{k+1} = f(x_{k-n+1},\dots,x_k)$ is not well defined for some $k \geq 0$. When $n=1$ and $\mathbb{A}=\mathbb{R},\mathbb{C}$, in some cases it is possible to define a bijection between the forbidden set of the difference equation and a set of symbols. Some topological corollaries and numerical estimates derive from the previous remark.

Cocycles of point-distal minimal flows

Gernot Greschonig
Vienna University of Technology, Austria
greschg@fastmail.net

In this talk we generalise results on the structure of real-valued cocycles of distal minimal compact metric flows for cocycles of a class of point-distal minimal flows (i.e. minimal flows with at least one point distal to any distinct point, which is sufficient for a residual set of points with this property). Since the general case of a point-distal minimal flow according to the Veech structure theorem seems illusive, we will consider minimal compact metric flows without strong Li-Yorke pairs (i.e. proximal pairs recurrent in the product space) which are almost 1-1 extensions of a distal flow with connected fibres. For this class of flows, which includes the point-distal flows on the torus constructed by Mary Rees, we can understand the structure of real-valued cocycles under a condition on recurrent points in the skew product of the cocycle. This condition requires that every non-distal point in the point-distal minimal compact flow is proximal to a point which lifts to recurrent points in the skew product.

Extending group actions on separable metric spaces

James Keesling
University of Florida, USA
kees@ufl.edu
David Wilson, James Maissen

Let G be a complete separable metric group and X be a separable metric space. Suppose that $G \times X \to X$ is a continuous action by G on X. Let C be a metric compactification of X. Suppose that for each $g \in G$ the homeomorphism associated with g has an extension to a homeomorphism on C. Then the action $G \times C \to C$ is also continuous. This theorem has implications for the Hilbert-Smith Conjecture. There are also applications to topological group actions in many settings. We will discuss these in this talk.

Dynamics on inverse limits generated by set-valued functions

Judy Kennedy
Lamar University, USA
kennedy9905@gmail.com

We investigate entropy and other dynamical properties in inverse limits generated by set-valued functions on intervals.

Generalized flow boxes

Krystyna Kuperberg
Auburn University, USA
kuperkm@auburn.edu

Let $\phi : \mathbb{R} \times M \to M$ be a non-singular continuous dynamical system defined on a differentiable n-manifold M. If ϕ is generated by a C^1 vector field, then M can be covered with flow boxes $D^{n-1} \times I$ on which ϕ is a local flow along the I-fibers and D^{n-1} is the $(n-1)$ disk. This in general is not the case for $n > 3$. In 1957, R.H. Bing gave an example of a generalized 3-manifold, which is not homeomorphic to $\mathbb{R}^3$, but its product with $\mathbb{R}^1$ is homeomorphic to $\mathbb{R}^4$. Thus there is a C^0 dynamical system defined on $\mathbb{R}^4$ not admitting flow boxes. We consider the feasibility of defining generalized flow boxes and their applications.

Smooth chaotic interval maps and indecomposable planar attractors

Jiri Kupka
IRAFM - University of Ostrava, Czech Rep
Jiri.Kupka@osu.cz
J. P. Boronski

We show that for every positive integer k there exists an interval map $f : I \to I$ such that: (1) f is Li-Yorke chaotic, (2) the inverse limit space $\lim_{\leftarrow}\{f, I\}$ does not contain an indecomposable subcontinuum, (3) f is C^k-smooth, and (4) f is not C^{k+1}-smooth. We also show that there exists a C^{∞}-smooth f that satisfies (1)-(2). This answers a recent question of P. Oprocha and J. P. Boronski from [*On indecomposability in chaotic attractors*, preprint 2013], where the result was proved for $k = 0$.

Expansion subshifts of iterative systems

Petr Kurka
Charles University in Prague, Czech Rep
kurka@cts.cuni.cz

An iterative system (F, W) consists of a compact metric space X, a finite alphabet A, homeomorphisms $F = \{F_a : X \to X\}_{a \in A}$, and a regular open almost-cover $W = \{W_a : \ a \in A\}$ of X. This means $\overline{W_a}^{\circ} = W_a$ and $\bigcup_a \overline{W_a} = X$. For $u \in A^*$ set $F_u = F_{u_{n-1}} \circ \cdots F_{u_0}$. A sequence $x_i \in X$ is a trajectory with itinerary $u \in A^N$, if $x_i \in W_{u_i}$ and $x_{i+1} = F_{u_i}(x_i)$. Denote by

$$W_u = W_{u_0} \cap F_{u_0}^{-1} W_{u_1} \cap \cdots \cap F_{u_{[0,n)}}^{-1} W_{u_n}$$

the set of points with itinerary u. The expansion subshift of (F, W) is defined by

$$S_{F,W} = \{u \in A^N : \ \forall n, W_{u_{[0,n)}} \neq \emptyset\}.$$

Proposition. If $F_a : W_a \to X$ are expansions then there exists a continuous surjective function $\Phi : S_{F,W} \to X$ such that

$$\{\Phi(u)\} = \bigcap_{n>0} \overline{W_{u_{[0,n)}}}, \ u \in S_{F,W}.$$

Theorem. $S_{F,W}$ is an SFT of order $m+1$ iff $\forall a \in A$, $\forall u \in A^m$,

$$W_u \cap F_a W_a \neq \emptyset \Rightarrow W_u \subseteq F_a W_a$$

Theorem. $S_{F,W}$ is sofic iff there exists a partition $V = \{V_p : \ p \in B\}$ with
1. $V_p \cap W_a \neq \emptyset \Rightarrow V_p \subseteq W_a$
2. $V_p \subseteq W_a, V_q \cap F_a V_p \neq \emptyset \Rightarrow V_q \subseteq F_a V_p$.

In this case $S_{F,W}$ is the subshift of the labelled graph

$$p \xrightarrow{a} q \Leftrightarrow V_p \subseteq W_a \ \& \ V_q \subseteq F_a V_p$$

A structure of full groups of minimal homeomorphisms of Cantor systems and Hopf-equivalence relation

Jan Kwiatkowski
University of Warmia and Mazury in Olsztyn, Poland
jkwiat@mat.uni.torun.pl

In 1932 Eberhard Hopf introduced a notion of an equivalent relation (called a Hopf-equivalence) for measurable sets with positive measure and for a non-singular and bimeasurable transformation T on a Lebesgue space. He

showed that T has an equivalent finite invariant measure iff there exists no measurable subset of the space which is Hopf-equivalent under T to its proper subset. Topological versions of the Hopf-equivalences were considered in many authors, i.e. B. Weiss, E. Glasner, T. Giordano, I. Putnam, C. Skau, H. Yuasa. Let T be a homeomorphism of a Cantor space X and let $C, D \subset X$ be nonempty clopen sets. We say that C and D are finitely Hopf-equivalent [countable Hopf-equivalent] if there are integers $(n_i, 1 \leq i \leq k)$ $[n_i, i \geq 1]$ and decompositions $C = \bigcup C_i$, $D = \bigcup D_i$ into nonempty pairwise disjoint clopen sets [into nonempty pairwise disjoint closed sets] such that $T^{n_i}(C_i) = D_i,\ i = 1, \ldots, k \quad [i = 1, 2, \ldots]$. The topological Hopf-equivalences completely determine orbit structure of a Cantor minimal system. Let (X, T) and (Y, S) be Cantor minimal systems. Then

- (X, T) and (Y, S) are strong orbit equivalent [orbit equivalent] iff there is a homeomorphism $F\colon X \to Y$ which respects the finite Hopf-equivalence [the countable Hopf-equivalence].

A criterion for two clopen sets are countable equivalent was found by Glasner and Weiss (1995) using the invariant measures and the topological full groups. We give some information on a structure of full groups in Cantor dynamics and also we present another criterion for two nonempty clopen sets C and D to be countable Hopf-equivalent without the use of the invariant measures.

Chain transitivity and variations of the shadowing property

Jonathan Meddaugh
Baylor University, USA
Jonathan_Meddaugh@baylor.edu
Will Brian and Brian Raines

We will discuss a number of variations of the shadowing property in the context of a continuous map f on a compact metric space X. We will also show that if f is chain recursive, then several of these variations, including that of thick shadowing, are equivalent to the usual shadowing property.

No semiconjugacy to a map of constant slope

Michal Misiurewicz
Indiana University-Purdue University Indianapolis, USA
mmisiure@math.iupui.edu
Samuel Roth

We study countably piecewise continuous, piecewise monotone interval maps. We establish a necessary and sufficient criterion for the existence of a nondecreasing semiconjugacy to a map of constant slope in terms of the existence of an eigenvector of an operator acting on a space of measures. Then we give sufficient conditions under which this criterion is not satisfied. Finally, we give examples of maps not semiconjugate to a map of constant slope via a nondecreasing map. Our examples are continuous and transitive.

Some invariant strongly mixing measures for operators

Alfred Peris
Universitat Politecnica de Valencia, Spain
aperis@mat.upv.es
M. Murillo-Arcila

Some strongly mixing invariant measures with full support are constructed for operators on metrizable and complete topological vector spaces which satisfy the Frequent Hypercyclicity Criterion, a sufficient condition for the existence of points x so that the hitting time set N(x,U) has positive lower density for any non-empty open set U. For unilateral backward shifts on sequence spaces, a slight modification shows that one can even obtain exact invariant measures.

Several problems of the modern shadowing theory

Sergei Pilyugin
St.Petersburg State University, Russia
sergeipil47@mail.ru

Problem 1. It is known [1] that for a diffeomorphism, Lipschitz shadowing property is equivalent to structural stability. One can naturally define Lipschitz shadowing for homeomorphisms. Find conditions under which a homeomorphism has the Lipschitz shadowing property.

Problem 2. Does the C^1 interior of the set of vector fields whose flows have the regular shadowing property (see [2]) coincide with the set of structurally stable vector fields? Note that a similar statement (with regular shadowing replaced by oriented shadowing) is false [2].

Problem 3. How are related the sets of dynamical systems having the shadowing property and the inverse shadowing property (see [3])?

Problem 4. For a two-dimensional Axiom A diffeomorphism, both the shadowing property and the inverse shadowing property are equivalent to the so-called C^0 transversality condition [4]. There exist three-dimensional Axiom A diffeomorphisms having the shadowing property that do not satisfy the C^0 transversality condition [5]. How to characterize multidimensional Axiom A diffeomorphisms having the shadowing property?

References

[1] S.Yu.Pilyugin and S.B.Tikhomirov: Lipschitz shadowing implies structural stability, Nonlinearity, vol. 23, 2509-2515 (2010).

[2] S.Yu.Pilyugin and S.B.Tikhomirov: Vector fields with the oriented shadowing property, J. Diff. Equat., vol. 248, 1345-1375 (2010).

[3] S.Yu.Pilyugin: Spaces of Dynamical Systems, Walter de Gruyter, Berlin/Boston (2012).

[4] S.Yu.Pilyugin and K.Sakai: C^0 transversality and shadowing properties, Proc. Steklov Math. Inst., vol. 256, 290-305 (2007).

[5] A.A.Petrov: Shadowing in the case of nontransverse intersection, Algebra Analiz (submitted).

Representation of Markov chains by random maps: existence and regularity conditions

Christian Rodrigues
Max Planck Institute for Mathematics in the Sciences, Germany
christian.rodrigues@mis.mpg.de
Juergen Jost and Martin Kell

Amongst the main concerns of Dynamics one wants to decide whether asymptotic states are robust under random perturbations. Considering the iteration of $f : M \to M$, such randomness are represented by a family $\{p_\varepsilon(\cdot|x)\}$ of Borel probability measures, such that every $p_\varepsilon(\cdot|x)$ is supported inside the ε-neighbourhood of $f(x)$. Alternatively, the orbit is given by the iteration $x_j = g_j \circ \cdots \circ g_1(x_0)$, where each measurable g_j is picked at random ε-close from the original map f. Endowing the collection of maps $\{g_j\}$ with a probability distribution ν_ε, we say that the sequence of random maps is a representation of that Markov chain if for every Borel subset U, $p_\varepsilon(U|x) = \nu_\varepsilon(\{g : g(x) \in U\})$. In this talk we systematically investigate the problem of representing Markov chains by families of random maps, and which regularity of these maps can be achieved depending on the properties of the probability measures and on the manifold. Our key idea is to use techniques from optimal transport to select optimal such maps. From this scheme, we not only deduce the representation by measurable and continuous random maps, but also obtain conditions to construct random diffeomorphisms from a given Markov chain.

The Many Facets of Chaos

Evelyn Sander
George Mason University, USA
esander@gmu.edu
James A. Yorke

There are many different definitions of chaos, and a lot of controversy as to which is the correct definition. The goal of this talk is to suggest that there is no one correct definition of chaos; although all of these definitions are different aspects of the same concept, taking one definition as canonical results in a loss of some facets of chaotic behavior.

Topological ergodicity of cylinder transformations and discrete orbits

Artur Siemaszko
University of Warmia and Mazury in Olsztyn, Poland
artur@uwm.edu.pl
Jan Kwiatkowski

Let X be a compact metric space and $T : X \longrightarrow X$ be a homeomorphism of X. Let $\varphi : X \longrightarrow \mathbb{R}$ be a continuous function (called *a cocycle*). By a *cylinder transformation* we mean a homeomorphism $T_\varphi : X \times \mathbb{R} \longrightarrow X \times \mathbb{R}$ (or rather a $\mathbb{Z}$–action generated by it) given by the formula

$$T\varphi(x, r) = (Tx, \varphi(x) + r).$$

Such a transformation cannot be itself minimal (Besicovitch 1951, Le Calvez and Yoccoz 1997). If a base transformation is a rotation on a compact metric group then T_φ is topologically ergodic iff φ has zero mean with respect to the Haar measure and is not a coboundary. Frączek and Lemańczyk (2010) asked whether there exist topologically ergodic transformations having either dense or discrete orbits. We are able to construct such an example with (X, T) being an odometer.

Sets of probability distribution functions generated by distributionally chaotic maps

Jaroslav Smital
Silesian University, Czech Rep
Jaroslav.Smital@math.slu.cz
Francisco Balibrea, Marta Stefankova

For a topological dynamical system (X, f) we consider the structure of the set $\mathcal{F}(f)$ of probability distribution functions of the distances between two trajectories. If f has the weak specification property then $\mathcal{F}(f)$ is closed and convex with a unique minimal element, and it can contain all nondecreasing functions $[0, \mathrm{diam}X] \to [0.1]$. Based on this we propose a new notion of chaos. Note that these problems cannot be easily studied by standard tools like the ergodic theorem.

Transitivity without (relative) specification in dendrites

Vladimir Spitalsky
Matej Bel University, Banska Bystrica, Slovak Rep
vspitalsky@gmail.com

By a result of Blokh from 1984, every transitive map $f : X \to X$ of a tree X has the relative specification property. That is, there is a regular periodic decomposition $(D_0, \ldots, D_{m-1})$ of X such that the restrictions $f^m|_{D_i} : D_i \to D_i$ have the specification property. This is not true for transitive dendrite maps, as was shown by Hoehn and Mouron. They constructed a weakly mixing dendrite map which is not mixing. Recently, we gave another example showing that Blokh's theorem cannot be extended to dendrites; we constructed a transitive map of a dendrite with infinite decomposition ideal. In the talk we review these results and we consider some of the related questions.

Rotation Sets for Maps of Tori

Sonja Stimac
University of Zagreb, Croatia
sonja@math.hr
Tobias Oertel-Jäger, Alejandro Passeggi

In this talk I will recall the notion of the rotation number of an orientation preserving homeomorphism of a circle and generalizations to many-dimensional cases. Then I will focus on the rotation sets of homeomorphisms of the two-dimensional torus onto itself isotopic to the identity and discus some properties and open problems. Finally I will show constructions of certain minimal sets of some of these homeomorphisms which rotation sets have very interesting properties.

Complexity estimates of Orthogonal Matching Pursuit under RIP conditions

Grzegorz Swirszcz
IBM T.J. Watson Research, USA
swirszcz@us.ibm.com
Aurellie Lozano, Tomasz Nowicki

The study of complexity of algorithms can be very difficult and pose significant scientific challenge. We present a study of complexity of the Orthogonal Matching Pursuit algorithm under Restricted Isometry Property conditions using the estimates of convergence rate of iterations of a family of functions. Restricted Isometry Property was introduced by Terrence Tao et. al. and is considered a very important topic in compressed sensing and Machine Learning. Orthogonal Matching Pursuit is a classical feature-selection algorithm in Machine Learning.

Chaotic behavior of non-autonomous systems with randomly perturbated trajectories

Leszek Szala
Silesian University in Opava, Czech Rep
leszek.szala@math.slu.cz

We study non-autonomous discrete dynamical systems generated by a sequence of continuous functions, which converges uniformly, with randomly perturbed trajectories . We give conditions, under which a recurrent point of a discrete dynamical system is also recurrent for a non-autonomous discrete dynamical system with randomly perturbed trajectories. We also present a necessary condition for a non-autonomous discrete dynamical system to be non-chaotic with respect to small random perturbations.

Special Session 8: Emergence and Dynamics of Patterns in Nonlinear Partial Differential Equations from Mathematical Science

Danielle Hilhorst, University of Paris-Sud, Orsay, France
Yoshihisa Morita, Ryukoku University, Japan

The solution structures of many nonlinear partial differential equations reveal the emergence and the evolution of very exciting patterns. Such nonlinear models come from various fields of mathematical science, including material science as well as life sciences. In this session, we will bring together recent studies on solutions of nonlinear partial differential equations related to pattern formation, dynamics, and bifurcations, presenting new aspects of solutions capturing nonlinear phenomena together with underlying solution structures.

Climate shift's effect on a species submitted to dispersion, evolution, growth and nonlocal competition

Matthieu Alfaro
Univ. Montpellier 2, France
matthieu.alfaro@univ-montp2.fr
Henri Berestycki and Gael Raoul

We consider a population structured by a space variable and a phenotypic trait, submitted to dispersion, mutations, growth and nonlocal competition. We introduce the climate shift due to Global Warming and discuss the future of the population by studying the long time behavior of the solution of the Cauchy problem. We consider different forms of growth function and determine threshold climate speeds for extinction, confined survival or invasion.

Reaction-diffusion equations from Biology with integral terms

Narcisa Apreutesei
"Gheorghe Asachi" Technical University of Iasi, Romania
napreut@gmail.com

Some models of integro-differential equations from Biology will be analyzed. Here the integral term describes the nonlocal consumption of resources. Fredholm property of the corresponding linear operators are useful to prove the existence of travelling wave solutions. For some models, this can be done only when the support of the integral is sufficiently small. In this case, the integro-differential operator is close to the differential one. One uses a perturbation method which combines the Fredholm property of the linearized operators and the implicit function theorem. For some other models, Leray-Schauder method can be applied. This implies the construction of a topological degree for the corresponding operators and the establishment of a priori estimates for the solution.

A gradient flow interpretation of the Keller-Segel systems

Adrien Blanchet
Toulouse School of Economics, France
adrien.blanchet@ut-capitole.fr

This talk is dedicated to recent results on the Keller-Segel model in 2d, and on its variants in higher dimensions where the diffusion is of critical porous medium type. These models have a critical mass M_c such that the solutions exist globally in time if the mass is less than M_c and above which there are solutions which blowup in finite time. The main tools, in particular the free energy and the Jordan-Kinderlherer-Otto's minimising scheme in the Monge-Kantorovich metric, and the idea of the methods are set out.

Standing Pulse Solutions to FitzHugh-Nagumo Equations

Chao-Nien Chen
National Changhua University of Education, Taiwan
macnchen@cc.ncue.edu.tw

We are concerned with standing pulse solutions for the FitzHugh-Nagumo equations. Since the reaction terms are coupled in a skew-gradient structure, a standing pulse solution is a homoclinic orbit of a second order Hamiltonian system. We use variational approach to study the existence question. If time permits, related stability results will be discussed.

Fast-reaction limits for some reaction-diffusion systems on unbounded domains

Elaine Crooks
Swansea University, Wales
e.c.m.crooks@swansea.ac.uk
Danielle Hilhorst

This talk is concerned with a unified approach to fast-reaction limits of systems of either two reaction-diffusion equations, or one reaction-diffusion equation and one ODE, on unbounded domains. The equations considered have the form

$$(P^k)\begin{cases} u_t &= d_u u_{xx} - kF(u,v), \\ v_t &= d_v v_{xx} - kF(u,v), \\ u(x,0) &= u_0^k(x), \\ v(x,0) &= v_0^k(x), \end{cases}$$

where $x \in D = \mathbb{R}$ or $[0,\infty)$, $t \in (0,T)$, the initial data $u_0^k(x)$ and $v_0^k(x)$ are non-negative and bounded, and in the case when $D = [0,\infty)$, $u(0,t) = U_0$ and $v_x(0,t) = 0$. Such systems can arise, for example, in modelling fast chemical reactions where there could be either two mobile reactants ($d_u > 0$ and $d_v > 0$) or a mixture of mobile and immobile reactants ($d_u > 0$ and $d_v = 0$). The function F is non-negative and increasing in both components, the positive parameter k controls the rate of interaction, and interest is in non-negative solutions because u and v typically correspond to concentrations. Under appropriate conditions on the initial data u_0^k and v_0^k, we show that solutions (u^k, v^k) of (P^k) converge in the fast-reaction limit as $k \to \infty$ to a self-similar limit that has one of four forms, depending on whether $d_v > 0$ or $d_v = 0$ and on whether $D = \mathbb{R}$ or $D = [0,\infty)$.

Pulse dynamics in FitzHugh-Nagumo systems on heterogeneous media

Shin-Ichiro Ei
Hokkaido University, Japan
Eichiro@math.sci.hokudai.ac.jp

We consider the motions of front or pulse solutions for Fitzh-Hugh Nagumo equations on one-dimensional heterogeneous media. We derive equations describing the motions of single front or single pulse solution depending on the heterogeneity and also consider the interaction of multi-front solutions. Through the analysis, we show opposite influences for a forward and a backward front solution from the heterogeneity in FitzHugh-Nagumo equations. Some part of this research is a joint work with Chao-Nien Chen and Shyuh-yaur Tzeng.

Bistable and monostable transition fronts

Francois Hamel
Aix Marseille University, France
francois.hamel@univ-amu.fr

The standard notions of reaction-diffusion waves and fronts can be viewed as examples of generalized transition waves. These notions involve uniform limits, with respect to the geodesic distance, to a family of hypersurfaces which are parametrized by time. The existence of transition waves has been proved in various contexts where the standard notions of waves make no longer sense. Even for homogeneous equations, fronts with various non-planar shapes or with varying speeds are known to exist. In this talk, I will report on some recent existence results and qualitative properties of transition fronts for monostable and bistable homogeneous and heterogeneous equations. I will also discuss their mean speed of propagation.

Spatio-temporal patterns in a chemotaxis-growth system

Hirofumi Izuhara
University of Miyazaki, Japan
izuhara@cc.miyazaki-u.ac.jp
Shin-Ichiro Ei, Masayasu Mimura

A chemotaxis system with a growth term is discussed from the spatio-temporal-oscillation point of view. This system exhibits two different types of spatio-temporal oscillations in certain distinct parameter regimes. In this talk, we report the difference between the two types of spatio-temporal oscillations. In particular, the characteristic properties of the behaviors become clear in a limiting system when a certain parameter value tends to zero. Moreover, we reveal that the onset of one of the spatio-temporal oscillatory patterns is an infinite-dimensional relaxation oscillation that consists of slow and fast dynamics.

Dynamical spike patterns in reaction-diffusion-ode models with Turing instability

Anna Marciniak-Czochra
University of Heidelberg, Germany
anna.marciniak@iwr.uni-heidelberg.de
Grzegorz Karch and Kanako Suzuki

In this talk we explore a mechanism of pattern formation arising in processes described by a system of a single reaction-diffusion equation coupled to ordinary differential equations. Such systems of equations arise, for example, in modeling of interactions between cellular processes and diffusing growth factors. Our theory applies to a wide class of pattern formation models with an autocatalytic non-diffusing component. We show that the lack of diffusion in some model components may lead to singularities which result in instability of all regular stationary patterns. Interestingly, the degeneration of the system yields a continuous spectrum of the linearization operator, which contains positive values. We show that, under some conditions, also all discontinuous stationary solutions are unstable. However, in numerical simulations, solutions having the form of periodic or irregular spikes are observed. We explain this phenomenon using a shadow-type reduction of the reaction-diffusion-ode model. For the resulting system of integro-differential equations, we prove convergence of the model solutions to singular unbounded spike patterns, which location depends on the initial condition.

Propagating terrace for semilinear diffusion equations in higher dimensions

Hiroshi Matano
University of Tokyo, Japan
matano@ms.u-tokyo.ac.jp
Yihong Du

Propagating terrace for semilinear diffusion equations in higher dimensions In this talk we will consider semilinear diffusion equations of the form

$$u_t = \Delta u + f(u) \quad \text{on } \mathbf{R}^N,$$

where f is a multi-stable nonlinearity satisfying $f(0) = 0$ and discuss the long-time behavior of solutions with compactly supported non-negative intial data. For this type of equations, under some mild non-degeneracy assumptions, it was proved recently by P. Polacik and Y. Du that the solution converges to a stationary solution as $t \to \infty$. What we are interested in is the transition process, that is, the behavior of the solutions in the intermediate time range. More precisely, we will show that every such solution behaves asymptotically like what we call a "radially symmetric propagating terrace", which roughly means a multiple layer of radially symmetric spreading fronts that expand at different speeds.

Gradient-like property and spectral comparison in a mass-conserved reaction-diffusion system

Yoshihisa Morita
Ryukoku University, Japan
morita@rins.ryukoku.ac.jp

We are concerned with a two-component reaction-diffusion system with conservation of mass, which is proposed as a simple cell polarity model. We first provide a Lyapunov function of the system. Then we reduce the system of stationary problem to a scalar equation with a nonlocal term and bring an energy functional whose critical points in a function space are given by solutions to the reduced scalar equation. We further show that the dimension of the unstable manifold of each equilibrium solution to the system coincides with the Morse index of the corresponding critical point of the energy functional. The proof for the last part is done by applying the spectral comparison argument to the linearized eigenvalue problems. This talk is based on a joint work with S. Jimbo (2013) and a recent joint work with E. Latos and T. Suzuki.

Mathematical models of cell-cell adhesion: diffusion vs. advection

Hideki Murakawa
Kyushu University, Japan
murakawa@math.kyushu-u.ac.jp

Cell adhesion is the binding of a cell to another cell or to an extracellular matrix component. This process is essential in organ formation during embryonic development and in maintaining multicellular structure. Armstrong, Painter and Sherratt [J. Theor. Biol. 243 (2006), pp.98–113] proposed a nonlocal advection-diffusion system as a possible continuous mathematical model for cell-cell adhesion. Although the system is attractive and challenging, it gives biologically unrealistic numerical solutions. We identify and remedy the problems, and provide a new continuous model for cell-cell adhesion. Our model replicates some phenomena which can not be captured at all by Armstrong-Painter-Sherratt model.

Threshold Phenomena for Symmetric Decreasing Solutions of Reaction-Diffusion Equations

Cyrill Muratov
NJIT, USA
muratov@njit.edu
X. Zhong

We study the long time behavior of solutions of the Cauchy problem for nonlinear reaction-diffusion equations in one space dimension with the nonlinearity of bistable, ignition or monostable type. We prove a one-to-one relation between the long time behavior of the solution and the limit value of its energy for symmetric decreasing initial data in L^2 under minimal assumptions on the nonlinearities. The obtained relation allows to establish sharp threshold results between propagation and extinction for monotone families of initial data in the considered general setting.

The deep quench obstacle problem

Amy Novick-Cohen
Technion IIT, Israel
amync@tx.technion.ac.il
L. Banas, R. Nurnberg

The deep quench obstacle problem can exhibit phenomena such as spinodal decomposition and coarsening, much like the Cahn-Hilliard equation. Using a number of benchmarks, we try to understand the details of the transition from the linear spinodal decomposition regime to the late time coarsening regime.

Bifurcation analysis of periodic traveling wave solutions to an excitable RD system

Toshiyuki Ogawa
Meiji University, Japan
togw@meiji.ac.jp
M Osman Gani

We introduce an excitable RD system to imitate cardiac cell activities and observe the stabilities of periodic traveling wave solutions. There are two families of wave trains, fast and slow. The fast family is basically stable in the case of FitzHugh-Nagumo system which is one of the typical excitable systems. However, we can observe that the fast wave train becomes unstable in our model and as a result bifurcates to an oscillatory wave. We shall explain this phenomena by calculating the essential spectrum numerically.

Convergence results in order-preserving systems and its applications to reaction-diffusion systems

Toshiko Ogiwara
Josai University, Japan
toshiko@josai.ac.jp

In this talk I shall investigate reaction-diffusion systems that satisfy the comparison principle and possess a mass conservation property. Motivated from mathematical analysis of transport models by molecular motors and chemical reversible reaction models, recently we have obtained some fundamental results on the structure of stationary and time-periodic solutions in a rather general framework of order-preserving dynamical systems. More precisely, our general results state that: (1) if there exists at least one fixed point, then there exist infinitely many of them, and the set of fixed points (which correspond to stationary or time-periodic solutions of the model equations) is totally ordered, connected and unbounded; (2) any bounded orbit converges to some element of this continua of fixed points as time tends to infinity. In particular, our general results imply that if the model equation possesses a trivial stationary or time-periodic solution (such as zero), then there are automatically infinitely many nontrivial stationary or time-periodic solutions. In the present talk I shall present some new applications of our general theory to other problems of reaction-diffusion systems. This is joint work with Danielle Hilhorst and Hiroshi Matano.

Hopf Bifurcation in a Gene Regulatory Network Model: Molecular Movement Causes Oscillations

Mariya Ptashnyk
University of Dundee, Scotland
mptashnyk@maths.dundee.ac.uk
Mark Chaplain, Marc Sturrock

Gene regulatory networks lie at the heart of many important intracellular signal transduction processes. In this paper we analyse a mathematical model of a canonical gene regulatory network consisting of a single negative feedback loop between a protein and its mRNA. The model consists of two partial differential equations describing the spatio-temporal interactions between the protein and its mRNA. Such intracellular negative feedback systems are known to exhibit oscillatory behaviour. Our results show that the diffusion coefficient of the protein/mRNA acts as a bifurcation parameter and gives rise to a Hopf bifurcation. This shows that the spatial movement of the mRNA and protein molecules alone is sufficient to cause the oscillations. Applying linearized stability analysis we study the stability of a spatially inhomogeneous steady state of the model and prove the existence of two Hopf bifurcation point. The local stability of periodic solutions, bifurcating from the steady state, is shown using a weakly nonlinear analysis and normal form theory.

Two-scale homogenization of reaction-diffusion systems with small diffusion

Sina Reichelt
WIAS Berlin, Germany
reichelt@wias-berlin.de
Marita Thomas and Alexander Mielke

Many reaction-diffusion processes arising in civil engineering, biology, or chemistry take place in media with underlying microstructure, for instance concrete carbonation or the spread-out of substances in biological tissues. Such microstructures are assumed to be periodically distributed with period length $\varepsilon > 0$, which is much smaller than the overall size of the domain. It is the aim to derive effective equations, independent of ε, which are ideally simpler and qualitatively describe the properties of the original system. In my talk I deal with a system of two coupled reaction-diffusion equations, where one species diffuses much slower than the other one and the coupling arises via nonlinear reaction terms. Using the method of two-scale convergence, I derive effective equations which are defined on a two-scale space. The two-scale space consists of the macroscopic domain and the microscopic unit cell attached to each point of the macroscopic domain.

Sudden directional diffusion: counting and watching facets

Piotr Rybka
The University of Warsaw, Poland
rybka@mimuw.edu.pl
T.Asai, M.Matusik, P.B.Mucha

We study a parabolic problem of the form

$$u_t = (\mathcal{L}(u_x))_x \quad \text{in } (0,b)\times(0,T).$$

The characteristic trait of function $\mathcal{L}$, which we consider is that it is increasing and it has jump(s). A jump of $\mathcal{L}$ at p_0 leads to creation of facets with slope p_0.

We concentrate on two instances of $\mathcal{L}$:

1. $\mathcal{L}(p) = \operatorname{sgn}(p+1) + \operatorname{sgn}(p-1)$;
2. $\mathcal{L}(p) = \operatorname{sgn} p + \epsilon p$.

The first case appears to admit infinite oscillations. We address this issue, showing that we have always only a finite number of facets with non-zero curvature. Moreover, with the help of the comparison principle for viscosity solutions we estimate the extinction time.

The second case is interesting because it is an instant of two types of competing diffusion. Counting facets becomes a more subtle job. We also show that their number is decreasing.

Diffusion Equation under dynamical boundary conditions

Kunimochi Sakamoto
Hiroshima University, Japan
kuni@math.sci.hiroshima-u.ac.jp

We consider a system of diffusion equations under nonlinear dynamic boundary conditions and investigate the Turing type destabilization of stable uniform states. We found that stable uniform states destabilize even when the diffusion rates are equal.

Travelling fronts and patterns for kinetic equations with delay

Hartmut Schwetlick
University of Bath, England
h.schwetlick@bath.ac.uk
J Mueller

We study a model for the morphology and growth of the fungus Phytophthora, a plant pathogen with a highly damaging impact on forestry economy with features related to the Ash die back disease. The model is based on a kinetic equation for the spread of the density of hyphen tips, whereas the branching of tips induces a delayed nonlinear growth term. We present how to prove the existence of travelling front solutions for this model and study the effect of the delay on the speed of these fronts. We also investigate the question if the nonlinear term is able to impose non-monotone wave profiles. This is joint work with J Mueller from TU Munich

Mathematical Understanding on Nuclear Pattern of Eukaryotic Cells by Phase-Field Method

Sungrim Seirin Lee
Hiroshima University, Japan
seirin.lee@gmail.com
Ryo Kobayashi

The nuclear architecture plays an important role in organizing a function of nucleus that is composed of two chromatin domains called heterochromatin and euchromatin. Heterochromatin is more condensed chromatin in the lower level of transcript state and typically distributed at nuclear envelope and around the nucleoli. In contrast, euchromatin is less condensed in active transcriptional state and distributed in the nuclear interior between the perinuclear and heterochromatin domains. The nuclear architecture described above is called Conventional Architecture and which is nearly universal in the vast majority of eukaryotic cells. On the other hand, there is very different nuclear architecture called Inverted Architecture, in which heterochromatin is gathered in the center of nuclear and euchromatin is enriched at nuclear envelope. Interestingly, rod photoreceptor cell of mouse has inverted architecture induced by very dynamical remodeling of conventional architecture during terminal differentiation. In this presentation, we regenerate the remodeling dynamics of chromatin architectures by novel approach of phase field method. We show how nuclear structures can change dynamically and create long-range spatial patterns without complex details of genetic information.

On a multiscale model involving cell contractivity and its effects on tumor invasion

Christian Stinner
University of Kaiserslautern, Germany
stinner@mathematik.uni-kl.de
Gülnihal Meral, Christina Surulescu, Michael Winkler

Invasion of tumor cells is an important step for metastasis and is governed by several subcellular processes. A number of them affect the contractivity, by which we describe the ability of the cancer cells to adapt their shape and orientation according to the surrounding tissue. We derive a multiscale model focusing on the influence of the cell contractivity on tumor cell migration. It takes into account both the subcellular level, where changes of contractivity are initiated, and the macroscopic level of the cell population. We provide the local existence of a unique solution in a general framework, prove the global existence of solutions for a slightly more specific setting and present numerical simulations to illustrate the effect of contractivity on the migration of cancer cells in our model.

Multiscale modeling of glioma spread

Christina Surulescu
University of Kaiserslautern, Germany
surulescu@mathematik.uni-kl.de
Christian Engwer, Markus Knappitsch, Thomas Hillen

Tumor cell invasion is an essential stage in the development of cancer. Tumor cells migrate through the surrounding tissue (normal cells, extracellular matrix, interstitial fluid) towards blood or lymph vessels which they penetrate and thus access the blood flow. They are carried by blood circulation to distant locations where they extravasate and develop new tumours, a phenomenon known as metastasis. The invasive spread of cancer cells is highly complex, as it is influenced by various dynamics ranging from the subcellular level (microscale) through the mesoscopic level of individual cells and up to the macroscale of a cell population, the latter involving processes like diffusion, chemotaxis or haptotaxis, separately or in a conjugate way. The mathematical modeling of these features leads to multiscale settings interconnecting two or all three of these scales and allowing to assess the effects of subcellular events on the behavior of an entire cell population. We present a class of multiscale models characterizing glioma invasion in white brain matter. Gliomas are rarely curable brain tumors arising from abnormal glial cells in the brain. In particular, the most agressive type, glioblastoma multiforme, has a poor prognosis with a median survival rate less than one year. Complex therapy approaches including surgical resection of neoplastic tissue, radio- and chemotherapy can still not ensure a complete healing of the patient and are part of ongoing research. Currently, Diffusion Tensor Imaging (DTI) is the preferred radiological method in glioma prognosis, which also allows to infer the white matter fibre structure of the brain in a noninvasive way. Kinetic transport equations provide an appropriate framework to include such patient specific DTI data into a micro-meso model for glioma growth.

Comparison method for Parabolic-Elliptic systems with chemotactic terms

J. Ignacio Tello
Universidad politecnica de Madrid, Spain
jtello@eui.upm.es

We will present results on a general system of reaction-diffusion equations and introduce a comparison method to obtain qualitative properties of its solutions. The comparison method is applied to study the stability of homogeneous steady states and the asymptotic behavior of the solutions of different systems with chemotactic terms.

Thermomechanical modeling of dissipative processes in elastic media via energy and entropy

Marita Thomas
WIAS Berlin, Germany
Marita.Thomas@wias-berlin.de
Alexander Mielke

This contribution deals with the framework of metriplectic systems introduced by P.J. Morrison and further developed under the acronym GENERIC (General Equation for Non-Equilibrium Reversible-Irreversible Coupling) by H.C. Oettinger and M. Grmela for fluid mechanics. Recently, this concept has been transferred to solid mechanics by A. Mielke and co-workers. This framework allows for the derivation of thermodynamically consistent models, which may couple both reversible and irreversible, dissipative effects. The GENERIC approach to thermomechanics is based on functionals and geometric structures rather than PDEs, which are in fact obtained as a result of this modeling ansatz. The GENERIC structure highlights the role of the energy as the driving potential for reversible processes and the role of the entropy as the driving potential for irreversible processes, and thus helps to characterize physical effects. In this talk, GENERIC is used to model the interaction of reaction-diffusion processes, temperature changes, and mechanical stresses in solids.

On the effects of the piwheel network symmetries on orientation tuning.

Romain Veltz
INRIA Sophia Antipolis, France
romain.veltz@inria.fr
Pascal Chossat, Olivier Faugeras

The goal of this work is to study how long-range connections in the primary visual cortex of mammals can influence local cortical activity as seen in optical imaging experiments. It is known that the perception of contours in the primary visual cortex is locally organized around "pinwheels" and possess an approximate Euclidean invariance when only local connections are considered. Moreover recent experimental evidences support that the local circuitry operates at the edge of an instability where the network shows self-sustained stationary/oscillatory activity. Assuming (to simplify the analysis) that the pinwheels are organized in a square lattice, we consider the effects of long-range (non local) connections as modeled in Bressloff:03. It produces a forced symmetry-breaking in the equations which, as we have shown, can lead to oscillatory and/or intermittent dynamics around the static square pattern produced by the local connections. This dynamics is closely related to the occurrence of heteroclinic cycles for the system. The tools are equivariant dynamical systems theory and degree theory. Numerical experiments support the theoretical analysis.

Special Session 9: Dissipative Systems and Applications

Georg Hetzer, Auburn University, USA
Wenxian Shen, Auburn University, USA
Jose Ignacio Tello, Universidad Politecnica de Madrid, Spain

Dissipative systems arise in many applications. The special session will feature a broad spectrum of research from infinite dynamical systems theory and random dynamical systems to evolutionary partial differential equations and numerical simulation. The scope of applications reaches from reaction-diffusion systems with local and nonlocal dispersal, from ecology to climate modeling.

Unsteady Non-Isothermal problem with Friction Law.

Mahdi Boukrouche
Institut Camille Jordan umr-5208 ST-Etienne University, France
Mahdi.Boukrouche@univ-st-etienne.fr
Imane Boussetouan , Laetitia Paoli

We consider an unsteady non-isothermal fluid flow subjected to non-homogeneous Dirichlet conditions on a part of the boundary and the Tresca friction law on the other part. For this problem an existence result has been proved recently in Boukrouche et al. (2014) but uniqueness has been left as an open question. Starting from the approximation of the problem based on a regularization of the free boundary condition due to friction combined with a special penalty method, we establish some sharp a priori estimates leading to better regularity properties for the velocity field and to the uniqueness of the solutions. Finally we study the regularity of the pressure field and of the stress tensor.

Evolution of Dispersal in Patchy Environments: Continuous and Discrete Time Models

Robert Stephen Cantrell
University of Miami, USA
rsc@math.miami.edu
Chris Cosner, Yuan Lou and Sebastian Schreiber

In previous work, Chris Cosner, Yuan Lou and I (and others) have considered the question of when dispersal strategies that can be viewed as leading to an ideal free distribution at equilibrium are evolutionarily advantageous among some class of strategies. We have done so in numerous mathematical frameworks including reaction-diffusion, integro-difference and for this discussion most particularly, discrete diffusion of an arbitrary number of competing species in an arbitrary number of patches. A common feature in all these models is that dispersal and growth appear in separate terms. Of course, not all patch models (in discrete or continuous time) have such forms. In many cases dispersal and growth are intermingled. Kirkland, Schreiber and Li considered the evolution of conditional and unconditional dispersers for a general such class of multi-patch difference equations. In this work, we build on the work of Kirkland, Schreiber and Li in the discrete time multi-patch case utilizing the approach of Cantrell, Cosner and Lou in discrete-diffusion case and then address some continuous time multi-patch models in which dispersal and growth are similarly intermingled, namely the Mouquet-Loreau metapopulation model and a model with density dependent growth and dispersal studied by Schoener . The discrete results cover a generalization of Kirkland, Schreiber and Li but for this talk I will focus on the model from Kirkland, Schreiber and Li and on the Mouquet-Loreau model. This work is in collaboration with Chris Cosner, Yuan Lou and Sebastian Schreiber.

Asymptotic behaviour of dissipative lattice dynamical systems with delays

Tomas Caraballo
Universidad de Sevilla, Spain
caraball@us.es
F. Morillas and J. Valero

The aim of this talk is to analyze a class of lattice differential equations containing some hereditary characteristics (delays). First we establish some results ensuring existence of solutions, and uniqueness when we impose stronger assumptions. Next we prove the existence of an attractor for the dynamical system generated by the model. We will use techniques from set-valued or multivalued dynamical systems since we will only impose rather general hypotheses which do not guarantee, in principle, the uniqueness of solutions. Finally, some illustrative applications will be shown.

Is it possible to maintain the global existence after the blowing up of a solution of a reaction-diffusion equation?

Alfonso Casal
Universidad Politecnica de Madrid, Spain
alfonso.casal@upm.es
Jesus Ildefonso Diaz, Jose Manuel Vegas

We consider blowing-up solutions $y^0(t)$, $t \in [0, T_{y^0})$, of some problems of ODEs, $\frac{dy}{dt}(t) = f(y(t)), y(0) = y_0$, where $f : \mathbb{R}^d \to \mathbb{R}^d$ is a locally Lipschitz function and $d \geq 1$. The controllability question we analyze in this work is the following: Given $\epsilon > 0$, can we find a continuous deformation of the trajectory $y^0(t)$, built as solution of the control perturbed problem obtained by replacing $f(y(t))$ by $f(y(t)) + u(t)$, for a suitable control $u \in L^1_{loc}(0, +\infty : \mathbb{R}^d)$ such that $y(t) = y^0(t)$ for any $t \in [0, T_{y^0} - \epsilon]$ and such that $y(t)$ is continued to the whole $[0, +\infty)$? We shall see that the answer is positive, by improving some previous work by the authors. The key ingredient is the following powerful nonlinear variation of constants formula, extending previous results [Alekseev (1961), as in Laksmikantham and Leela (1969), for nonlinear terms of class C^2] to more general ones: let f be a globally Lipschitz function and let $\beta(t, y)$ a family of maximal monotone operators on the space $H = \mathbb{R}^d$, with $\beta(t, .) \in L^1_{loc}(0, +\infty : \mathbb{R}^d)$, such that the solutions of the perturbed problem, $P(f, \beta)$, $\frac{dy}{dt}(t) \in f(y(t)) +$

$\beta(t,y(t)), y(t_0) = \xi$, are well defined as absolutely continuous functions on $[0,T]$, for a given $T > 0$. Let $y = \phi(t,t_0,\xi)$ be the unique solution of the ODE in $\mathbb{R}^d$, $y' = f(y(t)), y(t_0) = \xi$, and let $\Phi(t,t_0,\xi) = \partial_\xi \phi(t,t_0,\xi)$, where ∂_ξ denotes partial differentiation. We shall show then that ϕ is Lipschitz continuous (a generalization of the differentiability Peano's theorem), that Φ is absolutely continuous and that the solution $y(t)$ of the perturbed problem $P(f,\beta)$, for any $t \in [0,T]$, has the integral representation $y(t) = y^0(t) + \int_{t_0}^t \Phi(t,s,y(s))\beta(s,y(s))ds$, where $y^0(t) = \phi(t,t_0,y_0)$ is the unperturbed solution. A suitable similar expresion can be obtained if $\beta(t,.)$ is multivalued (as for some variational inequalities). We will give applications to some nonlinear blowing-up parabolic problems.

Some mathematical problems on the Doi-Edwards equation for polymer melts

Sorin I. Ciuperca
University of Lyon, France
ciuperca@math.univ-lyon1.fr
A. Heibig and L.I. Palade

We consider the so called Doi-Edwards equation which gives the density distribution of the molecules of melted polymers. We first give results of existence, uniqueness and positivity of the solution of the evolution problem, as well as the existence of at least a stationnary solution. Secondly we consider a coupled problem between the Doi-Edwards equation and a Navier-Stokes type equation in the particular case of a "shear-flow" fluid. We show that this problem falls into the category of K-BKZ models for visco-elastic fluids and we give a global in time existence result.

Parabolic Monge-Ampere equations giving rise to a free boundary: the worn stone model

Gregorio Diaz
Universidad Complutense de Madrid, Spain
gdiaz@ucm.es
J.I. Diaz

The idealized wearing process for a convex stone, isotropic with respect to wear, can be described by an abstract Cauchy problem related to some power p of the Gauss curvature flow. We prove the existence and uniqueness of a mild solution by solving implicit Euler schemes. A free boundary can be generated due to the degeneracy of the nonlinear equation once we assume that the initial datum u_0 has some flat region. A suitable balance between the dimension N and the power p is also required. Other contributions will be related to the shape of such worn stones. We prove that if $Np \geq 1$ then the initial flat region persists for small times under some conditions on u_0 (finite waiting time). By means of self–similar solutions we show also that any flat region must disappear after a time large. Concerning the asymptotic behavior for large t, we prove that if a flat obstacle does not coincide with u_0 in an open set the same occurs in any $t > 0$ (flattened retention).

Dissipative impulsive semidynamical systems

Jaqueline Ferreira
Universidade de Sao Paulo, Brazil
jaque1988@gmail.com
Everaldo de Mello Bonotto

The study of dissipative systems occurs in several applications due its natural dissipation. This theory has been studied in various directions. For example, Cheban dedicated his book [3] to the study of abstract nonautonomous dissipative dynamical systems and their applications to differential equations. In many natural phenomena, the real deterministic models are often described by systems which involve impulses. Thus we apply the theory of impulsive semidynamical systems to study topological properties of discontinuous dissipative systems as presented in [1] and [2].

References

[1] *E. M. Bonotto and D. P. Demuner*: Non-autonomous dissipative semidymanical systems with impulses.

[2] *E. M. Bonotto and D. P. Demuner*: Attractors of impulsive dissipative semidynamical systems. Bulletin des Sciences Mathématiques. In Press.

[3] *D. N. Cheban*: Global attractors of non-autonomous dissipative dynamical systems. Interdiscip. Math. Sci., Vol. 1, World Scientific Publishing, Hanckensack, NJ, 2004.

Multiplicative controllability of nonlinear degenerate parabolic equations

Giuseppe Floridia
Istituto Nazionale di Alta Matematica (INdAM), Italy
floridia.giuseppe@icloud.com

In this talk, we show the global approximate multiplicative controllability for a class of nonlinear degenerate 1-D reaction-diffusion equations. First, we obtain embedding results for weighted Sobolev spaces, that proved decisive in reaching well-posedness for nonlinear degenerate problems. Then, we show that the above systems can be steered in L^2 from any nonzero, nonnegative initial state into any neighborhood of any desirable nonnegative target-state by bilinear piecewise static controls. Moreover, we extend the above result relaxing the sign constraint on the initial date. We note that, for a special choise of the diffusion coefficient, the linear part of the above equations reduces to the one of the Budyko-Sellers climate model. This model is an energy balance model which attempts to study the evolution of the temperature on the Planet Earth, as a result of the interaction between large ice masses and solar radiation, see, for instance, [J.I.Diaz, G.Hetzer, L.Tello, An Energy Balance Climate Model with Hysteresis, Nonlinear Analysis 64 (2006), 2053–2074].

On a singular perturbation problem arising in the theory of Evolutionary Distributions

Gonzalo Galiano
University of Oviedo, Spain
galiano@uniovi.es
Yosef Cohen

Evolution by Natural Selection is a process by which progeny inherit some properties from their progenitors with small variation. These properties are subject to Natural Selection and are called adaptive traits and carriers of the latter are called phenotypes. The distribution of the density of phenotypes in a population is called Evolutionary Distributions (ED). We analyze mathematical models of the dynamics of a system of ED. Such systems are anisotropic in that diffusion of phenotypes in each population (equation) remains positive in the directions of their own adaptive space and vanishes in the directions of the other's adaptive space. We prove that solutions to such systems exist in a sense weaker than the usual. We develop an algorithm for numerical solutions of such systems. Finally, we conduct numerical experiments—with a model in which populations compete—that allow us to observe salient attributes of a specific system of ED.

Uppers bounds on the coarsening rates for the Allen-Cahn equation

Dmitry Glotov
Auburn University, USA
dglotov@auburn.edu
Nan Jiang

The coarsening rate is a statistical property of solutions that is readily observable in models of coarsening phenomena. We will present one-sided estimates on the coarsening rate for solutions of the Allen-Cahn equation. Our method follows the framework introduced by Kohn and Otto and employs two different length scales.

A high order numerical scheme for an atherosclerosis model

Arturo Hidalgo
Universidad Politecnica de Madrid, Spain
arturo.hidalgo@upm.es

We study an diffusion – reaction mathematical model for the evolution of atherosclerosis as an inflammation disease. A high order finite volume formulation in the context of ADER approach is developed to approximate the time-dependent solutions of the model proposed by El Khatib et al. (2007). Concerning the asymptotic behaviour of the solutions, the numerical examples show that a small perturbation of a healthy steady state makes the system evolve to a disease equilibrium for some choice of the parameters. We apply our numerical scheme to determine if each initial datum in a huge family of initial data is attracted by a disease equilibrium or by a healthy steady state. Simultaneously we compute the steady states. This is a joint work with L. Tello (Universidad Politécnica de Madrid) and E.F. Toro (Università degli Studi di Trento).

Soret effect in a closed-loop thermosyphon with a viscoelastic fluid

Angela Jimenez-Casas
Pontificia Comillas University, Spain
ajimenez@upcomillas.es
Justine Yasappan and Mario Castro

Chaotic motion of a viscoelastic fluid confined in a closed loop thermosyphon and subject to the Soret effect is investigated. Various thermal gradients, viscoelastic coefficients and solute gradients produce different types of complex dynamical behaviors on the system. We study the dynamics of the system and the competition/cooperation of these mechanisms to provide different outcomes, from chaotic to stable behavior by means of inertial manifold techniques and numerical integration of the reduced dynamics in the manifold.

On the stability of steady states of a free-boundary granuloma model

King-Yeung Lam
Ohio State University, USA
lam.184@math.ohio-state.edu
Avner Friedman and King-Yeung Lam

We consider a free boundary problem for a system of two semilinear parabolic equations. The system represents a simple model of granuloma, a collection of immune cells and bacteria filling a 3-dimensional domain $\Omega(t)$ which varies in time. We prove the existence of stationary spherical solutions and study their linear asymptotic stability as time increases to infinity.

Radial solutions of polyharmonic equations with power nonlinearities

Monica Lazzo
University of Bari, Italy
monica.lazzo@uniba.it
J.I. Diaz and P.G. Schmidt

A vast amount of literature on second-order semilinear elliptic equations with power-like nonlinearities is concerned with the existence, uniqueness or multiplicity, and qualitative behavior of solutions to various types of boundary-value problems. Lacking a maximum principle, higher-order analogues of such problems require entirely new methods, even if expected results are similar to what is known in the second-order case. As a first step toward a better understanding of the higher-order case, we have been using dynamical-systems methods to study radially symmetric solutions of polyharmonic equations with pure power nonlinearities. We give a survey of our results and their implications.

Global and Blow Up Solutions to Cross Diffusion Systems on 3D Domains

Dung Le
University of Texas at San Antonio, USA
Dung.Le@utsa.edu

We will discuss necessary and sufficient conditions for global existence of classical solutions to a class of cross diffusion systems on 3-dimensional domains. Examples of blow up solutions will be given and compared with the case when no cross diffusion is present.

Global attractors for multivalued semiflows with weak continuity properties

Grzegorz Lukaszewicz
University of Warsaw, Poland
glukasz@mimuw.edu.pl
Piotr Kalita

A method is proposed to prove the global attractor existence for multivalued semiflows in Banach spaces with weak continuity properties. We introduce the condition (NW), "norm-to-weak", that generalizes to the multivalued case the norm-to-weak continuity property introduced earlier for semigroups. Equivalently, the condition states that the multivalued semiflow has weakly compact values and is strong-weak upper semicontinuous, and is weaker then the classical upper semicontinuity property used in the proof of the existence of the global attractor. Condition (NW) is natural to check e.g. for problems governed by differential inclusions where the multivalued term has the form of Clarke subdifferential.

Recent advances in pattern formation on growing and evolving surfaces: Theory, numerics and applications

Anotida Madzvamuse
University of Sussex, England
a.madzvamuse@sussex.ac.uk

In this talk I will present theoretical results on the stability analysis of non-autonomous reaction-diffusion systems on evolving domains. There are two fundamental biological differences between the Turing conditions on fixed and growing domains, namely: (i) we need not enforce cross nor pure kinetic conditions and (ii) the restriction to activator-inhibitor kinetics to induce pattern formation on a growing biological system is no longer a requirement. Our theoretical findings are confirmed and reinforced by numerical simulations for the special cases of isotropic linear, exponential and logistic growth profiles. In particular we illustrate an example of a reaction-diffusion system which cannot exhibit a diffusively-driven instability on a fixed domain but is unstable in the presence of slow growth. Generalisations to reaction-diffusion systems with cross-diffusion as well as results on evolving surfaces will be presented.

Uniform persistence for monotone skew-product semiflows with applications to neural networks

Sylvia Novo
Universidad de Valladolid, Spain
sylnov@wmatem.eis.uva.es
Rafael Obaya and Ana M. Sanz

Uniform persistence and other related notions have been extensively studied for autonomous and non-autonomous dynamical systems. We introduce a concept of uniform persistence above and below a minimal set of an abstract monotone skew-product semiflow. When the minimal set has a continuous separation we characterize the uniform persistence in terms of the principal spectrum. Applications to the case in which the flow is generated by the solutions of a family of non-autonomous functional differential equations will be shown, as well as a method for the calculus of the upper Lyapunov exponent of the minimal set. Finally, we will illustrate our theory with the study of the behavior of a family of almost periodic neural networks, in a situation where persistence actually implies permanence.

The relative rearrangement: applications in Plasma Physics and Image Processing

Juan Francisco Padial Molina
Univ. Politecnica de Madrid, Spain
jf.padial@upm.es
J.I. Díaz

By using the notion of relative rearrangement (see, e.g., J.M. Rakotoson;*Réarrangement Relatif: Un instrument d'estimations dans les problèmes aux limites*, Mathématiques et Applications, SMAI, Springer, Paris 2008.) we introduce a new formulation for some mathematical model arising in Nuclear Fusion and Image Processing. In particular, we review some mathematical models related to the stationary regime of a plasma magnetically confined in nuclear fusion devices such as Tokamaks and Stellarators. We show that these models, expressed through a nonlinear partial differential equations (the Grad–Safranov equation), can be reformulated as inverse problems where several terms of the equations are not a priori known (non local terms). For the case of Stellarators devices, by using the current balance within each flux magnetic we reformulate again the problem as a non local one (see, e.g., J. I. Díaz, J. F. Padial and J. M. Rakotoson; *Mathematical treatment of the magnetic confinement in a current carrying Stellarator*, Nonlinear Analysis Theory Methods and Applications **34,** (1998), 857–887). We will prove some existence and uniqueness result by using Mini-Max methods. An other new appplication of the relative rearrangement notion appears in the study of neighbordhood filters in Image Processing. We shall consider some particular cases of Yaroslavsky filters (L.P. Yaroslavsky and M. Eden, *Fundamentals of digital optics*, Birkhäuser, Boston, 2003). We take special attention to the ocurrence of flat regions given rised by the solutions to our problems.

On the existence of boundary blow-up solutions for a general class of quasilinear elliptic systems

Paul Sauvy
Université Toulouse 1 - Capitole / Institut Mathématique de Toulouse, France
paul.sauvy@ut-capitole.fr
Rym Chemmam, Sonia Ben Othman

In this talk, we investigate the following type of quasilinear elliptic system (P) with explosive boundary conditions on a smooth and bounded domain :

$$\Delta_p u = f_1(x, u, v) \text{ in } \Omega\,;\ u|_{\partial\Omega} = +\infty,\ u > 0 \text{ in } \Omega,$$
$$\Delta_q v = f_2(x, u, v) \text{ in } \Omega\,;\ v|_{\partial\Omega} = +\infty,\ v > 0 \text{ in } \Omega.$$

Under suitable conditions on f_1 and f_2, we first give a general result relating the existence of large solutions to (P) to the existence of a sub and supersolutions pair to (P). Next, we give some applications considering particular systems arising in Biology.

On some Inverse problems arising Industrial Lubrication

J. Ignacio Tello
Universidad politecnica de Madrid, Spain
jtello@eui.upm.es

In this work we present results concerning equilibrium positions in Lubricated systems . The problem consists of two surfaces in relative motion separated by a small distance filled with a compressible fluid. We assume that both surfaces have a regular geometry, If the position of the surfaces are known, the pressure of the fluid is determined by Reynolds equation. The case we study the position of the surface is unknown and to complete the problem a load of the device is given (the external forces applied upon the surface). we will present result concerning existence of equilibrium of solutions fro the both, the journal bearing system and for a two dimensional slide.

On a simple biosphere energy balance model

Lourdes Tello
Universidad Politecnica de Madrid, Spain
l.tello@upm.es

We study a global climate energy balance model (EBM) which spatial domain is the Earth surface with vegetation cover. The evapotranspiration effect is included in the energy balance of two layers: canopy and ground layers. We analyze the existence of solutions for a parabolic system with nonlinear diffusion.

Special Session 10: Nonlinear Elliptic Partial Differential Equations and Systems

Wenxiong Chen, Yeshiva University, USA
Congming Li, University of Colorado at Boulder, USA

In this session, we will study nonlinear elliptic partial differential equations and systems, including the ones involving fractional Laplacians. We will investigate the existence and qualitative properties of solutions, such as symmetry, monotonicity, classification, uniqueness, non-existence, and regularity. We will also discuss various applications of these equations to Physics, Chemistry, Biology, Finance, Differential Geometry, and Convex Geometry.

Behavior of strong solutions to the degenerate oblique derivative problem for elliptic quasi-linear equations in a neighborhood of a conical boundary point.

Michail Borsuk
University of Warmia and Mazury in Olsztyn, Poland
borsuk@uwm.edu.pl
Mariusz Bodzioch

In this talk, we study the behavior of strong solutions to the degenerate oblique derivative problem for quasi-linear second-order elliptic equations in a neighborhood of a conical boundary point $\mathcal{O}$ of an n-dimensional domain:

$$\begin{cases} a^{ij}(x,u,u_x)u_{x_ix_j} + a(x,u,u_x) = 0, \ a^{ij} = a^{ji}, \ x \in G, \\ \frac{\partial u}{\partial \vec{n}} + \chi(\omega)\frac{\partial u}{\partial r} + \frac{1}{|x|}\gamma(\omega)u = g(x), \ x \in \partial G \backslash \mathcal{O}, \end{cases}$$

where $\vec{n}$ denotes the unit exterior normal to $\partial G\backslash\mathcal{O}$, (r,ω) are spherical coordinates in $\mathbb{R}^n$ with pole $\mathcal{O}$. Assuming that the equation is uniformly elliptic, $a(x,u,\nabla u) = O(|\nabla u|^2)$ and some other conditions are satisfied, we have established an exact exponent λ of the solution's decreasing rate near the conical boundary point, i.e. we have shown that $|u(x)| = O(|x|^\lambda)$. Here λ is the smallest positive eigenvalue of the corresponding eigenvalue problem for the Laplace - Beltrami operator on the unit sphere. We have also proved the existence of the smallest positive solution λ of equation

$$(n-2)\sin\frac{\omega_0}{2}\cdot\mathcal{C}_{\lambda-1}^{\frac{n}{2}}\left(\cos\frac{\omega_0}{2}\right) = (\lambda\chi_0+\gamma_0)\cdot\mathcal{C}_{\lambda}^{\frac{n-2}{2}}\left(\cos\frac{\omega_0}{2}\right)$$

and that this solution is the smallest positive eigenvalue of our eigenvalue problem; here $\mathcal{C}_\lambda^\alpha$ is the Gegenbauer function.

Existence, Uniqueness and Nonexistence of Positive Solutions for a Class of Semilinear Elliptic Systems

Shaohua Chen
Cape Breton University, Canada
george_chen@cbu.ca

In this talk, we study an elliptic system $-Du_i = f_i(x,u_1,\cdots,u_n)$ with homogeneous Dirichlet conditions in a bounded domain. We obtain a new result on existence and uniqueness under some sub-linear conditions. We also discuss nonexistence of a special system.

A Liouville Theorem for the Fractional Laplacian and its applications

Wenxiong Chen
Yeshiva University, USA
wchen@yu.edu
Ran Zhuo, Xuewei Cui, Zixia Yuan

In this talk, we consider the following fractional Laplace equation:

$$\begin{cases} (-\Delta)^{\alpha/2}u(x) = 0, & \text{in } R^n, \\ u(x) \geq 0, & \text{in } R^n, \end{cases} \tag{1}$$

where $n \geq 2$ and α is any real number between 0 and 2. We prove that the only solution for (1) is constant. Or equivalently, *Every α-harmonic function bounded either above or below in all of R^n must be constant.* This extends the classical Liouville Theorem from Laplacian to the fractional Laplacian. As an immediate application, we use it to obtain an equivalence between a semi-linear pseudo-differential

$$(-\Delta)^{\alpha/2}u = u^p(x), \ \ x \in R^n \tag{2}$$

and the corresponding integral equation

$$u(x) = \int_{R^n} \frac{1}{|x-y|^{n-\alpha}}dy, \ \ x \in R^n.$$

Combining this with the existing results on the integral equation, one can obtained much more general results on the qualitative properties of the solutions for (2).

On the prescribed scalar curvature problem on the standard sphere

Hichem Chtioui
Mathematics,Faculty of Sciences of Sfax, Tunisia
Hichem.Chtioui@fss.rnu.tn

Various prescribing curvature problems on a manifold can be expressed as follows: given a smooth positive function K defined on M with the metric g, can one find a conformal metric such that the aforementioned curvature is equal to K ? The typical example is the prescribing scalar curvature problem on the standard sphere. In this talk we give some existence results under flatness and non degeneracy conditions. In particular we extend the well known existence results of Bahri-Coron and Y.Y. Li.

Existence results for some systems of coupled Fractional Nonlinear Schrodinger Equations

Eduardo Colorado
Carlos III de Madrid University, Spain
ecolorad@math.uc3m.es

We will talk about the existence of solutions for a system of coupled Fractional Nonlinear Schrödinger Equations, proving the existence of bound and ground state solutions provided the coupled parameter is small, respectively, large.

Stability of solutions to the surface Quasi-Geostrophic equations

Mimi Dai
University of Illinois Chicago, USA
mdai@uic.edu

We consider the steady-state Surface Quasi-Geostrophic equation in the whole space $\mathbb{R}^2$ driven by a forcing function f. The class of source functions f under certain assumptions yield the existence of at least one solution with finite L^2 norm. These solutions are unique among all solutions with finite energy. The solutions are also shown to be stable in the following sense: If Θ is such a solution then any viscous, incompressible flow in the whole space, driven by f and starting with finite energy, will return to Θ.

Dislocation dynamics in crystals

Serena Dipierro
University of Edinburgh, Scotland
serydipierro@yahoo.it

We consider an evolution equation arising in the Peierls-Nabarro model for crystal dislocation. We study the evolution of such dislocation function and show that, at a macroscopic scale, the dislocations have the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the natural periodicity of the medium. These dislocation points evolve according to the external stress and an interior repulsive potential. The results presented were obtained in collaboration with A. Figalli, G. Palatucci and E. Valdinoci.

Existence results for a pseudorelativistic Hartree equation without the Ambrosetti-Rabinowitz condition

Mauro Francesconi
University of Perugia, Italy
mauro.francesconi@dmi.unipg.it
Dimitri Mugnai

Using variational methods based on critical point theory, we study a class of pseudo-relativistic Hartree equations, with general nonlinearities not satisfying the Ambrosetti-Rabinowitz condition. We find the existence of at least three non trivial solutions.

Layer solutions for the fractional Laplacian

Maria del Mar Gonzalez
Universitat Politecnica de Catalunya, Spain
mar.gonzalez@upc.edu

We consider a semilinear equation for the fractional Laplacian on hyperbolic space with a nonlinarity that comes from a double well potential. We prove existence and uniqueness of layer solutions, together with some qualitative and symmetry properties. One of the difficulties is to show a priori regularity for the equation, for which we need to derive a singular integral formula for the fractional Laplacian on hyperbolic space. If time permits, I will mention a few words about the construction on the Heisenberg group.

Existence of non-topological solutions for a skew-symmetric Chern-Simons system

Genggeng Huang
Shanghai Jiao Tong University, Peoples Rep of China
genggenghuang@sjtu.edu.cn
Chang-Shou Lin

In this talk, we investigate the existence of non-topological solutions (u_1, u_2) satisfying

$$u_i(x) = -2\beta_i \ln|x| + O(1), \quad \text{as } |x| \to +\infty,$$

such that $\beta_i > 1$ and

$$(\beta_1 - 1)(\beta_2 - 1) > (N_1 + 1)(N_2 + 1),$$

for a skew-symmetric Chern-Simons system. By the bubbling analysis and the Leray-Schauder degree theory, we get the existence results except for a finite set of curves:

$$\frac{N_1}{\beta_1 + N_1} + \frac{N_2}{\beta_2 + N_2} = \frac{k-1}{k},$$

$k = 2, \cdots, \max(N_1, N_2)$. This generalizes a previous work by Choe-Kim-Lin.

Asymptotic symmetry for a class of nonlinear fractional reaction-diffusion equations

Sven Jarohs
Goethe-University Frankfurt, Germany
jarohs@math.uni-frankfurt.de
Tobias Weth

We study the nonlinear fractional equation $(-\Delta)^s u = f(x, u)$, $s \in (0, 1)$ in a bounded open set Ω together with Dirichlet boundary conditions on $R^N \setminus \Omega$. We prove symmetry of nonnegative bounded solutions in the case where the underlying data obeys some symmetry and monotonicity assumptions. This result, which is obtained via a series of new estimates for antisymmetric supersolutions of a corresponding family of linear equations, implies a strong maximum type principle which is not available in the non-fractional case $s = 1$. Moreover, we will give possible extensions of this statement for more general nonlocal operators than the fractional Laplacian.

An Integral Identity and Measure Estimates for Stationary Fokker-Planck Equations

Min Ji
Chinese Academy of Sciences, Peoples Rep of China
jimin@math.ac.cn
Wen Huang, Yingfei Yi, Zhenxin Liu

We consider a Fokker-Planck equation in a general domain in $\mathbb{R}^n$ with L^p_{loc} drift term and $W^{1,p}_{loc}$ diffusion term for any $p > n$. By deriving an integral identity, we give several measure estimates for regular stationary solutions in an exterior domain with respect to diffusion and Lyapunov-like or anti-Lyapunov-like functions. These estimates will be useful to problems such as the existence and non-existence of stationary solutions in a general domain as well as the concentration and limit behaviors of stationary solutions as diffusion vanishes

Boundary blow-up in polyharmonic equations with power nonlinearities

Monica Lazzo
University of Bari, Italy
monica.lazzo@uniba.it
J.I. Diaz and P.G. Schmidt

We consider semilinear elliptic equations involving the polyharmonic operator Δ^m, power nonlinearities, and positive radial weights. Specifically, we classify all unbounded radial solutions on open balls by their behavior near the boundary and study the asymptotics of solutions that diverge to infinity ("large solutions"). Extending a classical result for the case $m = 1$, we obtain the first-order asymptotics (the "blow-up profile") of large radial solutions in the biharmonic case, $m = 2$. For arbitrary m, we prove the weaker assertion that large radial solutions remain between positive multiples of the expected blow-up profile.

Analysis of Hardy-Littlewood-Sobolev type systems

Congming Li
Shanghai Jiao Tong University, Peoples Rep of China
congmingli@gmail.com
Chen, Ze

In this presentation, we provide some existence, nonexistence, and classification of positive solutions for Hardy-Littlewood-Sobolev type systems:

$$\begin{cases} (-\Delta)^{\gamma/2}u = v^q, \;\; u > 0, \text{ in } R^n, \\ (-\Delta)^{\gamma/2}v = u^p, \;\; v > 0, \text{ in } R^n. \end{cases}$$

In deriving these results, some useful methods are created. We will briefly introduce the degree theory approach for shooting method and some other related methods.

Differentiabilty of Elliptic Equations on the Boundary

Dingsheng Li
Xi'an Jiaotong University, Peoples Rep of China
lidsh@mail.xjtu.edu.cn

We will investigate the influence of geometric properties of domains to the regularity of solutions of elliptic equations. Optimal geometric conditions of domains will be given that guarantee the differentiabilty of solutions of elliptic equations on the boundary.

Global regularity and existence of very weak solutions to certain quasilinear equations

Phuc Nguyen
Louisiana State University, USA
pcnguyen@math.lsu.edu
Karthik Adimurthi

We discuss several a priori estimates and the existence of very weak solutions, i.e., solutions with possibly infinite energy, to equations whose prototype is given by $\Delta_p u = \mathrm{div}|F|^{p-2}F$. Here $\Delta_p u = \mathrm{div}|\nabla u|^{p-2}\nabla u$ is the p-Laplacian, $p > 1$. The results are global and are obtained over domains whose complements are uniformly thick with respect to the p-capacity. As an application, a characterization of existence in the framework of Morrey spaces is obtained for a class of quasilinear Riccati type equations. This talk is based on joint work with Karthik Adimurthi.

Hodge-theoretic methods in nonlinear analysis

Thomas Otway
Yeshiva University, USA
otway@yu.edu
Antonella Marini

New analytic methods for a large class of quasilinear exterior elliptic–hyperbolic systems are discussed. These include a method for generating Bäcklund transformations and a method for contructing explicit solutions on both sides of the parabolic transition. In addition, we prove the well-posedness of boundary value problems for steady flows having nonzero vorticity; such problems are characterized by a degeneration of ellipticity near the sonic speed. All these methods use properties of the Hodge duality operator in an essential way.

Optimal Liouville theorems for supersolutions of elliptic equations with the Laplacian S

Alexander Quaas
Universidad Santa Maria, Chile
alexander.quaas@usm.cl
Salomon Alarcon, Jorge Garcia-Melian

In this paper we consider the question of nonexistence of positive supersolutions of the equation $-\Delta u = f(u)$ in exterior domains of $\mathbb{R}^N$. When $N \geq 3$, we find that positive supersolutions exist if and only if

$$\int_0^\delta \frac{f(t)}{t^{\frac{2(N-1)}{N-2}}} dt0.$$

A similar condition is found for $N = 2$:

$$\int_M^\infty e^{at} f(t) dt0.$$

The proofs are extended to consider some more general operators, which include the Laplacian with gradient terms, the $p-$Laplacian or uniformly elliptic fully nonlinear operators with radial symmetry, like the Pucci's maximal operators $\mathcal{M}^{\pm}_{\lambda,\Lambda}$, with $\Lambda > \lambda > 0$.

Higher-order effects in the boundary behavior of large radial solutions of polyharmonic equations with power nonlinearities

Paul Schmidt
Auburn University, USA
pgs@auburn.edu
J.I. Diaz and M. Lazzo

We study details of the boundary behavior of large radial solutions of semilinear elliptic equations involving the polyharmonic operator Δ^m, power nonlinearities, and positive radial weights. Extending a known result for the case $m = 1$, we obtain the first and second terms in the asymptotic expansions of all such solutions in the biharmonic case, $m = 2$. For general m, we establish analogous expansions under additional conditions. We also describe situations in which these conditions fail and somewhat unexpected boundary behavior ensues.

Sharp existence criteria for positive solutions to a class of weighted integral systems

John Villavert
University of Oklahoma, USA
john.villavert@gmail.com

In this talk, we will examine a family of integral equations involving Riesz potentials and their closely related family of differential equations. This class of problems includes weighted variants of Hardy–Littlewood–Sobolev systems and we shall discuss the optimal criteria for the existence and non-existence of positive solutions for these systems. Namely, we discuss some Liouville type theorems established using iteration methods and Pohozaev type identities in integral form, and we discuss existence results via a topological degree framework.

Some new results on Nonlinear Schrödinger system arising from Bose-Einstein condensates

Zhitao Zhang
the Chinese Academy of Sciences, Peoples Rep of China
zzt@math.ac.cn

We introduce some new results on Nonlinear Schrödinger system arising from Bose-Einstein condensates. We establish the limit system for the Gross-Pitaevskii equations when the segregation phenomenon appears, and shows this limit is the one arising from the competing systems in population dynamics. This verifies a conjecture of S. Terracini et.al., both in the parabolic case and the elliptic case. We also study a class of nonlinear Schrödinger system with external sources terms, as these external sources terms are positive functions and small in some sense, we use Nehari manifold to get the existence of a positive ground state solution and a positive bound state solution.

Special Session 11: Dynamics of Fluids and Nonlinear Waves

Zhiwu Lin, Georgia Tech, USA
Chongchun Zeng, Georgia Tech, USA

This special session focuses on the dynamic properties of fluids and nonlinear waves arsing in systems such as Euler equations, kinetic equations, nonlinear Schrodinger equations, nonlinear wave equations etc. The main aspects include the global well-posedness of solutions, existence, stability/instability and nearby local qualitative structures of special states etc. The theory and the methods applied on these problems address some of the most advanced development in the field.

Enhanced dissipation in the context of Taylor Dispersion

Margaret Beck
Boston University, USA
mabeck@math.bu.edu
Osman Chaudhary and C. Eugene Wayne

We consider the problem of fluid flowing in a channel with uniform cross section, subject to a shear flow. As observed by Taylor in the 1950s, even if fluid itself has viscosity ϵ, it can exhibit diffusion at a rate that is $\mathcal{O}(1/\epsilon)$ - much faster than one would naively expect. We provide a mathematical explanation for this using similarity variables and invariant manifolds.

Inviscid damping and the asymptotic stability of planar shear flows in 2D Euler

Jacob Bedrossian
New York University, USA
Jacob@cims.nyu.edu
Nader Masmoudi

We prove the asymptotic stability of periodic nearly-Couette shear flows in the 2D Euler equations. Specifically we prove that a sufficiently smooth (specifically better than Gevrey-2) perturbation converges as time goes to infinity to a shear flow. The vorticity converges weakly and the velocity converges strongly in L2.

A steady nonlinear and singular vortex Rossby wave within a rapidly rotating vortex

Philippe Caillol
University of Bio-Bio, Chile
filoudevede@hotmail.com

This study wishes to predict the evolution of a rapidly rotating vortex when a neutral wave is propagating therein. We consider a singular and nonlinear vortex Rossby helical wave within a linearly stable, barotropic, axisymmetric and dry vortex in the f-plane. The wave enters resonance with the vortex at a certain radius, where the wave angular speed is equal to the rotation frequency. The singularity in the modal equation at this radius strongly modifies the flow in the critical layer, the region where the wave/vortical flow interaction occurs. We resolve the singularity by reintroducing nonlinearity in the inner-flow equations in the steady-régime assumption. In this régime, the neutral mode is weakly singular. We indeed assume that the critical layer forms at the outer edge of the vortex where the basic axial vorticity has become small compared to the inertial frequency, owing to vortex erosion. Through matched asymptotic expansions, we determine the critical-layer induced distorted flow characterized by a secondary mean flow, whose amplitude is larger then the wave amplitude, which diffuses in an asymmetric way at either side of the critical layer that winds up around the vortex axis. The analysis shows that, after intensifying in the transition stage, the vortex is weakening in the steady stage, but in a negligible way with the slow homogenization time scale. The critical layer extent is nevertheless evolving with this same time all the more so since the ratio vertical wavenumber over azimuthal one is large.

Time-averaging and error estimates for reduced fluid models

Bin Cheng
University of Surrey, England
b.cheng@surrey.ac.uk

I will discuss the application of time-averaging in getting rigorous error estimates of some reduced fluid models, including the quasi-geostrophic approximation, incompressible approximation and zonal flows. The spatial boundary can be present as a non-penetrable solid wall. I will show a very recent (and somewhat surprising) result on the ϵ^2 accuracy of incompressible approximation of Euler equations, thanks to several decoupling properties.

Validity of the KdV approximation for the water wave problem

Wolf-Patrick Duell
University of Stuttgart, Germany
duell@mathematik.uni-stuttgart.de

We consider the 2D water wave problem in an infinite long canal of finite depth both with and without surface tension. It has been proven by several authors that long-wavelength solutions to this problem can be approximated over a physically relevant timespan by solutions of the Korteweg-de Vries equation or, for certain values of the surface tension, by solutions of the Kawahara equation. These proofsare formulated either in Langrangian or in Eulerian coordinates. In this talk, we provide an alternative proof, which is simpler, more elementary and shorter. Moreover, the rigorous justification of the KdV approximation can be given for the cases with and without surface tension together by one proof. In our proof, we parametrize the free surface by arc length and use some geometrically and physically motivated variables with good regularity properties.

Dimension-Breaking Phenomena for Solitary Gravity-Capillary Water Waves

Mark Groves
Universitaet des Saarlandes, Germany
groves@math.uni-sb.de
Erik Wahlen, Shu-Ming Sun

The water-wave problem has small-amplitude line solitary-wave solutions which to leading order are described by the Korteweg-deVries equation (for strong surface tension) or nonlinear Schrödinger equation (for weak surface tension). We present an existence theory for three-dimensional *periodically modulated solitary-wave solutions* to the water-wave problem which have a solitary-wave profile in the direction of propagation and are periodic in the transverse direction. They emanate from the line solitary waves in a dimension-breaking bifurcation and are described to leading order by the Kadomtsev-Petviashvili equation (for strong surface tension) or Davey-Stewartson equation (for weak surface tension). The term *dimension-breaking phenomenon* describes the spontaneous emergence of a spatially inhomogeneous solution of a partial differential equation from a solution which is homogeneous in one or more spatial dimensions.

Stability of the FLRW Solutions to the Dust-Einstein System with a > Positive Cosmological Constant

Mahir Hadzic
King's College London, England
mahir.hadzic@kcl.ac.uk
Jared Speck

The dust-Einstein system models the evolution of a spacetime containing a pressureless fluid, i.e. dust. We will show nonlinear stability of the well-known Friedman-Lemaitre-Robertson-Walker (FLRW) family of solutions to the dust-Einstein system with positive cosmological constant. FLRW solutions represent initially a quiet fluid evolving in a spacetime undergoing accelerated expansion. We work in a harmonic-type coordinate system, inspired by prior works of Rodnianski and Speck on Euler-Einstein system, and Ringstroem's work on the Einstein-scalar-field system. The main new mathematical difficulty is the additional loss of one degree of differentiability of the dust matter. To deal with this degeneracy, we commute the equations with a well-chosen differential operator and derive a family of elliptic estimates to complement the high-order energy estimates.

Out-of-equilibrium behavior of nonlinear dispersive equations

Zaher Hani
Courant Institute of Mathematical Sciences, USA
hani@cims.nyu.edu
Benoit Pausader, Nikolay Tzvetkov, and Nicola Visciglia

We will discuss some recent progress in the mathematically rigorous study of out-of-equilibrium behavior of nonlinear dispersive equations. The main focus will be on the phenomena of energy cascade as measured in terms of the growth of Sobolev norms of solutions. We study this cascade phenomenon for the cubic nonlinear Schrodinger equation and report on some first rigorous results in this direction of research (joint work with Benoit Pausader, Nikolay Tzvetkov, and Nicola Visciglia).

Navier-Stokes-Fourier Limits from Boltzmann Equation

Ning Jiang
Tsinghua University, Peoples Rep of China
njiang@tsinghua.edu.cn

In this talk I will present two results on the Navier-Stokes-Fourier (NSF) limit from Boltzmann equation. The first is joint work with Masmoudi, in a bounded domain, we justify NSF with Dirichlet and Neumann boundary condition from Boltzmann equation with Maxwell reflection boundary condition. The boundary conditions of NSF depends on the relative size of accommodation coefficient and Knudsen number. Moreover, by analyzing the viscous and kinetic boundary layers, we prove the strong L^1 convergence for Dirichlet condition. The second is joint work with Chao-jiang Xu and Hui-jiang Zhao, in the whole space, we prove the global classical solutions around the equilibrium uniform in Knudsen number for non-cutoff kernel Boltzmann equation, then taking fluid limit, then give a kinetic proof of the classical solutions of NSF around the equilibrium.

Nonlinear stability of source defects in general oscillatory media

Toan Nguyen
Penn State University, USA
nguyen@math.psu.edu
M Beck, B. Sandstede, K. Zumbrun

Consider a system of one-dimensional reaction-diffusion equations that models a general oscillatory medium. Such a model exhibits coherent structures or defect solutions that are time-periodic in an appropriate co-moving frame and are connecting two spatially periodic traveling waves at each infinity. Source-type solutions are those having their asymptotic group velocities pointing outward away from the core of the defect. As perturbations will be transported by the group velocities, perturbed source solutions will create non-localized responses in the phase dynamics, and hence analyzing nonlinear dynamics near a source solution is technically complicated. I will present a joint work with Beck, Sandstede, and Zumbrun that establishes nonlinear stability of general spectrally stable sources. This involves the introduction of new analytical techniques that both illuminate and unify previous results on modulational stability of shocks and patterns and expand the range of possible future applications.

Multi-solitons for the water-waves system

Frederic Rousset
Université Paris-Sud, France
Frederic.Rousset@math.u-psud.fr
Mei Ming, Nikolay Tzvetkov

We shall present a construction of multi-solitons solutions that is to say solutions that behave as time goes to infinity as a sum of decoupling solitary waves for the full water waves system with surface tension.

Validity and Non-validity of the NLS approximation for the water wave problem

Guido Schneider
Universität Stuttgart, Germany
guido.schneider@mathematik.uni-stuttgart.de

We consider the 2D water wave problem in case of finite depth with and without surface tension. We are interested in the validity of the NLS approximation for the description of surface water waves. After giving an overview about positive results we explain that in case of small surface tension there are situations where the NLS approximation fails to describe the water wave problem correctly.

Resonances in PDEs

Jalal Shatah
Courant Institute of Mathematical Sciences
shatah@cims.nyu.edu
N. Masmoudi, P. Germain

In this talk we will consider the different type of resonances that exist in nonlinear dispersive equations and their effects on the long time behavior of solutions. We will use the water waves equations as well as other dispersive equations to illustrate the different effects of resonance.

Well-posedness of an Euler-like scalar model and nonlocal maximum principle

Roman Shvydkoy
University of Illinois at Chicago, USA
shvydkoy@uic.edu
F. Vigneron, C. Imbert

We study a conservative scalar model that shares many similar properties with the classical Euler equation (energy conservation, structure of the nonlinearity). In the viscous case it has been proposed as a model for the Navier-Stokes system. We show that the inviscid model is globally well-posed for positive initial data using the nonlocal maximum principle developed recently by Constantin and Vicol in the context of the critical SQG.

Solitary water waves with weak surface tension in 3D

Erik Wahlen
Lund University, Sweden
ewahlen@maths.lth.se
Boris Buffoni, Mark Groves

I will present an existence result for solitary waves on a three-dimensional layer of water of finite depth. The waves are fully localised in the sense that they converge to the undisturbed state of the water in every horizontal direction. The surface tension is assumed to be weak but non-zero. The solitary-wave solutions are to leading order described by the Davey-Stewartson equation. The proof is variational in nature and relies on reducing the original water wave problem to a perturbation of the Davey-Stewartson equation. In the end, the solutions are constructed by minimising a certain functional on its natural constraint.

Mathematical theory of wind-generated water waves

Samuel Walsh
University of Missouri, USA
walshsa@missouri.edu
Oliver Buhler, Jalal Shatah, and Chongchun Zeng

It is easy to see that wind blowing over a body of water can create waves. But this simple observation leads to a more fundamental question: Under what conditions on the velocity profile of the wind will persistent surface water waves be generated? This problem has been studied intensively in the applied fluid dynamics community since the first efforts of Kelvin in 1871. In this talk, we will present a mathematical treatment of the predominant model for wind-wave generation, the so-called quasi-laminar model of J. Miles. Essentially, this entails determining the (linear) stability properties of the family of laminar flow solutions to the two-phase interface Euler equation. We give a rigorous derivation of the linearized evolution equations about an arbitrary steady solution, and, using this, a complete proof of the celebrated instability criterion of Miles. In particular, our analysis incorporates both the effects of surface tension and a vortex sheet on the air–sea interface. We are thus able to give a unified equation con- necting the Kelvin–Helmholtz and quasi-laminar models of wave generation.

Stability of traveling waves of nonlinear Schrödinger equation with nonzero condition at infinity

Zhengping Wang
Chinese Academy of Sciences, Peoples Rep of China
wangzp@wipm.ac.cn
Zhiwu Lin and Chongchun Zeng

In this talk, we present some recent results on the stability of traveling waves of nonlinear Schrödinger equation with nonzero condition at infinity. Two important physical models are Gross-Pitaevskii equation and NLS with cubic-quintic nonlinearity. These results are obtained in collaboration with Zhiwu Lin and Chongchun Zeng.

Uniqueness for the 2-D Euler equations on domains with corners

Chao Wang
Peking University, Peoples Rep of China
wangchao@amss.ac.cn
Christophe Lacave, Evelyne Miot

We prove uniqueness of the solution of the Euler equations with bounded vorticity for bounded simply connected planar domains with corners forming acute angles. Our strategy consists in mapping such domains on the unit disk via a biholomorphism. We then establish log-Lipschitz regularity for the resulting push-forward of the velocity field, which leads to uniqueness thanks to a Gronwall estimate involving the Lagrangian trajectories on the unit disk.

Break-down criterion of the water wave equations

Zhifei Zhang
Peking university, Peoples Rep of China
zfzhang@math.pku.edu.cn
Wang Chao

Beale-Kato-Majda proved that if smooth solution u of the incompressible Euler equations satisfies

$$\int_0^T \|\nabla \times u(t)\|_{L^\infty} dt < +\infty,$$

then the solution can be continued after $t = T$. We will prove a Beale-Kato-Majda breakdown criterion to the free boundary problem for the incompressible Euler equations in terms of $W^{1,\infty}$ norm of the velocity, the mean curvature of the free surface and Taylor sign condition.

Special Session 12: Complexity in Reaction-Diffusion Systems

Chiun-Chuan Chen, National Taiwan University , Taiwan
Hirokazu Ninomiya, Meiji University, Japan
Masaharu Taniguchi, Okayama University, Japan
Toshiyuki Ogawa, Meiji University, Japan

Reaction-diffusion systems sometimes exhibit complicated dynamics. Especially they are observed by numerics. Recently theoretical studies of reaction-diffusion systems have been developed and our mathematical understanding of dynamics of reaction-diffusion systems are getting deeper. We will bring the recent development of our researches related to the reaction-diffusion systems and exchange our ideas.

Traveling waves for nonlocal reaction diffusion equations (without comparison)

Matthieu Alfaro
Univ. Montpellier 2, France
matthieu.alfaro@univ-montp2.fr

We consider a nonlocal effect in the reaction term of classical reaction diffusion equations (Fisher-KPP case, ignition case, bistable case) so that the comparison principle does not hold. We review known facts and present recent developments on such issues. The talk is based on joint works with J. Coville and G. Raoul.

Traveling wave solutions of a free boundary problem for a two-species competition model

Chiun-Chuan Chen
National Taiwan University, Taiwan
chchchen@math.ntu.edu.tw
Chuen-Hsin Cahang

We study a diffusive logistic system with a free boundary in ecology proposed by Mimura, Yamada and Yotsutani. Motivated by the spreading vanishing dichotomy obtained by Du and Lin, we suppose the spreading speed of the free boundary tends to a constant as time tends to infinity and consider the corresponding travelling wave problem. We establish the existence and uniqueness of a travelling wave solution for this free boundary problem.

Study of the rotating spiral wave and rotating spot by the wave front interaction model

Yan-yu Chen
Tamkang University, Taiwan
chenyanyu24@gmail.com
Jong-Sheng Guo, HiroKazu Ninomiya

Recently, there are many kinds of patterns appeared in our nature. For example, the spiral waves, traveling spots and rotating spots can be observed from the experiment of the photosensitive BZ reaction. In this talk, we focus on the spiral wave and rotating spot and treat them by the wave interaction model proposed by Zykov in 2007. First, we give the equations to describe the boundary of these two patterns. Then we prove the existence, uniqueness and asymptotic behavior for the spiral wave under certain condition for the core radius and angular speed. Finally, we show the existence of the rotating spot. These two works are joint works with professors J.-S. Guo and H. Ninomiya

Collective motions of particles with diffusive interactions

Kota Ikeda
Graduate School of Advanced Mathematical Sciences, Meiji University, Japan
ikeda@isc.meiji.ac.jp
Shin-Ichiro Ei, Akiyasu Tomoeda, Masaharu Nagayama

The collective motion of camphor boats in the water channel exhibits both a homogeneous and an inhomogeneous state, depending on the number of boats, when unidirectional motion along an annular water channel can be observed even with only one camphor boat. In a theoretical research, the unidirectional motion is represented by a traveling wave solution in a model. Since the experimental results described above are thought of as a kind of bifurcation phenomena, we would like to investigate a linearized eigenvalue problem in order to prove the destabilization of a traveling wave solution. However, the eigenvalue problem is too difficult to analyze even if the number of camphor boats is 2. Hence we need to make a reduction on the model. In this talk, we apply the center manifold theory and reduce the model to an ordinary differential system rigorously. We also show that the reduced system has a periodic solution corresponding to an inhomogeneous state in the experiment via numerical simulations.

Diffusion-induced blowup in reaction-diffusion-ODE systems

Grzegorz Karch
Uniwersytet Wroclawski, Poland
karch@math.uni.wroc.pl
Marciniak-Czochra and Kanako Suzuki

Mathematical models of a pattern formation arising in processes described by a system of a single reaction-diffusion equation coupled with ordinary differential equations will be discussed. In such models, a certain natural (autocatalysis) property of the system leads to the instability of all inhomogeneous stationary solutions. We have proved, moreover, that space inhomogeneous solutions of these models become unbounded in either finite or infinite time, even if space homogeneous solutions are bounded uniformly in time.

On fast-moving criminals and chemotaxis

Theodore Kolokolnikov
Dalhousie University, Canada
tkolokol@gmail.com

In the first part of the talk, I will discuss an extension of the UCLA crime model to incorporate more realistic locomotion of criminals. Instead of assuming that the criminals follow a biased brownian motion [as in the original model], we ask what happens when they follow a biased Levy flight. It turns out that from the point of view of the criminal, there is an "optimal" Levy flight exponent which maximizes the criminal's chance of committing a burglary. In the second part of the talk, I will discuss the Keller-Segel model with logistic self-production terms that exhibits complex spatio-temporal dynamics of spikes. Unlike the "classical" KS model, we show that it is possible to stabilize a single interior spike, and we compute analytically a critical threshold which is responsible for spike stabilization.

Stability of traveling waves for time-delayed reaction-diffusion equations: (I) Non-critical oscillating waves

Ming Mei
Champlain College & McGill University, Canada
ming.mei8@gmail.com
Chi-Kun Lin, Chi-Tien Lin, Yaping Lin

This talk is concerned with Nicholson's blowflies equation, a kind of time-delayed reaction-diffusion equations. It is known that, when the ratio of birth rate coefficient and death rate coefficient satisfies $1e$, the equation losses its monotonicity, and its traveling waves are oscillatory when the time-delay r or the wave speed c is large, which causes the study of stability of these non-monotone traveling waves to be challenging. In this paper, we use the technical weighted energy method to prove that, when $ec_* > 0$ are exponentially stable, where $c_* > 0$ is the minimum wave speed. Here, we allow the traveling wave to be any, monotone or non-monotone with any speed $c > c_*$, and any size of the time-delay $r > 0$; while, when $\frac{p}{d} > e^2$ with a small time-delay $rc_* > 0$ are exponentially stable, too. As a corollary, we also prove the uniqueness of traveling waves in the case of $\frac{p}{d} > e^2$, which was open as we know. Finally, some numerical simulations are carried out. When ee^2, if the time-delay is small, then the solution numerically behaves like a monotone/non-monotone traveling wave, but if the time-delay is big, then the solution is numerically demonstrated to be chaotically oscillatory but not an oscillatory traveling wave. These either confirm and support our theoretical results, or open up some new phenomena for future research.

Fast reaction limit of a two-components system with different reaction terms

Harunori Monobe
Meiji University, Japan
te12001@meiji.ac.jp
Masato Iida, Hideki Murakawa and Hirokazu Nionomiya

The fast reaction limit of systems has been studied intensively during the last two decades. In 1996, Hilhorst, van der Hout and Peletier considered a simple two-components system with a common reaction term and showed that the fast reaction limit of the system is written as the one-phase Stefan problem. Moreover, in 2000, they studied the fast reaction limit of more general systems which have a common reaction term as the previous system. The idea of Hilhorst et al. forms the basis of the fast reaction limit analysis. In this talk, we consider the fast reaction limit of a two-components system which corresponds approximately to the system studied by Hilhorst et al., where the system has "different reaction terms" . Our goal is to show that the propagation speed of free boundary arising from the fast reaction limit become zero, finite or infinity, depending on the combination of two reaction terms in the system. This talk is based on a joint work with M. Iida, H. Murakawa and H. Ninomiya.

Traveling spots and obstacle-induced spirals in an excitable medium

Hirokazu Ninomiya
Meiji University, Japan
hirokazu.ninomiya@gmail.com
Y.-Y. Chen, Y. Kohsaka

In this talk, traveling spots observed in two-dimensional excitable media are explored. First, we introduce the singular limit problem of the FitzHugh-Nagumo equations. Then we show the existence of the traveling spot including the front and the back. Using this traveling spots, I will explain some mathematical understanding of the formation of spirals which is induced by obstacles. The formation of the spirals depends on the shapes of the obstacle. This spiral formation is deeply related to the ventricular fibrillation.

Stabilization of unstable standing waves in reaction-diffusion systems

Toshiyuki Ogawa
Meiji University, Japan
togw@meiji.ac.jp
Yusuke Umezu and Kenji Kashima

Oscillatory patterns are known to bifurcate in RD systems as well as stationary patterns. The purpose of this talk is to show the possibility to effectively generate various desired spatial oscillating patterns by guiding such patterns suitably. We first introduce the 3-component RD system which exhibits wave instability and prepare feedback stabilization control in terms of spatial spectrum consensus so that we can stabilize unstable standing waves.

Concentration on compact manifolds for some nonlinear elliptic equations

Yoshihito Oshita
Okayama University, Japan
oshita@okayama-u.ac.jp
Jaeyoung Byeon, Ohsang Kwon

We consider the equation $-\varepsilon^2\Delta u + V(x)u - u^p = 0$ in $\mathbb{R}^N$ where for each $k \in \{1, \cdots, K\}$, M_k is a q_k-dimensional smooth compact manifold in $\mathbb{R}^N$ with $q_k \in \{1, \cdots, N-1\}$, and $V(x) = |\text{dist}(x, M_k)|^{m_k} + O(|\text{dist}(x, M_k)|^{m_k+1})$ as $\text{dist}(x, M_k) \to 0$ for some $m_k > 0$. For a sequence of ε converging to zero, we will find a positive solution u_ε of the equation which concentrates on $M_1 \cup \cdots \cup M_K$.

On a free boundary problem for the curvature flow with driving force

Masahiko Shimojo
Okayama University of Science, Japan
shimojo@xmath.ous.ac.jp
Jong-Shenq Guo, Hiroshi Matano, Chang-Hong Wu

We study a free boundary problem associated with the curvature dependent motion of planar curves in the upper half plane whose two endpoints slide along the horizontal axis with prescribed fixed contact angles. Our first main result concerns the classification of solutions; every solution falls into one of the three categories, namely, area expanding, area bounded and area shrinking types. We then study in detail the asymptotic behavior of solutions in each category. Among other things we show that solutions are asymptotically self-similar both in the area expanding and the area shriknking cases, while solutions converge to either a stationary solution or a traveling wave in the area bounded case. We also prove results on the concavity properties of solutions.

Slow motion of interfaces for a system of reaction-diffusion equations

Marta Strani
University Paris-Diderot, Paris 7, France
martastrani@gmail.com

We consider a system of reaction-diffusion equations in a bounded interval of the real line, with emphasis on the metastable dynamics, whereby the time-dependent solution approaches its stable equilibrium configuration in an asymptotically exponentially long time interval as the viscosity coefficient $\varepsilon > 0$ goes to zero. In particular, we describe the phenomenon of the slow convergence of a layered solution into a patternless steady state. To rigorous describe such behavior, we analyze the dynamics of solutions in a neighborhood of a one-parameter family of approximate steady states, and we derive an ODE for the position of the internal interfaces.

Convex compact sets in $\mathbb{R}^{N-1}$ give traveling fronts in $\mathbb{R}^N$ in cooperative diffusion systems

Masaharu Taniguchi
Okayama University, Japan
taniguchi-m@okayama-u.ac.jp

This paper studies traveling fronts to cooperative diffusion systems in $\mathbb{R}^N$ for $N \geq 3$. We consider $(N-2)$-dimensional smooth surfaces as boundaries of strictly convex compact sets in $\mathbb{R}^{N-1}$. We prove that there exists a traveling front associated with a given surface and that it is asymptotically stable for given initial perturbation. The associated traveling fronts coincide up to phase transition if and only if the given surfaces satisfy an equivalence relation.

Mean first passage time for a small rotating trap inside a reflective disk

Justin C. Tzou
Dalhousie University, Canada tzou.justin@gmail.com
Theodore Kolnikov

We compute the mean first passage time (MFPT) for a Brownian particle inside a two-dimensional disk with reflective boundaries and a small interior trap rotating at a constant angular velocity. For a given angular velocity, we determine the optimal radius of rotation that minimizes the average MFPT over the entire disk. Several distinct regimes are observed, depending on the ratio between the angular velocity and trap size, and several intricate transitions are analyzed using tools of asymptotic analysis and Fourier series. This simple geometry provides a good test case for future studies of MFPT with more complex trap motion.

Accessibility of singular homoclinic orbits for a semilinear parabolic equation

Eiji Yanagida
Tokyo Institute of Technology, Japan
yanagida@math.titech.ac.jp
Marek Fila

We show the existence of at least three different continuations beyond blow-up for a backward self-similar solution of a supercritical Fujita equation. One of these extended solutions cannot be approximated by classical entire solutions in a specific way given by the scaling invariance of the equation, while the minimal continuation is known to be accessible by a family of such entire solutions.

Special Session 13: Nonlocally Coupled Dynamical Systems: Analysis and Applications

Yuri Maistrenko, National Academy of Sciences of Ukraine, Ukraine
Georgi Medvedev, Drexel University, USA

Nonlocally coupled dynamical systems arise as models of diverse phenomena throughout physics, biology, and engineering. They feature a variety of remarkable spatio-temporal patterns such as twisted states, waves, chimera states, and coherence-incoherence transition, to name a few. The talks in this special session highlight recent developments in the mathematical theory of nonlocally coupled systems and their applications to phenomena in natural sciences.

Limits for chimera states

Daniel Abrams
Northwestern University, USA
dmabrams@northwestern.edu
Mark Panaggio

"Chimera states" are surprising patterns that can be found in systems of identical coupled oscillators, where synchrony and incoherence seem to stably coexist. In this talk I'll review results on the existence and stability of chimera states in a variety of dimensions and coupling topologies. I'll specifically look at interesting limits including small and large-N, small and large phase lag, as well as near-local and near-global coupling.

Delayed coupling and spatio-temporal patterns in networks

Fatihcan Atay
Max Planck Institute for Mathematics in the Sciences, Germany
fatay@mis.mpg.de

I will discuss dynamical patterns in non-locally coupled networks arising from the presence of time delays. The motivation comes from recent results in chimera states, which were originally observed in simple ring configurations under symmetric coupling conditions. I will present a systematic way of searching for such nontrivial patterns in delayed networks using a combination of symmetry analysis and bifurcation theory, and will give several examples of novel dynamics and their stability analysis. Particular emphasis will be devoted to patterns combining synchrony and amplitude death.

A group-theoretic approach to metastability in networks of interacting SDEs

Nils Berglund
University of Orleans, France
nils.berglund@univ-orleans.fr
Sébastien Dutercq, Barbara Gentz, Bastien Fernandez

We consider systems of coupled overdamped ODEs perturbed by weak Gaussian white noise. Depending on coupling strength and noise intensities, their behaviour can range from Ising-model-like to globally synchronized dynamics. The long-time behaviour of such systems is determined by the local minima and saddles of the associated potential energy landscape. In totally asymmetric cases, the so-called metastable hierarchy determines expected transition times via the Eyring–Kramers formula. However the systems we are interested in admit large symmetry groups, due to the symmetries of couplings and local interactions. An approach based on representation theory for finite groups allows to determine a corrected formula for metastable transition times in symmetric situations.

References
[1] Nils Berglund and Sébastien Dutercq, The Eyring–Kramers law for Markovian jump processes with symmetries, arXiv/1312.0835 (2013).
[2] Nils Berglund and Barbara Gentz, The Eyring–Kramers law for potentials with nonquadratic saddles, Markov Processes Relat. Fields 16:549-598 (2010).
[3] Nils Berglund, Bastien Fernandez and Barbara Gentz, Metastability in interacting nonlinear stochastic differential equations I: From weak coupling to synchronisation, Nonlinearity 20:2551-2581 (2007).
[4] Metastability in interacting nonlinear stochastic differential equations II: Large-N behaviour, Nonlinearity 20:2583-2614 (2007).

Controlling Chimeras

Chris Bick
Rice University, USA
bick@rice.edu
Erik A. Martens

A curious feature of non-locally coupled phase oscillator is the emergence of chimera states. These localized pattern are characterized by localized phase synchrony while the remaining oscillators move incoherently. The position of synchronized region however depends strongly on the initial condition and is subject to pseudo-random fluctuations. Here we apply the idea of control to chimera states; through a new dynamic control scheme that exploits drift, a chimera will attain any desired target position. Our control approach extends beyond chimera states as it may also be used to optimize more general objective functions.

Dynamics of networks of randomly connected spiking neurons

Nicolas Brunel
University of Chicago, USA
nbrunel@uchicago.edu

I will review mathematical approaches to the study of the dynamics of networks of randomly connected excitatory and inhibitory spiking neurons. I will discuss the linear stability analysis of the asynchronous state; the zoo of

different types of states that are reached beyond various instabilities; and compute power spectra of population activity in finite size networks. These power spectra will be shown to provide good fits to experimental data recorded in monkey primary visual cortex.

Multistable behavior above synchronization in a locally coupled Kuramoto model

Hilda Cerdeira
Instituto de Fisica Toerica - UNESP, Brazil
cerdeira@ictp.it
Paulo F. C. Tilles and Fernando F. Ferreira

A system of nearest neighbors Kuramoto-like coupled oscillators placed in a ring is studied above the critical synchronization transition. We find a richness of solutions when the coupling increases, which exists only within a solvability region (SR). We also find that the solutions possess different characteristics, depending on the section of the boundary of the SR where they appear. We study the birth of these solutions and how they evolve when the coupling strength increases, and determine the diagram of solutions in phase space. We also show how analytical solutions may be obtained from symmetry assumptions, and while we proceed on our endeavor we show some unexpected phenomena resulting from the symmetry properties: the existence of local attractors in the synchronized region; stability exchange for crossing fixed points; fixed point stability dependent on the manifold dimension; chaotic period and intermittent phase slips. As a result of our analysis, we show that stable fixed points in the synchronized region may be obtained with just a small amount of the existent solutions, and for a class of natural frequencies configuration we show analytical expressions for the critical synchronization coupling as a function of the number of oscillators, both exact and asymptotic.

Localised pattern formation with a nonlinear nonlocal term

Jonathan Dawes
University of Bath, England
J.H.P.Dawes@bath.ac.uk
David Morgan

The formation and dynamics of localised patterns has been investigated in considerable detail in recent years. I will describe the influence of a nonlocal nonlinear term added to the well-known Swift-Hohenberg model equation. Although in some sense a nonlocal term allows a smooth transition between purely local and completely global coupling, the transition between these two limiting states turns out not to be as straightforward as one might imagine.

Different types of chimera states in coupled bi-stable oscillators

Dawid Dudkowski
Lodz University of Technology, Poland
dawdud@o2.pl
Yuri Maistrenko, Tomasz Kapitaniak

We describe chimera state phenomenon for networks of non-locally coupled oscillators, in which individual systems are bi-stable having one regular attractor (e.g. equilibrium) and one chaotic. By analyzing the dependence of the network dynamics on the range and strength of coupling, we obtain parameter regions for various chimera types, which are characterized by different type of chaotic behavior at the irregular part of chimera state. Peculiar cases are (i) pure temporal chaos which means the chaotic synchronization of the irregular oscillators and (ii) pure spatial chaos when temporal dynamics remain regular (stationary or periodic) but the oscillators are 'randomly situated' along the spatial network coordinate(s). More typical is the situation (iii) when both evolutional and translational dynamical systems are chaotic, then space-temporal chaotic behavior for the irregular oscillators emerges; e.g. for the original chimeras found in complex GLU and the Kuramoto model. Various combinations of (i)-(iii) are also possible. We demonstrate the phenomenon for networks of bi-stable units with different types of individual dynamics, including coupled maps (piece-wise linear and smooth) and coupled time-continuous systems of the Van der-Pol type. Parameter regions for various types of the chimera states are obtained and scenarios for transitions between them are analyzed.

Can laminar-turbulent coexistence in shear flows be modelled by chimera states?

Yohann Duguet
LIMSI-CNRS, University Paris-Sud, France
duguet@limsi.fr

The subcritical transition from laminar to turbulence in flows such as pipe flow, channel flow, Couette flows is one of the classical puzzles of hydrodynamics and is currently an area of intense research. The onset of turbulence in such flows is usually connected with the spatiotemporal dynamics of isolated patches of disordered fluid motion surrounded by more quiescent (laminar) flow. Physical scenarios for the spontaneous spatial localisation of the turbulent fluctuations are still lacking and there is hope that low-order models can shed light on this problem. In this talk I will first give an overview of subcritical transitional flows, focusing both on the local (at the scale of an elementary sub-unit of turbulence) and the global (i.e. spatially extended) dynamics. Next I will demonstrate the strong analogies between localised turbulence and the chimera states emerging in arrays of one- and two-dimensional identical non-locally (attractively) coupled oscillators, where the laminar flow is interpreted as the only possible fully synchronised state: spatial localisation of course but also fractal basin boundaries, scale separation and time-averaged properties.

Chimera states in networks of nonlocally coupled neural oscillators

Johanne Hizanidis
National Center for Scientific Research "Demokritos", Greece
ioanna@chem.demokritos.gr
V. G. Kanas, A. Bezerianos, T. Bountis, A. Vüllings, I. Omelchenko, P. Hövel

Chimera states is a peculiar phenomenon of coexisting coherent and incoherent behaviour discovered in networks of nonlocally coupled identical phase oscillators over ten years ago. Since then, chimeras were found to occur in a variety of theoretical and experimental studies. In this work, we are interested in the existence of chimera states in systems modelling neuron excitability and dynamical behaviour. First, we consider a generic model for a saddle-node bifurcation on a limit cycle representative for neuron excitability type I. We obtain multichimera states depending on the distance from the excitability threshold and the range of nonlocal coupling. A detailed study of the effect of all dynamical parameters on the stability of the chimera states is presented. Next, we consider a system of nonlocally coupled Hindmarsh-Rose oscillators which is a prototype system for type-I and type-II neuron excitablity that can reproduce many dynamical features of real neurons. We find various interesting synchronization patterns including chimera states in both spiking and bursting regimes. The system is investigated for various coupling schemes including electrical and chemical coupling.

Synchronization extends the life time of the desired behavior of globally coupled systems

Tomasz Kapitaniak
Technical University of Lodz, Poland
tomaszka@p.lodz.pl
Marcin Kapitaniak, Mateusz Lazarek, Michal Nielaczny, Krzysztof Czolczynski, Przemyslaw Perlikowski

Synchronization occurs widely in natural and technological world, but it has not been widely used extend the life time of the desirable behavior of the coupled systems. Here we consider the globally coupled system consisting of n units and show that the initial synchronous state extends the lifetime of desired behavior of the coupled system in the case when the excitation of one or few units is suddenly (breakdown of energy supply) or gradually (as the effect of aging and fatigue) switched off. We give evidence that for the properly chosen coupling the energy transfer from the excited units allows unexcited unit to operate in the desired manner. As proof of concept, we examine the system of coupled externally excited rotating pendula. After the partial excitation switch off the initial complete synchronization of all pendula is replaced by phase synchronization with a constant phase shift between the clusters of excited and unexcited pendula. Our results show that the described extension of the system's life time occurs for the wide range of coupling parameters and is robust to the external perturbations.

Multi-cluster and traveling chimera states in nonlocal phase-coupled oscillators

Edgar Knobloch
University of California at Berkeley, USA
knobloch@berkeley.edu
Jianbo Xie and Hsien-Ching Kao

Networks of coupled oscillators have been studied extensively because of their wide applications in physics, chemistry, and biology. When the coupling is weak, phase reduction leads to a simplified description of the dynamics. Among the phase-coupled models that result, Kuramoto-type models are best known. Numerous studies with either local or global coupling have been made, but the role of nonlocal coupling remains poorly understood. In 2002 Kuramoto and Battogtokh [*Nonlinear Phenom. Complex Syst.* **5** 380–385 (2002)] investigated a system of identical phase-coupled oscillators with a nonlocal coupling and found a solution consisting of coherent, phased-locked oscillators embedded in a background state of incoherent oscillators, later referred to as a chimera state [D.M. Abrams and S.H. Strogatz, *Phys. Rev. Lett.* **93** 174102 (2004)]. These states usually coexist with a stable fully synchronized state and are therefore hard to find. Here, we propose a nonlocal phase-coupled model in which chimera states can be found easily. By adjusting the parameters of the model, the number of coherent clusters can be controled. Stability properties and bifurcations of these states are described, with particular emphasis on the appearance of a traveling chimera state.

Chimera states of coupled pendula

Patrycja Kuzma
Lodz University of Technology, Poland
pat.kuzma@gmail.com
Yuri Maistrenko, Tomasz Kapitaniak

Systems of non-locally coupled oscillators may display remarkable spatial pattern, called chimera state, which consists of the coexisting domains of coherence and incoherence. Considering coupled pendulum-like nodes, we find wide region in parameter space, in which chimera states appear. We show that chimera state can coexist with some other pattern, the so-called *imperfect chimera state*, which is characterized by a certain small number of oscillators escaped from synchronized chimera's cluster. Escaped elements oscillate with different average frequencies (Poincare rotation number). This new type of behavior is not observed for classical Kuramoto model. We also report a novel mechanism for the creation of chimera states via the appearance of the so-called *solitary states*. Investigating the transition between complete synchronization and chimera state, we have observed regions in parameter space, where one or few oscillators escaped from the main synchronized state, which becomes a "fuzzy" cluster. This behavior represents the phenomenon of spatial chaos, i.e., sensitive dependence on the space coordinates. With further increase of the control parameter, more and more oscillators split, resulting in the appearance of the chimera state. This work is supported by the Foundation for Polish Science, Team Programme under project TEAM/2010/5/5

Chimera in delays dynamics: theory, experiments, and related application

Laurent Larger
FEMTO-ST / Optics Dpt., France
laurent.larger@univ-fcomte.fr
Bogdan Penkovskyi, Yuri Maistrenko

Delay differential equations, as infinite dimensional dynamics, are known since the 90s as having a virtual spatio-temporal representation. This raised recently interest to investigate conditions under which Chimera-like motion could be obtained in a such purely temporal dynamics. Inspired by recent independent discovery of conditions allowing for stable period-1 solution, a particular class of delay dynamics called integro-differential delay dynamics, has successfully led to the observation of Chimera-like patterns, as they can be visualized in the virtual space-time representation. In such a representation, space expands as a temporal functional over the delay, whereas time occurs as discrete variable labeling the delay interval. Chimera motions consist then, as discrete is iterated, of partition of space, each partition consisting of well identified and incongruent amplitude variations (e.g. islands of constant amplitude surrounded by chaotic fluctuations, with globally stationary frontiers). We will report on numerical simulations, experimental demonstrations, and theoretical analysis of the phenomena. Beyond this Chimera-like discovery, we will also report on a recently addressed application making use of the spatio-temporal analogy of the same class of delay dynamics: a virtual network of nodes can be emulated physically, in order to demonstrate brain-inspired computing machine.

Cascades of chimera states

Yuri Maistrenko
National Academy of Sciences of Ukraine, Ukraine
y.maistrenko@biomed.kiev.ua

Chimera state is a remarkable space-temporal phenomenon in nonlocally coupled dynamical systems that displays a self-organized pattern of co-existing coherence and incoherence. Intriguingly, this kind a hybrid behavior can develops robustly as a spontaneous symmetry breaking without any influence of asymmetry. Originally, chimera state was discovered for nonlocally coupled complex Ginzburg-Landau equation and Kuramoto model. Currently, this is an area of intense theoretical and experimental research, and for now, chimeras have already been found in many systems from various fields. In this talk, I report new discovered cascades of chimera states which are characterized by increasing number of the regions of irregularity, so-called chimera's heads, typically arising in networks with attractive and with repulsive coupling. For the cascades, parameter regions for multi-headed chimera states are obtained. Neighboring chimera's regions appear to be heavily intersecting which means that chimeras with different head number can coexist. Finally, I will present the very first observation of three-dimensional chimera states in the form of spirally-rotating streaks with incoherent core and an incoherent ball within regular flow, which also emerge as cascades of multi-headed chimera states.

Synchronization transitions in the Kuramoto model with inertia

Simona Olmi
Istituto dei Sistemi Complessi, Italy
simona.olmi@gmail.com
Adrian Navas, Stefano Boccaletti, Alessandro Torcini

We have analyzed the classical Kuramoto model with inertia for fully coupled and diluted networks. In particular, we numerically characterized the first order transition from desynchronized to synchronized state in terms of different system sizes and different values of the mass Furthermore, we have studied the conditions for the emergence of metastable clusters and their stability in finite systems. Finally, we will discuss how the scenario modifies in presence of random dilution in the network.

Bifurcation analysis of chimera states

Oleh Omel'chenko
Weierstrass Institute, Germany
Oleh.Omelchenko@wias-berlin.de

Chimera states are coherence-incoherence patterns observed in homogeneous discrete oscillatory media with a non-local coupling. Despite their nontrivial dynamical nature, such patterns can be effectively analyzed in the framework of the continuum limit formalism. Based on the statistical physics concept of local mean field and the Ott-Antonsen invariant manifold reduction, one can explain typical bifurcation scenarios leading to the appearance of chimera states. This provides a natural classification of known coherence-incoherence patterns, which also can be applied to predict their new types.

Chimeras with multiple coherent regions

Ram Ramaswamy
University of Hyderabad, India
rr@uohyd.ac.in
Sangeeta Rani Ujjwal

Collective phenomena in nonlocally coupled systems has attracted considerable attention due to the existence of interesting states such as the chimeras. Given the ubiquity of such states, one need to explore the relationship between the form of the nonlocal coupling and the nature of resulting chimera. We investigate the chimeras in a system of phase oscillators with piecewise linear nonlocal coupling. We show that it is possible to design chimera states with any desired number of coherent (or incoherent) regions by suitably constructing the coupling kernel. Chimeras with odd numbers of coherent regions will have all clusters in-phase with one another, while chimeras with even numbers of clusters can have clusters that are out-of-phase with each other.

Stabilization of incoherence in the disordered Hamiltonian Mean Field model

Juan Restrepo
University of Colorado at Boulder, USA
juanga@colorado.edu
James D. Meiss

The Hamiltonian Mean Field model is an iconic model for the study of Hamiltonian systems with long-range interactions. We study the Hamiltonian Mean Field model with a heterogeneous distribution of the rotors' moments of inertia and coupling strengths. We show that when the parameters of the rotors are heterogeneous, finite size fluctuations can greatly modify the coupling strength at which the incoherent state loses stability by inducing correlations between the momentum and parameters of the rotors. For unimodal parameter distributions, we find an analytical expression for the modified critical coupling strength in terms of statistical properties of the parameter distributions and confirm our results with numerical simulations. We find numerically that these effects disappear for strongly bimodal parameter distributions.

Linear response theory for quasi-stationary states in long-range systems

Stefano Ruffo
Florence University Italy, Italy
stefano.ruffo@gmail.com
Aurelio Patelli

N-body long-range interacting systems get trapped in long-living out-of-equilibrium states, called quasi-stationary states (QSS), whose lifetime diverges algebraically with system size N. We have studied the influence of a small external perturbation acting on a stationary stable QSS. In the large N limit, the time evolution of a QSS is well described by the Vlasov equation, which has an infinite number of stationary solutions that can be either homogeneous or inhomogeneous in space. We have developed a linear response theory for homogeneous QSS using the Fourier-Laplace techniques usually applied in the theory of linear Landau damping. The theory allows us to compute the time evolution of a generic observable when a small perturbing field is added to the Hamiltonian of the system. We have also developed an "approximate" linear response theory for inhomogeneous states, for which the standard Fourier-Laplace approach cannot work because of the coupling between different mean-field Fourier modes and the presence of an infinite number of conserved quantities, the Casimirs of the Vlasov equation. The theory is applied the Hamiltonian Mean Field (HMF) model, which describes the motion of particles on a unitary circle which interact all-to-all, and is tested against numerical simulations.

Noninvasive Control of Synchrony in Networks Coupled with Heterogeneous Delays

Eckehard Schoell
TU Berlin, Germany
schoell@physik.tu-berlin.de
Chol-Ung Choe, Ryong-Son Kim, Philipp Hövel

We consider a network of Hopf normal forms (Stuart-Landau oscillators) coupled with heterogeneous delays. By tuning the coupling parameters, different synchronization patterns, i.e., in-phase, splay, and clustering, can be selected. Our coupling scheme allows for arbitrary delays independent of the period of the synchronized periodic orbit. The characteristic equation for Floquet exponents of the heterogeneous delay network is derived in an analytical form, which reveals the coupling parameters for successful stabilization. The equation takes a unified form for both subcritical and supercritical Hopf bifurcations regardless of the synchronization patterns. The analysis of Floquet exponents and direct numerical simulations show that the heterogeneity in the delays drastically facilitates stabilization and provides an enlarged parameter region for successful control. Finally, we consider the thermodynamic limit in the framework of a mean field approximation and show that heterogeneous delays offer an enhanced performance of control.

Synchronization in Populations of Chemical Oscillators: Phase Clusters and Chimeras

Kenneth Showalter
West Virginia University, USA
kenneth.showalter@mail.wvu.edu
M. R. Tinsley, S. Nkomo

We have studied heterogeneous populations of chemical oscillators to characterize different types of synchronization behavior. The formation of phase clusters in stirred suspensions of Belousov-Zhabotinsky oscillators is described, where the (global) coupling occurs through the medium. We then describe the formation of phase clusters and chimera states in populations of photosensitive oscillators. The nonlocal coupling occurs via illumination intensity that is dependent on the state of each oscillator. The behavior of oscillators in ring configurations as a function of the number of oscillators is described, including traveling cluster states.

References

[1] A. F. Taylor et al., Angewandte Chemie Int. Ed. 50, 10161 (2011).

[2] M. R. Tinsley et al., Nature Physics 8, 662 (2012).

[3] S. Nkomo, et al., Phys. Rev. Lett. 110, 244102 (2013).

Linear Stability in networks of pulse-coupled neurons

Alessandro Torcini
Istituto dei Sistemi Complessi, Italy
alessandro.torcini@cnr.it
Simona Olmi, Antonio Politi

In a first step towards the comprehension of neural activity, one should focus on the stability of the various dynamical states. Even the characterization of idealized regimes, such as a perfectly periodic spiking activity, reveals unexpected difficulties. In this talk we discuss a general approach to linear stability of pulse-coupled neural networks for generic phase-response curves and post-synaptic response functions. In particular, we present: (i) a mean-field approach developed under the hypothesis of an infinite network and small synaptic conductances; (ii) a microscopic approach which applies to finite but large networks. As a result, we find that no matter how large is a neural network, its response to most of the perturbations depends on the system size. There exists, however, also a second class of perturbations, whose evolution typically covers an increasingly wide range of time scales. The analysis of perfectly regular, asynchronous, states reveals that their stability depends crucially on the smoothness of both the phase-response curve and the transmitted post-synaptic pulse. The general validity of this scenario is confirmed by numerical simulations of systems that are not amenable to a perturbative approach.

Mesoscopic limits of spatially extended stochastic networks and the emergence of synchrony

Jonathan Touboul
College de France, France
jonathan.touboul@college-de-france.fr

The cortex is a very large network characterized by a complex connectivity including at least two scales: a microscopic scale at which the interconnections are non-specific and very dense, while macroscopic connectivity patterns connecting different regions of the brain at larger scale are extremely sparse. This motivates to analyze the behavior of networks with multiscale coupling, in which a neuron is connected to its $v(N)$ nearest-neighbors where $v(N) = o(N)$, and in which the probability of macroscopic connection between two neurons vanishes. These are called singular multi-scale connectivity patterns. We introduce a class of such networks and derive their continuum limit. We show convergence in law and propagation of chaos in the thermodynamic limit. The limit equation obtained is an intricate non-local McKean-Vlasov equation with delays which is universal with respect to the type of micro-circuits and macro-circuits involved. The dynamics of these systems will be briefly investigated. We will show that these equations relate to the classical nonlocal neural field equation, in which noise acts as a parameter. Bifurcation analysis will show that these limit support spatially homogeneous solutions that are destabilized by the level of noise, yielding synchronized oscillations in law.

Dynamics of coupled oscillators with plasticity

Serhiy Yanchuk
Humboldt University, Germany
yanchuk@math.hu-berlin.de
Oleksandr Popovych, Peter Tass, Leonhard Lücken

In this talk, I would like to review our recent results on oscillatory neural networks with spike timing-dependent plasticity. In particular, we studied the effects of independent noise and coupling on the stability of the synchronized as well as the desynchronized states. We found that the mean synaptic coupling in such systems increases dynamically in response to the increase of the noise intensity, and there is an optimal noise level, where the amount of synaptic coupling gets maximal. We discuss also the conditions, under which the spike timing-dependent plasticity (STDP) can be reduced to the phase-difference dependent plasticity.

Chimera-like states in an ensemble of globally coupled oscillators

Azamat Yeldesbay
University of Potsdam, Germany
azayeld@gmail.com
Arkady Pikovsky, Michael Rosenblum

We demonstrate emergence of a complex state in a homogeneous ensemble of globally coupled identical oscillators, reminiscent of chimera states in nonlocally coupled oscillator lattices. In this regime some part of the ensemble forms a regularly evolving cluster, while all other units irregularly oscillate and remain asynchronous. We argue that chimera emerges because of effective bistability which dynamically appears in the originally monostable system due to internal delayed feedback in individual units. Additionally, we present two examples of chimeras in bistable systems with frequency-dependent phase shift in the global coupling.

Chimera death: a bridge between chimera states and oscillation death

Anna Zakharova
TU Berlin, Germany
Anna.Zakharova@tu-berlin.de
Marie Kapeller, Eckehard Schoell

For a network of generic oscillators with nonlocal topology and symmetry-breaking coupling we establish novel partially coherent inhomogeneous spatial patterns, which combine the features of chimera states (coexisting incongruous coherent and incoherent domains) and oscillation death (oscillation suppression), which we call chimera death. We show that due to the interplay of nonlocality and breaking of rotational symmetry by the coupling two distinct scenarios from oscillatory behavior to a stationary state regime are possible: a transition from an amplitude chimera to chimera death via in-phase synchronized oscillations, and a direct abrupt transition for larger coupling strength. Since we consider a generic model of coupled self-sustained oscillators, and networks are found in diverse areas of physics, chemistry, biology, and engineering, our results can be applied to a wide class of systems ranging from laser models, communication networks and power grids to biological networks. We believe that they are of particular importance for the life sciences. For instance, these peculiar hybrid states may account for the observation of partial synchrony in neural activity, like unihemispheric sleep etc.

Special Session 14: Reaction Diffusion Equations and Applications

Jerome Goddard II, Auburn University Montgomery, USA
Ratnasingham Shivaji, University of North Carolina Greensboro, USA

Recent developments in reaction diffusion equations have greatly increased their importance and usefulness in modeling physical and biological phenomena in many disciplines. The application of reaction diffusion is seemingly endless with their use naturally arising in areas such as biology, ecology, chemistry, geology, physics, and engineering. Investigation of the structure of steady states for such models yields interesting nonlinear elliptic boundary value problems of varied types. Even with the study of elliptic boundary value problems having such a rich mathematical history dating back to the 1960s, much is still unknown about the structure of solutions to such problems. This session will facilitate the exploration of current applications of reaction diffusion, proof techniques, and open questions of nonlinear elliptic boundary value problems.

Positive solutions of superlinear semipositone problems

Maya Chhetri
UNC Greensboro, USA
maya@uncg.edu

We will summarize the results concerning the solvability of classes of superlinear semipositone problems for positive solutions. Both scalar equation and system of equations will be discussed.

On quasilinear Sturm-Liouville problem with weights

Pavel Drabek
University of West Bohemia, Czech Rep
pdrabek@kma.zcu.cz
Komil Kuliev

We consider Sturm-Liouville problem for the quasilinear second order equation with weights on finite or, possibly, infinite interval. We prove that conditions on weights, which are equivalent to the compactness of the embedding of certain weigted Sobolev and Lebesgue spaces, are also equivalent to the fact that the eigenvalues and eigenfunctions have the usual properties as those for a "standard" Sturm-Liouville problem on a finite interval. Some consequences for the radial problem on the entire R^N will be discussed as well. This is a joint work with Komil Kuliev and it was published in Bull. Belg. Math. Soc. Simon Stevin 2012.

Halo-shaped bifurcation curves in ecological systems

Jerome Goddard II
Auburn University Montgomery, USA
jgoddard@aum.edu
R. Shivaji

We examine the structure of positive steady state solutions for a diffusive population model on a one-dimensional patch with logistic growth and negative density dependent emigration on the boundary. In particular, negative density dependent emigration is modeled via a nonlinear boundary condition that depends on both the population density and the diffusion coefficient. We will discuss the structure of positive steady state solutions, including the existence of Halo-shaped bifurcation curves.

Diffusion-driven Instability for Nonautonomous Problems

Georg Hetzer
Auburn University, USA
hetzege@auburn.edu

Reaction-diffusion systems on domains evolving in time can be studied by transforming the original system into a reaction-diffusion system on a fixed domain with time-dependent diffusion coefficients and non-autonomous reaction terms. If the original system satisfies no flux boundary conditions, a spatially homogeneous solution can be stable in the absence of diffusion (as a solution of the resulting system of ordinary differential equations), but unstable in the presence of diffusion. I will present conditions for this type of diffusion-driven instability based on results of joint work with A. Madzvamuse and Wenxian Shen and discuss work in progress.

Integrodifference Models for Persistence in Temporally Varying River Environments

Jon Jacobsen
Harvey Mudd College, USA
jacobsen@g.hmc.edu
Yu Jin and Mark Lewis

We consider integrodifference population models for growth and dispersal in the presence of advective flow and study population persistence in the context of both periodic and random kernel parameters. For the random setting we consider two persistence metrics and show they are mathematically equivalent.

Singular quasilinear elliptic problems on unbounded domains

Lakshmi Kalappattil
University of West Bohemia, Czech Rep
lakshmi@kma.zcu.cz
Pavel Drabek

We study the existence of positive solutions to problems of the form

$$\begin{cases} -\Delta_p u = \lambda K(x) f(u) & \text{in } B_1^c, \\ u = 0 & \text{on } \partial B_1, \\ u(x) \to 0 & \text{as } |x| \to \infty, \end{cases}$$

where

$$B_1^c = \{x \in \mathbb{R}^n \mid |x| > 1\},$$
$$\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u), p \in (1, n),$$

λ is a positive parameter, K belongs to a class of functions which satisfy certain decay assumptions and f belongs to a class of $(p-1)-$subhomogeneous functions which may be singular at the origin, namely $\lim_{s\to 0^+} f(s) = -\infty$. We use the method of sub and super solutions to prove our result. Our methods can be also applied to establish a similar existence result when the domain is entire $\mathbb{R}^n$.

Positive solutions for elliptic equations with nonlinear boundary conditions arising in a theory of thermal explosion

Eunkyung Ko
Tata Institute Fundamental Research Center for Applicable Mathematics, India
ekko1115@gmail.com
Peter Gordon, R. Shivaji

We consider a model of thermal explosion which is described by positive solutions to the boundary value problem

$$\begin{cases} -\Delta u = \lambda f(u), & x \in \Omega, \\ \mathbf{n}\cdot\nabla u + c(u)u = 0, & x \in \partial\Omega, \end{cases}$$

where $f, c : [0,\infty) \to (0,\infty)$ are C^1 and $C^{1,\gamma}$ non decreasing functions satisfying $\lim_{u\to\infty} \frac{f(u)}{u} = 0$, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$ and $\lambda > 0$ is a parameter. Using method of sub and super-solutions we show that solution of this problem is unique for large and small vales of parameter λ, whereas for intermediate values of λ solutions are multiple provided nonlinearity f satisfies some natural assumptions. An example of such nonlinearity which is most relevant to applications and satisfies all our hypotheses is $f(u) = \exp[\frac{\alpha u}{\alpha+u}]$ for $\alpha \gg 1$. Also, we extend our results to the problem on a exterior domain

$$\begin{cases} -\Delta u = \lambda K(|x|)f(u), & x \in \Omega_E, \\ \mathbf{n}\cdot\nabla u + c(u)u = 0, & |x| = r_0, \\ u(x) \to 0, & |x| \to \infty \end{cases}$$

where $\Omega_E = \{x \in \mathbb{R}^{\mathrm{N}} : |x| > r_0, N > 2, r_0 > 0\}$ and $K : [r_0,\infty) \to (0,\infty)$ is a continuous function such that $\lim_{r\to\infty} K(r) = 0$.

Solvability for p-Laplacian boundary value problems on a real line with dim (ker M)=2

Eun Kyoung Lee
Pusan National University, Korea
eunkyoung165@gmail.com
Chan-Gyun Kim, Jeongmi Jeong

The existence of at least one solution to the p-Laplacian nonlocal boundary value problems on a real line is investigated by using the Extention of Mawhin's continuation theorem.

Energy space in the study of linear problem

Marcello Lucia
College of Staten Island, CUNY, USA
marcello.lucia@gmail.com
V. Moroz, S. Prashanth

For Schrodinger operators with a regular potential it is well known that the non-negativity of the operator is equivalent to the existence of a positive supersolution. The aim of this talk, is to discuss this equivalence when the potential is only locally integrable.

Bifurcation and multiplicity for elliptic equations with nonlinear boundary conditions

Nsoki Mavinga
Swarthmore College, USA
mavinga@swarthmore.edu
M.N. Nkashama

We present multiplicity results for solutions of second order elliptic partial differential equations with nonlinear boundary conditions. We establish a priori estimates and use topological degree and bifurcation from infinity arguments.

A priori bounds for positive solutions of subcritical elliptic equations

Rosa Pardo
Complutense University, Spain
rpardo@ucm.es
Alfonso Castro

We provide a-priori L^∞ bounds for positive solutions to a class of subcritical elliptic problems in bounded C^2 domains. Our arguments rely on the *moving planes method* applied on the *Kelvin transform* of solutions. We prove that, locally, the image through the inversion map of a neighborhood of the boundary contains a convex neighborhood; applying the moving planes method, we prove that the transformed functions have no extremal point in a neighborhood of the boundary of the inverted domain. Retrieving the original solution u, the maximum of any positive solution in the domain Ω, is bounded above by a constant multiplied by the maximum on an open subset strongly contained in Ω. The constant and the open subset depend only on geometric properties of Ω, and are independent of the non-linearity and on the solution u. Our analysis answers a longstanding open problem.

On the large-time behavior of solutions of semilinear heat equations on the entire space

Peter Polacik
University of Minnesota, USA
polacik@math.umn.edu

We consider semilinear heat equations on R^N. We summarize recent results concerning the large-time behavior of bounded solutions. In particular, we mention several examples of nonconvergent bounded solutions.

Resonance problems for the p-Laplacian Fucik Spectrum

Stephen Robinson
Wake Forest University, USA
sbr@wfu.edu
Pavel Drabek

We consider the boundary value problem

$$-\Delta_p u = \lambda u^+ - \mu u^- + g(u) + h, \text{ in } \Omega,$$
$$u|_{\partial\Omega} = 0,$$

where $-\Delta$ is the p-Laplacian, (λ, μ) is a point in the associated Fucik Spectrum, g is a bounded continuous function, $h \in L^\infty$, and Ω is a smooth bounded domain. We prove that if an appropriate Landesman-Lazer condition is satisfied, then this resonance problem has at least one weak solution. The novelty lies in the fact that (λ, μ) lies outside of the range of values considered in previous papers.

Antimaximum principle in exterior domains

Sarath Sasi
University of West Bohemia, Czech Rep
sasi@kma.zcu.cz
Anoop T. V. , Pavel Drabek, Lakshmi Sankar

We discuss the antimaximum principle for

$$-\Delta_p u = \mu K(x)|u|^{p-2}u + h(x) \quad \text{in } B_1^c,$$
$$u = 0 \quad \text{on } \partial B_1,$$

where Δ_p is the p-Laplace operator with $p > 1$, and B_1^c is the exterior of the closed unit ball in $\mathbb{R}^N$ with $N \geq 1$ and $h \geq 0$. The weight function K is such that suppK^+ is of non-zero measure and $|K| \leq w$ for some appropriate choice of a positive weight function $w \in L^1(1, \infty)$.

Positive solutions for a class of superlinear semipositone systems on exterior domains

Ratnasingham Shivaji
University of North Carolina at Greensboro, USA
shivaji@uncg.edu

We study the existence of a positive radial solution to the nonlinear eigenvalue problem:

$$\begin{aligned} -\Delta u(x) &= \lambda k_1(|x|) f(v(x)); \; x \in \Omega_e \\ -\Delta v(x) &= \lambda k_2(|x|) g(u(x)); \; x \in \Omega_e \end{aligned}$$

$$\begin{aligned} u = 0 = v \quad &; \quad |x| = r_0 (> 0) \\ u \to 0, v \to 0 \quad &; \quad |x| \to \infty \end{aligned}$$

where $\lambda > 0$ is a parameter, $\Delta u = \text{div}(\nabla u)$ is the Laplacian operator, $\Omega_e = \{x \in \mathbb{R}^N \,|\, |x| > r_0, N > 2\}$ and $k_i \in C^1([r_0, \infty), (0, \infty))$: $i = 1, 2$ are such that $k_i(|x|) \to 0$ as $|x| \to \infty$. Here $f : [0, \infty) \to \mathbb{R}$ and $g : [0, \infty) \to \mathbb{R}$ are C^1 nondecreasing functions such that they are negative at the origin (semipositone) and superlinear at infinity. We establish the existence of a positive solution for $\lambda \approx 0$ via degree theory and rescaling arguments.

Analytic global bifurcation and infinite turning points for very singular problems

Prashanth Srinivasan
TIFR, India
pras@math.tifrbng.res.in
B. Bougherara, J. Giacomoni and S. Prashanth

Let Ω be a domain in $\mathbb{R}^N, N \geq 2$, $\lambda > 0$ a bifurcation parameter and $f : \mathbb{R} \to \mathbb{R}$ a "real analytic" type map such that $f(t)$ has superlinear growth as $t \to \infty$. We consider semilinear elliptic PDEs with the presence of a strong singular term as below:

$$(P_\lambda) \qquad \begin{cases} -\Delta u = \lambda(u^{-\delta} + f(u)) & \text{in } \Omega, \\ u > 0 \quad \text{in } \Omega, \; u|_{\partial\Omega} = 0. \end{cases}$$

Here the singular exponent δ is allowed to be any positive number. We are interested in this work to analyse the problem (P_λ) using the framework of analytic bifurcation theory as developed in the works of Buffoni, Dancer and Toland. We obtain an analytic global unbounded path of solutions to (P_λ) for any $\delta > 0$ using this framework.

Travelling waves in a Fisher-Kolmogorov-type model with degenerate diffusion and nonsmooth reaction

Peter Takac
University of Rostock, Germany
peter.takac@uni-rostock.de

We consider the nonlinear evolutionary problem for an unknown function $u = u(x, t)$,

$$\begin{cases} \partial_t u = \partial_x \left(|\partial_x u|^{p-2} \partial_x u \right) + f(u), \; x \in \mathbb{R}^1, \; t > 0; \\ u(\,\cdot\,, 0) = u_0 \quad \text{in } \mathbb{R}^1. \end{cases}$$

Here, $1 < p < \infty$,$f \colon \mathbb{R} \to \mathbb{R}$ is of Fisher-Kolmogorov-type, and $u_0 \colon \mathbb{R}^1 \to \mathbb{R}$ are given data between the two extremal zeros 0 and 1 of f. Under some "natural" hypotheses on the nonlinearities, we first derive the existence and uniqueness of a *travelling wave* (valued in $(0, 1)$ or $[0, 1]$), then discuss applications to (i) the existence and uniqueness of weak (semigroup) solutions and (ii) the existence and stability of monotone travelling waves connecting the equilibrium states 0 and 1. The monotone travelling waves connecting the equilibrium states 0 and 1 can either only approach these equilibria at $\pm\infty$, or else attain them at finite points, depending on the interaction between the degenerate / singular diffusion and the nonsmooth reaction function. Then we discuss the approach to such travelling waves by solutions with rather general initial data that are sqeezed between two travelling waves (that are each other's shift).

Weighted quasilinear eigenvalue problems in exterior domains

Anoop Thazhe Veetil
University of West Bohemia, Czech Rep
anoop@kma.zcu.cz
P. Drabek, Sarath Sasi

We consider the weighted eigenvalue problem in the exterior domain

$$-\Delta_p u = \lambda K(x)|u|^{p-2}u \quad \text{in } B_1^c,$$
$$u = 0 \quad \text{on } \partial B_1,$$

where Δ_p is the p-Laplace operator for $p \in (1, \infty)$, and B_1 is the closed unit ball in $\mathbb{R}^N$, $N \geq 1$. We consider both the cases $N \in (p, \infty)$ and $N \in [1, p]$. For some appropriate choice of $w \in L^1(1, \infty)$ with $w \geq 0$, we prove that the Beppo-Levi space $\mathcal{D}_0^{1,p}(B_1^c)$(:= the completion of $\mathcal{C}_c^\infty(B_1^c)$ with respect to $||\nabla u||_p$ norm) is continuously and compactly embedded into $L^p(B_1^c; w(|x|))$. Using this embedding, we prove the existence of a positive principal eigenvalue for K such that $K^+ \not\equiv 0$ and $|K(x)| \leq w(|x|)$.

Quasilinear elliptic equations via perturbation methods

Zhi-Qiang Wang
Utah State University, USA
zhi-qiang.wang@usu.edu

For a class of quasilinear elliptic problems we developed a perturbation approach which allows us to overcome difficulties of both smoothness and compactness involved.

Special Session 15: Geometric and Variational Techniques in the N-Body Problem

Vivina L. Barutello, Universitá degli studi di Torino, Italy
Alessandro Portaluri, Universitá degli studi di Torino, Italy

Recently there have been significant mathematical breakthroughs in Celestial Mechanics, concerning difficult problems: always more precise information about the linear stability and instability of some relative equilibrium configurations as well as of some collisionless periodic solutions, discovery of new quasiperiodic or periodic orbits with new kinds of symmetries, new results on non-integrability, sharper results from averaging methods and KAM theory. The purpose of this special session is to bring the main specialists in the N-body problem together with the leaders of different areas in dynamical systems (Variational Methods, Mather Theory, Hamiltonian dynamics, Ergodic Theory) trying to build a theoretical framework suitable to attack some of the many unsolved aspects of the N-body problem.

A new integrable extension of the 2 fixed centre problem

Alain Albouy
Observatoire de Paris, CNRS, France
albouy@imcce.fr

The two fixed centres problem was proposed and solved by Euler around 1760. Lagrange was the first, in 1769, to propose a force field that can be added to Euler's one while keeping the integrability. Since that time, other extensions have been proposed. I will propose a new one. The integrability may be discovered through simple remarks about the quadratic first integrals.

Variational approach to Poincaré's second species solutions of the nonrestricted 3 body problem

Sergey Bolotin
Department of Mathematics, University of Wisconsin, USA
bolotin@math.wisc.edu

We consider the plane 3 body problem with 2 of the masses small. Periodic solutions with near collisions of small bodies were named by Poincaré second species periodic solutions. Such solutions shadow chains of collision orbits of 2 uncoupled Kepler problems. It is generally accepted that Poincaré's arguments do not provide a proof of the existence of second species solutions. Rigorous proofs appeared much later and only for the restricted body problem. We develop a variational approach to the existence of second species periodic solutions for the nonrestricted 3 body problem. It turns out that Poincaré's proof is essentially correct. The talk is based on a work with Piero Negrini.

Non collision periodic solutions for the planar N-center problem with mild topological assumptions

Roberto Castelli
VU University Amsterdam, Netherlands
r.castelli3@gmail.com
Alessandro Portaluri

We consider the planar N-centre problem with α-homogeneous potential, $\alpha \in [1, 2)$ and, for any $N \geq 3$ we prove the existence of infinitely many topologically distinct periodic solutions without collisions. Topologically distinct means that the solutions are not homotopy equivalent in the punctured plane. Indeed we provide sufficient (and weak) conditions on the classes of the fundamental group of $\mathbb{R}^2 \setminus \{c_1, \ldots, c_N\}$ which ensure the existence of non collision solutions. In particular the solutions are allowed to self intersect and to design complicated paths around the singularities. The proof is based on variational techniques and relies on the so-called obstacle problem.

Fixed point indices of central configurations

Davide L. Ferrario
University of Milano-Bicocca, Uruguay
davide.ferrario@unimib.it

We consider under which conditions on the potential equivalence classes of central configurations are homeomorphic to fixed points of suitable quotient maps, and relate fixed point indices of the latter with critical point indices of CCs.

Periodic and chaotic motions in the N-body problem with non-Newtonian forces

Giovanni Federico Gronchi
Università di Pisa, Italy
gronchi@dm.unipi.it
Giorgio Fusco

We consider the N-body problem with interaction potential $U_\alpha = \frac{1}{|x_i - x_j|^\alpha}$ for $\alpha > 1$. We assume that the particles have all the same mass and that N is the order $|\mathcal{R}|$ of the rotation group $\mathcal{R}$ of one of the five Platonic polyhedra. We study motions that, up to a relabeling of the N particles, are invariant under $\mathcal{R}$. By variational techniques we prove the existence of periodic and chaotic motions.

Linear stability and Morse index of relative equilibria in n-body-type problems

Riccardo Jadanza
Polytechnic University of Torino, Italy
riccardo.jadanza@polito.it
Vivina Barutello, Alessandro Portaluri

We consider a planar mechanical system consisting of n point particles subject to a potential which is invariant under rigid rotations. We address the question of *linear and spectral stability* for a particular class of periodic solutions termed *relative equilibria*, providing a rather general sufficient condition for their instability. Notable applica-

tions are represented by two generalisations of the n-body problem involving two kinds of singular potentials: the α-homogeneous one, with $\alpha \in (0, 2)$, which includes the gravitational case, and the logarithmic one. More specifically, we focus on the 3-body-type problem (governed by the aforementioned potentials) and compute the Morse index of the Lagrangian circular orbit and of its iterates by availing ourselves of some symplectic techniques and an index theorem involving the Maslov index. This is a joint work with Vivina Barutello and Alessandro Portaluri.

KAM stability of the Eight

Tomasz Kapela
Jagiellonian University, Poland
kapela@ii.uj.edu.pl
Carles Simó

KAM Theorem is a fundamental result for Hamiltonian systems because it ensures the existence of a set, nowhere dense but of positive measure, of points of the phase space which behave in a regular, quasi-periodic way. We present a methodology for computer assisted proofs of the existence and the KAM stability of an arbitrary periodic orbit for Hamiltonian systems. In short, using interval arithmetics and rigorous ODE solvers we are able to get verified bounds for coefficients of the Birkhoff normal form of a suitable Poincaré map. These estimates allow to check the KAM conditions. We applied this algorithm to the 3-body problem and we proved the KAM stability of the well-known figure eight orbit and two selected orbits of the so called family of rotating Eights. They are examples of the so called choreographic solutions in which all bodies travel along the same curve with constant phase shift (in case of rotating Eights in suitable rotating frame).

On the integrability of the three-body problem

Ezequiel Maderna
Universidad de la Republica, Uruguay
emaderna@cmat.edu.uy

According to Poincare, the Newtonian three-body problem admits no other first integrals in addition to those we know within a class of algebraic functions. In this lecture we will prove that the critical Hamilton-Jacobi equation admits no smooth homogeneous solutions. As we shall see, this result quite obstructs the possibility of smooth complete integrability of the problem in the sense of Liouville.

Complex analytic aspects of the 3-tangent theorem on the planar 3-body problem

Katsumi Matsuda
Tokai University, Japan
monk@tokai-u.jp

We reformulate, in the complex analytic context, the 3-tangent theorem of Fujiwara-Fukuda-Ozaki; if planar three body orbits has zero linear momentum and zero angular momentum, then three tangent lines at bodies meet at a point. Considering the singularities, points at infinity on projective plane, and ramificaiton, we extend their study of equal mass three-body choreography on the real lemniscate $(x^2 + y^2)^2 = x^2 - y^2$ to complex projective context, and show that the intersection points of three tangent lines in the lemniscate case draw a conic, which is the rectangular hyperbola $z^2 - w^2 = 1$ in C^2 with coordinate system (z, w). And we also describe relate topics.

Classification of Symmetries of Planar Choreographies

James Montaldi
University of Manchester, England
j.montaldi@manchester.ac.uk

A choreography is a periodic solution of the N-body problem where the N bodies follow each other at regular intervals around a single closed path. Since Chenciner and Montgomery's (re)discovery of the figure-8 choreography in 2000, there has been considerable interest in finding new ones. The approach is usually to use variational methods on the appropriate loop space and the proofs have been a combination of symmetry arguments and analysis. In this talk I will describe a complete classification of all the possible symmetry groups arising in this problem. It is joint work with my PhD student Katrina Steckles.

Minimization and Instability for families of homographic solutions

Daniel Offin
Queen's University, Canada
offind@mast.queensu.ca
Abdalla Mansur, Mark Lewis

For certain families of homographic periodic solutions in the N-body problem, we can use a variational principle to characterize the geometric structure of these solutions. As a by product of this approach we can also make predictions concerning their stability type. We use a variant of the Maslov index together with a minimization argument to describe unstable periodic solutions of the four body problem.

Three tangents theorem in three-body motion in three-dimensional space

Hiroshi Ozaki
Tokai University, Japan
ozaki@tokai-u.jp
Toshiaki Fujiwara, Hiroshi Fukuda and Tetsuya Taniguchi

The three-body motions with zero angular momentum is restricted to be the planar motion. The equal mass three-body figure eight choreography which was found by Moore, Chenciner and Montgomery is one of the three-body planar motions. The figure eight choreography follows that the three tangent lines from the three bodies meet at a point (infinity is allowed) at any time. Actually, this property holds for any zero-angular momentum three-body motion and we can show that "If the linear momentum and the angular momentum are zero in the three-body motion, three-tangent lines meet at a point or three tangent lines are parallel." About ten years ago, this was proved by Fujiwara, Fukuda, and Ozaki, and they called it three-tangents theorem. The three-tangent theorem can be extended to the three-body motion with non-vanishing constant angular momentum in 3 dimension. If we project three tangent lines into any plane parallel to the total angular momentum, the projected three tangent lines intersect with each other at a point or three tangent lines are parallel. The proof will be given in this talk, and we will show an example.

The N-body problem in spaces of constant curvature

Ernesto Perez-Chavela
Universidad Autonoma Metropolitana Iztapalapa, Mexico
epc@xanum.uam.mx
F. Diacu, L. Garcia-Naranjo, J.C. Marrero

We consider N–point positive masses moving on a space of constant curvature K. We describe the extension of the Newtonian potential to these spaces, show some geometric aspects af this problem and study some families of relative equilibria (solutions where the mutual distances among the particles remain constant for all time), in both cases with positive or negative curvature.

Index theory in Celestial Mechanics: recent results and new perspectives

Alessandro Portaluri
University of Turin, Italy
alessandro.portaluri@unito.it

In the last decades a zoo of new symmetric periodic collisionless orbits for the N-body problem appeared in the literature as minimizers of the Lagrangian action functional. Certainly one of the key features of such orbits is their Morse index, as well as their linear (in)stability properties. A central device useful to deeply understand these questions is based on a well-known symplectic invariant: the Maslov index. In this talk we revise some important achievements in the field, stressing the importance of a Bott-type iteration formula for periodic solutions which are the object of a finite group action, and we present some interesting perspectives necessary to try to penetrate the intricate dynamics of the problem.

Stability of Relative Equilibria in the Planar N-Vortex Problem: A Topological Approach

Gareth Roberts
College of the Holy Cross, USA
groberts@holycross.edu

In the weather research and forecasting models of certain hurricanes, vortex crystals are found within a polygonal-shaped eyewall. These special configurations can be interpreted as relative equilibria (rigidly rotating solutions) of the point vortex problem introduced by Helmholtz. Their stability is thus of considerable importance. Adapting Moeckel's approach for the n-body problem, we present some theory and results on the linear and nonlinear stability of relative equilibria in the planar n-vortex problem. A topological approach is taken to show that for the case of positive circulations, a relative equilibrium is linearly stable if and only if it is a nondegenerate minimum of the Hamiltonian restricted to a level surface of the angular impulse (moment of inertia). Using a criterion of Dirichlet's, this implies that any linearly stable relative equilibrium with positive vorticities is also nonlinearly stable. Two symmetric families, the rhombus and the isosceles trapezoid, are analyzed, with stable solutions found in each case. As time permits, we will discuss how some of these ideas, at least numerically, can be applied to the n-body problem.

Variational proof of the existence of the super-eight orbit in the four-body problem

Mitsuru Shibayama
Osaka University, Japan
shibayama@sigmath.es.osaka-u.ac.jp

Using the variational method, Chenciner and Montgomery proved the existence of an eight-shaped periodic solution of the planar three-body problem with equal masses. Just after the discovery, Gerver have numerically found a similar periodic solution called "super-eight" in the planar four-body problem with equal mass. In this talk I will prove the existence of the super-eight orbit by using the variational method. The difficulty of the proof is to eliminate the possibility of collisions. In order to solve it, we apply the scaling technique established by Tanaka and investigate the asymptotic behavior of a binary collision.

Parabolic trajectories of the N-dody problem. A variational approach.

Susanna Terracini
University of Torino, Italy
susanna.terracini@unito.it
V. Barutello, G. Verzini

Zero energy entire solutions to the full N-body problem with diverging radii are called parabolic solutions. In spite of of their natural structural instability, these orbits act as connections between different central configurations and can be used as carriers from one to the other region of the phase space. In the recent papers, we have linked the presence of minimal parabolic orbits with the existence of minimal collision trajectories and the detection of unbounded families of noncollision periodic orbits. The topologically non-trivial parabolic orbits are of interest also from the point of view of weak KAM theory, as they are homoclinic to the infinity, which represents the Aubry-Mather set of our system. They can be used to construct multiple viscosity solutions of the associated Hamilton-Jacobi equation. We shall is to deal with the existence of homoclinic and heteroclinic trajectories linking central configurations of the full N-body problem, starting with the case when the configuration space is reduced by symmetries (platonic, dihedral).

Special Session 16: Optimal Control and its Applications

Alexander J. Zaslavski, The Technion - Israel Institute of Technology, Israel

A special session on "Optimal Control and its applications" will bring together a selected group of experts in this area. The growing importance of optimal control has been realized in recent years. This is due not only to theoretical developments in this area, but also because of numerous plications to engineering, economics and life sciences. Approximately 25 participants from Australia, France, Germany, Great Britain, Israel, Russian and USA will participate in the session. The topics which will be discussed include infinite horizon control problems, turnpike phenomenon, averaging in optimal control, optimality conditions in control problems, qualitative and quantitative aspects of optimal control, control and stabilization of PDEs.

Positional Strategies in a Motion Correction Problem for the Linear System with Integral Constraints

Boris Ananyev
Institute of Mathematics, Yekaterinburg, Russia
abi@imm.uran.ru

Consider a motion correction problem for the linear system with integral constraints on disturbances. It is supported that the phase vector $x(t)$ of the system is unknown, but one can observe some linear transformation $y(t)$ of this phase vector with an additive noise. Due to measurements, the information set $X(t, y(\cdot))$ containing the true vector $x(t)$ can be built. The goal of the controller (1-st player) is to minimize the terminal cost $\Phi(X(T, y(\cdot)))$. The control strategies is formed by functions of the form $u(t, X(t, y(\cdot)))$ depending on the position $(t, X(t, y(\cdot)))$. The initial minimax problem of motion correction can be reduced to the differential game with complete information, in which the second player may choose the disturbances. We prove the existence of the saddle point of the game and suggest a constructive method of building of optimal strategies. The results of N.N. Krasovski, A.B. Kurzhanski, and A.I. Subbotin are used. Some results of computer simulations are given. An application to the problem of coordinate alignment of navigation devices for a transport ship-airplane system is also considered.

Comparison of topological methods for solving nonlocal multivalued problems in abstract spaces

Irene Benedetti
University of Perugia, Italy
irene.benedetti@dmi.unipg.it

The interest for multivalued equations in abstract spaces is motivated by the study of control problems. Different nonlocal boundary conditions, including periodic, anti-periodic, mean value and multi-point conditions, are needed for different applications. The existence of solutions for these problems is frequently studied with topological techniques based on fixed point theorems for a suitable solution operator. This requires strong compactness conditions, which are very hard to check in an infinite dimensional framework. To weaken these hypotheses, several approaches can be used. A first technique is based on the concept of measure of non-compactness combined with a topological degree theory. Alternatively, weak topologies can be exploited, joined with the classical Ky Fan Fixed Point Theorem. Finally, one can consider the multivalued problem in an Hilbert space compactly embedded into a Banach space, in connection with Hartman-type conditions. The purpose of this talk is to compare these different approaches, discussing their positive sides and drawbacks.

Infinite-dimensional Discrete-time Pontryagin principles

Joel Blot
SAMM University Paris 1, France
blot@univ-paris1.fr

We present extensions to the infinite-dimensional framework of Pontryagin principles for infinite-horizon discrete-time problems of Optimal Control. These extensions notably use technics of ordered Banach spaces.

Optimal control of individual's health investments

Christine Burggraf
Leibniz IAMO, Germany
burggraf@iamo.de
Wilfried Grecksch, Thomas Glauben

Grossman's health investment model has been an important development in health economics. However, the model's derived demand function for medical care predicts the demand for medical care to increase if the individual's health status increases. Yet, empirical studies indicate the opposite relationship. In order to improve the informative value of the health investment model, this study introduces a reworked Grossman model by assuming a more realistic Cobb-Douglas investment function with decreasing returns to scale. The resulting dynamic utility maximization problem is tackled by optimal control theory; with individual's health capital and wealth being the two state variables and the amounts of consumed medical care and non-medical market goods being the two control variables. The optimization problem is solved in a deterministic but also in a more realistic stochastic model setting where uncertainty is introduced surrounding individual's health status. For this purpose, health capital is modelled as a linear generalized Brownian motion with drift. The reworked health investment model generates a demand function for medical care that predicts an increasing demand for medical care if the individual's stock of health capital decreases, a relationship that is substantially supported by empirical results. Finally, our empirical analysis highlights the derived theoretical predictions.

On Nonsmooth Maximum Principles for problems with state and mixed constraints

Maria do Rosario de Pinho
Universidade do Porto, Portugal
mrpinho@fe.up.pt
Md. Haider Ali Biswas

Here we discuss a variant of the nonsmooth maximum principle for optimal control problems with both pure state and mixed state and control constraints. Such result includes a Weierstrass condition together with an Euler adjoint inclusion involving the joint subdifferentials with respect to both state and control, generalizing previous results. This is accomplished with the help of well known penalization techniques for state constrained problem together with an appeal to a recent nonsmooth maximum principle for problems with mixed constraints.An important feature of our result is the bounded slope condition. Of particular interest is our discussion on the applicability of the main results done with some examples.

On worst-case optimal investment and consumption under a stochastic interest rate

Tina Engler
Martin Luther University Halle-Wittenberg, Germany
tina.engler@mathematik.uni-halle.de

We investigate an investment-consumption problem under the threat of a market crash, where the interest rate of the bond is stochastic. Inspired by the recent work of Desmettre et al. (2013), we model the market crash as an uncertain event (τ, l), where τ denotes the crash time and l the crash size. While the stock price is driven by a linear SDE at normal times $t\tau$, it loses a fraction l of its value at the crash time τ. The investor wants to maximize the expected discounted utility of consumption over an infinite time horizon in the worst-case scenario. We solve the problem by applying the following three ideas. First, we determine the optimal post-crash strategy by solving a classical stochastic control problem. Then, we reformulate the worst-case problem into a 'controller-vs-stopper' game in order to obtain the optimal pre-crash strategy. Finally, we apply a martingale approach and characterize the worst-case optimal investment-consumption strategy.

A dynamic domain decomposition method for optimal control problems

Maurizio Falcone
SAPIENZA - Universita di Roma, Italy
falcone@mat.uniroma1.it
S. Cacace, E. Cristiani

We present a new algorithm for the solution of Hamilton-Jacobi-Bellman equations related to optimal control problems . The main idea is to divide the domain of computation into subdomains following the dynamics of the control problem. This can result in a rather complex geometrical subdivision, but has the advantage that every subdomain is invariant with respect to the optimal dynamics. Exploiting this invariance speeds up the computation of the value function and also allows for an efficient parallelization, since the classical transmission conditions on the boundaries of the subdomains can be avoided. For this specific feature the subdomains are patches in the sense introduced by Ancona and Bressan. We present some properties of the method as well as several examples in dimension two and three.

Optimal control of the Saint-Venant equations coupled to a multibody system

Matthias Gerdts
University of the Federal Armed Forces Munich, Germany
matthias.gerdts@unibw.de
Sven-Joachim Kimmerle

We consider an optimal control problem subject to a coupled system of nonlinear hyperbolic partial differential equations and ordinary differential equations. The partial differential equation is given by the Saint-Venant equations (shallow water equations), which are hyperbolic transport equations modelling the motion of a fluid. The Saint-Venant equations are strongly coupled to the equations of motion of a mechanical multibody system through boundary conditions and generalized force terms. We discuss properties of the optimal control problem and employ a discretization scheme using a Lax-Friedrich scheme in space for its numerical solution. To this end a reduced optimization problem is derived and solved by a sequential quadratic programming method. For the computation of gradients an adjoint equation is being solved. Numerical results for an optimal braking maneuver of a truck with a basin filled with a fluid are discussed.

Optimal Epidemic Control at Changing Population Size

Ellina Grigorieva
Texas Woman's University, USA
egrigorieva@twu.edu
E. Khailov and A. Korobeinikov

The SIR (susceptible-infected-removed) model for the spread of epidemics in a population of a variable size N is considered. We control the spread of the infection by three bounded controls: $u(t)$ is vaccination of the susceptible, $w(t)$ is treatment of the infected, and $v(t)$ is other "indirect" strategies aimed at a reduction of the incidence rate. The model equations are:

$$\begin{cases} S'(t) &= \sigma N(t) - \dfrac{\beta}{1+v(t)}\dfrac{S(t)I(t)}{N(t)} \\ & \quad -(\mu+u(t))S(t), \quad t\in[0,T], \\ I'(t) &= \dfrac{\beta}{1+v(t)}\dfrac{S(t)I(t)}{N(t)} \\ & \quad -(\gamma+\delta+\mu+w(t))I(t), \\ R'(t) &= u(t)S(t)+(\gamma+w(t))I(t)-\mu R(t), \end{cases}$$

where σ and μ are birth and natural mortality rates, δ is infection induced mortality rate, β and γ are effective contact and recovery rates, respectively. Since $N(t) = S(t) + I(t) + R(t)$, then the equation for a size of a population

$$N'(t) = (\sigma - \mu)N(t) - \delta I(t),$$

is added into the considered system. Excluding $R(t)$, we then study properties of the system of the equations for $N(t)$, $S(t)$, and $I(t)$. This very nonlinear system is more complex than one for a constant population size. We state the optimal control problem of minimizing the number of

the infected at the terminal time T. The corresponding optimal solutions are obtained with the use of the Pontryagin maximum principle. The types of optimal controls are investigated analytically. Numerical results illustrate the optimal solutions.

Infinite-horizon optimal control problems for bounded processes

Nalia Hayek
Paris 2 University, France
naila.hayek@u-paris2.fr
Joel Blot

We provide necessary conditions and sufficient conditions of optimality for infinite-horizon optimal control problems for bounded processes in the discrete time case. These problems are governed by difference equations or inequations. Pontryagin maximum principles are established in the strong form and in the weak form. An application to a management problem for Internet quality services is given.

Optimal Control and Stability Analysis of an Epidemic model with Education Campaign and Treatment

Sanjukta Hota
Fisk University, USA
sanjuktahota@gmail.com
Folashade Agusto, Hemraj Joshi, Suzanne Lenhart

In this paper we investigated a SIR type epidemic model in which education campaign and treatment are both important for the disease management. Optimal control theory was used on the system of differential equations to achieve the goal of minimizing the infected population and slow down the epidemic outbreak. We did the stability analysis of the system and determined the basic reproduction number. The optimality system was solved numerically using the forward-backward sweep method and the corresponding results were presented with interpretation and comments.

Program control with probability one

Elena Karachanskaya
Pacific National University, Russia
ekarachanskaya@mail.khstu.ru

Consider the stochastic dynamical non linear system with the Wiener process and the Poisson jumps:

$$\begin{aligned} d\mathbf{x}(t) &= \Big(P(t;\mathbf{x}(t)) + Q(t;\mathbf{x}(t))\cdot \mathbf{s}(t;\mathbf{x}(t))\Big)dt \\ &+B(t;\mathbf{x}(t))d\mathbf{w}(t) + \int_\Gamma \Xi(t;\mathbf{x}(t);\gamma)\nu(dt;d\gamma), \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbf{R^n}$, $n \geq 2$; $\mathbf{w}(t)$ is the $m-$dimensional Wiener process; $\nu(t;\Delta\gamma)$ is the homogeneous with respect to t non centered Poisson measure. For such systems we construct the program control $\mathbf{s}(t;\mathbf{x}(t))$ with probability one (PCP1), which allows the system (1) to move on the given manifold $\{g(t;\mathbf{x}(t)) = 0\}$ for each $t \in [0;T]$, $T \leq \infty$. The control program $\mathbf{s}(t;\mathbf{x}(t))$ is solution for the algebraic system of linear equations. Considered method is based on the theory of the first integrals for stochastic differential equations systems.

On control synthesis for uncertain dynamical discrete-time systems through polyhedral techniques

Elena Kostousova
Institute of Mathematics and Mechanics, Russian Academy of Sciences, Ekaterinburg, Russia
kek@imm.uran.ru

Problems of feedback terminal target control for linear dynamical discrete-time systems without and with uncertainties are considered. There are known approaches to solving problems of this kind, including ones for differential systems, based on construction of solvability tubes. Since practical construction of such tubes may be cumbersome, the different numerical methods are devised. Among them computation schemes for linear systems based on the ellipsoidal techniques were proposed by A.B. Kurzhanski and then expanded to the polyhedral techniques by the author. Here we continue the development of methods of control synthesis for discrete-time systems using polyhedral (parallelotope-valued) solvability tubes. The cases without uncertainties, with additive parallelotope-valued uncertainties, and also with interval uncertainties in coefficients of the system are considered. Also polyhedral methods of control synthesis for the same systems under constraints on the state are proposed. The state constraints are described in terms of zones (i.e., intersections of strips). Control strategies, which can be calculated by explicit formulas on the base of polyhedral solvability tubes, are proposed. Recurrence relations, which describe the mentioned tubes, are presented for each of the mentioned cases. Results of computer simulations are presented.

Decomposition of Discrete Linear-Quadratic Optimal Control Problems for Two-Steps Systems

Galina Kurina
Voronezh State University, Russia
kurina@math.vsu.ru
Shahlar Meherrem

A discrete linear-quadratic optimal control problem for two sequentially acting controlled systems is considered. Matching conditions for trajectories at the switch point are absent, however the minimized functional depends on values of a state trajectory in the left and right sides from the switch point. State trajectories have fixed left and right points. Control optimality conditions in the maximum principle form are derived. The solvability of the considered problem is established. The algorithm for solving the problem is given, which is based on sequential solving some initial value problems. The formula for the minimal value of the performance index is also obtained. The similar result for continuous problems has been presented in [1].

References

[1] G.A. Kurina and Y. Zhou, Decomposition of Linear-Quadratic Optimal Control Problems for Two-Steps Systems, Doklady Mathematics, vol. 83, no. 2, 2011, pp. 275–277.

Optimal decay rates for the energy in systems arising in nonlinear mechanics with memory effects.

Irena Lasiecka
University of Memphis, USA
lasiecka@memphis.edu
Xiaojun Wang

We consider viscoelastic nonlinear plate equation with memory kernel quantified by the following differential inequality: $g' + H(g) \leq 0$ where $H(s)$ is a given continuous, positive, increasing, and convex function such that H(0) = 0. We shall show that the energy of the nonlinear PDE is driven by the same inequality. The results presented provide an uniform framework for obtaining optimal decay rates for the energy of nonlinear mechanical systems which contain memory effects. The study of PDE with a memory will be reduced to solving an appropriate nonlinear ODE systems. The method is based on the idea introduced in Lasiecka and Tataru in 1993 for determining decay rates of the energy given in terms of the function H(s). This new method allows to optimize the benefits (both geometric and topological) secured by a combination of dissipative mechanisms generated by both frictional and memory damping. The obtained results are quantitative and allow for optimal design of materials arising in nonlinear mechanical structures.

Lavrentiev phenomenon: its non occurrence for some multidimensional scalar problems of the calculus of variations

Carlo Mariconda
Università di Padova, Italy
maricond@math.unipd.it
Pierre Bousquet, Giulia Treu

We consider an autonomous integral functional defined in the space of Sobolev functions on an open and bounded set, and that agree with a prescribed Lipschitz function on the boundary of the domain. We show, without assuming growth conditions, that the Lavrentiev gap does not occur for a wide class of Lagrangeans.

About singular infinite horizon calculus of variation problems

Eladio Ocana
Instituto de Matematca y Ciencias Afines, Peru
eocana@imca.edu.pe

Throughout this talk we present some results concerning the optimal solution for this class of problems. For that we associate an auxiliary calculus of variations problem whose solutions connect as quickly as possible the initial conditions to some constant solutions. Then we deduce the optimality of these curves, called MRAPs (Most Rapid Approach Path), for the original problem.

New Contributions to Theory and Numerics for State-Constrained Elliptic Optimal Control Problems

Hans Josef Pesch
University of Bayreuth, Germany
hans-josef.pesch@uni-bayreuth.de
Michael Frey, Simon Bechmann, Armin Rund

Based on different reformulations of state-constrained elliptic optimal control problems with distributed control and a hypothesis on the structure of the active set, new necessary conditions are obtained which exhibit higher regularity of the multiplier associated with the state constraint. Moreover, we obtain also new jump and sign conditions. Measures are no longer an issue, so that regularization techniques become superfluous. Finally, the method can be fully described in function spaces which is an essential element to obtain a mesh-independent numerical method. The new approach mimics the well-known Bryson-Denham-Dreyfus indirect adjoining method which is the preferred ansatz in solving state-constrained optimal control problems with ordinary differential equations numerically. However, in the context of PDE constrained optimization this approach turns out to be extremely involved. Mathematically the reformulations lead to a new kind of set optimal control problem, where the active set of the state constraint, resp. the interface between the inactive and the active set are to be determined as part of the solution as the switching points in multipoint boundary value problems based on first-order necessary conditions for ODE constrained optimal control problems. Various formulations of this type of PDE optimization problem as shape and/or topology as well as bilevel optimization problem or, concerning the numerical solution, as free boundary value problem are discussed. Moreover, parallels can be drawn to optimization on vector bundels which seems to be essential for the design of an appropriate Newton-type method, since the optimization over sets of admissible active sets have a nonlineare structure. This requires an answer to the question how does a Newton method looks on a nonlinear manifold where no Banach space structure is present. Numerical results will demonstrate the performance of the new method. This presentation will open new research directions in PDE constrained optimal control as indicated in the final outlook.

Sufficient second-order optimality conditions for optimal control problems

Laurent Pfeiffer
Graz University, Austria
lrt.pfeiffer@gmail.com

In optimization, second-order sufficient optimality conditions consist in the existence of a Lagrange multiplier and the definite positivity of a certain quadratic form on a cone, called critical cone. They ensure the local optimality of a feasible point. Proving the local optimality in the case of an infinite number of constraints is difficult. In particular, for nonlinear optimal control problems with control contraints, the local optimality is difficult to obtain without a supplementary assumption, called strengthend Legendre-Clebsch assumption, implying that the Hessian of the Hamiltonian is definite positive for almost all time. In this talk, I will prove the local optimality without the Legendre-Clebsch assumption, with a new technique combining a decomposition principle and a natural extension of the critical cone in the space of Young measures.

Asymptotic controllability and optimal control

Franco Rampazzo
Padova University, Italy, Italy
rampazzo@math.unipd.it
M. Motta

We consider a control problem where the state must approach asymptotically a target C while paying an integral cost with a *non-negative* Lagrangian l. The dynamics f is just continuous, and no assumptions are made on the zero level set of the Lagrangian l. Through an inequality involving a positive number $\bar{p}_0$ and a Minimum Restraint Function $U = U(x)$ –a special type of Control Lyapunov Function– we provide a condition implying that **(i)** the system is asymptotically controllable, and **(ii)** the value function is bounded by $U/\bar{p}_0$. The result has significant consequences for the uniqueness issue of the corresponding Hamilton-Jacobi equation. Furthermore it may be regarded as a first step in the direction of a feedback construction.

Lie algebraic criteria for approximate ensemble controllability

Andrey Sarychev
University of Florence, Italy
asarychev@unifi.it

One considers ensembles (parameterized families) of nonlinear control systems $\dot{x} = f^\theta(x, u)$ with a parameter $\theta \in \Theta$ - finite-dimensional manifold. One is interested in controlling all the systems of the ensemble by means of a single θ-independent control $u(t)$. It is known, that achieving exact controllability in such setting would require, in general, infinite-dimensional controls. On the contrast we aim at criteria of *approximate ensemble controllability* by means of controls, which take their values in a space of fixed finite dimension. We approach this infinite-dimensional problem setting by the tools of *geometric/Lie algebraic control* theory. We manage to provide sufficient approximate controllability criteria for ensembles of distributions as well as for Bloch system. The criteria are formulated in terms of Lie span condition, analogous to Lie rank controllability/acessibilty conditions, known in finite-dimensional case.

Sufficient Conditions for Strong Local Optimality for Multi-Input Bilinear Control Systems Arising in Cancer Chemotherapy

Heinz Schaettler
Washington University, USA
hms@wustl.edu
Urszula Ledzewicz

We consider optimal control problems over a fixed interval for multi-input bilinear dynamical systems with both linear and quadratic objectives on the controls in the presence of control constraints. Problems of this type arise as mathematical models for cancer chemotherapy over an a priori specified fixed therapy horizon. Conditions are given that allow to embed extremals (controlled trajectories that satisfy the conditions of the Pontryagin maximum principle) into a field of broken extremals leading to easily verifiable sufficient conditions for strong local optimality. For a linear objective, flows of extremal bang-bang trajectories arise and a simple algorithm will be formulated that allows us to determine their local optimality. For a quadratic objective, sufficient conditions are based on the existence of a bounded solution to a matrix Riccati differential equation will be formulated. Upper and lower bounds on the solution will be formulated that for some systems allow us to guarantee the existence of such a solution. The theory will be illustrated using a 3-compartment model for multi-drug cancer chemotherapy with cytotoxic and cytostatic agents.

Method of Finite-Dimensional Approximations in Optimal Control

Ilya Shvartsman
Penn State Harrisburg, USA
ius13@psu.edu

In this talk we will show how the method of finite-dimensional approximations can be used to derive maximum principle type conditions for various optimal control problems.

Consistent Approximations of Impulsive Optimal Control Problems

Geraldo Silva
UNESP - Univ Estadual Paulista, Brazil
gnsilva03@gmail.com
Daniella Porto, Heloisa Silva

In this work we consider an optimal impulsive control problem in which the system dynamics is excited by a regular Borel measure. Using a recent notion of impulsive control solution introduced in the literature we show that the general optimal impulsive control can discretized by Euler method so that a subsequence of optimal solutions of the discretized problems do converge to an optimal solution of the original problem. For the convergence analysis we will introduce a metric taking into account the state and impulsive control spaces jointly. The convergence is then of the entire control processes, states and impulsive controls. This notion of convergence used here is neither weak star nor in the graph as previously studied in the literature. In the end we discuss an example illustrating the features of the Euler discretization for impulsive systems as well as the convergence of the optimal solutions of the discrete problems to the continuous one.

Application of Optimal Control to the Reduction of Co-Infected Tuberculosis-HIV/AIDS Individuals

Delfim F. M. Torres
University of Aveiro, Portugal
delfim@ua.pt
Cristiana J. Silva

Tuberculosis (TB) is a major cause of death among people living with human immunodeficiency virus (HIV), and HIV presents a big challenge to TB control. Each disease speeds up the progress of the other, and TB considerably shortens the survival of people with HIV/AIDS. An estimate one-third of the 40 million people living with HIV/AIDS worldwide are co-infected with TB. We propose and analyze a model for TB-HIV/AIDS co-infection

which considers single disease (TB or HIV/AIDS) and co-infection TB-HIV/AIDS treatment. The basic reproduction number of the system model is computed and stability is analyzed. Optimal control treatment procedures are derived for the reduction of co-infected TB-HIV/AIDS individuals.

On the optimal control on infinite horizon

Vladimir Veliov
Vienna University of Technology, Austria
veliov@tuwien.ac.at

The first part of the talk will present a recently obtained (jointly with S. Aseev) form of the Pontryagin maximum principle for ODE optimal control on infinite horizon. In the second part, the approach will be extended to optimal control problems on infinite horizon of a class of first order PDEs with non-local dynamics and non-local boundary conditions. The second part is based on a joint work with B. Skritek. In both cases it is remarkable that the solution of the adjoint equation, for which the condition for maximization of the Hamiltonian holds, is not determined by transversalty conditions, rather it is defined explicitly.

Stability of the turnpike phenomenon

Alexander Zaslavski
The Technion - Israel Institute of Technology, Israel
ajzasl@tx.technion.ac.il

We study the structure of solutions of a discrete-time control system with a compact metric space of states which arises in economic dynamics. This control system is described by a constraint set and an objective function. We show the stability of the turnpike phenomenon under small perturbations of the objective function and the constraint set.

Rauch and Bonnet-Myers type comparison theorems in sub-Riemannian geometry

Igor Zelenko
Texas A&M University, USA
zelenko@math.tamu.edu

We will give estimates for the number of conjugate points along extremals of a general sub-Riemannian metric in terms of curvature-type invariants of this metric. These estimates generalize the classical Rauch and Bonnet-Myers comparison theorems in Riemannian Geometry and they are based on the differential geometry of curves in Lagrangian Grassmannians developed in my previous works with Chengbo Li. The special emphasis will be given to the case of sub-Riemannian metrics on distributions of rank 2 where the formulation of the comparison theorems is especially simple.

Special Session 17: Direct and Inverse Problems in Abstract Spaces and Applications

Angelo Favini, University of Bologna, Department of Mathematics, Bologna, Italy
Davide Guidetti, University of Bologna, Department of Mathematics, Bologna, Italy

In this session the most recent results on evolution problems for degenerate differential equations and related inverse problems, new controllability results for degenerate problems, optimal control problems shall be described.Some attention is also devoted to regularity of solutions to different degenerate evolution equations.

Dynamical analysis of impulsive functional differential equations

Syed Abbas
Indian Institute of Technology Mandi, India
sabbas.iitk@gmail.com
Lakshman Mahto

In this work we study the existence and uniqueness of solution of impulsive fractional differential equations. We use various fixed point techniques and semigroup theory of linear operators to establish our results. Further we study almost periodicity and controllability of the problem under consideration. At the end we give an example to illustrate our analytical findings.

On uniqueness and stability for an inverse problem in elasticity

Elena Beretta
Politecnico di Milano, Italy
elena.beretta@polimi.it
E. Francini, S. Vessella

We consider the problem of determining an unknown pair of piecewise constant Lame parameters λ and μ inside a three dimensional body from the Dirichlet to Neumann map. We prove uniqueness and Lipschitz continuous dependence of λ and μ from the Dirichlet to Neumann map.

Decay rates of solutions to a fluid-structure interaction problem

Francesca Bucci
Università di Firenze, Italy
francesca.bucci@unifi.it
George Avalos

The talk will focus on a system of Partial Differential Equations (PDE) which describes the interaction of a fluid flow in a three-dimensional bounded domain, with the transversal displacement of a (fixed) part of its boundary. The mathematical model comprises a Stokes system for the fluid velocity field and a classical fourth order PDE for the elastic deformation of the plate; both the Euler-Bernoulli and the Kirchhoff models are specifically taken into consideration. The two distinct models give rise to two different stability results: namely, while uniform exponential stability holds true in the former case, rational decay rates of strong solutions have been established in the latter. An interesting feature of the obtained results is that the corresponding proofs are based on a frequency domain analysis rather than on energy/multiplier methods. (The talk is based on joint work with George Avalos (University of Nebraska-Lincoln, USA))

Identification of a source factor in a control problem for the heat equation with a boundary memory term

Cecilia Cavaterra
Università degli Studi di Milano, Italy
cecilia.cavaterra@unimi.it
Davide Guidetti

We consider a material with thermal memory occupying a bounded region Ω with boundary Γ. The evolution of the temperature $u(t,x)$ is described by an integrodifferential parabolic equation containing a heat source of the form $f(t)z_0(x)$. We formulate an initial and boundary value control problem based on a feedback device located on Γ and prescribed by means of a quite general memory operator. Assuming both u and the source factor f unknown, we study the corresponding inverse and control problem on account of an additional information. We prove a result of existence and uniqueness of the solution (u,f).

A general approach to identification problems and applications

Angelo Favini
University of Bologna, Italy
favini@dm.unibo.it

An abstract method to study a general identification problem for differential inclusions in Banach spaces is introduced.It is shown that it allows to handle related problems concerning degenerate evolution equations. Various concrete applications are described.

Controllability of nonlinear reaction-diffusion equations governed by bilinear control

Giuseppe Floridia
Istituto Nazionale di Alta Matematica (INdAM), Italy
floridia.giuseppe@icloud.com

In this talk, based on joint work with P. Cannarsa and A.Y. Khapalov, we study the global approximate controllability properties of a one dimensional semilinear reaction-diffusion equation governed via the coefficient of the reaction term. It is assumed that both the initial and target states admit no more than finitely many changes of sign. We also extend our 1-D results to higher dimensional reaction-diffusion equations on a disc, provided that all involved parameters have radial symmetry.

Singular parabolic equations with interior degeneracy

Genni Fragnelli
University of Bari, Italy
genni.fragnelli@uniba.it

We shall present new results for parabolic equations with a singular term and with a leading term which degenerates at the interior of the spatial domain. Previous analogous results for degeneracy at the boundary of the spatial domain cannot be applied directly. New Carleman estimates will also be presented.

Control approach to an ill-posed variational inequality

Gabriela Marinoschi
Romanian Academy, Romania
gmarino@acad.ro

We are concerned with the proof of a generalized solution to an ill-posed variational inequality. This is determined as a solution to an appropriate minimization problem involving a nonconvex functional, treated by an optimal control technique.

Controllability of systems with persisten memory and cosine operators

Luciano Pandolfi
Dipartimento di Scienze Matematiche "G.L. Lagrange"
Politecnico di Torino, Italy
luciano.pandolfi@polito.it

Systems of interest in viscoelasticity are often described by the equation

$$w'' = \Delta w + \int_0^t M(t-s)\Delta w(s)\mathrm{d}s \qquad (\mathbf{A})$$

where $w = w(x,t)$ and x belongs to a region Ω with smooth boundary. The control problem consists on finding a control f (acting on the boundary of Ω, or on an interior subregion Ω_1) which forces the pair $(w(T), w'(T))$ to hit a prescribed target $(\xi,\eta) \in L^2(\Omega) \times H^{-1}(\Omega)$. Several methods can be used to prove the existence of such control (at a suitable time T). The common idea is to see system **(A)** as a perturbation of the memoryless wave equation. It is known that the solution of the wave equation can be represented using cosine operator theory, and this fact suggests the use of cosine operators in the study of controllability of system **(A)**. The goal of this talk is both to review existing results and to present new results on the application of cosine operator theory to the study of controllability for system **(A)**.

Global uniqueness and stability in determining the electric potential of an inverse problem for the Schrodinger equation on a Riemannian manifold from one boundary measurement

Roberto Triggiani
University of Memphis, USA
rtrggani@memphis.edu
Zhifei Zhang

We consider an inverse problem for a Schrodinger equation defined on an open, bounded, connected set of a complete n-dimensional Riemannian manifold, with non-homogeneous Dirichlet boundary conditions. The goal is to recover the electric potential by means of a Neumann boundary measurement on an explicit sub portion of the boundary. Both uniqueness and stability of the recovery are obtained in terms of sharp conditions on the data. A key ingredient of the investigation are the Carleman estimates in Triggiani-Xu (2007) of the Schrodinger equation on a Riemannian manifold.

An inverse problem for a transport-like equation and some computational experiments

Mustafa Yildiz
Bulent Ecevit University, Turkey
mustafayildiz2002@hotmail.com

We deal with the solvability and numerical solution of a two-dimensional integral geometry problem for a family of curves of given curvature. The problem is related with a two-space-dimensional inverse problem for a transport-like equation. The uniqueness and existence of the solution of the problem are proven and a numerical approximation method is developed.

Special Session 18: Nonlinear Elliptic and Parabolic Problems

J. Lopez-Gomez, Complutense University of Madrid, Spain

Nonlinear elliptic and parabolic problems model a great variety of real systems in theoretical physics, chemistry, biochemistry, biology, economics, ecology and engineering. Actually, its study is a categorical imperative in many of these disciplines. The huge advances experienced by linear and nonlinear analysis during the last five decades have tremendously facilitated the development of new mathematical devices to analyse them. It is the main purpose of this session to gather some of the top World experts in this area to enjoy the last advances and the amazing perspectives of this emerging area.

Existence and multiplicity results of Fourth-Order Semilinear Parabolic Equations of Cahn–Hilliard Type.

Pablo Alvarez Caudevilla
University Carlos III of Madrid, Spain
pacaudev@math.uc3m.es

Assuming fourth-order semilinear parabolic equations of the Cahn–Hilliard-type

$$u_t + \Delta^2 u = \gamma u \pm \Delta(|u|^{p-1}u) \quad \text{in} \quad \Omega \times \mathbb{R}_+,$$

we will discuss several aspects regarding existence and multiplicity results of classic steady states when $\Omega \subset \mathbb{R}^N$ is a bounded domain under Navier boundary conditions and, also, considering the whole $\mathbb{R}^N$ and in a class of functions properly decaying at infinity,

$$\lim_{|x|\to\infty} u(x) = 0.$$

Moreover, for the different cases presented here we will show global existence of solutions as well as different blow-up patterns. These discussions will be supported with some numerical results.

Boundary value problems for second order ODEs with indefinite weight

Alberto Boscaggin
University of Milano-Bicocca, Italy
alberto.boscaggin@unito.it
Vivina Barutello, Maurizio Garrione, Gianmaria Verzini, Fabio Zanolin

We discuss existence, multiplicity and chaotic-dynamics results for some classes of nonlinear second order ODEs with indefinite weight. Based on recent joint works with Vivina Barutello (University of Torino) and Gianmaria Verzini (Polytechnic of Milano), Maurizio Garrione (University of Milano-Bicocca) and Fabio Zanolin (University of Udine).

Positive solutions for logistic problems with spatial heterogeneities, nonlinear mixed boundary conditions and a bifurcation parameter on the boundary conditions

Santiago Cano-Casanova
Universidad Pontificia Comillas, Spain
scano@upcomillas.es

In this talk will be analyzed the existence, uniqueness and stability of the positive solutions of a very general class of logistic problems with spatial heterogeneities and nonlinear mixed boundary conditions with potentials which may change of sign on the boundary. The results will be obtained in term of the balance of the exponents of the nonlinearities in the PDE and in the boundary conditions, in term of the signs of the potentials on the boundary and in term of the size of the superlinear part of the nonlinearity in the boundary conditions.

Avoidance behavior in intraguild communities: A cross-diffusion model

Robert Stephen Cantrell
University of Miami, USA
rsc@math.miami.edu
Daniel Ryan

A cross-diffusion model of an intraguild predation community where the intraguild prey employs a fitness based avoidance strategy is examined. The avoidance strategy employed is to increase motility in response to negative local fitness. Global existence of trajectories and the existence of a compact global attractor is proved. It is shown that if the intraguild prey has positive fitness at some point in the habitat when trying to invade, then it will be uniformly persistent in the system if its avoidance tendency is sufficiently strong. This type of movement strategy can lead to coexistence states in which the intraguild prey is marginalized to areas with low resource productivity while the intraguild predator maintains high densities in regions with abundant resources, a pattern observed in many real world intraguild predation systems.

A reaction-diffusion model for producers and scroungers

George Cosner
University of Miami, USA
gcc@math.miami.edu
Andrew Nevai

This talk will present a reaction-diffusion model for interacting populations of producers that obtain resources directly from the environment and scroungers that obtain resources by stealing them from producers. (The technical term for this is kleptoparasitism.) That situation arises frequently in nature. Like many ecological models, the model can be analyzed in terms of permanence/persistence theory, with invasibility conditions determined by the principal eigenvalues of linearized problems, but the model is different in detail from standard competition and predator-prey models.

Qualitative results for parabolic p-Laplacian equations under dynamical boundary conditions

Mabel Cuesta
Universite du Littoral Cote d'Opale ULCO, France
cuesta@lmpa.univ-littoral.fr
J. von Below and G. Pincet

We discuss global existence results and the occurrence of blow up phenomena for the nonlinear parabolic problem involving the p–Laplace operator of the form

$$\begin{cases} \partial_t u = \Delta_p u + F(t,x,u) & \text{in } \Omega \text{ for } t>0, \\ \sigma\partial_t u + |\nabla u|^{p-2}\partial_\nu u = 0 & \text{on } \partial\Omega \text{ for } t>0, \\ u(0,\cdot) = u_0 & \text{in } \overline{\Omega}, \end{cases}$$

where Ω is a bounded domain of $\mathbb{R}^N$ with Lipschitz boundary, and where

$$\Delta_p u := \div \left(|\nabla u|^{p-2}\nabla u\right)$$

is the *p–Laplacian* defined for any $u \in W^{1,p}(\Omega)$ and $p > 1$. As for the *dynamical* time lateral boundary condition $\sigma\partial_t u + |\nabla u|^{p-2}\partial_\nu u = 0$ the coefficient σ is assumed to be a nonnegative constant. The parameter dependent non linearity $F(\cdot,\cdot,u) = \lambda|u|^{p-2}u$ will be of particular interest, as well as the case $F(\cdot,\cdot,u) = \lambda|u|^{q-2}u$ with $q > p > 2$.

The results presented here stem from a joint work with Joachim von Below and Gaëlle Pincet Mailly.

Analysis of the Ericksen-Leslie System for Liquid Crystals

Matthias Hieber
TU Darmstadt, Germany
hieber@mathematik.tu-darmstadt.de

In this talk we consider the Ericksen-Leslie system describing the nematic phase of liquid crystals. We discuss local and global wellposedness results for strong solutions for various subclasses of this system. This is joint work with J. Pruss and K. Schade.

On a reaction-diffusion system for an unstirred chemostat with internal storage

Sze-bi Hsu
National Tsing-Hua University, Taiwan, Taiwan
sbhsu@math.nthu.edu.tw
Junping Shi, Feng-Bin Wang

The dynamics of a reaction-diffusion system for two species of microorganism in an unstirred chemostat with internal storage is stuided. It is shown that the diffusion coefficient is a key parameter of determining the asymptotic dynamics, and there exists a threshold diffusion coefficient above which both species become extinct. On the other hand, for diffusion coefficient below the threshold, either one species or both species persist, and in the asymptotic limit, a steady state showing competition exclusion or coexistence is reached.

Some existence results of solutions for φ-Laplacian systems

Yong-Hoon Lee
Pusan National University, Korea
yhlee@pusan.ac.kr
Gyeong-Mi Cho

First we consider the existence of non-trivial solutions of problem (P_λ);

$$\begin{cases} \psi_p(u')' + \lambda h(t)\cdot f(u) = 0, & t \in (0,1), \\ u(0) = 0 = u(1), \end{cases} \qquad (P_\lambda)$$

where $\lambda > 0$ a parameter, $\psi_p : \mathbb{R}^N \to \mathbb{R}^N$ is defined by $\psi_p(x) = |x|^{p-2}x$. $h : (0,1) \to \mathbb{R}^N$ may be singular at 0 or/and 1 and changes a sign. Moreover h may not be in $L^1(0,1)$. $f \in C(\mathbb{R}^N, \mathbb{R}^N)$ and $x \cdot y := (x_1y_1, x_2y_2, \cdots, x_Ny_N)$. Mainly motivated by the pioneering work of Manásevich-Mawhin [1], we derive a new integral operator of (P_λ) given as

$$T_\lambda(u)(t) = \begin{cases} \int_0^t \psi_p^{-1}(\alpha(\lambda h\cdot f(u)) + \lambda \int_s^{\frac{1}{2}} h(r)\cdot f(u(r))dr)ds, \\ 0 \le t \le \frac{1}{2}, \\ \int_t^1 \psi_p^{-1}(-\alpha(\lambda h\cdot f(u)) + \lambda \int_{\frac{1}{2}}^s h(r)\cdot f(u(r))dr)ds, \\ \frac{1}{2} \le t \le 1, \end{cases}$$

where $\alpha = \alpha(\lambda h\cdot f(u)) \in \mathbb{R}^N$ is the unique zero of the following integral equation

$$\int_0^{\frac{1}{2}} \psi_p^{-1}\left(\alpha + \lambda\int_s^{\frac{1}{2}} h(r)\cdot f(u(r))dr\right)ds = \int_{\frac{1}{2}}^1 \psi_p^{-1}\left(-\alpha + \int_{\frac{1}{2}}^s h(r)\cdot f(u(r))dr\right)ds.$$

This operator T extends cases introduced by Manásevich-Mawhin [1] of $g \in L^1(0,1)$ and Agarwal-Lü-O'Regan [2] of $g \ge 0$ to case of $g \notin L^1(0,1)$ and sign-changing. Second, we apply similar argument to the following φ-Laplacian systems;

$$\begin{cases} -\Phi(u')' = h(t)\cdot f(u), & t \in (0,1), \\ u(0) = 0 = u(1), \end{cases} \qquad (P)$$

where $\Phi(u') = (\varphi(u_1'), \cdots, \varphi(u_N'))$ with $\varphi : \mathbb{R} \to \mathbb{R}$ an odd increasing homeomorphism,
$h(t) = (h_1(t), \cdots, h_N(t))$ with $h_i : (0,1) \to \mathbb{R}_+$, $h_i \not\equiv 0$ on any subinterval in $(0,1)$ and $f(u) = (f^1(u), \cdots, f^N(u))$ with $f^i : \mathbb{R}_+^N \to \mathbb{R}_+$. Under suitable assumption on φ and a singularity condition on h, we introduce several existence results of positive solutions of problem (P).

References

[1] *R. Manásevich and J. Mawhin*: Periodic solutions of nonlinear systems with p-Laplacian-like operators. JDE, *145* (1998), 367-393.

[2] *R. P. Agarwal and H. Lü and D. O'Regan*: Eigenvalues and the one-dimensional p-Laplacian. J. Math. Anal. Appl. *266* (2002), 383-400.

Some astonishing results in spatially heterogeneous landscapes

Julian Lopez-Gomez
Complutense University, Spain
Lopez_Gomez@mat.ucm.es

In this talk we discuss a number of rather astonishing results in the context of spatially heterogeneous RD systems. Among them we will force some classical multiplicity results in the context of the calculus of variations up to get some really shocking results.

Radial positive solutions of elliptic systems modelling burglary of houses

Jean Mawhin
University of Louvain, Belgium
jean.mawhin@uclouvain.be
M. Garcia-Huidbro, R. Manásevich

Using Leray-Schauder degree arguments, existence results for positive solutions are proved for ordinary differential systems of the form

$$\begin{cases} \eta(r)[r^{n-1}(A - A^0(r))']' + \\ \quad r^{n-1}[-A + A^0(r) + Nf(r,A)] = 0, \\ (r^{n-1}N')' + [r^{n-1}g(r,A,A')N]' \\ \quad -r^{n-1}\omega^2(N-1) = 0, \end{cases}$$

with Neumann boundary conditions on $[l, L]$, coming from the study of radial solutions in an annulus of elliptic systems coming from some model for the burglary of houses. The requested a priori estimates are obtained by some unusual combination of pointwise and L^1-estimates.

A collocation spectral method to solve the heterogeneous logistic equation in circular domains

Marcela Molina Meyer
Universidad Carlos III, Spain
mmolinam@math.uc3m.es
F. Prieto-Medina

In this talk, the logistic equation without radial symmetries in circular is solved numerically. Our methods enjoy a great versatility, up to allow us to incorporate spatial heterogeneities and computing classical positive solutions, large solutions and metasolutions. We develop some of the ideas of Trefethen to discretize the Laplace operator in polar coordinates in the unit disk B1(0) and the circular annulus A(R0; R1). This is joint work with F. Prieto-Medina.

Existence of solutions in peridynamics and convergence to classical elasticity

Carlos Mora-Corral
Univeristy Autonoma of Madrid, Spain
carlos.mora@uam.es
J. C. Bellido and P. Pedregal

Peridynamics is a model in Solid Mechanics formulated by S. Silling in 2000. Its main difference with the usual Cauchy–Green elasticity relies in its non-locality, which reflects the fact that particles at a positive distance exert an interaction force upon each other. Mathematically, the deformations are not assumed to have weak derivatives, in contrast with classical Continuum Mechanics, and in particular, hyperelasticity, where they are assumed to be Sobolev. This makes peridynamics a suitable framework for mechanical problems where discontinuities appear naturally, such as fracture, dislocation or general multiscale materials. In this talk we study the variational theory of time-independent problems in peridynamics. In this formulation, the energy of a deformation is expressed as a double integral, and resembles the energy functional of a nonlocal singular p-Laplacian. In particular, we will explain optimal conditions over the integrand guaranteeing that the energy admits a minimum, and, hence, that the problem has an equilibrium solution. Moreover, we will see how the Cauchy–Green elasticity theory can be recovered from this nonlocal theory as a Γ-limit passage when the interaction distance tends to zero.

Global existence of solutions to a two-dimensional parabolic-elliptic system of chemotaxis with critical mass

Toshitaka Nagai
Hiroshima University, Japan
nagai@math.sci.hiroshima-u.ac.jp

In this talk, we consider the Cauchy problem of a parabolic-elliptic system of chemotaxis in two dimensions, which is a simplified version of chemotaxis system derived from the original Keller-Segel model. The total mass of the nonnegative solutions to the Cauchy problem is conserved, and the global existence and large-time behavior of the solutions heavily depend on the total mass of the initial data. We focus on the critical mass case and discuss the global exitence of nonnegative solutions in time.

Asymmetric Poincarè inequalities and applications to capillarity problems

Pierpaolo Omari
University of Trieste, Italy
omari@units.it
Franco Obersnel and Sabrina Rivetti

We first establish an asymmetric version of the Poincarè inequality in the space of bounded variation functions, then, basically relying on this result, we discuss the existence, the non- existence and the multiplicity of bounded variation solutions of a class of capillarity problems with asymmetric perturbations.

Coupling population and price dynamics

Duccio Papini
Università di Siena, Italy
papini@dii.unisi.it
Sylvain Arlot, Stefano Marmi

There is evidence of cyclic behavior and fluctuations in prices of livestock commodities (pork cycle) which have drawn the attention of economists for a long time. Possible causes of the persistence of such cycles could be endogenous instable dynamics due to underlying nonlinearities and specific features of livestock commodities production such as the significant time lag that usually occurs between herd building and meat production. A recent model of population dynamics by Yoccoz and Birkeland includes some of these features. The model is formulated as a delayed integral equation, where the delay takes into account the maturation time of pups, and the fertility function decreases non-linearly as the fertile population increases. Such deterministic model can produce complex dynamics with a high sensitivity to initial conditions. On the other hand, the effect of production delays on price fluctuations was considered by Bélair and Mackey in 1989 by means of an integro-differential equation modeling the price dynamics of a single commodity market depending on supply and demand. Having in mind livestock commodities production (meat), we propose and study a model that couples a population dynamics model similar to Yoccoz-Birkeland's one and a market dynamics model inspired by that of Bélair and Mackey.

Landesman-Lazer conditions at half-eigenvalues of the p-Laplacian

Bryan Rynne
Heriot-Watt University, Scotland
b.p.rynne@hw.ac.uk
Francois Genoud

We study the existence of solutions of the Dirichlet problem

$$\begin{aligned} -\phi_p(u')' - a_+\phi_p(u^+) + a_-\phi_p(u^-) \\ -\lambda\phi_p(u) = f(x,u),\ x \in (0,1), \end{aligned} \tag{1}$$

$$u(0) = u(1) = 0, \tag{2}$$

where $p > 1$, $\phi_p(s) := |s|^{p-1}\text{sgn}\ s$ for $s \in \mathbb{R}$, the coefficients $a_\pm \in C^0[0,1]$, $\lambda \in \mathbb{R}$, and $u^\pm := \max\{\pm u, 0\}$. We suppose that $f \in C^1([0,1]\times\mathbb{R})$ and that there exists $f_\pm \in C^0[0,1]$ such that $\lim_{\xi\to\pm\infty} f(x,\xi) = f_\pm(x)$, for all $x \in [0,1]$. With these conditions the problem (1)-(2) is said to have a 'jumping nonlinearity'. We also suppose that the problem

$$\begin{aligned} -\phi_p(u')' = a_+\phi_p(u^+) - a_-\phi_p(u^-) + \lambda\phi_p(u) \\ \text{on } (0,1), \end{aligned} \tag{3}$$

together with (1), has a non-trivial solution u. That is, λ is a 'half-eigenvalue' of (2)-(3) and the problem (1)-(2) is said to be 'resonant'. Combining a shooting method with so called 'Landesman-Lazer' conditions, we show that the problem (1)-(2) has a solution. Most previous existence results for jumping nonlinearity problems at resonance have considered the case where the coefficients $a_\pm$ are constants, and the resonance has been at a point in the 'fuc spectrum'. Even in this constant coefficient case our result extends previous results. In particular, previous variational approaches have required strong conditions on the location of the resonant point, whereas our result applies to any point in the fuc spectrum.

Some properties of radial stationary solutions to a parabolic-elliptic system related to Keller-Segel system.

Takasi Senba
Kyushu Institute of Technology, Japan
senba@mns.kyutech.ac.jp

In this talk, we treat behavior of radial solutions to a parabolic-elliptic system which is a generalized version of so called Keller-Segel system. There are many results about behavior of solutions to a simplified version of Keller-Segel system which is a parabolic-elliptic system. The behavior is closely related to properties of stationary solutions. In this talk, we talk about some properties of radial stationary solutions to a generalized version of Keller-Segel system and the relation between behavior of radial solutions and properties of radial stationary solutions to the system.

Uniqueness of the positive solution to coupled cooperative systems on an interval

Junping Shi
College of William and Mary, USA
jxshix@wm.edu
Jann-Long Chern, Changshou Lin

Oscillatory behavior of the solutions of linearized equations for semilinear elliptic systems of two equations on one-dimensional domains are proved, and it is shown that the stability of the positive solutions for such semilinear system is closely related to the oscillatory behavior. These properties are used to prove the uniqueness of positive solutions to some semilinear elliptic systems with variational structure.

Inverse and Direct Bifurcation Problems for Nonlinear Elliptic Equations

Tetsutaro Shibata
HIroshima University, Japan
shibata@amath.hiroshima-u.ac.jp

We consider the nonlinear elliptic equations, which contain the unknown nonlinear term. In this talk, we study the inverse bifurcation problems from two points of view. Firstly, we determine the unknown nonlinear term from the asymptotic behavior of the bifurcation diagrams of the equation. Secondly, we establish the uniqueness of the unknown term from the shape of the bifurcation diagrams. For the direct problems, we discuss the global behavior of the bifurcation diagrams.

Niche Partitioning in a Continuous Environment

Hal Smith
Arizona State University, USA
halsmith@asu.edu
Isaac Klapper and Jack Dockery

Biological systematics studies suggest that species are discretized in niche space. That is, rather than seeing a continuum of organism types with respect to continuous environmental variations, instead observers find discrete species or clumps of species, with one clump separated from another in niche space by a gap. Here, using a simple one dimensional reaction-diffusion model with a smoothly varying environmental condition, we investigate conditions for a discrete speciation instability of a continuously varying species structure in the context of asexually reproducing microbes. We find that significant perturbation of heterogeneity is required for instability, but that conditions for such perturbations might reasonably occur, for example through influence of boundary conditions.

Bernstein type theorem for some fully nonlinear PDEs

Kazuhiro Takimoto
Hiroshima University, Japan
takimoto@math.sci.hiroshima-u.ac.jp

In the early 20th century, Bernstein proved that if $f \in C^2(\mathbb{R}^2)$ and the graph of $z = f(x, y)$ is a minimal surface in $\mathbb{R}^3$, then f is necessarily a linear function of x and y. For Monge-Ampère equation, Pogolerov proved the Bernstein type theorem of the following form: Let $u \in C^4(\mathbb{R}^n)$ be a convex function of $\det D^2u = 1$ in $\mathbb{R}^n$. Then u is necessarily a quadratic polynomial. In this talk, we shall obtain the Bernstein type theorem for some fully nonlinear equations.

The effect of two roads with fast diffusion on Fisher-KPP propagation

Andrea Tellini
Universidad Complutense de Madrid, Spain
andrea.tellini@mat.ucm.es
Luca Rossi, Enrico Valdinoci

Many biological and ecological phenomena occur in which organisms or diseases diffuse faster in some parts of the environment, typically along roads or rivers. Recently, H. Berestycki, J.-M. Roquejoffre and L. Rossi have introduced a new PDE model to study the effect on the asymptotic speed of propagation of such diffusion heterogeneity produced by a line which bounds a half plane where a classical logistic reproduction of Fisher-KPP type takes place. The novelty consists in the fact that the equations are posed in different spatial dimensions. In this talk we will analyze the effect that two roads with different diffusion have on the propagation in a strip bounded by them. We will study the limits as the diffusion on the roads goes to 0 and ∞ and the limit as the width of the strip goes to 0 and ∞. In the latter case we recover the asymptotic speed of propagation of the half plane. This is a joint work with L. Rossi (Univ. Padova, Italy) and E. Valdinoci (WIAS, Berlin).

Two-sex populations with short reproduction season that diffuse under Dirichlet boundary conditions

Horst Thieme
Arizona State University, USA
hthieme@asu.edu
Wen Jin

Persistence results will be presented for discrete semiflows arising from models for two-sex populations that have short reproductive seasons and diffuse under Dirichlet boundary conditions. A threshold separating persistence from local stability of the extinction equilibrium is provided by the spectral radius of a homogeneous order preserving map that is not additive.

The effect of a nonlinear boundary condition with an indefinite weight on the positive solution set of the logistic elliptic equation

Kenichiro Umezu
Ibaraki University, Japan
uken@mx.ibaraki.ac.jp
Humberto Ramos Quoirin

In this talk, we consider the positive solutions of the logistic elliptic equation having a nonlinear boundary condition with an indefinite weight, according to a parameter, and investigate the role played by the weight in determining the structure of the positive solution set. Using variational and bifurcation techniques, we prove the existence of at least three positive solutions in some range of the parameter and the asymptotic behavior. Eventually, we discuss the existence of a CS-shaped as well as an S-shaped bifurcation diagrams. This is a joint work with Humberto Ramos Quoirin.

Long-time behavior and Turing patterns in a three species food chain model with a Holling type-II functional response

Zhifu Xie
Virginia State University, USA
zxie@vsu.edu
Dawit Haile

In this paper, we study a strongly coupled reaction-diffusion system describing three interacting species in a food chain model, where the third species preys on the second one and simultaneously the second species preys on the first one. We first show that the model is very rich dynamically. We conduct a one parameter analysis which illustrate the existence of chaos, limit cycle, and stable equilibrium while the parameter changes. Some interesting dynamical phenomena occur when we perform analysis of interactions in term of self-production of prey and self-competition of the middle predator. Then we conduct linear stability analysis and various phenomena exist such as Turing instability, as well as diffusion induced chaos. By numerical simulations,it shows the existence of various patterns such as stripe pattern, spot pattern, and labyrinth patterns.

Logistic diffusion equations with nonlocal effects in population biology

Yoshio Yamada
Waseda University, Japan
yamada@waseda.jp

My talk is concerned with a certain class of logistic diffusion equations with nonlocal effects, which appear in population biology. In particular, we are interested in the structure of positive solutions of the corresponding stationary problem. We will give an elementary idea to find positive stationary solutions. Generally, it is difficult to study the stability of stationary solutions for reaction-diffusion equations with nonlocal terms. We will also show some stability results on our positive stationary solutions under suitable conditions.

Optimal decay rates for convection diffusion equations in the whole space

Tetsuya Yamada
Fukui National College of Technology, Japan
yamada@fukui-nct.ac.jp

In this talk we consider the Cauchy problem for the convection diffusion equation $\partial_t u = \Delta u + a \cdot \nabla(|u|^{q-1}u)$ in $(0,\infty) \times \mathbb{R}^n$ with $a \in \mathbb{R}^n$ and $q > 1 + 1/n$, $n \geq 1$, and give more precise asymptotic profiles of the solutions that behaves like the heat kernel as $t \to \infty$, introducing a suitable spatial shift and correction term. As a corollary we also prove that the decay rates of the difference between the solution and the heat kernel are optimal.

Multiplicity results for one-dimensional nonlinear Schrödinger equations with stepwise potential

Fabio Zanolin
University of Udine, Italy
fabio.zanolin@uniud.it

We prove the existence and multiplicity of solutions presenting a precise nodal behavior for two classes one-dimensional nonlinear Schrödinger equation as

$$-\varepsilon^2 u'' + V(x)u = f(u)$$

and

$$u'' + ku - a(x)f(u) = 0,$$

for some special forms of the potential $V(x)$ or the coefficient $a(x)$. The term $f(u)$ generalizes the typical p-power nonlinearity considered by several authors in this context. We discuss the existence and multiplicity of periodic, Neumann, homoclinic and heteroclinic solutions. This talk is based on recent joint works with Chiara Zanini and Elisa Ellero.

Special Session 19: Nonautonomous Dynamics

Russell Johnson, Università di Firenze, Italy
Sylvia Novo, Universidad de Valladolid, Spain
Rafael Obaya, Universidad de Valladolid, Spain

Nonautonomous dynamical systems are defined by differential or difference systems whose coefficients vary with time. The time dependence may exhibit a wide range of behavior, from periodic to stochastic. Such dynamical systems have been intensively studied from the theoretical and computational viewpoints in the last 40 years. A body of results has been obtained which has permitted extensive applications to spectral theory, control theory, fluid dynamics, neural networks, to name some of many areas. Naturally this work on applied problems has had a feedback effect on efforts in theory and computation. The aim of this session is to present an overview of activities in this challenging and rapidly developing field.

A model for the nonautonomous Hopf bifurcation

Vasso Anagnostopoulou
Imperial College London, England
vanagnos@imperial.ac.uk
Tobias Jaeger, Gerhard Keller

We study a class of model systems which exhibit the full two step scenario for the nonautonomous Hopf bifurcation, as proposed by Ludwig Arnold. The specific structure of these models allows for a rigorous and thorough analysis of the bifurcation pattern. In particular, we show the existence of an invariant 'torus' splitting off a previously stable central manifold after the second bifurcation point.

On the existence of orbits terminating in finite time in nolinear RLC circuits

Flaviano Battelli
Marche Polytechnic university, Italy
battelli@dipmat.univpm.it
Michal Feckan

We apply dynamical system methods and Melnikov theory to study small amplitude perturbation of some implicit differential equations exhibiting I–singularities. In particular we show persistence of such I–singularities and orbits connecting them in finite time provided a Melnikov like condition holds.

Global bifurcation for periodic Dirac-type systems

Anna Capietto
University of Torino, Italy
anna.capietto@unito.it
Alessandro Marchino

It is proved the existence of a continuum of solutions to a Dirac-type system of first order ODEs with periodic coefficients. To this end, it is applied an abstract bifurcation theorem developed in the framework of degree theory for Fredholm maps. This is a joint work with Alessandro Marchino.

Reducibility to a normal form for quasiperiodic cocycles with a resonant frequency

Claire Chavaudret
University of Nice-Sophia Antipolis, France
chavaudr@unice.fr

Given a linear differential system with quasiperiodic coefficients, whose fundamental solution is a cocycle over a rotation in a torus, a long studied problem is the reducibility of the system, that is the possibility of conjugating it to a constant one, by a quasiperiodic transformation which preserves the Lyapunov exponents and the rotational properties. In the case of a rationally independent frequency vector, the system can be reducible under arithmetical conditions which ensure that the conjugation is smooth, analytic or Gevrey. Now if the frequency vector has integer relations (which happens if the system depends on a parameter varying in a torus), one cannot expect the system to be reducible to a constant. Instead, one will look for reducibility to a normal form, that is, to a new system which is constant on the orbits of the base dynamics (a "resonant cocycle"). We give algebraic and arithmetical conditions to ensure analytic conjugacy to such a normal form.

Growth rates for persistently excited systems

Fritz Colonius
University of Augsburg, Germany
fritz.colonius@math.uni-augsburg.de
Y. Chitour, Mario Sigalotti

We consider a family of linear control systems $\dot{x}(t) = Ax(t) + \alpha(t)Bu(t)$ on $\mathbb{R}^d$, where $\alpha(\cdot)$ belongs to a given class of persistently exciting signals taking values in $[0, 1]$. Thus the average value of $\alpha(\cdot)$ must be bounded away from zero. The interpretation of this setting is that the average transmission of data from the controller to the system is restricted. We seek maximal α-uniform stabilization and destabilization by means of linear feedbacks $u = Kx$. Using an associated linear flow and controllability properties in projective space it is shown that the existence of a feedback K such that the Lie algebra generated by A and BK is equal to the set of all $d \times d$ matrices implies that the maximal rate of convergence of (A, B) is equal to the maximal rate of divergence of $(-A, -B)$. This generalizes the classical result for linear control systems (with $\alpha \equiv 1$).

Existence of globally attracting solutions of the viscous Burgers equation on the line with periodic boundary conditions and nonautonomous forcing

Jacek Cyranka
Warsaw University, Poland
jacek.cyranka@ii.uj.edu.pl
P. Zgliczynski

We prove the existence of globally attracting solutions of the viscous Burgers equation with periodic boundary conditions on the line for some particular choices of viscosity and non-autonomous forcing

$$u_t + u \cdot u_x - \nu u_{xx} = f(t, x).$$

The attracting solution is periodic if the forcing is periodic. The convergence towards attracting solution is exponential. The proof is computer assisted. The method is general and can be applied to other similar partial differential equations including the Navier-Stokes equations. The technique we use is not restricted to some particular type of equation nor to the dimension one, as we are not using any maximum principles, nor unconstructive functional analysis techniques. We need some kind of 'energy' decay as a global property of our dissipative PDEs and then if the system exhibits an attracting orbit, then we should in principle be able to prove it independent of the dimensionality of of the system. At the present state our technique strongly relies on the existence of good coordinates, the Fourier modes in the considered example. We hope that the further development of the rigorous numerics for dissipative PDEs based on other function bases, e.g. for example the finite elements, should allow to treat also different domains and boundary conditions in near future.

The Fibonacci Hamiltonian

David Damanik
Rice University, USA
david.damanik@yahoo.com

We discuss recent results concerning the spectrum, the density of states measure, and the transport exponents associated with the Fibonacci Hamiltonian. This is joint work with Anton Gorodetski and William Yessen.

Bifurcation results for singular Dirac systems

Walter Dambrosio
Dipartimento di Matematica - Universita di Torino, Italy
walter.dambrosio@unito.it
Anna Capietto, Duccio Papini

We present a bifurcation result for a singular planar Dirac system; linear and nonlinear eigenvalue problems are studied. An application to a nonlinear Dirac PDE is given.

Periodic orbits for planar piecewise smooth dynamical systems

Cinzia Elia
University of Bari, Italy
cinzia.elia@uniba.it
Dieci Luca

In this talk we will examine the existence of periodic orbits for planar piecewise smooth dynamical systems with a line of discontinuity. We will investigate both bifurcation phenomena that give birth to the periodic orbits and bifurcations of the periodic orbits themselves that do not occur in smooth dynamical systems. Linear and nonlinear models are considered.

Spectral properties for the quasi-periodic Schroedinger equation

Roberta Fabbri
Dipartimento Matematica e Informatica "Ulisse Dini", Italy
roberta.fabbri@unifi.it
Cinzia Elia

In the talk the spectral properties for the one-dimensional quasi-periodic Schrödinger operator and the two-dimensional Schrödinger equation are considered. In particular, the relations between the concepts of the rotation number and the exponential dichotomy for a family of linear nonautonomus Hamiltonian systems are used to obtain information on the structure of the spectrum of the corresponding operators.

Stability in Aperiodic Nearly-Integrable Hamiltonian Systems

Alessandro Fortunati
Bristol University, England
alessandro.fortunati2012@gmail.com
Stephen Wiggins

Hamiltonian Perturbation Theory is a well established subject for a wide class of nearly integrable systems with a periodic or quasi-periodic time dependence. Nevertheless, very little is known for systems in which the dependence on time is only aperiodic. As a consequence, several basic questions concerning their stability are still open. A Nekhoroshev result for such a class of systems will be discussed in detail, and we will also address some further topics related to the Aperiodic Perturbation Theory.

Melnikov theory for discontinuous system.

Matteo Franca
Universit'a Politecnica delle Marche, Italy
franca@dipmat.univpm.it
A. Calamai, J. Diblík, M. Pospíšil

In this talk we present some recent development on Melnikov theory for non-smooth ODE. We consider a system having a critical point O on a discontinuity surface $\mathcal{S}$, and a trajectory homoclinic to O. We assume that the system is subject to a non-autonomous perturbation and we look for conditions which are sufficient for the persistence of the homocline and for the existence of a chaotic pattern.

An important issue will be to control exactly the position of chaotic trajectories close to 0 and $\mathcal{S}$. Surprisingly we find a new geometrical condition (always verified in the continuous case) which is not needed for the persistence of homoclinic trajectories, but is necessary for chaos. This setting of assumptions finds natural applications in many physical examples, such as dry friction pendulum, where the critical point lies on the discontinuity surface.

The geometry of finite-time coherent sets in nonautonomous dynamics

Gary Froyland
University of New South Wales, Australia
g.froyland@unsw.edu.au

The study of transport and mixing processes in dynamical systems is particularly important for the analysis of mathematical models of physical systems. In the autonomous setting, the use of transfer operators to identify invariant and almost-invariant sets has been particularly successful. In the nonautonomous setting, coherent sets, a time-parameterised family of minimally dispersive sets, are a natural extension of almost-invariant sets. Here we detail new results concerning the geometric characterisation of finite-time coherent sets, and describe an alternative construction based on transfer operators to find optimal coherent sets.

Non-Smooth Saddle-Node Bifurcations of Quasi-Periodically Forced Interval Maps

Gabriel Fuhrmann
TU Dresden, Germany
gabrielfuhrmann@googlemail.com
Tobias Jäger and Maik Gröger

This talk deals with quasi-periodically forced interval maps of the form

$$f : \mathbb{T} \times \mathbb{R} \to \mathbb{T} \times \mathbb{R},$$
$$(\theta, x) \mapsto (\theta + \omega, g(\theta, x)),$$

with $\mathbb{T} := \mathbb{R}/\mathbb{Z}$ and $\omega \in \mathbb{R} \setminus \mathbb{Q}$. In particular, we focus on bifurcations of the corresponding invariant graphs of such systems, that is, bifurcations of measurable functions $\psi : \mathbb{T} \to \mathbb{R}$ which satisfy

$$\psi(\theta + \omega) = g(\theta, \psi(\theta)).$$

We consider the case where the functions $g(\theta, \cdot)$ are monotonously increasing and concave. In this situation, we provide sufficient conditions for a family of forced maps to undergo a non-smooth saddle-node bifurcation. In contrast to former results in this direction, our conditions are $\mathcal{C}^2$-open in the set of families with fixed Diophantine rotation number ω. Further, we answer a question by Herman on the topological structure of the minimal set at the bifurcation. A similar result has earlier been derived by Bjerklöv for the special case of a projective dynamical system associated to a quasi-periodic Schrödinger cocycle. We provide a simplified proof of an extension of his result to systems of the above form. Finally, we compute the Hausdorff dimension of the upper bounding graph of the minimal set at the bifurcation.

Two stability results for non-autonomous dynamical systems

Cecilia Gonzalez Tokman
University of New South Wales, Australia
ceciliagt@unsw.edu.au
Gary Froyland and Anthony Quas

Stability properties of dynamical systems are of fundamental interest to applied scientists, because models are imperfect representations of reality. In this talk, we will discuss recent stability results which are relevant for the study of transport phenomena in non-autonomous dynamical systems. They cover perturbations arising from numerical approximation schemes and random noise. The first result concerns stability of non-autonomous counterparts of stationary distributions or physical invariant measures – so-called random acims (absolutely continuous invariant measures) – in the context of piecewise expanding interval maps. The second one concerns stochastic stability of Oseledets splittings and Lyapunov exponents for semi-invertible matrix cocycles.

Quasi-periodic long-range Schrödinger operators: spectral properties from a dynamical perspective.

Alex Haro
Universitat de Barcelona, Spain
alex@maia.ub.es
Joaquim Puig

This talk is devoted to quasi-periodic Schrodinger operators beyond the Almost Mathieu, with more general trigonometric potentials and finite range interactions. The links between the spectral properties of these operators and the dynamical properties of the associated quasi-periodic linear skew-products rule the game. In particular, we present a Thouless formula and some consequences of Aubry duality.

Characterization of uniform attractors

Jose Langa
University of Seville, Spain
langa@us.es
Matheus Bortolan and Alexandre Carvalho

A global attractor related to autonomous ordinary or partial differential equation is an invariant compact set attracting bounded sets forwards in time. However, when the system under consideration is non-autonomous, during the last twenty years two more or less disconnected approaches have been developed in order to study attractors for the associated non-autonomous dynamical systems. On the one hand, the pullback attractor, an invariant set for the evolution process which is pullback (but, in general, not forwards) attracting. On the other hand, the uniform attractor, a non-invariant compact set attracting uniformly forwards in time. The characterization of pullback attractors in terms of their internal structures and connecting dynamics has received an intensive research in the last years. This study, join to a careful description of the relationship between pullback and uniform attractors, leads to a detailed description of the uniform attractor, and to some results on the upper and lower semicontinuity, topological and structural stability of the uniform attractors associated to non-autonomous perturbations of a semigroup.

Principal spectrum estimates in linear nonautonomous order-preserving dynamical systems: A survey

Janusz Mierczyński
Wroclaw University of Technology, Poland
mierczyn@pwr.edu.pl

For linear nonautonomous strongly order-preserving dynamical systems the principal spectrum plays a role in establishing stability and/or persistence similar to that played by the principal eigenvalue in their autonomous counterparts. Unfortunately, even in the time-periodic case generally there is no relation between the principal spectrum and the principal eigenvalues of the linear operators generating the system. The purpose of this talk is to give a survey of known partial results, as well as present open problems and possible directions of future research.

Fractional Lyapunov exponent for solutions of linear fractional differential equations

Dinh Cong Nguyen
Institute of Mathematics, VAST, Vietnam
ndcong@math.ac.vn
Doan Thai Son, Hoang The Tuan

We investigate the asymptotic behavior of solutions of linear fractional differential equations. Firstly, we show that the classical Lyapunov exponent of an arbitrary nontrivial solution of a bounded linear fractional differential equation is always nonnegative. Next, using the Mittag-Leffler function, we introduce an adequate notion of fractional Lyapunov exponent for an arbitrary function. We show that for a linear fractional differential equation, the fractional Lyapunov spectrum which consists of all possible fractional Lyapunov exponents of its solutions provides a good description of asymptotic behavior of this equation. Consequently, stability of a linear fractional differential equation can be characterized by its fractional Lyapunov spectrum.

Linear-quadratic dissipative control systems

Carmen Nunez
Universidad de Valladolid, Spain
carnun@wmatem.eis.uva.es
Russell Johnson

We analyze the concept of dissipativity in the sense of Willems for nonautonomous linear-quadratic (LQ) control systems. A nonautonomous system of Hamiltonian ODEs is associated with such an LQ system by way of the Pontryagin Maximum Principle. The concepts of exponential dichotomy and weak disconjugacy for this Hamiltonian ODE are related to that of dissipativity for the LQ system in order to investigate the relationship between the strict dissipativity condition and the conditions arising in the frequency theorem. We also investigate LQ systems which are dissipative but not strictly so. We find sufficient conditions for the existence of a storage function and of a stabilizing feedback control. And we see that the concept of weak disconjugacy can be used to formulate such conditions. The work is based on previous results obtained in collaboration with Roberta Fabbri, Sylvia Novo and Rafael Obaya.

Period 3 and Chaos for Unimodal Maps

Kenneth Palmer
Providence University, Taiwan
remlapk@gmail.com
Kaijen Cheng

This is joint work with Kairen Cheng (Chung Yuan Christian University, Taiwan). We study unimodal maps on the closed unit interval, which have a stable period 3 orbit and an unstable period 3 orbit, and give conditions under which all points in the open unit interval are either asymptotic to the stable period 3 orbit or land after a finite time on an invariant Cantor set Λ on which the dynamics is conjugate to a subshift of finite type and is, in fact, chaotic. For the particular value of $\mu = 3.839$, Devaney, following ideas of Smale and Williams, shows that the logistic map $f(x) = \mu x(1-x)$ has this property. In this case the stable and unstable period 3 orbits appear when $\mu = \mu_0 = 1 + \sqrt{8}$. We use our theorem to show that the property holds for all values of $\mu > \mu_0$ for which the stable period 3 orbit persists.

Nonautonomous Dynamics at work: Analytical and numerical analysis of a population-dynamical model

Christian Poetzsche
Alpen-Adria University Klagenfurt, Austria
christian.poetzsche@aau.at
Thorsten Huels

We apply various numerical and analytical tools to obtain an insight into the local and global dynamics of a planar model from population dynamics with aperiodic coefficients. Due to the lack of equilibria and the insignificance of eigenvalues, we employ numerical schemes to approximate entire solutions, their dichotomy spectra, as well as the corresponding invariant manifolds. Moreover, we aim to illustrate the qualitative theory for nonautonomous equations.

Superstable periodic orbits of 1d maps under quasi-periodic forcing and reducibility loss

Pau Rabassa
Queen Mary, University of London, England
p.sans@qmul.ac.uk
Jorba, À. and Tatjer, J. C.

Let g_α be a one-parameter family of one-dimensional maps with a cascade of period doubling bifurcations. Between each of these bifurcations, a superstable periodic orbit is known to exist. An example of such a family is the well-known logistic map. We will focus on the effect of a quasi-periodic perturbation on this cascade. It is

known that, if ε is small enough, the superstable periodic orbits of the unperturbed map become attracting invariant curves the perturbed system. This talk will focus on the reducibility of these invariant curves. We will show that, under generic conditions, there are both reducible and non-reducible invariant curves depending on the values of α and ε. The curves in the space (α, ε) separating the reducible and the non-reducible regions are called reducibility loss bifurcation curves. We will discuss on the existence of this bifurcation curves along the cascade of the one-dimensional map and its asymptotic behavior.

Bifurcations of random dynamical systems

Martin Rasmussen
Imperial College London, England
m.rasmussen@imperial.ac.uk
M. Callaway, T.S. Doan, J.S.W Lamb, C.S. Rodrigues

Despite its importance for applications, relatively little progress has been made towards the development of a bifurcation theory for random dynamical systems. In this talk, I will demonstrate that adding noise to a deterministic mapping with a pitchfork bifurcation does not destroy the bifurcation, but leads to two different types of bifurcations. The first bifurcation is characterized by a breakdown of uniform attraction, while the second bifurcation can be described topologically. Both bifurcations do not correspond to a change of sign of the Lyapunov exponents, but I will explain that these bifurcations can be characterized by qualitative changes in the dichotomy spectrum and collisions of attractor-repeller pairs.

Dichotomy spectrum in infinite dimensions

Evamaria Russ
Alpen-Adria Universität Klagenfurt, Austria
evamaria.russ@aau.at

The dichotomy spectrum (also known as Sacker-Sell or dynamical spectrum) is a crucial spectral notion in the theory of dynamical systems. In this talk we study the dichotomy spectrum for linear difference equations with an infinite-dimensional state space. In general we cannot expect a nice structure of the dichotomy spectrum like in the finite dimensional case, but compactness properties of the transition operator provide a more regular spectrum. Finally, we have a look at various evolutionary differential equations in order to illustrate possible applications.

Continuous separations in monotone skew-product semiflows: some theory and computation

Ana Maria Sanz
Universidad de Valladolid, Spain
anasan@wmatem.eis.uva.es
Juan A. Calzada, Sylvia Novo, Rafael Obaya

The talk deals with the classical concept of a continuous separation over a compact positively-invariant set K of a monotone and C^1 skew-product semiflow, as well as with a recently introduced new concept which is useful in applications to delay equations. Sufficient conditions are given in terms of the linearized operators over K and some practical criteria for non-autonomous recurrent differential equations are presented. When a continuous separation over K exists, the uniform persistence of the semiflow in the areas situated strongly above and strongly below K can be determined by means of the principal spectrum. Besides, we show how to manage general cooperative systems when in principle there is no continuous separation. Finally, we show how to numerically compute the leading direction of the continuous separation.

A new notion of entropy which describes transient dynamics

Stefan Siegmund
TU Dresden, Germany
stefan.siegmund@tu-dresden.de
Luu Hoang Duc

We introduce a new concept of entropy called Finite-time Metric Entropy (FTME), which is a transient-in-time and local-in-space version of the classical concept of metric entropy. Based on that, a transient version of Pesin's entropy formula and also an explicit formula for finite-time metric entropy for 2-D systems are derived. We also discuss how to apply the finite-time metric entropy field to detect special transient dynamics such as Lagrangian coherent structures.

Morse decomposition of global attractors with infinite components

Jose Valero
Universidad Miguel Hernandez de Elche, Spain
jvalero@umh.es
T. Caraballo, J.A. Langa

In this talk we describe some dynamical properties of a Morse decomposition with a countable number of sets. We prove that the dynamically gradient dynamics of Morse sets together with a separation assumption is equivalent to the existence of an ordered Lyapunov function associated to the Morse sets and also to the existence of a Morse decomposition (that is, the global attractor can be described as an increasing family of local attractors and their associated repellers). This theory generalizes the well known classical results, in which a finite number of components is considered, and is illustrated with a suitable application.

Invariant Manifolds and Decoupling for Nonautonomous Systems

Erik Van Vleck
University of Kansas, USA
erikvv@ku.edu
Yu-Min Chung, Michael Jolly, Andrew Steyer

We consider the decoupling of dissipative initial value differential equations by recasting as nonautonomous systems where the linear term in the equation corresponds to the coefficient matrix function of the linear variational equation. We investigate iterative methods for determining the decoupling transformation that result in different spectral gap conditions. We outline the theoretical results we obtain, the consequences of these different spectral gap conditions, and illustrate the utility of these techniques numerically for some challenging problems including Lorenz '96 models.

Floquet bundles for tridiagonal competitive systems with applications

Yi Wang
University of Science and Technology of China, Peoples Rep of China
wangyi@ustc.edu.cn
Chun Fang and Mats Gyllenberg

In this talk, we consider a general time-dependent linear competitive tridiagonal system of differential equations in the framework of skew-product flows and obtain canonical Floquet invariant bundles which are exponentially separated. Such Floquet bundles naturally reduce to the standard Floquet space when the system is assumed to be time-periodic. We apply the Floquet theory so obtained to study the dynamics on the hyperbolic omega-limit sets for the nonlinear competitive tridiagonal systems in time-recurrent structures including almost periodicity and almost automorphy. This is a joint work with Chun Fang and Mats Gyllenberg.

Stabilizability of linear time-varying systems

Fabian Wirth
IBM Research Ireland, Ireland
fabwirth@ie.ibm.com
B.D.O. Anderson, A. Ilchmann

In this talk we consider nonautonomous linear control systems of the form

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)\,.$$

For such linear time-varying systems with bounded system matrices we discuss the problem of stabilizability by linear state feedback, that is, the existence of a time-varying feedback F such that

$$\dot{x}(t) = A(t)x(t) + B(t)F(t)x(t)$$

is stable in a suitable sense. For example, it is shown that complete controllability implies the existence of a feedback so that the closed-loop system is asymptotically stable. We also show that the system is completely controllable if, and only if, the Lyapunov exponent is arbitrarily assignable by a suitable feedback. For uniform exponential stabilizability and the assignability of the Bohl exponent this property is known. Also, it is shown that dynamic feedback does not provide more freedom to address the stabilization problem. The unifying tools for our results are two finite $L^2 - L^2$ cost conditions. The distinction of exponential and uniform exponential stabilizability is then a question of whether the finite cost condition is uniform in the initial time or not.

Construction of generalized reflectionless potentials for the Sturm-Liouville operator

Luca Zampogni
University of Perugia, Italy
zampoglu@dmi.unipg.it
Russell Johnson

Techniques of Nonautonomous Dynamics are applied to the study of the inverse spectral problem of the Sturm-Liouville eigenvalue equation $E\varphi := -(p\varphi')' + q\varphi = \lambda y\varphi$ with $\lambda \in \mathbb{C}$ and potential (p, q, y) belonging to a certain functional space. We focus on the problem of the construction and of the properties of potentials (p, q, y) which are limits of the so-called algebro-geometric potentials. We also emphasize the relation between these potentials and the solutions of a hierarchy of highly nonlinear evolution equations.

Special Session 20: Dynamics with Fractional and Time Scale Derivatives

Martin Bohner, Missouri University of Science and Technology, USA
Natalia Martins, University of Aveiro, Portugal
Delfim F. M. Torres, University of Aveiro, Portugal

Nonautonomous dynamical systems are defined by differential or difference systems whose coefficients vary with time. The time dependence may exhibit a wide range of behavior, from periodic to stochastic. Such dynamical systems have been intensively studied from the theoretical and computational viewpoints in the last 40 years. A body of results has been obtained which has permitted extensive applications to spectral theory, control theory, fluid dynamics, neural networks, to name some of many areas. Naturally this work on applied problems has had a feedback effect on efforts in theory and computation. The aim of this session is to present an overview of activities in this challenging and rapidly developing field.

Approximation formulas for fractional derivatives of variable order

Ricardo Almeida
University of Aveiro, Portugal
ricardo.almeida@ua.pt
Delfim F. M. Torres

Expansion formulas for fractional derivatives of Riemann–Liouville and Marchaud types with variable fractional order are obtained, using integer-order derivatives only. An estimation for the error is given. As applications, we show how the obtained results are useful to solve differential equations and problems of the calculus of variations that depend on fractional derivatives of Marchaud type.

Simplification of the structure of analytic systems on time scales

Zbigniew Bartosiewicz
Bialystok University of Technology, Poland
z.bartosiewicz@pb.edu.pl

We study analytic control systems on time scales, with output. Under certain conditions such a system may be immersed into another system with simpler structure: affine, polynomial or rational with respect to the state variables. Though the state space of the simpler system is usually higher dimensional than the state space of the original system, but the simpler structure may have some advantage. We construct the observation space, observation algebra and observation field of the original analytic system and show that these objects play fundamental role in simplification of the structure of the system. We show that the system may be immersed into an affine system if and only if the observation space of the system is finitely dimensional. Similarly, the system may be immersed into a polynomial system if and only if the observation algebra of the system is contained in some finitely generated algebra, and the system may be immersed into a rational system if and only if the observation field of the system is a finitely generated extension of the field of reals. We also show that the observation algebra and the observation field are a skew differential algebra and a skew differential field, respectively, with respect to some skew derivation.

A discretization of the Hadamard fractional derivative

Nuno Bastos
Polytechnic Institute of Viseu, Portugal
nbastos@estv.ipv.pt
Ricardo Almeida

We present a discrete-time Hadamard fractional derivative. We then apply to solve fractional differential equations and fractional variational calculus with dependence of Hadamard fractional derivatives.

Diamond-alpha Opial dynamic inequalities

Martin Bohner
Missouri S&T, USA
bohner@mst.edu

In this talk, we present some new diamond-alpha dynamic inequalities of Opial type with different weight functions on time scales.

A time-scale nonsymmetric and symmetric fractional calculus

Artur Brito da Cruz
Escola Superior de Tecnologia de Setubal, Portugal
artur.cruz@estsetubal.ips.pt
Nadia Benkhettou, Delfim F. M. Torres

We introduce a nabla, a delta, and a symmetric fractional calculus on arbitrary nonempty closed subsets of the real numbers. These fractional calculi provide a study of differentiation and integration of noninteger order on discrete, continuous, and hybrid settings. Main properties of the new fractional operators are investigated, and some fundamental results presented, illustrating the interplay between discrete and continuous behaviors.

Lyapunov fractional differential inequalities

Rui Ferreira
Lusophone University of Humanities and Technologies, Portugal
ruiacferreira@ulusofona.pt

In this talk we will present Lyapunov-type inequalities for boundary value problems depending on fractional derivatives. We start with what is known so far in the literature and then point out some directions for future research.

A necessary condition of viability for fractional equations with the Caputo derivative

Ewa Girejko
Bialystok University of Technology, Poland
e.girejko@pb.edu.pl
Dorota Mozyrska, Malgorzata Wyrwas

In this paper viability results for nonlinear fractional differential equations with the Caputo derivative are proved. We give a necessary condition for fractional viability of a locally closed set with respect to a nonlinear function. An illustrative example is also contained.

Power functions and essentials of fractional calculus on time scales

Tomas Kisela
Brno University of Technology, Czech Rep
kisela@fme.vutbr.cz

The contribution deals with a recently suggested axiomatic definition of power functions on a general time scale and its consequences to fractional calculus. In particular, it discusses existence and uniqueness of such functions, ways of their computation as well as their Laplace transform. A comparison of the results with relevant literature and some examples is presented as well.

Fractional variational calculus: an application to the fractional Sturm–Liouville problem

Agnieszka B. Malinowska
Bialystok University of Technology, Poland
a.malinowska@pb.edu.pl
Magorzata Klimek and Tatiana Odzijewicz

In the last few decades, fractional differential equations have been found to be successful models of real life phenomenon. We wish to investigate the regular fractional Sturm–Liouville eigenvalue problem. Applying methods of fractional variational analysis we prove existence of countable set of orthogonal solutions and corresponding eigenvalues. Base on this result we can apply the method of separating variables to space– and time–fractional diffusion equations.
This is joint work with Małgorzata Klimek and Tatiana Odzijewicz.

Noether's second theorem for variational problems involving multiple delta integrals

Natalia Martins
University of Aveiro, Portugal
natalia@ua.pt
Agnieszka B. Malinowska

We extend the second Noether theorem to multiple integral variational problems on time scales. Our result provides as corollaries the classical second Noether theorem, the second Noether theorem for the h-calculus and the second Noether theorem for the q-calculus.

Applications of the fractional Sturm–Liouville problem to the space– and time–fractional diffusion equation

Tatiana Odzijewicz
University of Aveiro, Portugal
tatianao@ua.pt
Malgorzata Klimek, Agnieszka B. Malinowska

We study space– and time–fractional diffusion equations. Our approach strongly depends on the fractional Sturm–Liouville theory, precisely on the problem of finding eigenvalues and corresponding eigenfunctions to the certain fractional differential equation. Using the method of separating variables and applying theorem ensuring existence of solutions to the fractional Sturm–Liouville problem we solve several types of fractional duffusion equations.

Exponentials and transforms on regular time scales

Manuel Ortigueira
UNINOVA, Portugal
mdo@fct.unl.pt
Delfim Torres and Juan Trujillo

In this paper we formulate a coherent approach to signals and systems theory on regular time scales. The two derivatives - nabla (forward) and delta (backward) are used and the corresponding eigenfunctions computed. They are the so-called nabla and delta exponentials. With these exponentials the nabla and delta Laplace transforms are deduced and their properties studied. These transforms are back compatible with the current Laplace and Z transforms. They are used to study the nabla linear systems defined by differential equations. These equations mimmic the usual continuous-time equations that are uniformly approximated when the jump interval becomes small. Impulse response and transfer function notions are introduced and obtained. This implies a unified mathematical framework that allows us to approximate the classic continuous-time case when the sampling rate is high or obtain the current discrete-time case based on difference equation when the jump becomes constant.

Local controllability of nonlinear discrete-time systems with n fractional orders

Ewa Pawluszewicz
Bialystok University of Technology, Poland
e.pawluszewicz@pb.edu.pl
D. Mozyrska

The Riemann-Liouville, Caputo and Grünwald-Letnikov fractional order difference operators are discussed and used to state and solve the local controllability problem of local controllability in a finite number of steps of h-difference nonlinear control systems with n fractional orders. There are used three types of forward differences: the fractional Riemann-Liouville type difference, fractional Caputo type difference and fractional Grünwald-Letnikov type difference. It is shown that independently of the type of fractional order difference, such a system is locally controllable in q steps if its linear approximation is globally controllable in q steps.

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Variational problems with dependence on Caputo fractional derivatives of variable order

Dina Tavares
Polytechnic Institute of Leiria, Portugal
dina.tavares@gmail.com
Ricardo Almeida, Delfim F. M. Torres

We show necessary conditions of optimality for functionals depending on a combined Caputo derivative of variable fractional order. The objective is to obtain Euler–Lagrange equations as well transversality conditions on the end-point. Using numerical approximations, we show how to solve such problems.

Necessary condition for a dynamic Euler-Lagrange integro-differential equation

Delfim F. M. Torres
University of Aveiro, Portugal
delfim@ua.pt
Monika Dryl

We prove a necessary condition for a dynamic integro-differential equation to be an Euler–Lagrange equation. New and interesting results for the discrete and quantum calculus are obtained as particular cases. An example of a second order dynamic equation, which is not an Euler–Lagrange equation on an arbitrary time scale, is given.

Special Session 21: Variational, Topological, and Set-Valued Methods for Differential Problems

Gabriele Bonanno, University of Messina, Italy
Siegfried Carl, University of Halle, Germany
Salvatore A. Marano, University of Catania, Italy
Dumitru Motreanu, University of Perpignan, France

Lifespan estimates for solutions of p-Kirchhoff systems.

Giuseppina Autuori
Polytechnical University of Marche, Italy
autuori@dipmat.univpm.it

In this talk we shall present some perturbed evolution systems governed by the p-Kirchhoff operator in bounded domains. These models are characterized by time dependent nonlinear driving forces and dynamic boundary conditions. The question of non-continuation of maximal solutions will be discussed and some a priori estimates for the lifespan of solutions will be given.

Existence and multiplicity of solutions for second order periodic systems with a nonsmooth potential

Giuseppina Barletta
University of Reggio Calabria, Italy
giuseppina.barletta@unirc.it
Nikolaos S. Papageorgiou

We present some existence and multiplicity results for the following second order periodic system with a nonsmooth potential

$$\begin{cases} -x''(t) - A(t)x(t) \in \partial j(t, x(t)) \text{ a.e. on } T = [0, b], \\ \qquad\qquad x(0) = x(b),\ x'(0) = x'(b)\,. \end{cases}$$

Here $A : T \to R^{N\times N}$ is a continuous map and for every $t \in T$, $A(t)$ is a symmetric $N \times N$-matrix. Also $j : T \times R^N \to R$ is a measurable function, which is locally Lipschitz and in general nonsmooth in the $x \in R^N$ variable. We provide different sets of verifiable hypotheses on $j(t, x)$ ensuring the existence of at least one or two nontrivial solutions of the problem above. In particular, in an existence theorem the Euler functional is coercive and bounded below, in others it is unbounded and in still others it is bounded below but not coercive. Furthermore, in some cases, the analytical framework incorporates strongly resonant periodic systems.

A local minimum theorem and applications

Gabriele Bonanno
Messina University, Italy
bonanno@unime.it

A local minimum theorem is presented and applications to nonlinear problems, as elliptic Dirichlet problems with critical growth, are given. Relations between the mountain pass theorem and local minima are pointed out. As a consequence, multiple critical points theorems, as a theorem of two nonzero critical points, are obtained. Finally, multiplicity results for wide classes of nonlinear problems are established.

Periodic solutions of second order ODEs: a symplectic approach

Alberto Boscaggin
University of Milano-Bicocca, Italy
alberto.boscaggin@unito.it
Rafael Ortega, Fabio Zanolin

We present some recent applications of the Poincaré-Birkhoff fixed point theorem to the search of periodic (harmonic and subharmonic) solutions of second order ordinary differential equations. Joint works with Rafael Ortega (University of Granada) and Fabio Zanolin (University of Udine).

Optimization of the principal eigenvalue under mixed boundary conditions

Lucio Cadeddu
Dept. of Mathematics and Computer Science, Italy
cadeddu@unica.it
G. Porru, M.A. Farina

We investigate minimization and maximization of the principal eigenvalue of the Laplacian under mixed boundary conditions in case the weight has indefinite sign and varies in a class of rearrangements. In special cases, we prove results of symmetry and results of symmetry breaking for the optimizers (minimizers).

Some multiplicity results for p-Laplacian type problems with an asymptotically p-linear term

Anna Maria Candela
University of Bari, Italy
annamaria.candela@uniba.it

Let us consider the p-Laplacian type equation

$$\begin{cases} -\operatorname{div}(A(x,u)\ |\nabla u|^{p-2}\ \nabla u) + \\ \dfrac{1}{p}\ A_t(x,u)\ |\nabla u|^p \ = \ f(x,u) \text{ in } \Omega, \\ \qquad\qquad u \ = \ 0 \text{ on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $N \geq 2$, $p > 1$, $A : \Omega\times\mathbb{R} \to \mathbb{R}$ is a given function which admits the partial derivative $A_t(x,t) = \frac{\partial A}{\partial t}(x,t)$ and $f : \Omega\times\mathbb{R} \to \mathbb{R}$ is asymptotically p-linear at infinity. Under suitable hypotheses both at the origin and at infinity and by using variational tools and index theories we prove some multiplicity results in the non-resonant case when $A(x,\cdot)$ is even and $f(x,\cdot)$ is odd. Joint works with Giuliana Palmieri and Kanishka Perera.

Constant-sign solutions for nonlinear discrete problems

Pasquale Candito
University of Reggio Calabria, Italy
pasquale.candito@unirc.it

The talk makes the point on some recent existence and multiplicity results for difference equations obtained by requiring nonstandard conditions on the nonlinearities. The approach adopted is based on variational methods on finite dimensional Banach spaces, truncations techniques and the discrete maximum principle. In particular, it points out that, in some cases, such results are not the analogous of those achieved by using the same tools for the corresponding differential equations.

Linear and nonlinear eigenvalue problems for Dirac systems in unbounded domains

Anna Capietto
University of Torino, Italy
anna.capietto@unito.it
Walter Dambrosio, Duccio Papini

It is proved the existence of a continuum of solutions of a special form to a nonlinear partial differential equation of Dirac type. To this end, we use the notions of "partial wave subspace", in connection with the well-known concept of rotation number.

Multi-valued variational inequalities versus variational-hemivariational inequalities

Siegfried Carl
University Halle, Germany
siegfried.carl@mathematik.uni-halle.de

We present recent existence and enclosure results for multi-valued variational inequalities and related variational-hemivariational inequalities.

Existence and multiplicity results for elliptic problems with variable exponent

Antonia Chinni'
DICIEAMA, University of Messina, Italy
achinni@unime.it

We present a collection of results on the existence and multiplicity of weak solutions for some elliptic problems involving the $p(\cdot)$-Laplace operator where $p \in C(\bar{\Omega})$ and $\Omega \subset \mathbb{R}^N$ is an open bounded domain with smooth boundary. In particular, under an appropriate oscillating behaviour of the nonlinearity f, the Neumann and Dirichlet problems will be studied in the case $1 < p^- \leq p(x) \leq p^+ < +\infty$. The results are based on variational methods.

Some existence results for a semilinear elliptic problem with a singular term

Francesca Faraci
Catania University, Italy
ffaraci@dmi.unict.it
G. Anello, Cs. Farkas

We discuss some existence results for a semilinear elliptic equation involving a singular term of the type $-\Delta u = f(u) + u^{q-1}$, coupled with Dirichlet boundary conditions. We assume that $q \in]0,1[$ and f is a continuous function. Under different assumptions on f we will prove the existence of a positive solution for our problem. Our approach is variational and combines methods from classical critical point theory.

Comparison and positive solutions for a class of dirichlet problem involving the (p,q)-laplacian

Luiz Fernando Faria
Universidade Federal de Juiz de Fora, Brazil
luiz.faria@ufjf.edu.br
Olimpio Hiroshi Miyagaki, Dumitru Motreanu

The aim of this paper is to prove the existence of a positive solution for a quasi-linear elliptic problem involving the (p,q)-Laplacian and a convection term, which means an expression which is not in the principal part and depends on the solution and its gradient. The solution is constructed through an approximating process based on gradient bounds and regularity up to the boundary. The positivity of the solution is shown by applying a new comparison principle which is established here.

Optimization of the principal eigenvalue under mixed boundary conditions

Maria Antonietta Farina
University of Cagliari, Italy
mafarina@unica.it
G. Porru, L. Cadeddu

We investigate biologically-oriented problems, motivated by the question of determining the most convenient spatial arrangement of favorable and unfavorable resources for a species to survive or to decline. We prove existence and uniqueness results, and present some features of optimizers.

A quasilinear elliptic equations involving critical Sobolev exponents

Csaba Farkas
Babes Bolyai University, Cluj Napoca, Romania
farkas.csaba2008@gmail.com
Francesca Faraci

In the present talk we consider the following quasilinear elliptic equation $-\Delta_p u = |u|^{p^*-2}u + g(u)$, in Ω coupled with Dirichlet boundary condition, where Ω is a bounded domain of $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, g is a continuous function with suitable growth condition. We will prove the existence of a weak solution for problem by combining semicontinuity argument with direct methods of calculus of variations. The existence of a local minimum for the energy functional is ensured provided a suitable algebraic inequality is fulfilled.

Multiple positive solutions for a superlinear problem: a topological approach

Guglielmo Feltrin
SISSA, Italy
gfeltrin@sissa.it
Fabio Zanolin

We study the multiplicity of positive solutions for a two-point boundary value problem associated to the nonlinear second order equation $u'' + f(x,u) = 0$. We allow $x \mapsto f(x,s)$ to change its sign in order to cover the case of scalar equations with indefinite weight. Roughly speaking, our main assumptions require that $f(x,s)/s$ is below λ_1 as $s \to 0^+$ and above λ_1 as $s \to +\infty$. In particular, we can deal with the situation in which $f(x,s)$ has a superlinear growth at zero and at infinity. We propose a new approach based on topological degree which provides the multiplicity of solutions. Applications are given for $u'' + a(x)g(u) = 0$, where we prove the existence of $2^n - 1$ positive solutions when $a(x)$ has n positive humps and $a^-(x)$ is sufficiently large.

Nodal solutions for Neumann problems with a nonhomogeneous differential operator

Michael Filippakis
University of Piraeus, Dept. of Digital Systems, Greece
mfilip@unipi.gr
Nikolaos S. Papageorgiou

In this paper, we consider a nonlinear elliptic equation driven by the p-Laplacian and with a parameter $\lambda > 0$. Using a combination of variational and degree theoretic methods, we show that there exists $\lambda^* > 0$ such that, if $\lambda > \lambda^*$, then the problem has two positive smooth solutions. Our result extends earlier ones by Rabinowitz (semilinear equations) and Guo (nonlinear equations)

An abstract existence result for solitons and applications

Donato Fortunato
University of Bari Italy, Italy
donato.fortunato@uniba.it
Vieri Benci

We state an abstract existence theorem for solitons and we apply this theorem to some field equations.

A recent result about existence of periodic solutions to the Brillouin equation

Maurizio Garrione
Università di Milano-Bicocca, Italy
maurizio85.g@gmail.com
Manuel Zamora

We will deal with the scalar second order equation (known as Brillouin electron beam focusing equation)

$$x'' + b(1 + \cos t)x = \frac{1}{x},$$

where b is a positive constant. A classical problem related to this equation is to determine the values of b for which there exists a 2π-periodic positive solution. This gave rise to an unproven conjecture, motivated by some numerical experiments, saying that this always happens if $b \in (0, 1/4)$. In this talk, we present a new range of values of b for which the Brillouin equation has a 2π-periodic solution, disjoint from the conjectured one $(0, 1/4)$. Precisely, we will prove 2π-periodic solvability for $b \in [0.4705, 0.59165]$. The technique of proof relies on careful estimates of the rotation numbers of the solutions in the phase-plane, under suitable nonresonance assumptions which will be briefly commented.

Periodic solutions for a class of second-order differential systems with impulses

Shapour Heidarkhani
Razi University, Iran
sh.heidarkhani@yahoo.com

Based on variational methods and critical point theory, under suitable assumptions on the nonlinear terms, we establish the existence of multiple periodic solutions for a class of perturbed second-order differential systems with impulses. We illustrate the results by giving convenient examples.

Weyl-type laws for fractional p-eigenvalues

Antonio Iannizzotto
University of Verona, Italy
antonio.iannizzotto@univr.it
Marco Squassina

Weyl's law yields an estimate of the asymptotic behavior of variational eigenvalues of a differential operator. First proved by Weyl (1912) for the Laplacian, it was extended by Garcia Azorero-Peral Alonso (1988) and Friedlander (1989) to the p-Laplacian. We define a sequence of variational min-max eigenvalues for the fractional p-Laplacian and prove a two-side asymptotic estimate for it. The method relies on the cohomological index and Krasnoselskii genus.

On some boundary value problems with singular Laplacians

Petru Jebelean
West University of Timisoara, Romania
jebelean@math.uvt.ro

We survey recent results on multiplicity of periodic solutions for some N-dimensional relativistic equations. Also, we present existence and multiplicity results for Dirichlet problems involving the mean extrinsic curvature operator in Minkovski space. The talk is based on joint work with Cristian Bereanu, Jean Mawhin, Călin Şerban and Pedro Torres.

Existence Results and Inverse Problems for Evolutionary Quasi Variational Inequalities

Akhtar Khan
Rochester Institute of Technology, USA
aaksma@rit.edu
D. Motreanu

This paper gives new existence results for parabolic and elliptic quasi variational inequalities. We will also study an inverse problem for quasi variational inequalities.

Topological invariants of almost Hamiltonian systems with singularities of symplectic structure

Mikhail Kharlamov
Presidential Academy NEPA, Russia
mikeh@inbox.ru

In a non-reducible integrable Hamiltonian system with three degrees of freedom, we can present the set of the critical points of the momentum mapping as a union of the phase spaces of Hamiltonian systems with less number of degrees of freedom. In such critical subsystems, manifolds of co-dimension 1 can exist, on which the induced symplectic structure degenerates. Sometimes the topology of a critical subsystem does not notice such degeneration. In this case the theory of the Fomenko-Zieschang topological invariants can be applied. The corresponding example is the integrable system with two degrees of freedom found by O.I.Bogoyavlensky in the dynamics of a heavy magnet. In other problems we come across bifurcations which are impossible in the systems without singularities of the symplectic form. Moreover, we present an example of a critical subsystem with two degrees of freedom having non-orientable phase space. In this system, new types of loop molecules, non-orientable 3-atoms and bifurcations arise. We classify 3-dimensional isoenergetic manifolds which are S^1-bundles over orientable or non-orientable 2-dimensional surfaces. The investigation is carried out with the help of algebraic separation of variables found for this system. Such separating gives rise to a universal algorithm of calculating exact topological invariants of the system in the form of gluing matrices.

A variational approach to Nash equilibria on Riemannian manifolds

Alexandru Kristaly
Babes-Bolyai University, Romania
alexandrukristaly@yahoo.com

The concept of Nash-Stampacchia equilibrium points is introduced via variational inequalities on Riemannian manifolds. Characterizations, existence, and stability of Nash-Stampacchia equilibria are studied when the strategy sets are compact/noncompact geodesic convex subsets of Hadamard manifolds, exploiting two well-known geometrical features of these spaces both involving the metric projection map. Our analytical approach exploits various elements from set-valued and variational analysis, dynamical systems, and non-smooth calculus on Riemannian manifolds. The talk is based on the paper: A. Kristaly, Nash-type equilibria on Riemannian manifolds: a variational approach, J. Math. Pures Appl., in press.

Infinitely many solutions for a nonlinear Klein-Gordon-Maxwell System

Lin Li
Southwest University, China, Peoples Rep of China
lilin420@gmail.com
Chun-Lei Tang

In this paper, a nonlinear Klein-Gordon-Maxwell System with solitary waves solution is considered. Using critical point theory, we establish sufficient conditions for the existence of infinitely many solitary waves solutions. Results obtained improve and unifies the existing ones for the Klein-Gordon-Maxwell System with non-constant potential even for the Schrödinger–Poisson System with non-constant potential.

Nodal and multiple solutions for a Dirichlet problem involving the p-Laplacian

Roberto Livrea
Reggio Calabria University, Italy
roberto.livrea@unirc.it
P. Candito and S. Carl

The talk deals with some recent multiplicity results, jointly obtained with P. Candito and S. Carl, for a parametric Dirichlet quasilinear problem. In particular, it will be shown how the combined use of variational methods and the sub-supersolution techniques assures the existence of explicit intervals of parameters for which the problem under examination admits multiple solutions satisfying some sign conditions, as well an a priori estimate. An alternative approach based on the pseudomonotone operator theory will be also outlined.

An approximation solvability method for nonlocal differential problems in Hilbert spaces

Luisa Malaguti
University of Modena and Reggio Emilia, Italy
luisa.malaguti@unimore.it
I. Benedetti, N.V. Loi and V. Obukhovskii

A new approach is developed for the solvability of nonlocal problems in Hilbert spaces associated to nonlinear differential equations. Periodic, anti-periodic, mean value and multipoint conditions are included in this study. The investigation is based on a joint combination of the degree theory with the approximation solvability method. Hartman-type inequalities are involved. No compactness or condensivity conditions on the nonlinearities are assumed. Applications to the study of integro-differential equations and systems of integrodifferential equations are showed. The method is then extended to a multivalued setting and a feedback control problem is discussed.

Non-smooth critical point theory on closed convex sets and applications

Salvatore Marano
University of Catania, Italy
marano@dmi.unict.it
Sunra J.N. Mosconi

The existence of critical points for a locally Lipschitz continuous functional F on a closed convex set C of a Banch space X is investigated. The problem of finding extra conditions under which critical points for F on C turn out to be critical on X is also addressed. Some applications concerning elliptic variational-hemivariational inequalities are then worked out.

Bounds for blow-up time in nonlinear parabolic problems under various boundary conditions

Monica Marras
University of Cagliari, Italy, Italy
mmarras@unica.it
Stella Vernier-Piro

The question of blow-up of solutions to nonlinear parabolic equations and systems has received considerable attention in the recent literature.In practical situations one would like to know among other things whether the solution blows up and, if so, at which time blow-up occurs. When the solution does blow up at some finite time T, this time can seldom be determined explicitly, so much effort has been devoted to the calculation of estimates for T. Most of the methods used until recently have yielded only estimates from above for T, so that in particular problems in which blow-up has to be avoided, they are of little value. We are mainly interested in estimates from below. In particular, we investigate the question of blow-up for nonnegative classical solutions of some nonlinear parabolic problems defined in bounded domains. Under conditions on data and geometry of the spatial domain, explicit estimates from below for the blow-up time are derived.

On a class of the nonhomongeneous eigenvalue problems and applications

Olimpio Miyagaki
UFJF-Universidade Federal de Juiz de Fora, Brazil
ohmiyagaki@gmail.com
J.M. Do O, S. Moreira Neto

in this work we establish the existence of standing wave solutions for quasilinear Schrodinger equations involving subcritical growth at resonance. By using a change of variables, the quasilinear equation is reduced to semilinear one, which associated functional is well defined in the usual Sobolev space. The “first” eigenvalue type of a nonnhomogeneous operator was studied. Using this fact and a variant of the monotone operator theorem, we show that the problem at resonance has at least one nontrivial solution.

Nonlinear elliptic problem driven by a nonhomogeneous operator

Dumitru Motreanu
University of Perpignan, France
motreanu@univ-perp.fr

The obtained results ensure existence of multiple solutions, under Dirichlet and Neumann boundary conditions, for nonlinear elliptic equations whose differential part is expressed through a nonhomogeneous operator. The presence of the non-homogeneity raises serious difficulties, especially related to the eigenvalue problems. Through our approach, information regarding the sign of solutions is available.

A quasilinear singular elliptic system with superhomogeneous condition

Abdelkrim Moussaoui
A. Mira Bejaia University, Algeria
remdz@yahoo.fr
Claudianor O. Alves

In the present contribution we establish the existence of at least two distinct (positive) smooth solutions of a singular quasilinear system of elliptic equations with superhomogeneous condition. The proof of existence of the first solution is based on the sub-supersolution methods for systems of quasilinear equations combined with perturbation arguments involving singular terms. The structure of the singular terms in the system is essentially used to construct the sub-supersolution. The second solution is obtained via topological degree argument combined with a priori bounds of solutions.

A multiplicity result for the scalar field equation

Kanishka Perera
Florida Institute of Technology, USA
kperera@fit.edu

We prove the existence of $N-1$ distinct pairs of nontrivial solutions of the scalar field equation in $\mathbb{R}^N$ under a slow decay condition on the potential near infinity, without any symmetry assumptions. Our result gives more solutions than the existing results in the literature when $N \geq 6$. When the ground state is the only positive solu-

tion, we also obtain the stronger result that at least $N-1$ of the first N minimax levels are critical, i.e., we locate our solutions on particular energy levels with variational characterizations. Finally we prove a symmetry breaking result when the potential is radial. To overcome the difficulties arising from the lack of compactness we use the concentration compactness principle of Lions, expressed as a suitable profile decomposition for critical sequences.

Existence of entire solutions for a class of variable exponent elliptic equations

Patrizia Pucci
University of Perugia, Italy
patrizia.pucci@unipg.it
Qihu Zhang

The talk deals with the existence of entire solutions of a quasilinear equation in $\mathbb{R}^N$, which involves a general variable exponent elliptic operator $\mathbf{A}$ of the $p(x)$-Laplacian type in divergence form and two main nonlinearities of growth $q = q(x)$ and $r = r(x)$, involving two coefficients. The results we present extend the previous work in several directions. We first weaken the condition $\max\{2, p\} < q < \min\{r, p^*\}$ to the simpler request that $1 << q << r$. We also ask milder assumptions on the coefficients of the nonlinearities, as well as a very weak ellipticity condition on $\mathbf{A}$. The results we present are new even in the case of constant exponents and even in the semilinear case $p \equiv 2$.

Bifurcations of First Integrals in the Goryachev case

Pavel Ryabov
Financial University, Russia
orelryabov@mail.ru

The Chaplygin case (1903) of the problem of rigid body motion in fluid was generalized by D.N.Goryachev (1916) to the problem with the potential that has a singularity in the equatorial plane of the inertia ellipsoid. In this talk we represent the results on the phase topology of particular case of D.N.Goryachev integrability. To study the phase topology for the Goryachev case, we use the explicit real separation of variables. This fact helped us to obtain the explicit form of the Abel-Jacobi equations with the sixth power polynomial under the radical and the algebraic expression of all phase variables in terms of real separated variables. The analytic formulas obtained allow us to study phase topology, in particular, bifurcations of Liouville tori. The investigation is carried out with the help of the method of Boolean functions developed by M.P.Kharlamov for algebraically separable systems. We found the bifurcation diagram of the moment mapping and calculate the Fomenko invariant which makes it possible to classify the system up to rough Liouville equivalence.

Multiplicity results for some perturbed elliptic problems in unbounded domains

Addolorata Salvatore
University of Bari, Italy
addolorata.salvatore@uniba.it

We study the following semilinear elliptic problem

$$\begin{cases} -\Delta u + mu = |u|^{p-2}u + f(x) & \text{in } \Omega = \mathbb{R}^N \setminus B_R \\ u = \xi & \text{on } \partial\Omega = \partial B_R \\ u \to 0 & \text{as } |x| \to +\infty, \end{cases}$$

where $m > 0$, $N \geq 3$, $\xi \in \mathbb{R}$ and $p > 2$ but subcritical. If $f : \Omega \to \mathbb{R}$ is a radial function, we prove the existence of infinitely many radial solutions by using variational tools, perturbative methods and suitable growth estimates on min-max critical levels. No restriction on the exponent p is required for N large enough. Joint work with Sara Barile.

Numerical extremal solutions for a mixed problem with singular ϕ-Laplacian

Calin Serban
West University of Timisoara, Romania
cserban2005@yahoo.com

We are concerned with extremal solutions for the mixed boundary value problem

$$-(r^{N-1}\phi(u'))' = r^{N-1}g(r,u), \qquad u'(0) = 0 = u(R),$$

where $g : [0, R] \times \mathbb{R} \to \mathbb{R}$ is a continuous function and $\phi : (-\eta, \eta) \to \mathbb{R}$ is an increasing homeomorphism with $\phi(0) = 0$. We prove the existence of minimal and maximal solutions in presence of well-ordered lower and upper solutions and we develop a numerical algorithm for theirs approximation. The talk is based on joint work with Petru Jebelean and Constantin Popa.

Some distinct phenomena in $p(x)$-Laplacian problems

Inbo Sim
University of Ulsan, Korea
ibsim@ulsan.ac.kr
Ky Ho

In this talk, we show the existence results of nontrivial nonnegative solutions for degenerate $p(x)$-Laplacian problems with weighted convex and cocave terms which have a parameter. The proofs are mainly based on variational methods, Ekeland principle, Mountain pass lemma and Fountain theorem. We investigate the relationship between the growth rates of principal part, convex and concave part which can not be appeared in p-Laplacian problems.

Positive solutions for nonlinear elliptic equations with a singular term

George Smyrlis
National Technical University of Athens, Greece
gsmyrlis@math.ntua.gr
Nikolaos S. Papageorgiou

In this contribution we study nonlinear elliptic problems with a singular term of the form

$$\begin{cases} -\text{div}\, a(Du(z)) = \beta(z)u(z)^{-\gamma} + f(z,u(z)), \\ \qquad\qquad\qquad \text{for almost all } z \in \Omega, \\ u\mid_{\partial\Omega} = 0, \quad \gamma \in (0,1), \end{cases}$$

where $\Omega \subseteq \mathbb{R}^N$ $(N \geq 1)$ is a bounded domain with a C^2-boundary $\partial\Omega$, $a : \mathbb{R}^N \to \mathbb{R}^N$ is strictly monotone with certain regularity properties and $\beta \in C(\Omega) \cap L^\infty(\Omega)_+ \setminus \{0\}$. Moreover, $f \in C(\Omega \times \mathbb{R})$ being either superlinear near $+\infty$ (without satisfying the Ambrosetti-Rabinowitz condition) or sublinear near $+\infty$. We prove three multiplicity theorems producing at least two smooth positive solutions for problems of the above form. In the first one, the leading differential operator is not in general homogeneous and the result is obtained for "small" $||b||_\infty$. The other two results concern problems driven by the p-Laplacian differential operator. In these cases, no restrictions are imposed on $||b||_\infty$. Our approach is variational employing suitable truncation and comparison techniques.

A logarithmic Schrödinger equation with periodic potential

Andrzej Szulkin
Department of Mathematics, Stockholm University, Sweden
andrzejs@math.su.se
Marco Squassina

We consider the logarithmic Schrödinger equation

$$-\Delta u + V(x)u = Q(x)u \log u^2, \quad u \in H^1(\mathbb{R}^N),$$

where V, Q are periodic in $x_1, \ldots, x_N$, $Q > 0$ and $V + Q > 0$. We show that this equation has infinitely many geometrically distinct solutions and that one of these solutions is positive. The main difficulty here is that the functional associated with this problem is lower semicontinuous and takes the value $+\infty$ for some $u \in H^1(\mathbb{R}^N)$.

Generalized eigenvalue problems for (p, q)-Laplacian with indefinite weight

Mieko Tanaka
Tokyo University of Science, Japan
tanaka@ma.kagu.tus.ac.jp

In this talk, I will talk existence and non-existence results on a positive solution for quasilinear elliptic equations of the form $-\Delta_r u - \mu\Delta_{r^*} u = \lambda m_r(x)|u|^{r-2}u$ in Ω.

Positive solutions for a system of nonlinear integral boundary value problems

Rodica Luca Tudorache
Technical University of Iasi, Romania
rlucatudor@yahoo.com
Johnny Henderson

We investigate a system of nonlinear second-order ordinary differential equations with two eigenvalues, subject to integral boundary conditions. Under some assumptions on the eigenvalues and nonlinearities, we prove the existence and nonexistence of positive solutions. In a special case, we also study the multiplicity of positive solutions by applying the fixed point index theory.

Sharp pointwise estimates from above and from below for solutions to a class of singular parabolic problems

Stella Vernier-Piro
Universiy of Cagliari, Italy, Italy
svernier@unica.it
Vincenzo Vespri , Francesco Ragnedda

This is a joint research with Vincenzo Vespri e Francesco Ragnedda. We deal with the Cauchy problem associated to a class of quasilinear singular parabolic equations with L^∞ coefficients, which prototypes are the p-laplacian and the Porous medium equation (in the supercritical range). Estimates for the p-Laplacian and Porous medium equations (both for the slow diffusion and fast diffusion case) have been considered by several authors. To prove estimates for more general operators, we are forced to use a completely different and more sophisticated approach, based on DiBenedetto's techniques, recent Harnack inequalities and De Giorgi estimates. Sharp pointwise estimates from above and from below for the fundamental solutions are derived, assuming as initial data the Dirac mass. Our results can be extended to general nonnegative L^1 initial data.

Blowing up and global solutions of a nonlinear parabolic problem

Giuseppe Viglialoro
University of Cagliari, Italy, Italy
giuseppe.viglialoro@unica.it
M. Marras, S. Piro-Vernier

This session deals with the blowing up and global solutions of a nonlinear and weakly coupled parabolic system, containing gradient terms, under Dirichlet boundary conditions. The blow-up phenomena of its positive solutions are analyzed and, in particular, an analytical estimate of the lower bound of the blow-up time is obtained. Moreover, we propose a resolution algorithm capable to solve the original problem. The numerical examples both confirm the theoretical result and allow to observe other interesting phenomena connected to the behavior of the solutions, as for instance their global existence.

Resonant (p,2)-equations with concave terms

Patrick Winkert
Technische Universitaet Berlin, Germany
winkert@math.tu-berlin.de
Nikolaos S. Papageorgiou

We consider a nonlinear, nonhomogeneous parametric elliptic Dirichlet equation driven by the sum of a p-Laplacian and a Laplacian (so-called $(p, 2)$-equation) and with a nonlinearity involving a concave term which enters with a negative sign. By applying variational methods along with truncation and comparison techniques as well as Morse theory, we show that the problem under consideration has at least five nontrivial solutions (four of them have constant sign) for all sufficiently small values of the parameter.

A fixed point theorem on topological cylinders, with applications

Fabio Zanolin
University of Udine, Italy
fabio.zanolin@uniud.it

We present a fixed point theorem for completely continuous operators in Banach spaces, which makes use of a "stretching along the paths" property and we also show some applications to existence and multiplicity of solutions for nonlinear equations in different situations (both in the case of finite and infinite dimensional spaces). The main abstract results are based on joint work with Duccio Papini (University of Siena) and recent developments by Guglielmo Feltrin (SISSA, Trieste). Some applications to periodic boundary value problems are proposed.

Special Session 22: Modeling and Dynamic Analysis of Complex Patterns in Biological Systems and Data

Jianzhong Su, University of Texas at Arlington, USA
Qishao Lu, Beihang Univ., China
Miguel A. F. Sanjuan, Universidad Rey Juan Carlos, Spain

Many biological systems, such as neuronal systems, genomic systems, tissue systems and disease systems, are featured by certain nonlinear and complex patterns in elements and networks. These phenomena carry significant biological information and regulate down-stream mechanism in many instances. The common themes of this session include mathematical models and theoretical analysis, computational and statistical methods of dynamical systems and differential equations, as well as applications, in neurodynamics, systems biology, inflammatory responses, diseases, biophysics and biochemistry, with diversity of couplings and noises and in several scales. The topics may include but not restrict to: 1. Modeling, dynamical analysis and computation of electrical , chemical and cognitive activities in neuronal systems and nervous disorders. 2. Modeling and dynamics of biomedical processes , healing and inflammatory responses in diseases and immune systems. 3. Dynamics and computation of biological circuitry and bioinformics in protein structures, gene regulation networks and evolutionary biology.

Strategy for modeling the growth and degradation of the fibrous cap of an atherosclerotic plaque

Jon Bell
University of Maryland Baltimore County (UMBC), USA
jbell@umbc.edu

Atherosclerosis is an inflammatory disease of major arteries due to fatty lesions forming in arterial walls, causing stenosis (constricting blood flow) or thrombosis (blood clots, blockage). Certain lesions, called vulnerable plaques, are responsible for most deaths from atherosclerosis. The growth and degradation of these plaques is dynamic, involving complex biochemical, hemodynamic, and mechanical interactions. Present experimental means for studying arterial plaque development is limited, calling for augmenting such studies by mathematical modeling, analysis, and simulation. I will outline a strategy for model development, starting with an ODE model of principle chemical processes, and progressing to more complicated, more mechanistic PDE models. This is an early stage investigation, so the talk is a possible roadmap for approaching a variety of questions.

Change detection in the dynamics of an intracellular protein synthesis model using nonlinear Kalman Filtering

Jean-Daniel Djida
Ngaoundéré University, Cameroon
jdjida@yahoo.fr
Gerasimos Rigatos

A method for early diagnosis of parametric changes in intracellular protein synthesis models, is developed with the use of a nonlinear Kalman Filtering approach (Derivative-free nonlinear Kalman Filter) and of statistical change detection methods. The intracellular protein synthesis dynamic model is described by a set of coupled nonlinear differential equations. It is shown that such a dynamical systems satisfies differential flatness properties and this allows to transform it, through a change of variables (diffeomorphism), to so-called linear canonical form. For the linearized equivalent of the dynamical system, state estimation can be performed using the Kalman Filter recursion, while by applying an inverse transformation based on the previous diffeomorphism it becomes also possible to obtain estimates of the state variables of the initial nonlinear model. By comparing the output of the Kalman Filter (which is assumed to correspond to the undistorted dynamical model) with measurements obtained from the monitored protein synthesis system, a sequence of differences (residuals) is obtained. The statistical processing of the residuals with the use of χ^2 change detection tests, can provide indication within specific confidence intervals about parametric changes in the considered biological system (e.g. the p53 protein-mdm2 inhibitor model) and consequently indications about the appearance of specific diseases (e.g. malignancies).

Studies on firing patterns of excitatory neurons within the Pre-Botzinger complex based on the first recurrence map

Lixia Duan
North China Univ. of Tech., Peoples Rep of China
lxduan2012@gmail.com
Xi Chen and Jing Liu

The pre-Botzinger complex of the mammalian brain stem has been discovered to play a crucial role in generating respiratory rhythms. Excitatory neurons within the pre-Botzinger complex can exhibit different bursting patterns. In this paper, we study different bursting patterns of a self-coupled single cell network of Butera model based on first recurrence map. By the first recurrence map the original model will be reduced to a one-dimensional map. With the sodium conductance changed, the bursting pattern switch from one to another, and the properties of the one-dimensional map are studied. Through the analysis of the map, some analytical conditions for bursting patterns transition from one type to another are confirmed, and numerical results are also presented.

Synchronization and noise-induced phase slips

Barbara Gentz
University of Bielefeld, Germany
gentz@math.uni-bielefeld.de
Nils Berglund

Synchronization of biological clocks is a well studied topic. We will address the question how noise in general affects synchronization before turning to a system of two coupled oscillators. In the absence of noise and for sufficiently

strong coupling, synchronization is observed (the oscillators are “in phase”). In the presence of small noise, the system will still spend most of the time in metastable equilibrium, i.e., the difference in phase will show only small fluctuations. However, occasionally transitions to other metastable states are observed. Such transitions can by viewed as a “phase slips”. We will derive an expression for probability distribution of phase slips, thereby uncovering universal properties of the distribution.

Traveling pulses in a neural network with asymmetric coupling and non-saturating gain

Yixin Guo
Drexel University, USA
yixinguo@gmail.com
Aijun Zhang

We consider a neural field coupled by an asymmetric off-centered connection. We first construct the traveling pulses using a system of delayed differential equations derived from the neural field model. We then investigate the necessary conditions for traveling pulses to exist in such a neural network. We find that the asymmetry in the coupling function is crucial for traveling pulses to exist. We explore when the traveling pulses cease to exist in terms of the off-center shift presented in the asymmetric coupling. We further study other dynamical properties of the traveling pulses and how the propagating speed depends on the gain and coupling parameters.

Burst Synchronization of Chemically Coupled Hodgkin-Huxley Neuronal Networks with Synaptic Plasticity

Fang Han
Donghua University, Peoples Rep of China
yadiahan@163.com
Zhijie Wang, Bin Zhang

Burst synchronization of a globally coupled neuronal network composed of Hodgkin-Huxley neurons with delayed and plastic chemical synapses is studied in this paper. First a modified Hodgkin-Huxley neuronal model exhibiting bursting behaviors is proposed and used to construct neuronal systems, with a short-term synaptic plasticity considered in delayed chemical synapses. Then the effect of the learning rate, the coupling strength, and the synaptic delay on burst synchronization of the network is explored for excitatory and inhibitory neuronal networks, respectively. It is found that burst synchronization is sensitive to these parameter values and might be meaningful for information processing in human brain.

Noise and adaptation in multistable perception: a case study with tristable visual plaids

Gemma Huguet
Universitat Politecnica de Catalunya, Spain
gemma.huguet@upc.edu
John Rinzel, Jean-Michel Hupé

We study the dynamics of perceptual switching in ambiguous visual scenes that admit more than two interpretations. We focus on visual plaids that are tristable and we present both experimental and computational results. We developed a firing-rate model based on mutual inhibition and adaptation that involves stochastic dynamics of multiple-attractor systems. The model can account for the dynamical properties (transition probabilities, distributions of percept durations, etc) observed in the experiments. Noise and adaptation have been shown to both play a role in the dynamics of bistable perception. Here, tristable perception allows us to specify the role of noise and adaptation in our model. Noise is critical when considering the time of a switch. However, adaptation mechanisms are critical when considering perceptual choice (in tristable perception, each time a percept ends, there is a possible choice between two new percepts).

Analysis of the coupled inflammatory response in the lumen and sub-endothelial layer

Akif Ibragimov
Texas Tech University, Lubbock, Texas, USA
akif.ibraguimov@ttu.edu
L.Ritter, J. Walton

In this talk I will discuss joint work with Lake Ritter and Jay Walton on modeling inflammatory instability in coupled in the lumen and subendothelia layer system. First we will introduce a steady state equilibrium in the lumen which will generate non-standard condition on the interface between lumen and subendothelia. Nest we will investigate structural stability of linearized system depending on the input parameters and boundary conditions. The notion of a “healthy state” is then introduced as an equilibrium state in which inflammatory markers are absent. The mathematical question studied is the stability of this healthy state to small perturbations in the inflammatory markers. It is shown that the healthy state can be stabilized through hydro-diffusion processes and sufficiently strong anti-oxidant mitigation and destabilized through strong chemotactic and hydrodynamic effects which promote local lesion formation and growth and low anti-oxidant levels.

Reduction of a Kinetic Model of Active Export of Importins

Sarbaz Hamza Abdullah Khoshnaw
University of Leicester, Department of Mathematics, England
sarbazmath@yahoo.com

We study a kinetic model of active export of importins. The model was developed earlier. The kinetic model is written as a system of ordinary differential equations. We apply model reduction techniques to simplify the system. The techniques are: removal of slow reactions and quasi-steady state approximation. CellDesigner software is used to draw the chemical reaction network of the model, and Systems Biology Toolbox (SBedit) for Matlab is applied to simulate the concentration species. After model reduction, the numbers of reactions and species are reduced from 28 reactions and 29 species to 22 reactions and 21 species. The reduced model and original model are compared in numerical simulation. The methods of further model reduction are discussed.

Networks that learn to precisely encode the timing of sequences

Zachary Kilpatrick
University of Houston, USA
zpkilpat@math.uh.edu
Alan Veliz-Cuba, Kresimir Josic

We discuss a neuronal network model capable of learning the timing of a sequence of events, each of which lasts milliseconds to seconds. Short term facilitation is a second-timescale accumulation process that controls switching between events. Long term plasticity allows the network to learn event timings quickly and accurately. Time scale separations, between the plasticity processes and neuronal activity dynamics, allow us to describe how long term plasticity parameters should depend upon short term facilitation parameters for the network to be able to learn **any** sequence of timings.

Mixed-Mode Bursting Oscillations (MMBOs): slow passage through spike-adding canard explosion

Maciej Krupa
INRIA, France
maciej.krupa@inria.fr
Mathieu Desroches, Tasso J. Kaper

In this talk, I will present the phenomenon of Mixed-Mode Bursting Oscillations (MMBOs). These are solutions of fast-slow systems of ordinary differential equations that exhibit both small amplitude oscillations (SAOs) and bursts consisting of one or multiple large-amplitude oscillations (LAOs). The name MMBO is given in analogy to Mixed-Mode Oscillations (MMOs), which consist of alternating SAOs and LAOs, without the LAOs being organized into burst events. I will show how MMBOs are created naturally in systems that have a spike-adding bifurcation, or spike-adding mechanism, and in which the dynamics of one (or more) of the slow variables causes the system to pass slowly through that bifurcation. Canards are central to the dynamics of MMBOs, and their role in shaping the MMBOs is two-fold : saddle-type canards are involved in the spike-adding mechanism of the underlying burster and permit one to understand the number of LAOs in each burst event, and folded-node canards arise due to the slow passage effect and control the number of SAOs. The analysis is carried out for a prototypical fourth order system of this type, which consists of the third-order Hindmarsh-Rose (H-R) system, known to have the spike-adding mechanism, and in which one of the key bifurcation parameters also varies slowly.

A mathematical model of P53/MDM2 oscillation with regulations with PDCD5

Jinzhi Lei
Tsinghua University, Peoples Rep of China
jzlei@tsinghua.edu.cn
Changjing Zhuge

P53/MDM2 oscillation is known to be an important regulation in cell response to DNA damage, and PDCD5 is recently found to interaction with the P53/MDM2 network. In this talk we introduce our recent work on a mathematical model of PDCD5 regulating P53/MDM2 oscillation. The dynamical patterns of the P53/MDM2 oscillation under various level of PDCD5 and DNA repairing during are studied based on the proposed model. Our results suggest that the effect of PDCD5 in regulating the p53 oscillation on cell fate decision is non-monotonous and can relate to other functions of PDCD5 in various cell processes.

The Effect of Systemic Estrogen and Cortisol on the Inflammatory Phase of Wound Healing

Angela Reynolds
Virginia Commonwealth University, USA
areynolds2@vcu.edu
Racheal Cooper, Rebecca Segal, Robert Diegelmann

A complex combination of interactions initiates and regulates the inflammatory phase of the wound healing response. Many chronic wounds arise due to an improper transition out of this phase. To understand regulation of this transition, we developed a model of key interactions involving neutrophils and macrophages during wound healing. This model also accounts for the effects of cortisol, (a stress hormone, elevated post trauma), and estrogen (protective, treatment option) on the activity of these cells. Latin Hypercube sampling was performed to determine biologically feasible parameter sets for this model, which satisfy conditions for stability of the healthy outcome and peak times for these key cell types. Experimental data from Broughton et al (2006) and Lebovich & Ross (1975) was used to validate model results. This model forms the inflammatory subsystem of a wound healing model that accounts for wound dynamics post trauma.

Burst Synchronization and Rhythm Dynamics in a Two-layer Neuronal Network

Xia Shi
Beijing University of Posts and Telecommunications, Peoples Rep of China
shixiabupt@163.com
Yongbing Lv

In-phase burst synchronization and rhythm dynamics in a two-layer neuronal network are studied. A quantitative characteristic, the width factor, is introduced to describe the rhythm dynamics of an individual neuron, and the average width factor is used to characterize the rhythm dynamics of a neuronal network. In-phase synchronization can be reached by changing the value of the inter-coupling strengths for higher inter-connection probability of different sub-networks. And the bursting type of the network can transform from the short bursting to the long one for higher inter-connection probability of different sub-networks.

Modeling, Simulation and Analysis for Foreign Body Fibrotic Reactions in 2D

Jianzhong Su
University of Texas at Arlington, USA
su@uta.edu
Larrissa Owens, Akif Ibraguimov, Jichen Yang, Liping Tang

The implantation of medical devices often triggers several immune responses, one kind of which is categorized as foreign body reactions. It is well established that macrophages and many other cells participate in the complex processes of foreign body reactions, and cause severe inflammations and fibrotic capsule formation in surrounding tissues. However, the detailed mechanisms of macrophage responses, recruitment and activation, in foreign body reactions are not totally understood. In the meantime, mathematical models have been proposed to systematically decipher the behavior of this complex system of multiple cells, proteins and biochemical processes in wound healing responses. This talk introduces a kinetics-based model for analyzing reactions of various cells/proteins and biochemical processes as well as their transient behavior during the implant healing in 2-dimensional space. In particular, we provide a detailed modeling study of different roles of macrophages and their effects on fibrotic reactions. The mathematical result indicates that the stability of the inflamed steady state depends primarily on the reaction dynamics of the system. However, if the said equilibrium is unstable by its reaction-only system, the spatial diffusion and chemotactic effects can help to stabilize when the model is dominated by classical and regulatory macrophages over the inflammatory macrophages. We find that the simulation results are consistent with the experimental observations. These findings support that the model can reveal quantitative insights for studying foreign body reaction processes.

Effects of time-periodic intercoupling strength on burst synchronization of a clustered neuronal network

Xiaojuan Sun
Bejing University of Posts and Telecommunications, Peoples Rep of China
sunxiaojuan@bupt.edu.cn
Xia Shi, Fang Han

In this paper, we investigate the effect of time-periodic inter-coupling strength on burst synchronization of a clustered neuronal network. We mainly focus on discussing the effects of amplitude and frequency of the time-periodic inter-coupling strength on burst synchronization. We found that, by tuning the frequency, burst synchrony of the clustered neuronal network could change from higher synchronized states to low synchronized or unsynchronized states, and vice verse. While for the amplitude, we surprisingly found that with increasing of the amplitude, burst synchrony of the clustered neuronal network is not always enhanced. We know that synchronization has close relationship with cognitive activities and brain disorders. Thus, our results showed in this paper could give us some important implications on understanding the important role of time-dependent couplings in neuronal systems.

A Latently Infected Cell Inclusive Model for HIV Treatment with Time-varying Antiretroviral Therapy

Nicoleta Tarfulea
Purdue University Calumet, USA
ntarfule@purduecal.edu

We present a mathematical model to investigate theoretically and numerically the effect of latently infected cells in the presence of immune effectors in modeling HIV pathogenesis. Additionally, by introducing drug therapy, we assess the effect of treatments consisting of a combination of several antiretroviral drugs. A periodic model of bang-bang type and a pharmacokinetic model are employed to estimate the drug efficacies. We study the effectiveness of the treatment with respect to the time when it is introduced exploring whether immediate antiretroviral therapy can reduce HIV infection of resting CD4 T-cells. We also investigate numerically how time-varying drug efficacy due to drug dosing regimen and/or suboptimal adherence affects the antiviral response and the emergence of drug resistance.

Decreased firing time of given spikes by alteration of input strategies in leaky integrate-and-fire model

Jiaoyan Wang
Tianjin University of technology and education, Peoples Rep of China
jiaoyanwang@163.com

In this paper, the constrained optimization of excitatory synaptic input patterns to fastest generate given number of spikes is considered in leaky integrate-and-fire neuron model. Different input strategies are investigated by using phase plane arguments for discrete input kicks with a given total magnitude. Furthermore, analytical results are conducted to estimate the firing time of given number of spikes resulting from a given input train. Optimal input timings and strengths are identified to fastest generate given number of spikes. When total input size is sufficiently large, big kick is the fastest strategy. In addition, we establish an optimal value for the dependent variable, v, where each input should be delivered in a non-threshold-based strategy to fastest achieve given output of subsequent spikes.

Gamma oscillations with frequency volatility in an excitatory-inhibitory network

Zhijie Wang
Donghua University, Peoples Rep of China
wangzj@dhu.edu.cn
Hong Fan

Gamma rhythms may involve in many functional roles such as acting as reference signals for temporal encoding, sensory binding of features into a coherent percept, and storage and recall of information. Many experimental studies found that gamma oscillation appears in V1 after the presentation of the visual stimuli. A characteristic property of the gamma oscillation in V1 is that the peak frequency of the oscillation is sensitive to the strength, the size, or the contrast of the visual stimuli. However, according to the mechanism of the interneuron gamma(ING) and pyramidal-interneuron gamma (PING), the network frequency is sensitive to the decay time constant of the inhibitory synapses and is insensitive to other parameters. To explore the underlying mechanism of the frequency volatility of the gamma oscillation in V1, a simple excitatory/inhibitory (E/I) neural network, which simulates the structure of that of V1, is constructed. Gamma oscillation appears in the E/I network, the frequency of which is sensitive to the synaptic delay and the external drives. The frequency of the oscillation even varies in the two runs of the simulation of the same network with different initial conditions.

Understanding electromagnetic radiation and cortical information processing through a mathematical framework

Ying Wu
Xi'an Jiaotong University, Peoples Rep of China
wying36@163.com
Jiajia Li

Electromagnetic radiation appears to have a significant impact on the neural system, especially when the telephone is widely used, yet the mechanisms underlying this effect are unclear. Based on the model proposed by a neuroscientist Wachtel Howard, We established a mathematical framework that models the process of electromagnetic energy's translation into the form of neuronal current, and found that the firing frequency of neuron was decreased by increasing electromagnetic radiation intensity, which is well consistent with experimental data. Then, we developed such a model into neural network, and found that electromagnetic radiation have much effect on the dynamic characteristics of neural network. This phenomenon shows that electromagnetic radiation may act as obstacles for cortical information processing of brain.

Traveling Waves and Its Evolution in Species Reaction-Diffusion Model Considering Spatiotemporal Delay

Jian Xu
Tongji University,, Peoples Rep of China
xujian@tongji.edu.cn
Gao Xiang Yang

In this paper, the geometric singular perturbation method is employed to present existence of traveling wave in single species reaction-diffusion model with spatiotemporal delay and spatially homogeneous states. A periodic traveling wave in small amplitude is solved analytically in terms of the Turing bifurcation. Then, various patters of starting from the periodic traveling wave related to two spatially homogeneous states are displayed when the delay is considered as a variable parameter. The results show that the spatiotemporal delay has a substantial effect on the pattern such that distribution of the species can be changed form the homogeneous to complex states.

The transcriptional amplitude ratio modulates the mammalian circadian clock

Ling Yang
Soochow University, Peoples Rep of China
lyang@suda.edu.cn
Jie Yan, Guangsen Shi, Zhihui Zhang, Ying Xu

The circadian oscillator consists of a primary feedback loop and is associated with one or more auxiliary loops . We constructed a conceptual model of period dynamics in which the REV-ERB/ROR response element (RORE)-mediated auxiliary loop coordinates with the E-box-driven primary loop to modulate mammalian circadian period length. Using a combined mathematical-experimental approach, we found that the amplitude ratio of E-box activity to RORE activity can regulate the period length, even when posttranslational feedback is fixed. This transcriptional modulation system ensures period robustness through amplitude-locked (transmitting) loops, and therefore provides a possible means of regulating circadian period length by targeting both the E-box and RORE to disrupt the interlocking amplitude ratio. The ratio rule highlights the transcriptional architecture of the circadian clock and allows us to probe the design principles of its complex natural multi-loops.

The projective synchronizaiton of a neural network with community structure

Li-Xin Yang
Xian Jiaotong University, Peoples Rep of China
jiaodayanglixin@163.com
Jun Jiang, Xiao-Jun Liu

Synchronization in the brain can be related to cognitive capacities as well as to pathological conditions. Therefore, there has been tremendous interest in the study of synchronization in neural networks. In this paper, the synchronization of a neural network where the nodes are modeled as FitzHugh-Nagumo (FHN) systems with community structure is investigated. Cluster projective generalizes previously existing synchronization schemes. In comparison with the existing synchronization, the cluster projective synchronization is more general that includes projective synchronization and cluster synchronization, as its special cases. The cluster projective synchronization of these networks is discussed via some pinning control strategy. Only the nodes in one community which have direct connections to the nodes in other communities are controlled. Several sufficient conditions for the network to achieve cluster projective synchronization are derived based on Lyapunov stability theory. Numerical simulations are used to demonstrate the effectiveness and feasibility of the proposed scheme.

Noise-induced switching kinetics in multiple interlinked positive feedback loops

Zhuoqin Yang
Beihang University, Peoples Rep of China
yangzhuoqin@buaa.edu.cn
Yuanhong Bi

Positive feedback, especially interlinked positive feedback loops, may play a key role in bistability over a range of biological conditions at the heart of many celluar processes. In this work, a new interlinked positive feedback loops model is set up by adding two positive feedbacks between two species, based on the two-loop models[1,2]. At a certain stimulus level, the positive feedback strength is mainly applied to survey bistablility in the interlinked positive feedback loops, by means of codimension-1 and 2 bifurcations analysis. Furthermore, stochastic dynamics including noise-induced switching between two stable steady states, fractions of transition at the higher stable steady states and potential energy landscape [3] are investigated when the fluctuation of external noise in the feedback strength is introduced into the rate equations.

Random dynamics and robustness of stochastic reaction-diffusion systems

Yuncheng You
University of South Florida, USA
you@mail.usf.edu

For a class of the stochastic multi-component reaction-diffusion systems with additive colored noises, which serves as mathematical models of many chemical and biochemical autocatalytic reactions on 2D and 3D bounded domains with Dirichlet or Neumann boundary conditions, the longtime and asymptotic dynamics of the solutions are investigated. It is proved that for the cubic autocatalysis there exists a random attractor in the L^2 phase space and with the H^1 attracting regularity. Moreover, the robustness is shown that when the strength coefficients of additive noises tend to zero the random attractors converge to the deterministic global attractor in terms of Hausdorff distance.

Special Session 23: Recent Progress in the Mathematical Theory of Compressible Fluid Flows

Eduard Feireisl, Czech Acad. Sci. Prague, Czech Rep
Milan Pokorny, Charles University in Prague, Czech Rep

The session focuses on recent results in the mathematical theory of compressible viscous fluids. The main topics include the fundamental questions of well-posedness for both stationary and evolutionary problems, qualitative properties of solutions, singular limits and other relevant issues. Contributions related to numerical analysis and implementations in the real world problems will be also discussed.

Diffuse Interface Models for Two-Phase Flows with Surfactants

Helmut Abels
University of Regensburg, Germany
helmut.abels@mathematik.uni-regensburg.de
Harald Garcke, Josef Weber

We will present a recent diffuse interface model for a two-phase flow of viscous incompressible fluids taking the effect of a surfactant into account, which diffuses through the bulk phases and along the interface. The model was developed by Garcke, Lam and Stinner. We will discuss the existence of weak solutions for this model. If time permits, we wll comment on the sharp interface limits.

On asymptotic behavior of solutions to the compressible Navier-Stokes equation around a time-periodic parallel flow

Jan Brezina
IMI Kyushu University, Japan
b-jan@imi.kyushu-u.ac.jp

Under appropriate smallness conditions on Reynolds and Mach numbers we show the global in time existence of strong solutions to the compressible Navier-Stokes equation around time-periodic parallel flows in R^n, $n \geq 2$. Furthermore, we study the asymptotic behavior of these solutions and prove that the cases $n = 2$ and $n > 2$ are considerably different.

Global solutions with critical regularity for a model of radiating flows

Raphaël Danchin
Université Paris-Est Créteil, France
danchin@univ-paris12.fr
Bernard Ducomet

We consider a simplified model arising in radiation hydrodynamics which is based on the barotropic Navier-Stokes system describing the macroscopic fluid motion, and the so-called P1-approximation of the transport equation modeling the propagation of radiative intensity. We establish global-in-time existence of strong solutions for the associated Cauchy problem when initial data are close to a stable radiative equilibrium, and local existence for large data with no vacuum.

We all also discuss the low mach number limit and various diffusive asymptotics.

All our results are stated in the so-called critical Besov spaces.

On the dynamical Rayleigh-Taylor instability in compressible viscous flows without heat conductivity

Song Jiang
Institute of Applied Physics & Computational Mathematics, Beijing, Peoples Rep of China
jiang@iapcm.ac.cn
Fei Jiang

We investigate the instability of a smooth Rayleigh-Taylor steady-state solution to compressible viscous flows without heat conductivity in the presence of a uniform gravitational field in a bounded domain with smooth boundary. We show that the steady-state is linearly unstable by constructing a suitable energy functional and exploiting arguments of the modified variational method. Then, based on the constructed linearly unstable solutions and a local well-posedness result of classical solutions to the original nonlinear problem, we further reconstruct the initial data of linearly unstable solutions to be the one of the original nonlinear problem and establish an appropriate energy estimate of Gronwall-type. With the help of the established energy estimate, we show that the steady-state is nonlinearly unstable in the sense of Hadamard by a careful bootstrap argument. As a byproduct of our analysis, we find that the compressibility has no stabilizing effect in the linearized problem for compressible viscous flows without heat conductivity. (Joint work with Fei Jiang.)

On bounded solutions to the compressible isentropic Euler system

Ondřej Kreml
Czech Academy of Sciences, Czech Rep
kreml@math.cas.cz
Elisabetta Chiodaroli

We analyze the Riemann problem for the compressible isentropic Euler system in the whole space $\mathbb{R}^2$. Using the tools developed by De Lellis and Székelyhidi for the incompressible Euler system we show that for every Riemann initial data yielding the self-similar solution in the form of two admissible shocks there exist in fact infinitely many admissible bounded weak solutions. Moreover for some of these initial data such solutions dissipate more total energy than the self-similar solution which might be looked at as a natural candidate for the "physical" solution.

Asymptotic Limits of the Full Compressible Magnetohydrodynamic Equations

Fucai Li
Nanjing University, Peoples Rep of China
fli@nju.edu.cn

In this talk we shall report our recent results on the asymptotic limits of the full compressible magnetohydrodynamic (MHD) equations in the whole space R^3. We shall show that, for the general initial data, the weak solutions of the full compressible MHD equations converge to the strong solution of the ideal incompressible MHD equations as the Mach number, the viscosity coefficients, the heat conductivity, and the magnetic diffusion coefficient go to zero simultaneously. Furthermore, the convergence rates are also discussed.

Mathematical cocktails: construction of solutions

Piotr Mucha
University of Warsaw, Poland
p.mucha@mimuw.edu.pl
Ewelina Zatorska, Milan Pokorny

The aim of the talk is analysis of a model describing motion of chemically reacting heat-conducting gaseous mixtures, based on a modification of the compressible Navier-Stokes equations. The key point is thermodynamical well posedness of the studied model. I would like to focus on presenting the detailed approximation scheme for the full system. At least on the first steps.

Relative energy method in the thermodynamics of compressible fluids

Antonin Novotny
University of Toulon, France
novotny@univ-tln.fr
Eduard Feireisl

We will talk about several issues related to the notions of weak solutions, dissipative solutions and stability properties to the compressible Navier–Stokes system and its approximations.

Compressible Navier-Stokes Equations and A Radon Transform

Pavel Plotnikov
Lavrentyev Institute of Hydrodynamics RAS, Russia
plotnikov@hydro.nsc.ru
W. Weigant

Estimates of the Radon transform of solutions to the compressible Navier-Stokes equations are derived. The results obtained are applied to boundary value problems for the compressible Navier-Stokes equations with the critical adiabatic exponents and to Navier-Stokes-Fourier equations. The existence of weak solutions is proved. We also discuss the question on existence of strong solutions to 2D Navier-Stokes-Fourier equations with the viscosity depending on the density.

On some two phase free boundary problem for compressible viscous fluid flow

Yoshihiro Shibata
Full Professor, RISE and Math. Dept, Waseda University, Japan
yshibata@waseda.jp

I would like to talk about a local well-posedness and also a global well-posedness of the evolution of two compressible fluids separated by a sharp interface without surface tension. One of the key issues is to prove the maximal regularity for the linearized problem. I used the R bounded solution operator to the corresponding resolvent problem and the Weis operator valued Fourier multiplier theorem. This talk is based on a joint work with Takayuki Kubo, Tsukuba University.

Blowup Criteria for Strong Solutions to Navier-Stokes-Fourier System with Variable Viscosities

Yongzhong Sun
Nanjing University, Peoples Rep of China
sunyz@nju.edu.cn

For a local-in-time strong solution to the initial-boundary value problem of compressible Navier-Stokes(-Fourier) equations, some blowup criteria in terms of the velocity gradient are discussed. We focus on the case that the viscous (and heat-conducting) coefficients possibly depend on the density or temperature, which make the problem more involved than the case of constant coefficients.

On qualitative studies of solutions for several related Radiation Hydrodynamics Models

Feng Xie
Shanghai Jiao Tong University, Peoples Rep of China
tzxief@sjtu.edu.cn

In this talk we will investigate several compressible Radiation Hydrodynamics mathematical models. We mainly focus on the global in time existence, blowup phenomena and stability of elementary waves of solutions to these Radiation Hydrodynamics models. This talk is based on joint works with C. Rohde and J. Wang.

On the Coupling between the Navier-Stokes and the Maxwell-Stefan Systems

Ewelina Zatorska
CMAP, Ecole Polytechnique, France
e.zatorska@mimuw.edu.pl
Piotr Mucha, Milan Pokorny

I will present a model of motion of compressible mixture of chemically reacting species. Mathematical description of such flow leads to a hyperbolic deviation in the species mass conservation equations (the full Maxwell-Stefan system). The thermodynamics implies that the diffusion terms are non-symmetric, non positively defined, and cross-diffusion effects must be strongly marked. We consider a special form of density-dependent viscosity coefficients and a singular behavior of the cold component of the internal pressure near vacuum. Under these hypotheses we prove global-in-time existence of weak solutions.

Special Session 24: Qualitative Analysis of Reaction Diffusion Systems

Junping Shi, College of Williams and Mary, USA
Xingfu Zou, University of Western Ontario, Canada

Reaction-diffusion models arise from various applications such as physics, chemistry, biology and engineering. This special session focuses on recent advances in theory, methods/techniques and applications in quantitative study of dynamical and stationary patterns of reaction-diffusion systems. In particular, models in ecology, epidemics, and material science will be discussed.

Pathogen spread in a wave-like environment

Jian Fang
Harbin Institute of Technology, Peoples Rep of China
jfanghit@gmail.com
Yijun Lou, Jianhong Wu

We consider the issue whether pathogen can keep pace with the invasion of its host. This is modeled as a scalar reaction diffusion equation with logistic nonlinear term involvoing the wave-like carrying capacity. The obtained threshold result is then used to analyze two generalized eigenvalues of an elliptic operator in unbounded domain.

Multiple steady-state in phytoplankton population induced by photoinhibition

Sze-bi Hsu
National Tsing-Hua University, Taiwan, Taiwan
sbhsu@math.nthu.edu.tw
Yihong Du, Yuan Lou

We study the effect of photo-inhibition in a non-local reaction-diffusion- advection equation, which models the dynamics of a single phytoplankton species in a water column where the growth of the species depends solely on light. Our results show that, in contrast to the case of no photo-inhibition, there might exist alternative stable steady-states for the persistence of a single phytoplankton species due to photo-inhibition.

Bifurcation analysis of a diffusive plankton system with nonlinear harvesting

Weihua Jiang
Harbin Institute of Technology, Peoples Rep of China
jiangwh@hit.edu.cn
Yong Wang

In this talk, I will report our recent research on dynamical analysis of a diffusive phytoplankton-zooplankton model with nonlinear harvesting and subject to homogeneous Neumann boundary conditions. We discuss the problem of non-constant positive steady state solution's existence, which can identify the ranges of parameters of spatial pattern formation. For the positive constant steady-state solution, stability, Hopf and steady-state bifurcation analysis are considered in detail. Moreover, we investigate whether the appearance of harvesting and the diffusion can lead to the appearance of some codimension two bifurcations, such as Bagdanov-Takens bifurcation, which can not occur in corresponding local system (ODEs). These results show that the impact of harvesting essentially increases the system spatiotemporal complexity.

A system of nonlinear Volterra equations with blow-up solutions

Colleen Kirk
California Polytechnic State University, USA
ckirk@calpoly.edu
W.E. Olmstead, C.A. Roberts

A pair of coupled nonlinear Volterra equations are examined for solutions that can have either global or blow-up behavior. The system is motivated by certain models of explosion phenomena in diffusive media. In the case of blow-up solutions, bounds on the blow-up time are derived. To demonstrate the applicability of the results, two examples are analyzed.

Limiting structure of steady-states to the Lotka-Volterra system with large diffusion and advection

Kousuke Kuto
University of Electro-Communications, Japan
kuto@e-one.uec.ac.jp
Tohru Tsujikawa

This talk is concerned with the Neumann problem of a stationary Lotka-Volterra population model with diffusion and advection. First we give sufficient conditions of the existence/nonexistence of nonconstant solutions. Next we derive a shadow system of the model as both diffusion and advection of one of the species tend to infinity. The shadow system can be reduced to a semilinear elliptic equation with nonlocal constraint. For the simplified 1D case, the bifurcation structure of nonconstant solutions of the shadow system can be classified according to the coefficients. For the competition case, this structure involves a simple curve of nonconstant solutions which connects two different singularly perturbed states (the boundary layer solutions and the internal layer solutions).

Traveling Waves in a Nonlocal Dispersal SIR Model

Wan-tong Li
Lanzhou University, Peoples Rep of China
wtli@lzu.edu.cn
Fei-Ying Yang

This talk is concerned with traveling wave solutions of a nonlocal dispersal SIR epidemic model. We show that our results on existence and nonexistence of traveling wave solutions are determined by the basic reproduction number of the corresponding ordinary differential model and the minimal wave speed. This threshold dy-

namics is proved by constructing an invariant cone and applying Schauder's fixed point theorem on this cone and the Laplace transform. The main difficulties are that a lack of regularizing effect occurs and the order-preserving property of this model loses.

Traveling waves solutions of isothermal diffusion systems with decay

Yuanwei Qi
University of Central Florida, USA
yuanwei.qi@ucf.edu
Xinfu Chen

In this talk recent results on existence and non-existence of propagating traveling waves in a class of reaction-diffusion systems which include a model of isothermal autocatalytic systems in chemical reaction of order m with a decay order n, where m and n are positive integers and $m \neq n$, as well as a well-known model in microbial growth and competition in a flow reactor. will be reported and related questions discussed.

A double bubble assembly as a new phase of a ternary inhibitory system

Xiaofeng Ren
George Washington University, USA
ren@gwu.edu
Juncheng Wei

A ternary inhibitory system is a three component system characterized by two properties: growth and inhibition. A deviation from homogeneity has a strong positive feedback on its further increase. In the meantime a longer ranging confinement mechanism prevents unlimited spreading. Together they lead to a locally self-enhancing and self-organizing process. The model considered here is a planar nonlocal geometric problem derived from the triblock copolymer theory. An assembly of perturbed double bubbles is mathematically constructed as a stable critical point of the free energy functional. Triple junction, a phenomenon that the three components meet at a single point, is a key issue addressed in the construction. Coarsening, an undesirable scenario of excessive microdomain growth, is prevented by a lower bound on the long range interaction term in the free energy. The proof involves several ideas: perturbation of double bubbles in a restricted class; use of internal variables to remove nonlinear constraints, local minimization in a restricted class formulated as a nonlinear problem on a Hilbert space; and reduction to finite dimensional minimization. This existence theorem predicts a new morphological phase of a double bubble assembly.

Reaction diffusion systems with different types of delays

Alexander Rezounenko
Kharkiv National University & UTIA, Ukraine
rezounenko@yahoo.com

We are interested in the well-posedness and qualitative properties of solutions to reaction diffusion systems with different types of delays. We will discuss systems in a bounded domain and with distributed, descrete and mixed delays. We also distinguish constant, time-dependent and state-dependent delays.

Higher Dimensional Solitary Waves Generated by Second-Harmonic Generation in Quadratic Media

Junping Shi
College of William and Mary, USA
jxshix@wm.edu
Leiga Zhao, Fukun Zhao

It is known that Second Harmonic Generation (SHG) can occur when the optical material has a $\chi^{(2)}$ (i.e. quadratic) nonlinear response instead of conventional Kerr $\chi^{(3)}$ material. Here we consider the soliton solutions of nonlinear SHG Schrödinger system in a higher dimensional space. We prove the existence of a positive ground state solution for all parameter range, and we also study the continuity of the ground states and the asymptotic behavior of the ground state when the parameter approaches zero or infinity. The uniqueness of positive solution is also proved in some cases. We also consider the multiplicity of the solutions and the case of system on a bounded domain.

Well-posedness and exponential equilibration of a volume-surface reaction-diffusion system with nonlinear boundary coupling

Bao Tang
Institute for Mathematics and Scientific Computing, University of Graz, Austria
baotangquoc@gmail.com
Klemens Fellner, Evangelos Latos

We consider a model system consisting of two reaction-diffusion equations, where one specie diffuses in a volume while the other specie diffuses on the surface which surrounds the volume. The two equations are coupled via nonlinear reversible Robin-type boundary conditions for the volume specie and a matching reversible source term for the boundary specie. As a consequence the total mass of the species is conserved. The considered system is motivated by models for asymmetric stem cell division. We first prove the existence of a unique weak solution via an iterative method of converging upper and lower solutions to overcome the difficulties of the nonlinear boundary terms. Secondly, we show explicit exponential convergence to equilibrium via an entropy method after deriving a suitable entropy entropy-dissipation estimate.

Dynamics and bifurcation analysis of a host-pathogen model in spatial habitats

Feng-Bin Wang
Chang Gung University, Taiwan
fbwang@mail.cgu.edu.tw
Junping Shi and Xingfu Zou

In classical epidemiological models, the host population is divided into infected and susceptible classes, with one differential equation representing each class in homogeneous environments. Anderson and May introduced an additional class representing the population of infectious pathogen particles in the environment. These particles are found in invertebrate pathogens. Dwyer further proposed a mathematical disease model that includes two realistic complications: density-dependent host reproduction and host movement behavior. Then he studied the spatial spread of the PDE system. In the real world, a domain in which populations habitat is bounded, and this motivates us to study a host-pathogen model in a spatially bounded domain. In this talk, the habitat we consider is a closed environment in the sense that the fluxes for each of these subpopulations are zero. Corresponding to this, we shall propose the Neumann boundary conditions to the equations on the boundary. This is a joint work with Drs. Junping Shi and Xingfu Zou.

Exact multiplicity and bifurcation diagrams of positive solutions of a one-dimensional multiparameter prescribed mean curvature problem

Shin-Hwa Wang
National Tsing Hua University, Taiwan
shwang@math.nthu.edu.tw
Yan-Hsiou Cheng, Chia-Hao Chuang and Kuo-Chih Hung

We study the exact multiplicity and bifurcation diagrams of positive solutions $u \in C^2(-L,L) \cap C[-L,L]$ of the one-dimensional multiparameter prescribed mean curvature problem $-\left(\frac{u'(x)}{\sqrt{1+(u'(x))^2}}\right)' = \lambda(u^p + u^q)$, $x \in (-L,L)$, $u(-L) = u(L) = 0$, where $\lambda > 0$ is a bifurcation parameter, $L > 0$ is an evolution parameter, and $0 \leq p < q < \infty$ are two constants. We prove that the problem has at most two positive solutions for any $0 \leq p < q < \infty$ and $\lambda, L > 0$. In addition, if $0 \leq p < q \leq \tilde{q}(p) = p + 1 + 2\sqrt{p+1}$, we give a classification of totally three qualitatively different bifurcation diagrams on the $(\lambda, \|u\|_\infty)$-plane for any $L > 0$. For any fixed $p \geq 0$ and $q \geq \bar{q}(p) = p + 2 + 2\sqrt{2p+3}$, we prove that there exist positive $L_* < L^*$ such that the bifurcation diagrams on the $(\lambda, \|u\|_\infty)$-plane are individually qualitatively different for the cases (i) $L \in (0, L_*)$ and $L > L^*$, (ii) $L = L^*$, (iii) $L \in [L_*, L^*)$.

Dynamics in a diffusive plankton system with delay and toxic substances effect

Junjie Wei
Harbin Institute of Technology, Peoples Rep of China
weijj@hit.edu.cn
Jiantao Zhao

The dynamics of a reaction-diffusion plankton system with delay and toxic substances effect is considered. Existence and priori bound of solution for model without delay are shown. Globally asymptotically stability of the axial equilibrium is obtained. The stability of the positive equilibrium and the existence of Hopf bifurcation are investigated by analyzing the distribution of the eigenvalues. And the properties of Hopf bifurcation is determined by the normal form theory and the center manifold reduction for partial functional differential equations. Some numerical simulations are carried out for illustrating the theoretical results.

Spatiotemporal patterns in a reaction-diffusion Seelig model

Fengqi Yi
Harbin Engineering University, Peoples Rep of China
fengqi.yi@gmail.com
Siyu Liu

A kind of reaction-diffusion Seelig model is considered. Firstly, we show that the parabolic system has an invariant region in the phase plane which attracts all solutions of this system, regardless of the initial values; Secondly, we discuss the basic properties of the non-constant steady state solutions, including the a priori estimates. Then, we prove the existence and non-existence of the non-constant steady state solutions. Finally, we perform multiple bifurcation analysis to this particular reaction diffusion system. These results allow for the clearer understanding of the critical role of the system parameters in leading to the formation of spatiotemporal patterns.

Global dynamics of an in-host viral model with spatial variation

Xingfu Zou
University of Western Ontario, Canada
xzou@uwo.ca
Feng-Bin Wang

We are interested in the dynamics of virus infection within host in a bounded domain. Under the assumption that only the free virions diffuse but the host cells (infected and uninfected) do not diffuse, we propose a model which is coupled by two ODEs and a PDE. The parameters are assumed to be location dependent, and the homogeneous Neumann boundary condition is posed. We will discuss the global dynamics of this model. This is a joint work with Feng-Bin Wang

Special Session 25: Dynamics of Chaotic and Complex Systems and Applications

Jesus M. Seoane, Universidad Rey Juan Carlos, Spain
Miguel A.F. Sanjuan, Physics, Universidad Rey Juan Carlos, Spain

This special session will focus on current research related to dynamics of chaotic and complex systems in a broadest way, paying a special attention to applications in physics, engineering, biology, neurosciences and others. The main goal of this special session is to join together physicists, mathematicians and other scientists interested in new developments of chaotic dynamics in its broader sense, the different available methods of controlling and synchronization of chaotic systems and their applications in complex systems arising in many fields of engineering and sciences, and in particular to life sciences such as the analysis of the dynamics of neurons and genetic regulation networks. The topics included, but not limited to, in this session, are: new developments of chaotic dynamics, novel methods of controlling chaotic and complex dynamics, Hamiltonian and dissipative chaotic systems, fractal structures in phase space, chaotic dynamics of neuronal and genetic models, nonlinear dynamics of biological systems, control and synchronization in neuronal and genetic networks.

Solitary Wave Formation and Dynamics of Piecewise Autonomous Systems

Anastasios Bountis
University of Patras, Greece
tassosbountis@gmail.com
Tassos Bountis and Yannis Kominis

We study the dynamics of a class of non-autonomous Hamiltonian systems consisting of a periodic sequence of linear and nonlinear autonomous parts and focus on their control capabilities in terms of altering the global phase space structure of the motion. One degree of freedom systems are studied which model solitary wave formation in transversely inhomogeneous configurations such as planar nonlinear optical structures or Bose-Einstein condensates. When one of the two alternating parts is linear, the solutions of the system are closely related to those of the nonlinear autonomous part. When the nonlinear parts have alternating signs, all types of solitary waves can be formed, including bright, dark, anti-dark and kink solitons as well as bound states. We also study multi-degree of freedom systems such as a nonautonomous Toda lattice with pulsating coupling, whose breathers are directly related to the solitons of the corresponding autonomous Toda lattice, while a “ratchet” effect provides a mechanism for their velocity and collision control. Finally, we consider a Klein-Gordon particle chain with long range interactions and switch on and off the inter-particle harmonic coupling to examine oits effect on breather formation, stability and the dynamics of wave packet diffusion.

Finite-time synchronization of tunnel diode based chaotic oscillators

Hilda Cerdeira
Instituto de Fisica Toerica - UNESP, Brazil
cerdeira@ictp.it
P. Louodop, H. Fotsin, M. Kountchou, E. B. Megam Ngoukandie and S. Bowong

This work addresses the problem of finite-time synchronization of tunnel diode based chaotic oscillators. After a brief investigation of its chaotic dynamics, we propose an active adaptive feedback coupling which accomplishes the synchronization of tunnel diode based chaotic systems with and without the presence of delay(s), basing ourselves on Lyapunov and on Krasovskii-Lyapunov stability theories. This feedback coupling could be applied to many other chaotic systems. A finite horizon can be arbitrarily established by ensuring that chaos synchronization is achieved at a pre-established time. An advantage of the proposed feedback coupling is that it is simple and easy to implement. Both mathematical investigations and numerical simulations followed by Pspice experiment are presented to show the feasibility of the proposed method.

On the integrability of a system describing the stationary solutions in Bose–Fermi mixtures

Ognyan Christov
Sofia University, Bulgaria
christov@fmi.uni-sofia.bg
G. Georgiev

We study the integrability of a Hamiltonian system describing the stationary solutions in Bose–Fermi mixtures in one dimensional optical lattices. We prove that the system is integrable only when it is separable. The non-integrability proof is based on the Differential Galois approach. In one case we use Ziglin’s approach about the self-intersection of complex separatrices, which is related to the above approach.

Phase synchronisation and control by cell environmental variables in a cell division cycle model

Jean Clairambault
INRIA & UPMC, Paris, France
jean.clairambault@inria.fr
F. Billy, F. Delaunay, C. Feillet, O. Fercoq, S. Gaubert, T. Lepoutre, T. Ouillon, N. Robert, S. Saito

We present a McKendrick-like model of the division cycle in proliferating cell populations and we identify its parameters, using the fluorescence ubiquitination based cell cycle indicator (FUCCI) technology, on biological data recorded in two different conditions of a nutritive medium for NIH3T3 cells. Furthermore, we show how theoretical periodic inputs allow cell populations to be more or less synchronised with respect to the phases of the division cycle. Possible consequences for coupling between circadian clocks and the cell division cycle, and for cancer therapeutics with drug delivery optimisation principles are also proposed.

References

[1] Billy, F., Clairambault, J., Fercoq, O., Gaubert, S., Lepoutre, T., Ouillon, T., Saito, S. Synchronisation and control of proliferation in cycling cell population models with age structure. Mathematics and Computers in Simulation, 96:66-94, 2014.

[2] Billy, F., Clairambault, J., Delaunay, F., Feillet, C., Robert, N. Age-structured cell population model to study the influence of growth factors on cell cycle dynamics. Mathematical Biosciences and Engineering, 10(1):1-17, 2013.

[3] Billy, F., Clairambault, J., Fercoq, O. Optimisation of cancer drug treatments using cell population dynamics. pp. 257-299 in "Mathematical Models and Methods in Biomedicine", A. Friedman, E. Kashdan, U. Ledzewicz and H. Schättler eds., Eds., Part 4, pp. 265-309, Springer, New-York, 2013.

New generalization for the intermittency theory for type I, II and III.

Ezequiel del Rio
UPM, Spain
ezequiel.delrio@upm.es
Sergio Elaskar

The concept of intermittency has been introduced by Pomeau and Maneville in the context of the Lorenz system and are usually classified in three classes called I, II, and III. Intermittency is a specific route to the deterministic chaos when spontaneous transitions between laminar and chaotic dynamics occur. The main attribute of intermittency is a *global reinjection mechanism* described by the corresponding reinjection probability density (RPD), that maps trajectories of the system from the chaotic region back into the *local* laminar phase. We generalize the classical analytical expressions for the RPD in systems showing Type-I, II, or III intermittency. As a consequence, the classical intermittency theory is a particular case of the new one. we present an analytical approach to the noise reinjection probability density. It is also important to note that from the RPD, obtained from noisy data, we have a complete description of the noiseless system. Pathological cases of intermittency described in the literature are known by their significant deviation of the main characteristics from those predicted by the classical theory. In this work we have shown that the use of generalized RPD provides faithful description of anomalous and standard intermittencies in the unified framework.

Spatio-temporal dynamics of p53 and nonlinear reaction-diffusion equations

Jan Elias
Université Pierre et Marie Curie - Paris 6, France
jan.elias@inria.fr
Jean Clairambault, Luna Dimitrio, Natalini Roberto

In a protein spatio-temporal dynamics in a single cell one can easily arrive to coupled non-linear reaction-diffusion systems defined for the nuclear and cytoplasmic concentrations of species involved in the protein signaling network (for example, by using Michaelis-Menten and Hill kinetics to describe reactions between the species), connected with the so-called Kedem-Katchalsky boundary conditions on the nuclear membrane simulating transport of the species between the compartments. We are particularly interested in spatio-temporal signaling of the p53 protein in response to DNA damage. Among other things, we show that oscillatory patterns (and instabilities) in p53 concentration can be driven by spatial and compartmental representation of processes occurring in a cell, and by a membrane permeability property that can impose physiological delays into protein responses. The semi-implicit Rothe method is used to (constructively) prove existence and uniqueness of non-negative solutions of such reaction-diffusion systems.

Kinetic theory of two-species coagulation

Carlos Escudero
Universidad Autónoma de Madrid & ICMAT, Spain
cel@icmat.es

We will outline our study on the stochastic process of two-species coagulation. This process consists in the aggregation dynamics taking place in a ring. Particles and clusters of particles are set in this ring and they can move either clockwise or counterclockwise. They have a probability to aggregate forming larger clusters when they collide with another particle or cluster. We study the stochastic process by means of Boltzmann equations that do not conserve momentum and therefore give rise to an interesting dynamics. We determine the long time behavior of such a model, making emphasis in one special case in which it displays self-similar solutions. In particular these calculations answer the question of how the system gets ordered, with all particles and clusters moving in the same direction, in the long time.

Effect of the Central Bank on the Stability of a Bank Network System

Hong Fan
Donghua University, Peoples Rep of China
fan_honghong@hotmail.com

In the China interbank market, the transaction behaviors of banks are similar and the interbank borrowing is short of regulation, which may result in a big problem. In this case, when the bank system faces liquidity shocks, it is difficult for the bank system to resolve the crisis only with the help of the interbank market. Therefore, how to develop monetary policies by the central bank to ease the system risk is a big issue. Considering the issue of system risk, the present paper constructs a dynamical bank network model based on agents' behavior with the method of the agent-based computational finance, and then studies the effects of the central bank on the stability of the bank system. The results show that the central bank can eliminate financial panic and achieve stability and security of the bank system with a range of appropriate monetary policy tools. Furthermore, when the bank system becomes unstable due to the short of regulation, it is difficult for the central bank to improve the stability of the bank system by adjusting the money policy of the deposit reserve rate.

"Mixed dynamics" as a new type of dynamical chaos

Sergey Gonchenko
Nizhny Novgorod State University, Russia
gonchenko@pochta.ru

We say that a system possesses a mixed dynamics if 1) it has infinitely many hyperbolic periodic orbits of all possible types (stable, unstable,saddle) and 2) the closures of the sets of orbits of different types have nonempty intersections. Recall that Newhouse regions are open domains (from the space of smooth dynamical systems) in which systems with homoclinic tangencies are dense. Newhouse regions in which systems with mixed dynamics are generic (compose residual subsets) are called absolute Newhouse regions. Their existence was proved in the paper [1] for the case of 2d diffeomorphisms close to a diffeomorphism with a nontransversal heteroclinic cycle containing two fixed (periodic) points with the Jacobians less and greater than 1. Fundamentally that "mixed dynamics" is the universal property of reversible chaotic systems. Moreover, in this case generic systems from absolute Newhouse regions have infinitely many stable, unstable, saddle and elliptic periodic orbits [2,3]. As well-known, reversible systems are often met in applications and they can demonstrate a chaotic orbit behavior. However, the phenomenon of mixed dynamics means that this chaos can not be associated with "strange attractor" or "conservative chaos". Attractors and repellers have here a nonempty intersection containing symmetric orbits (elliptic and saddle ones) but do not coincide, since periodic sinks (sources) do not belong to the repeller (attractor). Therefore, "mixed dynamics" should be considered as a new form of dynamical chaos posed between "strange attractor" and "conservative chaos". These and related questions will be discussed in the talk.

References

[1] S.Gonchenko, D.Turaev, L.Shilnikov *Proc. Steklov Inst. Math.*, 1997, **216**, 70-118.

[2] J.S.W.Lamb, O.V.Stenkin *Nonlinearity*, 2004, v.17(4), 1217-1244.

[3] A.Delshams *et al Nonlinearity*, 2013, v.26(1), 1-33.

Computer simulation of macro system nonlinear dynamics

Paul Inchin
Institute of Mathematics and Mathematical Modeling, Kazakhstan
paul.inchin@yahoo.com

We propose a class of the dynamical systems as nonlinear matrix models for macro system dynamics in the presence of limiting factor. The dynamical systems are generated by non-invertible generally, map F of the form $Fy = \Phi(\|y\|)Ay$ in some compact $\mathrm{X} \subset \mathbb{R}^n$. Here R^n is n- dimensional real vector-space, A is a matrix of n order, $\|\cdot\|$ is a vector norm in R^n and $\Phi(\|y\|)$ is a (scalar) function playing a role of a limiting factor. The results of qualitative theory we develop for this class of the dynamical systems are essentially used in modelling. In this talk we present an application package for Matlab which we elaborate for the numerical realization of models. A computer method is developed for determining the macro system asymptotic behavior. We call it the method of one-dimensional superpositions. The advantages of applying the results of the qualitative research and using the method of one-dimensional superpositions in computer simulation of the dynamics of macro systems governed by this class of models are discussed. In particular, the method proved to be very useful for a large number of macro system's components (n) and big periods ($> n$ and $> n^2$) of asymptotically stabilized macro system structure if to denote the macro system structure (at the time m) by the vector $\|F^m y\|^{-1} F^m y$.

Physics based modelling of friction

Krzysztof Jankowski
Lodz University of Technology, Poland
krzysztof.m.jankowski@gmail.com
Andrzej Stefanski, Ashesh Saha

In this paper, an overview of physics based modelling of friction phenomenon, emphasizing the transition from microscale to macroscale is presented. The asperities in the contacting surfaces are replaced by spring-mass systems. Coupling is introduced between the dynamic loading alterations in normal and tangential directions. The forces acting on a single asperity, considered as spring-mass system, are then combined statistically to produce the friction force over the entire surface. The present paper proposes an innovative approach to connect the pre-sliding and the pure sliding regimes of the friction force, starting from the dynamics of a single asperity.

Chaos in a fractional Duffing's equation

Salvador Jiménez
ETSIT, Universidad Politécnica de Madrid, Spain
s.jimenez@upm.es
Pedro J. Zufiria

We characterize the chaos in a fractional Duffing's equation computing the Liapunov exponents and the dimension of the strange attractor in the effective phase space of the system.

Dynamical systems generated by multivalued functions

Judy Kennedy
Lamar University, USA
kennedy9905@gmail.com

In 2003, topologists discovered a new way of viewing multi-valued functions. They began studying them as inverse limits on set-valued functions. These new inverse limits behave quite differently than do standard inverse limits. That theory has developed quite a bit since that time. Such a system still admits a shift map, which in this situation is a continuous surjective function from the space onto itself. Hence, it is possible to study the dynamics admitted by these spaces. We will discuss entropy and other dynamical properties on inverse limits with set-valued functions.

On analytic proof of existence of Lorenz attractor

Ivan Ovsyannikov
Imperial College London, England
Ivan.I.Ovsyannikov@gmail.com

The main goal on the current work is the proof of the existence of a geometrical Lorenz attractor in the Lorenz-Yudovich model:

$$\dot{X} = Y,$$
$$\dot{Y} = X - \lambda Y - XZ - X^3,$$
$$\dot{Z} = -\alpha Z + BX^2,$$

where parameters (α, λ, B) can take arbitrary finite values. The proof is based on the Shilnikov criterion [1]. According to it, in a system possessing two homoclinic orbits that tend to the same saddle equilibrium tangent to each other (a homoclinic butterfly-figure-eight) a Lorenz attractor is born if a saddle value σ is zero and a separatrix value A satisfies inequalities $0 < |A| < 2$ and $|A| \neq 1$. For the Lorenz-Yudovich model the separatrix value was calculated and the existence of Lorenz attractor was established near the integrable case $(\alpha, \lambda, B) = (1, 0, 0)$.

References
[1] Shilnikov L.P. Theory of bifurcations and quasihyperbolic attractors. Rus. Math. Surv., 36:4, 1981.

Dynamical systems as mathematical models with a limiting factor

Irina Pankratova
Institute of Math and Math Modelling, Kazakhstan
inpankratova@gmail.com
P.A. Inchin

We consider a class of the dynamical systems generated by non-invertible generally, map F of the form $Fy = \Phi(y)Ay$ in some compact $\mathrm{X} \subseteq \mathbb{R}^n$. Here R^n is n- dimensional real vector-space, $\Phi(y)$ is a scalar function and A is a matrix of n order. The dynamical systems are proposed as mathematical models with a limiting factor (nonlinear matrix models). Let n be a number of macro system's components, y be a vector of components' characteristics, A be a matrix of components' interrelations and $\Phi(y)$ be a limiting function (limiting factor). Then the systems describe the dynamics of model and real macro systems in the presence of limiting factors. There are some problems with the nonlinear matrix modelling one of which concerns reducing the system's dimension or the matrix order. Since the number of the system's components may be very large or the matrix structure may be very sparse then constructing appropriate algorithms for determining asymptotic behavior of the system is essential. In this talk we discuss the algorithm which we propose for determining the asymptotic behavior of macro systems governed by the class of the dynamical systems considered. It is based on the qualitative theory which we develop for this class of the systems. As the fist step of the algorithm, an asymptotically stabilized macro system structure is obtained by matrix A, i.e. using linear dynamical system. As the second step, the dynamics of macro system with the stabilized structure is determined by the one-dimensional nonlinear dynamical system with many parameters to which n- dimensional dynamical system reduces. Some problems of the qualitative theory are under consideration as well. In particular, we define the number of parameters by which the dynamics of n- dimensional systems' family depending on A is described.

Quantifying uncertainty in state and parameter estimation

Ulrich Parlitz
Max Planck Institute for Dynamics and Self-Organization, Germany
ulrich.parlitz@ds.mpg.de
Jan Schumann-Bischoff, Stefan Luther

Prediction and analysis of complex dynamics requires a suitable representation of the underlying dynamical structure in terms of a mathematical model (ODEs, PDEs, ...) and methods for estimating relevant model parameters and the current state of the system. Whether this task can be solved depends on the observability of the required quantities given the available (time series) data and the efficacy of the estimation algorithm chosen. We shall present methods based on delay embedding to address the observability problem and algorithms for parameter and state estimation employing nonlinear optimization and synchronization.

Desynchronization by weak stimulation

Oleksandr Popovych
Juelich Research Center, Germany
o.popovych@fz-juelich.de
Markos Xenakis, Peter A. Tass

We develop and optimize neuromodulation techniques for counteracting of abnormal neuronal synchronization characteristic for several neurological disorders like Parkinson's disease or tinnitus. An efficient stimulation has to be of sufficient strength and duration, and too weak stimulation may fail to desynchronize the stimulated neurons. For this case we propose a novel stimulation approach which can sustainably shift the neuronal population to a desynchronized regime by even weak stimulation, where the conventional stimulation protocol does not work. We suggest to stimulate the neuronal target population in an intermittent stop-and-go manner, which explores multistable regimes induced in the neuronal network by spike timing-dependent plasticity. The suggested approach is illustrated on coordinated reset stimulation that has successfully been tested in several experimental and clinical studies. Our findings can provide an opportunity to reduce the stimulation intensity without affecting the efficacy of the stimulation method, which may be of clinical relevance.

Phase-locked solutions with time-delay coupling: system size dependence

Ram Ramaswamy
University of Hyderabad, India
rr@uohyd.ac.in

We consider an ensemble of globally coupled oscillators with time–delayed interactions. A general relation for different frequencies of synchronized dynamics corresponding to different phase behaviors is obtained. In general, solutions are either all in–phase or mixed-phase (arbitrary as well as splay) configurations. In the strong coupling limit, however, and for larger networks, only the in–phase synchronized configuration remains. When the coupling strength or the number of oscillators is increased in the network a frequency discontinuity occurs, accompanied by a transition from an in–phase state to a mixed-phase state, or even between two in–phase configurations. The results presented here for coupled Landau–Stuart oscillators are found to hold for any system wherein oscillator phases are properly defined.

Kuramoto model of synchronization: Equilibrium and nonequilibrium aspects

Stefano Ruffo
University of Florence, Italy
stefano.ruffo@gmail.com
Alessandro Campa, Shamik Gupta

Recently, there have been considerable interests in the study of spontaneous synchronization, particularly within the framework of the Kuramoto model. The model comprises oscillators with distributed natural frequencies interacting through a mean-field coupling, and serves as a paradigm to study synchronization. In this talk, I will outline recent progress. In particular, I put forward a general framework in which I wll discuss in a unified way known results with more recent developments obtained for a generalized Kuramoto model that includes inertial effects and noise. I describe the model from a different perspective, emphasizing the equilibrium and out-of-equilibrium aspects of its dynamics from a statistical physics point of view: i) I discuss the phase diagram for unimodal frequency distributions; ii) I analyze the dynamics on a lattice where the coupling decays algebraically with separation between lattice sites, and discuss for specific cases how the long-time transition to synchrony is essentially governed by the dynamics of the mean-field mode.

The Dynamics of Nucleation

Evelyn Sander
George Mason University, USA
esander@gmu.edu

The Cahn-Hilliard equation is one of the fundamental models to describe phase separation dynamics in metal alloys. In this talk, I will focus on applying traditional dynamical tools, such as bifurcation theory and computational topology in order to gain a better understanding of droplet formation during nucleation for the stochastic Cahn-Hilliard equation for different variations of noise and of boundary conditions.

Dynamics of Partial Control of Chaotic Systems

Miguel A. F. Sanjuan
Physics, Universidad Rey Juan Carlos, Spain
miguel.sanjuan@urjc.es

In our chaotic lives we usually do not try to specify our plans in great detail, or if we do, we should be prepared to make major modifications. Our plans for what we want to achieve are accompanied with situations we must avoid. Disturbances often disrupt our immediate plans, so we adapt to new situations. We only have partial control over our futures. Partial control aims at providing toy examples of chaotic situations where we try to avoid disasters, constantly revising our trajectories. More mathematically, partial control of chaotic systems is a new kind of control of chaotic dynamical systems in presence of disturbances. The goal of partial control is to avoid certain undesired behaviors without determining a specific trajectory. The surprising advantage of this control technique is that it sometimes allows the avoidance of the undesired behaviors even if the control applied is smaller than the external disturbances of the dynamical system. A key ingredient of this technique is what we call safe sets. Recently we have found a general algorithm for finding these sets in an arbitrary dynamical system, if they exist. The appearance of these safe sets can be rather complex though they do not appear to have fractal boundaries. In order to understand better the dynamics on these sets, we introduce in this paper a new concept, the asymptotic safe set. Trajectories in the safe set tend asymptotically to the asymptotic safe set. I will present two algorithms for finding such sets. I will illustrate all these concepts for a time-2π map of the Duffing oscillator. Furthermore I will show two examples of applications to a cancer model and to a species extinction model in ecology.

Effects of periodic forcing in chaotic scattering

Jesus M. Seoane
Universidad Rey Juan Carlos, Spain
jesus.seoane@urjc.es
F. Blesa, R. Barrio and M. A. F. Sanjuán

The effects of a periodic forcing on chaotic scattering are relevant in certain situations of physical interest. We investigate the effects of the forcing amplitude and the external frequency in both, the survival probability of the particles in the scattering region and the exit basins associated to phase space. We have found an exponential decay law for the survival probability of the particles in the scattering region. A resonant-like behavior is uncovered where the critical values of the frequencies $\omega \simeq 1$ and $\omega \simeq 2$ permit to escape the particles faster than for other different values. On the other hand, the computation of the exit basins in phase space reveals the existence of Wada basins depending of the frequency values. We provide some heuristic arguments that are in good agreement with the numerical results. Our results are expected to be relevant for physical phenomena such as the effect of companion galaxies, among others.

Lyapunov exponents of an impact oscillator with Hertz's and Newton's model of contact

Wioleta Serweta
Lodz University of Technology, Poland
w.serweta@wp.pl
Andrzej Okolewski, Barbara Blazejczyk Okolewska, Krzysztof Czolczynski, Tomasz Kapitaniak

The paper is concerned with the analysis of motion of a harmonically excited one - degree - of - freedom mechanical system having an amplitude constraint. The contact between the oscillated mass and the barrier is modeled on the basis of Newton's law as well as Hertz's law with nonlinear damping. The influence of the frequency of excitation force of the system's behavior is studied. The spectrums of Lyapunov exponents are calculated in a wide range of control parameter. The dynamical behaviors of two systems with impacts: a system with hard impacts and system with Hertz's impacts, which are equivalent in the sense of the same rate of impact energy dissipation, are compared and strong qualitative and quantitative similarities are observed.

Quasi-Particle Dynamics of Linearly Coupled Systems of Nonlinear Schrodinger Equations

Michail Todorov
Technical University of Sofia, Bulgaria
mtod@tu-sofia.bg

We investigate numerically by a conservative difference scheme in complex arithmetic the head-on and taking over collision dynamics of the solitary waves as solutions of linearly Coupled Nonlinear Schrodinger Equations for various initial phases. The initial conditions are superposition of two one-soliton solutions with general polarization. The quasi-particle behavior of propagating and interacting solutions in conditions of rotational polarization is examined. We find that the total mass, pseudomomentum and energy are conserved while the local masses, individual and total polarization depend strongly on the initial phase difference. We also find out that the polarization angle of the quasi particles can change independently of the interaction. The investigation is partially supported by the Science Fund of Ministry of Education, Science and Youth of Republic Bulgaria under grant DDVU02/71.

On the persistence of degenerate lower-dimensional tori in reversible systems

Dongfeng Zhang
Southeast University, Peoples Rep of China
zhdf@seu.edu.cn
Wang Xiaocai, Xu Junxiang

This work focuses on the persistence of lower-dimensional tori with prescribed frequencies and singular normal matrices in reversible systems. By KAM method and the special structure of unperturbed nonlinear terms in the differential equation, we prove that the invariant tori with given frequencies can persist under small perturbation.

Special Session 26: Dynamical Systems and Spectral Theory

David Damanik, Rice University, USA

The talks in this special session will cover recent work on spectral theory and/or dynamics.

Spectra of Random Operators with absolutely continuous Integrated Density of States

Rafael del Rio
IIMAS-UNAM, Mexico
delriomagia@gmail.com

The structure of the spectrum of random operators is studied. It is shown that if the density of states measure of some subsets of the spectrum is zero, then these subsets are empty. In particular follows that absolute continuity of the IDS implies singular spectra of ergodic operators is either empty or of positive measure. Our results apply to Anderson and alloy type models, perturbed Landau Hamiltonians, almost periodic potentials and models which are not ergodic.

On the density of uniform hyperbolicty for SL(2,R)-valued cocycles

Roberta Fabbri
Dipartimento Matematica e Informatica “Ulisse Dini”, Italy
roberta.fabbri@unifi.it

The question of the density of the uniform hyperbolicity for SL(2,R)-valued cocycles is considered with application to the study of the spectral properties of the quasi-periodic Schrödinger equation

Schrödinger Operators with a Thue-Morse Potential

Jacob Fillman
Rice University, USA
jake.fillman@gmail.com
Paul Munger

We will discuss the spectral theory of discrete Schrödinger operators whose potentials are described by the Thue-Morse substitution. In particular, we will discuss bounds on the Hausdorff dimension of the spectrum and the Hölder continuity of the integrated density of states.

Spectral analysis of V-variable Sierpinski gaskets

Uta Freiberg
University of Stuttgart, Germany
uta.freiberg@mathematik.uni-stuttgart.de

Self similar fractals are often used in modeling porous materials. However, the assumption of strict self similarity could be too restricting. So, we present several models of random fractals which could be used instead. After recalling the classical approaches of random homogenous and recursive random fractals, we show how to interpolate between these two models with the help of so called V-variable fractals. This concept (developed by Barnsley, Hutchinson & Stenflo) allows the definition of new families of random fractals, hereby the parameter V describes the degree of “variability” of the realizations. We discuss how the degree of variability influences the spectral asymptotics of corresponding Dirichlet forms. Moreover, on–diagonal heat kernel estimates are presented.

Thermodynamic formalism in hyperbolic dynamics and applications to spectral theory

Anton Gorodetski
University of California Irvine, USA
asgor@math.uci.edu
David Damanik, William Yessen

Thermodynamic formalism is a powerful technique that provides many insights to the dynamical properties of hyperbolic maps. Using the connections between spectral characteristics of Fibonacci Hamiltonian (such as optimal Hölder exponent of the integrated density of states, dimensions of the spectrum and of the density of states measure, and upper transport exponents) and dynamical characteristics of the Fibonacci trace map, we apply the thermodynamic formalism techniques to establish strict inequalities between the spectral characteristics for all non-zero values of the coupling constant. In particular, this proves a conjecture on non-coincidence of the dimension of the spectrum and the dimension of the density of states measure stated by Barry Simon. The results are joint with David Damanik and William Yessen.

Almost automorphic generalized reflectionless potentials

Russell Johnson
Università di Firenze, Italy
johnson@dsi.unifi.it
Luca Zampogni, Università di Perugia

The set GR of generalized reflectionless Schroedinger potentials was introduced and studied by Lundina in 1985. It contains the classical reflectionless potentials and (translations of) the algebro-geometric potentials. The class GR has been studied further by Marchenko, Gesztesy, Kotani and other scientists. The stationary ergodic elements of GR are of interest for several reasons. It seems that all such elements which have been constructed till now are Bohr almost periodic. Our goal is to construct stationary ergodic, generalized reflectionless potentials which are almost automorphic in the sense of Bochner-Veech, but not almost periodic. To do so, we make systematic use of results concerning Parreau-Widom domains by Sodin-Yuditskii and by Hasumi.

The spectral properties of the strongly coupled Sturm Hamiltonian of constant type

Yanhui Qu
Tsinghua University, Peoples Rep of China
yanhui.qu@gmail.com

We will study the spectral property of the strongly coupled Sturm Hamiltonian of constant type. Especially we will study the Hausdorff dimensions of the spectrum and the related density states measure.

Cantor spectra of zero Lebesgue measure for continuum models

Christian Seifert
Technische Universität Hamburg-Harburg, Germany
christian.seifert@tuhh.de
Daniel Lenz, Peter Stollmann

We study Schrödinger operators on $\mathbb{R}$ with measures as potentials. Choosing a suitable dynamical system of measures we can relate properties of it with spectral properties of the associated operators. This enables us to prove Cantor spectra of zero Lebesgue measure for a large class of operator families, including many families generated by aperiodic subshifts.

Spectral Properties of Random Schrodinger Operators With Small Coupling Constants

Mihai Stoiciu
Williams College, USA
Mihai.Stoiciu@williams.edu
Peter Hislop

We consider one-dimensional Schrodinger operators $H = -\Delta + \lambda V$, defined by the random potential $V = V(\omega)$ and the coupling constant $\lambda > 0$. We investigate the spectral properties of these operators for small values of λ. In particular, we describe the behavior of the density of states and the transition in the microscopic eigenvalue statistics, as the coupling constant λ approaches 0.

Recurrence in quantum dynamical systems, Schur functions and spectral theory

Luis Velázquez
Universidad de Zaragoza and IUMA, Spain
velazque@unizar.es
J. Bourgain, F.A. Grünbaum, A.H. Werner, R.F. Werner, J. Wilkening

A relation between the theory of Schur functions and a notion for recurrence in discrete time quantum systems has been recently discovered. This is the origin of a rich interplay between spectral theory, complex analysis, orthogonal polynomials theory and the issue of quantum recurrence. The above connection not only provides new analytical techniques for quantum mechanical problems, but also reveals an unexpected geometrical and a topological meaning of some recurrence properties of quantum systems. We will review some of these results and their surprising physical consequences. The results that will be reviewed are the fruit of joint works with: Jean Bourgain (IAS Princeton) Alberto Grünbaum (UC Berkeley) Albert Werner (Freie Universität Berlin) Reinhard Werner (Leibniz Universität Hannover) Jon Wilkening (UC Berkeley)

References

[1] Recurrence for discrete time unitary evolutions, F.A. Grünbaum, L. Velázquez, A.H. Werner, R.F. Werner, Commun. Math. Phys. 320 (2013) 543-569.

[2] Quantum recurrence of a subspace and operator-valued Schur functions, J. Bourgain, F.A. Grünbaum, L. Velázquez, J. Wilkening, Commun. Math. Phys. (in press), arXiv:1302.7286 [quant-ph].

On large deviations estimates for the Dirichlet determinants of the Anderson model

Mircea Voda
University of Toronto, Canada
voda.mircea@gmail.com
Ilia Binder, Michael Goldstein

We develop a large deviations estimate for the Dirichlet determinants of the Anderson model on $\mathbb{Z}^d$, and we consider its application to estimating the sum of non-negative Lyapunov exponents on strips.

Lyapunov Exponent and spectrum in 1-dim quasi-periodic Schrodinger operators

Yiqian Wang
Nanjing University, Peoples Rep of China
yiqianw@nju.edu.cn
Jiangong You, Zhenghe Zhang

One of central tasks in the study of Schrodinger Cocycles is to understand the properties of the spectrum. With the help of dynamical method, a lot of results have been obtained on the spectrum of one dimensional quasi periodic Schrodinger operators. In particular, the Lyapunov Exponent (LE) plays a key role. In this talk, we first introduce some facts on the relation between LE and spectrum. Then we review the results on the properties of LE for analytic cases and their applications on the study of spectrum. Finally, we report recent advances on smooth cases, from which one may find that dynamical method has its own advantages for studying one dimensional Schrodinger operators.

Bound states and propagation properties of Quantum Walks

Albert H. Werner
FU Berlin, Germany
albert.werner@fu-berlin.de
A. Ahlbrecht, A. Alberti, C. Cedzich, M. Genske, D. Meschede, T. Rybar, V.B. Scholz, R.F. Werner

We consider the effects of interactions and simulated homogeneous electric fields on the propagation properties of one dimensional quantum walks. For interacting two particles quantum walks we show the existence of stable bound states, in the spectral gap of the free Evolution Operator and exponentially decay of the corresponding eigen-

function with respect to the relative position of the two particles. Furthermore, for a certain class of interactions, we develop an effective theory and find that the dynamics of the molecule is described by a quantum walk in its own right. Homogeneous electric fields are introduced by an additional local phase which depends linearly on position and is applied after each step. The long time propagation properties of this system, such as revivals, ballistic expansion and Anderson localization, depend very sensitively on the value of the electric field Φ, e.g., on whether $\Phi/(2\pi)$ is rational or irrational. We relate these properties to the continued fraction expansion of the field. When the field is given only with finite accuracy, the beginning of the expansion allows analogous conclusions about the behavior on finite time scales.

On reducibility of 2-dimensional linear quasi-periodic system with small parameter

Junxiang Xu
Southeast University, Peoples Rep of China
xujun@seu.edu.cn
Lu Xuezhu

In this paper we consider linear analytic quasi-periodic system of two differential equations, whose coefficient matrix analytically depends on a small parameter and closes to constant. Under some non-resonance condition about the basic frequencies and the eigenvalues of the constant matrix, without any non-degeneracy assumption with respect to the small parameter, we prove that the system is reducible for most of the sufficiently small parameters in the sense of Lebesgue measure.

Jacobi flow on SMP matrices and Killip-Simon problem

Peter Yuditskii
University of Linz, Austria
petro.yudytskiy@jku.at

We give a free parametric representation for coefficient sequences of Jacobi matrices whose spectral measure satisfy the Killip-Simon conditions. This parametrization is given by means the Jacobi flow on SMP matrices, which we introduce here.

CMV Matrices with Super Exponentially Decaying Verblunsky Coefficients

Maxim Zinchenko
University of New Mexico, USA
maxim@math.unm.edu

We establish one-to-one correspondences between CMV matrices with super exponentially decaying coefficients and the corresponding spectral data associated with the Jost function.

Special Session 27: Mathematical Problems in Economics, Materials and Life Science: Analysis and Simulation of Nonlinear Multiscale Dynamics

Toyohiko Aiki, Japan Women's Univeristy, Japan
Nobuyuki Kenmochi, Bukkyo University, Japan
Adrian Muntean, Department of Mathematics/TU Eindhoven, Netherlands

This special session considers impact problems from economics, materials science and life science from a mathematical perspective. Besides well-posedness studies of nonlinear PDEs and related particle systems, the session also includes presentations on averaging processes in porous media, non-periodic homogenization, statistical mechanics of particle systems, large-time behavior of weak solutions, multiscale dynamics of pedestrian flows.

Large time behavior of a solution to a free boundary problem describing adsorption phenomenum

Toyohiko Aiki
Japan Women's University, Japan
aikit@fc.jwu.ac.jp
Yusuke Murase

We consider a drying and wetting process in a porous media. In this talk we focus one hole of the media and regard the hole as a one-dimensional interval $[0, L]$. Here, at $x = 0$ the wall exists and $x = L$ the air comes from outside. Also, the intervals $[0, s(t)]$ and $[s(t), L]$ indicate the water-drop region and the air region in the hole, respectively, and u is the humidity in the pore. Then u satisfies a diffusion equation in the non-cylindrical domain given by the free boundary s, and s satisfies the free boundary condition. This problem was proposed in order to overcome some difficulties in our study of hysteresis appearing in concrete carbonation phenomena. The aims of the present talk are to give our modeling process for the free boundary problem, and to establish the well-posedness of the problem and a large time behavior of the solution. Moreover, we show some numerical results of the problem and discuss about possibilities of this free boundary problem as a mathematical description for hysteresis.

Large scale modelling of tumour dynamics

Maciej Cytowski
ICM, University of Warsaw, Poland
m.cytowski@icm.edu.pl
Zuzanna Szymanska

The main objective of our work was to devise a large scale individual-based modelling and computational methodology allowing for simulations of tumour growth and dynamics in clinically detectable scale. In our model each cell is treated as an individual and hence can posses its individual (e.g. biomechanical) properties. Tumour environment such as oxygen and glucose is modelled with the use of PDE based description. We define the clinically detectable scale of the model as an ability to consider significant volumes of tissue which can be detected and operated by clinical doctors. Thanks to efficient implementation on modern massively parallel computer architectures, our model is capable of simulating 10^9 cells i.e. approximately $1cm^3$ volume of human tissue. Such simulation scale enables us to consider avascular tumours consisting of approximately 10^6 cells together with surrounding healthy tissue and cellular environment.

Parabolic quasi-variational inequalities arising from economic growth models

Nobuyuki Kenmochi
Bukkyo University, Japan
kenmochi@bukkyo-u.ac.jp

It is very important to have various collaborations between several companies or regions or countries in order to make their production systems effectively progressive. In this talk we propose a regional economic growth model of two regions having a technological collaboration in which two regions provide their respective great fortes to cultivate a common knowledge-field. Each region selects some technologies obtained in the common knowlege-field and makes use of them to progress in the production. Of course, the technological cooperation is maintained by the investiments from both regions whose ammounts depend on their (unknown) economic growth. In this point one can find a quasi-variational structure of the problem. Our model consists of two parabolic PDEs describing the economic growth under a mutual help system in both regions and a parabolic quasi-variational inclusion of the unknown-dependent subdifferentials describing the cultivation of knowledge-field. In this talk an existence result is given.

A thermal diffusion system with Smoluchowski interactions: well-posedness and homogenization

Oleh Krehel
Technical University of Eindhoven, Netherlands
o.krehel@tue.nl
Toyohiko Aiki and Adrian Muntean

We study the solvability and homogenization of a thermal-diffusion reaction problem posed in a periodically perforated domain. The system describes the motion of populations of hot colloidal particles interacting together via Smoluchowski production terms. The upscaled system, obtained via two-scale convergence techniques, allows the investigation of deposition effects in the presence of thermal gradients. This is joint work with Toyohiko Aiki (Tokyo) and Adrian Muntean (Eindhoven).

Uniqueness of a solution for some parabolic type equation with hysteresis in three dimensional case

Kota Kumazaki
Tomakomai National College of Technology, Japan
k.kumazaki@gt.tomakomai-ct.ac.jp
Toyohiko Aiki, Pavel Krejči

In this talk, we consider the system consisting of some nonlinear parabolic equation and an ordinary differential equation which describes a hysteresis operator. This problem is proposed as a mathematical model of moisture transport in concrete carbonation process, and we already proved the existence of a time global solution of our problem with an inhomogenious Dirichlet boundary condition and initial condition. The uniqueness of a solution is only obtained in one dimensional case, and is not be obtained in three dimensional case. The difficulties of the uniqueness of a solution in three dimensional case is a treatment of a nonlinear term in our parabolic type equation and a topology of a solution of the ordinary differential equation with a multivalued mappings. The aim of this talk is to prove the uniqueness of a solution of our problem in three dimensional case under the smooth boundary and initial data.

Mathematical Modeling of Mechanochemical Pattern Formation in Biology

Moritz Mercker
Heidelberg University, Germany
mmercker_bioscience@gmx.de
Anna Marciniak-Czochra

One of the most fascinating characteristics of biological systems is their ability of self-organization and pattern formation: Many biological processes lead to emergence of various chemical and mechanical, spatially heterogeneous structures from homogenous or chaotic systems. One prominent example is embryonic development, where a tissue sphere develops step by step into an organism of complex shape and function. Another example are biomembranes, which emerge as complex structures from initially chaotically distributed molecules. During the last decades, advanced molecular biological methods allowed to identify various molecules involved in the biological processes of pattern formation. However, the question of how these molecules create patterns and structures often remains unanswered. An increasing amount of experimental results suggests an interplay between chemical and mechanical processes in biological patterning. We develop new mathematical models to investigate biological patterning processes, taking into account biomechanical aspects of tissues and membranes. Models are based on minimization of free energy leading to nonlinear PDE systems of fourth order, related to the Willmore flow. Parametric finite element simulations allow us to avoid several limitations of experiments and to test different biological hypotheses. We demonstrate that even simple mechanochemical interplays may lead to spontaneous pattern formation in biological surfaces.

Homogenization for a kinetic description of self-assembly of fibrous materials

Adrian Muntean
Department of Mathematics/TU Eindhoven, Netherlands
a.muntean@tue.nl
Kees Storm, Bart van Lith

We present a continuum PDE-ODE model for collagen self-assembly describing the interplay between the change in the polymer distribution and the evolution of monomers. We endow the model with periodic coefficients, where the small parameter ϵ is interpreted as the ratio of lengths of monomers and fibrils. After applying a fixed-point homogenization argument and proving corrector estimates, we use information from the first-order corrections to explain the so-called *turbidity measurement*.

Solvability and optimal control problems on mathematical modeling for brewing process of Japanese Sake

Yusuke Murase
Meijo University, Japan
ymurase@meijo-u.ac.jp

Our mathematical modeling for brewing Sake is constructed by 14 reaction-diffusion equations, heat equation, and a constraint condition. This mathematical model can be expressed by the quasi-variational inequality. We study this model with homogeneous Neumann boundary conditions for reaction-diffusion equations, and Robin boundary condition for heat equation. In phenomena, Brewing Sake has 5 different fermenting stages, we provide the model so that represents several stages. In this talk, we discuss the solvability of our model in 1st stage, and its optimal control problems.

Stochastic Homogenization: An optimal quantitative two-scale expansion

Stefan Neukamm
WIAS Berlin, Germany
stefan.neukamm@wias-berlin.de
Antoine Gloria and Felix Otto

Consider a discrete elliptic equation on the discrete torus of size L, with iid conductivities. We show that the L^2-norm in probability of the discrete H^1-norm in space of the first two terms of the two-scale expansion decays with the same rate as in the case of deterministic, periodic coefficients (up to a logarithm in dimension 2). The proof relies on moment bounds on the corrector and its gradient and an optimal estimate of the error on the approximation of the homogenized coefficients by periodization.

A two obstacles coupled problem

Lisa Santos
University of Minho, Portugal
lisa@math.uminho.pt
Assis Azevedo

We consider a system of a evolutive variational inequality of two obstacles type depending on the temperature, coupled with the heat equation. We prove existence of solution of this system and we present examples that mo-

tivated this work. In particular, with assumptions on the data, we prove that solutions of this problem are also solutions of a similar problem where the convex set is of gradient constraint type (that depends on the temperature), improving a previous result.

On the travelling wave problem for elastic phase transitions in the Fermi-Pasta-Ulam chain

Hartmut Schwetlick
University of Bath, England
h.schwetlick@bath.ac.uk
M Herrmann, K Matthies, J Zimmer.

We analyse the travelling wave problem for a lattice model of elastic phase transitions. In particular we consider a bistable model with a piecewise quadratic interaction potential and a smoothed variant with small spinodal region. A number of authors have been able to prove in both cases the existence of families of subsonic travelling waves, so that special interest is focusing on relevant selection criteria to identify meaningful solutions. Following ideas from vanishing viscosity approach to conservation laws we present ideas to analyse the stabilisation of waves in the model with dissipation and discuss how this might help to set up a framework for the interpretation of the various types of wave solutions existing for the FPU lattice problem. This is joint work with M Herrmann (Saarbruecken), K Matthies, J Zimmer (Bath).

Energy dissipations for mathematical models of grain boundary motions with isothermal solidifications

Ken Shirakawa
Chiba University, Japan
sirakawa@faculty.chiba-u.jp
Hiroshi Watanabe, Noriaki Yamazaki

This study is based on the line of jointworks with Prof. S. Moll (Univ. Valencia, Spain), Prof. H. Watanabe (Salesian Polytechnic, Japan) and Prof. N. Yamazaki (Kanagawa Univ., Japan). In this talk, a coupled system of parabolic type variational inequalities is considered. On the basis of the modelling method of [Kobayashi, RIMS Kokyuroku, 1210 (2001), 68-77], this system is derived as a gradient system of a governing free-energy. So, roughly summarized, our system is largely consists of two gradient flows: the Allen-Cahn type equation for the isothermal solidification; the Kobayashi-Warren-Carter type system for the grain boundary motion, originated from [Kobayashi-Warren-Carter, Phys. D, 140 (2000), 141-150]. The focus in this talk is on a special kind of solution, named as "energy-dissipative solution", which realizes the dissipation of the free-energy in time. Consequently, the existence of energy-dissipative solution and some related topics will be presented as the main results of this talk.

The existence of weak solution for mean curvature flow with transport term

Keisuke Takasao
Hokkaido University, Japan
takasao@math.sci.hokudai.ac.jp
Yoshihiro Tonegawa

In this talk we consider a family of hypersurfaces $\{\Gamma(t)\}_{t\in[0,\infty)}$ in $\mathbb{R}^n$ whose velocity is

$$V_\Gamma = H + (u \cdot \nu)\nu \qquad \text{on} \qquad \Gamma(t), \qquad t \geq 0.$$

Here H and ν are the mean curvature vector and the unit normal vector of $\Gamma(t)$ respectively and $u : \mathbb{R}^n \times [0,\infty) \to \mathbb{R}^n$ is a given vector valued function. In 1978, Brakke proved the existence of a weak solution for the mean curvature flow by using the geometric measure theory, and the weak solution is called Brakke's mean curvature flow. In 2010, Liu, Sato and Tonegawa proved that there exists Brakke's mean curvature flow with transport term when $n = 2, 3$ by using the phase field method and the monotonicity formula. Regarding the results, we consider the existence of mean curvature flow with transport term for $n \geq 2$. This is a joint work with Yoshihiro Tonegawa (Hokkaido University).

Solvability of p-Laplacian parabolic equations with constraints coupled with Navier-Stokes equations in 3D domains

Yutaka Tsuzuki
Tokyo University of Science, Japan
yutack1296@gmail.com

We consider the system of p-Laplacian parabolic equations with constraints coupled with Navier-Stokes equations in 3D domains. The solvability of the system in 2D domains has already been studied by Fukao and Kubo (2008), Sobajima, Tsuzuki and Yokota (2012), and Tsuzuki (2013). However, the solvability in 3D domains was left as an open problem. Recently, Fukao, Tsuzuki and Yokota solved it for a simplified system. In this talk, we deal with more generalized system including logistic terms in 3D domains.

Strongly degenerate parabolic equations with diffusion coefficients depending on the spatial variable

Hiroshi Watanabe
Salesian Polytechnic, Japan
h-watanabe@salesio-sp.ac.jp

Strongly degenerate parabolic equations are regarded as a linear combination of the time-dependent conservation laws (quasilinear hyperbolic equations) and the porous medium type equations (nonlinear degenerate parabolic equations). Thus, these equations have both properties of hyperbolic equations and those of parabolic equations and describe various nonlinear convective diffusion phenomena such as filtration problems, Stefan problems and so on. In this talk we consider strongly degenerate parabolic equations with diffusion coefficients depending on the spatial variable. In particular, we formulate entropy solutions associated with the equations and prove the existence and uniqueness of its.

Numerical experiments of Allen-Cahn equation with constraints via Lagrange multiplier

Noriaki Yamazaki
Kanagawa University, Japan
noriaki@kanagawa-u.ac.jp
M. H. Farshbaf-Shaker and T. Fukao

Let us consider Allen-Cahn equation with constraints. Our constraint is the subdifferential of the indicator function on the closed interval, which is the multivalued function. Recently, we consider the singular limit of our equations. Then, we gave the characterization of the Lagrange multiplier to our equation. But, by our constraint, It is very difficult to consider our equation numericaly. So, in this talk, we gave numerical experiments of Allen-Cahn equation with constraints via Lagrange multiplier.

Special Session 28: Functional Analytic Techniques for Evolutionary Equations Arising in the Natural Sciences

Jacek Banasiak, University of KwaZulu-Natal, So Africa
Wilson Lamb, University of Strathclyde, Scotland

Models of phenomena occurring in the natural environment are becoming increasingly complex and hence their practical implementation and numerical treatment must be preceded by careful theoretical analysis so that, at the very least, some basic information about their properties is available. This theoretical and rigorous analysis, which usually relies heavily on sophisticated techniques from functional analysis, enables unrealistic and weak models to be discarded and can often lead to a simplification of calculations involved in good models. The session will be devoted to the development and applications of such techniques to models that arise in any field of natural science and technology, including transport equations coupled with Boltzmann type models, reaction-diffusion equations and fragmentation-coagulation type equations.

Delayed stability switches in singularly perturbed dynamical systems

Jacek Banasiak
University of KwaZulu-Natal, So Africa
banasiak@ukzn.ac.za

In the classical applications of the geometric singular perturbation theory or of the Tikhonov theorem to dynamical systems with a small parameter, it is required that the limit manifold (quasi steady state) is isolated and attracting in a specific domain. However, often the quasi steady states intersect and switch stability. It is the expected that solutions to such systems with the parameter small enough, having followed the attracting branch of one of the quasi steady states, after passing close to the intersection, will switch stability and start following the attracting branch of the other quasi steady state. It turns out, however, that in many cases the solutions will continue to follow the repelling branch of the first quasi steady state for a considerable time before switching to the new, attracting one. In this talk we shall discuss generalizations and applications of the Butuzov theorem which deals with such problems but only in one dimension.

Some mathematical aspects of 'cluster eating' equations

Fernando da Costa
Universidade Aberta, Portugal
fcosta@math.ist.utl.pt
João T. Pinto, Rafael Sasportes

We present some results on the basic existence, uniqueness, regularity and long-time behaviour of solutions to a system of differential equations modelling "cluster eating". The equations, one version of which can be written as

$$\frac{dc_j}{dt} = \sum_{k=1}^{\infty} a_{j+k,k}c_{j+k}c_k - c_j \sum_{k=1}^{\infty} c_k, \quad j \in \mathbf{N},$$

were introduced in the late 1980s but only recently have been subject to a more detailed study. Connections with other models of cluster equations will be discussed.

Positive steady states of evolution equations motivated by structured population dynamics

Jozsef Farkas
University of Stirling, Scotland
jozsef.farkas@stir.ac.uk
Angel Calsina

We describe a general framework for studying the question of existence of positive steady states of some nonlinear evolution equations. In particular, we cast the steady state problem in the form of eigenvalue problems for a parametrised family of unbounded linear operators, which are generators of strongly continuous semigroups; and a fixed point problem. In case of irreducible governing semigroups we consider evolution equations with non-monotone nonlinearities of dimension two, and we establish a new fixed point theorem for set-valued maps. In case of reducible governing semigroups we establish results for monotone nonlinearities of any finite dimension n. We illustrate our theoretical results with examples of partial differential equations arising in structured population dynamics. In particular, we establish existence of positive steady states of a size-structured juvenile-adult and a structured consumer-resource population model, as well as for a selection-mutation model with distributed recruitment process. This talk is based on joint work with Angel Calsina (Universitat Autonoma de Barcelona).

Coagulation and Fragmentation Processes with Evolving Size and Shape Profiles.

Wilson Lamb
University of Strathclyde, Scotland
w.lamb@strath.ac.uk
A.C. McBride, A.L. Smith

A class of bivariate coagulation-fragmentation equations are discussed. These equations describe the evolution of a system of particles that are characterized not only by a discrete size variable but also by a shape variable which can be either discrete or continuous. Existence and uniqueness of strong solutions to the associated abstract Cauchy problems are established by using the theory of substochastic semigroups of operators.

Molecular ion channels

Henryk Leszczynski
University of Gdansk, Poland
hleszcz@mat.ug.edu.pl
Boguslaw Bozek, Elzbieta Puzniakowska-Galuch

Living bodies have hundreds of molecular ion channels. We start from the Nerst-Planck-Poisson description in a 3D-environment. Our main goal is the analysis of stationary solutions and feasible aproximations of evolutionary problems.

Dimension reduction in a model of morphogen transport

Marcin Malogrosz
University of Warsaw, Poland
malogrosz@mimuw.edu.pl

Morphogen is a protein substance which by the mechanism of positional signalling determines the process of cell differentiation. To model morphogen transport, a PDE-ODE system of reaction-diffusion type was proposed in [Huf]. The system consists of 1 parabolic semilinear PDE, posed on a rectangle $\Omega_h = I \times (0, h)$, which is coupled via nonlinear reaction terms, with 1 semilinear PDE and 3 ODE's posed on $I \times \{0\}$. We prove that the system has a unique steady state and that the evolution problem is well-posed in the appropriately chosen function setting. Moreover we analize limit of solutions as $h \to 0$. The presence of singular source term (a Dirac delta) in the boundary condition for the 1st equation, causes problems with regularity in the ODE part of the sytem.

Symmetrizable operators in Hilbert spaces and spectra of neutron transport

Yahya Mohamed
Universitede Franche compte, France
yahyajidou@yahoo.fr

A linear bounded operator G in a complex Hilbert space H is said to be symmetrizable by a bounded positive self-adjoint operator S if SG is self-adjoint. We study the spectrum of non- compact symmetrizable operators G with non injective symmetrizers S. We analyze its essential spectrum and also its isolated eigenvalues outside its essential disc. In particular, we give variational characterizations of such eigenvalues. Finally, we show how these tools apply to spectral analysis of neutron transport equations with partly elastic scattering operators.

Compactness properties of perturbed substochastic semigroups on L1 with applications to discreteness and spectral gaps

Mustapha Mokhtar-Kharroubi
University of Franche-Comte Besancon, France
mmokhtar@univ-fcomte.fr

We deal with positive semigroups of contractions on L1 spaces over abstract measure spaces and provide a systematic approach of compactness properties of perturbed semigroups induced by perturbing the generator by singular and bounded below potentials. The results are precised further on L1 spaces over metric measure spaces. This new theory relies on several ingredients: new a priori estimates peculiar to L1-spaces, local weak compactness assumptions on unperturbed operators, Dunford-Pettis arguments and the assumption that the sublevel sets of the potential are "thin at infinity". We show also how spectral gaps occur when the sublevel sets are not "thin at infinity". Various applications, in particular to convolution semigroups, are given.

Dishonesty in natural science and engineering

Suares Clovis Oukouomi Noutchie
North-West University, So Africa
23238917@nwu.ac.za

In this talk, we review dishonest processes in natural sciences and engineering. Examples will include shattering in fragmentation models, gelation in coagulation models and non-conservative random motion. We discuss the physical structures of these models and propose a unifying framework for investigating their dynamics.

A problem of population dynamic

Julie Sauzeau
Universite Rennes 1, France
julie.sauzeau@univ-rennes1.fr
Francois Castella, Philippe Chartier

We study the dynamics of a prey-predator system, with the particularity that the species are spread out over N sites, each site possessing its own characteristics (birth/death rates, predation pressure, etc.). The evolution is governed by two phenomena. On the one hand populations tend to migrate from one site to another, on a fast time scale, and we assume that the migration rates themselves oscillate on the same time scale (so as to reproduce migrations on a daily scale, say). On the other hand, the predator-prey dynamics itself takes place, yet on a much longer time-scale. We model this situation through a Lotka- Volterra-like system, modified by a fast oscillating, periodic, migration term, whose typical dimensionless time-scale is a small parameter epsilon. We completely describe the asymptotic model that is relevant as epsilon goes to zero, and analyse the qualitative properties of the limiting model. Our approach provides approximations at any order of the original equations. Our strategy relies on an original combination of a central manifold approach (so as to smooth out the rapid trend to equilibrium involved during the migration process), and of an averaging procedure (so as average out the fast oscillating coefficients involved in the migration rates themselves). Technically, we make use of a Floquet-Magnus approach, combined with a specific version of the central manifold in the case when the central manifold has oscillating coefficients, our last tool being the use of a high order version of periodic averaging for evolution equations.

Positive Equilibrium Solutions in Structured Population Dynamics

Christoph Walker
University of Hanover, Germany
walker@ifam.uni-hannover.de

The talk focuses on positive equilibrium (i.e. time-independent) solutions to mathematical models for the dynamics of populations structured by age and spatial position. This leads to the study of quasilinear parabolic equations with nonlocal and possibly nonlinear initial conditions. We shall see in an abstract functional analytic framework how bifurcation techniques may be combined with optimal parabolic regularity theory to establish the existence of positive solutions. As an application of these results we give a description of the geometry of coexistence states in a two-parameter predator-prey model.

Special Session 29: Stochastic and Deterministic Dynamical Systems and Applications

Tomas Caraballo, University of Sevilla, Spain
Maria J. Garrido-Atienza, University of Sevilla, Spain
Jose Valero, University Miguel Hernandez, Spain
Yuncheng You, University of South Florida, USA

The aim of this session is to offer an overview on recent results concerning the asymptotic behaviour of solutions of stochastic and deterministic partial and ordinary differential equations. Some of the main topics, but not the only ones, in this sessions are: asymptotic dynamics of dynamical systems, in particular, existence and properties of attractors for autonomous, non-autonomous and stochastic equations, stability, stabilization, attractors for equations without uniqueness of solutions, dynamics of equations with delay, and finite-dimensional dynamics for infinite dimensional dynamical systems.

Stochastic models for Chladni figures

Jaime Arango
Universidad del Valle, Colombia
jaime.arango@correounivalle.edu.co
Carlos Reyes

Chladni figures are formed when particles scattered throughout a plate move upon an external harmonic force resonating with one of the natural frequencies of the plate. Chladni figures are precisely the nodal set of the vibration mode corresponding to the frequency resonating with the external force. We propose a plausible model for the movement of the particles that explains the formation of Chladni figures in terms of the stochastic stability of the equilibrium solutions of stochastic differential equations.

Some results on set valued discrete dynamical systems

Francisco Balibrea
Universidad de Murcia, Spain
balibrea@um.es

Given a discrete dynamical system by the pair (X, f), where X is a continuum and f a continuous map of X into itself, we will deal with some dynamical properties of the corresponding hyperspaces $C(X)$ and 2^X endowed with the Hausdorff topology, such as Li-Yorke sensitivity, weakly mixing, mixing and also ergodicity. We will give some results and examples chosen in the continuum theory and develop in this setting a non-autonomous theory.

Global variational solutions for the shell model driven by a multiplicative fBM

Hakima Bessaih
University of Wyoming, USA
bessaih@uwyo.edu
Maria J. Garrido Atienza & Bjorn Schmalfuss

We consider some shell models of turbulence in a very general form. These are phenomenological approximations of the Navier-Stokes equations, with a viscous linear part that is dissipative and a nonlinear part that is not globally Lipschitz. We assume that this model is driven by a multiplicative noise (fBM with Hurst parameter $H > 1/2$). We will assume that the nontrivial diffusion term which is a nonlinear operator will satisfy some Lipschitz property and some other differentiability conditions. We will prove the existence and uniqueness of a global variational solution. The proof will be achieved in two steps. The first step is to prove that the variational solutions exist and are unique for a smooth noise. Moreover, some a priori estimates will be obtained in some functional spaces. The second step uses a compactness argument. In fact, we prove that these solutions have a limit when the smooth noise converges to the fBM and that the limit is a variational solution for the shell model. All these statements and proofs are based on a pathwise argument. We hope to extend these results to the 2d Navier-Stokes equations.

Asymptotic dynamics for non-autonomous differential equations

Matheus Bortolan
Universidade de São Paulo, Brazil
mbortolan@gmail.com
Alexandre N. Carvalho, José A. Langa, Tomás Caraballo

In this talk we will discuss the asymptotic dynamics of non-autonomous differential equations. We point out that this is a vague term, since we have to give meaning to the term "asymptotic dynamics" when dealing with non-autonomous equation, since it may have several distinct (and at first sight, unrelated) meanings. We will clarify theses various meanings and give the connection between them, leading to a detailed analysis on the long-time behavior of the problem.

Mathematical modeling of Indirect Effects of Predation

Renato Colucci
Pontificia Universidad Javeriana, Colombia
renatocolucci@hotmail.com

We present several models describing indirect effect of predation in presence of two preys. We study the asymptotic behavior of the systems by the analysis of global/pullback attractors. We discuss the differences between the various models proposed.

Minimality properties of set-valued autonomous and non-autonomous dynamical systems

Michele Coti Zelati
Indiana University, USA
micotize@indiana.edu

From the point of view of longterm dynamics, we study multivalued dynamical systems acting on complete metric spaces. We relax the classical definition of global and pullback attractors and provide a full range of examples and counterexamples to investigate the relations between different notions of asymptotic compactness and dissipativity.

Pullback exponential attractors with applications to reaction-diffusion equations

Radoslaw Czaja
Instituto Superior Tecnico, Lisbon, Portugal
czaja@math.ist.utl.pt

In this talk results on the existence of a pullback exponential attractor for an evolution process are presented. This positively invariant family of compact subsets has a uniformly bounded fractal dimension and pullback attracts all bounded subsets at an exponential rate. The novelty is that the construction admits the exponential growth in the past of the sets forming the family and generalizes the known approaches. It also allows to substitute the smoothing property by a weaker requirement without auxiliary spaces. The theory is illustrated with examples of nonautonomous reaction-diffusion equations including the time-dependent version of the well-known Chafee-Infante equation.

Stabilization of long-time asymptotics by noise

Benjamin Gess
University of Chicago, USA
gess@math.tu-berlin.de

In this talk we are going to survey some methods and results proving stabilization of long-time asymptotics of PDE by noise, in terms of the collapse of the (random) attractor to a single point. We will then lay out some recent results obtained in [Gess, JDDE, 2013] where it is shown that the infinite-dimensional attractor of degenerate, perturbed p-Laplace equations collapses if enough additive noise is included.

Nonlocal Cahn-Hilliard-Navier-Stokes systems

Maurizio Grasselli
Politecnico di Milano, Italy
maurizio.grasselli@polimi.it

We consider a diffuse interface model for incompressible isothermal mixtures of two immiscible fluids with matched densities. This model consists of the Navier-Stokes system coupled with a convective nonlocal Cahn-Hilliard equation. We discuss some recent results in dimension two obtained in collaboration with C.G. Gal (Florida International University) and S. Frigeri (WIAS, Berlin).

A Linearization Methode of a Nonlinear Stochastic Schrödinger Equation

Wilfried Grecksch
Martin-Luther-University Halle-Wittenberg, Germany
wilfried.grecksch@mathematik.uni-halle.de

We consider a one dimensional stochastic Schrödinger equation with Lipschitz nonlinearities on a bounded domain with homogeneous Neumann boundaries conditions. The equation contains a multiplicative noise which is defined by a stochastic integral with respect to a cylindrical Wiener process. The solution is defined in the variational sense. We discuss a solution approximation by means of a method of successive approximation where linear stochastic Schrödinger equations with additive noise are solved step by step.

Chemostats with time-dependent inputs and wall growth

Xiaoying Han
Auburn University, USA
xzh0003@auburn.edu
Tomas Caraballo, Peter Kloeden

Traditional assumptions in the simple chemostat model include fixed availability of the nutrient and its supply rate, and fast flow rate to avoid wall growth. However, these assumptions become unrealistic when the availability of a nutrient depends on the nutrient consumption rate and input nutrient concentration and when the flow rate is not fast enough. We relax these assumptions and study the chemostat models with a variable nutrient supplying rate or a variable input nutrient concentration, with or without wall growth. This leads the models to nonautonomous/random dynamical systems and requires new concepts of nonautonomous/random attractors from the recently developed theory of nonautonomous/random dynamical systems. Our results provide sufficient conditions for existence of nonautonomous/random attractors and singleton attractors.

On a parabolic equation with nonlocal diffusion and sublinear terms

Marta Herrera-Cobos
Universidad de Sevilla, Spain
mhc@us.es
Tomás Caraballo, Pedro Marín-Rubio

We study the asymptotic behaviour of a non-autonomous parabolic equation with nonlocal diffusion and sublinear terms. Due to the presence of non-autonomous terms, in order to analyze the problem we will turn to the pullback attractor theory to prove the existence of attractors in different universes and a relation between them.
This is a joint work with Tomás Caraballo and Pedro Marín-Rubio, from Universidad de Sevilla.

Well-posedness and dynamics of stochastic generalized fractional Benjiamin-Ono equation

Jianhua Huang
National University of Defense Technology, Peoples Rep of China
jhhuang32@nudt.edu.cn
Wei Yan, Boling Guo

In this talks, the well-posedness and dynamics of the Cauchy problem for the stochastic generalized Benjamin-Ono equation are presented. The Cauchy problem for the stochastic generalized Benjamin-Ono equation is locally well-posed for the initial data $u_0(x, w) \in L^2(\Omega; H^s(R))$ with $s \geq \frac{1}{2} - \frac{\alpha}{4}$, where $0 < \alpha \leq 1$. In particular, when $u_0 \in L^2(\Omega; H^{\frac{\alpha+1}{2}}(R)) \cap L^{\frac{2(2+3\alpha)}{\alpha}}(\Omega; L^2(R))$, the global well-posedness of the solution $u \in L^2(\Omega; H^{\frac{\alpha+1}{2}}(R))$ with $0 < \alpha \leq 1$ is also established. Our main results shows that the well-posedness depends on the order of fractional operator, the norm of Hilbert-Schmidt operator, and regularity of the initial value. Finally, the random attractor of random dynamical systems generated by the global solution is also obtained.

The dynamics of reaction-diffusion equations with α-stable noise

Peter Imkeller
Humboldt-Universitaet zu Berlin, Germany
imkeller@math.hu-berlin.de
Arnaud Debussche, Jan Gairing, Claudia Hein, Michael Högele, Ilya Pavlyukevich

Dynamical systems of the reaction-diffusion type with small noise have been instrumental to explain basic features of the dynamics of paleo-climate data. For instance, a spectral analysis of Greenland ice time series performed at the end of the 1990s representing average temperatures during the last ice age suggest an $\alpha-$stable noise component with an $\alpha \sim 1.75$. We model the time series as a dynamical system perturbed by α-stable noise, and develop an efficient testing method for the best fitting α. The method is based on the observed p-variation of the residuals of the time series, and their asymptotic $\frac{\alpha}{p}$-stability established in local limit theorems.

Generalizing the solution of this model selection problem, we are led to a class of reaction-diffusion equations with additive α-stable Lévy noise, a stochastic perturbation of the Chafee-Infante equation. We study exit and transition between meta-stable states of their solutions. Due to the heavy-tail nature of an α-stable noise component, the results differ strongly from the well known case of purely Gaussian perturbations.

Existence and Upper Semicontinuity of Pullback Attractors for Non-autonomous Stochastic Degenerate Parabolic Equations on Unbounded Domains

Andrew Krause
New Mexico Tech, USA
akrause@alumni.nmt.edu
Bixiang Wang

We prove the existence and uniqueness of random attractors for non-autonomous stochastic P-laplace equations on R^n driven by white noise. The periodicity of random attractors is obtained when external forcing is time periodic. We also establish the upper semicontinuity of random attractors as the intensity of noise approaches zero. The pullback asymptotic compactness of solutions is proved by combining the compactness of Sobolev embeddings in bounded domains and the uniform smallness of solutions outside a sufficiently large ball.

A dynamical system approach to complex networks in Biology

Jose Langa
University of Seville, Spain
langa@us.es
Gyovanny Guerrero and Antonio Suarez

We study N-dimensional mutualistic models in Biology with competitive-cooperative relations among participants. We compare the dynamical properties of this type of systems, described as complex networks. We observe how cooperation is a common fact to increase biodiversity, which it is known that, generically, holds for general mutualistic dynamical systems in Ecology. We also give mathematical evidence on how a cooperative species induces an increased biodiversity, even if the species is push to extinction. For this fact, we propose a necessary change in the model formulation which could explain this kind of phenomenon. We will point out some interesting open problems on dynamics and complex networks related to this kind of models.

On a heat equation with non-local diffusion

Pedro Marin-Rubio
Universidad de Sevilla, Spain
pmr@us.es
Tomás Caraballo, Marta Herrera-Cobos

In this contribution we will discuss on a heat equation with non-local diffusion and other terms. Well-posedness and asymptotic behaviour of an associated dynamical system will be stated. Namely, and since the problem contains non-autonomous terms, we will use the approach of pullback attractors.
This is a joint work with Tomás Caraballo and Marta Herrera-Cobos, from Universidad de Sevilla.

Asymptotic behavior of variants of the Cahn-Hilliard equation

Alain Miranville
Universite de Poitiers, France
miranv@math.univ-poitiers.fr

Our aim in this talk is to discuss the qualitative behavior (existence of finite-dimensional attractors and blow up in finite time) of variants of the Cahn-Hilliard equation. Such equations arise in the context of image inpainting and biology.

Exponential ordering for nonautonomous neutral functional differential equations with applications

Rafael Obaya
University of Valladolid, Spain
rafoba@wmatem.eis.uva.es
Sylvia Novo and Victor M. Villarragut

We use the skew-product formalism to study the structure of the omega-limit set of relatively compact solutions of non-autonomous neutral functional differential equations. We assume some recurrence in the temporal variation of the vector field in order that our flow in the base was minimal. We present recent results which prove that the equation generates a semiflow which is monotone for some exponential ordering and prove in these conditions that these omega limit sets are minimal and define a copy of the base.

Effects of strong convection on the cooling process for a long or thin pipe

Igor Pazanin
University of Zagreb, Croatia
pazanin@math.hr
Eduard Marušić-Paloka, Sanja Marušić

We consider a heat flow through a thin (or long) pipe filled with incompressible viscous fluid. The process is described by a non-stationary convection-diffusion equation. The fluid in the pipe is cooled by the exterior medium and we describe the heat exchange simply by the Newton's cooling law . Assuming that the flow through a pipe has a given Poiseuille profile, we study the thermodynamic part of the system. Depending on the ratio between the pipe's thickness ε and the Reynolds number Re^{ε} , we obtain three different macroscopic models via rigorous asymptotic analysis. For small Re^{ε} the fluid in the pipe is perfectly cooled, i.e. it assumes the temperature of the surrounding medium. For large Re^{ε}, the fluid is not cooled at all, i.e. it maintains the same temperature as it had when it entered the pipe. Between those two cases there is a critical value of Re^{ε} when the macroscopic model is described by an ODE keeping the effects of the surrounding medium as well as the entering temperature.

Decomposition of Lévy flows in manifolds according to complementary foliations

Paulo Ruffino
University of Campinas, Brazil
ruffino@ime.unicamp.br
Leandro Morgado

Let M be a differentiable manifold endowed locally with two complementary foliations, say horizontal and vertical. I.e. in a neighbourhood of each point $x \in M$ there are two submanifolds passing through x whose intersection is $\{x\}$ and have complementary dimensions in M. We consider the two subgroups of (local) diffeomorphisms of M generated by vector fields in each of of these foliation. Let φ_t be a stochastic flow of diffeomorphisms in M generated by Lévy noise (Marcus equation). We prove that in a neighbourhood of an initial condition, up to a stopping time one can decompose $\varphi_t = \xi_t \circ \psi_t$ where the first component is again a solution of a Marcus equation (autonomous vector fields) in the group of horizontal diffeomorphisms and the second component is a process in the group of vertical diffeomorphisms. Further decompositions considering more than two foliations will include more than two components: it leads to a maximal cascade decomposition in local coordinates where each component acts only in the corresponding coordinate. This decomposition extends the results in Catuogno, da Silva and Ruffino, Stochastics and Dynamics (2013).

Numerical Analysis and Simulation of Random Partial Differential Equations with Boundary Excitations

Florian Rupp
German University of Technology in Oman (GUtech), Oman
fhhrupp@web.de
Tobias Neckel and Alfredo Parra Hinojosa

Important physical phenomena can be described in terms of partial differential equations that are perturbed by random forces acting on the boundaries of the domain under consideration, like the ground-motion excitation of multistory buildings or the top-layer thermal excitation of the earth's atmosphere. In particular, many of these random forces are given as filtered white noise processes (colored noise) or power law noises. This leads to the mathematical study of random partial differential equations (RPDEs) with path-wise continuous or path-wise differentiable solution stochastic processes. From the numerical point of view RPDEs are simulated by polynomial chaos methods or stochastic Galerkin schemes. Here, we focus on the theoretical and numerical properties of a different approach tailored for boundary excited problems: a method of lines that reduces RPDEs to a finite-dimensional system of random ordinary differential equations (RODEs). Novel numerical schemes allow the effective and efficient computation of solutions of RODEs such that this reduction provides a very practical alternative to traditional numerical schemes for boundary excited RPDEs. We illustrate our RPDE-RODE reduction method with the aid of ground-motion induced oscillations of solid structures where we discuss existence of weak solution as well as the convergence properties of the numerical scheme.

Asymptotic behavior of a semilinear heat equation on time-varying domains

Chunyou Sun
Lanzhou University, Peoples Rep of China
sunchy@lzu.edu.cn

In this talk we will discuss the asymptotic behavior of a semilinear heat equation defined on time-varying domains. Since this kind of equation is intrinsically non-autonomous even if the terms in the equation do not depend explicit on time, we will use the approach of pullback attractors.

Regularity of random attractors for stochastic reaction diffusion equations in unbounded domains

Bao Tang
Institute for Mathematics and Scientific Computing, University of Graz, Austria
baotangquoc@gmail.com

The aim of this talk is to study the regularity of random attractors for a class of stochastic reaction diffusion equations with time-dependent external forces in unbounded domains. The regularity of attractors in deterministic case, e.g. global attractors, uniform attractors, pullback attractors, is well established by many mathematicians. However, their methods seem not applicable to the stochastic case due to the irregularity of the random terms. To overcome this difficulty, we introduce new ideas which combine tail estimates of solutions and careful estimates of the nonlinearity as well as the time derivative of the solution.

On uniform attractors for non-autonomous p-Laplacian equation with a dynamic boundary condition

Meihua Yang
Huazhong University of Science and Technology, Peoples Rep of China
yangmeih@hust.edu.cn

In this talk we will discuss the dynamical behavior of solutions of the non-autonomous p-Laplacian equation with nonlinear dynamic boundary condition. While the nonlinearity f and the boundary nonlinearity g are dissipative for large values without restriction on the growth order of the polynomial.

Stationary Solutions of Stochastic Partial Differential Equations

Qi Zhang
Fudan University, Peoples Rep of China
qzh@fudan.edu.cn
Huaizhong Zhao

The stationary solution is a natural extension of the fixed point in a deterministic dynamical system to the stochastic case. However, due to the existence of external random force in the stochastic system generated by SPDE, the construction of stationary solution is difficult to realize. In our joint work with Huaizhong Zhao, we use the backward stochastic differential equations to construct the stationary solutions of SPDEs, which was proved to be a feasible method to this issue. In this talk, i would like to introduce the process of our work on the stationary solutions of SPDEs.

Special Session 30: Discrete Dynamics and Applications

Eduardo Liz, Universidad de Vigo, Spain
Daniel Franco, UNED, Spain
Christian Pötzsche, Alpen-Adria Universität Klagenfurt, Austria

This session focuses on Difference Equations and Discrete Dynamical Systems. Topics include stability, bifurcations, complex dynamics, periodicity and global dynamics both for autonomous and nonautonomous equations. Applications are particularly welcome, with special attention to Mathematical Biology.

Global attractivity in difference equations of the form $x_{n+1} = x_n f(x_{n-1}) \pm h$

Ziyad Al-Sharawi
Sultan Qaboos University, Oman
alsha1zm@alsharawi.info
Asma Al-Gassani, Nasser Salti

In this talk, we discuss the global attractivity of a steady state in equations of the form $x_{n+1} = x_n f(x_{n-1}) \pm h$, where h is a parameter that can denote constant stocking or harvesting in population models. We establish a connection between Pielou's model with harvesting/stocking, Lyness difference equations and the Y2K problem. The established connection simplifies proving the global attractivity of the positive steady state in the Y2K problem. The Y2K problem is extended to cover negative values of the parameters and results on persistence are provided. This work is about an ongoing research in which some open questions will be posed.

Rotation Numbers for Planar Attractors of Equivariant Homeomorphisms

Begona Alarcon
Universidade Federal Fluminense, Brazil
balarcon@id.uff.br

Given an integer $n > 1$ we consider Zn-equivariant and orientation preserving homeomorphisms of the plane with an asymptotically stable fixed point at the origin. We present examples without periodic points and having some complicated dynamical features. The key is a preliminary construction of Zn-equivariant Denjoy maps of the circle.

Invariant quadrics for certain systems of rational difference equations

Ignacio Bajo
Universidad de Vigo, Spain
ibajo@dma.uvigo.es

Systems of difference equations defined by linear fractionals sharing denominator are studied. Such systems can be nicely described in terms of a square matrix by the use of homogeneous coordinates. The talk will focus on the existence of invariant affine varieties and quadrics. In particular, we show that there is a correspondence between invariant non-degenerate quadrics and solutions to certain matrix equation involving the matrix defining the system. Further, when such matrix is semisimple, it is proved that every orbit is contained either in an invariant affine variety or in an invariant quadric.

The effect of harvesting on Pielou's equation for $k = 2$

Mohamed Ben Haj Rhouma
Qatar University, Qatar
rhouma@qu.edu.qa

In this talk, we will discuss the effect of harvesting on a population governed by Pielou's equation. In particular, we will discuss the stability and the basin of attraction of the positive fixed point of the difference equation

$$x_{n+1} = \frac{\beta x_n}{1 + x_{n-k}} - h$$

The effect of both the delay k and the harvesting level h will be examined.

Stabilization of prescribed values and periodic orbits with regular and pulse target oriented control

Elena Braverman
University of Calgary, Canada
maelena@math.ucalgary.ca
Brian Chan

Investigating a method of chaos control for one-dimensional maps, where the intervention is proportional to the difference between a fixed value and a current state, we demonstrate that stabilization is possible in one of the two following cases: 1) for small values, the map is increasing and the slope of the line connecting the points on the line with the origin is decreasing; 2) the chaotic map is locally Lipschitz. Moreover, in the latter case we prove that any point of the map can be stabilized. In addition, we study pulse stabilization when the intervention occurs each m-th step and illustrate that stabilization is possible for the first type of maps. In the context of population dynamics, we notice that control with a positive target, even if stabilization is not achieved, leads to persistent solutions and prevents extinction in models which experience the Allee effect.

Control of chaos in discrete spatial systems

Pablo Carmona
pcarmonal@clh.es
Daniel Franco

We consider the evolution of a metapopulation which we model, as it is common in the literature, by a discrete dynamical system. This evolution consists of a phase of reproduction followed by a phase of dispersion or migration among the regions that the species occupies. It is known that depending on the intrinsic parameters of the system, the long term behaviour could either tend towards an equilibrium point, a cycle or a chaotic attractor. In

this ongoing work, we simulate and study the effect of some control strategies applied to these systems with the objective of avoiding the irregular behaviour and, ideally, leading the response to an equilibrium point. We will see that it is possible to take advantage of the dispersion for the application of a selective control in some regions instead of applying it to the whole system. Moreover, we will discuss the consequences of the previous choice regarding whether it is possible to get an extended interval in which the control parameter produces the stabilization of the system or even to avoid the Allee effect.

Enveloping and Population Models

Paul Cull
Oregon State University, USA
pc@cs.orst.edu

One dimensional nonlinear difference equations are commonly used to model population growth. Although such models can display wild behavior including chaos, the standard population models have the interesting property that they are globally stable if they are locally stable. We show that models with a single positive equilibrium are globally stable if they are *enveloped* by a self-inverse function. In particular, we show that the standard population models are enveloped by linear fractional functions which are self-inverse. Although enveloping by a linear fractional is sufficient for global stability, we show by example that such enveloping is not necessary. We extend our results by showing that *enveloping implies global stability* even when $f(x)$ is a discontinuous multifunction, which may be a more reasonable description of real biological data. We also show that our techniques can be applied to situations which are not population models. Finally, we give examples of population models which have local stability but not global stability.

Matrix models, strong Allee effects, and adaptive changes in biological populations to environmental change

Jim Cushing
University of Arizona, USA
cushing@math.arizona.edu

Component Allee effects are low density positive feedback mechanisms that affect individual fitness. They can, under certain circumstances, lead to a strong Allee effect, that is, the simultaneously occurrence of an extinction attractor and a non-extinction (or positive) attractor. Motivated by issues concerning endangered species, especially in the light of climate change, there is a rapidly growing interest in strong Allee effects and the resulting extinction thresholds. I will discuss strong Allee effects in matrix models that describe the discrete time dynamics of a structured population. From a bifurcation theory point of view, I will give some general criteria under which strong Allee effects occur in nonlinear matrix models. One key is a backward bifurcation of positive equilibria. I will also apply this approach to evolutionary versions of matrix models. I will give a specific application arising from my collaborative research with field ecologists who study marine birds at the US National Wildlife Refuge on Protection Island, Washington State, USA. The model, with its strong Allee effect, offers an hypothesis that explains certain life history strategy changes recently observed in breeding colonies that relate to climate change during the last half century (specifically, a mean sea surface temperature increase).

Global stability of some second order difference equations

Abel Garab
University of Szeged, Hungary
garab@math.u-szeged.hu
Ferenc A. Bartha, Tibor Krisztin

Consider the second order difference equation

$$x_{k+1} = x_k e^{\alpha - x_{k-d}}$$

where α is a positive parameter and d is a nonnegative integer. The case $d = 0$ was introduced by W. E. Ricker in 1954. For the delayed version $d \geq 1$ of the equation S. Levin and R. May conjectured in 1976 that local stability of the nontrivial equilibrium implies its global stability. Based on rigorous, computer-aided calculations and analytical tools, we prove the conjecture for $d = 1$. We also apply our method to give necessary and sufficient conditions for the global stability of the trivial equilibrium of the difference equation $x_{k+1} = mx_k + \alpha \tanh x_{k-1}$, where m and α are real parameters.

Integrability and non-integrability of some difference equations

Armengol Gasull
Universitat Autonoma de Barcelona, Spain
gasull@mat.uab.cat
Anna Cima and Víctor Mañosa

Consider second order difference equation $x_{n+2} = f(x_n, x_{n+1})$. An invariant is a non-constant function $I(x, y)$ such that $I(x_n, x_{n+1}) = I(x_{n+1}, x_{n+2})$. It is clear that it is equivalent to have invariants for a difference equation that to have first integrals for its associated discrete dynamical system $F(x, y) = (y, f(x, y))$. Similar concepts can also be introduced for higher order or for non-autonomous difference equations. In this talk we will introduce several results about the existence or non-existence of first integrals of discrete dynamical systems. We will apply them to several know difference equations.

Homoclinic trajectories in non-autonomous systems and their discretization

Alina Girod
Bielefeld University, Germany
agirod@uni-bielefeld.de
Thorsten Huels

We consider a continuous time non-autonomous dynamical system having two hyperbolic bounded trajectories that converge towards each other. Applying a one-step-method with sufficiently small step size we get hyperbolic bounded trajectories of the discretized system. They lie in a small neighborhood of the original trajectories and are also homoclinic. For verifying our error estimates, we construct an example in continuous time with known

homoclinic trajectories. An illustration of homoclinic dynamics can be achieved by computing stable and unstable fiber bundles. For this task, an algorithm of England, Krauskopf and Osinga is introduced that we generalize to the non-autonomous case.

Selections and their Absolutely Continuous Invariant Measures

Pawel Gora
Concordia University, Canada
pawel.gora@concordia.ca
Abraham Boyarsky and Zhenyang Li

Let $I = [0,1]$ and let P be a partition of I into a finite number of intervals. Let τ_1, $\tau_2; I \to I$ be two piecewise expanding maps on P. Let $G \subset I \times I$ be the region between the boundaries of the graphs of τ_1 and τ_2. Any map $\tau : I \to I$ that takes values in G is called a selection of the multivalued map defined by G. There are many results devoted to the study of the existence of selections with specified topological properties. However, there are no results concerning the existence of selection with measure-theoretic properties. In this paper we prove the existence of selections which have absolutely continuous invariant measures (acim). By our assumptions we know that τ_1 and τ_2 possess acims preserving the distribution functions $F^{(1)}$ and $F^{(2)}$. The main result shows that for any convex combination F of $F^{(1)}$ and $F^{(2)}$ we can find a map η with values between the graphs of τ_1 and τ_2 (that is, a selection) such that F is the η-invariant distribution function. Examples are presented. We also study the relationship of the dynamics of our multivalued maps to random maps.

Global bifurcations in a non-invertible model of asset pricing

Thorsten Huels
Bielefeld University, Germany
huels@math.uni-bielefeld.de
Volker Boehm

We introduce a canonical form for a standard model of asset pricing. The resulting (non-invertible) two-dimensional dynamical system depends on two parameters only, a rate of geometric decay and a mean reversion parameter. We detect parameter regimes for which homoclinic and heteroclinic orbits exist and illustrate corresponding intersections of stable and unstable sets. Finally, autocorrelations of prices and returns before and after these global bifurcations are discussed.

Stability, periodicity, and global dynamics in a class of multi-dimensional maps

Anatoli Ivanov
Pennsylvania State University, USA
afi1@psu.edu

We study dynamics of multi-dimensional maps F_i of the form

$$F_i : [u_1, u_2, \ldots, u_{N-1}, u_N] \mapsto [u_1 + a_i f(x_N), u_1, u_2, \ldots, u_{N-1}],$$

where f is a fixed one-dimensional map on $\mathbb{R}$ and $a_i, i = 1, \ldots, K$, are real numbers. Such maps result from exact reduction of dynamics in periodic differential delay equations with piece-wise constant argument of the form

$$x'(t) = a(t)g(x([t-N])),$$

where $a(t)$ is a K-periodic function, g is a real valued function, and $[\cdot]$ is the integer part function. We address the problems of global stability of a unique fixed point, existence and stability of cycles (periodic points), and other questions of global dynamics in such maps and their compositions.

Stability, negative Schwarzian derivative, and control of chaos

Victor Jimenez Lopez
Universidad de Murcia, Spain
vjimenez@um.es
Enrique Parreno

We say that a map $h : I \to I$ belongs to the class S if , roughly speaking, it is monotone or unimodal, has a unique fixed point u and the Schwarzian derivative of h is negative. These maps are important in dynamics because i if u is a local attractor for (1): $x_{n+1} = h(x_n)$ (which is equivalent to $|h'(u)| \leq 1$), then it is a global attractor of (1) as well. In this work we address the question of whether the local stability of u for the higher order difference equation (2): $x_{n+1} = \alpha x_{n-k} + (1-\alpha)h(x_n)$ may imply global attraction whenever h belongs to the class S. This equation has been widely used in the literature in population dynamics and control of chaos models. Our main results: a) If k is odd, then u is locally attracting for (1) if and only if it is locally attracting for (2); moreover, in this case u is globally attracting for (2). b) If k is even, local attraction need not imply global attraction for (2); a counterexample for $k = 2$ is the Ricker function $h(x) = px^{qx}$. Yet we give some arguments supporting the validity of the statement when k is large enough.

Folding and unfolding in periodic difference equations

Antonio Linero Bas
Universidad de Murcia, Spain
lineroba@um.es
Z. AlSharawi, J.S. Cánovas

Periodic difference equations of the form

$$x_{n+1} = f_{n\, mod p}(x_n),$$

where each f_j is a continuous interval map, $j = 0, 1, \ldots, p-1$, appear in a natural way in technical and social sciences, related to processes involving two or more interactions so that for the knowledge of the behaviour of these systems it is necessary to alternate different discrete dynamical systems corresponding to each period of the process. In this sense, it is interesting to stress that the above equations can model, for instance, certain populations in a periodically fluctuating environment. In the present talk we discuss the notion of folding and unfolding related to this type of non-autonomous equations. It is possible to glue certain maps of this equation to shorten its period, which we call folding. On the other hand, we can unfold the glued maps so the original structure can be recovered or understood. We focus on the periodic

structure under the effect of folding and unfolding: we analyze the relationship between the periods of periodic sequences of the p-periodic difference equation and the periods of the corresponding subsequences related to the folded systems.

Exponential Stability Criteria for Discrete Time-Delay Systems

Rigoberto Medina
Universidad de Los Lagos, Chile
rmedina@ulagos.cl
Carlos Martinez

In this talk we will examine the exponential stability of linear discrete time-delay systems with slowly varying coefficients and nonlinear perturbations. We will establish the robustness of the exponential stability in Hilbert spaces, in the sense that the exponential stability for a given linear equation persists under sufficiently small perturbations. As an application of the main results, we will discuss the exponential stability of general nonlinear systems. We always consider the exponential behavior of solutions with respect to an specific ball, thus ensuring reasonable dynamics.

On Qualitative Identification of Linear Discrete Dynamical Systems arising in Nondestructive Testing using Infrared Thermography

Juan Peran
UNED, Spain
jperan@ind.uned.es
Laura Vega

Infrared thermography provides a non-destructive technique for testing of the surface or near-surface defects in certain materials. The part to be inspected is illuminated with infrared light and the thermal response is recorded by an infrared detector to obtain a three dimensional array (*frame width (number of pixels)* $\times$ *frame length (number of pixels)* $\times$ *time (number of frames)*) which can be visualized as a greyscale video clip. We model this process as a first order linear discrete dynamical system (discrete anisotropic diffusion equation) whose unknown coefficients have to be estimated to localize defects. The amount of information of the greyscale video clip (a *slice* of the total evolution of the dynamical system) is summarized into a RGB colour image to which we apply edge detection techniques.

Nonnegative iterations in Banach spaces

Mihály Pituk
University of Pannonia, Hungary
pituk

In this talk we will consider orbits of compact linear operators in a real Banach space. It will be shown that if the orbit is nonnegative with respect to the partial ordering induced by a given cone then its local spectral radius is an eigenvalue of the operator with a positive eigenvector. The result can be extended to certain nonhomogeneous linear difference equations in a Banach space.

Analysis of dispersal effects in metapopulation models

Alfonso Ruiz-Herrera
University of Szeged, Hungary
alfonsoruiz@ugr.es

The interplay between local dynamics and the movement of the population in discrete metapopulation models is studied. In considering homogeneous landscapes, we characterize when there is synchronization independently of the strategy of dispersal. In particular, our analytic results support the numerical studies of Allen et al in Chaos reduces special extinction by amplifying local population noise, Nature 364 (1993), 229–232 ; and Heino et al in Synchronous dynamics and rates of extinction in spatially structured populations, Proc. R. Soc. Lond. B 264 (1997), 481–486 where these authors prove that a chaotic behavior in the local dynamics reduces the degree of synchrony. In considering heterogeneous landscapes, we study global attractivity, compensating role of dispersal, chaotic dynamics, and some counterintuitive phenomena involving Allee's effect. Relative to these counterintuitive phenomena, on the one hand we provide some insights to guarantee the salvage effect introduced by Gyllenberg et al in Bifurcation Analysis of a metapopulation model with sources and sinks, J. Nonlinear Sci. 6 (1996), 329–366. On the other hand, we discuss when dispersal can produce global extinction independently of the dynamics within the patches.

Competitive exclusion and coexistence in an n-species Ricker model

Paul Salceanu
University of Louisiana at Lafayette, USA
salceanu@louisiana.edu
Azmy S. Ackleh

We analyze a discrete-time Ricker competition model with n competing species and give sufficient conditions, which depend on the competition coefficients only, for one species to survive (not necessarily at an equilibrium) and to drive all the other species to extinction. Our results complement and extend similar existing results from the literature. For the model reduced to three species ($n = 3$), we also investigate various scenarios under which all species coexist, in the sense that each species is robustly uniformly persistent. We provide a few numerical simulations to illustrate that coexistence does not necessarily mean convergence to the interior equilibrium, and that the interior dynamics can be quite complex.

Some linear fractional maps with zero entropy

Sundus Zafar
Autonomous University of Barcelona, Spain
sunduszafar@gmail.com
Anna Cima

Given complex numbers α_i, γ_i and $\delta_i, i = 0, ..., 2$, consider the family of birational maps $f : \mathbf{C}^2 \to \mathbf{C}^2$ of the following form

$$f(x, y) = \left(\alpha_0 + \alpha_1 x + \alpha_2 y, \frac{\gamma_0 + \gamma_1 x + \gamma_2 y}{\delta_0 + \delta_1 x + \delta_2 y} \right). \qquad (1)$$

This family (1) is dynamically classified completely in [1]. For all the values of parameters for which the determinants $(\gamma\delta)_{12}$ and $(\alpha\delta)_{12}$ are zero, it is called a *degenerate case*. In general the family (1) has dynamical degree $D = 2$. The main interest is to identify the possible subcases of (1) for all the parameter values. By the help of the associated characteristic polynomial of each subcase/subfamily it is possible to know their growth rate. Therefore finding the dynamical degree D for all the subcases helps to locate the subfamilies with entropy *zero* and the ones where $1 < D < 2$. The subfamilies with zero entropy have rather simpler dynamics than the other subfamilies which have non zero entropy. This talk will focus on providing information of all the existing subcases/subfamilies of (1) when it is degenerate. Then the dynamics of the families with zero entropy will be discussed. The family 1 includes a subfamily dynamically studied in [2] and also this work provides examples for the theoretical results stated in [3, 4].

References

[1] Cima, A. and Zafar, S. *Classification of a family of birational surface maps via dynamical degree*; Preprint

[2] Bedford, E. and Kim, K. *Periodicities in Linear Fractional Recurrences: Degree Growth of Birational Surface Maps* Michigan Math. J. **54** (2006), 647-670.

[3] Diller, J. and Favre, C. *Dynamics of bimeromorphic maps of surfaces* Amer. J. of Math., **123** (2001), 1135-1169.

[4] Fornaes, J-E and Sibony, N. *Complex dynamics in higher dimension. II* Modern methods in complex analysis, Ann. of Math. Stud. 137, Princeton Univ. Press, 1995, pp. 135-182.

Special Session 31: Variational Energy and Entropy Approaches in Non-Smooth Thermomechanics

Elena Bonetti, University of Pavia, Italy
Elisabetta Rocca, WIAS, Germany

Non-smooth phenomena in thermo-mecanics, deeply studied both from modelling and analytical points of view, recently introduced new mathematical challenges. Indeed new formulation and solution notion need to be developed in order to handle the mathematical difficulties and to ensure the thermodynamical consistency of the associated models.

Shape memory alloys: a new flexible 3D constitutive modeling direction

Ferdinando Auricchio
University of Pavia, Italy
auricchio@unipv.it
Elena Bonetti, Giulia Scalet, Francesco Ubertini

Among the broad class of smart materials, *shape-memory alloys* (SMAs) have unique features due to their ability to regain the original shape either during unloading or through a thermal cycle. Thanks to such properties, SMAs are exploited in innovative applications. A substantial research effort has been conducted with the aim of developing reliable constitutive models to be used as design tools for SMA devices. Among the others, macroscopic models appear to be a powerful tool for SMA behavior simulation. Accordingly, the present work aims to develop a more flexible and general 3D constitutive model, along the lines of what recently proposed by Auricchio and Bonetti (2013). The proposed model introduces volume proportions of different configurations of crystal lattice (i.e., austenite, single- and multiple-variant martensites) as scalar internal variables and the direction of single-variant martensite as tensorial one. The theoretical framework allows for a completely independent description of phase transformations, leading to a very flexible frame in terms of model features and allowing to capture several physical phenomena, involving martensite reorientation, different kinetics between forward/reverse phase transformations, smooth thermomechanical response, low-stress phase transformations, as well as transformation-dependent elastic properties. The model and its numerical implementation are tested on several boundary-value problems.

Analysis of a model for adhesive contact in thermoviscoelasticity

Giovanna Bonfanti
DICATAM, University of Brescia (Italy), Italy
giovanna.bonfanti@unibs.it
Elena Bonetti, Riccarda Rossi

The present talk concerns the analysis of a model for adhesive contact between a thermoviscoelastic body and a rigid support. The related PDE system features the quasi-static momentum balance, the equation governing the evolution of a surface damage parameter and the equations for the temperature in the bulk domain and on the contact surface. The main difficulties connected to the analytical investigation are due to the presence of multivalued operators rendering physical constraints on the variables, as well as to the contact conditions, to the singular character of the temperature equations, and to the highly nonlinear coupling between the equations themselves. Global-in-time existence results are proved. The presented results have been obtained in collaboration with Elena Bonetti and Riccarda Rossi.

On the energy dissipation rate of solutions to the compressible isentropic Euler system

Elisabetta Chiodaroli
Ecole Polytechnique federale de Lausanne, Switzerland
elisabetta.chiodaroli@epfl.ch
Camillo De Lellis and Ondrej Kreml

In this talk we discuss the well-posedness problem for weak solutions of the compressible isentropic Euler system in 2 space dimensions with particular attention to the role of the maximal dissipation criterion proposed by Dafermos. The results we present in collaboration with C. De Lellis and O. Kreml are in the line with the program of investigating the efficiency of different selection criteria proposed in the literature in order to weed out nonphysical solutions to more-dimensional systems of conservation laws. Specifically we will illustrate how some nonstandard (i.e. constructed via convex integration methods) solutions to the Riemann problem for the isentropic Euler system in 2 space dimensions have greater energy dissipation rate than the classical self-similar solution emanating from the same Riemann data. We therefore show that the maximal dissipation criterion proposed by Dafermos does not favour in general the self-similar solutions.

Optimal control of Allen-Cahn equations with dynamic boundary conditions and singular potentials

Pierluigi Colli
University of Pavia, Italy
pierluigi.colli@unipv.it

The talk is concerned with an optimal control problems for an Allen-Cahn equation with nonlinear dynamic boundary condition involving the Laplace–Beltrami operator. The nonlinearities both in the bulk and on the boundary are assumed to be singular, i.e., they may range from the derivative of logarithmic potentials confined in $[-1, 1]$ to the subdifferential of the indicator function of the interval $[-1, +1]$ up to a concave perturbation. We address both the cases of distributed and boundary controls. We first examine the case of logarithmic nonlinearities: in a recent paper by Colli and Sprekels the corresponding control problems were studied, and results concerning existence and first-order necessary and second-order sufficient optimality conditions were shown. Then, in the case of double obstacle potentials, a recent joint work with M.

H. Farshbaf-Shaker and J. Sprekels investigated a "deep quench" approximation (i.e., approximating the indicator function by logarithmic nonlinearities) and led us to establish both the existence of optimal control and first-order necessary optimality conditions.

Stability issues in complete fluid systems

Eduard Feireisl
Czech Acad. Sci. Prague, Czech Rep
feireisl@math.cas.cz

We discuss the issues of stability of the set of solutions to the complete Navier-Stokes-Fourier system. The main topics include the weak-strong uniqueness, the relative entropies, ans stability of the associated numerical schemes.

An existence result for a model for solid-solid phase transition

Gianni Gilardi
University of Pavia, Italy
gianni.gilardi@unipv.it

A model for solid-solid phase transition has been recently introduced by Mauro Fabrizio et al. The physical quantities that are involved are the absolute temperature, the order parameter, and the scalar displacement along with the associated stress vector. In the corresponding PDE system, two parabolic equations and a second order hyperbolic equation are coupled. As the problem is highly nonlinear, even the existence of a solution is not clear at all, at first sight. The present talk provides the outline of an existence result recently obtained by the speaker in collaboration with Elena Bonetti, Pierluigi Colli and Mauro Fabrizio.

On the Γ-convergence of damage models to cohesive fracture models

Flaviana Iurlano
Bonn University, Italy
iurlano@iam.uni-bonn.de
Sergio Conti, Matteo Focardi

We obtain a cohesive fracture model as Γ-limit of damage models. The elastic coefficient in these damage models is computed from the damage variable v through a function f_k of the form $f_k(t) = \min\{1, \epsilon_k^{1/2} f(t)\}$, with f diverging for v close to the value describing undamaged material. The resulting fracture energy is linear in the opening s at small values of s and has a finite limit as $s \to \infty$, and can be determined by solving a one-dimensional vectorial optimal profile problem.

Global spatial regularity results for elasticity models with friction or damage

Dorothee Knees
Weierstrass Institute, Germany
knees@wias-berlin.de

For the analysis of strongly coupled material models it is useful to have deeper insight into the spatial regularity properties of the involved quantities like displacement fields or internal variables. In this lecture we will discuss some recent results for non-smooth situations with a special focus on Tresca friction models and on damage models as they will be investigated in the lecture by C. Zanini. The talk relies on joint results with R. Rossi (University of Brescia), C. Zanini (Politecnico di Torino) and A. Schroeder (University of Salzburg).

A thermomechanical mesoscopic model of SMA thin films

Martin Kruzik
Academy of Sciences of the Czech Republic, Czech Rep
kruzik@utia.cas.cz
B. Benesova, G. Patho

We design a new mesoscopic thin-film model for shape-memory materials which takes into account thermomechanical effects. Starting from a microscopic thermodynamical bulk model we perform a suitable dimension-reduction procedure followed by a scale transition valid for specimen large in area up to a limiting model which describes microstructure by means of parametrized measures. All our models obey the second law of thermodynamics and possess suitable weak solutions. This is shown for the resulting thin-film models by making the procedure described above mathematically rigorous. The main emphasize is, thus, put on modeling and mathematical treatment of joint interactions of mechanical and thermal effects accompanying phase transitions and on reduction of specimen dimensions and transition of material scales.

The Penrose-Fife phase-field system with dynamic boundary conditions

Alain Miranville
Universite de Poitiers, France
miranv@math.univ-poitiers.fr

Our aim in this talk is to discuss the well-posedness and longtime behavior of the Penrose-Fife system in phase transition with dynamic boundary conditions.

Free fall of one-dimensional bodies in hyperviscous fluids

Alessandro Musesti
Università Cattolica del Sacro Cuore, Italy
alessandro.musesti@unicatt.it
Giulio G. Giusteri, Alfredo Marzocchi

We discuss the free fall of very slender rigid bodies in a hyperviscous incompressible fluid, showing the global-in-time existence and uniqueness of the solution. Moreover, in the case of a low Reynolds number we study sufficient conditions on the geometry of the body in order to get purely translational steady motions. Finally, we address the issue of the free fall of an elastic beam.

Energy based BV evolutions: existence and convergence.

Matteo Negri
University of Pavia, Italy
matteo.negri@unipv.it

We consider rate-independent evolutions for energy functionals F of class C^1 in infinite dimensional reflexive separable Banach spaces. Evolutions are characterized by means of their graph parametrization (as in Efendiev & Mielke) in terms of a couple of equations which give stationarity and energy balance. Similarities and differences with the formulation of energetic and BV evolutions by Mielke will be shown. The proofs of existence are based on incremental problems based on the Euler scheme (both backward and forward), they share common features with the theory of minimizing movements for gradient flows and make reference to a suggested problem by De Giorgi. In the spirit of Sandier & Serfaty, considering a sequence of functionals F_n and its Γ-limit F we provide a convergence result for the associated quasi-static evolutions. An application to phase-field models for brittle fracture will be presented.

A singular heat equation with dynamic boundary conditions

Giulio Schimperna
University of Pavia, Italy
giusch04@unipv.it
Antonio Segatti, Sergey Zelilk

We address a singular parabolic equation of the form $\theta_t+\Delta\theta^{-1}=0$ on a smooth bounded domain $\Omega\subset\mathbf{R}^3$. The equation is complemented with a dynamic boundary condition of the form $\theta_t-\Delta_{\partial\Omega}\theta=\partial_\nu\theta^{-1}$ on $\partial\Omega$, where $\Delta_{\partial\Omega}$ is the Laplace-Beltrami operator and ν is the outer normal unit vector to $\partial\Omega$. We discuss existence, uniqueness, and regularity of solutions for the initial-value problem for this equation.

L^2-asymptotic stability of mild solutions to Navier-Stokes system in $\mathbb{R}^3$

Maria Schonbek
University of California Santa Cruz, USA
schonbek@ucsc.edu
Grzergorz Karch, Dominika Pilarczik

The classical theory of viscous incompressible fluid flow is governed by the celebrated Navier-Stokes equations. There are two main approaches for the construction of solutions to the Navier-Stokes equations. In the 1934 pioneering paper by Leray, weak solutions are obtained for all divergence free initial data $u_0\in L^2(\mathbb{R}^3)^3$ and $F=0$. The second approach leads to mild solutions. These solutions are given by an integral formulation using the Duhamel principle and they are obtained by means of the Banach contraction principle.
I will describe a link between these two approaches. That is I will show that the initial value problem with data $V(x,0)$ perturbed by an arbitrarily large divergence free L^2-vector field has a global-in-time weak solution in the sense of Leray and this weak solution converges as $t\to\infty$ in the energy L^2-norm towards the mild solution $V=V(x,t)$. That is sufficiently small mild solutions of the Navier-Stokes equations are asymptotically stable weak solutions under all divergence free initial perturbations from $L^2(\mathbb{R}^3)^3$.

The GENERIC approach to the Souza-Auricchio model for SMAs

Ulisse Stefanelli
University of Vienna, Austria
ulisse.stefanelli@univie.ac.at

I will discuss the opportunity of reformulating the celebrated Souza-Auricchio phenomenological model for shape-memory alloys within the GENERIC variational frame. In particular, the thermomechanical evolution of the medium is rephrased as a generalized gradient flow of its entropy. This variational structure can be efficiently exploited in connection with approximations. I will comment of this perspective both from the theoretical and the numerical viewpoint.

Viscous and rate-independent damage systems in non-smooth domains

Chiara Zanini
Politecnico di Torino, Italy
chiara.zanini@polito.it
Dorothee Knees, Riccarda Rossi

We address both a rate-independent system for damage, and its rate-dependent, or viscous, regularization. These systems consist of a system of elliptic PDEs that is coupled to a doubly nonlinear evolution equation of either rate-independent or parabolic type with velocity constraints. The analysis of the latter PDE system presents remarkable difficulties, due to its highly nonlinear character. We tackle it by combining a variational approach to a class of abstract doubly nonlinear evolution equations, with careful regularity estimates tailored to this specific system, relying on a q-Laplacian type gradient regularization of the damage variable. Hence for the viscous problem we conclude the existence of weak solutions, satisfying a suitable energy-dissipation inequality that is the starting point for the vanishing viscosity analysis. Then for the latter we obtain the global-in-time existence of weak solutions, exploiting refined regularity techniques which do not require a smooth domain. This talk is based on a joint collaboration with Dorothee Knees (WIAS, Berlin) and Riccarda Rossi (University of Brescia).

On defects with half-integer degree in the Q-tensor theory of nematic liquid crystals

Arghir Zarnescu
University of Sussex, England
A.Zarnescu@sussex.ac.uk
G. di Fratta, J. Robbins and V. Slastikov

Some of the most interesting aspects of nematic liquid crystals concern the defect patterns, and among these some of the most mysterious ones are the defects of index one-half (and higher), which cannot be described in the simpler, director theories, but only using a tensorial description. I will report on recent progress on their description and qualitative properties of them, in particular their local stability.

Special Session 32: Applied Analysis and Dynamics in Engineering and Sciences

Thomas Hagen, University of Memphis, USA
Florian Rupp, German University of Technology in Oman, Oman

The goal of this session is to bring together mathematicians who work in different areas of applied mathematics and might thus not meet and exchange ideas and points of view. Consequently, the session program addresses a cross section of theoretical and computational developments and their applications to fluid dynamics, solid mechanics and life sciences. Areas of analytical interest include the theory of linear/nonlinear differential equations, the qualitative behavior of solutions, stability and asymptotics, control-theoretic issues, and related aspects. The areas of application range from fluid dynamics and wave phenomena to industrial flows and applications in mathematical biology and material science. A key aspect of this session is its focus on the impact of theoretical results on the study of real-world problems.

Predator-Prey Interactions, Age Structures and Delay Equations

Maria Vittoria Barbarossa
University of Szeged, Hungary
barbarossamv@gmail.com
Marcel Mohr, Christina Kuttler

In the context of population dynamics, DDEs with constant delay can be obtained, e.g., from the balance laws of age-structured population dynamics, assuming that birth rates and death rates, as functions of age, are piece-wise constant. The delay arises naturally from biology as the age-at-maturity of individuals. A general framework for age-structured predator-prey systems is introduced. Individuals are distinguished into two classes, juveniles and adults, and several possible interactions are considered. The initial system of partial differential equations is reduced to a system of (neutral) delay differential equations with one or two delays. Thanks to this approach, physically correct models for predator-prey with delay are provided. Previous models are considered and analysed in view of the above results. A Rosenzweig-MacArthur model with delay is presented as an example. The basic model is finally extended to obtain a predator-prey model with SIS dynamics and delay.

Optimal Control in Free Boundary Fluid-Elasticity Interactions

Lorena Bociu
NC State University, USA
lvbociu@ncsu.edu
Jean-Paul Zolesio

The talk addresses the problem of minimizing turbulence inside fluid flow in the case of free boundary interaction between a viscous fluid (modeled by the Navier-Stokes equations) and a moving and deforming elastic body (modeled by the nonlinear equations of elastodynamics). Reducing and controlling turbulence flow is particularly relevant in the design of small-scale unmanned aircrafts, and is also of great interest in the medical community. The issue of minimizing vorticity in the flow is addressed from the point of view of optimal control. Due to the moving domains and the nonlinearity of the state equations, the optimality conditions must be derived from differentiability arguments that involve strategies from sensitivity and shape differentiability analysis.

Classes of operators on some spaces of analytic functions

Fernanda Botelho
University of Memphis, USA
mbotelho@memphis.edu
J. Jamison

In this talk we present a characterization of the surjective linear isometries and hermitian operators on some spaces of vector valued analytic functions . This talk is based on joint work with J. Jamison.

Existence and Regularity in the Oval Problem

Jochen Denzler
University of Tennessee, USA
denzler@math.utk.edu

The oval problem asks to determine, among all closed loops in $\mathbf{R}^n$ of fixed length, carrying a Schrödinger operator $\mathbf{H} = -\frac{d^2}{ds^2} + \kappa^2$ (with curvature κ and arclength s), those loops for which the principal eigenvalue of $\mathbf{H}$ is smallest. A 1-parameter family of ovals connecting the circle with a doubly traversed segment (digon) is conjectured to be the minimizer. Whereas this conjectured solution is an example that proves a lack of compactness and coercivity in the problem, it is proved in this talk (via a relaxed variation problem) that a minimizer exists; it is either the digon, or a strictly convex planar analytic curve with positive curvature. While the Euler-Lagrange equation of the problem appears daunting, its asymptotic analysis near a presumptive singularity gives useful information based on which a strong variation can exclude singular solutions as minimizers.

Sharp interface models for concrete carbonation

Jonathan Evans
University of Bath, England
masjde@bath.ac.uk

We investigate the fast-reaction asymptotics for a reaction-diffusion (RD) system describing the penetration of the carbonation reaction in concrete. The technique of matched-asymptotics is used to show that the RD system leads to two distinct classes of sharp-interface models, that correspond to different scalings in a small parameter ϵ representing the fast-reaction. We explore three conceptually different scaling regimes (in terms of the small parameter ϵ) of the effective diffusivities of the driving chemical species. The limiting models include one-phase and two-phase generalised Stefan moving-boundary problems as well as a nonstandard two-scale (micro-macro) moving-boundary problem – the main result.

Modelling rapid evolution in structured predator-prey systems

Jozsef Farkas
University of Stirling, Scotland
jozsef.farkas@stir.ac.uk
A. Yu Morozov

In this talk we will explore the mathematical properties of a predator-prey model, where the prey population is structured according to a certain life history trait. The trait distribution within the prey population in the model is the result of interplay between genetic inheritance and mutation, as well as selectivity in the consumption of the predator. The evolutionary processes are considered to take place on the same time scale as ecological dynamics, i.e. we consider the evolution to be rapid. We investigate the existence of a coexistence stationary state in the model and carry out stability analysis of this state. We establish a number of biologically significant results. Amongst them we prove that the coexistence stationary state is stable when the saturation in the predation term is low. For a class of kernels describing genetic inheritance and mutation we show that stability of the predator-prey interaction would require a selectivity of predation according to the life trait. Finally, we derive expressions for the Hopf-bifurcation curve which can be used for constructing bifurcation diagrams in the parameter space without the need for a direct numerical simulation of the underlying integro-differential equations. This talk is based on joint work with A. Yu Morozov (University of Leicester).

Biological principles

Tor Fla
University of Tromsoe, Norway
tor.fla@uit.no

The classical theory evolution of biological systems and populations is based on that robust and distinct, functional equlibria are selected for. Even the transition to the equilibria was supposed to be based on the so called minimum frustration principle and funneling landscape. The noise in these models are included in a thermodynamic, funneling landscape model for molecular folding/binding and an analogous landscape in (cell) populations. Recent research both on the molecular level, signalling/decision level of single cell and on the (cell) population level indicates that these landscape models are not necessarily minimal frustrated, but that quasistationary states modulated by noise change the properties of the biological system. One of the most popular approaches to include noise modulation are through bayesian prior statistics of the noise variance (superstatistics) which can be specialized to the Tsallis statistics. We formulate a two scale protein landscape model for protein folding which includes stochastics variations in the contact number both at a short range scale and a long range scale. We show that this lead to an interesting model for a nonextensive mean free energy difference which gives a possible bridge between ordered and disordered protein folding. Both a dynamic Langevin type model with multiplicative contact number noise and long range interactions modelled by a bayesian prior model are discussed. Consequences for the protein folding landscape in terms of nonextensive folding rates and stability are discussed. The evolutionary theory of stem cell decision rates is also discussed together with possible consequences for diseases.

Macroscopic interaction of small amplitude-modulated 3D water waves

Ioannis Giannoulis
University of Ioannina, Greece
giannoul@uoi.gr
Walter H. Aschbacher

Starting from the nonlinear 3D gravity water wave problem of finite depth (without surface tension), we consider two small amplitude-modulated plane wave solutions of the linearized problem with different phase in the asymptotic regime where the smallness parameter and the scaling coefficient of macroscopic time and space are the same. In order to observe the macroscopic interaction of two arbitrary carrier waves we derive the corresponding system of modulation equations up to second order in the scaling coefficient and discuss its justification.

Numerical methods to construct and verify Lyapunov functions in Dynamical Systems

Peter Giesl
University of Sussex, England
p.a.giesl@sussex.ac.uk
Sigurdur Hafstein

Lyapunov functions are an important tool to determine the basin of attraction of equilibria in Dynamical Systems. A Lyapunov function is a function which is decreasing along trajectories; sublevel sets of the Lyapunov function are subsets of the basin of attraction. The explicit construction of Lyapunov functions for a given system is a difficult problem. Recently, several numerical construction methods have been proposed, among them the RBF and CPA methods. In this talk, we will combine the advantages of both. The RBF method formulates the decreasing property as a linear PDE and solves it approximately using meshless collocation, in particular Radial Basis Functions (RBF). Error estimates show that the method always constructs a (smooth) Lyapunov function if the collocation points are dense enough and placed in the appropriate area. So far, however, the method lacks a verification of whether a given approximation is decreasing along trajectories. The CPA method triangulates the phase space and constructs a continuous Lyapunov function, which is piece-wise affine (CPA) on each simplex of the triangulation, using linear optimization. The method includes error estimates, which guarantee that the CPA function is indeed decreasing along trajectories. In this talk, we propose a combination of these two methods: we use the RBF method to construct a Lyapunov function. Then we interpolate this function and thus construct a CPA Lyapunov function. Checking a finite number of inequalities, we are able to verify that this interpolation is indeed decreasing along trajectories. Moreover, sublevel sets get arbitrarily close to the basin of attraction.

Mathematical Models related to Muscles and Biomechanics

Thomas Goetz
University Koblenz, Germany
goetz@uni-koblenz.de
Robert Rockenfeller

We will present models describing the activation and action of muscles in the human body. Nonlinear systems of differential equations relate the electrical stimulation of the muscle to its motion. These -often heuristic- models typically contain several phenomenological parameters, whose values vary widely throughout literature. To quantify the influence of the parameters, a sensitivity analysis is carried out and we present numerical results compared to available experimental data.

Dynamics in droplet networks

Thomas Hagen
University of Memphis, USA
thagen@memphis.edu

The Florida Palm Beetle has an interesting defense mechanism: When attacked, it exerts a strong adhesive force to suck itself to the ground. This adhesion mechanism consists of a series of connected, fluid-filled channels opening to the outside. The adhesive force is generated by controlling the flow through these channels. Inspired by this adhesion strategy, van Lengerich, Vogel and Steen (Physica D, 2009) studied networks of channels (pipes) filled with a Newtonian liquid. Each channel ends in a hole where the liquid forms a partial droplet due to interaction with the ambient air. The pressures acting on the droplets depends on the size of each droplet. Interesting dynamical behavior ensues. Pressure-driven, non-turbulent pipe flow is, of course, well-understood and adequately modeled by the Hagen-Poiseuille law in case of viscous liquids. In our investigation we will concentrate on shear-thinning behavior. In order to study the stability of steady droplet configurations, we devise Lyapunov and Chetaev function techniques since equilibria are generally not hyperbolic. Numerical simulation will be given in support of our conclusions. Part of this presentation is based on work done with Ben Jenkins.

Optimal control of system governed by the Gao beam equation

Jitka Machalova
Palacky University Olomouc, Czech Rep
jitka.machalova@upol.cz
Horymir Netuka

In our contribution we want to mathematically correct formulate and analyze several optimal control problems for the nonlinear beam which was introduced in 1996 by David Y. Gao. The beam model is given by a static nonlinear fourth-order differential equation with some boundary conditions. The beam is here subjected to a vertical load and possibly to an axial load as well. A cost functional we will construct in such a way that the lower its value is, the better model we obtain. We will study the existence and eventually the uniqueness for the solution of our control problems. Another task is to derive optimality conditions. Next we want to study the finite element approximation of the considered problems as exact solutions are not available. Finally we proceed to a numerical realization of our problems. Some results including numerical examples are offered to illustrate justification of our previous considerations.

RPDE-RODE Reduction for Earthquake-induced Oscillations of Solid Structures

Tobias Neckel
TU Muenchen, Germany
neckel@in.tum.de
Florian Rupp, Alfredo Parra

Partial differential equations (PDEs) are frequently used to model important phenomena. In many application scenarios, such as the top-layer thermal excitation of the earth's atmosphere or ground-motion excitation of multistorey buildings, PDEs are perturbed by random influences acting on the boundaries of the geometrical domain. The underlying random effects are often modeled as filtered white noise processes (coloured noise) or power law noises, resulting in the mathematical study of random partial differential equations (RPDEs). In this contribution, we focus on the theoretical and numerical properties of an approach tailored to boundary-excited problems: We apply a method of lines that reduces RPDEs with boundary noise to a finite-dimensional system of random ordinary differential equations (RODEs). The resulting RODEs can be solved efficiently by recent numerical schemes. Hence, this reduction provides a very useful alternative to traditional numerical schemes for boundary-excited RPDEs. We apply our RPDE-RODE reduction method to earthquake-induced oscillations of solid structures.

Solution of contact problem for nonlinear beam and elastic obstacle

Horymir Netuka
Palacky University Olomouc, Czech Rep
horymir.netuka@upol.cz
Jitka Machalova

This contribution will deal with the issue related to contact problems. We want to formulate, analyze and numerically solve a contact of a large deformed beam with an elastic obstacle. The beam model is governed by a nonlinear fourth-order differential equation developed by D.Y. Gao, while the obstacle is considered as the elastic foundation of the Winkler's type in some distance under the beam. The contact is static and modeled by using the contact conditions with normal compliance and without a friction. In contrast to usual formulations based on variational inequalities we can infer for our problem a nonlinear variational equation. The problem under consideration is then reformulated as an optimal control problem what is useful both for theoretical aspects as well as for solution methods. Discretization is based on using the mixed finite element method with independent discretization and interpolations for foundation and beam elements. Numerical examples demonstrate usefulness of the presented solution method. Results for the nonlinear Gao beam are compared with results for the classical Euler-Bernoulli beam model.

Bifurcation Patterns in a Generalized Stem Cell Cancer System with Signaling

Florian Rupp
German University of Technology in Oman (GUtech), Oman
fhhrupp@web.de
Tor Fla and Clemens Woywod

Based on a discrete Markovian birth-death model including regulated symmetric and asymmetric cell division, we formulate a continuous four-dimensional stochastic (ordinary) differential equation model for the dynamics of Chronic Myelogenous Leukaemia (CML) stem cells in a bone marrow niche involving signaling and competition between active stem cells. Invoking stochastic-deterministic correspondence we then investigate several subsystems. By totally analytic means we discuss the existence and stability of the equilibria of these systems in the deterministic small noise limit, and establish, by numerical means, connections between these classical results and the original stochastic setting. The robust, stable finite population equilibria can be interpreted as homeostatic equilibria of normal and leukaemic stem cell populations, in the case of the four-dimensional model for the scenario of treatment of the wild-type CML clone with a CML suppressing agent, e.g., imatinib, which leads to the emergence of a resistant CML strain. The four-dimensional model thus represents a common clinical picture.

Rogue waves and enhanced downshifting in a wind driven sea

Constance Schober
University of Central Florida, USA
drschober@gmail.com

In this paper we investigate the effects of nonlinear damping with and without linear damping/forcing in a sea state modeled by higher-order NLS in a two unstable mode regime. In particular, we are interested in how the linear term affects downshifting, rogue wave formation, and the number of rogue waves. We find that irreversible downshifting occurs when the nonlinear damping is the dominant damping effect. In particular, when only nonlinear damping is present, permanent downshifting occurs for all values of the nonlinear damping parameter β, appearing abruptly for larger values of β. We find that including linear damping weakens the nonlinear damping effect of downshifting while linear forcing enhances downshifting.

A Free Boundary Problem for MEMS

Christoph Walker
University of Hanover, Germany
walker@ifam.uni-hannover.de
Philippe Laurencot, Joachim Escher

Idealized microelectromechanical systems (MEMS) consist of a fixed ground plate above which a membrane is suspended that deforms due to a voltage difference that is applied between the two components. The mathematical model involves the harmonic electrostatic potential in the free domain between ground plate and membrane along with a singular evolution equation for the membrane displacement, the coupling term being the trace of the potential gradient on the membrane. The number of steady-state solutions and the possible phenomenon of a touchdown of the membrane on the ground plate are analyzed.

Enhanced biogas production in regions of bistability

Marion Weedermann
Dominican University, USA
mweederm@dom.edu
Gail Wolkowicz

We present a model for anaerobic digestion, a complex naturally occurring process during which organic matter is broken down to biogas and various byproducts in an oxygen-free environment. In waste treatment facilities the biogas is captured before it escapes into the atmosphere and can then be used as renewable energy. The model consists of differential equations describing the interactions of microbial populations involved in three of the four main stages of anaerobic digestion: acidogenesis, acetogenesis, and methanogenesis. Due to various inhibitory effects, the system possesses regions of bistability in parameter space. We argue that the highest biogas production is achieved in regions where control parameters are likely to give a bistable state. Surprisingly, the optimal biogas production does not always occur at a steady state where all the different classes of microorganisms coexist. In some regions of bistability biogas production occurs at only one of the steady states, while in others both steady-states result in biogas production with one state being more productive than the other. We show which control parameters and changes in initial conditions can move the system to or from the optimal state.

Flashover and flashback—fighting fires forth and back in time

Tobias Weinzierl
Durham University, England
tobias.weinzierl@mytum.de
Florian Rupp

In 2012, the Societe de Calcul Mathematique and the Federation Francaise des Jeux Mathematiques published a mathematical open competition riddling what the best way is to command fire fighting units over a vast spatial map over a long time period to fight wildland fire with minimal economic effort and maximal impact. While the original challenge narrative obviously roots in cellular games, one can translate the task into a coupled system of convection-diffusion partial differential equations modeling the fire spreading, time-dependent control functions modeling the fire fighters, and ordinary differential reaction equations modeling the burning. Within such a setting, the captain of the fire fighters faces an optimal control problem: His units have to minimize a cost functional. The present talk first applies standard finite element techniques on adaptive Cartesian meshes to transform the reaction-diffusion problem into a discretised one-step time stepping formulation, and it applies a textbook adjoint formulation to translate the impact of unit movements into defect variables yielding an optimal control. This gives a second parabolic differential equation running backward in time as well as an additional evolution equation per finite element. With a parallel, memory-efficient

realisation of simple relaxation solvers of all the equations on a space-time adaptive Cartesian grid, we are able to solve all equations simultaneously which in turn allows us to study some interesting techniques: local time stepping resulting from adaptivity in space-time grids, successive adoption of the control states in both space and time, and the multiscale interaction of control and the forward problem, e.g. Particular interesting is the opportunity to tailor hierarchically the unit movement in time to the fire spreading.

Exact and approximate wave packet dynamics with quantum trajectories

Clemens Woywod
University of Tromso, Norway
woywod@ch.tum.de

Quantum fluid dynamics (QFD) is an alternative to conventional methods for the solution of the Schroedinger equation for nuclear motion on one or more electronic potential energy surfaces. Integration of the QFD equations via time propagation of quantum trajectories corresponds to a calculation of the full nuclear wave function, the only approximation being discretization. The bottleneck of this approach is clearly the numerically difficult computation of the quantum force term. The performance of different implementations of QFD is analyzed. QFD is also interesting as starting point for the development of mixed quantum-classical methods. One particular version is applied to the dynamics of coupled anharmonic oscillators and compared to the mean field ansatz.

Special Session 33: Bifurcations and Asymptotic Analysis of Solutions of Nonlinear Models

Jann-Long Chern, National Central University, Taiwan
Yoshio Yamada, Waseda University, Japan
Shoji Yotsutani, Ryukoku University, Japan

The aim of this special session is to exchange recent results, ideas and techniques on nonlinear elliptic and parabolic PDEs, including reaction-diffusion systems and free boundary problems, from mathematical physics, chemical reactions, mathematical biology, medical science and some other fields. In particular, we are interested in the global bifurcation structure for such models. Combinations of numerical simulations and theoretical approaches with asymptotic analysis will be very useful to understand the nonlinear phenomena together with underlying structure of solutions. We will give opportunities to both established and junior researchers working in the related area to present their recent results.

Non-compact global attractor structures for PDEs with infinite-time blow-up

Nitsan Ben-Gal
Institute for Mathematics and its Applications (IMA), USA
ben-gal@ima.umn.edu
Kristen Moore, Juliette Hell

We consider reaction-diffusion equations characterized by the presence of infinite-time blow-up and the absence of finite-time blow-up. Such PDEs are ensured non-compact global attractors, but the structural decomposition of these attractors has been an open problem until recently. We present results on how to use bifurcation structure and asymptotic analysis to determine the structure of the non-compact global attractor, including in the case of asymptotically asymmetric growth rates.

Uniqueness of Topological Multivortex Solutions for the Self-Dual Maxwell-Chern-Simons U(1) Model

Zhi-you Chen
National Central University, Taiwan
zhiyou@math.ncu.edu.tw
Jann-Long Chern

n this paper, we prove the uniqueness of topological multivortex solutions for the self-dual Maxwell-Chern-Simons $U(1)$ model if the Chern-Simons coupling parameter is ufficiently large and the charge of electron is sufficiently small or large. On the other hand, we also establish the sharp region of the flux for non-topological solutions and provide the classification of radial solutions of all types in the case of one vortex point.

Caffarelli-Kohn-Nirenberg inequality with boundary singularities

Jann-long Chern
National Central University, Taiwan
chern@math.ncu.edu.tw
X. Fang and C. Hsia

In this talk, we are interested in how the geometry of boundary singularities can affect the attainability of the respective best Caffarelli-Kohn-Nirenberg and Hardy-Sobolev constant.

A Phase Field Limit of Grouting Models

Mykola Demchuk
National University of Water Management and Natural Resources Use, Ukraine
nbdemch@gmail.com
N. Saiyouri and E. Vashai

For several sophisticated grouting models that assume different set ups, it is checked numerically that the ratio of the distance covered by the injection front to the width of the zone of the transition from the soil with maximal value of cement concentration in the liquid phase to the soil where this concentration is negligible increases with time at sufficiently high injection pressure.

Influence of nonlinear incidence rates with delays for asymptotic stability of epidemic models

Yoichi Enatsu
University of Tokyo, Japan
yenatsu@ms.u-tokyo.ac.jp

We present recent results for the asymptotic behavior of positive solutions of delayed epidemic models with a class of nonlinear incidence rates. Here the delay denotes the length of incubation period in the vector population. By means of a threshold parameter R_0, known as the basic reproduction number, we establish a characterization for the incidence rate, which shows that non-monotonicity with delay in the incidence rate is necessary for destabilization of an endemic equilibrium E_*. This enables us to improve a stability condition obtained in Y. Yang and D. Xiao (2010). It is proven that as we increase the value of a parameter measuring saturation effect, the number of infective individuals at the endemic steady state decreases, while the equilibrium can be unstable via Hopf bifurcation. Two-parameter plane analysis together with an application of the implicit function theorem facilitates us to obtain an exact stability condition.

On the time periodic solutions and the asymptotic stabilities of GFDs and the related equations

Chun-Hsiung Hsia
National Taiwan University, Taiwan
willhsia@math.ntu.edu.tw
Ming-Cheng Shiue, Bongsuk Kwon, Chang-Yeol Jung

In this lecture, we shall introduce time periodic solutions of differential equations arised from different occasions. We then turn our attention to the GFDs and viscous Burgers' type equations and study the asymptic stabilities of the solutions. In particular, two different approaches will be introduced to prove the existence of time periodic solutions of GFDs with time periodic focrings.

A new geometrical concept in the finding bifurcations. Generalized Collatz-Wielandt formula

Yavdat Ilyasov
Institute of Mathematics RAS, Ufa, Russia, Russia
ilyasov02@gmail.com

We present a method for the finding bifurcations of turning point type for nonlinear equations, including partial differential systems. The method is based on a generalization of the Collatz-Wielandt formula. A number of examples, including system of elliptic and parabolic problems with p- Laplacian, will be discussed. In addition, we present a numerical algorithm for the computation of the turning point, which is actually a new geometrical concept in the theory of numerical calculation of the bifurcation points.

Behavior of solutions to degenerate Keller-Segel systems

Sachiko Ishida
Tokyo University of Science, Japan
s-ishida@rs.tus.ac.jp

As to the power-type degenerate Keller-Segel system, we know that there is a possibly grow-up solution in the subcritical case and that there is a bounded solution with the small initial data and an unbounded solution in finite or infinite time with the large negative energy initial data in the super-critical case. However, from research on the non-degenerate Keller-Segel system, we expect that the solution is bounded in the sub-critical case and moreover that the solution blows up in finite-time for the large negative energy initial data in the super-critical case. So, we'd like to discuss these themes in this talk.

Singular solutions to a nonlinear elliptic equation on the whole sphere

Yoshitsugu Kabeya
Osaka Prefecture Univeristy, Japan
kabeya@ms.osakafu-u.ac.jp

We consider a nonlinear elliptic equation on the whole sphere and discuss properties of singular solutions to the equation. We show that properties of singular solutions are quite different from those on the whole Euclidean space.

Free boundary problems modeling the spreading of species in multi-dimensional domains

Yuki Kaneko
Waseda University, Japan
kaneko.y5oda@toki.waseda.jp
Yoshio Yamada

We discuss free boundary problems for reaction-diffusion equations in multi-dimensions, where unknown functions are the population density of invasive or new species and the spreading front of the species which is represented as a free boundary. Such a model was first proposed by Du-Lin (2010) in one dimension and a multi-dimensional case was studied by Du-Guo (2012). We consider both radially and non-radially symmetric solutions for the free boundary problems which admit exterior domains. I will present some results on the unique existence of solutions and asymptotic behaviors of solutions as $t \to \infty$.

On a shadow system of the Lotka-Volterra competition model with cross-diffusion

Kousuke Kuto
University of Electro-Communications, Japan
kuto@e-one.uec.ac.jp

This talk is concerned with the limiting behavior of the coexistence steady-states to the Lotka-Volterra competition model as one of the cross-diffusion terms tends to infinity. Under the Neumann boundary condition, Lou-Ni (1999) derived a couple of shadow systems which characterize the limiting behaviors of the coexistence steady-states. One of the shadow system charcterizing the segregation of the competing species has been studied by Lou-Ni-Yotsutani and the detailed bifurcation structure for the 1D case was revealed. This talk focuses on the other shadow system characterizing the shrinking of the species not endowed with the cross-diffusion effect. The bifurcation structure of the positive solutions to the shadow system under the Dirichlet/Neumann boundary condition will be stated.

Spreading speed and sharp asymptotic profiles of solutions in free boundary problems for nonlinear advection-diffusion equations

Hiroshi Matsuzawa
Numazu National College of Technology, Japan
hmatsu@numazu-ct.ac.jp
Yuki Kaneko

In this talk, we concern with a free boundary problem of nonlinear advection-diffusion problem in one space dimension. We assume that the nonlinear term is monostable, bistable or combustion type. Such problems may be used to describe the spreading of a biological or chemical species under an advective environment. Du and Lou(to appear) considered the problem without advection term and studied the long-time dynamical behavior of solutions (**spreading** and **vanishing**) and determined the asymptotic spreading speed of the free boundaries when spreading happens. Du, Matsuzawa and Zhou (2014) obtained a sharper estimate for the spreading speed of the fronts than that in [Du-Lou], and they show that the solution approaches the semi-wave when spreading happens. For the problem with the advection term, Gu, Lin and Lou(to appear) considered the problem in the case where the nonlinear term is the logistic type and showed that when spreading happens, the rightward and leftward asymptotic spreading speeds are different due to the advection term. The aim of this talk is to give a much sharper estimate for the spreading speed of the fronts than that in [Gu-Lin-Lou], and obtain how the solution approaches the semi-wave when spreading happens.

Computer-assisted analysis on a four-leaf orbit of Craik's 3D dynamical system

Tomoyuki Miyaji
Kyoto University, Japan
tmiyaji@kurims.kyoto-u.ac.jp

The following system of equations is studied:

$$\begin{cases} \dot{x} = ayz + bz + cy \\ \dot{y} = dzx + ex + fy \\ \dot{z} = gxy + hy + kx, \end{cases}$$

where $x(t)$, $y(t)$, and $z(t)$ are real-valued functions, $\dot{x}$, $\dot{y}$, and $\dot{z}$ are their derivatives with respect to the independent variable t, and the coefficients a to k are real constants. This system arises several contexts in mechanics and fluid mechanics. Especially, Craik has shown that the equations of the form describe a class of exact solutions of the full incompressible Navier-Stokes equations. Most of solution orbits for the system are unbounded. We can, however, observe characteristic behavior. A typical solution orbit draws a helical curve, which changes amplitude in a vicinity of the origin. Some solutions change only the amplitude, while some solutions change not only the amplitude but also the axis along which they go to infinity as $t \to \infty$. Craik and Okamoto have found a four-leaf structure and a periodic orbit, which play an important role in controlling the solution orbits. We prove the existence and bifurcation of such a periodic orbit by a method of numerical verification.

Global bifurcation analysis of wave-pinning in a reaction-diffusion model for cell polarization

Tatsuki Mori
Ryukoku University, Japan
t13d003@mail.ryukoku.ac.jp
K. Kuto, T. Tsujikawa, M. Nagayama, S. Yotsutani

We are interesting in wave-pinning in a reaction-diffusion model for cell polarization proposed by Y. Mori, A. Jilkine and L. Edelstein-Keshet in SIAM J. Appl. Math (2011). Wave-pinning means a phenomenon that a wave of activation of one of the species is initiated at one end of the domain, moves into the domain, decelerates, and eventually stops inside the domain, forming a stationary front. Several mathematical bifurcation results of stationary solutions are obtained by Kuto and Tsujikawa in DCDS Supplement (2013). We propose a new method to represent a bifurcation sheet of a shadow-system. It determines the global bifurcation structure of stationary solutions of the shadow-system completely including even secondary bifurcation branches. Moreover, we numerically investigate the global bifurcation structure and stability of the original reaction-diffusion model to understand the wave-pinning.

Exact solutions for the derivative nonlinear Schrödinger equation with periodic boundary condition

Minoru Murai
Ryukoku University, Japan
a96249@mail.ryukoku.ac.jp
K. Sakamoto, S. Yotsutani

We are interested in exact solutions for the derivative nonlinear Schrödinger equation with periodic boundary condition. We investigate solutions $u(x,t) = \mathrm{e}^{-i\omega t}\phi(x-ct)$ with $\omega \in \mathbf{R}$ and $c \in \mathbf{R}$. The equation is reduced to the nonlocal nonlinear second order differential equation by introducing the polar coordinate. We will talk about representation theorem and the global structure for the equation.

The collective motion of camphor papers in a cylindrical channel

Masaharu Nagayama
Hokkaido University, Japan
nagayama@es.hokudai.ac.jp
Ken Wakai, Yasuaki Kobayashi, Kei Nishi, Yumihiko Ikura and Satoshi Nakata

Billiard and jamming like motions of camphor papers placed over water have recently been observed in cylindrical channels. We investigate the mechanisms of these motions by constructing a mathematical model for the camphor system. In particular, we study the motion of two camphor papers by means of numerical simulation and mathematical analysis. As a result of our investigations, we have uncovered various morphologies of the camphor paper motions. Moreover, we were able to obtain the existence and stability of rotational and cluster motions numerically, by means of computer aided analysis. We have found that the billiard-like phenomenon is caused by the coexistence of uniform rotational and symmetrical oscillatory motions, and that the jamming phenomenon arises from a Hopf bifurcation of the uniform rotating motion.

On the long-time stability of implicit Euler schemes for Primitive equations

Ming-Cheng Shiue
National Chiao-Tung University, Taiwan
mingcheng.shiue@gmail.com
Chun-Hsiung Hsia

In this talk, we consider the three dimensional viscous primitive equations which are governing equations of geophysical fluid dynamics. We study the implicit Euler approximation for the three dimensional viscous Primitive equations. The long-time stability for this approximation can be proven provided the initial data is small.

The basis property of generalized Jacobian elliptic functions

Shingo Takeuchi
Shibaura Institute of Technology, Japan
shingo@shibaura-it.ac.jp

The Jacobian elliptic functions are generalized to functions including the generalized trigonometric functions. This talk deals with the basis property of the sequence of generalized Jacobian elliptic functions in any Lebesgue space. In particular, it is shown that the sequence of the classical Jacobian elliptic functions is a basis in any Lebesgue space if the modulus k satisfies $0 \leq k \leq 0.99$.

Classification and sharp range of flux-pairs for radial solutions to a coupled Liouville-type system

Yong-Li Tang
National Central University, Taiwan
tangyl@math.ncu.edu.tw
Zhi-You Chen

In this talk, we study a coupled system of two nonlinear partial differential equations in the plane, which is related to Liouville equations, non-abelian Higgs BPS vortex equations or two Higgs electroweak model, with singularities at the origin. In addition to deriving the uniqueness of the so-called topological solutions, we also clarify the structure of all types of solutions, including blow-up ones, under various conditions on coefficients and parameters appearing in the system. Furthermore, the sharp range of flux-pairs associated with specific types of solutions is considered as well.

Traveling waves and their stability of a tumour growth model

Tohru Wakasa
Kyushu Institute of Technology, Japan
wakasa@mns.kyutech.ac.jp
Michiel Bertsch, Danielle Hilhorst, Hirofumi Izuhara and Masayasu Mimura

In 2010 Bertsch, Dal-Passo and Mimura has proposed a tumour growth model for populations of normal and abnormal cells. It is known that this model admit a segregated traveling wave solution, which implies contact inhibition of cells. On the other hand, it also admits a usual smooth traveling wave solutions under a suitable choice of parameters in the equations. In this talk we fill focus on the smooth traveling wave solutions and discuss their stability.

Blow-up in H^1-norm for a special type of complex Ginzburg-Landau equations

Tomomi Yokota
Tokyo University of Science, Japan
yokota@rs.kagu.tus.ac.jp
Takuya Tomidokoro

This talk investigates the blow-up of solutions to complex Ginzburg-Landau equations of the special form $u_t = e^{i\theta}\Delta u + e^{i\theta}|u|^\alpha u$, $x \in \mathbb{R}^N$, $t > 0$, where $\alpha > 0$ and $-\pi/2 < \theta < \pi/2$. Cazenave, Dickstein and Weissler (2013) proved that the solution blows up in L^2-norm in finite time for negative energy initial data. The purpose of this talk is to show that the solution blows up in H^1-norm in finite time.

Special Session 34: Variational Methods for Discrete and Continuous Boundary Value Problems (With Applications)

Antonio Iannizzotto, University of Verona, Italy
Giovanni Molica Bisci, University of Reggio Calabria, Italy
Vicentiu D. Radulescu, University of Craiova, Romania

This Special Session is intended to develop some recent advances in the qualitative theory of discrete and continuous nonlinear problems, with a particular emphasis to the role of variational methods in the mathematical description of these models. The abstract results will be illustrated by relevant applications in mathematical physics, mechanics, non-Newtonian fluids, and other fields.

A sublinear elliptic problem with nonlinearity indefinite in sign

Giovanni Anello
Messina University, Italy
ganello@unime.it

Let Ω be a nonempty connected bounded open set in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. Moreover, let $s \in]1,2[$ and $r \in]1,s[$. We present some existence and multiplicity results of nonzero and nonnegative solutions for the following elliptic problem involving a nonlinearity indefinite in sing

$$\begin{cases} -\Delta u = \lambda u^{s-1} - u^{r-1} & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega. \end{cases}$$

Here, λ is a positive parameter. Particular emphasis is devoted to the question of finding positive solutions to the above problem. The main difficulty, in dealing with this question, comes from the particular structure of the right hand side which prevents us to use the classical Strong Maximum Principle in order to obtain the positivity of every nonzero and nonnegative solution. Further results related to the singular case $r \in]0,1[$ are also presented jointly to some open problems concerning both the singular and non-singular case.

Existence results for quasilinear elliptic eigenvalue problems in unbounded domains

Giuseppina Autuori
Polytechnical University of Marche, Italy
autuori@dipmat.univpm.it
Patrizia Pucci, Csaba Varga

In this talk we shall present existence results for a nonlinear eigenvalue problem depending on a real parameter λ, driven by a weighted p-Laplacian operator and involving subcritical nonlinearities in unbounded domains, under Robin boundary conditions. Different approaches and methods will be presented, according to the size λ as well as to the assumptions on the nonlinearities, which could be either of Ambrosetti-Rabinowitz type or of Szulkin-Weth type.

Nontrivial solutions for p-Laplace equations

Rossella Bartolo
Polytechnic of Bari, Italy
r.bartolo@poliba.it

We study on bounded domains of $\mathbf{R}^N$ the multiplicity of weak solutions of quasilinear elliptic problem involving the p-Laplace operator when the nonlinearity behaves as $|u|^{p-2}u$ at infinity. Both the non–resonant and the resonant case are analyzed. Furthermore, we consider an analogous problem on $\mathbf{R}^N$.

Variable exponent problems involving generalized operators

Maria-Magdalena Boureanu
University of Craiova, Romania, Romania
mmboureanu@yahoo.com

The study of problems with variable exponents is becoming more and more popular due to a wide range of applications to various domains. In this context, we are concerned with elliptic problems involving generalized operators that are related to two well known classes of operators, that is, the Laplace-type operators and the mean curvature-type operators. Our discussion is conducted in the framework of the spaces with variable exponents and the main argumentation is made by means of the critical point theory.

Acknowledgement
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Multiplicity results for fourth-order nonlinear problems

Alberto Cabada
University of Santiago de Compostela, Spain, Spain
alberto.cabada@usc.es

In this talk, we consider a fourth order two-point boundary value problem describing elastic deflections of a rod with clamped ends, depending on two real parameters. We prove that there exists a set of parameters for which the problem has at least two nontrivial solutions. The proof is based on a recent three critical point theorem due to Cabada and Tersian.

Singular nonlinearities in PDE and applications to MEMS

Daniele Cassani
Università degli Studi dell'Insubria, Italy
daniele.cassani@uninsubria.it

Nonlinear eigenvalue problems in which the nonlinearity may develop singularities come up in a natural fashion in designing micro electromechanical actuators. We present existence and regularity results of stationary as well as dynamic solutions to nonlocal MEMS equations.

Positive periodic solutions for a second order ODE related with valveless pumping

Jose Angel Cid Araujo
University of Vigo, Spain
angelcid@uvigo.es
G. Infante, M. Tvrdy and M. Zima

We will give sufficient conditions for the existence of positive periodic solutions for $x''(t) + ax'(t) = r(t)x^{\alpha}(t) - s(t)x^{\beta}(t)$ by means of topological arguments. Our main motivation comes from a problem related with the valveless pumping phenomenon [see J. A. Cid, G. Propst and M. Tvrdy, On the pumping effect in a pipe/tank flow configuration with friction, to appear in Phys. D].

Phase transitions in almost Kähler manifolds and boundary energies

Eleonora Cinti
Universitá di Bologna, Italy
eleonora.cinti5@unibo.it
B.Franchi, M.d.M. Gonzalez

In this talk I will present a Γ-convergence type result for a boundary phase transition model in an almost Kähler manifold M. The functional that we consider involves a Dirichlet energy in the interior of the manifold and a double-well potential term on its boundary. Locally, and close to the boundary, the manifold M is diffeomorphic to $\mathbb{H}^n \times \mathbb{R}$, and therefore the model case for our functional is of the form

$$\int_{\mathbb{H}^n \times \mathbb{R}} (|\nabla_{\mathbb{H}} u(\xi, z)|^2 + |\partial_z u|^2) d\xi dz + \int_{\mathbb{H}^n} W(u) d\xi,$$

where $(\xi, z) \in \mathbb{H}^n \times \mathbb{R}$ and $\nabla_{\mathbb{H}}$ denotes the horizontal gradient in $\mathbb{H}^n$. In our main result we establish that, after a suitable rescaling, this energy functional Γ-converges to the (intrinsic) perimeter functional on the boundary of M. This is the analogue, for almost Kähler manifolds, of a (Euclidean) Γ-convergence type result by Alberti, Bouchitté and Seppecher.

Positive solutions to some systems of coupled nonlinear Schrodinger equations

Eduardo Colorado
Carlos III de Madrid University, Spain
ecolorad@math.uc3m.es

We study the existence of positive ground state solutions for a system of more than two coupled nonlinear time-independent Schrodinger equations. Precisely, we will show that there exists positive (in whole the component) standing wave solutions which are ground state for suitable conditions on the coupled parameter.

On an inequality for absolutely continuous functions with an application to a class of non-local B.V.P.'s

Francesca Faraci
Catania University, Italy
ffaraci@dmi.unict.it
D. Puglisi

In the present talk we show an inequality for absolutely continuous functions with a suitable summability property. As an application, we prove a multiplicity result for a non-local boundary value problem in a bounded domain of R^N involving an exponential nonlinearity. The approach is variational and combines results from critical point theory.

Global Solvability of Cauchy-Dirichlet Problem for Fully Nonlinear Parabolic Systems

Luisa Fattorusso
University of Reggio Calabria, Italy
luisa.fattorusso@unirc.it
Antonio Tarsia

By using Campanato's near operator theory we show some existence results of global strong solutions for a suitable class of Cauchy-Dirichlet problems.

Symmetry and multiple solutions for certain quasilinear elliptic equations

Roberta Filippucci
University of Perugia, Italy
roberta.filippucci@unipg.it
P. Pucci and C. Varga

In this talk we present a symmetric version of the Pucci and Serrin three critical points theorem, which we apply to an abstract eigenvalue problem in order to show the existence of three different symmetric solutions. Furthermore we illustrate the existence of nontrivial nonnegative solutions, which are invariant by k-spherical cap symmetrization, of quasilinear elliptic Dirichlet problems in either a ball of $\mathbb{R}^N$ or an annulus of $\mathbb{R}^N$, both centered at 0.

The geodesic connectedness problem in Lorentzian manifolds: variational methods

Jose Luis Flores
Universidad de Malaga, Spain
floresj@uma.es

Given a Lorentzian manifold (M,g), the geodesic connectedness problem consists on searching some geodesic connecting any two prescribed points of the manifold. In the last decades, variational methods and critical point theory have turned to be extremely useful for solving this problem in many cases. In this talk we will briefly review this topic, illustrating it with some recent results.

On anisotropic Neumann problems with indefinite potential

Genni Fragnelli
University of Bari, Italy
genni.fragnelli@uniba.it
Dimitri Mugnai

We consider nonlinear Neumann problems driven by the $p(x)$–Laplacian plus an indefinite potential. First we develop the spectral properties of such differential operators. Then, using variational methods, we prove existence results.

Measure Type and L^p Lagrange Multipliers in Elastic-Plastic Torsion

Sofia Giuffré
Mediterranea University of Reggio Calabria, Italy
sofia.giuffre@unirc.it
Antonino Maugeri, Daniele Puglisi

Aim of the talk is to present some new results concerning the existence of Lagrange multipliers in elastic-plastic torsion. Moreover, the relationships between elastic-plastic torsion problem and the obstacle problem are investigated. Finally, an example of the so-called "Von Mises functions" is provided.

Nehari manifolds for positive and compact support solutions to some semilinear elliptic problems related with nonlinear Schrödinger equations

Jesus Hernandez
Universidad Autonoma de Madrid, Spain
jesus.hernandez@uam.es
J.I.Díaz and Y. Ilyasov

We study existence of positive and compact support solutions to a class of semilinear elliptic equations related with nonlinear Schrödinger equations. First we prove existence of non-negative solutions by using a Nehari manifold argument. We also prove that solutions are actually positive for a bounded interval of values of a parameter and then study the existence of compact support solutions with the help of a Pohozaev identity. Bifurcation at infinity and the asymptotic behavior of the solution branches are studied as well. This is joint work with J.I.Díaz and Y. Ilyasov.

Discrete Dirichlet Problems and Nonlinear Algebraic Systems

Maurizio Imbesi
University of Messina, Italy
maurizio.imbesi@unime.it
Giovanni Molica Bisci

In this talk we are interested in the existence of infinitely many solutions for a partial discrete Dirichlet problem depending on a real parameter. More precisely, we determine unbounded intervals of parameters such that the treated problems admit either an unbounded sequence of solutions, provided that the nonlinearity has a suitable behaviour at infinity, or a pairwise distinct sequence of solutions that strongly converges to zero if a similar behaviour occurs at zero. Finally, the attained solutions are positive when the nonlinearity is supposed to be nonnegative thanks to a discrete maximum principle.

Domain perturbation problems for the Hardy constant

Pier Domenico Lamberti
University of Padova, Italy
lamberti@math.unipd.it
Gerassimos Barbatis and Pier Domenico Lamberti

We consider the celebrated L^p-Hardy inequality involving the distance d_Ω to the boundary of a domain Ω in $\mathbb{R}^n$

$$\int_\Omega |\nabla u|^p dx \geq c \int_\Omega \frac{|u|^p}{d_\Omega^p} dx\,, \text{ for all } u \in C_c^\infty(\Omega).$$

The L^p-Hardy constant is the best constant in the inequality above and is denoted by $H_p(\Omega)$. We study the dependence of $H_p(\Omega)$ upon perturbation of Ω and we prove stability results. Since for convex domains it is well-known that $H_p(\Omega) = ((p-1)/p)^p$, the focus is mainly on nonconvex domains.

Existence and multiplicity of self-similar solutions for the heat equation with indefinite weight functions

Lin Li
Southwest University, China, Peoples Rep of China
lilin420@gmail.com
Chun-Lei Tang

In this talk, we introduce the Nehari manifold for selfsilimar solution of the heat equation. We use it to establish the existence of solutions and multiple solutions for some nonlinear elliptic problem with sign-changing weight with subcritical growth and critical growth, respectively. By pass, we also establish some bifurcation results, nonexistence results and near resonance results.

Some remarks about singularly perturbed elliptic problems with nonautonomous nonlinearities

Eugenio Montefusco
Roma University, Italy
montefusco@mat.uniroma1.it
Liliane Maia, Benedetta Pellacci

We consider some class of singularly perturbed elliptic problems with nonhomogeneous nonlinearities. We investigate the existence of nontrivial positive solutions concentrating around points or curves.

Wang's multiplicity result for superlinear $(p, q)-$equations without the Ambrosetti-Rabinowitz condition

Dimitri Mugnai
University of Perugia, Italy
dimitri.mugnai@unipg.it
Nikolaos S. Papageorgiou

We consider a nonlinear elliptic equation driven by the sum of a p–Laplacian and a q–Laplacian with a nonlinear term which doesn't satisfy the usual Ambrosetti–Rabinowitz condition. Using variational methods based on critical point theory together with techniques from Morse theory we show that the problem has at least three nontrivial solutions.

Optimal profiles in a phase-transition model with a saturating flux

Franco Obersnel
Università di Trieste, Italy
obersnel@units.it
Denis Bonheure and Pierpaolo Omari

We prove the one-dimensional character of minimizers of the relaxation $\mathcal{J}_\omega$ of the functional

$$\int_{\mathbb{R}\times\omega}(\sqrt{1+|\nabla v|^2}-1)\,dx+\int_{\mathbb{R}\times\omega}F(v)\,dx$$

on the set $\mathcal{E}_\omega$ of all functions $v : \mathbb{R}\times\omega\to\mathbb{R}$ having bounded variation in any cylinder $]-T,T[\times\omega$ and satisfying

$$\lim_{x_1\to\pm\infty} v(x_1,x_2,\ldots,x_N)=\pm 1$$

uniformly a.e. with respect to $(x_2,\ldots,x_N)\in\omega$. Here $\omega\subset\mathbb{R}^{N-1}$ is an open bounded set and $F:\mathbb{R}\to[0,+\infty[$ is a double-well potential, a typical example being $F(s)=\frac{1}{4}(1-s^2)^2$. The result gives a partial positive answers to Gibbons' conjecture with the Laplace operator replaced by the curvature operator (Gibbon's conjecture is a variant of De Giorgi's conjecture on the rigidity of the solutions of the stationary Allen-Cahn equation $\Delta v = F'(v)$ in $\mathbb{R}^N$). Minimizers of $\mathcal{J}_\omega$ represent the transitions between phase states of a substance within the van der Waals-Cahn-Hilliard gradient theory of phase transitions, when the model is characterized for small gradients by linear gradient-flux relations and the increase of gradients is expected to slow down and ultimately to approach saturation at large gradients.

Neumann boundary value problems under the action of fractional diffusion

Benedetta Pellacci
Universita di Napoli Parthenope, Italy
pellacci@uniparthenope.it
Eugenio Monefusco, Gianmaria Verzini

The spectral square root of the Laplacian in bounded domains with Neumann homogeneous boundary conditions will be introduced and studied. Existence and uniqueness results for positive solutions are investigated in the case of indefinite nonlinearities of logistic type by means of bifurcation theory.

Existence and regularity of higher critical points in elliptic free boundary problems

Kanishka Perera
Florida Institute of Technology, USA
kperera@fit.edu
David Jerison

Existence and regularity of minimizers in elliptic free boundary problems have been extensively studied in the literature. We initiate the corresponding study of higher critical points by considering a superlinear free boundary problem related to plasma confinement. The associated energy functional is nondifferentiable and therefore standard variational methods cannot be used directly to prove the existence of critical points. First we obtain a nontrivial generalized solution u of mountain pass type as the limit of mountain pass points of a suitable sequence of C^1-functionals approximating the energy. Next we show that u minimizes the energy on the associated Nehari manifold and use this fact to prove that it is nondegenerate. Finally we use the nondegeneracy of u to show that it satisfies the free boundary condition in the viscosity sense and that near regular points the free boundary is a smooth surface and hence this condition holds in the classical sense.

Critical stationary Kirchhoff equations in $\mathbb{R}^N$ involving nonlocal operators

Patrizia Pucci
University of Perugia, Italy
patrizia.pucci@unipg.it
S. Saldi

Lately Fiscella and Valdinoci in [*A critical Kirchhoff type problem involving a nonlocal operator*, Nonlinear Anal. **94** (2014), 156–170] proposed a stationary Kirchhoff variational model, in bounded regular domains of $\mathbb{R}^N$, which takes into account the *nonlocal* aspect of the tension arising from nonlocal measurements of the fractional length of the string. In this talk, inspired by the above paper and by recent results of the speaker, we extend in several directions the above paper in the entire $\mathbb{R}^N$. Since interesting questions arise from the search of nontrivial non–negative (weak) solutions, we deal with existence and multiplicity of nontrivial non–negative entire solutions of a Kirchhoff eigenvalue problem, involving critical non–linearities and nonlocal elliptic operators for equations for equations in $\mathbb{R}^N$.

On an initial value problem modeling evolution and selection in living systems

Maria Cesarina Salvatori
University of Perugia, Italy
mariacesarina.salvatori@unipg.it
Patrizia Pucci

Evolution and mutation phenomena have an important role in developmental biology and life sciences in general. Recently the study and modeling of these phenomena became one of the most challenging new frontiers of applied mathematics. We obtain and study a new broad class of nonlinear integro-differential equations, derived by the mathematical tools of the kinetic theory for active particles, suitable to model the dynamics of large interacting populations. These equations, which offer the basis for the derivation of specific examples, can model mutations, namely generation of new populations with a phenotype different form the origin genotype that could be more (or less) fitted to an environment that evolves in time with known dynamics, and subsequently proliferative/distructive events that can bring to growth of new populations and, in some cases, of extinction events. We present a detailed qualitative analysis of the related initial value problem associated to applications.In particular, we prove that the problem admits a unique non–negative maximal solution. However, the solution cannot be in general global in time, due to the possibility of blow–up. The blow–up occurs when the biological life system is globally proliferative.

Nonlocal nonlinear problems

Raffaella Servadei
Università della Calabria, Italy
servadei@mat.unical.it

Fractional and non-local operators appear in concrete applications in many fields such as, among the others, optimization, finance, phase transitions, stratified materials, anomalous diffusion, crystal dislocation, soft thin films, semipermeable membranes, flame propagation, conservation laws, ultra-relativistic limits of quantum mechanics, quasi-geostrophic flows, multiple scattering, minimal surfaces, materials science and water waves. This is one of the reason why, recently, non-local fractional problems are widely studied in the literature. Aim of this talk will be to present some recent results for nonlocal problems driven by the fractional Laplace operator $(-\Delta)^s$, which (up to normalization factors) may be defined as

$$-(-\Delta)^s u(x) = \int_{\mathbb{R}^n} \frac{u(x+y)+u(x-y)-2u(x)}{|y|^{n+2s}}\, dy\,,$$

for $x \in \mathbb{R}^n$. These results were obtained through variational and topological methods and extend the validity of some theorems known in the classical case of the Laplacian to the non-local fractional framework.

Existence results for nonlocal problems at critical or subcritical growth

Marco Squassina
University of Verona, Italy
marco.squassina@univr.it

We establish existence of solutions for semi-linear and quasi-linear problems involving fractional operators and with a nonlinearity at critical or subcritical growth.

On the Bolza problem

Miguel Sánchez
Universidad de Granada, Spain
sanchezm@ugr.es

Given a Riemannian manifold (M,g) and a potential V, the Bolza problem searchs for the existence and multiplicity of trajectories connecting any two prescribed points. Our purpose is to make a short review on this topic with prospective results, including its extension to Lorentzian and Finslerian geometries.

References

[1] R. Bartolo: Trajectories connecting two events of a Lorentzian manifold in the presence of a vector field. J. Differential Equations 153 (1999), no. 1, 82-95.

[2] R. Bartolo, E. Caponio, A.V. Germinario, M. Sánchez: Convex domains of Finsler and Riemannian manifolds. Calc. Var. Partial Differential Equations 40 (2011), no. 3-4, 335-356.

[3] P. Bolle: On the Bolza problem. J. Differential Equations 152 (1999), no. 2, 274-288.

[4] A. M. Candela, J. L. Flores, M. Sánchez: Global hyperbolicity and Palais-Smale condition for action functionals in stationary spacetimes. Adv. Math. 218 (2008), no. 2, 515-536.

[5] A. M. Candela, J. L. Flores, M. Sánchez: A quadratic Bolza-type problem in a Riemannian manifold. J. Differential Equations 193 (2003), no. 1, 196-211.

A global regularity result for some classes of second-order degenerate elliptic systems on Morrey's spaces

Antonio Tarsia
University of Pisa, Italy
tarsia@dm.unipi.it
Luisa Fattorusso and Giovanni Molica Bisci

We establish a Caccioppoli-type inequality for a class of second-order degenerate elliptic systems. As a direct consequence we prove a global regularity result on Morrey's spaces for the weak solutions of the treated problem.

Parametric nonlinear nonhomogeneous Neumann equations involving a nonhomogeneous differential operator

Patrick Winkert
Technische Universitaet Berlin, Germany
winkert@math.tu-berlin.de
Said El Manouni, Nikolaos S. Papageorgiou

This talk is concerned with the existence of solutions to parametric elliptic equations driven by a nonhomogeneous differential operator with a nonhomogeneous Neumann boundary condition. The assumptions on the operator involve the p-Laplacian, the (p, q)-Laplacian and the generalized p-mean curvature differential operator. Based on variational tools combined with truncation and comparison techniques we prove the existence of at least three nontrivial solutions provided the parameter is sufficiently large.

On non-local fractional equations with asymmetries

Binlin Zhang
Heilongjiang Institute of Technology, Peoples Rep of China
zhbinlin@gmail.com
Giovanni Molica Bisci

In this talk we consider the existence of weak solutions for equations driven by a non-local integrodifferential operator with an asymmetric nonlinear term. As a particular case, we derive existence results for the asymmetric fractional Laplace equations. By using variational methods in an appropriate abstract framework developed by Servadei and Valdinoci, we investigate the existence, nonexistence and uniqueness of weak solutions for the aforementioned equations. Our results extend some classical results for asymmetric elliptic boundary value problems to the non-local fractional setting.

Special Session 35: Direct and Inverse Problems in Wave Propagation

Fioralba Cakoni, University of Delaware, USA
Francisco-Javier Sayas, University of Delaware, USA

The main goal of this special session is bringing together specialists in analysis and computation of direct and inverse problems in different wave propagation phenomena (acoustic, elastic, electromagnetic) in the time and frequency domains. More specifically, for direct problems, the session will focus in new methods for simulation of waves, including, but not restricted to, boundary integral equation methods and volume based schemes such as the finite element method. Regarding inverse problems, optimization based techniques and qualitative inversion methods, both for time-harmonic and transient waves, will be considered.

Oblivious quadrature for long-time computation of waves

Lehel Banjai
Heriot-Watt University, Edinburgh, Scotland
l.banjai@hw.ac.uk
Maria Lopez Fernandez, Achim Schaedle

Propagation of waves in 2 dimensions, or in the presence of damping, c.f. viscoelastodynamics, follows the weak Huygen's principle. This makes long time computations expensive when using time-domain boundary integral equations as the complete history needs to be stored . We will show how the smoothness of this tail can be exploited to perform fast computations. In particular, we will show that the late time propagation is goverened by a parabolic operator and that consequently oblivious quadrature can be applied. Oblivious quadrature requires the storage to increase logarithmically rather than linearly in the number of time-steps. The method will be illustrated by numerical examples.

Shape reconstruction of non-convex elastic scatterers using a regularized Newton-type method

Hélène Barucq
Inria, France
helene.barucq@inria.fr
Rabia Djellouli, Elodie Estecahandy

The determination of the shape of an elastic obstacle immersed in water from some measurements of the scattered field is an important problem in many technologies such as sonar, geophysical exploration, and medical imaging. This inverse obstacle problem (IOP) is very difficult to solve, especially from a numerical viewpoint, because of its non-linear and ill-posedness characters. We present a work pertaining to the mathematical and numerical analysis of the elasto-acoustic IOP. We have developed an efficient numerical simulation code for wave propagation based on a DG-type method using higher-order finite elements and curved edges at the interface and we have applied it to the reconstruction of objects with the implementation of a regularized Newton method involving a Jacobian matrix which must be evaluated at each iteration. The Fréchet derivative of the elasto-acoustic scattered field with respect to the shape of the obstacle is characterized as the solution to the initial direct elasto-acoustic problem except a change in the transmission conditions set on the fluid-structure interface. We therefore show that the evaluation of the Jacobian matrix requires the solution of the direct problem with multiple right-hand sides. We have performed a set of experiments that illustrate the interest of our approach.

An inverse boundary value problem for the reduced wave equation with multi-frequency data

Elena Beretta
Politecnico di Milano, Italy
elena.beretta@polimi.it
M. de Hoop, L. Qiu, O.Scherzer

We study the inverse problem for the reduced wave equation using the Dirichlet to Neumann map at selected frequencies. We establish Lipschitz stability estimates for wavespeeds in the form of finite linear combinations of known functions defined on a Lipschitz partition $\mathcal{D}_N = \cup_{j=1}^N D_j$ of the underlying domain Ω. A crucial role for the effective reconstruction is played by the Lipschitz constant appearing in the estimates, in particular its dependence on the mesh size of the partition $\mathcal{D}_N$. We establish an explicit optimal bound of the Lipschitz constant with respect to the mesh size and this is used to derive a multifrequency convergent iterative method for the reconstruction of the wavespeed.

Size Estimates in Inverse Problems

Michele Di Cristo
Politecnico di Milano, Italy
michele.dicristo@polimi.it

Detection of inclusions or obstacles inside a body by boundary measurements is an inverse problems very useful in practical applications. When only finite numbers of measurements are available, we try to detect some information on the embedded object such as its size. In this talk we review some recent results on several inverse problems. The idea is to provide constructive upper and lower estimates of the area/volume of the unknown defect in terms of a quantity related to the work that can be expressed with the available boundary data.

Arbitrary high order time scheme for wave equation

Julien Diaz
Inria, France
julien.diaz@inria.fr
Hélène Barucq, Henri Calandra, Florent Ventimiglia

High-order Discontinuous Galerkin Methods (DGM) are now routinely used for simulation of wave propagation, especially for geophysical applications. However, to take full advantage of the high-order space discretization, it is relevant to use a high-order time discretization. Hence, DGM are currently coupled with ADER schemes, which leads to high-order explicit time schemes, but requires the intro-

duction of auxiliary unknowns. The memory can thus be considerably cluttered up. That is why we propose a new time scheme which requires less memory than DG-ADER methods for a given level of accuracy. The construction of the new scheme is based on the fact that DGMs are well-suited for the approximation of high-order space operators. By exploiting this property, it is relevant to construct high-order time schemes which involve high-order space operators. This can be done by following the same approach than for the Modified Equation technique but by working with the continuous problem directly. By this way, we address the time discretization directly while the classical used technique consists in applying the space discretization before the time discretization. The proposed time scheme demonstrates a high level of accuracy while requiring acceptable computational costs.

Regularized integral equations for acoustic transmission problems in smooth domains

Victor Dominguez
Universidad Publica de Navarra, Spain
victor.dominguez@unavarra.es
Yassine Boubendir, Cataline Turc

In this work we present a novel integral equation formulation for the transmission problem for the Helmholtz equation. We start introducing the admittance operators, which map the transmission boundary conditions to the exterior and respectively interior Cauchy data on the interface between the media. With such mapping, the solution of the transmission problem is straightforward since the densities needed to construct the acoustic fields are the result of applying these admittance operators to the jumps of the trace and normal derivative. However, since these mappings are usually difficult to compute, we propose instead to construct suitable approximations of such operators based on approximating linear combinations of Dirichlet-to-Neumann mappings, assuming that they exist, by boundary layer operators with pure imaginary wave-numbers. We then prove that the interior and exterior acoustic fields can be evaluated in terms of layer potentials whose densities are the solution of a system of integral equations, that the suggested approximations of the admittance operator make this system a compact perturbation of the identity in Sobolev spaces (property which hold without needing to assume that the above mentioned combination of the Dirichlet-to-Neumann operators exist), and that this system is uniquely solvable.

High order transmission conditions at the interface between homogeneous and periodic media

Sonia Fliss
POems (UMR 7231 CNRS-INRIA-ENSTA), France
sonia.fliss@ensta-paristech.fr
Xavier Claeys, Valentin Vinoles

The mathematical modelling of electromagnetic metamaterials and the homogenization theory are intimately related because metamaterials are precisely constructed by a periodic assembly of small resonating micro-structures involving dielectric materials presenting a high contrast with respect to a reference medium. We wish to look carefully at the treatment of boundaries and interfaces that are generally poorly taken into account by the first order homogenization. This question is already relevant for standard homogenization (ie without high contrast) for which taking into account the presence of a boundary induces a loss of accuracy due to the inadequateness of the standard homogenization approach to take into account the boundary layers induced by the boundary. The objective of this work is to construct approximate effective boundary conditions that would restore the desired accuracy. We have first considered a plane interface between a homogeneous and a periodic media in the standard case without high-contrast. We obtain high order transmission conditions between the homogeneous media and the periodic media. The technique we use involves matched asymptotic expansions combined with standard homogenization ansatz. Those conditions are non standard : they involve Laplace-Beltrami operators at the interface and requires to solve cell problems in infinite periodic waveguides. The analysis is based on a original combination of Floquet-Bloch and a periodic version of Kondratiev technique.

High-order uncertainty quantification algorithms for multiple component wave propagation configurations

Mahadevan Ganesh
Colorado School of Mines, USA
mganesh@mines.edu
S.C. Hawkins

We develop efficient high-order algorithms for simulation of the statistical properties of quantities of interest (QoI) in a class of stochastic configurations comprising multiple wave propagation components. In particular, the moments of the QoI play an important role in quantifying random uncertainty in the multiple component structure wave scattering and absorption models. The stochastic nature of configurations governing the models may include randomness in location and orientations of the obstacles, their shapes, and their material properties.

Enhanced approximate cloaking by optimal change of variables

Roland Griesmaier
Universität Leipzig, Germany
griesmaier@math.uni-leipzig.de
Michael Vogelius

The aim of (passive) cloaking with respect to electromagnetic (or acoustic) sensing is to surround a region of space with a material layer — the cloak — that renders its contents and even the existence of the layer undetectable by such measurements. At least theoretically this can be achieved using the coordinate invariance of the underlying wave equation, through so-called cloaking by mapping. However, a practical realization of the cloaking by mapping schemes discussed in the literature frequently requires the design of highly anisotropic materials with extreme dielectric properties. In this talk we consider, in the electrostatic case, a regularized, approximate cloaking by mapping scheme and discuss the problem of optimal choice of radial changes of variables, that determine the conductivity distribution inside the cloak. We consider two different optimality criteria: minimal maximal anisotropy and minimal mean anisotropy of this conductivity distribution. Using both criteria we show that it is possible to achieve significantly lower anisotropy (for a prescribed level of invisibility) or significantly lower visibility (for a prescribed level of anisotropy). For example, in two dimensions one may achieve exponentially small visibility with a cloak, that in terms of anisotropy (and lowest and highest conductivity) is no worse than the traditional affine map cloak, which only yields quadratically small visibility.

Hardy Space Infinite Elements for Time-Harmonic Wave Equations with Phase Velocities of Different Signs

Thorsten Hohage
University of Goettingen, Germany
hohage@math.uni-goettingen.de
Martin Halla, Lothar Nannen, Joachim Schöberl

We consider time harmonic wave equations in cylindric waveguides with physical solutions for which the signs of group and phase velocities differ. In particular, we will consider a one-dimenisonal fourth order model problem and two-dimensional elastic waveguides for which this phenomenon occurs. Standard transparent boundary conditions, e.g. the Perfectly Matched Layers (PML) method select modes with positive phase velocity, whereas physical modes are characterized by positive group velocity. Hence these methods yield stable, but unphysical solutions for such problems. We derive an infinite element method for a physically correct discretization of such waveguide problems which is based on a Laplace transform in propagation direction. In the Laplace domain the space of transformed solutions can be separated into a sum of a space of incoming and a space of outgoing functions where both function spaces are curved Hardy spaces. The curved Hardy space is constructed such that it contains a simple and convenient Riesz basis with moderate condition numbers. Our method does not use a modal separation and works on an interval of frequencies. In particular, it is well-adapted for the computation of resonances. Numerical experiments exhibit super-algebraic convergence and moderate condition numbers.

Non-destructive eddy current inspection of highly conductive thin layer deposits via asymptotic models

Zixian Jiang
Universitaet Bremen, Germany
jiang@math.uni-bremen.de
Houssem Haddar

Highly conductive thin layer deposits may blind the eddy current probes in non-destructive inspections. In this talk, we study several asymptotic models using different rescaling techniques to represent the thin layer by some transmission conditions on an interface. We choose a pertinent model from which we develop the inversion methods to reconstruct the layer thickness using eddy current signals. We give some numerical examples showing the modeling and the identification of thin layers.

Characterization of Transmission Eigenvalues

Armin Lechleiter
University of Bremen, Germany
lechleiter@math.uni-bremen.de
Marcel Rennoch

It is well-known that the interior eigenvalues of a bounded domain share connections to scattering problems in the exterior of this domain. For instance, certain boundary integral equations for exterior scattering problems fail at interior eigenvalues. Similar connections also exist for inverse exterior scattering problems - for instance, if zero is an eigenvalue of the far-field operator at a fixed wave number, then the squared wave number is an interior eigenvalue. Despite it is in general wrong that interior eigenvalues correspond to zero being an eigenvalue of the far field operator, one can prove a pretty direct characterization of interior eigenvalues via the behavior of the phases of the eigenvalues of the far-field operator. We present such a characterization of transmission eigenvalues for penetrable media governed by the H-mode equations $\mathrm{div}(A\nabla u) + k^2 u = 0$ or the Maxwell's equations $\mathrm{curl}(A\mathrm{curl}u) - k^2 u = 0$.

Variable time stepping for wave scattering problems

Maria Lopez-Fernandez
University of Zurich, Switzerland
maria.lopez@math.uzh.ch
Stefan Sauter

We present a generalization of Lubich's Convolution Quadrature which allows for variable time steps. The main application of our method is the time integration of retarded potentials arising in wave scattering problems. The algorithmic realization of the new method relies on contour integral techniques in the complex plane. Numerical experiments are provided to show the potential of our approach.

Asymptotic Expansions for Transmission Eigenvalues in the presence of small inhomogeneities

Shari Moskow
Drexel University, USA
moskow@math.drexel.edu
Fioralba Cakoni

We consider the transmission eigenvalue problem for an inhomogeneous medium containing a finite number of diametrically small inhomogeneities of different refractive index. We prove a convergence result for the transmission eigenvalues and eigenvectors corresponding to media with small homogeneities as the diameter of small inhomogeneities goes to zero. In addition we derive rigorously a formula for the perturbations in the real transmission eigenvalues caused by the presence of these small inhomogeneities.

Domain reconstruction using multifrequency topological sensitivity

María-Luisa Rapún
Universidad Politecnica de Madrid, Spain
marialuisa.rapun@upm.es
José Manuel Perales, José Manuel Vega

In this work we propose a non-iterative method based on topological derivative computations to detect structural defects or inclusions (determine their location, size, shape, orientation) by combining multifrequency near-field observations of the total acoustic wave field at a few observation points. We will show some numerical tests illustrating the viability of this technique in different situations including the simultaneous reconstruction of objects of different sizes and the identification of poorly illuminated defects.

Numerical simulation of wave propagation on unbounded domains using transparent boundary conditions

Jeronimo Rodriguez
Universidad de Santiago de Compostela, Spain
jeronimo.rodriguez@usc.es
Toufic Abboud and Patrick Joly

We are interested in the numerical simulation of wave propagation phenomena modeled by symmetric hyperbolic Friedrichs systems such as scalar acoustics or aeroacoustics. The unbounded nature of the computational domain is treated through the retarded potential method which is coupled with an explicit in time FDTD-DG volume method. The coupling formula that allows local time stepping on the domain discretized with the DG method, ensures a positivity property that guarantees the well posedness of the discrete algorithm and energy conservation. Numerical experiments for the scalar wave equation using this general methodology will be shown.

Scattering of acoustic waves by homogeneous penetrable obstacles

Francisco-Javier Sayas
University of Delaware, USA
fjsayas@math.udel.edu
Tianyu Qui

In this talk we present some new results on an integral formulation for transmission problems related to the transient linear wave equation. The resulting system of equations is based on a direct formulation and mimics in the time-domain what the Costabel-Stephan system did in the frequency domain. Using techniques based on evolution equations we show well-posedness of the problem after Galerkin semidiscretization in space and estimates on the error for Galerkin semidiscretization. Finally we show the effect of full discretization and some numerical experiments for smooth and non-smooth scatterers in two dimensions.

Spiral Laser Beams in Inhomogeneous Media

Erwin Suazo
Arizona State University/University of Puerto Rico, USA
esuazo@asu.edu
Alex Mahalov and Sergei K. Suslov

Explicit solutions of the inhomogeneous paraxial wave equation in a linear and quadratic approximation are applied to wave fields with invariant features, such as oscillating laser beams in a parabolic waveguide and spiral light beams in varying media. A similar effect of superfocusing of particle beams in a thin monocrystal film, harmonic oscillations of cold trapped atoms, and motion in magnetic field are also mentioned.

References
[1] A. Mahalov, E. Suazo and S. K. Suslov, Spiral Laser Beams in Inhomogeneous Media , Optics Letters, Vol. 38, (2013) Issue 15, pp. 2763-2766.

Well-conditioned boundary integral equation formulations for the solution of high-frequency electromagnetic scattering problems

Catalin Turc
New Jersey Institute of Techmology, USA
catalin.c.turc@njit.edu
Yassine Boubendir

We present several versions of Regularized Combined Field Integral Equation (CFIER) formulations for the solution of three dimensional frequency domain electromagnetic scattering problems with Perfectly Electric Conducting (PEC) boundary conditions. Just as in the Combined Field Integral Equations (CFIE), we seek the scattered fields in the form of a combined magnetic and electric dipole layer potentials that involves a composition of the latter type of boundary layers with regularizing operators. The regularizing operators are of two types: (1) modified versions of electric field integral operators with complex wavenumbers, and (2) principal symbols of those operators in the sense of pseudodifferential operators. We show that the boundary integral operators that enter these CFIER formulations are Fredholm of the second kind, and invertible with bounded inverses in the classical trace spaces of electromagnetic scattering problems. We present a spectral analysis of CFIER operators with regularizing operators that have purely imaginary wavenumbers for spherical geometries. Under certain assumptions on the coupling constants and the absolute values of the imaginary wavenumbers of the regularizing operators, we show that the ensuing CFIER operators are coercive for spherical geometries. These properties allow us to derive wavenumber explicit bounds on the condition numbers of certain CFIER operators that have been proposed in the literature. When regularizing operators with complex wavenumbers with non-zero real parts are used, we show numerical evidence that those complex wavenumbers can be selected in a manner that leads to CFIER formulations whose condition numbers can be bounded independently of frequency for spherical geometries. In addition, the Regularized Combined Field Integral Equations that employ as regularizers electric field integral operators with carefully chosen complex numbers possess excellent spectral properties in the high-frequency regime for strictly convex scatterers. We provide numerical evidence that our solvers based on fast, high-order Nyström discretization of these equations converge in very small numbers of GMRES iterations, and the iteration counts are virtually independent of frequency for strictly convex scatterers.

Special Session 36: Analytical Aspects of the Dynamics of Nonlinear Schrodinger Equations

Francois Genoud, University of Vienna, Austria

This session will be devoted to recent analytical results about the dynamics of NLS, from a broad perspective. Issues of interest may include well-posedness, scattering, stability analysis, singularity formation, semiclassical limit. Connections with physical models presenting challenging mathematical problems are particularly welcome.

Spatial localization of some NLS

Pascal Bégout
Institut de Mathématiques de Toulouse, France
Pascal.Begout@math.cnrs.fr
Jesús Ildefonso Díaz

We will give a review about spatial localization and finite time extinction of some PDEs. We will expose a sharper energy method and will explain how it can be applied to some complex-valued solutions, thereby extending preceding known results. We will illustrate our method on some nonlinear Schrödinger equations.

Scattering for nonlinear Schrödinger equation under partial harmonic confinement

Remi Carles
CNRS Montpellier, France
Remi.Carles@math.cnrs.fr
Paolo Antonelli, Jorge Drumong Silva

We consider the nonlinear Schödinger equation under a partial quadratic confinement. We show that the global dispersion corresponding to the direction(s) with no potential is enough to prove global in time Strichartz estimates, from which we infer the existence of wave operators thanks to suitable vector-fields. Conversely, given an initial Cauchy datum, the solution is global in time and asymptotically free, provided that confinement affects one spatial direction only. This stems from anisotropic Morawetz estimates, involving a marginal of the position density.

Local dynamics near unstable branches of NLS solitons

Vianney Combet
Université Lille 1, France
vianney.combet@math.univ-lille1.fr
Tai-Peng Tsai, Ian Zwiers

In this talk, we will consider a branch of unstable solitons of NLS whose linearized operators have one pair of simple real eigenvalues in addition to the zero eigenvalue. Under radial symmetry and standard assumptions, we will show that solutions to initial data from a neighbourhood of the branch either converge to a soliton, or exit a larger neighbourhood of the branch transversally. The qualitative dynamic near a branch of unstable solitons is irrespective of whether blowup eventually occurs, which has practical implications for the description of blowup of NLS with supercritical nonlinearity that will be discussed.

Stability in the energy space for chains of solitons of the Landau-Lifshitz equation

André de Laire
Université Lille 1, France
andre.de-laire@math.univ-lille1.fr
Philippe Gravejat

In this talk we will consider the one-dimensional Landau-Lifshitz equation, a model describing the dynamics for the spin in ferromagnetic materials. This equation is related to the Schrodinger maps onto the sphere and also to the cubic nonlinear Schrodinger equation. We prove the orbital stability of sums of solitons with an easy-plane anisotropy, under the assumptions that the speeds of the solitons are different, and that their initial positions are sufficiently separated and ordered according to their speeds.

A bifurcation analysis for the Lugiato-Lefever equation

Cyril Godey
Universite de Franche Comte, France
cyril.godey@univ-fcomte.fr
Mariana Haragus

The Lugiato-Lefever equation is a cubic nonlinear Schrödinger equation with damping, detuning and driving force arising as a model in nonlinear optics. We focus on the existence of steady waves which are found as solutions of a four-dimensional reversible dynamical system in which the evolutionary variable is the space variable. Relying upon tools from bifurcation theory and normal forms theory, we classify the local bifurcations and then discuss the codimension 1 bifurcations. We show the existence of various types of steady solutions, including spatially localized, periodic, or quasi-periodic solutions.

Multiple normalized solutions for quasi-linear Schrodinger equations

Louis Jeanjean
University of Franche Comte, France
louis.jeanjean@univ-fcomte.fr
Tingjian Luo, Zhi-Qiang Wang

For a quasilinear Schrodinger equation arising in Plasma Physics we prove the existence of two solutions having a prescribed L^2 norm. These solutions are obtained as constrained critical points of the associated functional. One of these solutions is a mountain pass solution relative to the constraint and the other one a minimum either local or global. The orbital stability/instability of the associated standing waves, which is widely open, will also be discussed.

Dynamical Aspects near Bifurcation Points in NLS

Eduard Kirr
University of Illinois, USA
ekirr@math.uiuc.edu
Vivek Natarajan

I will first summarize a rigorous theory on existence of bifurcation points for ground states and excited states of nonlinear Schrödinger/Gross-Pitaevskii equations in both weakly and strongly nonlinear regimes. A few particular cases where all bifurcation points can be found will also be presented. Then I will discuss dynamical properties of solutions starting near the bifurcation points, including the orbital and asymptotic stability of the bound state branches emerging from the bifurcation point. The first part is joint work with V. Natarajan (Tel Aviv U.).

On the blow-up speed for modified critical nonlinear Schrodinger equations

Stefan Le Coz
Toulouse III University, France
slecoz@math.univ-toulouse.fr
Yvan Martel and Pierre Raphael

So far, only two blow-up regimes have been studied for NLS equations: the pseudo-conformal regime, where the blow-up speed is like $|t|^{-1}$ and the log-log regime where the blow-up speed is like $|t|^{-1/2}$ with a log-log correction. In this talk, we consider the nonlinear Schrodinger with a double power nonlinearity where one of the power is L2 critical and the other one is L2-subcritical. We construct a minimal mass blowing up solution whose blow-up speed is neither the log-log speed nor the pseudo-conformal speed, but is of the type $|t|^{-s}$ with s varying between 1/2 and 1 depending on the subcritical power. This is based on a joint work with Yvan Martel and Pierre Raphael.

Modeling quantum dissipation with nonlinear Doebner-Goldin and logarithmic Schrödinger equations

José Luis López
Universidad de Granada, Spain
jllopez@ugr.es
Pilar Guerrero, Jesús Montejo–Gámez, Juanjo Nieto

We are intended to deal with the modeling of quantum dissipation/diffusion effects at the level of Schrödinger systems, in connection with the corresponding phase space and fluid formulations of such kind of phenomena, especially in what concerns the role of the Fokker–Planck mechanism in the description of open quantum systems and the macroscopic dynamics associated with some viscous hydrodynamic models of Euler and Navier–Stokes type

On the eigenvalues of Aharonov-Bohm operators with varying poles

Manon Nys
Universite Libre de Bruxelles - Universita di Milano-Bicocca, Belgium
manonys@gmail.com
V. Bonnaillie-Noel, B. Noris, S. Terracini

We consider a magnetic operator of Aharonov-Bohm type with Dirichlet boundary conditions in a planar, bounded and simply connected domain Ω

$$(i\nabla + A_a)^2 u = -\Delta u + 2iA_a \cdot \nabla u + i\nabla \cdot A_a u + |A_a|^2 u,$$

where for each $a \in \Omega$ the magnetic potential is given by

$$A_a(x_1, x_2) = \alpha \left(-\frac{x_2 - a_2}{|x-a|^2}, \frac{x_1 - a_1}{|x-a|^2} \right).$$

We analyse the behavior of its eigenvalues as the singular pole a moves in the domain. For any value of the circulation α of the potential, we prove that the k-th magnetic eigenvalue converges to the k-th eigenvalue of the Laplacian as the pole approaches the boundary. We show that the magnetic eigenvalues depend in a smooth way on the position of the pole, as long as they remain simple. In case of half-integer circulation, we show that the rate of convergence depends on the number of nodal lines of the corresponding magnetic eigenfunction. In addition, we provide several numerical simulations both on the circular sector and on the square, which find a perfect theoretical justification within our main results.

A Theorem of Paley-Wiener type for Schrödinger evolutions

Gustavo Ponce
University of California-Santa Barbara, USA
ponce@math.ucsb.edu
Carlos E. Kenig, Luis Vega

We prove unique continuation principles for solutions of evolution Schodinger equations with time dependent potentials. These correspond to uncertainly principles of Paley-Wiener type for the Fourier transform. Our results extends to a large class of semi-linear Schrodinger equation.

Mathematical study of a nonlinear model for nucleons

Simona Rota Nodari
Université Lille 1, France
simona.rota-nodari@univ-lille1.fr
Maria J. Esteban, Loïc Le Treust

In this talk we consider a model for a nucleon interacting with the σ and ω mesons in the atomic nucleus. This model is described by a nonlinear Schrödinger-type equation, where the mass depends on the solution itself. I will present some existence results for this equation and for the associated minimization problem. I will discuss some perspectives on the time-dependent version of this model. The talk is based on joint works with Maria J. Esteban and Loïc Le Treust

Nondispersive dynamics of the Maxwell-Schrödinger-Poisson system.

Oscar Sanchez
University of Granada, Spain
ossanche@ugr.es
I. Catto, J. Dolbeault, J. Soler

This talk is intended to summarize recent results and open problems concerning the existence of steady states to the Maxwell-Schrödinger system. A combination of tools, proofs and results are presented in the framework of the concentration–compactness method.

Wellposedness for nonlinear Schrodinger equations with inverse-square potentials

Toshiyuki Suzuki
Tokyo University of Science, Japan
t21.suzuki@gmail.com

We consider the nonlinear Schrödinger equations with inverse-square potentials $a|x|^{-2}$:

$$i\frac{\partial u}{\partial t} = -\Delta u + \frac{a}{|x|^2}u + f(u) \text{ in } \quad \mathbb{R} \times \mathbb{R}^N$$

where $i = \sqrt{-1}$, $N \geq 3$ and $a \geq -(N-2)^2/4$. The feature is the presence of a strongly singular potential $a|x|^{-2}$; note that $-\Delta$ and $a|x|^{-2}$ are the same scale symmetry and hence scaling argument can not be applied to $P_a := -\Delta + a|x|^{-2}$. The restriction on a follows from the selfadjointness of P_a in the sense of form-sum in $L^2(\mathbb{R}^N)$. In this talk we show the global existence and blow-up in finite time.

Two-component nonlinear Schrodinger system with linear coupling

Rada Maria Weishaeupl
University of Vienna, Austria
rada.weishaeupl@univie.ac.at

We consider a system of two nonlinear Schrödinger equations, which are coupled through a linear term in addition to the nonlinearity. We are interested in the long-time behavior and blow-up alternative of such systems. Numerical simulations in one and two space dimensional equations verify and complement the theoretical results. Finally, we want to understand the effect of the linear coupling in this setting and perform the asymptotics for large linear coupling.

Special Session 37: Global or/and Blowup Solutions for Nonlinear Evolution Equations and Their Applications

Shaohua George Chen, Cape Breton University, Canada
Ming Mei, Champlain College & McGill University, Canada

This session is devoted to the recent developments in global or/and blowup solutions for nonlinear evolution equations and their applications, include reaction-diffusion equations, fluid dynamics, delay, localized, nonlocal, degenerate evolution equations, steady states and their properties.

Hyperbolic equations with variable nonlinearity: existence and blow-up

Stanislav Antontsev
CMAF, University of Lisbon, Portugal
antontsevsn@mail.ru

We study the Dirichlet problem for a class of nonlinear hyperbolic equations with p(x,t)- Laplacian relatively spatial variables and with damping term. Under suitable conditions on the data, we prove local and global existence theorems and study the finite time blow-up of the energy solutions. Also we consider Young measure solutions of such equations. The analysis relies on the methods developed in [1-6].

References

[1] Antontsev S.N., Diaz J.I., Shmarev S.I., Energy Methods for Free Boundary Problems : Applications to Nonlinear PDEs and Fluid Mechanics. Bikhauser, Boston, 2002. Progress in Nonlinear Differential Equations and Their Applications, Vol. 48.

[2] Antontsev S.N., Wave equation with p(x,t)-Laplacian and damping term: Blow-up of solutions, C.R. Mecanique, 339, 12(2011),751–755.

[3] Antontsev S.N., Wave equation with p(x,t)- Laplacian and damping term: Existence and blow-up, Differ. Equ. Appl., 3 (2011), pp. 503–525.

[4] Antontsev S.N., Shmarev S., Blow-up of solutions to parabolic equations with nonstandard growth conditions. J. Comput. Appl. Math., 234 (2010), pp.2633–2645.

[5] Antontsev S., Amorim P., Young measure solutions for wave equation with p(x,t)-Laplacian: Existence and blow up, Nonlinear Analysis: Theory, Methods and Applications, 92(2013), pp.153-167.

[6] Antontsev S., Ferreira J., Existence, uniqueness and blow up for hyperbolic equations with nonstandard growth conditions, Nonlinear Analysis Series A: Theory, Methods and Applications, 93(2013), pp.62-77.

Convergence to Steady State for Degenerate Parabolic Equations

Shaohua Chen
Cape Breton University, Canada
george_chen@cbu.ca
Xu Runzhang

In this talk, we first discuss large time behavior of the homogeneous Dirichlet problem to the degenerate parabolic equation $u_t = g(u)\Delta u + f(u)$ in a bounded domain $\Omega \subset R^n$ with smooth boundary $\partial\Omega$. Under suitable conditions on $f(u)$ and $g(u)$, we show that all solutions will converge to the steady state exponentially. Next, we study the degenerate parabolic system $u_t = u\Delta u + u(a_1 - b_1 u + c_1 v)$ and $v_t = v\Delta v + v(a_2 + b_2 u - c_2 v)$ with the same boundary condition. We show that any positive solutions converge to a unique steady state exponentially if the coefficients satisfy some conditions.

Blow-up for possible singular solutions of the Navier-Stokes equations

Jean Cortissoz
Universidad de los Andes, Colombia
jcortiss@uniandes.edu.co
Julio A. Montero, Carlos E. Pinilla

In this talk we will present blow-up rates for possible singular solutions to the Navier-Stokes equations in the homogeneous Sobolev spaces $\dot{H}^{\frac{3}{2}}(X)$ and $\dot{H}^{\frac{5}{2}}(X)$, where $X = \mathbb{T}^3$, the 3-dimensional torus, or $X = \mathbb{R}^3$.

Global in time weak solutions for a nonlinear model for tumor growth.

Donatella Donatelli
University of L'Aquila, Italy
donatell@gmail.com
K. Trivisa

We investigate a free boundary problem modeling the growth of tumors cells. The model is given by a multiphase flow and the tumor is described as a growing continuum Ω with boundary $\partial\Omega$ both of which evolve in time. In particular the model consists of a nonlinear second-order parabolic equations describing the diffusion of nutrient, and three nonlinear first-order hyperbolic equations describing the evolution of proliferative cells, quiescent cells and dead cells. Global-in-time weak solutions are obtained using an approach based on penalization of the boundary behavior, diffusion and viscosity in the weak formulation.

Problems with singularity in the u variable: nonnegative solutions

Daniela Giachetti
Università di Roma Sapienza, Italy
daniela.giachetti@sbai.uniroma1.it

We deal with the existence of nonnegative solutions to parabolic problems which are singular in the u variable whose model is

$$\begin{cases} u_t - \Delta_p u = f(x,t)(\frac{1}{u^\theta} + 1) & \text{in } \Omega \times (0,T) \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,T) \\ u(x,0) = u_0(x) & \text{in } \Omega. \end{cases}$$

Here Ω is a bounded open subset of $\mathbb{R}^N$, $N \geq 2$, $0 < T < +\infty$, $\theta > 0$, $\Delta_p u = -\text{div}(|\nabla u|^{p-2}\nabla u)$ with $p > 1$. As far as the data, we assume $f(x,t) \in L^r(0,T;L^m(\Omega))$, with $\frac{1}{r} + \frac{N}{pm} < 1$, $f(x,t) \geq 0$ a.e. in $\Omega \times (0,T)$ and $u_0(x) \geq 0$ a.e. in Ω. We consider also the case where the right hand side depends on the gradient of the solution. In this last case the model of the right hand side is $F(x,t,u,\nabla u) = \frac{f(x,t)+D|\nabla u|^q}{u^\theta}$, with $\theta > 0$, $D > 0$, $1 < q < p$ and $f(x,t)$ as before.

Analysis on the initial-boundary value problem of a full bipolar hydrodynamic model for semiconductors

Haifeng Hu
Northeast Normal University, Peoples Rep of China
huhf836@nenu.edu.cn
Kaijun Zhang

In this paper, we study the initial boundary value problem of the one dimensional full bipolar hydrodynamic model for semiconductors. The existence and uniqueness of the stationary solution are established by the theory of strongly elliptic systems and the Banach fixed point theorem. The exponentially asymptotic stability of the stationary solution is given by means of the energy estimate method.

Existence and blow-up of solutions for semilinear filtration problems

Evangelos Latos
Karl-Franzens University of Graz, Austria
evangelos.latos@uni-graz.at
D. Tzanetis

We first examine the local existence and uniqueness of solutions $u = u(x,t;\lambda)$ to the semi linear filtration equation $u_t = \Delta K(u) + \lambda f(u)$, for $\lambda > 0$, with initial data $u_0 \geq 0$ and appropriate boundary conditions. Our main result is the proof of blow-up of solutions for some λ. Moreover, we discuss the existence of solutions for the corresponding steady-state problem. It is found that there exists a critical value λ^* such that for $\lambda > \lambda^*$ the problem has no stationary solution of any kind, while for $\lambda \leq \lambda^*$ there exist classical stationary solutions. Finally, our main result is that the solution u, for $\lambda > \lambda^*$, blows-up in finite time independently of $u_0 \geq 0$. The functions f, K are mostly positive, increasing and convex and K'/f is integrable at infinity.

Existence and stability of traveling waves for an Allen-Cahn model with relaxation

Corrado Lattanzio
L'Aquila University, Italy
corrado@univaq.it
Corrado Mascia, Ramon G. Plaza, Chiara Simeoni

We investigate an hyperbolic variation of the Allen–Cahn equation (bistable reaction–diffusion equation), where the Fick's law of diffusion is replaced by a relaxation term, thus introducing a delay in the process. The main feature of the model is the combination of the dissipation coming from the relaxation term, which play the role of the diffusive transport mechanism of the classical Allen–Cahn equation, and a zero–order reactive term, which determines the presence of two stable constant states. Some rigorous results concerning existence and stability of traveling waves for this hyperbolic model are provided, together with numerical experiments, also in connection with the standard parabolic Allen–Cahn equation.

Smooth Solutions to Strongly Coupled Elliptic Systems on 2D Domains

Dung Le
University of Texas at San Antonio, USA
Dung.Le@utsa.edu

We discuss the existence of smooth solutions to a class of strongly coupled elliptic systems consisting of two or more equations. Since maximum principles for such systems are not available, the solutions are not known to be bounded a priori and the ellipticity constants can be unbounded. The theories of BMO functions and A_p weights are used here to bypass these obstacles to provide the existence of classical solutions to the systems.

On a hyperbolic equation in MEMS

Jingyu Li
Northeast Normal University, Peoples Rep of China
lijy645@nenu.edu.cn
Chuangchuang Liang and Kaijun Zhang

We consider a damped wave equation with singular nonlinearity and Dirichlet boundary condition in a bounded domain, which describes an electrostatic micro-electro-mechanical system (MEMS) device. We show that the pull-in voltage λ^* is the critical threshold for global existence and quenching in this wave equation. More precisely, if the applied voltage $\lambda\lambda^*$, then any solution quenches in finite time. Finally, we analyze the relation between the hyperbolic model and the parabolic model through the viscosity dominated limit.

Numerical Results on Asymptotic Stability of Travelling Wave for Nicholson's Blowflies Equation

Chi-tien Lin
Providence University, Taiwan
ctlin@gm.pu.edu.tw
Ming Mei, Chi-Kun Lin, Yanping Lin

In this talk, we numerically study the asymptotic stability of travelling wave solutions for Nicholson's Blowflies equation, a time-delayed reaction diffusion equation, with local or nonlocal nonlinearity. It is known that, when the ratio of birth rate coefficient and death rate coefficient p/d lies between 1 and e, the equation is monotone and possesses monotone traveling wave solutions. However, when the rate is larger than e, the equation losses its monotonicity and may possess non-monotone traveling waves when the delay time r is large, which causes the study of stability of these non-monotone traveling waves to be challenging. In this talk, for the case p/d lies between e and e^2, we numerically show that monotone and non-monotone travelling waves are exponentially stable. For the case that p/d

is larger than e^2, monotone or non-monotone travelling waves are exponentially stable for some small delay time, and are unstable for large delay time. Several interesting numerical results will be demonstrated too. Joint work with Ming Mei, Chi-Kun Lin and Yanping Lin.

Long-time dynamics of a thermoelastic plate with second sound

To Fu Ma
ICMC - University of Sao Paulo, Brazil
matofu@icmc.usp.br
Rodrigo Nunes Monteiro

We consider a nonlinear thermoelastic plate where the heat flux is given by the Cattaneo's law. In the presence of a rotational inertia term, we prove the existence of a finite-dimensional global attractor under the sole dissipation given by the heat flux. On the other hand, without rotational inertia, the existence of a global attractor is done by adding a frictional damping in the displacement equation.

Metastable dynamics in conservation laws

Corrado Mascia
Sapienza University, Rome, Italy
mascia@mat.uniroma1.it

The presence of small parameters, such as viscosity coefficients or relaxation time, induces in the dynamics of evolutive PDEs the presence of different timescales. As a consequence, systems may exhibit an initial fast transient leading the solutions close to a low-dimensional approximately invariant manifold, followed by a slow evolution in the vicinity of the manifold itself. The aim of this talk is to present the mathematical quantification of the slow motion relative to the long-time scale, starting from the basic example of viscous scalar conservation laws and discussing some possible extensions to more realistic models.

Stationary solutions to symmetric hyperbolic-parabolic systems in half space

Shinya Nishibata
Tokyo Institute of Technology, Japan
shinya@is.titech.ac.jp
Tohru Nakamura

In this talk, we consider the large-time behavior of solutions to hyperbolic-parabolic coupled systems in the half line. Assuming that the systems admit the entropy function, we may rewrite them to symmetric forms. For these symmetrizable hyperbolic-parabolic systems, we first prove the existence of the stationary solution. In the case where one eigenvalue of Jacobian matrix appeared in a stationary problem is zero, we assume that the characteristics field corresponding to the zero eigenvalue is genuine non-linear in order to show the existence of a degenerate stationary solution. We also prove that the stationary solution is time asymptotically stable under a smallness assumption on the initial perturbation. The key to the proof is to derive the uniform a priori estimates by using the energy method in half space developed by Matsumura and Nishida as well as the stability condition of Shizuta-Kawashima type. These theorems for the general hyperbolic-parabolic system cover the compressible Navier-Stokes equation for heat conductive gas.

Solvability and long time behavior of nonlinear Reaction-Diffusion equations with Robin Boundary Condition

Eylem Ozturk
Hacettepe University, Turkey
eyturk1983@gmail.com
Kamal Soltanov

We investigated the long-time behavior and solvability of the reaction-diffusion equation, which has a polynomial growth nonlinearity of arbitrary order, with Robin boundary condition on the bounded domain. The problem that we investigate as the following:

$$\begin{cases} u_t - \Delta u + a(x,t)|u|^\rho u - b(x,t)|u|^\nu u = h(x,t), \\ \hfill (x,t) \in Q_T \quad (1) \\ \hfill (\frac{\partial u}{\partial \eta} + k(x',t)u)|_{\partial\Omega} = \varphi(x',t), \\ \hfill (x',t) \in \Sigma_T \quad (2) \\ \hfill u(x,0) = u_0(x), x \in \Omega \quad (3) \end{cases}$$

here $\Omega \subset \mathbb{R}^n$ $(n \geq 3)$, is a bounded domain such that $\partial\Omega$ sufficiently smooth boundary, $T > 0$, $Q_T = \Omega \times (0,T)$ and $\Sigma_T = \partial\Omega \times [0,T]$. In super linear case, for the existence and uniqueness of the generalized solution of problem (1)-(3), we obtain sufficient conditions for functions a, b and k and relations between ρ and ν and under these conditions we show the existence of generalized solution of problem (1)-(3) and the uniqueness of the solution in corresponding spaces, by applying a general existence theorem. For the long-time behavior, firstly we prove that solution has an absorbing set in $L_2(\Omega)$. Secondly assuming that functions h and φ do not depend on the variable t, we prove the existence of an absorbing set in $W_2^1(\Omega) \cap L_{\rho+2}(\Omega)$. Also when the coefficients functions depend only on x as well as h and φ, we prove some asymptotic regularity and the existence of global attractor in $W_2^1(\Omega) \cap L_{\rho+2}(\Omega)$.

Decay estimates for unbounded solutions

Maria Michaela Porzio
Sapienza University of Rome, Italy
porzio@mat.uniroma1.it

It is well known that the solution of the heat equation with summable initial datum u_0 becomes "immediately bounded" and satisfies the decay estimate

$$\|u(\cdot,t)\|_{L^\infty} \leq C \frac{\|u_0\|_{L^1}}{t^{\frac{N}{2}}}, \quad t > 0.$$

This "strong regularizing effect" is not a peculiarity of the heat equation since it appears also for a lot of other parabolic problems, also nonlinear, degenerate or singular like the p-Laplacian equation, the porous medium equation, the fast diffusion equation, etc. In a recent paper we have proved that this phenomenon occurs when the solutions satisfy certain integral inequalities. We investigate here what happens when this regularizing effect does not appear and which is the solutions' behavior in this case.

Bohmenian type boundary conditions for quantum hydrodynamics

Bruno Rubino
Univeristy of L'Aquila, Italy
bruno.rubino@univaq.it
P. Marcati

The quantum hydrodynamical model (QHD) coupled with the Poisson equation is able to describe the quantum effect appearing at nanoscale in many modern semiconductor devices. It can be obtained adding the Bhöm potential to the classical hydrodynamical equations (HD) or directly from the Schroedinger equation. Here we derive a physically reasonable set of boundary conditions (BCs) for the QHD-Poisson system. We just consider the unipolar case, thus the holes concentration is neglected. These new BCs have two interesting explanations from the physical viewpoint. Firstly, if we consider the Bohm term as a correction for the pressure functional, it implies the conservation of the generalized enthalpy at the interface metal-semiconductor. Alternatively, assuming the Bohm term works together to the electrical potential, the BCs imply the equilibrium between diffusive and quantum forces. The existence and the uniqueness of a regular solution for the QHD-Poisson system is then discussed using these new BCs. The model is tested numerically on a toy device and the linear stability of the solution is discussed in a special case. The same consideration can be used to derive interface conditions between QHD and HD in the contest of the hybrid models.

Localization of solutions of doubly nonlinear parabolic equations with anisotropic variable growth

Sergey Shmarev
University of Oviedo, Spain
sergey.shmarev@gmail.com
S. Antontsev

We present results on the properties of localization in time and space for solutions of doubly nonlinear parabolic equations with anisotropic and variable growth conditions. We derive the ranges of the variable exponents of nonlinearity where the solutions vanish or blowup in a finite time and study the asymptotic behavior of solutions for large time. Sufficient conditions of finite speed of propagation in space are established. We also study the effect of nonpropagation of disturbances from the data in certain directions due to the anisotropy of the diffusion operator. The results were obtained in collaboration with S.Antontsev.

References

[1] S.Antontsev, S.Shmarev. Doubly degenerate parabolic equations with variable nonlinearity II: Blow-up and extinction in a finite time. Nonlinear Anal. 95 (2014), 483–498.

[2] S.Antontsev, S.Shmarev. Localization of solutions of anisotropic parabolic equations. Nonlinear Anal. 71 (2009), no.12, e725–e737.

Global existence of the singularly perturbed Boussinesq-type equation

Changming Song
College of Science,Zhongyuan University of Technology, Peoples Rep of China
cmsongh@163.com

We are concerned with the singularly perturbed Boussinesq-type equation including the singularly perturbed sixth-order Boussinesq equation, which describes the bi-directional propagation of small amplitude and long capillary-gravity waves on the surface of shallow water for bond number (surface tension parameter) less than but very close to 1/3. The existence and uniqueness of the global generalized solution and the global classical solution of the initial boundary value problem for the singularly perturbed Boussinesq-type equation are proved. The nonexistence of global solution of the above-problem is discussed and two examples are given

Boundary layers to the Euler-Poisson equations for a multicomponent plasma

Masahiro Suzuki
Tokyo Institute of Technology, Japan
masahiro@is.titech.ac.jp

In this talk, we study a boundary layer, called a sheath, which occurs on the surface of materials with which a multicomponent plasma contacts. For the sheath formation, the generalized Bohm criterion demands that the ions enter the sheath region with a high velocity. The motion of the multicomponent plasma is governed by the Euler-Poisson equations. The sheath is mathematically understood as the stationary solution to the equations. We show the unique existence and the asymptotic stability of the stationary solution under the the generalized Bohm criterion.

Decay structure of the regularity-loss type and the asymptotic stability for the Euler-Maxwell system

Yoshihiro Ueda
Associate Professor/ Faculty of Maritime Sciences, Kobe University, Japan
ueda@maritime.kobe-u.ac.jp

In this talk, we consider the Cauchy problem of the Euler-Maxwell system. The Euler-Maxwell system describes the dynamics of compressible electrons in plasma physics under the interaction of the magnetic and electric fields via the Lorentz force. Our purpose is to study the large-time behavior of solutions to the initial value problem for the Euler-Maxwell system in whole space. This system verifies the decay property of the regularity-loss type. Under smallness condition on the initial perturbation, we show that the solution to the problem exists globally in time and converges to the equilibrium state (and the stationary solution). Moreover we derive the corresponding convergence rate of the solutions.The key to the proof of our main theorems are to derive a priori estimates of solutions by using the energy method.

On the thin film approximation for the flow of a viscous incompressible fluid down an inclined plane

Hiroki Ueno
Keio University, Japan
mathematics@a7.keio.jp
Tatsuo Iguchi and Akinori Shiraishi

We consider two-dimensional motion of liquid film of a viscous incompressible fluid down an inclined plane in the influence of the gravity and the surface tension. In order to investigate such a motion, a method of the thin film approximation is often used. It is the approximation by the perturbation expansion of the solution for the nondimensional parameter δ defined by ratio between the thickness of the liquid film and the typical wave length. In this study, we will give uniform estimates of the solution to the original Navier–Stokes equations in δ when the Reynolds number, the angle of inclination, and the initial date are sufficiently small.

Blow-up solutions in a Keller-Segel type system modelling the chemotaxis phenomenon

Stella Vernier-Piro
Universiy of Cagliari, Italy, Italy
svernier@unica.it
M. Marras and G. Viglialoro

We study a Keller-Segel parabolic system, with time dependent coefficients. This system models a chemotaxis phenomenon. Sufficient conditions on data are introduced to obtain a lower bound for the blow - up time. This is a joint work with Monica Marras and Giuseppe Viglialoro.

Global existence and asymptotic behavior of solution for sixth order Boussinesq equation with damped term

Shubin Wang
Zhengzhou University, Peoples Rep of China
wangshubin@zzu.edu.cn
Xiao Su

We investigate the small-data Cauchy problem for sixth order Boussinesq equation with damped term. We prove the global existence and asymptotic behavior of a solution. Our method and techniques rely upon contracting mapping principle, the dyadic decomposition and some properties of Bessel function.

Global existence and finite time blow up for a class of semilinear pseudo-parabolic equations

Runzhang Xu
Harbin Engineering University, Peoples Rep of China
xurunzh@163.com
Jia Su

In this paper, we study a class of semilinear pseudo-parabolic equations. By introducing a family of potential wells, we prove the invariance of some sets, global existence, nonexistence and asymptotic behavior of solutions with initial energy $J(u_0) \leq d$. Moreover, we obtain finite time blow up with high initial energy $J(u_0) > d$ by comparison principle.

Longtime dynamics for the strongly damped wave equation with critical and supercritical nonlinearities

Zhijian Yang
Zhengzhou University, Peoples Rep of China
yzjzzvt@zzu.edu.cn

We are concerned with the longtime dynamics of the Kirchhoff equation with strong dissipation

$$u_{tt} - M(\|\nabla u\|^2)\Delta u - \Delta u_t + h(u_t) + g(u) = f(x).$$

We prove that the IBVP of the equation admits a unique global weak solution, the related dynamical system has a finite dimensional global attractor and an exponential attractor provided that $h(s)$ is quasi-monotone and both growth exponents p and q of the nonlinearities $h(s)$ and $g(s)$ are up to critical range, that is, $1 \leq p, q \leq p^* = \frac{N+2}{(N-2)^+}$. Moreover, the optimal regularity of the global attractor is established within further restrictions that the above-mentioned growth exponents are equal and up to subcritical range. In particular, when $M(s) = 1$, we further show that the IBVP of the equation possesses a global weak solution provided that the growth exponents p and q are fully supercritical, that is, $p = q > p^*$, and the related generalized semiflow has in natural energy space endowed with strong topology a global attractor.

The global existence of solution to the damped wave equations

Xiongfeng Yang
Shanghai Jiao Tong University, Peoples Rep of China
xf-yang@sjtu.edu.cn

We consider the Cauchy problem for the system of weakly coupled semi-linear damped wave equations with small initial data. The global existence of the solution has been proved by constructing a contract mapping in some suitably function space in the supercritical case, which generalizes the results to the system in dimensional space $n \leq 5$. The proof is based on the estimates of solution operator for the linear damped wave equation.

Blow up in coupled parity-time symmetric nonlinear Schrödinger equations

Dmitry Zezyulin
University of Lisbon, Portugal
d.zezyulin@gmail.com
J.-P. Dias, M. Figueira, V. V. Konotop

We consider the Cauchy problem for a system of two self-focusing nonlinear Schrödinger equations with linear and nonlinear coupling. The system is PT symmetric, which means that one of the equations includes the gain term while another equation accounts for the damping. We find sufficient conditions for the finite time blow up in the supercritical case (three or more spatial dimensions). The proof is based on the virial technique arguments. Several other physically relevant particular cases are also discussed and illustrated numerically.

Special Session 38: Recent Trends in Nonlinear Schrodinger Systems

Benedetta Pellacci, Università degli Studi di Napoli Parthenope, Italy
Gianmaria Verzini, Politecnico di Milano, Italy

In the last twenty years much interest has been devoted to the study of systems of coupled nonlinear Schrödinger equations. This interest is motivated by many physical experiments, e.g. in the theory of Bose-Einstein condensates and in nonlinear optics. In a ultracold dilute Bose gas, condensation in different hyperfine spin states is commonly modeled by the Gross-Pitaevskii system. Different qualitative properties arise, depending on the interactions, which can be attractive or repulsive. The passage of rays along materials induces nonlinear effects. Whereas the single Schrödinger equation well represents the auto-interaction of the beam, coupled Schrödinger equations take into account also interactions with the material. Other models analyze different interactions, such as Liouville and Toda systems, or Schrödinger-Poisson-Slater problems, as well as relativistic versions of the Schrödinger equation, involving non local diffusion. In these contexts, spatial solitary waves have attracted much attention due to their good properties, for instance they are usually least energy solutions, but also the study of solutions at higher energy level is an interesting issue. Different related topics include concentration phenomena in semiclassical limits, singularly perturbed problems for strong competition, classification of entire solutions of elliptic systems.

On a semilinear equation involving the curl-curl operator

Thomas Bartsch
University of Giessen, Germany
Thomas.Bartsch@math.uni-giessen.de
Jaroslaw Mederski

We present recent results on finite energy solutions $A : \Omega \to \mathbb{R}^3$ of the equation

$$\nabla \times (\nabla \times A) + V(x)A = \partial_A f(x, A)$$

on a smooth bounded domain Ω of $\mathbb{R}^3$ with boundary condition $n \times A = 0$ on $\partial\Omega$. Here "$\nabla\times$" denotes the curl operator, $V \in L^\infty_{loc}(\Omega)$ is bounded below, and $f : \Omega \times \mathbb{R}^3 \to \mathbb{R}$ is a superlinear and subcritical nonlinearity; n is the exterior normal to the boundary.

Semi-classical standing waves for nonlinear Schroedinger systems

Jaeyoung Byeon
KAIST, Korea
byeon@kaist.ac.kr
Kazunaga Tanaka

For $N \leq 3$, we consider the following singularly perturbed elliptic system

$$\begin{cases} \varepsilon^2 \Delta u_1 - W_1(x)u_1 + \mu_1(u_1)^3 + \beta u_1(u_2)^2 = 0, \\ \hfill u_1 > 0 \text{ in } \mathbb{R}^N, \\ \varepsilon^2 \Delta u_2 - W_2(x)u_2 + \mu_2(u_2)^3 + \beta u_2(u_1)^2 = 0, \\ \hfill u_2 > 0 \text{ in } \mathbb{R}^N. \end{cases}$$

For certain minmax values of a limiting problem, we show that there exist one bump vector solutions corresponding to the minimax values. This is a joint work with Kazunaga Tanaka in Waseda University.

Asymptotic analysis and sign-changing bubble towers for Lane-Emden problems

Francesca De Marchis
University of Rome 2, Italy
francesca.demarchis@gmail.com
Isabella Ianni, Filomena Pacella

We consider the semilinear Lane-Emden equation on a smooth bounded domain of $\mathbb{R}^2$ when the exponent p is large. In particular we analyze the asymptotic behavior of sign changing solutions as $p \to +\infty$. Among other results we show, under some symmetry assumptions on the domain, that the positive and negative parts of a family of symmetric solutions concentrate at the same point, as $p \to +\infty$, and the limit profile looks like a tower of two bubbles given by a superposition of a regular and a singular solution of the Liouville problem in $\mathbb{R}^2$. This is a joint work with I. Ianni and F. Pacella.

On the sharp effect of attaching a thin handle on the spectral rate of convergence

Veronica Felli
Milano-Bicocca University, Italy
veronica.felli@unimib.it
Laura Abatangelo and Susanna Terracini

Consider two domains connected by a thin tube: it is well known that the resolvent of the Dirichlet Laplacian is continuous with respect to the channel section parameter. This in particular implies the continuity of isolated simple eigenvalues and the corresponding eigenfunctions with respect to domain perturbation. Under an explicit nondegeneracy condition, we improve this information providing a sharp control of the rate of convergence of the eigenvalues and eigenfunctions in the perturbed domain to the relative eigenvalue and eigenfunction in the limit domain. This is a joint paper with L. Abatangelo and S. Terracini.

Soliton dynamics for the a fourth order nonlinear Schroedinger equation

Marco Ghimenti
University of Pisa, Italy
marco.ghimenti@dma.unipi.it
Denis Bonheure

We study the soliton dynamics for a class of nonlinear Schrodinger equations with a biharmonic term singularly perturbed with a parameter epsilon. We prove that soliton solutions under the effect of an external potential exhibit in the semi-classical limit a particle-like behavior when epsilon is small. This problema arises in the context of nonlinear optics

Sign-changing solutions for the Lane-Emden problem with interior nodal line.

Isabella Ianni
Seconda Universita di Napoli, Italy
isabella.ianni@unina2.it
Francesca De Marchis, Filomena Pacella

We consider the semilinear Lane-Emden problem

$$\begin{cases} -\Delta u = |u|^{p-1}u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \qquad (\mathcal{E}_p)$$

where $p > 1$ and Ω is a smooth bounded simply connected domain of $\mathbb{R}^2$. By imposing some symmetry on the domain we show the existence, for p sufficiently large, of sign-changing solutions u_p having two nodal regions and whose nodal line doesn't touch the boundary. The results presented are obtained in collaboration with F. De Marchis (Universita Tor Vergata, Roma) and F. Pacella (Universita Sapienza, Roma).

Normalized solutions for nonlinear Schrodinger sytems

Louis Jeanjean
University of Franche Comte, France
louis.jeanjean@univ-fcomte.fr
T. Bartsch

We consider the existence of solutions in $H^1(R^N) \times H^1(R^N)$ for systems of the form

$$\begin{cases} -\Delta u_1 - \lambda_1 u_1 = \mu_1 |u_1|^{p_1-2}u_1 + r_1\beta|u_1|^{r_1-2}|u_2|^{r_2}u_1 \\ -\Delta u_2 - \lambda_2 u_2 = \mu_2 |u_2|^{p_2-2}u_2 + r_2\beta|u_1|^{r_1}|u_2|^{r_2-2}u_2. \end{cases}$$

Here $N \geq 1$, μ_1, μ_2, r_1, r_2 are given positive constants and $\beta \in R$. For $a_1 > 0, a_2 > 0$ given, we look for solutions satisfying $||u_1||_2^2 = a_1$ and $||u_2||_2^2 = a_2$. In the system λ_1 and λ_2 are unknown and they will appear as Lagrange parameters. This talk describes joint work with T. Bartsch

Multi-Speeds Solitary Waves Solutions For Nonlinear Schrodinger Systems

Stefan Le Coz
Toulouse III University, France
slecoz@math.univ-toulouse.fr
Fanny Delebecque, Isabella Ianni, Rada Maria Weishaeupl

We prove the existence of a new type of solutions to a nonlinear Schrodinger system. These solutions, which we call multi-speeds solitary waves, are behaving at large time as a couple of scalar solitary waves traveling at different speeds. The proof relies on the construction of approximations of the multi-speeds solitary waves by solving the system backwards in time and using energy methods to obtain uniform estimates. We also study numerically the dynamical behavior of these multi-speeds solitary waves

Asymptotically Linear Fractional Schrodinger Equations

Raquel Lehrer
UNIOESTE, Brazil
rlehrer@gmail.com
Liliane de Almeida Maia and Marco Squassina

By exploiting a variational technique based upon projecting over the Pohozaev manifold, we prove existence of positive solutions for a class of nonlinear fractional Schrödinger equations having a nonhomogenous nonautonomous asymptotically linear nonlinearity.

Saturable weakly coupled nonlinear Schrödinger systems

Liliane Maia
University of Brasília, Brazil
lilimaia.unb@gmail.com
Eugenio Montefusco and Benedetta Pellacci

The existence of solutions for a class of weakly coupled Schrödinger system with saturation effect is presented. In most cases it is shown that least energy solutions have necessarily one trivial component. In addition, sufficient conditions for the existence of a solution with both positive components are found. Moreover, new results on semi classical limits of standing waves of equations with a variable saturation function are discussed.

Multiple Constraints and Nonlinear Schrodinger Systems

Eugenio Montefusco
Roma University, Italy
montefusco@mat.uniroma1.it
Liliane Maia, Benedetta Pellacci

We want to discuss the use of multiple constraints in order to describe the set of the non-negative solutions of the model system

$$\begin{cases} -\Delta u_1(x) + \omega_1 u_1(x) = \left(\mu_1 u_1^2(x) + \beta u_2^2(x)\right) u_1(x) \\ -\Delta u_2(x) + \omega_2 u_2(x) = \left(\mu_2 u_2^2(x) + \beta u_1^2(x)\right) u_2(x) \end{cases}$$

where $x \in \mathbb{R}^n$, $n = 1, 2, 3$ and β, μ_i, ω_i (for $i = 1, 2$) are real positive numbers.
Existence and multiplicity of non-negative and positive solutions will be discussed. In particular we will try to characterize the set of the parameters for which solutions, with both positive components, exist.
Moreover we will present also some generalizations and open problems.

On a pseudorelativistic Hartree equation

Dimitri Mugnai
University of Perugia, Italy
dimitri.mugnai@unipg.it

We study a class of pseudo-relativistic Hartree equations, whose prototype in R^3 is the usual Schroedinger-Poisson-Slater system. We show some existence and nonexistence results according to different assumptions on the nonlinearity.

Thomas-Fermi approximation for coexisting two component Bose-Einstein condensates and nonexistence of vortices for small rotation

Benedetta Noris
INdAM COFUND Marie Curie Fellow, Italy
benedettanoris@gmail.com
Amandine Aftalion, Christos Sourdis

We study minimizers of a Gross-Pitaevskii energy describing a two component Bose-Einstein condensate confined in a radially symmetric harmonic trap and set into rotation. We consider the case of coexistence of the components in the Thomas-Fermi regime, where a small parameter ε conveys a singular perturbation. The minimizer of the energy without rotation is determined as the positive solution of a system of coupled PDE's for which we show uniqueness. The limiting problem for $\varepsilon = 0$ has degenerate and irregular behavior at specific radii, where the gradient blows up. By means of a perturbation argument, we obtain precise estimates for the convergence of the minimizer to this limiting profile, as ε tends to zero. For low rotation, based on these estimates, we can show that the ground states remain real valued and do not have vortices, even in the region of small density.

Large mass boundary condensation patterns in the stationary Keller-Segel system

Angela Pistoia
La Sapienza Università di Roma, Italy
angela.pistoia@uniroma1.it

We consider the stationary Keller-Segel system from chemotaxis in a domain of the plane. We establish the existence of a solution which exhibits a sharp boundary layer along the entire boundary of the domain. This is a joint work with Manuel del Pino and Giusi Vaira.

Ground states of a nonlinear curl-curl problem

Wolfgang Reichel
Karlsruhe Institute of Technology, Germany
wolfgang.reichel@kit.edu
Thomas Bartsch, Tomas Dohnal, Michael Plum

We are interested in ground states for the nonlinear curl-curl equation

$$\nabla \times \nabla \times U + V(x)U = \Gamma(x)|U|^{p-1}U \text{ in } \mathbb{R}^3,$$

where $U : \mathbb{R}^3 \to \mathbb{R}^3$. A basic requirement is to find scenarios, where 0 does not belong to the spectrum of the operator

$$\mathcal{L} = \nabla \times \nabla \times +V(x).$$

Under suitable assumptions on V, Γ we construct ground states both for the defocusing case ($\Gamma \leq 0$) and the focusing case ($\Gamma \geq 0$). The main tools are variational methods and the use of symmetries.

Proportionality of components, Liouville theorems and existence results for a class of elliptic systems

Boyan Sirakov
PUC-Rio de Janeiro, Brazil
bsirakov@yahoo.com
Alexandre Montaru, Philippe Souplet

We study qualitative properties of positive solutions of noncooperative, possibly nonvariational, elliptic systems such as

$$\begin{cases} -\Delta u - \mu(x)u &= u^r v^p \left[a(x)v^q - c(x)u^q\right] \\ -\Delta v - \nu(x)v &= v^r u^p \left[b(x)u^q - d(x)v^q\right]. \end{cases}$$

We obtain new classification and Liouville type theorems in the whole Euclidean space, as well as in half-spaces, and deduce a priori estimates and existence of positive solutions for related Dirichlet problems. We significantly improve the known results for a large class of systems involving a balance between repulsive and attractive terms. This class contains systems arising in biological models of Lotka-Volterra type, in physical models of Bose-Einstein condensates and in models of chemical reactions.

References

[1] Arch. Rat. Mech. Anal. (2014), arXiv:1312.1380

Soliton dynamics for nonlocal Schrodinger equations

Marco Squassina
University of Verona, Italy
marco.squassina@univr.it

By means of suitable modulational stability estimates, we investigate soliton dynamics for the Schrodinger-Newton problem, for the generalized Choquard equation as well as the fractional Schrodinger equation.

Singularly perturbed nonlinear elliptic problems in expanding tubular type domains

Kazunaga Tanaka
Waseda University, Japan
kazunaga@waseda.jp
Jaeyoung Byeon

We consider the following nonlinear elliptic problems:

$$-\Delta u + u = u^p \text{ in } \Omega_\varepsilon, \quad u = 0 \text{ on } \partial\Omega_\varepsilon.$$

Here p is subcritical and Ω_ε is a bounded domain with a smooth boundary which is expanding as $\varepsilon \to 0$. We consider a situation where a cylindrical domain $\mathbf{R}^k \times D$ appears as a limit of Ω_ε. We show the existence of a family of solution $(u_\varepsilon)_\varepsilon$ which converges to a solution of the limit problem in a cylindrical domain $\mathbf{R}^k \times D$ after suitable translations. This talk is based on my joint works with Jaeyoung Byeon (KAIST, Korea).

Existence and regularity of solutions to optimal partition problems involving Laplacian eigenvalues

Hugo Tavares
IST, University of Lisbon, Portugal
htavares@math.ist.utl.pt
Miguel Ramos, Susanna Terracini

Let $\Omega \subset \mathbb{R}^N$ be a open bounded domain and $m \in \mathbb{N}$. Given $k_1, \ldots, k_m \in \mathbb{N}$, we consider the following optimal partition problem

$$\inf \big\{\Phi(\omega_1, \ldots, \omega_m) := \sum_{i=1}^{k} \lambda_{k_i}(\omega_i) : (\omega_1, \ldots, \omega_k) \in \mathcal{P}_m(\Omega)\big\},$$

where $\lambda_{k_i}(\omega_i)$ denotes the k_i–th eigenvalue of $(-\Delta, H_0^1(\omega_i))$ counting multiplicities, and $\mathcal{P}_m(\Omega)$ is the set of all sub-partitions of Ω, namely

$$\mathcal{P}_m(\Omega) = \{(\omega_1, \ldots, \omega_m) : \ \omega_i \subset \Omega \text{ open}, \omega_i \cap \omega_j = \emptyset \ \forall i \neq j\}.$$

We prove the existence of an optimal partition $(\omega_1, \ldots, \omega_m)$, proving as well its regularity in the sense that the free boundary $\Omega \cap \cup_{i=1}^m \partial\omega_i$ is, up to a residual set, locally a $C^{1,\alpha}$ hyper-surface. In order to prove this result, we first show existence and regularity for a general optimal partition problem involving eigenvalues where there is an underlying invariance with respect to orthogonal transformations. This class of problems includes the one with cost function

$$\Phi_p(\omega_1, \ldots, \omega_m) = \sum_{i=1}^{m} \left(\sum_{j=1}^{k_i} (\lambda_j(\omega_i))^p \right)^{1/p},$$

whose solutions approach the solutions of the original problem as $p \to \infty$. The study of this new class of problems is done via a detailed study of a Schrodinger-type system which models competition between different groups of possibly sign-changing components. An optimal partition appears as the nodal set of the components, as the competition parameter becomes large. The proofs also involve new boundary Harnack principles on NTA and Reifenberg flat domains, as well as an extensive use of Almgren's monotonicity–type formulas.

Geometric aspects and a priori estimates for strongly competing systems with non standard diffusiona

Susanna Terracini
University of Torino, Italy
susanna.terracini@unito.it
G. Verzini, A. Zilio

Fractional laplacians arise in some models of enhanced anomalous diffusion, when the Gaussian statistics of the classical Brownian motion is replaced by a different one, giving rise to the Lévy jumps (or flights). As such operators are of real interest both in population dynamics and in relativistic quantum electrodynamics, we plan to extend the theory in this direction. We shall report about the asymptotic analysis and the study of the nodal set in case of fractional laplacians, New tools, involving different extremality conditions and new monotonicity formulas, are needed to attack this problem and raise new challenging spectral problems. Contrary to the usual competition-diffusion cases, we observe the emergence of dramatically different phenomena depending on the type of competitive interaction.

Standing waves with prescribed mass for the Gross-Pitaevsii system: existence and stability issues.

Gianmaria Verzini
Politecnico di Milano, Italy
gianmaria.verzini@polimi.it
Benedetta Noris, Hugo Tavares

We consider the problem of finding $(\lambda_1, \lambda_2, u_1, u_2)$ such that:

$$\begin{cases} \Delta u_1 + V_1(x)u_1 + \mu_1 u_1^3 + \beta u_1 u_2^2 = \lambda_1 u_1, \\ \Delta u_2 + V_2(x)_2 + \mu_2 u_2^3 + \beta u_2 u_1^2 = \lambda_2 u_2, \\ \int_\Omega u_1^2 \, dx = \rho_1, \quad \int_\Omega u_2^2 \, dx = \rho_2, \end{cases}$$

on the whole $\mathbb{R}^N$, $N = 2, 3$ (with trapping potentials), or in some bounded domain with Dirichlet boundary conditions. We study existence of solutions for different ranges of the parameters (focusing/defocusing, competitive/cooperative, weak/strong interaction, small/big mass). For some selected family of solutions we prove orbital stability.

Sign-changing solutions for nonlinear elliptic systems

Zhi-Qiang Wang
Utah State University, USA
zhi-qiang.wang@usu.edu
Jiaquan Liu and Xiangqing Liu

We present recent work on multiplicity results on sign-changing solutions for a class of nonlinear Schrodinger type systems in the repulsive case.

Positive solutions for an asymptotically linear Schrödinger-Poisson system

Fubao Zhang
Southeast University, Peoples Rep of China
zhangfubao@seu.edu.cn
Miao Du

In this talk, we study the following Schrödinger-Poisson system

$$\begin{cases} -\Delta u + V(x)u + K(x)\phi u = f(x,u), & x \in \mathbb{R}^3, \\ -\Delta \phi = K(x)u^2, & x \in \mathbb{R}^3, \end{cases}$$

where $f(x,s)$ is asymptotically linear with respect to s at infinity. Under appropriate assumptions on V and K, the existence and nonexistence results are obtained via variational methods.

Uniform Lipschitz bounds for strongly competing systems

Alessandro Zilio
Politecnico di Milano, Italy
alex.zilio86@gmail.com
Nicola Soave

We present some new optimal regularity results regarding the family of systems

$$-\Delta u_{i,\beta} = f_{i,\beta}(x, u_{1,\beta}, \dots, u_{k,\beta}) - \beta u_{i,\beta} \sum_{j \neq i} a_{ij} u_{j,\beta}^p$$

in the case $p = 1$ and $p = 2$. For such systems, of interest in the study of phase-separation and pattern-formation phenomena, we show that under very mild assumptions on the non-linear terms $f_{i,\beta}$, uniform L^∞ bounds imply corresponding uniform Lipschitz bounds. These results extend the regularity theory available in the literature to the optimal case.

Special Session 39: Interfaces in Fluid Mechanics

Helmut Abels, University of Regensburg, Germany
Harald Garcke, University of Regensburg, Germany

Interfaces separating different fluids and the interaction of a fluid with an elastic material arise in many situations. This leads to a coupling of fundamental equations in fluid dynamics with geometric evolution equations for the interface or the free boundary of the domain. The goal of this session is to bring together experts and young researchers working on various models in this field ranging e.g. from diffuse and sharp interface models for two phase flows to fluid-elastic structure interaction.

Sphere-like free surfaces in three dimensions

Frederic Abergel
Ecole Centrale Paris, France
frederic.abergel@ecp.fr

In the absence of surface tension, severe difficulties arise when trying to solve the stationary free surface equations for a Navier-Stokes flow, in particular when the free surface is supposed to be a compact, connected hypersurface in $\mathbf{R}^3$. Let ρ be a function defining a graph above the unit sphere S_0 (or any reasonable, smooth, sphere-like surface Σ_0), so that the domain Ω_ρ occupied by the fluid is just the interior of a smooth, compact and connected surface. In this talk, I will focus on a local approach, assuming that there exists a parameter (e.g. a force term, an angular velocity...) such that, for a given parameter value, there exists a particular solution $\rho_0 \equiv 0$ to the free surface problem. The question of interest is then the existence of a solution in a neighbourhood of ρ_0. The main object will be the linearized free surface operator $\mathcal{L}$, and the main question is that of the invertibility of $\mathcal{L}$. Unfortunately, the method introduced in a previous work (Abergel, Bailly) fails in this context, as the vectorfield u has to vanish at some points (Poincaré-Bendixon's theorem). In the light of results by Holcman and Kupka, I will give some sufficient conditions ensuring that the linearized operator $\mathcal{L}$ is invertible and hence, that the free surface problem under scrutiny has solutions.

The Entropy Principle for Fluid Systems

Hans Wilhelm Alt
Technical University Munich, Germany
alt@ma.tum.de

The entropy inequality in fluids has its counterpart on interfaces, for example, in the positivity of the coefficient in the mean curvature flow. We give an unified distributional formulation of this principle which is consistent with phase field approximations. We refer also to cases where the entropy principle in the limit splits into several inequalities. Therefore, the entropy principle for regular solutions is the only statement one has to postulate.

Bounded traveling waves for a thin film with gravity and insoluble surfactant

Joachim Escher
Leibniz University Hannover, Germany
escher@ifam.uni-hannover.de
Matthieu Hillairet, Philippe Laurencot, Christoph Walker

Lubrication equations for a surfactant-driven flow of a thin layer of fluid are considered with a singular surfactant-dependent surface tension. It is shown that there exists a bounded traveling wave solution which connects a fully surfactant-coated film to an uncoated film. The regularity of the traveling wave depends on whether or not surface diffusion of the insoluble surfactant is included.

The coupling of Poisson-Boltzmann and Stokes equations modelling a drop of electrolyte solutions in an electric field

Marco Fontelos
ICMAT, Spain
marco.fontelos@icmat.es
Lucia B. Gamboa

In an electrolyte solution, such as salt in water, ions are responsible for an increase of the electrical conductance of the liquid. When the ions' mobility is large, the distribution of the concentration of ions in the fluid is modelled by Poisson-Botzmann equation, which is a nonlinear and nonlocal elliptic equation. We prove the existence of solutions to such equation and, more interestingly, deduce asymptotic expansions for the distribution of ions near the interface separating the liquid mass from the outer medium. This allows us to couple in a suitable manner the bulk concentration of ions with Stokes equations to produce an effective model for the evolution of the surface of a drop filled with an ionic solution and subject to electric fields. As an application, we numerically compute the onset of Rayleigh jets emerging from a drop supercritically charged and compute important physical quantities such as their velocity, diameter and total charge. The numerical procedure consists of a Boundary Integral Method.

A diffuse interface model related to tumor dynamics

Sergio Frigeri
WIAS-Berlin, Italy
SergioPietro.Frigeri@wias-berlin.de
Maurizio Grasselli, Elisabetta Rocca

A diffuse interface model related to tumor growth dynamics and due to Hawkins Daarud, van der Zee and Oden will be discussed. The analysis will concern existence and uniqueness of weak solutions, strong solutions and asymptotic behavior (existence of the global attractor).

On finite speed of propagation and waiting time phenomena for the stochastic porous medium equation

Günther Grün
Erlangen University, Germany
gruen@am.uni-erlangen.de
Julian Fischer

We study finite speed of propagation and waiting time phenomena for the stochastic porous-media equation with linear multiplicative noise. Based on a novel iteration technique and on stochastic counterparts of weighted integral estimates used in the deterministic setting, we formulate a sufficient criterion on the growth of initial data which locally guarantees a waiting time phenomenon to occur almost surely. Up to a logarithmic factor, this criterion coincides with the optimal criterion known from the deterministic setting. Our technique can be modified to prove finite speed of propagation as well.

Reduced models for the propagation of fluids in the presence of a contact point

Hans Knuepfer
University of Heidelberg, Germany
knuepfer@uni-heidelberg.de

The propagation of a liquid drop on a plate is characterized by the evolution of the three-phase contact line where air, liquid and solid meet. The region occupied by the liquid is given by time-dependent domain with non-smooth boundary. We consider well-posedness and regularity for certain evolution models describing the propagation of the liquid. We focus on simplified models we focus on model for fluid propagation in the lubrication approximation.

Recent results to stability of stationary solutions to compressible diffuse interface models

Matthias Kotschote
University Konstanz, Germany
matthias.kotschote@uni-konstanz.de

In this talk we consider the so-called Navier-Stokes-Allen-Cahn as well as Navier-Stokes-Cahn-Hilliard system. These equations are combinations of the compressible Navier-Stokes equations with a Allen-Cahn or Cahn-Hilliard phase field description. These models admit of describing two-phase patterns in a flowing liquid with or without phase transformations. The main focus of this talk relies on investigating linear equations that are obtained by considering the deviation from equilibrium. The spectrum of the associated linear operators is analysed. The purpose is then to conclude linear stability/instability by using the localization of the spectrum.

Dynamics of Membrane Configurations in Incompressible Newtonian Fluids

Matthias Köhne
Heinrich-Heine-Universität Düsseldorf, Germany
koehne@math.uni-duesseldorf.de
Daniel Lengeler

We study a model that describes the quasi-stationary evolution of a membrane formed by a lipid bilayer in a surrounding incompressible Newtonian fluid. The membrane is thereby modeled as a sharp interface that encloses a part of the liquid - the vesicle. In contrast to fluid interfaces the dynamics of such a membrane are rather driven by elasticity than by surface tension. Moreover, the model incorporates the conservation not only of the vesicle volume but also of the membrane area, which leads to an incompressibility constraint on the interface. After a short introduction of the model we prove its (local-in-time) well-posedness in an L_p-setting. Our approach is based on the direct mapping technique (i.e. a Hanzawa transformation), maximal regularity results for a suitable linearization and a fixed-point argument.

Dynamics of fluid membranes

Daniel Lengeler
University of Regensburg, Germany
daniel.lengeler@mathematik.uni-regensburg.de
Matthias Köhne

I will present several models describing the dynamics of a fluid membrane immersed in a bulk fluid. Such a membrane consists of a two-layered sheet of phospholipid molecules and can be modelled mathematically as one or two Boussinesq-Scriven surface fluids living on a moving interface and subject to bending stresses of Willmore type. These models give rise to interesting mathematical problems in the analysis of elliptic, parabolic, and wave type pdes.

Initial-Boundary Value Problem for the Fully-Coupled Navier-Stokes/Q-Tensor System

Yuning Liu
Regensburg university, Germany, Germany
liuyuning850314@163.com
Helmut Abels Georg Dolzmann

We prove short-time well-posedness and existence of global weak solutions of the Beris–Edwards model for nematic liquid crystals in the case of a domain with inhomogeneous Dirichlet boundary conditions. The system consists of the Navier-Stokes equations coupled with an evolution equation for the Q-tensor. The solutions possess higher regularity in time of order one compared to the class of weak solutions with finite energy. This regularity is enough to obtain Lipschitz continuity of the non-linear terms in the corresponding function spaces. Therefore the well-posedness is shown with the aid of the contraction mapping principle using that the linearized system is an isomorphism between the associated function spaces.

The two-phase porous medium equation: asymptotic behavior of weak solutions

Bogdan Matioc
Leibniz University Hanover, Germany
matioc@ifam.uni-hannover.de
Philippe Laurencot

We discuss the asymptotic behavior of globally defined non-negative weak solutions of the two-phase porous medium equation

$$\begin{cases} \partial_t f &= \partial_x \left(f \partial_x \left((1+R) f + Rg \right) \right), \\ \partial_t g &= R_\mu \partial_x \left(g \partial_x \left(f + g \right) \right), \end{cases}$$

where $(t,x) \in (0,\infty) \times \mathbb{R}$ and R and R_μ are positive parameters. This strongly coupled degenerate parabolic system was obtained as the thin film approximation of the Muskat problem. The existence of globally defined non-negative weak solutions is established by interpreting the system of evolution equations as a gradient flow for the L_2-Wasserstein distance. This system possesses a rich landscape of self-similar symmetric and non-symmetric Barenblatt type profiles. We show that any non-negative global solution converges in the large towards a self-similar solution of the problem.

Generalized Riemann Solvers for Compressible Liquid-Vapour Flow with Phase Transition and Surface Tension

Christian Rohde
University of Stuttgart, Germany
crohde@mathematik.uni-stuttgart.de

The dynamics of compressible liquid-vapour flow with phase transition and surface tension can be described for ideal fluids by the Euler equations in the bulk and appropriate interface conditions. The interface conditions consist not only of the standard Rankine-Hugoniot relations but contain a dynamic version of the Young-Laplace law and –more important– a generalized form of the Gibbs-Thomson relation such that one gets a complex free boundary value problem.
For compressible multiphase flow lots of numerical methods that rely on e.g. front tracking or ghostfluid ideas have been suggested in recent years. The core ingredient of most of these methods are Riemann solvers at the interface. For the situation at hand exact Riemann solvers do not exist or are computationally much too expensive. To circumvent this difficulty we present new approximative solvers. The solvers are tested on various flow examples in one and two space dimensions. This is joint work with C. Chalons, F. Coquel, and C. Zeiler.

On multi-phase flow with surfactant

Bjorn Stinner
University of Warwick, England
bjorn.stinner@warwick.ac.uk

Recent results on phase field models for multi-phase flow with soluble surfactant will be presented. The diffuse interface approach is considered as an approximation to a free boundary problem. The focus lies on correctly recovering interfacial equations and conditions in the sharp interface limit, particularly with respect to the surfactant. Partial rigorous results have been obtained in the case that the motion of the interface is given. The presentation will be rounded off by some heuristics for the full system that are supported by numerical results.

Support splitting and non-splitting phenomena in the initial-boundary value problem for a porous media equation

Kenji Tomoeda
Osaka Institute of Technology, Japan
tomoeda@ge.oit.ac.jp

We are concerned with the dynamical behaviour of nonstationary seepage in the flow through porous media. It is expected that the seepage of the fluid exhibits "*support splitting and non-splitting phenomena*", which are caused by the interaction between the nonlinear diffusion and the penetration of the fluid from the boundary on which the flowing tide and the ebbing tide occur. Here the support means the region where the fluid exists. The model equation which describes such phenomena is written in the form of the initial-boundary value problem for a porous media equation. We treat it in the one-dimensional case, and demonstrate some numerical examples which show "*repeated support splitting and merging phenomena*" and "*non-splitting phenomena*". From mathematical points of view we show the stabilization theorem; that is, the solution converges to the unique stationary solution as the time tends to the infinity. Moreover, we state some sufficient conditions imposed on the boundary value under which such phenomena appear.

Well posedness of the linearized problem for MHD contact discontinuities

Paola Trebeschi
University of Brescia, Italy
paola.trebeschi@unibs.it
Alessandro Morando, Yuri Trakhinin

We study the free boundary problem for contact discontinuities in ideal compressible magnetohydrodynamics (MHD). They are characteristic discontinuities with no flow across the discontinuity for which the pressure, the magnetic field and the velocity are continuous whereas the density and the entropy may have a jump. Under the Rayleigh-Taylor sign condition on the jump of the normal derivative of the pressure, $[\partial p/\partial N]$ less than zero, satisfied at each point of the unperturbed contact discontinuity, we prove the well-posedness in Sobolev spaces of the linearized problem for 2D planar MHD flows. The result obtained is a joint work with Alessandro Morando and Yuri Trakhinin.

A Proof of a Sharp Interface Limit for Compressible Phase Change Flows

Gabriele Witterstein
Technical University Munich, Germany
gw@ma.tum.de

We consider diffusive interface models describing compressible two-phase flows with a mass exchange. The equations consist of the compressible Navier-Stokes system coupled with the Allen-Cahn equation. We prove in the distributional sense the convergence to the corresponding sharp interface model in the one dimensional case. Here a small parameter $\delta > 0$ is related to the interface thickness and in the limit a free boundary arises which in higher space dimensions is governed by the Gibbs-Thomson law.

Special Session 40: Qualitative Aspects of Nonlinear Elliptic and Parabolic Problems

Marta Garcia-Huidobro, Pontificia Universidad Catolica de Chile, Chile
Raul Manasevich, University of Chile, Chile

The aim of this session is to discuss properties of solutions to various types of elliptic and parabolic PDEs. Topics will include analysis of singularities, Liouville-type theorems, blow-up, convergence of solutions, etc.

Higher Dimensional Catenoid, Liouville Equation and Allen-Cahn Equation

Oscar Agudelo
West Bohemia University, Czech Rep
oiagudel84@gmail.com
Manuel del Pino, Juncheng Wei

In this lecture we present a family of entire solutions to the Allen-Cahn equation in $\mathbb{R}^{N+1}$ for $N \geq 3$, whose level set approaches the higher dimensional catenoid in a compact region and has two logarithmic ends governed by the solutions to the Liouville equation.

A priori estimates and initial trace for a Hamilton-Jacobi equation with gradient absorption terms

Marie-Francoise Bidaut-Veron
Universite de Tours, France, France
veronmf@univ-tours.fr

Here we consider the nonnegative solutions of the parabolic Hamilton-Jacobi equation

$$u_t - \Delta u + |\nabla u|^q = 0$$

in $\Omega \times (0,T)$ where $\Omega = \mathbb{R}^N$ or Ω is a bounded domain of $\mathbb{R}^N$, $q > 0$. We give new a priori local or global estimates for solutions, without conditions as $|x| \to \infty$ or $x \to \partial\Omega$, and corresponding existence results with initial data measure. We study the existence of an initial trace. We show that all the solutions admit a trace as a Borel measure (S, u_0) : there exist a set $\mathcal{S} \subset \Omega$ such that $\mathcal{R} = \Omega \backslash \mathcal{S}$ is open, and a (possibly unbounded) measure $u_0 \in \mathcal{M}^+(\mathcal{R})$, such that

$$\lim_{t \to 0} \int_{\mathcal{R}} u(.,t)\psi = \int_{\mathcal{R}} \psi d\mu_0, \quad \forall \psi \in C_c^0(\mathcal{R}),$$

$$\lim_{t \to 0} \int_{\mathcal{U} \cap \mathcal{S}} u(.,t)dx = \infty, \quad \forall \mathcal{U} \text{ open} \subset \Omega, \text{s.th. } \mathcal{U} \cap \mathcal{S} \neq \emptyset.$$

We give more generally existence results of solutions with such a trace, according to assumptions on u_0, and give their behaviour as $|x| \to \infty$. In particular we construct a solution with trace $(\mathbb{R}^{N+}, 0)$. When $q \leq 1$, we show that $\mathcal{S}$ is empty.

On sign-changing solutions for elliptic equations with complex nonlinearities

Vladimir Bobkov
University of Rostock, Germany
bobkovve@gmail.com

We consider the question of the existence of sign-changing solutions for elliptic problems of the following type

$$\begin{cases} -\Delta_p u = f_\lambda(x,u), & x \in \Omega, \\ u|_{\partial\Omega} = 0, \end{cases} \qquad (\mathcal{D})$$

where $f_\lambda(x,u)$ has two forms: 1) indefinite nonlinearity $f(x,u) = \lambda|u|^{p-2}u + f(x)|u|^{\gamma-2}u$, where $f \in L^\infty(\Omega)$ changes the sign and $p < \gamma < p^*$; 2) convex-concave nonlinearity $f_\lambda(x,u) = \lambda|u|^{q-2}u + |u|^{\gamma-2}u$, where $1 < q < p < \gamma < p^*$. Using the variational and topological approaches we obtain the continuous branch of sign-changing solutions to $(\mathcal{D})$ on non-local spectral interval $(-\infty, \lambda^*)$. The obtained solutions have precisely 2 nodal domains and least energy property.

Stationary solutions of a nonlocal, nonhomogenous equation.

Carmen Cortazar
U. Catolica de Chile, Chile
ccortaza@mat.puc.cl
M. Elgueta, J. Garcia- Melian and S. Martinez

We consider the following nonlocal equation

$$\int_{\mathbb{R}^N} J\left(\frac{x-y}{g(y)}\right)\frac{u(y)}{g(y)}dy - u(x) = 0$$

where J is an even, compactly supported, Hölder continuous probability kernel and g is a continuous and positive function. We study the solutions depending on the behavior of g.

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A one side superlinear critical resonant problem with small forcing term

Mabel Cuesta
Universite du Littoral Cote d'Opale ULCO, France
cuesta@lmpa.univ-littoral.fr
C. De Coster

Non homogeneous superlinear resonant problems were first studied by [4] and more recently by [3, 2, 1]. The techniques used in these papers are deeply related to the linear character of the differential operator as well as on

the simplicity of the first eigenvalue and the positivity of the corresponding eigenfunction. So the questions we are interested in are the extension of these results to other differential operators such as the p-Laplacian problem

$$\begin{cases} -\Delta_p u = \lambda_1 |u|^{p-2}u + (u^+)^q - f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $p > 1$ and $-\Delta_p u := -\text{div}\,(|\nabla u|^{p-2}\nabla u)$, or to the fourth order problem

$$\begin{cases} \Delta^2 u = \lambda_1 u + (u^+)^q - f & \text{in } \Omega, \\ u = 0, \quad \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases}$$

for which, depending on the domain, we can have sign changing first eigenfunction and multiple first eigenvalue.

References

[1] M. Cuesta and C. De Coster, *A resonant-superlinear elliptic problem revisited*, Advanced Nonlinear Studies 13 (2013), 97-114.

[2] M. Cuesta, D. de Figueiredo and P.N. Srikanth, *On a resonant-superlinear elliptic problem*, Calc. Var. Partial Differential Equations 17 (2003), 221-233.

[3] D. de Figueiredo and Yang Jianfu, *Critical Superlinear Ambrosetti-Prodi Problems*, Top. Meth. Nonlin. Anal. 14 (1999), 59-80.

[4] R. Kannan and R. Ortega, *Landesman-Lazer conditions for problems with "one-sided unbounded" nonlinearities*, Nonl. Anal. T.M.A. 9 (1985), 1313-1317.

An extension of a Theorem of V. Sverák to variable exponent spaces

Julian Fernandez Bonder
Universidad de Buenos Aires, Argentina
jfbonder@dm.uba.ar
C. Baroncini

In 1993, V. Sverák proved that if a sequence of uniformly bounded domains $\Omega_n \subset \mathbb{R}^2$ such that $\Omega_n \to \Omega$ in the sense of the Hausdorff complementary topology, verify that the number of connected components of its complements are bounded, then the solutions of the Dirichlet problem for the Laplacian with source $f \in L^2(\mathbb{R}^2)$ converges to the solution of the limit domain with same source. In this talk, we present the extension of Sveráks' result to variable exponent spaces.

On entire solutions of elliptic systems

Roberta Filippucci
University of Perugia, Italy
roberta.filippucci@unipg.it

In this talk we give a priori estimates and Liouville type results for solutions of elliptic inequalities and systems of inequalities with gradient dependence for the forcing terms via the test function method. Our results include both the p-Laplacian operator and the mean curvature operator. No assumptions on symmetry or behavior at infinity of the solutions are required.

Optimal Liouville theorems for semilinear equations in exterior domains

Jorge Garcia-Melian
Universidad de La Laguna, Spain
jjgarmel@ull.es
Salomon Alarcon, Alexander Quaas

We consider the question of nonexistence of positive supersolutions of the equation

$$-\Delta u = f(u) \qquad \text{in } \mathbb{R}^N \setminus B_R \tag{1}$$

where $N \geq 3$ and f is a continuous, positive function defined in $(0, \infty)$. For the model case $f(t) = t^p$, $p > 0$, it is well-known that positive supersolutions do not exist provided $p \leq \frac{N}{N-2}$, while they can be constructed if $p > \frac{N}{N-2}$ (see [2] for a simple proof). It has also been proved that this holds true if $f(t)$ is comparable to a power near $t = 0$, even with some more general fully nonlinear second order operators (cf. [1]). We extend this result to arbitrary nonlinearities, and show that if the condition

$$\int_0^\delta \frac{f(t)}{t^{\frac{2(N-1)}{N-2}}} dt = \infty$$

holds for some small positive δ, then no positive supersolutions of (1) exist. This condition is shown to be optimal. We also consider some generalizations of this result to problems with gradient terms and/or weights, and to more general operators like the $p-$Laplacian or the Pucci's maximal operators. The case $N = 2$ is also briefly considered.

References

[1] S. N. Armstrong, B. Sirakov, *Nonexistence of positive supersolutions of elliptic equations via the maximum principle.* Comm. Partial Differential Equations 36 (2011), no. 11, 2011–2047.

[2] P. Quittner, P. Souplet, *Superlinear parabolic problems. Blow-up, global existence and steady states.* Birkhäuser Advanced Texts: Basel Textbooks, Birkhäuser Verlag, Basel, 2007.

Solutions for a semilinear elliptic equation in dimension two with supercritical growth

Ignacio Guerra
Universidad de Santiago de Chile, Chile
ignacio.guerra@usach.cl
Manuel del Pino and Monica Musso

We consider the problem

$$-\Delta u = \lambda u e^{u^p}, \quad u > 0, \quad \text{in} \quad \Omega,$$
$$u = 0 \quad \text{on} \quad \partial\Omega,$$

where $\Omega \subset \mathbb{R}^2$ and $p > 2$. Let λ_1 be the first eigenvalue of the Laplacian. For each $\lambda \in (0, \lambda_1)$, we prove the existence of solutions for p sufficiently close to 2. In the case of Ω a ball, we also describe numerically the bifurcation diagram (λ, u) for $p > 2$.

Regularized nonlinearly elastic fractures

Duvan Henao Manrique
Pontificia Universidad Catolica de Chile, Chile
dhenao@mat.puc.cl
Carlos Mora-Corral, Xianmin Xu

In a series of papers (ARMA 197, 2010, p. 619; ARMA 201, 2011, p. 575; ACV 5, 2012, 355) we proposed, jointly with Carlos Mora-Corral, a variational model for the initiation of fracture in elastic incompressible materials such as rubber and other polymers, and in ductile metals like iron, steel or others commonly used in the industry. In our model the propagation of cracks is regarded as a free-discontinuity problem where the different fracture mechanisms result from the minimization of a highly non-convex elastic energy in the space of vector-valued special functions with bounded variation. In order to recover the weak continuity of the Jacobian determinant, lost in the analysis of incompressible materials, we add a surface energy term coming from geometric measure theory. The proposed model is satisfactory from the analytic and physical viewpoints, but it is important to be able to compute the solutions to the minimization problem numerically. In this talk we present a joint work with Carlos Mora-Corral and Xianmin Xu which shows how the theoretical model can be approximated by one in which the discontinuities associated to the fracture mechanisms are replaced by tubular zones of width ε where large gradients are allowed (as in the Ambrosio-Tortorelli regularized model for image segmentation in Computer Vision). It will be seen in the talk that the regularized model can indeed be numerically studied and that their solutions converge to those of the theoretical model as $\varepsilon \to 0$.

Uniqueness of the limit for an asymptotically autonomous semilinear equation on $\mathbb{R}^N$

Pilar Herreros
P. Universidad Catolica de Chile, Chile
pherrero@mat.puc.cl
Carmen Cortázar and Marta García-Huidobro

We consider a parabolic equation of the form

$$u_t = \Delta u + f(u) + h(x,t), \quad (x,t) \in \mathbb{R}^N \times (0,\infty)$$
$$u(x,t) \geq 0 \quad \text{for all } (x,t) \in \mathbb{R}^N \times (0,\infty).$$

where $f \in C^1(\mathbb{R})$ is such that $f(0) = 0$, $f'(0)$ negative and h is a suitable function on $\mathbb{R}^N \times (0,\infty)$. We show that under certain conditions, each globally defined and nonnegative bounded solution u converges to a single steady state.

Sign changing radial solutions on R^N for an equation with a p-Laplace operator and weights

Raul Manasevich
University of Chile, Chile
manasevi@dim.uchile.cl
C. Cortazar, J. Dolbeault, and M. Garcia-Huidobro

Radial solutions that change sign for a general elliptic equation involving a weighted p-Laplace operator with a subcritical nonlinearity are considered. We prove existence of solutions with any prescribed number of nodes based on a change of variables in the phase plane, a very general computation of an angular velocity and new estimates for the decay of an energy associated with an asymptotic Hamiltonian problem. We cover the case of solutions which are not compactly supported or have compact support. Joint work with C. Cortázar, J. Dolbeault, and M. García-Huidobro

Diffusion problems of concave–convex nature: existence and multiplicity of solutions.

Jose Sabina de Lis
Universidad de La Laguna, Spain
josabina@ull.es
J. Garcia–Melian, J. Rossi, S. Segura

In this talk several kinds of reaction–diffusion problems of concave–convex type are going to be discussed. In some of them, nonlinearity involves a variable exponent. In other ones, concave and convex effects are separately distributed in the reference domain and its boundary.

Entire solutions of polyharmonic equations with superlinear but subcritical growth

Paul Schmidt
Auburn University, USA
pgs@auburn.edu
M. Lazzo

Much of the literature on entire solutions for semilinear elliptic equations with superlinear growth focuses on positive solutions (or "ground states"). Typically, positive entire solutions do not exist if the growth of the nonlinearity is subcritical in a certain sense. A natural question, then, is whether there are sign-changing entire solutions in such cases. We will present recent and ongoing work on the existence, uniqueness up to scaling and symmetry, and asymptotic behavior of oscillatory entire radial solutions for a subcritical biharmonic equation with power nonlinearity and discuss possible generalizations to the polyharmonic case.

Uniqueness of a positive radial solutions of some elliptic problems and its nondegeneracy

Naoki Shioji
Yokohama National University, Japan
shioji@ynu.ac.jp

We consider the problem

$$u''(r) + \frac{f'(r)}{f(r)}u'(r) - g(r)u(r) + h(r)|u(r)|^{p-1}u(r) = 0,$$

$r \in (0, R)$ and $u(0) \in \mathbb{R}$, $u(R) = 0$, where $R \in (0, \infty]$, $p > 1$ and f, g, h are appropriate functions. In the case of $R = \infty$, $u(R) = \infty$ means $u(r) \to 0$ as $r \to \infty$. We study the uniqueness of positive solutions of the problem and we apply it to various examples. We also study its nondegeneracy not only in radial spaces but also in nonradial spaces. This is a joint work with Kohtaro Watanabe.

Uniqueness of sign-changing radial solutions for a semilinear elliptic equation with an exponent near 1 in some 2-dimensional annulus

Satoshi Tanaka
Okayama University of Science, Japan
tanaka@xmath.ous.ac.jp

The Dirichlet problem for a semilinear elliptic equation in a semilinear in 2-dimensional annulus is considered. It is shown that, for each positive integer k, the problem has a unique radial solution having exactly $k-1$ nodes on some annulus, provided the exponent is near 1.

Semilinear fractional elliptic equations involving measures

Laurent Veron
Universite Francois Rabelais, France
veronl@univ-tours.fr
Huyuan Chen

We study the existence of weak solutions to (E) $(-\Delta)^\alpha u + g(u) = \nu$ in a bounded regular domain Ω in $\mathbb{R}^N (N \geq 2)$ which vanish in $\mathbb{R}^N \setminus \Omega$, where $(-\Delta)^\alpha$ denotes the fractional Laplacian with $\alpha \in (0, 1)$, ν is a Radon measure and g is a nondecreasing function satisfying some extra hypotheses. When g satisfies a subcritical integrability condition, we prove the existence and uniqueness of weak solution for problem (E) for any measure. In the case where ν is Dirac measure, we characterize the asymptotic behavior of the solution. When $g(r) = |r|^{k-1}r$ with k supercritical, we show that a condition of absolute continuity of the measure with respect to some Bessel capacity is a necessary and sufficient condition in order (E) to be solved.

Special Session 41: Topological and Variational Methods for Multivalued Differential Equations

Irene Benedetti, Dipartimento di Matematica e Informatica, University of Perugia, Italy
Valentina Taddei, Dipartimento di Scienze Fisiche, Informatiche e Matematiche, University of Modena and Reggio Emilia, Italy

Multivalued differential equations arise naturally in various branches of modern mathematics, indeed they play a significant role in the description of processes in control theory and non-smooth analysis. Due to their wide applicability in physics, biology, chemistry, and economics, there has recently been an increasing interest in this field. The aim of this Special Session is to outline the recent progress on the field of differential inclusions, both in infinite and finite dimensional spaces. In particular, topics of interest are control problems, non-smooth variational analysis, differential equations with discontinuous nonlinearities, functional inclusions, nonlocal problems, and boundary value problems in bounded and unbounded domains. Due to the diversity of applications and the variety of problems, there is a wide range of methods and techniques available. In this Session, both variational methods (e.g. critical point theory, linking theorems) as well as topological methods (e.g. fixed points theorems, lower and upper solutions, topological degree) will be presented and discussed.

Hartman-type conditions for multivalued Dirichlet problem

Jan Andres
Palacky University, Olomouc, Czech Rep
jan.andres@upol.cz
Luisa Malaguti, Martina Pavlackova

The application of the classical Hartman-type conditions, as presented in [Trans. Amer. Math. Soc. 96 (1960), 493–509] i.e. sign conditions w.r.t. the first state variable and growth conditions (sometimes called as the Bernstein–Nagumo–Hartman conditions) w.r.t. the second state variable, will be discussed, on various levels of abstraction, for a multivalued Dirichlet problem. For instance, in abstract spaces, if the right-hand sides are Marchaud (i.e. globally upper-semicontinuous) and condensing, then the growth conditions can be very liberal. On the other hand, for multivalued upper-Carathéodory condensing right-hand sides, the situation is more delicate. Nevertheless, the related obtained criteria can be still not worse than those of Hartman, i.e. as for vector equations in finite-dimensional spaces. The approach is based on the combination of topological degree arguments, bounding (Liapunov-like) functions and a Scorza–Dragoni approximation technique. An illustrative application of the main existence and localization results can concern partial integro-differential equations involving discontinuities in state variables.

Topological methods and degree for compact multivalued perturbations of Fredholm maps in Banach spaces and applications

Pierluigi Benevieri
University of Sao Paulo, Brasil, Brazil
pluigi@ime.usp.br
Pietro Zecca

We present a construction of an oriented topological degree theory for locally compact multivalued perturbations of Fredholm maps of index zero in Banach spaces. The construction is based on an infinite dimensional nortion of orientation for nonlinear Fredholm maps and by an analogous concept of degree for Fredholm maps of index zero. This theory is applied to existence and bifurcation problems for differential inclusions.

Some qualitative properties of solution sets for a fractional integro-differential inclusion

Aurelian Cernea
University of Bucharest, Romania
acernea@fmi.unibuc.ro

We study fractional integro-differential inclusions of the form

$$D_c^\alpha x(t) \in F(t, x(t), V(x)(t)) \quad a.e.\ ([0,T]),$$
$$x(0) = x_0, \quad x'(0) = x_1,$$

where $\alpha \in (1,2]$, D_c^α is the Caputo fractional derivative, $F : [0,T] \times \mathbf{R} \times \mathbf{R} \to \mathcal{P}(\mathbf{R})$ is a set-valued map and $x_0, x_1 \in \mathbf{R}$, $x_0, x_1 \neq 0$. $V : C([0,T], \mathbf{R}) \to C([0,T], \mathbf{R})$ is a nonlinear Volterra integral operator defined by $V(x)(t) = \int_0^t k(t,s,x(s))ds$ with $k(.,.,.) : [0,T] \times \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ a given function. We prove the arcwise connectedness of the solution set this problem when the set-valued map is Lipschitz in the second and third variable. Moreover, under such type of hypotheses on the set-valued map, we establish a more general topological property of the solution set of our problem. Namely, we prove that the set of selections of the set-valued map F that correspond to the solutions of the problem is a retract of $L^1([0,T], \mathbf{R})$. Both results are essentially based on Fryszkowski, Bressan and Colombo results concerning the existence of continuous selections of lower semicontinuous multifunctions with decomposable values.

Multiplicity results for Neumann-type differential inclusion problems with variable exponent

Antonia Chinni'
DICIEAMA, University of Messina, Italy
achinni@unime.it

We study the existence of at least three distinct solutions for the following Neumann-type differential inclusion problem involving the $p(\cdot)$-Laplacian:

$$(N_\lambda) \begin{cases} -\Delta_{p(x)} u + a(x)|u|^{p(x)-2}u \in \lambda \partial F(x,u) \ \text{ in } \Omega \\ \dfrac{\partial u}{\partial \nu} = 0 \ \text{ on } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded domain with smooth boundary $\partial\Omega$, a is a suitable function belonging to $L^\infty(\Omega)$, $\Delta_{p(x)}u := \mathrm{div}(|\nabla u|^{p(x)-2}\nabla u)$ denotes the $p(x)$-Laplace operator related to a convenient function p of $C(\bar{\Omega})$, ν is the outward unit normal to $\partial\Omega$, λ is a positive parameter and $\partial F(x,\xi)$ is the generalized gradient with respect to ξ of a fixed function F defined on $\Omega \times \mathbb{R}$. The results are obtained by using a multiple critical points theorem for locally Lipschitz continuous functionals.

Fourth-order hemivariational inequality problem

Beatrice Di Bella
Messina University, Italy
bdibella@unime.it

In this talk we will deal with a fourth-order hemivariational inequality with boundary conditions. By using the nonsmooth critical point theory we will prove the existence of infinitely many solutions for our problem.

Variational differential inclusions arising in optimal control

Helene Frankowska
CNRS & University Pierre and Marie Curie, France
frankowska@math.jussieu.fr
D. Hoehener, D. Tonon

It is well known that (first order) variational inclusions can be used to derive first order necessary optimality conditions in optimal control. In this talk we discuss second order variational inclusions leading to second order necessary optimality conditions. More precisely, consider a differential inclusion under state constraints on a given time interval $[0,T]$ and its set of solutions S. Given $\bar{x} \in S$, it is possible to associate to it a variational differential inclusion whose solutions are tangent to S at $\bar{x}$. Denote by $V(\bar{x})$ the set of all such solutions and let $\bar{y}(\cdot) \in V(\bar{x})$. We introduce a second order variational differential inclusion along $(\bar{x},\bar{y})$ whose solutions are second order tangents to S at $(\bar{x},\bar{y})$. The presence of state constraints is handled thanks to a recent Neighbouring Feasible Trajectories Theorem. As a consequence, a new pointwise second order condition verified by the adjoint state of the maximum principle is obtained for the Mayer optimal control problem.

Viable periodic trajectories in totally leaky sets with barriers

Grzegorz Gabor
Nicolaus Copernicus University, Poland
ggabor@mat.umk.pl

The talk is devoted to the existence of solutions to the following single-valued or multivalued differential problem $\dot{x}(t) = f(x(t))$ [resp. $\in F(x(t))$] for a.e. $t \geq 0$, $x(t^+) := \lim_{s\to t^+} x(s) \in I(x(t))$ for $x(t) \in M \subset \partial K$, $x(t) \in K$ for every $t \geq 0$, and $x(0) = x(T)$ for some $T > 0$, where $K \subset \mathbb{R}^n$ is a closed set and $I : M \to 2^{\mathbb{R}^n}$ is an impulse function. Every solution to this problem is said to be viable periodic. Without tangency conditions on the whole set K, we allow trajectories for a non-impulsive problem, even all of them (then we say that K is totally leaky), to leave K through a so-called exit set. To prevent this, we place in an exit set a barrier M, and define an impulse function I possibly moving some trajectories back to the set K. We will look for topological sufficient conditions for the existence of viable periodic trajectories for an impulsive system. An 'impulsive index' will be provided as a suitable homotopy invariant, and its properties will be presented. It will be defined as a fixed point index of a corresponding multivalued (or single-valued) map on the exit set induced by a flow and an impulse function. The research is a continuation and development of the one presented in Gabor G.: The existence of viable trajectories in state-dependent impulsive systems. Nonlinear Anal. 72 (2010), 3828-3836.

The generalized Krasnosel'skii formula and bifurcations of closed orbits to semilinear inclusions

Dorota Gabor
Nicolaus Copernicus University, Poland
dgabor@mat.umk.pl
Wojciech Kryszewski

The classical Krasnosel'skii formula relates the Brouwer degree of the right-hand side of a differential equation in $\mathbb{R}^n$ with the Brouwer fixed point index of the corresponding Poincaré operator (of translations along trajectories). We apply the similar idea to the parameterized semilinear differential inclusion

$$(*) \qquad \dot{u} \in Au + F_\lambda(t,u), \qquad t \in J, u \in E$$

where $J = [0,\infty)$, $\lambda \in \Lambda \subset \mathbb{R}^k$, E is a Banach space, $A : D(A) \to E$ is the infinitesimal generator of the C_0-semigroup of bounded linear operators on E and $F : \Lambda \times J \times E \to 2^E$ is a set-valued weakly upper semicontinuous map with convex weakly compact values. Namely we compare the respective homotopy invariants of the right-hand side and of the operator Φ_t which assigns to the initial value $x \in E$ the set $\{u(t);\ u$ is a (mild) solution of $(*)$ starting at $x\}$. It allows to detect bifurcations of closed orbits to $(*)$. We also present how one can apply this method to a concrete reaction-diffusion inclusion.

Multiple solutions to a Neumann differential inclusion via Morse theory

Antonio Iannizzotto
University of Verona, Italy
antonio.iannizzotto@univr.it
Francesca Colasuonno, Dimitri Mugnai

We study a partial differential inclusion driven by the p-Laplacian operator, with a locally Lipschitz potential and a Neumann boundary condition. The potential is assumed to be p-superlinear at infinity. Following Wang (1991), we prove the existence of a positive and a negative solution via variational methods. Then, using the non-smooth critical groups and the non-smooth Morse identity introduced by Corvellec (1995), we prove the existence of a third non-zero solution.

The Krasnosel'skii formula for constrained semilinear differential equations

Wojciech Kryszewski
Nicolaus Copernicus University, Faculty of Mathematics and Computer Sciences, Poland
wkrysz@mat.umk.pl
Jakub Siemianowski

In the talk we will study the relations between the fixed point index of the Poincaré translation operator generated by the the following semilinear differential equation

$$\dot{u} \in Au + F(t,u),\ \ t \in J,\ ,u \in C,$$

where $J := [0,T]$, $T > 0$, $C \subset E$, E is a Banach space, is a closed convex set, $A : D(A) \to E$, $D(A) \subset E$, is the generator of a compact C_0-semigroup $S = \{S(t)\}_{t \geq 0}$ of bounded linear operators on E and $F : J \times E \to E$ is a continuous map or a set-valued weakly upper semicontinuous map with convex weakly compact values. It is to be noted that many partial differential equations (or systems of such equations), e.g. of parabolic (in particular nonlinear reaction-diffusion equations) or hyperbolic type (or inclusions), can be transformed so as to have this form. The presence of the constraining set C is justified by applications. With this problem one associates the Poincare translation operator Φ_t, where $t > 0$, which assigns to each initial value $x \in E$ the set of values $u(t)$, where u the (mild) solution starting at x. We show that under some natural assumptions C is viable with respect to Φ, i.e., solutions stay in C and then we show that, in the spirit of the Krasnosel'skii formula which relates the Brouwer degree of the right-hand side of an ordinary differential equation in R^n with the Brouwer fixed point index of the Poincaré operator, also in the considered situation the right-hand side of the equation is in an appropriate sense homotopic to Φ_t with sufficiently small $t > 0$, which gives the formula relating the Granas fixed point index of Φ_t on C with the appropriately defined constrained topological degree of the right hand side.

Non-smooth critical point theory on closed convex sets and applications

Salvatore Marano
University of Catania, Italy
marano@dmi.unict.it
Sunra J.N. Mosconi

The existence of critical points for a locally Lipschitz continuous functional F on a closed convex set C of a Banch space X is investigated. The problem of finding extra conditions under which critical points for F on C turn out to be critical on X is also addressed. Some applications concerning elliptic variational-hemivariational inequalities are then worked out.

A unified approach to some classes of evolution equations and systems with nonlocal conditions

Paola Rubbioni
University of Perugia, Italy
paola.rubbioni@unipg.it
Tiziana Cardinali, Radu Precup

We present the study of the existence of global solutions for a general semilinear evolution equation in a Banach space X under the effect of a nonlocal condition expressed by a linear continuous mapping $F : C([0,a];X) \to X$. A transition from Volterra to Fredholm integral operator associated to the problem appears as a consequence of the specific nature of the nonlocal map F. Further, both the classical Cauchy problem and the Byszewski one, where the nonlocal condition is dissipated on the entire interval $[0,a]$, are recovered as special cases. Thanks to a matrix approach, the results are extended to systems of equations in such a way that the system nonlinearities behave independently as much as possible.

Exact non-local controllability of semilinear differential inclusion

Krzysztof Rykaczewski
Nicolaus Copernicus University, Poland
mozgun@mat.umk.pl
Luisa Malaguti

The main aim of our presentation is to study exact nonlocal controllability of solutions for semilinear inclusion in a reflexive Banach space $\mathbb{E}$. More precisely, we shall establish sufficient conditions for controllability of non-linear evolution system with generalized non-local condition of the form:

$$\begin{cases} \dot{y}(t) \in A\big(t,y(t)\big)y(t) + F\big(t,y(t)\big) + Bu(t), \\ \quad \text{for } \ t \in \mathcal{J} := [0,T],\ \ T > 0, \\ y(0) = M(y), \end{cases}$$

where $F\colon \mathcal{J} \times \mathbb{E} \to \mathbb{E}$ is a bounded, closed, multivalued map with convex values, $A(t,y)$ is a linear continuous operator on $\mathbb{E}$, for each $(t,y) \in \mathcal{J} \times \mathbb{E}$, control $u(\cdot)$ is a given function from $L^2(\mathcal{J},\mathbb{U})$, a space of admissible control functions with $\mathbb{U}$ as reflexive Banach space, and B is a bounded linear operator from $\mathbb{U}$ to $\mathbb{E}$. The assumptions regarding the operator $M\colon \mathcal{C}(\mathcal{J},\mathbb{E}) \to \mathbb{E}$ will be stated explicitly in the presentation. Sufficient conditions are formulated and proved using a fixed point theorem. Finally, we present an example to illustrate application of the proposed method.

Delay evolution inclusions with general nonlocal initial conditions

Ioan Vrabie
Al. I. Cuza University, Iasi, Romania, Romania
ivrabie@uaic.ro
Mihai Necula

We prove and existence result for bounded C^0-solutions to a class of nonlinear delay differential evolution inclusions subjected to nonlocal implicit initial conditions. We assume that the m-dissipative part is the infinitesimal generator of a nonlinear compact semigroup of contractions, while the multi-valued forcing term is nonempty, convex, weakly compact valued and almost strongly-weakly u.s.c. The main difficulty here is that the history constraint function g, which is assumed to be nonexpansive, has affine, instead of linear, growth and thus $g(0)$ is allowed to be nonzero.

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Special Session 43: Harmonic Analysis Tools for Fluid Mechanics

Francesco Di Plinio, Brown University, USA
Roger Temam, Indiana University, USA
Djoko Wirosoetisno, University of Durham, UK

The more recent advances in the field of Fluid Mechanics, and more generally, of Partial Differential Equations arising from models of Physics and Continuum Mechanics are increasingly featuring, and motivating, novel Harmonic and Fourier analytical tools. A nonexhaustive list of significant examples are the breakthroughs in the circle of problems related to anomalous dissipation and Onsager's conjecture; the well-posedness and regularity results for the surface quasi-geostrophic equation and related problems; the blow-up problem and double exponential growth of Sobolev norms for the two-dimensional Euler system; Euler wellposedness and elliptic regularity theory in nonsmooth domains; Euler equations in mean oscillation spaces; new estimates for linear and multilinear oscillatory integrals and their applications to linear and nonlinear wave and Schrdinger-type equations. The aim of this special session is to provide an opportunity, for specialists of either discipline, of presenting novel results at the boundary between the fields of harmonic analysis and partial differential equations, with particular focus on the theory of fluids, and discussing open questions, possibly sparkling the need for new analytical tools and fostering future collaborations.

Absolute Maxwellian Eternal Solutions and Scattering

Claude Bardos
Laboratoire Jacques Lous Lions, France
claude.bardos@gmail.com
Irene Gamaba, Francois Golse and David Levermore.

When he derived the equations

$$\partial_t F + v \cdot \nabla_x F = \mathcal{B}(F,F)$$

Boltzmann identified special solutions which are both Maxwellian and which are solution of the advection equation:

$$F = F(v, x - vt) \Rightarrow \mathcal{B}(F,F) = 0 \text{ and } \partial_t F + v \cdot \nabla F = 0$$

Dave Levermore dubbed these l solutions "global Maxwellian" and identified them by their global moments. In this talk I want to show that the global Maxwellian generate global solutions both of the compressible Euler and Navier Stokes equations. Their stability and scattering properties can be studied following classical contributions of Kaniel-Shinbrot [1] or Wei and Zhang [2]. And eventually this perturbation analysis produces eternall solutions of the Boltzmann equation which do not coincide with global Maxwellian

References

[1] S. Kaniel and M. Shinbrot, *The Boltzmann Equation I: Uniqueness and Local Existence*, Commun. Math. Phys. **58** (1978), 65–84.

[2] J. Wei and X. Zang *Global Solution of the Initial Value Problem for the Boltzmann Equation near a Local Maxwellian*, Arch. Rational Mech & Anal. **102** (1988), 231–241.

Onsager's conjecture

Tristan Buckmaster
Leipzig University, Germany
tristan.buckmaster@gmail.com
Camillo De Lellis and László Székelyhidi Jr.

In 1949, Lars Onsager in his famous note on statistical hydrodynamics conjectured that weak solutions to the Euler equations belonging to the Hölder space with Hölder exponent greater than 1/3 conserve energy; conversely, he conjectured the existence of solutions belonging to any Hölder space with exponent less than 1/3 which dissipate energy. The first part of this conjecture has since been confirmed (cf. Eyink 1994, Constantin, E and Titi 1994). In this talk we discuss recent work related to resolving the second component of Onsager's conjecture. In particular, we present a proof of a weak version of the conjecture: there exists weak non-conservative solutions to the Euler equations whose $1/3 - \epsilon$ Hölder norm in space is Lebesgue integrable in time.

Commutator estimates and uniqueness of solutions of complex fluids

Peter Constantin
Princeton University, USA
const@math.princeton.edu

We show that the infinite dimensional group nature of the Lagrangian dynamics coupled with the singular integral nature of the hydrodynamic equations of state in Eulerian coordinates give rise to a commutator structure that enables the proof of uniqueness of solutions with minimal regularity.

Remarks on geometric properties of SQG sharp fronts and alpha-patches

Diego Córdoba
ICMat-CSIC, Spain
dcg@icmat.es
Angel Castro, Javier Gomez-Serrano, Alberto Martin Zamora

Guided by numerical simulations, we present the proof of two results concerning the behavior of SQG sharp fronts and *alpha*-patches. We establish that ellipses are not rotational solutions and we prove that initially convex interfaces may lose this property in finite time.

Partial regularity with angular integrability for the Navier–Stokes equation

Renato Luca
ICMAT, Spain
renato.luca@icmat.es
Piero D'Ancona

We focus on suitable weak solutions of the Navier–Stokes equation with small initial data in weighted Lebesgue spaces with angular integrability. We show how the size of the regular sets increases under higher angular integrability assumptions.

Spectral instability of characteristic boundary layer flows

Toan Nguyen
Penn State University, USA
nguyen@math.psu.edu
E. Grenier, Y. Guo

Boundary layers typically appear in the inviscid limit of the Navier-Stokes equations. There are ranges of spatial frequencies and large Reynolds number with which generic boundary layers are spectrally unstable. This includes boundary layers without an inflection point, and thus the instability is due to the presence of viscosity. I will introduce a new, operator-based approach to construct exact growing modes of the linearized Navier-Stokes about stationary boundary layers. Our approach avoids to deal with matching inner and outer asymptotic expansions, but instead involves a careful study of singularity in the critical layers by deriving pointwise bounds on the Green function of the corresponding Rayleigh and Airy operators. This is a joint work with Grenier and Guo.

Rayleigh-Bénard convection at finite Prandtl number: bounds on the Nusselt number

Camilla Nobili
Max-PIanck-Institute MIS, Germany
camilla.nobili@mis.mpg.de
Felix Otto and Antoine Choffrut

We consider Rayleigh-Bénard convection at finite Prandtl number as modelled by the Boussinesq equation. We are interested in the scaling of the average upward heat transport, the Nusselt number Nu, in terms of the Rayleigh number Ra, and the Prandtl number Pr. Physically motivated heuristics suggest the scaling $Nu \sim Ra^{\frac{1}{3}}$ and $Nu \sim Ra^{\frac{1}{2}}$ depending on Pr, in different regimes. In this talk I present a rigorous upper bound for Nu reproducing both physical scalings in some parameter regimes up to logarithms. This is obtained by a (logarithmically failing) maximal regularity estimate in L^∞ and in L^1 for the non-stationary Stokes equation with forcing term given by the buoyancy term and the nonlinear term, respectively. This is a joint work with Felix Otto and Antoine Choffrut.

Exponential Decay of the Power Spectrum and Finite Dimensionality for Solutions of the Three Dimensional Primitive Equations

Madalina Petcu
University of Poitiers, France
Madalina.Petcu@math.univ-poitiers.fr

The purpose of this talk is to estimate the number of modes, volumes and nodes, sufficient to describe well the solution of the three dimensional primitive equations; the physical meaning of these estimates is also discussed. We also study the exponential decay of the spatial power spectrum for the three dimensional primitive equations.

Global existence for coupled fluid models.

Frederic Rousset
Université Paris-Sud, France
Frederic.Rousset@math.u-psud.fr
T. Hmidi, S. Keraani, T. Elgindi

We shall present global well-posedness results for two-dimensional fluid models that couple the incompressible Euler equation with a dissipative equation in a "critical way". Our main examples will be Boussinesq and Oldroyd B type models.

Dispersion dynamics of the defocusing Korteweg - de Vries equation

Stefan Steinerberger
Yale University, USA
stefan.steinerberger@gmail.com

The defocusing Korteweg - de Vries equation is a nonlinear dispersive equation on the real line. The defocusing nature of the nonlinearity suggests that soliton-like solutions (keeping a lot of the preserved L^2 mass in a small intervall over long periods of time) should not exist. We describe a new approach to this problem, which complements earlier results of Tao and improves a result of Kwon & Shao.

The (discrete and continuous) fractional Laplacian

Pablo Stinga
The University of Texas at Austin, USA
stinga@math.utexas.edu
O. Ciaurri, L. Roncal, J. L. Torrea, J. L. Varona, C. Zhang

Important equations in fluid dynamics involve nonlocal diffusions typically modeled by the fractional Laplacian [5]. These kind of nonlocal operators received a great deal of attention in the past years, especially since the appearance of the works by Luis Caffarelli and collaborators [1, 2, 3]. In this talk we shall discuss a novel point of view to understand fractional Laplacians: the *semigroup language* approach. This method was introduced in [6]. With this method at hand we can generalize the Caffarelli–Silvestre extension problem to positive operators other than the Laplacian in the whole space [7] and then prove Harnack's inequalities for a large class of fractional operators [8]. Moreover, we are able to define the fractional powers of the discrete Laplacian on a mesh of size h and to show that it converges to the fractional Laplacian on the whole space in the discrete supremum norm as $h \to 0$ [4].

References

[1] L. Caffarelli, S. Salsa and L. Silvestre, Regularity estimates for the solution and the free boundary of the obstacle problem for the fractional Laplacian, *Invent. math.* **171** (2008), 425–461.

[2] L. Caffarelli and L. Silvestre, An extension problem related to the fractional Laplacian, *Comm. Partial Differential Equations* **32** (2007), 1245–1260.

[3] L. Caffarelli and A. Vasseur, Drift diffusion equations with fractional diffusion and the quasi-geostrophic equation, *Ann. of Math. (2)* **171** (2010), 1903–1930.

[4] Ó. Ciaurri, L. Roncal, P. R. Stinga, J. L. Torrea and J. L. Varona, The discrete fractional Laplacian, *preprint* (2014).

[5] P. Constantin, A. J. Majda and E. Tabak, Formation of strong fronts in the 2-D quasigeostrophic thermal active scalar, *Nonlinearity* **7** (1994), 1495–1533.

[6] P. R. Stinga, *Fractional Powers of Second Order Partial Differential Operators: Extension Problem and Regularity Theory*, PhD Thesis, Universidad Autónoma de Madrid, 2010.

[7] P. R. Stinga and J. L. Torrea, Extension problem and Harnack's inequality for some fractional operators, *Comm. Partial Differential Equations* **35** (2010), 2092–2122.

[8] P. R. Stinga and C. Zhang, Harnack's inequality for fractional nonlocal equations, *Discrete Contin. Dyn. Syst.* **33** (2013), 3153–3170.

Lagrangian Analyticity in Two Dimensional Fluids

Vlad Vicol
Princeton University, USA
vvicol@math.princeton.edu
P. Constantin and J. Wu

We consider two-dimensional fluid models, such as the Euler equations, the surface quasi-geostrophic equations, the incompressible porous media equation, and the Boussinesq equations. We prove that as long as the velocity field stays $C^{1,\alpha}$ for some $\alpha \in (0,1)$, the Lagrangian particle trajectories are real-analytic functions of time.

Special Session 44: Quasilinear Elliptic and Parabolic Problems and Their Applications

J.F. Padial, Universidad Politecnica de Madrid, Spain
P. Takac, Rostock University, Germany
L. Tello, Universidad Politecnica de Madrid, Spain

Quasilinear partial differential equations arise in many fields of Physics, Chemistry or Biology. The aim of this special session is to present recent progress in the theory of quasilinear elliptic and parabolic partial differential equations and their applications. This session will bring experts in this area to exchange ideas.

Extinction in a finite time for a class of nonlinear degenerate parabolic equations

Yves Belaud
Universite Francois Rabelais, France
belaud@lmpt.univ-tours.fr
Andrey Shishkov

Let $\Omega \subset \mathbb{R}^N$ be a bounded domain and u be a solution of

$$\begin{cases} \frac{\partial}{\partial t}(u^q) - \nabla(|\nabla u|^{p-2}\nabla u) + a(x)u^\lambda &= 0 \\ u(x,0) &= u_0(x) \end{cases}$$

for the Dirichlet boundary condition where $0 < \lambda < q < p-1$ and a is nonnegative on Ω. In what follows, the function a is degenerate, i.e., it vanishes on sets of positive measure.
The point is : is there a finite time T such that the solution u vanishes in whole Ω ?
We will deal with some results comming from so-called semi-classical methods. The first semi-classical method was initiated in 1997 by V.A. Kondratiev and L. Véron, improved in 2001 by Y.B., B. Helffer and L. Véron and optimized with A. Shishkov in 2007 for $q = 1$ and $p = 2$. In 2001, some results have been established when $q = 1$ and $p > 2$ or when $q < 1$ and $p = 2$. We present some new results when $q \neq 1$ and $p \neq 2$.

On maximum and comparison principles for parabolic problems with p-Laplacian

Vladimir Bobkov
University of Rostock, Germany
bobkovve@gmail.com
Peter Takac

We consider initial-boundary value problems for nonlinear parabolic equations with p-Laplacian

$$\partial_t u - \Delta_p u - \lambda |u|^{p-2}u = f(t,x),$$

where $\lambda \in \mathbb{R}$ is a spectral parameter. Problems of this type attract a lot of attention in the last decades. This is due to the fact that solutions to such problems possesses many unusual qualitative effects, which are not observed in the linear case ($p = 2$), e.g., extinction in finite time, simultaneously backward, elliptic and forward Harnack's estimates, violation of the strong maximum principle and Hopf's lemma, etc. In this talk we will speak about current state of affairs and recent progress in maximum and comparison principles for such kind of parabolic problems. Some outstanding issues will be discussed.

Local Behavior of continua of solutions for asymptotically linear systems near resonance

Maya Chhetri
UNC Greensboro, USA
maya@uncg.edu
Petr Girg

We will consider an asymptotically linear system involving weighted eigenvalue problem. We provide sufficient conditions for determining the direction of bifurcation of positive and negative solutions from infinity at the first eigenvalue of the associated linear eigenvalue problem. As a corollary to these results, we also provide sufficient conditions for the solvability of a class of asymptotically linear system at resonance satisfying *Landesman–Lazer* type conditions.

Travelling waves in a Fisher-Kolmogorov-type model with degenerate diffusion and nonsmooth reaction

Pavel Drabek
University of West Bohemia, Czech Rep
pdrabek@kma.zcu.cz
Peter Takac

We will discuss the existence and uniqueness of monotone travelling waves connecting the equilibrium states 0 and 1. They can either only approach these equilibria at plus or minus infinity, or else attain them at finite points, depending on the interaction between the degenerate and/or singular diffusion and the nonsmooth reaction function. Then we discuss the approach to such travelling waves by solutions with rather general initial data that are sqeezed between two travelling waves (that are each other's shift).

Some nonexistence results for nonlinear differential inequalities with gradient terms and singularities on unbounded sets

Evgeny Galakhov
Peoples' Friendship University of Russia, Russia
galakhov@rambler.ru
Olga Salieva

Let $S \subset \mathbb{R}^n$ be a closed unbounded set. We consider nonlinear elliptic partial differential inequalities of the form

$$(-\Delta)^k u \geq a(x)|Du|^q \quad (x \in \mathbb{R}^n \setminus S)$$

and

$$-\Delta_p u \geq a(x)|Du|^q \quad (x \in \mathbb{R}^n \setminus S)$$

where the coefficient $a(x)$ may have a singularity on the set S, and their generalizations. We establish sufficient conditions for nonexistence of solutions to such inequalities in appropriate functional classes extending and improving in a certain sense the results of our paper.

Quasilinear an singular parabolic problems

Jacques Giacomoni
LMAP, University of Pau, France
jacques.giacomoni@univ-pau.fr
B. Bougherara and P. Takac

In this talk, I will present recent joint contributions with P. Takac and B. Bougherara about the following quasilinear and singular parabolic equation:

$$\begin{cases} \partial_t u - \Delta_p u = u^{-\delta} + f(x,u,\nabla u) \\ \qquad \text{in } (0,T)\times\Omega = Q_T, \\ u = 0 \quad \text{on } (0,T)\times\partial\Omega, \\ u > 0 \text{ in } Q_T, \\ u(0,x) = u_0 \geq 0 \text{ in } \Omega, \end{cases}$$

where Ω stands for a regular bounded domain of $\mathbb{R}^N$, $2 \leq p$, $T > 0$ and $u_0 \in L^r(\Omega)$ with $r \geq 2$ large enough. The nonlinear term $f : \Omega\times\mathbb{R}\times\mathbb{R}^N \to \mathbb{R}$ is a Caratheodory function satisfying addtional growth conditions. I will present existence results to (P). In the case $\delta < 2 + \frac{1}{p-1}$, we give further results : uniqueness of the solution and regularity results. Proofs used some estimates based on logarithmic Sobolev inequalities to get ultracontractivity of the associated semi-group.

Diffusive logistic equation with constant yield harvesting and negative density dependent emigration on the boundary

Jerome Goddard II
Auburn University Montgomery, USA
jgoddard@aum.edu
R. Shivaji

The structure of positive steady state solutions of a diffusive logistic population model with constant yield harvesting and negative density dependent emigration on the boundary is examined. In particular, a class of nonlinear boundary conditions that depends both on the population density and the diffusion coefficient is used to model the effects of negative density dependent emigration on the boundary. In this presentation, we discuss existence results established via the well-known sub-super solution method.

Positive solutions for a nonlocal nonlinear elliptic system arising in desertification theory.

Jesus Hernandez
Universidad Autonoma de Madrid, Spain
jesus.hernandez@uam.es

We study existence of positive solutions for a 3 x 3 nonlinear nonlocal elliptic system previously studied by J.I. Diaz and P. Za. We consider different versions of the mathematical model in theis paper. We prove first existence of positive solutions for the full system both in the nonlocal version and for a simplified local one by using Schauder fixed point Then we prove existence for a simplified 2 x 2 local cooperative system by using sub and supersolutions. Finally, a reduced model for one single equation is considered giving conditions for existence and uniqueness, or multiplicity, depending on some parameters. This is joint work with J.I.Diaz.

Reaction-Diffusion Equations from Energy Balance Climate Models

Georg Hetzer
Auburn University, USA
hetzege@auburn.edu

Energy balance climate models are simple diagnostic models which describe the evolution of a long-term mean of temperature by employing the relevant balance equations for the heat fluxes involved. The mean horizontal heat flux is parameterized by a slow-diffusion operator, and important feedback processes appear in the net radiation flux. This talk focuses on incorporating a bio-feedback into the model which leads to a nonautonomous functional reaction-diffusion problem with a set-valued reaction term. A global existence result for nonnegative solutions and the existence of a trajectory attractor are discussed.

Precise range for stable and blow-up solutions to equations with p-Laplacian and supercritical nonlinearities

Yavdat Ilyasov
Institute of Mathematics RAS, Ufa, Russia, Russia
ilyasov02@gmail.com

We present an approach for the determining of the precise range of the existence stable and blow-up solutions for families of elliptic and heat equations with p-Laplacian and supercritical nonlinearities. The approach is based on the extended functional method where precise boundaries of the range of the existence stable and blow-up solutions are defined by direct and dual minimax type formulas. A number of examples, including system of elliptic and parabolic problems with p- Laplacian, will be discussed. In addition, we present a numerical algorithm for the computation of solutions of the minimax variational problems corresponding to the extended functional method.

A model of traffic flow in a isolated network

Angela Jimenez-Casas
Pontificia Comillas University, Spain
ajimenez@upcomillas.es
Aníbal Rodríguez-Bernal

We obtain a mathematical model which describes the dynamical given by the traffic of material objects in a network, (represented by a graph). This model generalizes several previous models that have been using for instance in networks of air traffic. We analyze existence and uniqueness of solutions and some particular cases.

Positivity of self-similar solutions to doubly nonlinear reaction-diffusion equations

Jochen Merker
University of applied science Stralsund, Germany
jochen.merker@uni-rostock.de
Aleš Matas

Doubly nonlinear reaction-diffusion equations allow to model reactions of non-Newtonian fluids inside a porous medium like catalytic pellets. In the talk, the prototypical doubly nonlinear reaction-diffusion equation

$$\frac{\partial u^{m-1}}{\partial t} - \Delta_p u = u^{q-1}$$

is considered. After a survey of extinction and blow up, positivity of solutions is discussed for L^1-data resp. measures as initial values. While the weak comparison principle guarantees non-negative solutions to L^1-data $u_0^{m-1} \geq 0$ and blow up occurs for $1 < m < p < q < q_c := p(1 + \frac{m-1}{n})$ (where q_c is Fujita's critical exponent of blow-up), we were able to show that in the same situation there exist self-similar solutions u which have a positive multiple of the Dirac measure at the origin as renormalized initial value and do not blow up in finite time, but instantly become sign-changing for $t > 0$.

A finite element approach to modelling tumor growth

Ana Munoz
Rey Juan Carlos University, Spain
anaisabel.munoz@urjc.es

We shall consider and solve numerically a model of tumor growth based on cancer stem cells (CSC) hypothesis which was proposed and studied mathematically in [2]. The model consists of four hyperbolic equations of first order to describe the evolution of different subpopulations of cells and a fifth equation to model the evolution of the moving boundary. The rates at which reactions occurs are represented by parameters, containing some of them non-local integral terms in their definition. A formulation in terms of total derivatives is posed in order to integrate the four hyperbolic equations. The discretization of these total derivatives leads to computing the corresponding ODE for the characteristics. To integrate the model equations in space a finite element discretization is applied. We present some numerical simulations and compare our results with the ones obtained in [1] where a model, also based on CSC, consisting of ODEs for the evolution of the densities of the different subpopulations of cells is considered. Finally, some conclusions are drawn.

References
[1] R. Molina-Peña and M. M. Alvarez *A simple mathematical model based on cancer stem cell hypothesis suggests kinetic commonalities in solid tumor growth.* PLoS ONE, 7 (2) (2012) e26233.

[2] J. I. Tello, *On a mathematical model of tumor growth based on cancer stem cells*, Mathematical Biosciences and Engineering, 10 (1) (2013), 263-278.

Comparison principle for doubly nonlinear parabolic systems in the $BV_t(Q)$ space revisted and new applications

Juan Francisco Padial Molina
Univ. Politecnica de Madrid, Spain
jf.padial@upm.es
J.I. Díaz and J.F. Padial

We present some uniqueness results for a class of weak solutions (the so called BV solutions) of the Cauchy-Dirichlet problem associated to doubly nonlinear diffusion systems. The results are established in the $BV_t(Q)$ space following the paper by Díaz and Padial (*Uniqueness and existence of solution in the $BV_t(Q)$ space to a doubly nonlinear parabolic problem*, Publicacions Matemàtiques, Vol. **40** (1996)) jointly with the doubling time variable method inspired in some ideas introduced by S.N. Kruzhkov (see, e.g., S.N. Kruzhkov, *First order quasilinear equations in several independent variables.* Math. USSR Sbornik, **10**, No. 2, 1970, pp. 217-243). We point out that, in contrast to the case of scalar equations, there are counterexamples showing that the comparison principle may fail for systems of equations. In particular, we shall consider, for instance, the problem that appear in chemical engineering and metallurgy involving the interaction of diffusing substances with immobile solid phases.

Some nonexistence results for nonlinear differential inequalities with gradient terms and singularities on unbounded sets

Olga Salieva
Moscow State Technological University “Stankin”, Russia
olga.a.salieva@gmail.com
Evgeny Galakhov

Let $S \subset \mathbb{R}$ be a closed unbounded set. We consider elliptic nonlinear partial differential inequalities of the form

$$(-\Delta)^k u \geq a(x)|Du|^q \quad (x \in \mathbb{R}^n \setminus S)$$

and

$$-\Delta_p u \geq a(x)|Du|^q \quad (x \in \mathbb{R}^n \setminus S)$$

where the coefficient $a(x)$ may have a singularity on S, and their generalizations. We establish sufficient conditions for nonexistence of solutions to such inequalities in appropriate functional classes extending and improving in a certain sense the results of our paper.

Infinite Semipositone Problems

Ratnasingham Shivaji
University of North Carolina at Greensboro, USA
shivaji@uncg.edu

We consider positive solutions to classes of steady state reaction diffusion equations of the form

$$\begin{aligned} -\Delta_p u &= \lambda f(u);\ \Omega \\ u &= 0;\ \partial\Omega \end{aligned}$$

where $\Delta_p z := \text{div}\left(|\nabla z|^{p-2}\nabla z\right)$; $p > 1$ is the p–Laplacian of z, Ω is a smooth bounded domain in $\mathbb{R}^N$; $N \geq 1$, λ is a positive parameter and $f : (0,\infty) \to \mathbb{R}$ is a class of C^1 functions such that $\lim_{s\to 0^+} f(s) = -\infty$ (infinite semipositone case). We establish existence results via the method of sub-super solutions. We also discuss results for classes of systems of equations with infinite semipositone structure.

Boundedness of extremal solutions of semilinear elliptic equations in dimension 4

Salvador Villegas
Universidad de Granada, Spain
svillega@ugr.es

We consider the following semilinear elliptic equation (P_λ), which has been extensively studied:

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega\,, \\ u \geq 0 & \text{in } \Omega\,, \\ u = 0 & \text{on } \partial\Omega\,, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N \geq 1$, $\lambda \geq 0$ is a real parameter and the nonlinearity $f : [0,\infty) \to \mathbb{R}$ satisfies (P_λ) f is C^1, nondecreasing and convex, $f(0) > 0$, and $\lim_{u\to+\infty} \frac{f(u)}{u} = +\infty$. It is well known that there exists a finite positive extremal parameter λ^* such that (P_λ) has a minimal classical solution $u_\lambda \in C^2(\overline{\Omega})$ if $0 \leq \lambda < \lambda^*$, while no solution exists, even in the weak sense, for $\lambda > \lambda^*$. The set $\{u_\lambda : 0 \leq \lambda < \lambda^*\}$ forms a branch of classical solutions increasing in λ. Its increasing pointwise limit $u^*(x) := \lim_{\lambda\uparrow\lambda^*} u_\lambda(x)$ is a weak solution of (P_λ) for $\lambda = \lambda^*$, which is called the extremal solution of (P_λ). Brezis and Vazquez raised the question of determining the regularity of u^*, depending on the dimension N, for general nonlinearities f satisfying (P_λ). We establish the boundedness of the extremal solution for general bounded smooth domains in dimension $N = 4$, not necessarily convex. In higher dimensions, we improve the previous results, obtaining that, for $N \geq 5$, the extremal solution $u^* \in W^{2,\frac{N}{N-2}}$. This gives $u^* \in L^{\frac{N}{N-4}}$, if $N \geq 5$ and $u^* \in H_0^1$, if $N = 6$.

Special Session 45: Hybrid Imaging Methods

Leonid Kunyansky, University of Arizona, USA
Shari Moskow, Drexel University, USA
John Schotland, University of Michigan, USA

Since the introduction of the X-ray computed tomography in the 60's, it became one of the most important methods of non-invasive biomedical imaging. Quite a few other modalities have been introduced since then, and become indispensable tools in medicine, biology, and other areas of science and engineering. These modalities differ by types of waves or particles used to scan the object of interest, which, in turn, determines which biological (or physical) features they can or cannot detect. In all of these techniques images are not formed directly, but are being reconstructed by solving inverse problems with respect to the coefficients of the partial differential equations describing propagation of the corresponding type of waves through the medium. These problems are frequently strongly unstable, thus disallowing reconstructions of high-quality images. In many cases, modalities that are capable of delivering high-resolution images (such as utrasound) cannot distinguish between healthy and cancerous tissues, and sensitive modalities (such as electrical impedance tomography) require solving an extremely unstable inverse problem. Recently, several novel "hybrid" or "multi-physics" modalities have been introduced and become an object of intensive research. Among these methods are thermo- and photo-acoustic tomography, acousto- electric tomography, ultrasound-modulated optical tomography, and others. These technique utilize physical coupling between two different types of waves (for example, infra-red radiation and ultrasound) in order to overcome the low sensitivity or high instability associated with particular types of radiation, thus, yielding information that cannot be obtained by traditional methods. The theoretical and algorithmic foundations of these multi-physics modalities are being actively developed. This session brings together leading experts on thermo- and photoacoustic tomography, acoustically modulated optical and electric impedance tomography, MRI assisted elastography and current density tomography, and other related techniques.

The spherical Radon transform-the injectivity problem revisited

Mark Agranovsky
Bar-Ilan University, Israel
agranovs@014.net.il

A known conjecture is that non-injectivity sets for the spherical Radon transform on compactly supported functions are contained in translates of the zero sets of spatial harmonics. Results towards the conjecture will be discussed in the talk.

Exterior problem of image reconstruction in intravascular PAT

Gaik Ambartsoumian
University of Texas at Arlington, USA
gambarts@uta.edu
Leonid Kunyansky

The talk discusses an exterior inverse problem for the circular means transform (CMT), which arises in intravascular photoacoustic tomography (PAT) and some other imaging modalities. In this problem one needs to recover an unknown image function supported outside of a disc with CMT data collected for circles centered along the boundary of that disc. The inversion process in this setup is known to be very unstable. We present a novel algorithm for CMT inversion in the exterior problem, and show that it recovers accurately all visible singularities.

Dynamic PhotoAcoustics

Simon Arridge
University College London, England
S.Arridge@cs.ucl.ac.uk
Marta Betcke and Ben Cox

PhotoAcoustic Tomography (PAT) in 3D is a challeging computational problem for which many solutions have been proposed. The computational overhead is compounded when considering dynamic (i.e. 4D) PAT. In this talk we consider methods based on time-reversal and discuss the application of time-series methods to accelerate the inversion scheme.

Spatial resolution for photoacoustic tomography in a fluctuating media with finite sized detectors

Peter Burgholzer
Research Center for Non-Destructive Testing, Austria
peter.burgholzer@recendt.at

Fluctuations have been widely studied for small systems composed of a limited, small number of particles. Fluctuation relations or theorems describe the non-equilibrium statistical behavior of such systems and apply on microscopic as well as macroscopic scales. In large systems fluctuations play an important role if chaotic behavior appears or inverse problems like in photo- or thermoacoustic imaging have to be solved. Then the spatial resolution turns out to be limited by fluctuations if real finite sized detectors are used. Detector arrays enable parallel detection for faster photoacoustic imaging than by moving a single detector. Spatial over-sampling is scanning with a step-size smaller than the size of the detector element and is possible only for a moving single detector. If the reconstruction is performed assuming point-like detection this over-sampling brings no advantage. For image reconstruc-

tion methods which can take the finite size of real detectors into account, this spatial over-sampling makes sense because it reduces the blurring significantly. The effect of step size on the reconstructed images is systematically examined using simulated and experimental data.

Microlocal analysis of artifacts in limited anlge x-ray tomography

Jürgen Frikel
Helmholtz Center Munich, Germany
j.frikel@gmx.de
Eric Todd Quinto

In many tomographic imaging scenarios (e.g. x-ray tomography, electron microscopy, photo- and thermoacoustic tomgraphy) only limited-view data is available. As a consequence, only specific features of the unknown object can be reconstructed reliable, and artifacts can be generated. In this talk, we will use microlocal analysis to explain that artifacts might appear only along lines that are tangent to singularities of the original object. Moreover, we will show that by smoothly truncating the data at the ends of the angular range, the artifacts can be reduced.

Universal Inversion Formulas for Recovering a Function from Spherical Means

Markus Haltmeier
University of Innsbruck, Austria
markus.haltmeier@uibk.ac.at

In this talk we derive universal back-projection type reconstruction formulas for recovering a function in arbitrary dimension from averages over spheres centered on the boundary of a bounded convex domain with smooth boundary. Provided that the unknown function is supported inside that domain, the derived formulas recover the unknown function up to an explicitly computed integral operator. For elliptical domains the integral operator is shown to vanish and hence we establish exact inversion formulas for recovering a function from spherical means centered on the boundary of elliptical domains in arbitrary dimension.

Why are hybrid techniques stable?

Peter Kuchment
Texas A&M UNiversity, USA
kuchment@math.tamu.edu
Dustin Steinhauer

The microlocal technique is presented that allows quick test of stability of a hybrid imaging method.

Photo- and thermo- acoustic tomography in the presence of acoustically reflecting boundaries

Leonid Kunyansky
University of Arizona, USA
leonk@math.arizona.edu
B. Holman, B. T. Cox

Most of the known theoretical and algorithmic results pertaining to the inverse problem of the thermo- and photoacoustic tomography are obtained under the assumption that the acoustic waves propagate in free space. However, neglecting the reflections of the waves from the detectors is not always possible. When optically scanned solid plates surrounding the object are used to measure the acoustic signal, one nas to account for the multiple reflections of the acoustic waves from these plates. In this case the forward problem is accurately modeled by the wave equation with the Neumann boundary conditions on the walls of the resonant cavity formed by the detecting surfaces. The energy of waves in such a cavity does not decay in time (if we neglect the absorption) which renders inapplicable the classical analytic results. In the talk I will discuss the possibility ofan approximate time reversal in the problems of photo/thermo- acoustic tomography within a reverberant cavity. I will also present a fast reconstruction algorithm for the practically important particular case of a rectangular cavity in 3D.

Local inversions in ultrasound modulated optical tomography

Shari Moskow
Drexel University, USA
moskow@math.drexel.edu
Guillaume Bal

We consider the problem of the simultaneous recovery of both the absorption and diffusion coefficients with internal data in a form that is of power density type. We assume we have some known background coefficients, and examine the corresponding linearized problem. This linearized problem can be expressed as a fourth order system, and we examine its stability and injectivity properties.

Micro - Electrical Impedance Tomography

Loc Nguyen
Ecole Polytechnique Federale de Lausanne, Switzerland
loc.nguyen@epfl.ch
Habib Ammari, Laure Giovangigli, Jin-Keun Seo

The aim of this talk is to propose an optimal control optimization algorithm for reconstructing admittivity distributions (i.e., both conductivity and permittivity distributions) from multi-frequency micro-electrical impedance tomography. It is shown that using multi-frequency data, one obtains a convergent and stable optimization scheme.

Exact reconstructions and parametrices in photoacoustic tomography

Victor Palamodov
Tel Aviv University, Israel
palamodo@post.tau.ac.il

Several closed exact formulae are known for reconstruction of a function from data of spherecal means for spheres centered at a (hyper)surface. The central sets are spheres, ellipsoids or paraboloids. There is an infinite series of unknown algebraic hypersurfaces that can serve as central sets for a reconstruction formula of the same analytical form. These surfaces look exotic but have a simple algebraic definition. It is much easier to construct a parametrix for this problem. A parametrix is a left inverse for a spherical mean operator modulo a smoothing operator in the Sobolev scale of spaces. Application of a parametrix to the integral data correctly reproduces up to a continuous function simple discontinuities like jumps or delta inclusion.

Microlocal analysis and limited data Thermoacoustic Tomography

Eric Todd Quinto
Tufts University, USA
todd.quinto@tufts.edu
Juergen Frikel

We will present the microlocal properties of a limited data transform in thermoacoustic tomography. We consider the case in which data are recovered on a small circular arc, and we use microlocal analysis to prove how the reconstruction operator R^*R adds artifacts unless one truncates smoothly at the boundaries of the arc. We prove generalizations to the three-dimensional case and provide a framework for other limited data problems.

Ultrasound modulated bioluminescence imaging

John Schotland
University of Michigan, USA
schotland@umich.edu

We propose a method to reconstruct the density of a luminescent source in a highly-scattering medium from ultrasound modulated optical measurements. Our approach is based on the solution to a hybrid inverse source problem for the diffusion equation.

Nanophosphor image reconstruction with selective excitation

Plamen Stefanov
Purdue University, USA
stefanov@math.purdue.edu

We will discuss the mathematics arising in modulated luminescence tomography. The talk is based on a joint work of the speaker, Ge Wang and R. Filkins.

Stabilizing Inverse Problems by Internal Data

Dustin Steinhauer
Texas A&M University, USA
dsteinha@math.tamu.edu

We will discuss a simple procedure that allows one to detect whether and explain why internal information arising in several novel hybrid imaging modalities could turn extremely unstable techniques, such as optical tomography or electrical impedance tomography, into stable, good-resolution procedures. Applications to local and nonlocal functionals will be shown, and generic linearized and nonlinear uniqueness results will be presented.

Iterative solutions to the p-Laplace equation arising in hybrid conductivity imaging

Alex Timonov
University of South Carolina, USA
atimonov@uscupstate.edu

The p-Laplace equation (usually for $p = 0$ or $p = 1$) with prescribed boundary conditions is considered as a mathematical model of hybrid conductivity imaging. Due to the absence of convexity (for $p < 1$) or strict convexity (for $p = 1$) in a variational formulation of the p-Laplace equation, as well as to the singularity at the critical points for $p < 2$, constructing the stable and computationally efficient algorithms is a challenging problem. In this talk I will present some results of the numerical study of an alternating split Bregman algorithm, as well as the regularised "simple" iterations, with the emphasis on the case $p = 1$ in two and, possibly, three dimensions. This research is a part of the collaborative project with Nachman, Tamasan and Moradifam.

Stability estimates for multi-wave inverse problems

Faouzi Triki
Universite Grenoble-Alpes, France
Faouzi.Triki@imag.fr

A major problem in solving mutli-wave inverse problems is the presence of critical points where the collected data vanishes. The set of these critical points depends on the choice of the boundary conditions, and can be directly determined from the data itself. In most existing stability results, the boundary conditions are assumed to be close to a set of CGO solutions where the critical points are controlled. In this talk I will present new stability estimates for some mutli-wave inverse problems without assumption on the boundary conditions.

Special Session 46: Qualitative Theory of Differential Equations and Applications

Yi Li, Wright State University and Xi'an Jiaotong University, USA
Wei Feng, UNCW, USA
Xiaojie Hou, UNCW, USA

The purpose of the session is to communicate the recent progress in the qualitative theory of differential equations and applications. In particular, the special session will emphasize the up to date development in the steady state solutions of the elliptic equations, the traveling wave solutions and their stability of the reaction diffusion equations and systems.

Topological characterization of omega-limit sets for analytic flows on open subsets of the sphere and the projective plane

Jose G. Espin Buendia
University of Murcia, Spain
josegines.espin@um.es
V. Jiménez López

In a recent paper by V. Jiménez López and J. Llibre (Adv. Math., 2007), topological characterizations of the ω-limit sets for analytic flows on open subsets of the sphere and the projective plane were given. An auxiliary lemma, essential in the proof, states that analytic flows on arbitrary analytic surfaces have the following property: if an orbit meets both sides of an arc of singular points contained in its ω-limit set, then the flow must be equally oriented in both sides. We will show that, despite the fact that the property is true for the plane, the sphere and the projective plane, the statement is not true in general. We shall present a couple of examples on open proper subsets of the plane and on the torus. Therefore, the characterizations given in the cited paper are incomplete; in this talk we present full, correct characterizations of these sets. (This is part of a joint work, still in progress, with V. Jiménez López).

Predator-prey models with time delay in the conversion

Guihong Fan
Columbus State University (GA), USA
fan_guihong@columbusstate.edu
Gail S.K. Wolkowicz

The dynamics of the classical predator-prey model and the predator-prey model based on the chemostat are studied and compared to see whether delay in the conversion process can lead to sustained oscillatory behavior, when no such behavior is possible when delay is ignored. A discrete delay is introduced to model the time between the capture of the prey and its conversion to biomass. We use Holling type I response functions so that no nontrivial oscillating solutions are possible in the absence of delay. We prove that as the parameter modeling the delay is varied Hopf bifurcation of the interior equilibrium can occur for both models. We plot the bifurcation diagram of predators as a function of delay. For the classical predator-prey model, the bifurcation diagram demonstrates that more complicated dynamics including a series of period doublings occur, leading to chaotic dynamics. This expands upon the results in a paper by Gourley and Kuang (J. Math. Biol., 2004).

Traveling Wave Solutions in Models of Complex Competition

Wei Feng
University of North Carolina Wilmington, USA
fengw@uncw.edu
Xin Lu

In this talk, we first review some existing results on a combined mathematical model of resource and sexual competition. The population dynamics (competitive exclusion and coexistence) in this model is analyzed through a coupled system of reaction-diffusion equations. Through construction of smooth upper-lower solutions, the traveling wave solutions of this complex competition system are shown to exist for a family of wave speeds. When strong sexual competition and low growth rate lead to competitive exclusion of the biological species, we obtain the traveling wave solution connecting the corresponding equilibria. Models with sexual competition affecting one or both species are considered and compared.

Analysis of an Extended Rosenzweig-MacArthur Model of a Tri-Trophic Food Chain

Michael Freeze
UNC Wilmington, USA
freezem@uncw.edu
Wei Feng, Nicole Rocco, Xin Lu

We study an extension of the Rosenzweig-MacArthur tri-trophic food chain model in which the super-predator consumes both the predator and the prey. After finding the ultimate bounds and conditions for exponential convergence for these populations, we identify all equilibria and investigate their stability or instability in relation with all the ecological parameters. We focus our investigation on determining conditions for the existence, uniqueness and stability of a coexistence equilibrium.

Topological atlases of integrable Hamiltonian systems

Mikhail Kharlamov
Presidential Academy NEPA, Russia
mikeh@inbox.ru
I.I.Kharlamova

Topological analysis of a completely integrable system supposes a description of the topological structure of all regular integral manifolds and critical integral surfaces and a possibility to find out the bifurcations occurring along any path in the phase space. Having a system with non-linear first integrals, we present an approach to calcu-

late explicitly all bifurcations called the method of critical subsystems. We also describe a general way of the most compact description of the topology of an integrable system in the form of the topological atlas of the system. To build topological atlases one can use contemporary computer algebra programs. We present an electronic realization of such an atlas for one of the most topologically complicated systems in the dynamics of a gyrostat with a fixed point generalizing the classical Kowalevski problem.

Persistence of Lower Dimensional Hyperbolic Tori in Reversible System

Yuedong Kong
Southesat University, Peoples Rep of China
kongyuedong@gmail.com
Junxiang Xu

In this paper we consider the persistence of hyperbolic lower dimensional invariant tori with prescribed frequency in reversible system, and prove that if the frequency mapping has nonzero topological degree at some diophantine frequency, then the hyperbolic lower invariant dimensional torus with this frequency persists under small perturbations.

The nested method and immuno-epidemiological models

Xue-Zhi Li
Xinyang Normal University, Peoples Rep of China
xzli66@126.com
Maia Martcheva

The results in the field of Mathematical modeling and researches in within-host virus infection systems (micro) and diseases transmission systems among hosts (Macro) are very plentiful. Since it has very closed relations between the virus munber within a host and the ability of disease transmission of this host, it is of great significance and realistic meaning to formulate and to investigate coupled systems between immunological and epidemiological systems to reveal the mechanism of disease transmission. In this talk, what is the nested method and how to use this mehtod to formulate the immuno-epidemiological models, are introduced. A two-strain immuno-epidemiological model of HIV with superinfection is formulated. The dynamical behavior of this model is analyized. How the with-host parameters affect the HIV transmission are also investigated by numerical simulations.

Multiple Solutions to an Elliptic Problem Related to Vortex Pairs

Yi Li
Wright State University and Xi'an Jiaotong University, USA
yi.li@wright.edu
Shuangjie Peng

We consider the problem

$$-\Delta u = \lambda(u-\phi)_+^{p-1}, \quad x \in \Omega,\ u|_{\partial\Omega} = 0$$

where Ω is a bounded domain in R^N, ϕ is a positive harmonic function in $\bar{\Omega}$. This problem is related to steady vortex pairs in an ideal fluid. Under the following condition: ϕ has k ($k \geq 1$) strictly local minimum points $\bar{z}_1, \cdots, \bar{z}_k \in \partial\Omega$, we are able to prove the existence of a solution pair (u_λ, A_λ) satisfying that the 'vortex core' (where $u_\lambda > \phi$) A_λ has exactly k components $A_{\lambda,j}$, $j = 1, 2, \cdots, k$ which shrink to the points $\bar{z}_1, \cdots, \bar{z}_k$ respectively as $\lambda \to +\infty$. Moreover, $A_{\lambda,j}$ is approximately a ball with very precise estimates of $z_j - \bar{z}_j$ and $diam(A_{\lambda,j})$. (Joint work with Shuangjie Peng).

Oscillation Results for Second Order Nonlinear Neutral Differential Equations with Several Delays

Saroj Panigrahi
University of Hyderabad, India
panigrahi2008@gmail.com

In this paper, oscillatory and asymptotic behavior of solutions of a class of nonlinear second order neutral differential equations in several delays with positive and negative coefficients of the form

$$\Big(r_1(t)\Big(x(t)+\sum_{i=1}^{k} p_i(t)x(\tau_i(t))\Big)'\Big)' + r_2(t)\Big(x(t)+\sum_{i=1}^{l} q_i(t)x(\sigma_i(t))\Big)' + \sum_{i=1}^{m} s_i(t)G\Big(x(\alpha_i(t))\Big) - \sum_{i=1}^{n} h_i(t)H\Big(x(\beta_i(t))\Big) = 0,$$

and

$$\Big(r_1(t)\Big(x(t)+\sum_{i=1}^{k} p_i(t)x(\tau_i(t))\Big)'\Big)' + r_2(t)\Big(x(t)+\sum_{i=1}^{l} q_i(t)x(\sigma_i(t))\Big)' + \sum_{i=1}^{m} s_i(t)G\Big(x(\alpha_i(t))\Big) - \sum_{i=1}^{n} h_i(t)H\Big(x(\beta_i(t))\Big) = f(t)$$

are studied for $p_i(t) \in C^2([t_0,\infty),\mathbb{R})$; $i = 1,..,k$, $q_i(t) \in C^1([t_0,\infty),\mathbb{R})$; $i = 1,...,l$. Moreover, using Banach fixed point theorem, sufficient conditions are obtained for the existence of bounded positive solutions of the forced equation.

Degenerate Quasilinear Parabolic Equations with Nonlinear Boundary Condition

Weihua Ruan
Purdue University Calumet, USA
ruanw@purduecal.edu
C.V. Pao

This paper is concerned with viscosity solutions for a class of degenerate quasilinear parabolic equations in a bounded domain under nonlinear boundary conditions. The equation under consideration arises from a number of practical model problems, including heat-transfer problems, chemical reactions, and reaction-diffusion processes in porous media. The goal of this paper is: to prove the

existence of a continuous viscosity solution and its comparison property with viscosity upper and lower solutions, to investigate the relationship between the viscosity solution and the classical and weak solutions, and to investigate the asymptotic behavior of the classical solution. Conditions are obtained to ensure that the viscosity solution coincides or evolves into a unique classical solution which leads to some dynamic property of the solution in relation to the positive steady-state solutions. An application of these results is given to a heat transfer problem where the thermal conductivity is temperature dependent and the boundary condition follows a fourth-power radiation law.

On Lyapunov functions method in the problem of stochastic stability of the integral manifold

Gulmira Vasilina
Institute of Mathematics and Mathematical Modeling, Kazakhstan
v_gulmira@mail.ru
Tleubergenov M.I.

Using Lyapunov functions method the sufficient conditions of stability and asymptotic stability in probability of the integral manifold of the first order Ito stochastic differential equations. In this case the random perturbations are assumed from the class processes with independent increments. The theorems about stability in probability of the integral manifold of the first approximation are proved. The problem of stability of motion's properties given analytically under constantly acting random perturbations, being small value on the average and damped perturbations, is solved. Earlier the stability in probability of the unperturbed motion in the presence of random perturbations from the class of Wiener processes (Khas'minskii R.Z., Nevelson M.B., Kushner H.J., Morozan T., Katz I.J., Krasovskii N.N., Stanzhytskyi A.N.) and also from the class of processes with independent increments (Gikhman I.I., Dorogovtsev A.Y.) were investigated. Also before this the stability of the integral manifold given analytically of stochastic differential equations with random perturbations in the class of Wiener processes (Tleubergenov M.I.) was studied.

Qualitative analysis of reaction diffusion systems

Weiqing Xie
CalPoly Pomona, USA
wxie@csupomona.edu

We investigate a class of nonlinear integro-differential equations, which is motivated from quasistatic contact problem in thermoelasticity. The existence and uniqueness of solution for the problem are examined. Some properties of the solution will be discussed.

Spatiotemporal patterns in a reaction-diffusion Lengyel-Epstein system modeling CIMA chemical reaction

Fengqi Yi
Harbin Engineering University, Peoples Rep of China
fengqi.yi@gmail.com
J. Wei, J. Shi, and J. Jin

We are concerned with the spatiotemporal patterns in a kind of reaction diffusion Lengyel-Epstein system, which accounts for the qualitative feature of the well-known Cholrite-Iodide-Malonic Acid and Starch reaction, through which the first experimental evidence of Turing patterns was observed by De Kepper et al. Firstly, we derived the precise conditions on the system parameters so that the spatially homogenous equilibrium solution becomes Turing unstable. Secondly, we constructed a Lyapunov function to show that the spatially homogenous equilibrium solution is globally asymptotically stable when the feeding rate of iodide is small. We also showed that for small spatial domains, all the solutions eventually converge to a spatially homogeneous and time periodic solution. Finally, we proved the existence of bifurcations of spatially non-homogeneous periodic solutions and steady state solutions. The existence of these patterned solutions shows the richness of the spatiotemporal dynamics including Turing instability and oscillatory behavior. Examples of numerical simulation are also shown to support and strengthen the analytical approach.

Special Session 47: Mathematical Modelling and Numerical Methods for Phase-Field Problems

Laurence Cherfils, La Rochelle University, France
Alain Miranville, Universite de Poitiers, France
Madalina Petcu, University of Poitiers, France

The modelling and the study of phase transition/separation processes play an important role both from the physical and mathematical point of view. The aim of this special session is to bring together a number of researchers who work on the modelling of phase field problems, as well as on the numerical resolution of such models and to discuss how new ideas and methods will advance the field in the next decade.

Robust error control for phase field models past topological changes

Soeren Bartels
University of Freiburg, Germany
bartels@mathematik.uni-freiburg.de

Phase field models are often used to describe the evolution of submanifolds, e.g., the Allen-Cahn equation approximates motion by mean curvature and more sophisticated phase field models provide regularizations of the Willmore flow and other geometric evolution problems. The models involve small regularization parameters and we discuss the dependence of a priori and a posteriori error estimates for the numerical solution of the regularized problems on this parameter. In particular, we address the question whether robust error estimation is possible past topological changes. We provide an affirmative answer for a priori error estimates assuming a logarithmic scaling law of the time averaged principal eigenvalue of the linearized Allen-Cahn or Ginzburg-Landau operator about the exact solution. This scaling law is confirmed by numerical experiments for generic topological changes. The averaged eigenvalue about the approximate solution enters a posteriori error estimates exponentially and therefore, critical scenarios are detected automatically by related adaptive finite element methods. The devised scheme extracts information about the stability of the evolution from the approximate solution and thereby allows for a rigorous a posteriori error analysis.

Segmentation and edge-enhancement with adaptive methods in Computer vision.

Zakaria Belhachmi
Mulhouse University, France
zakaria.belhachmi@uha.fr

We consider the segmentation problem for some motion analysis in computer vision. In the variational framework the segmentation of images mainly results from the minimization of the Mumford-Shah functional. This is performed by solving some nonlinear elliptic PDEs (e.g. the Ambrosio and Tortorelli approximation formulation). A second class of methods is based on TV-model where the segmentation is achieved with piecewise constant solution. For the optic flow estimation, such methods of segmentation extend with an additional difficulty relying to the vectoriel character of the problem. We develop a different approach based on the minimization of a linear functional with a spatially varying regularization parameter. The regularization varying parameter acts as a phase field function and is determined by an adaptive method which allows us to select locally its values and to achieve the segmentation of the optic flow. We analyze the method in the framework of the Γ-convergence to explain why such an approach leads to the minimization of the Mumford-Shah functional. In the same spirit we consider the problem of edge-enhancement with the total variation and show that our method allows us to obtain pre-segmented flow.

Finite-dimensional attractors for the Bertozzi-Esedoglu-Gillette-Cahn-Hilliard equation in image inpainting

Hussein Fakih
Poitiers University, France
Hussein.Fakih@math.univ-poitiers.fr
Laurence Cherfils and Alain Miranville

In this article, we are interested in the study of the asymptotic behavior, in terms of finite-dimensional attractors, of a generalization of the Cahn-Hilliard equation with a fidelity term (integrated over $\Omega\backslash D$ instead of the entire domain Ω, $D \subset\subset \Omega$). Such a model has, in particular, applications in image inpainting. The difficulty here is that we no longer have the conservation of mass, i.e. of the spatial average of the order parameter u, as in the Cahn-Hilliard equation. Instead, we prove that the spatial average of u is dissipative. We finally give some numerical simulations which confirm previous ones on the efficiency of the model.

Phase-Field-Crystal Modeling of Wavelength Selection of a Crystal Lattice

Peter Galenko
University of Jena, Germany
peter.galenko@uni-jena.de
Ken Elder

The problem of lattice parameter and velocity selection of a periodic crystal pattern invading a homogeneous liquid phase is analyzed. Special attention is paid to the lattice selection at the front of crystal pattern and in a bulk crystal phase. Analytical and numerical solutions of the phase-field-crystal model are discussed for a periodic pattern which emerges as a phase front sweeps through an unstable liquid.

Phase-field methods for shape and topology optimization

Harald Garcke
University Regensburg, Germany
harald.garcke@ur.de

We consider the problem of shape and topology optimization in fluid and solid mechanics. A phase field approach is introduced and discussed in terms of well-posedness and first order optimality conditions. We find that the minimizers of the diffuse interface model converge along subsequences to a minimizer of classical shape and topology optimization problems. Additionally, we can pass to the sharp interface limit in the phase field equations and obtain classical shape derivatives in the limit. Finally we present numerical simulations based on the phase field approach which demonstrate that the phase field approach can be used to solve shape and topology optimization problems.

Phase-field modeling with surfactants

Ludovic Goudenege
CNRS - Ecole Centrale Paris, France
goudenege@math.cnrs.fr

We study a phase field model for the mixture of two incompressible fluids, typically air and water. Moreover we add a species with surfactant properties. The incompressible Navier-Stokes equations coupled with Allen-Cahn or Cahn-Hilliard equation are presented with supplementary equations for the surfactant concentrations. The model is based on an energetic variational formulation. Moreover since the method is based on the use of a diffuse interface, it allows an implementation using finite element techniques. An application about bubble of soap is presented in collaboration with the physical institut of Rennes in France.

Efficient solution of nonsmooth multicomponent phase field models

Carsten Gräser
Freie Universität Berlin, Germany
graeser@mi.fu-berlin.de

Physically motivated energy functionals for phase field models often incorporate singular terms leading to degenerate nonlinear or nonsmooth problems. While its common to replace those terms by smooth approximations to avoid numerical difficulties, numerical examples shows that this has a strong impact on coarsening rates of solutions. We present numerical methods for phase field models under the presence of singular or nonsmooth terms. Combining nonsmooth Newton and multigrid techniques those methods are robust with respect to the nonlinearity and exhibit mesh independence and global convergence. Efficiency and robustness are illustrated for multicomponent Cahn–Hilliard and Allen–Cahn equations with logarithmic and obstacle potentials.

Consistent hierarchy of Cahn-Hilliard systems and applications to multiphase flows

Sebastian Minjeaud
Univ. Nice Sophia Antipolis, France
minjeaud@unice.fr
Franck Boyer

In this talk, we present recent developments of Cahn-Hilliard systems for an arbitrary number of components. The models are derived using the consistency principle: we require that the n-phase model exactly coincide with the well-known two-phase model when only two phases are present in the system. We give properties of such models in some cases and propose numerical illustrations. We also present simulations of multiphase flows based on the coupling of the n-phase Cahn-Hilliard model with the Navier-Stokes equation. This is a joint work with F. Boyer.

Numerical analysis of an iterative scheme of fractional steps type associated to the nonlinear phase-field equation in Caginalp's model endowed with nonhomogeneous dynamic boundary conditions

Costica Morosanu
Alexandru Ioan Cuza University, Romania
costica.morosanu@uaic.ro

The paper concerns with the existence, uniqueness, regularity and the approximation of solutions to the nonlinear phase-field (Allen-Cahn) equation, endowed with nonhomogeneous dynamic boundary conditions (depending both on time and space variables). It extends the already studied types of boundary conditions, which makes the problem to be more able to describe many important phenomena of two-phase systems, in particular, the interactions with the walls in confined systems. The convergence and error estimate results for an iterative scheme of fractional steps type, associated to the nonlinear parabolic equation, are also established. The advantage of such method consists in simplifying the numerical computation. On the basis of this approach, a conceptual numerical algorithm is formulated in the end.

A Finite-Volume scheme for the Cahn-Hilliard equation with dynamic boundary conditions

Flore Nabet
I2M, Aix-Marseille Université, France
flore.nabet@latp.univ-mrs.fr

This talk is devoted to the numerical analysis of a finite-volume scheme of the Cahn-Hilliard equation with dynamic boundary conditions. The finite-volume method is well adapted to the coupling of the dynamics in the domain and those on the boundary by a flux term. Furthermore, this scheme accounts naturally for the non-flat geometry of the boundary and for the associated Laplace-Beltrami operator. We prove existence and convergence results and we present various numerical simulations.

Convergence to equilibrium for discretized phase-field systems with gradient-like structure

Morgan Pierre
Poitiers University, France
Morgan.Pierre@math.univ-poitiers.fr
N.E. Alaa

A famous result of S. Lojasiewicz states that if $F : \mathbb{R}^d \to \mathbb{R}$ is real analytic, then every bounded solution U of the gradient flow $U'(t) = -\nabla F(U(t))$ converges to a critical point of F as $t \to +\infty$. This convergence result has been generalized to a large variety of finite or infinite dimensional gradient-like flows. In this talk, we show how some of these results can be adapted to time discretizations of gradient-like flows, in view of applications to Allen-Cahn, Cahn-Hilliard or phase-field crystal equations.

Relaxed Diffuse Interface Models for Liquid-Vapour Flow

Christian Rohde
University of Stuttgart, Germany
crohde@mathematik.uni-stuttgart.de

Phase field or diffuse interface models have became a popular modelling tool in recent years to describe the complex dynamics of two-phase flow. We are in particular interested in Navier-Stokes-Korteweg (NSK) models for compressible liquid-vapour phase transitions. The phase-field approach usually introduces higher-order derivatives into the governing system of equations. The classical NSK model contains e.g. third-order derivatives which complicates a fast numerical simulation. Moreover, the first-order part of the NSK model is of mixed hyperbolic-elliptic type which excludes modern Riemann-solver type methods. As a consequence computations for convection-dominated regimes are practically impossible. To avoid these difficulties we introduce a class of relaxed approaximations with purely hyperbolic first-order part. Moreover the relaxed systems do not contain higher-order space derivatives.
In the limit of infinite Korteweg parameter the relaxation approximation coincides with the original NSK model. We will present rigorous convergence results on the level of model problems. The talk concludes with multidimensional numerical experiments. The experiments show the numerical efficiency of the relaxation approximation. We will show furthermore that the relaxation approximation allows stable computations for convection-dominated regimes and in the sharp-interface limit. Finally we will give an outlook how to generalize the relaxation technique to other instances of phase field models. This is joint work with A. Corli, J. Neusser, and V. Schleper.

Crossover of coarsening rates in demixing binary viscous fluids

Christian Seis
University of Toronto, Canada
cseis@math.toronto.edu
Felix Otto and Dejan Slepcev

We consider the coarsening dynamics of a binary viscous fluid modeled by a Cahn-Hilliard/Stokes system. Heuristics, experiments and simulations suggest that early stages of the demixing process are governed by diffusive evaporation-recondensation dynamics leading to a growth of the typical length scale proportional to $t^{1/3}$, where t denotes time, while later stages are characterized by a viscous bulk flow leading to a linear coarsening rate, known as Siggia's growth. In joint work with F. Otto and D. Slepcev, we investigate the crossover between both growth mechanisms and prove a weak upper bound on the respective coarsening rates.

Diffuse Interface Models on Surfaces for Biophysical Applications

Bjorn Stinner
University of Warwick, England
bjorn.stinner@warwick.ac.uk

Some recent results on phase field equations of Allen-Cahn or Cahn-Hilliard type on moving surfaces will be presented. Both analytical questions regarding the asymptotic limit but also the numerical approximation using surface finite element methods will be addressed. While for rigorous results a given moving surface is assumed, the surface often is unknown in applications, i.e., its motion is governed by a free boundary problem which usually back-couples to the lateral processes. Biophysical applications comprise phase separation in biomembranes and the formation of adhesion patches during cell motility.

Boussinesq Systems and Internal Waves in Stratified Fluids

Shu-Ming Sun
Virginia Tech, USA
sun@math.vt.edu

The talk will discuss the internal waves moving under the gravity in stratified fluids. First, it is assumed the fluid is bounded above and below by horizontal rigid plates and the density is stratified vertically. Using Sturm-Liouville theory, it is shown that if the amplitudes of the waves are small and the wave lengths are long, then the approximating model equations are the classical Boussinesq systems of infinite dimensions. Similar systems can be derived for internal gravity waves in stratified fluids with free surfaces.

Long-time stability of the implicit Euler scheme for an incompressible two-phase flow model

Florentina Tone
University of West Florida, USA
ftone@uwf.edu
T. Tachim Medjo

In this talk we present results on the stability for all positive time of the fully implicit Euler scheme for an incompressible two-phase flow model. More precisely, we consider the time discretisation scheme and with the aid of the discrete Gronwall lemma and of the discrete uniform Gronwall lemma we prove that the numerical scheme is stable.

1D Cahn-Hilliard : non-linear growth, coarsening and pattern formation

Simon Villain-Guillot
University of Bordeaux, France
simon.villain-guillot@u-bordeaux1.fr

Many systems exhibit a phase where the order parameter is spatially modulated. These patterns can be the result of a frustration caused by the competition between interaction forces with opposite effects. In models with local interactions, these ordered phases disappear in the strong segregation regime (low temperature). It is expected however that these phases should persist in the case of long range interactions, which can't be correctly described by a Ginzburg-Landau type model with only a finite number of spatial derivatives of the order parameter. An alternative approach is to study the dynamics of the phase transition or pattern formation. While, in the usual process of Ostwald ripening, succession of doubling of the domain size leads to a total segregation, or macro-segregation, C. Misbah and P. Politi have shown using a phase-field approach that long-range interactions could cause an interruption of this coalescence process, stabilizing a pattern which then remains in a micro-structured state or super-crystal. We show that this is indeed the case for a modified Cahn-Hilliard dynamics due to Oono which includes a non local term and which is particularly well suited to describe systems with a modulated phase.

A second order numerical scheme for Cahn-Hilliard-Navier-Stokes equation

Xiaoming Wang
Florida State University, USA
xwang@fsu.edu

We present a novel second order in time algorithm for Cahn-Hilliard-Navier-Stokes system which models two phase flows. The new scheme enjoys several very desirable features: (1) it inherits the physically important energy law and hence is unconditionally stable; (2) it is also uniquely solvable at each time step; (3) the phase field and the velocity field are weakly coupled. Numerical results will be presented as well. This is a joint work with Daozhi Han.

Kinetics of pattern formation in solid-state reactions: mesoscopique and microscopic approaches

Helena Zapolsky
University of Rouen, France
helena.zapolsky@univ-rouen.fr
M. Lavrsky, R. Patte, A.G. Khachaturyan

A general approach to the evolution of complex atomic structures in terms of the Phase Field Field Theory is proposed. It is shown that this approach can be extended to atomic scale and can be used to get an insight on the atomic mechanisms of solid-solid phase transformation. This new approach is illustrated by examples of systems of different physical nature, scales and complexities. The link between the self-organisation in living and physical systems will be also discussed.

Special Session 48: Sparse Optimization and Optimal Control in Dynamical Systems and PDEs

Massimo Fornasier, Johann Radon Institute for Computational and Applied Mathematics, Austria
Boris Vexler, TU Munich, Germany

The dimension scale of problems arising in our modern information society became very large. A new area of science and engineering is now urgently needed in order to extract and interpret significant information from the universe of data collected from a variety of modern sources (Internet, physics experiments, medical diagnostics, etc.). Numerical simulations at the required scale will be one of the great challenges of the 21st century. In short, we need to become capable of organizing and understanding complexity. The most notable recent advances in data analysis and numerical simulation are based on the observation that in several situations, even for very complex phenomena, only a few governing components/degrees of freedom are required to describe the whole dynamics; a dimensionality reduction can be achieved by demanding that the solution be "sparse" or "compressible". Since the relevant degrees of freedom are not prescribed, and may depend on the particular solution, we need efficient optimization methods for solving the hard combinatorial problem of identifying them. Within this Special Session we are first addressing results in designing efficient algorithms which allow us to achieve sparse optimization in high-dimension. Secondly, such tools developed for achieving adaptive dimensionality reductions are used as building blocks for solving large-scale optimal control problems in dynamical systems, partial differential equations, and variational problems arising in various contexts. The novelty of the results presented in our session is precisely the combination of sparsity promoting optimizations, adaptive discretizations, and their use and application to model parsimonius control of complex dynamics. The search for the minimal amount of degrees of freedom to allow the control of systems opens as well interesting connections with information based complexity and information theory. The interest in insisting in combining optimal control of dynamical systems and PDEs is due to their intimate relationship, e.g., by means of discretization processes and finite dimensional approximations. One relevant scope of this session is to further emphasize and explore the relationship between finite and infinite dimensional optimal control problems, with special emphasis on sparse control. Finally, we will be interested in innovative applications in image processing, social-dynamics control, such as guided consensus models and controlled pattern formation, automatic learning and observability of dynamical systems and parameter identification in PDEs via sparse control. Moreover, sparsity based formulations allow for elegant approaches to attack problems of optimal sensor/actuator placement in the context of optimal control of PDEs as well as of point source reconstruction and related inverse problems with PDEs.

Sparse stabilization of dynamical systems driven by attraction and avoidance forces

Mattia Bongini
TU Munich, Germany
mattia.bongini@ma.tum.de
Massimo Fornasier

In this talk we address dynamical systems of agents driven by attraction and repulsion forces, modeling cohesion and collision avoidance. When the total energy, which is composed of a kinetic part and a geometrical part describing the balance between attraction and repulsion forces, is below a certain threshold, then it is known that the agents will converge to a dynamics where mutual space confinement is guaranteed. In this paper we question the construction of a stabilization strategy, which requires the minimal amount of external intervention for nevertheless inducing space confinement, also when the initial energy threshold is violated. Our main result establishes that if the initial energy exceeds the threshold mainly because of its kinetic component, then a sparse control instantaneously applied with enough strength on the most rowdy agent, i.e., the one with maximal speed, will be able to steer in finite time the system to an energy level under the threshold.

Sparse optimal control of the Korteweg de Vries equation

Anne-Celine Boulanger
T.U. Munich, Germany
boulanger@ma.tum.de
Philip Trautmann, Boris Vexler

We focus in this work on optimal control problems of the following form

$$\min_{q\in\mathcal{M}_I} J(y) = \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega,L^2(I))} + \alpha\|q\|_{\mathcal{M}_I}$$

where y is the solution of the nonlinear Korteweg de Vries equation with a time dependent measure valued source term acting as control

$$\begin{cases} \partial_t y + \partial_x y + \partial_{xxx} y + y\partial_x y = q \text{ in } \Omega, \\ y(.,0) = y(.,L) = \partial_x y(.,L) = 0 \text{ in } \Gamma, \\ y(0,.) = 0 \text{ on } \Omega. \end{cases}$$

which is known to have traveling wave solutions. The control space $\mathcal{M}_I$ is either the Bochner space $L^2(I,\mathcal{M}(\Omega))$ or the space of vector measures $\mathcal{M}(\Omega.L^2(I))$ with values in $L^2(I)$. For both choices the controls are sparse in space and distributed in time. However, the first space allows for moving dirac measures while the second does not. We will tackle the following questions: well posedness of the KdV equation with a non-smooth source term, existence and characterization of an optimal control, algorithmic treatment of the problem by a semi-smooth Newton method in function space. In the end, we present some numerical examples that motivate our work: sparse stabilization of the KDV equation and sparse inverse source problems for the KDV equation (reconstruction of the topography and/or topography changes).

Inverse problems in spaces of measures

Kristian Bredies
University of Graz, Austria
kristian.bredies@uni-graz.at
Hanna K. Pikkarainen

In the talk, the ill-posed problem of solving linear equations in the space of vector-valued finite Radon measures and Hilbert-space data is addressed. Well-posedness of Tikhonov-regularization with the Radon norm as well as further regularization properties and optimality conditions are discussed. Moreover, a flexible and convergent optimization algorithm in the space of measures is proposed. As an example, analysis and numerical experiments for sparse deconvolution are presented. For this problem, optimization in the space of Radon measures turns out to be a suitable and effective approach.

Sparse stabilization of multi-agents systems

Marco Caponigro
CNAM - Paris, France
marco.caponigro@cnam.fr

We study controlled alignment models in finite dimension and we explore how to enforce pattern formation or convergence to consensus in a group of interacting agents. In particular we focus on the design of control strategies requiring a small amount of external intervention: we want to minimize at each instant of time the number of leaders needed to steer the systems to the desired state. These sparsity features are desirable in view of practical issues.

Sparse Optimal Control of the FitzHugh-Nagumo System

Eduardo Casas
University of Cantabria, Spain
eduardo.casas@unican.es
Christopher Ryll and Fredi Tröltzsch

We investigate the problem of sparse optimal controls for the so-called FitzHugh-Nagumo system. In these reaction-diffusion equations, traveling wave fronts occur that can be controlled in different ways. The L^1-norm of the distributed control is included in the objective functional so that optimal controls exhibit effects of sparsity. We prove the differentiability of the control-to-state mapping, show the well-posedness of the optimal control problem and derive first- and second-order optimality conditions. Based on them, the sparsity of optimal controls is shown. The theory is illustrated by various numerical examples, where wave fronts or spiral waves are controlled in a desired way.

A convex analysis approach to hybrid binary-continuous optimal control problems with application to sparse and multi-bang controls

Christian Clason
University Duisburg-Essen, Germany
christian.clason@uni-due.de
Kazufumi Ito, Karl Kunisch

Convex relaxation of binary-continuous optimization problems and their numerical solution by semi-smooth Newton methods are discussed. The proposed framework involves L^0-type penalties that pointwise are zero on the admissible set and one otherwise. Such penalties can be used to promote controls that are sparse or that take values only from a given discrete set (called “multi-bang” controls) but are non-convex and lack weak lower-semicontinuity, application of Fenchel duality yields a formal primal-dual optimality system that admits a unique solution. Under appropriate conditions, it is possible to derive a generalized multi-bang principle, i.e., to prove that this solution is optimal and almost everywhere takes on values only from the admissible set. This is illustrated by numerical examples.

First- and Second-Order Optimality Conditions for Optimal Control Problems with Directional Sparsity Constraints

Roland Herzog
TU Chemnitz, Germany
roland.herzog@mathematik.tu-chemnitz.de
Eduardo Casas, Gerd Wachsmuth

We consider optimal control problems subject to semilinear parabolic equations. The objective features an L^1-norm term, which promotes the sparsity of the optimal control. Different types of iterated norms are considered, which give rise to different (directional) sparsity patterns. In this presentation, we consider first-order necessary as well as second-order necessary and sufficient optimality conditions for these problems. These constitute the basis of fast numerical solution methods and discretization error estimates.

Regularization based on all-at-once formulations of inverse problems for PDEs

Barbara Kaltenbacher
Alpen-Adria-Universitaet Klagenfurt, Austria
barbara.kaltenbacher@aau.at
Alana Kirchner, Boris Vexler

The common approach of using the coefficient-to-state map within the operator equation formulation of an inverse problem has certain drawbacks from a computational point of view. In particular, each step in a solution by iteratively minimizing some Tikhonov functional or applying a regularized Gauss-Newton method will require more or less exact solution of the PDE. This can be avoided by all-at once formulations, where the PDE and the measurement equation are considered as one large system of equations which is solved simultaneously. This

allows to safe a considerable amount of computational cost, especially in the context of nonlinear PDEs. In this talk we will particularly focus on all-at-once versions of regularized Newton type methods and their adaptive discretization using dual weighted residual estimators.

Inverse Medium Scattering and Sparse Reconstruction

Kamil Kazimierski
University of Graz, German
kamil.kazimierski-hentschel@uni-graz.at
Armin Lechleiter

We consider inverse medium scattering problem where the aim is to reconstruct a spatially dependent index of refraction form field measurements of waves scattered in a inhomogeneous medium. In particular we discuss the case, that the medium is sparse with respect to some a-priori known basis. Such inversion problems can be reformulated using Tikhonov-like functionals with L^p or ℓ^p penalty terms. In the talk we first discuss continuity and differentiability properties of the forward problem, then the implications for the reconstruction scheme and finally we present numerical results.

An accelerated and preconditioned forward-backward method for monotone inclusions

Dirk Lorenz
TU Braunschweig, Germany
d.lorenz@tu-braunschweig.de
Thomas Pock

Sparse optimization usually leads to non-smooth convex minimization problems. Often these problems can be conveniently formulated as convex-concave saddle-point problems. The solution of these saddle point problems is equivalent to the solution of a monotone operator equation. In this talk we augment the well know forward-backward method for monotone inclusions by preconditioning and an acceleration of Nesterov-type. The resulting method is very flexible in that it can tackle non-smooth primal and dual terms as well as exploit smooth terms in both the primal and the dual problem. It is successfully applied to various large scale non-smooth optimization problems.

A globalized semismooth Newton method for nonsmooth optimization problems

Andre Milzarek
Technical University Munich, Germany
milzarek@ma.tum.de
Michael Ulbrich

In this talk we present a globalized semismooth Newton method for solving a class of optimization problems involving smooth nonconvex and nonsmooth convex terms in the objective function. The approach is based on a prox-type fixed point equation that represents the first order stationarity conditions. In many important situations, including e.g. sparse optimization problems that arise from l_1-regularization or tree-/group-sparsity, the corresponding proximity operator can be shown to be semismooth. The method we investigate combines semismooth Newton steps for solving the fixed point equation, a filter, and an embedded basic globally convergent method, such as, e.g., a proximal gradient scheme. We present both global and local convergence results and conclude with numerical examples illustrating the efficiency of the proposed method.

Measure valued directional sparsity for parabolic optimal control problems

Konstantin Pieper
Technische Universitaet München, Germany
pieper@ma.tum.de
Karl Kunisch, Boris Vexler

We consider a parabolic optimal control problem with a directional sparsity functional, where the control variable is searched for in the space of vector measures $\mathcal{M}(\Omega_c, L^2(I))$.

$$\text{Minimize } \frac{1}{2}\|y - y_d\|^2_{L^2(I\times\Omega_o)} + \alpha\|u\|_{\mathcal{M}(\Omega_c,L^2(I))},$$

$$\text{subject to } \partial_t y + Ay = u \quad \text{in } I \times \Omega.$$

The optimal solutions of this problem are localized in space, where the spatial support is independent of time. We establish an appropriate function space setting for the problem and derive structural properties of the minimizer from the optimality conditions. A typical solution given by a sum of point sources with time dependent intensities, $u = \sum_i u_i(t)\delta_{x_i}$. We motivate this problem formulation by discussing an application to a deconvolution problem. Furthermore we discuss a suitable finite element discretization and an efficient solution method within a path-following Newton framework. We provide an a priori error analysis for the discretization of the optimal control problem.

Uncertainty quantification via compressive sensing

Holger Rauhut
RWTH Aachen University, Germany
rauhut@mathc.rwth-aachen.de
Christoph Schwab

Compressive sensing enables accurate recovery of approximately sparse vectors from incomplete information. We apply this principle to the numerical solution of parametric partial differential equations (PDEs with random coefficients). In fact, one can show that the solution of certain parametric PDEs is analytic in the parameters which can be exploited to show convergence rates for nonlinear (sparse) approximation. Building on this fact, we show that methods from compressive sensing can be used to compute approximations from samples (snapshots) of the parametric PDEs, which in turn can be computed by standard methods for usual (nonparametric) PDEs. We provide theoretical approximation rates for this scheme.

How to steer high-dimensional Cucker-Smale systems to consensus using low-dimensional information only

Benjamin Scharf
TU Munich, Germany
benjamin.scharf@ma.tum.de
Mattia Bongini, Massimo Fornasier, Oliver Junge

Dynamical systems of Cucker-Smale type can be used to describe the pattern formation of interacting agents. There are two situations: If the difference between the velocities of the agents is not two large in comparison to the distances of the agents, the system tends to consensus. Otherwise, when there is no self-organization, it was shown recently that one can steer the system to consensus using a sparse control acting only on the agent farthest away from the mean velocity. However, in real-life complex situations the dimension of the agents might be very large (thousands and more) and numerical simulations might be extremely expensive. In this talk we will present an idea to use Johnson-Lindenstrauss embeddings to reduce the system to a low-dimensional Cucker-Smale system. The main question is: Can we choose the agent on which we want to infer control only using the information from the low-dimensional system and thereby steer the high-dimensional system to consensus?

Mean field Optimal Control

Francesco Solombrino
Munich University of Technology, Germany
francesco.solombrino@ma.tum.de
Massimo Fornasier

We introduce the concept of *mean-field optimal control* which is the rigorous limit process connecting finite dimensional optimal control problems with ODE constraints modeling multi-agent interactions to an infinite dimensional optimal control problem with a constraint given by a PDE of Vlasov-type, governing the dynamics of the probability distribution of interacting agents. While in the classical mean-field theory one studies the behavior of a large number of small individuals *freely interacting* with each other, by simplifying the effect of all the other individuals on any given individual by a single averaged effect, we address the situation where the individuals are actually influenced also by an external *policy maker*, and we propagate its effect for the number N of individuals going to infinity. On the one hand, from a modeling point of view, we take into account also that the policy maker is constrained to act according to optimal strategies promoting its most parsimonious interaction with the group of individuals. This will be realized by considering cost functionals including L^1-norm terms penalizing a broadly distributed control of the group, while promoting its sparsity. On the other hand, from the analysis point of view, and for the sake of generality, we consider broader classes of convex control penalizations. In order to develop this new concept of limit rigorously, we need to carefully combine the classical concept of mean-field limit, connecting the finite dimensional system of ODE describing the dynamics of each individual of the group to the PDE describing the dynamics of the respective probability distribution, with the well-known concept of Γ-convergence to show that optimal strategies for the finite dimensional problems converge to optimal strategies of the infinite dimensional problem.

Sparse optimal control for wave equations

Philip Trautmann
University of Graz, Austria
philip.trautmann@uni-graz.at
Karl Kunisch, Boris Vexler

This talk deals with optimal control problems for instationary wave equations involving a sparsity functional as control cost term. In particular the norms of the Bochner space $L^2((0,T),\mathcal{M}(\Omega))$ or of the space of vector measures $\mathcal{M}(\Omega,L^2(0,T))$ are chosen to induce sparsity of the optimal control in space direction and to have L^2-regularity in time. One important difference between these two approaches is that the first one allows for a time dependent support of the optimal measure, e.g., a moving point source, whereas the second only allows for a time independent support of the optimal measure. Further differences regarding the well posedness of the state equation and the first order optimality conditions are discussed. The sparsity properties of the optimal controls are derived from these first order optimality conditions. A regularized version of the problem can be solved by a semismooth Newton-method. The talk is concluded with some numerical examples.

Optimal shape and location of sensors or actuators in PDE models

Emmanuel Trélat
Univ. Pierre et Marie Curie (Paris 6), France
emmanuel.trelat@upmc.fr
Yannick Privat, Enrique Zuazua

We investigate the problem of optimizing the shape and location of sensors and actuators for evolution systems driven by distributed parameter systems or partial differential equations (PDE). We consider wave, Schrödinger and heat equations on an arbitrary domain Ω, in any space dimension, and with suitable boundary conditions (if there is a boundary) which can be of Dirichlet, Neumann, mixed or Robin type. This kind of problem is frequently encountered in applications where one aims, for instance, at maximizing the quality of reconstruction of the solution, using only a partial observation. From the mathematical point of view, using probabilistic considerations we model this problem as that of maximizing the so-called randomized observability constant, over all possible subdomains of Ω having a prescribed measure. The spectral analysis of this problem reveals intimate connections with the theory of quantum chaos. More precisely, we provide a solution to this problem when the domain Ω satisfies suitable quantum ergodicity assumptions.

Weighted and reweighed l1 minimization for tracking sparse dynamical systems

Rachel Ward
UT-Austin, USA
rward@math.utexas.edu

We discuss the application of weighted and re-weighted l1 minimization for recovering sparse signals whose support set is known to have been drawn according to a non-uniform prior distribution over s-sparse support sets. In particular, we show that in this regime, weighted l1 minimization can outperform unweighted l1 minimization in terms of number of measurements needed to achieve a given reconstruction accuracy. Finally, we leverage this theory to provide recovery guarantees for reweighted l1 minimization as an effective tool for dynamic filtering to track time-varying sparse signals.

Special Session 49: Advances in the Numerical Solution of Nonlinear Evolution Equations

Thalhammer Mechthild, University of Innsbruck, Austria

The intention of this special session on "Advances in the numerical solution of nonlinear evolution equations" is to gather mathematicians interconnected through their field of application, the analytical tools, or the numerical methods used. The scope of topics in particular includes nonlinear Schrödinger type equations and nonlinear wave equations.

WKB-based finite difference schemes for highly oscillatory ODEs with turning points

Anton Arnold
Vienna University of Technology, Austria
anton.arnold@tuwien.ac.at
Jens Geier, Claudia Negulescu

We are concerned with the numerical integration of ODEs of the form $\epsilon^2\psi_{xx} + a(x)\psi = 0$ for given $a(x) \geq 0$ in the highly oscillatory regime 0

Fractional-in-space reaction-diffusion equations: Applications to biological tissues

Alfonso Bueno-Orovio
University of Oxford, England
alfonso.bueno@cs.ox.ac.uk
David Kay, Vicente Grau, Blanca Rodriguez, Kevin Burrage

Fractional differential equations are becoming increasingly used as a powerful modelling approach for understanding the many aspects of non-locality and spatial heterogeneity. However, the numerical approximation of these models is demanding and imposes a number of computational constraints. Here, we will introduce Fourier spectral methods as an attractive and easy-to-code alternative for the integration of fractional-in-space reaction-diffusion equations described by the fractional Laplacian in bounded rectangular domains. We will also provide a biophysical interpretation for the fractional Laplacian in problems related to excitable media, with broad applications in cardiac electrophysiology.

Error estimation and adaptive time stepping for nonlinear evolution equations

Winfried Auzinger
Institute for Analysis and Scientific Computing, Vienna University of Technology, Austria
w.auzinger@tuwien.ac.at

We consider one-step discretization methods for nonlinear evolution equations and address the issues of local error estimation, adaptive time step control, and global error estimation. Concerning local error estimation we consider a technique which is generally applicable, on the basis of evaluating the defect of the numerical solution. We explain the approach and its concrete realization for exponential splitting methods and partitioned Runge-Kutta methods. This way of estimating the local error can also be interpreted as a quadrature approximation of Hermite type for an integral representation of the local error. The order of this approximation is such that the error estimate is asymptotically correct, i.e., is deviation from the true error is of a higher order than the error itself. We present numerical results obtained with adaptive codes based on spectral discretization in space for a nonlinear Schrödinger equation and nonlinear wave equations as for instance the Klein-Gordon equation. The precise design of the stepsize controller and its impact on the behavior of the adaptive integrator are discussed in the pre-asymptotic and asymptotic regimes. Moreover, we demonstrate how a global error estimate can be computed in parallel at minimal extra computational cost.

Plane wave numerical stability of some explicit exponential methods for cubic Schrödinger equation

Begoña Cano
Universidad de Valladolid, Spain
bego@mac.uva.es
Adolfo González-Pachón

Plane wave numerical stability for cubic Schrödinger equation has already been studied in the literature when integrating in time with a first-order splitting and with some second-order implicit methods. On the other hand, it has recently been observed that exponential Lawson methods can be an efficient tool to integrate this equation. In particular, when this type of methods are based on explicit Runge-Kutta ones, after projecting on one invariant (norm), the numerical solution arrives at another invariant (momentum) for many solutions which include plane waves. As the projection is very cheap, some times these methods are competitive against splitting ones, which have also been proved to very efficient when intetrating this equation. In this talk, we will show the results of an exhaustive analysis on plane wave numerical stability when integrating with the following second-order explicit exponential methods: Strang splitting and Lawson methods based on a one-parameter family of 2-stage Runge-Kutta ones. For the latter, we will consider the projected and unprojected version onto the norm. We will show stability regions and numerical experiments which corroborate theoretical results.

Combined Newton and Sobolev gradient methods for Navier-Stokes-Boussinesq and Gross-Pitaevskii equations

Ionut Danaila
University of Rouen, France
ionut.danaila@univ-rouen.fr
Parimah Kazemi

Newton linearization has been successfully used in [1] for solving fluid dynamics and heat transfer (Navier-Stokes-Boussinesq) equations with the advantage to accelerate computations due to its rapid quadratic convergence. Sobolev gradient methods proved very effective in minimizing constrained Schrödinger type energy functionals, as the Gross-Pitaevskii energy [2]. Since the Newton method can be viewed as a steepest descent gradient method with variable metric, we can combine the Newton method with the Sobolev gradient method to solve unconstrained or constrained minimization problems. We illustrate the new method by computing various cases from fluid mechanics (phase-change materials) and condensed matter physics (Bose-Einstein condensates).

References

[1] I. Danaila, R. Moglan, F. Hecht, S. Le Masson, A Newton method with adaptive finite elements for solving phase-change problems with natural convection, submitted, 2013.

[2] I. Danaila, P. Kazemi, A new Sobolev gradient method for direct minimization of the Gross-Pitaevskii energy with rotation. SIAM J. of Scientific Computing, 32:2447-2467, 2010.

Exponential type integrators for abstract quasilinear parabolic equations with variable domains

Cesáreo González
University of Valladolid, Spain
ome@am.uva.es

In this talk, I propose an exponential explicit integrator for the time discretization of quasilinear parabolic problems. My numerical scheme is based on splitting methods. In an abstract formulation, the initial-boundary value problem is written as an initial value problem on a Banach space X

$$u'(t) = A\big(u(t)\big)u(t) + b(t), \quad 0 < t \leq T,\ u(0) \text{ given},$$

involving the sectorial operator $A(v) : D(v) \to X$ with variable domains $D(v) \subset X$ with regard to $v \in V \subset X$. Under reasonable regularity requirements on the problem, I analyze the stability, the convergence behaviour and some numerical examples of the numerical methods.

References

[1] C. González and M. Thalhammer, *A second-order Magnus type integrator for quasilinear parabolic problems.* Math. Comp., 76, 257, (2007) pp. 205-231.

[2] M. Hochbruck and Ch. Lubich, *On Magnus integrators for time-dependent Schrödinger equations.* SIAM J. Numer. Anal. 41 (2003) pp. 945-963.

[3] M. Hochbruck and A. Ostermann, *Exponential multistep methods of Adams-type.* BIT, vol. 51, pp. 889-908 (2011).

[4] M. Thalhammer, *Convergence analysis of high-order time-splitting pseudospectral methos for nonlinear Schrödinger equations.* SIAM J. Numer. Anal. 41, vol. 50, No. 6, (2012), pp. 3231-3258.

The Westervelt equation and its efficient time integration by operator splitting methods

Barbara Kaltenbacher
Alpen-Adria-Universitaet Klagenfurt, Austria
barbara.kaltenbacher@aau.at
Mechthild Thalhammer, Vanja Nikolić

In the simulation of high intensity ultrasound, a particular challenge due to nonlinearity and the presence of different wave lengths is efficient and robust time integration. For this purpose, a promising approach are operator splitting techniques exploiting the intrinsic structure of the equations. As a model problem we study the Westervelt equation in more detail, which is a nonlinear wave equation with potential degeneracy and strong damping. We will show several decomposition variants based on a reformulation as a first order system and, for the two most promising ones, show convergence results for the Lie Trotter splitting (for details on the convergence analysis we refer to the talk by Mechthild Thalhammer in ss108) These rely on some new spatial regularity results for the Westervelt equation. Finally, numerical experiments will illustrate the theoretical findings.

Stiff convergence of force-gradient operator splitting methods

Emil Kieri
Uppsala University, Sweden
emil.kieri@it.uu.se

We consider force-gradient, also called modified potential, operator splitting methods for problems with unbounded operators. We prove that force-gradient operator splitting schemes retain their classical orders of accuracy for time-dependent partial differential equations of parabolic or Schrödinger type, provided that the solution is sufficiently regular.

Micro-macro parareal algorithms

Frederic Legoll
Ecole des Ponts, France
legoll@lami.enpc.fr

Coarse-graining is ubiquitous in many application fields, including molecular dynamics, complex fluids, materials science, ... Starting from a fine model, one derives a coarse model, which is a good approximation of the fine model in some regimes. The coarse model is cheaper to simulate than the fine one, because it typically implies a simpler physics, and/or the dimensionality of the state variable is smaller. We will review recent advances using the parareal algorithm to address this context. The algorithm uses the coarse model as a predictor (which is iteratively corrected) to efficiently compute the trajectory of the fine model, using parallel-in-time computations.

Optimal non-reversible linear drift for the convergence to equilibrium of a diffusion

Tony Lelievre
Ecole des Ponts ParisTech, France
lelievre@cermics.enpc.fr
F. Nier and G. Pavliotis

We consider non-reversible perturbations of reversible diffusions that do not alter the invariant distribution and we ask whether there exists an optimal perturbation such that the rate of convergence to equilibrium is maximized. We solve this problem for the case of linear drift by proving the existence of such optimal perturbations and by providing an easily implementable algorithm for constructing them. We discuss in particular the role of the prefactor in the exponential convergence estimate. Our rigorous results are illustrated by numerical experiments. Reference: T. Lelièvre, F. Nier and G. Pavliotis, Optimal non-reversible linear drift for the convergence to equilibrium of a diffusion, Journal of Statistical Physics, 152(2), 237-274, (2013).

Numerical methods for highly oscillatory PDEs

Florian Mehats
IRMAR, University of Rennes 1, France
florian.mehats@univ-rennes1.fr
Philippe Chartier, Nicolas Crouseilles, Mohammed Lemou

This work is devoted to the numerical simulation of highly oscillatory nonlinear Schrödinger ou Klein-Gordon equations. I will present a general strategy to construct numerical schemes which are uniformly accurate with respect to the oscillation frequency. This is a stronger feature than the usual so called "Asymptotic preserving" property, the last being also satisfied by our scheme in the highly oscillatory limit. Our strategy enables to simulate the oscillatory problem without using any mesh or time step refinement, and the orders of our schemes are preserved uniformly in all regimes. In other words, since our numerical method is not based on the derivation and the simulation of asymptotic models, it works in the regime where the solution does not oscillate rapidly, in the highly oscillatory limit regime, and in the intermediate regime with the same order of accuracy.

Implicit-explicit schemes for nonlinear PDEs with multiple scales

Lorenzo Pareschi
University of Ferrara, Italy
lorenzo.pareschi@unife.it
Sebastiano Boscarino, Giovanni Russo

In this talk we consider the development of implicit-explicit schemes for nonlinear PDEs in presence of multiple space and time scales. Several issues will be addressed, in particular the development of asymptotic preserving methods and the problem of uniform accuracy of the resulting schemes with respect to the parameters characterizing the small scales. Applications to hyperbolic and kinetic equations n the diffusion limit will be presented.

Long time accuracy of numerical integrators for ergodic stochastic differential equations.

Gilles Vilmart
University of Geneva, Switzerland
Gilles.Vilmart@unige.ch
A. Abdulle, K.C. Zygalakis

We introduce new sufficient conditions for a numerical method to approximate with high order of accuracy the invariant measure of a nonlinear ergodic system of stochastic differential equations, independently of the standard weak order of accuracy of the method. We then present a systematic procedure based on the framework of modified differential equations for the construction of stochastic integrators that capture the invariant measure with a high order of accuracy, again independently of the weak order. Special attention is paid to the high order properties of Lie-Trotter splitting methods for Langevin dynamics, in spite of their standard weak order one.

References

[1] A. Abdulle, G. Vilmart, and K.C. Zygalakis, High order numerical approximation of the invariant measure of ergodic SDEs, preprint. 2013.

[2] A. Abdulle, G. Vilmart, and K.C. Zygalakis, Long time accuracy of Lie-Trotter splitting methods for second order stochastic dynamics, preprint. 2014.

Special Session 50: Evolution Equations and Inclusions With Applications to Control, Mathematical Modeling and Mechanics

Nasir U. Ahmed, University of Ottawa, Canada
Stanislaw Migorski, Jagiellonian University in Krakow, Poland
Ioan I. Vrabie, Al. I. Cuza University of Iasi, Romania

The session will focus on the recent developments in the theory of nonlinear evolution equations, optimal control theory and related topics including real life problems of mechanics, biology, economics, and medicine. The main topics of the session include, but are not limited to, analysis of solutions of evolution problems and partial differential equations, operator inclusions, evolution inclusions, control problems, mathematical modeling of natural systems, nonsmooth systems, variational methods, convex and nonconvex problems, optimization of systems and applications.

Rothe method for parabolic variational-hemivariational inequalities

Krzysztof Bartosz
Jagiellonian University, Poland
bartosz@ii.uj.edu.pl
Xiaoliang Cheng, Piotr Kalita, Yuanjie Yu, Cong Zheng

We deal with the convergence analysis of the semidiscrete Rothe scheme for the parabolic variational-hemivariational inequality with the nonlinear pseudomonotone operator. The problem involves both a discontinuous and nonmonotone multivalued term as well as a monotone term with potentials which assume infinite values and hence are not locally lipschitz. The proof can be viewed both as the proof of solution existence as well as of the convergence of a numerical semidiscrete scheme.

Nonlinear multi-valued reaction-diffusion systems with delay

Monica-Dana Burlica
"G. Asachi" Technical University, Romania
monicaburlica@yahoo.com
Mihai Necula, Daniela Rosu, Ioan I. Vrabie

The aim of this lecture is to present a sufficient condition for the existence of C^0-solutions for a class of nonlinear multi-valued reaction-diffusion systems with delay subjected to nonlocal implicit initial conditions. This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS–UEFISCDI, project number PN-II-ID-PCE-2011-3-0052.

On sensitivity of optimal solutions to control problems for systems governed by evolution subdifferential inclusions

Zdzislaw Denkowski
Jagellonian University - Cracow, Poland
zdzislaw.denkowski@gmail.com
Krzysztof Bartosz, Piotr Kalita

We consider a class of second order evolution subdifferential inclusions to which can be reduced some hemivariational inequalities. First we quote an existence result for such inclusions based on the theory of pseudomonotone operators. Next we formulate control problems and we present theorems concerning the existence of optimal solutions and their sensitivity under perturbations of state relations as well as the cost functionals. The sensitivity part, based on Γ-convergence theory, works when we are able to prove the Kuratowski-Painleve convergence of the solution sets for the perturbed state relations and simultaneously the appriopriate complementary Γ-convergence of the perturbed cost functionals to the unperturbed one.

Variational-hemivariational approach to a quasistatic viscoelastic problem with normal compliance, friction and material damage

Leszek Gasinski
Jagiellonian University, Poland
Leszek.Gasinski@ii.uj.edu.pl
Anna Ochal, Meir Shillor

This work studies a model for quasistatic frictional contact between a viscoelastic body and a reactive foundation. The constitutive law is assumed to be nonlinear and contains damage effects modeled by a parabolic inclusion. Contact is described by the normal compliance condition and by a subdifferential frictional condition. A variational-hemivariational formulation of the problem is provided and the existence and uniqueness of its weak solution is proved. The proof is based on a surjectivity result for pseudomonotone coercive operators and a fixed point argument.

Approximate and null controllability results for the heat equation with memory

Andrei Halanay
Politehnica of Bucharest, Romania
halanay@mathem.pub.ro
Luciano Pandolfi

The model to be studied is the heat equation with memory in the smooth domain $\Omega \subset \mathbb{R}^m$

$$\frac{\partial w}{\partial t}(t,x) = \Delta_x w(t,x) + \int_0^t M(t-s)\Delta_x w(s,x)ds$$

subject to initial conditions

$$w(0,x) = \xi(x), \quad x \in \Omega$$

and with a boundary control on $\Gamma \subset \partial\Omega$

$$w(t,x) = \begin{cases} v(t,x), & x \in \Gamma,\ t \in [0,T] \\ 0, & x \in \partial\Omega - \Gamma,\ t \in [0,T]. \end{cases}$$

or with an interior control

$$\frac{\partial w}{\partial t}(t,x) = \Delta_x w(t,x) + \int_0^t M(t-s)\Delta_x w(s,x)ds + u(t,x)\chi_\omega, \omega \subset \Omega$$

Under smoothness hypothesis on the kernel M, approximate and null controllability will be investigated using the reduction to a moment problem.

A global implicit function theorem and its applications

Dariusz Idczak
University of Lodz, Poland
idczak@math.uni.lodz.pl

The main result of the paper is a global implicit function theorem. In the proof of this theorem, we use a variational approach and apply Mountain Pass Theorem. An assumption guaranteing existence of an implicit function on the whole space is a Palais-Smale condition. Some applications to integro-differential equations are given.

Dynamic viscoelastic unilateral contact problem with normal compliance and nonmonotone friction

Piotr Kalita
Jagiellonian University, Poland
piotr.kalita@ii.uj.edu.pl
Mikael Barboteu, Krzysztof Bartosz

We formulate a dynamic problem that models contact of a viscoelastic body with a rigid foundation covered by a layer of an elastic material. The normal contact is governed, up to a certain threshold, by a normal compliance law and, once this threshold is reached, by a Signorini condition. As for the friction condition, we consider a generalized Tresca law, such that the dependance of the tangential stress on the tangential velocity can be nonmonotone. We provide a proof of the solution existence as well as of the convergence of solutions to the problems with infinite penetration to the one with finite penetration. The results of the numerical experiment are presented as well.

Estimates for large time controls

Alina Lazu
"Gh. Asachi" Technical University, Romania
vieru_alina@yahoo.com
O. Carja

We study the behavior of the minimum L^p-norm control, $p \in (1, \infty]$, needed to drive a linear system to zero as time duration goes to infinity. We apply the result to various examples.

This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS–UEFISCDI, project number PN-II-ID-PCE-2011-3-0154 .

Topological methods for semi-linear evolution equations in abstract spaces

Luisa Malaguti
University of Modena and Reggio Emilia, Italy
luisa.malaguti@unimore.it

This talk deals with a semi-linear evolution equation in a Banach space: x'(t) = Ax(t) + f(t, x(t)) which is the abstract formulation of many concrete differential models. The densely defined linear part A generates a strongly continuous semigroup of contractions; the nonlinear term f is continuos and possibly super-linear in x. A wide family of nonlocal associated boundary problems is studied, including Periodic, anti-periodic, mean value and multipoint conditions. The investigation is based on topological techniques and suitable Lyaponov-like functions for guaranteing the required transversality are introduced. Applications to the study of nonlocal population diffusion models complete this discussion.

Weak and strong solutions to stochastic inclusions and applications

Mariusz Michta
University of Zielona Gora, Poland, Poland
m.michta@wmie.uz.zgora.pl

In the talk we study the problem of existence of strong and weak solutions to stochastic inclusions. Further we present main topological properties of such solutions and discuss their applications to the theory of fuzzy-valued stochastic equations.

Existence results for first order evolution inclusions and variational-hemivariational inequalities

Stanislaw Migorski
Jagiellonian University in Krakow, Poland
migorski@ii.uj.edu.pl

We deal with an abstract first order evolution inclusion in a reflexive Banach space. The inclusion contains the sum of L-pseudomonotone operator and a maximal monotone operator. We provide an existence theorem which is a generalization of former results known in the literature. Next, we apply our result to the case of nonlinear variational-hemivariational inequalities considered in the setting of an evolution triple of spaces. We specify the multivalued operators in the problem and obtain existence results for several classes of variational-hemivariational inequality problems. Finally, we illustrate our existence result and study a class of quasilinear parabolic problems under nonmonotone and multivalued flux boundary conditions.

Stochastic delay inclusion with Carathéodory-upper separated multi-functions

Jerzy Motyl
University of Zielona Gora, Poland
j.motyl@wmie.uz.zgora.pl

The existence of strong solutions for stochastic delay inclusion $dx(t) \in F(t, x_t)dt + G(t, x_t)dW(t)$, $x_0 = \xi$ will be discussed for measurable multifunctions $F, G : [0, T] \times C(-r, 0) \to ClConvR^n$ being upper separated with respect to their second variable. The result will follow by the Carathéodory type selection method. Therefore, the problem of the existence of Carathéodory-convex selections in nonseparable Banach spaces for multifunctions of the type mentioned above will be discussed in detail.

Variational analysis of a diffusion-controlled model for describing the surfactant behavior at the air-water interface

Cristina Núñez García
University of Santiago de Compostela, Spain
cristina.nunez.garcia@usc.es
J. R. Fernández, M.C. Muñiz

The study of the dynamic surface tension of surfactant solutions at the air-water interface has been revealed an interesting issue since it plays an important role in several biological, biochemical and industrial processes. When a new surface is formed in a surfactant solution, surfactant molecules migrate from the bulk of the solution to the air-water interface and, consequently, they vary its surface properties. This process is modeled by the partial diffusion equation in one spatial dimension, together with suitable initial and boundary conditions, being the unknowns both the surface and bulk concentrations. Moreover, in order to close the problem, we consider an adsorption model, that is coupled to the system of equations as a boundary condition at the subsurface. There are two families of models for describing the adsorption dynamics: the diffusion-controlled models and the mixed kinetic-diffusion ones. In this work, we focus on a diffusion-controlled model considering the well-known Langmuir isotherm, for which we study the existence and uniqueness of weak solution. The existence is obtained by using the Rothe method, an intermediate problem, a priori error estimates and passing to the limit, and the uniqueness is proved integrating in time the weak equations and using adequate test functions.

Probing Fundamental Bounds in Hydrodynamics Using Variational Optimization Methods

Bartosz Protas
McMaster University, Canada
bprotas@mcmaster.ca
Diego Ayala

In the presentation we will review recent results and discuss some emerging research directions concerning application of the modern methods of PDE-constrained optimization to study a class of fundamental problems in mathematical fluid mechanics. These problems concern the sharpness of certain estimates, such as the bounds on the maximum enstrophy growth in 3D flows governed by the Navier-Stokes system, which are intimately related to the question of spontaneous singularity formation, known as the "blow-up" problem. We demonstrate how new insights concerning such problems can be obtained by formulating them as variational PDE optimization problems which can be solved computationally using discretized gradient flows. Vortex states determined in this way represent the most extreme flow behavior allowed for by the Navier-Stokes dynamics and are therefore natural candidates for blow-up. In offering a systematic approach to finding such flow solutions which saturate known estimates, the proposed paradigm provides a bridge between theory and computation. In the presentation we will show a number of new results concerning extreme vortex states in 2D and 3D Navier-Stokes flows, and will discuss their relation to the available theoretical bounds obtained with methods of mathematical analysis.

Bifurcation on a finite time interval in nonlinear hyperbolic-parabolic parameter dependent control system

Volker Reitmann
St. Petersburg State University, Russia
VReitmann@aol.com
Dina Kalinichenko

The aim of the talk is to discuss doubly nonlinear hyperbolic-parabolic parameter dependent control system. It will be shown that the coupled system of Maxwell's equation and a heat equation on finite time interval in one space dimension involving a phase transition property and Joule's heating effect can be written in such a way. This results into two multivalued nonlinearities in the system. Using frequency domain conditions for the hyperbolic-parabolic control system we derive sufficient conditions for bifurcation on a finite time interval. As an important part of the proof we show the existence of a cocycle of our nonautonomous parameter dependent control system.

Existence for a nonlinear delay reaction-diffusion system subjected to nonlocal initial conditions

Daniela Rosu
"G. Asachi" Technical University, Romania
rosudaniela100@yahoo.com
Monica-Dana Burlica, Mihai Necula, Ioan I. Vrabie

We present an existence result for a class of nonlinear delay reaction-diffusion systems subjected to nonlocal initial conditions having affine growth. We also include some sufficient conditions for the uniqueness and global asymptotic stability of the solution and some specific examples. This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS–UEFISCDI, project number PN-II-ID-PCE-2011-3-0052.

Consistent high-frequency damping for nonsmooth flexible multibody systems

Thorsten Schindler
Technische Universitaet Muenchen, Germany
thorsten.schindler@mytum.de

This talk is devoted to the consistent numerical integration of nonsmooth flexible multibody systems with impacts and friction. We bring together two ideas at the same time. First, we split non-impulsive and impulsive force propagation. In the context of time-discontinous Galerkin methods, we take care of possible impacts and allow velocity jumps at the end of each discretization interval. Second inside discretization intervals, we use sophisticated base integration schemes known from computational mechanics. For non-impulsive periods, the generalized-alpha, ED-alpha (energy-decaying) or Bathe method are motivated by automatically reducing artificial high-frequencies being in the numerical model due to standard space discretization schemes. This technique mixes non-impulsive and impulsive integration strategies, but from the beginning embeds non-impulsive discretizations consistently in a concept which allows velocity jumps and impacts. For the purpose of comparison, the integration schemes are applied to mechanical systems with impacts and Coulomb friction, e.g. imperfect slider-crank type mechanisms. We study convergence, computing time and vibrational behavior by this numerical experiment and discuss the representation of physical oscillations.

Exact controllability of evolution equations by smooth controls and applications

Benzion Shklyar
Holon Institute of Technology, Israel, Israel
shk_b@hit.ac.il

Let $V \subset X \subset V'$ are Hilbert spaces with continuous dense injections (see more details in [3] and references therein). Consider the control evolution equation [1]

$$\dot{x}(t) = Ax(t) + bu(t), x(0) = x^0, 0 \le t0$$

there exists a square integrable control $u(\cdot) \in L_2([0,t_1], \mathbb{R}^r)$ such that a mild solution $x(t, x^0, u(\cdot))$ of equation with initial condition x^0, generated by a control $u(\cdot)$, satisfies the condition $\|x(t, x^0, u(\cdot))\|$.

On noncompact fractional order differential inclusions with generalized boundary condition and impulses in a Banach space

Valentina Taddei
University of Modena and Reggio Emilia, Italy
valentina.taddei@unimore.it
Irene Benedetti, Valeri Obukhovskii

We consider a fractional semilinear evolution inclusion in a reflexive Banach space in the presence of impulse effects associated with a very general multivalued boundary condition including several nonlocal conditions as well as the attainability problem. We assume the regularities on the nonlinear terms by means of the weak topology. Thus no compactness is assumed, neither on the evolution operator generated by the linear part, or on the nonlinear term. This technique allows to consider both sublinear and superlinear growth condition on the nonlinear term. The existence of a solution is investigated by means of a fixed point technique. The talk ends with some applications to hyperbolic integro-differential equations arising from physics and biology.

A viability result for delay evolution equations with implicit nonlocal initial conditions

Ioan Vrabie
Al. I. Cuza University, Iasi, Romania, Romania
ivrabie@uaic.ro
Mihai Necula, Marius Popescu

A necessary and sufficient condition in order that a time-dependent set be viable with respect to a delay evolution equation subjected to an implicit nonlocal initial condition is established. Then, using this result which, as far as we know, is the first one referring to viability for such kind of problems, some concrete applications to nonlinear parabolic equations are derived.

Acknowledgement
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On the abstract evolution equations of hyperbolic type

Kentarou Yoshii
Tokyo University of Science, Japan
yoshii@ma.kagu.tus.ac.jp
Noboru Okazawa

In this talk, we consider existence and uniqueness of classical solution to the abstract Cauchy problem for linear evolution equations of the form

$$B(t)(d/dt)u(t) + A(t)u(t) = f(t); \quad u(0) = u_0,$$

where $\{A(t)\}$ and $\{B(t)\}$ are families of closed linear operators in Hilbert space.

Special Session 51: Variational Analysis and Applications to Equilibrium Problems

Patrizia Daniele, University of Catania, Italy
Sofia Giuffre, Mediterranean University of Reggio Calabria, Italy

The aim of this special session is to join together researchers coming from different parts of the world and to discuss and present results related to the variational inequality problems, equilibrium models, Nash equilibria and Optimization problems.

Evolutionary quasi-variational inequalities and applications to a general Cournot-Nash priciple

Annamaria Barbagallo
University of Naples, Italy
annamaria.barbagallo@unina.it
Paolo Mauro

The aim of the talk is to study the dynamic oligopolistic market equilibrium problem in the realistic case in which we allow the presence of capacity constraints and production excesses and, moreover, we assume that the production function depends not only on the time but also on the equilibrium distribution. As a consequence, we introduce the generalized dynamic Cournot- Nash principle in the elastic case and prove the equivalence between this equilibrium definition and a suitable evolutionary quasi-variational inequality. For completeness we make the analysis of existence, regularity, and sensitivity of the solution. In the end, a numerical example is provided.

References

[1] A. Barbagallo and P. Mauro, A quasi-variational approach for the dynamic oligopolistic market equilibrium problem, Abstr. Appl. Anal. 2013 (2013), art. no. 952915.

[2] A. Barbagallo and P. Mauro, A general quasi-variational formulation for an oligopolistic market equilibrium problem, submitted.

Supply chain relationship dynamics: Efficiency and risk

Jose Cruz
University of Connecticut, USA
jcruz@business.uconn.edu

This paper explores the dynamic evolution of supply chain relationships and their effects on product flows and prices. We investigate the interplay of the heterogeneous decision makers in the supply chain network (manufacturers, retailers, and demand markets) and compute the resultant equilibrium pattern of product outputs, transactions, product prices, and levels of relationship between decision makers. Manufacturers and retailers are multicriteria decision-makers who decide about their production and transaction quantities as well as the level of relationship they want to pursue in order to maximize net return and minimize risk over the multiperiod planning horizon. We focus on the following questions: (1) what are the effects of relationship on supply chain network transaction cost, risk and profitability? (2) What are the impacts of the time of relationship-specific investments on supply chain network? (3) what are the effects of relationships on supply chain network efficiency? The results show that high levels of relationship can lead to lower overall supply chain cost and risk and therefore lower price and higher product transaction.

Equilibrium models for end-of-life products with nonlinear constraints

Patrizia Daniele
University of Catania, Italy
daniele@dmi.unict.it
Fuminori Toyasaki, Tina Wakolbinger

Variational inequality theory facilitates the formulation of equilibrium problems in economic networks. Examples of successful applications include models of supply chains, financial networks, transportation networks, and electricity networks. Previous economic network equilibrium models that were formulated as variational inequalities only included linear constraints; in this case the equivalence between equilibrium problems and variational inequality problems is achieved with a standard procedure because of the linearity of the constraints. However, in reality, often nonlinear constraints can be observed in the context of economic networks. In this talk, we first highlight with an application from the context of reverse logistics why the introduction of nonlinear constraints is beneficial.

Variational Inequality Approach to Stochastic Nash Equilibrium Problems

Baasansuren Jadamba
Rochester Institute of Technology, USA
bxjsma@rit.edu
Akhtar Khan, Fabio Raciti

In this work we consider stochastic Nash equilibrum problems (SNEPs) by using stochastic variational inequalities (SVIs). As an application of the considered SNEP model, we investigate the oligopolistic market equilibria with uncertain data. Our objective is to establish a connection between general SNEPs and SVIs and propose a model of oligopolistic markets where the cost functions are not necessarily quadratic and the demand price is not restricted to be linear.

Variational Inequality Based Numerical Methods for Elliptic Inverse Problems

Akhtar Khan
Rochester Institute of Technology, USA
aaksma@rit.edu

This talk will focus on the variational based numerical methods to solve the inverse problem of parameter identification in elliptic partial differential equations.

Global optimization methods using space-filling curves

Daniela Lera
University of Cagliari, Italy
lera@unica.it
Yaroslav D. Sergeyev

Let us consider the global optimization problem of a function F defined in a hypercube of R^N, that satisfies the Lipschitz condition, with the constant L generally unknown. In this paper we consider an approach that uses numerical approximations of space-filling curves to reduce the original Lipschitz multi-dimensional problem to a univariate one satisfying the Holder condition, and we propose a new geometric method that uses, at each iteration, a number of possible Holder constants from a set of values varying from zero to infinity.

General Financial Equilibrium Problem

Cristina Mirabella
University of Catania, Italy
crimira@hotmail.it
Barbagallo A. Daniele P., Lorino M., Maugeri A.

We consider a general equilibrium model of financial flows and prices. The model is assumed evolving in time. Then equilibrium conditions are considered in dynamic sense. We present the governing variational inequality formulation and we study the dual Lagrange problem. From the Lagrange formulation we derive the Deficit Formula, Balance Law and Liability Formula which enable us to give some suggestion for the achievement of the world financial equilibrium.

References

[1] Barbagallo A. Daniele P., Lorino M., Maugeri A., Mirabella C., Further Results for General Financial Equilibrium Problems via Variational Inequalities, Journal of Mathematical Finance, 2013, 3, 33-52.

[2] Barbagallo A. Daniele P., Lorino M., Maugeri A., Mirabella C., Recent results on a general financial equilibrium problem, AIP Conference Proceedings Volume 1558, 2013, pages 1789-1792.

Variational Analysis for some nonlocal problems

Giovanni Molica Bisci
University of Reggio Calabria, Italy
gmolica@unirc.it

Aim of this talk is to present some recent existence and multiplicity properties for nonlocal elliptic equations. Our approach is based on variational methods. These contributions are obtained in collaboration with V. Radulescu and R. Servadei.

A qualitative property for a class on elliptic problems on the Sierpinski Gasket

Vicentiu Radulescu
University of Craiova, Romania
vicentiu.radulescu@math.cnrs.fr
Giovanni Molica Bisci

In this talk, we will report some recent results contained in the paper [G. Molica Bisci, V. Radulescu, A characterization for elliptic problems on fractal sets, Proc. Amer. Math. Soc., in press]. We are concerned with a characterization theorem on the existence of a non-zero strong solution for elliptic equations on the Sierpinski gasket. More generally, the validity of our result can be checked studying elliptic equations defined on self-similar fractal domains whose spectral dimension is less than 2.

A set-valued analysis approach to conical regularization of abstract optimization problems

Miguel Sama
UNED, Spain
msama@ind.uned.es

In this talk we deal with optimization problems in Banach spaces which represent a standard abstract model for many PDE control problems. By conical regularization we understand those methods which construct a family of approximate problems by replacing the constraint cone by an approximating family of cones. These methods are specially indicated for those problems where KKT conditions are not available or such that the associated multipliers exhibit low regularity. In [A.A. Khan, M. Sama, A new conical regularization for some optimization and optimal control problems: Convergence analysis and finite element discretization, Numer. Funct. Anal. Optim. 34(8), 861-895 (2013)] a set-valued model was proposed in order to prove the convergence in norm of the regularized solutions to the solution of the original problem. The aim of this talk is to apply set-valued analysis techniques to get an estimate of the order of convergence.

On the optimal pollution emission price problem

Laura Scrimali
University of Catania, Italy
scrimali@dmi.unict.it

In this paper we consider the problem of the optimal pollution emission price in a continuous-time context and give a formulation as a bilevel programming problem. In particular, we suppose that control policies are imposed to the end of regulating the pollution emissions at supply markets. Therefore, the policy-maker chooses the optimal price of the pollution emission with consideration to manufacturers' response to the price. On the other hand, the manufacturers choose the optimal quantities of production to maximize their profits, given the price of pollution emission. Using some new recent results on infinite dimensional variational analysis, we are able to reformulate the bilevel programming problem into a one level optimization problem by means of complementarity conditions. Moreover, the existence of solutions is investigated. Finally, a numerical example is provided.

Variational analysis for fractional elliptic equations

Raffaella Servadei
Università della Calabria, Italy
servadei@mat.unical.it

Recently in the literature a great attention has been devoted to the study of nonlocal problems driven by fractional Laplace type operators, not only for a pure academic interest, but also for the various applications in different fields. Aim of this talk is to present some recent existence and regularity results for nonlocal elliptic equations driven by the fractional Laplacian and by more general integro-differential operators.

Special Session 52: Nonlinear Evolution Equations

Petronela Radu, University of Nebraska-Lincoln, USA
John Stalker, Trinity College Dublin, Ireland

Proof of the cosmic no hair conjecture in the T^3-Gowdy symmetric Einstein-Vlasov setting

Haakan Andreasson
University of Gothenburg, Sweden
hand@chalmers.se
Hans Ringström

The currently preferred models of the universe undergo accelerated expansion induced by dark energy. One model for dark energy is a positive cosmological constant. It is consequently of interest to study Einstein's equations with a positive cosmological constant coupled to matter satisfying the ordinary energy conditions; the dominant energy condition etc. In this talk, the future asymptotic behaviour of T^3-Gowdy symmetric solutions to the Einstein-Vlasov equations with a positive cosmological constant will be discussed. In particular, the cosmic no-hair conjecture will be shown in this setting. This is a joint work with Hans Ringström.

A Frequency Domain Approach for the Rational Decay of Coupled PDE Dynamics of Hyperbolic-Parabolic Type

George Avalos
University of Nebraska-Lincoln, USA
gavalos@math.unl.edu

In this talk, we shall demonstrate how delicate frequency domain relations and estimates, associated with coupled systems of partial differential equation models (PDE's), may be exploited so as to establish results of uniform and rational decay. In particular, our focus will be upon decay properties of coupled PDE systems of different characteristics; e.g., hyperbolic versus parabolic characteristics. For such PDE systems of contrasting dynamics, the attainment of explicit decay rates is known to be a difficult problem, inasmuch as there has not been an established methodology to handle hyperbolic-parabolic systems. For uncoupled wave equations or uncoupled heat equations, there are specific Carleman's multiplier methods in the time domain, wherein the exponential weights in each Carleman's multiplier carefully take into account the particular dynamics involved, be it hyperbolic or parabolic. But for coupled PDE systems which involve hyperbolic dynamics interacting with parabolic dynamics, typically across some boundary interface, Carleman's multipliers are readily applicable. Given that such coupled PDE systems occur frequently in nature and in engineering applications; e.g., fluid-structure and structural acoustic interactions, there is a patent need to devise broadly implementable techniques by which one can infer uniform decay for a given PDE system. As one particular example, we shall work to conclude uniform decays for structural acoustic dynamics. In these PDE models, the structural component is subjected to a structural damping ranging from viscous (weak) to strong (Kelvin-Voight). The rational decay rates we derive for this problem explicitly reflect the extent of the damping which is in play. Since the damped elastic component of the coupled dynamics is present on only a portion of the boundary, there will necessarily be assumptions imposed upon the geometry.

Torsional instability in a fish-bone model for suspension bridges

Elvise Berchio
Politecnico di Torino, Italy
elvise.berchio@polito.it
Filippo Gazzola

Why do torsional oscillations appear suddenly in a suspension bridge? We give an answer to this long-standing question by analyzing a suitable mathematical model for suspension bridges named fish-bone model. More precisely, we obtain both theoretical and numerical estimates of the thresholds of the torsional instability. Our method is based on a finite dimensional projection of the phase space which reduces the stability analysis of the model to the stability of suitable Hill equations.

Shape Differentiability for the Wave Equation with Neumann Boundary Conditions

Lorena Bociu
NC State University, USA
lvbociu@ncsu.edu

This talk will provide a full analysis of shape differentiability for the solution to the second order hyperbolic equation with Neumann boundary conditions. This answers a fundamental question in shape optimization and control problems for the linear wave equations and coupled systems where the hyperbolic equation is coupled with other dynamics, and the matching conditions at the boundary are of Neumann type. While the shape derivative analysis has been solved for many classical linear problems, the hyperbolic situation is more delicate, due to the lack of good boundary regularity for the wave solution, which is a key ingredient in the differentiability analysis. Therefore the talk will also include a new hidden regularity result, obtained through a new pseudo-extractor technique.

A nonlinear fourth order evolution equation modeling a suspension bridge

Alberto Ferrero
Università degli Studi del Piemonte Orientale, Italy
alberto.ferrero@mfn.unipmn.it
Filippo Gazzola

A plate model describing the statics and dynamics of a suspension bridge is suggested. A partially hinged plate subject to nonlinear restoring hangers is considered. The whole theory from linear problems, through nonlinear stationary equations, ending with the full hyperbolic evolution equation is studied. We model the bridge by considering it as a rectangular plate subject both to live and dead loads. In order to recover the elastic energy of a deflected plate we exploit the Kirchhoff-Love theory. The action of the hangers on the bridge is modeled by a suitable nonlinear forcing term. We are then lead to consider a nonlinear evolution equation of the type

$$u_{tt} + \Delta^2 u + h(x, y, u) = f(x, y, t) \quad \text{in } \Omega \times (0, T)$$

where $\Omega \subset \mathbb{R}^2$ is the open rectangle $(0, L) \times (-\ell, \ell)$ with L and 2ℓ representing respectively length and width of the bridge.

A new mathematical explanation of the Tacoma Narrows Bridge collapse

Filippo Gazzola
Politecnico of Milano, Italy
filippo.gazzola@polimi.it
Gianni Arioli

The spectacular collapse of the Tacoma Narrows Bridge has attracted the attention of engineers, physicists, and mathematicians in the last 74 years. There have been many attempts to explain this amazing event, but none is universally accepted. It is however well established that the main culprit was the unexpected appearance of torsional oscillations. We suggest a mathematical model for the study of the dynamical behavior of suspension bridges which provides a new explanation of the Tacoma collapse. We show that internal resonances, which depend on the bridge structure only, are the source of torsional oscillations.

Long-time behaviour for nonlocal problems

Ignat Ioan liviu
Institute of Mathematics Simion Stoilow, Romania
liviu.ignat@gmail.com
Tatiana Ignat, Denisa Stancu-Dumitru

In this talk we will present some nonlocal evolution problems that involve operators of the type:

$$\mathcal{L}u(x) = \int_{\mathbf{R}^d} J(x-y)(u(y)-u(x))\,dy$$

We analyze the asymptotic behaviour of the solutions of the following nonlocal convection-diffusion equation

$$u_t = J * u - u + G * u^2 - u^2.$$

The results are mainly obtained by scaling arguments and a new compactness argument that is adapted to nonlocal evolution problems. The compactness tool is the following one: Let $\Omega \subset \mathbf{R}^d$ be an open set. Let $\rho : \mathbf{R}^d \to \mathbf{R}$ be a nonnegative smooth radial function with compact support, non identically zero, and $\rho_n(x) = n^d \rho(nx)$. Let $\{f_n\}_{n\geq 1}$ be a sequence of functions in $L^p((0,T)\times\Omega)$ such that

$$\int_0^T \int_\Omega |f_n|^p \leq \ M$$

and

$$n^p \int_0^T \int_\Omega \int_\Omega \rho_n(x-y)|f_n(t,x) - f_n(t,y)|^p dxdydt \leq M.$$

If $\{f_n\}_{n\geq 1}$ is weakly convergent in $L^p((0,T)\times\Omega)$ to f then $f \in L^p((0,T), W^{1,p}(\Omega))$ for $p > 1$ and $f \in L^1((0,T), BV(\Omega))$ for $p = 1$. Let $p > 1$. Assuming that Ω is a smooth bounded domain in $\mathbf{R}^d$, $\rho(x) \geq \rho(y)$ if $|x| \leq |y|$ and that

$$\|\partial_t f_n\|_{L^p((0,T),W^{-1,p}(\Omega))} \leq M$$

then $\{f_n\}_{n\geq 1}$ is relatively compact in $L^p((0,T)\times\Omega)$.

Asymptotic behaviour of cosmological solutions of the Einstein-Boltzmann system

Ho Lee
Kyung Hee University, Korea
holee@khu.ac.kr

In this talk we study the Einstein-Boltzmann system in the Robertson-Walker or the Bianchi type I spacetimes. We first assume that a spatially flat Robertson-Walker spacetime is given, and then show that classical solutions of the Boltzmann equation exist globally in time and have a certain asymptotic behaviour. To obtain differentiability of solutions of the Boltzmann equation we apply the splitting argument of Guo and Strain. This result is extended to the Einstein-Boltzmann case. The Robertson-Walker models are generalized to the Bianchi models of type I, and the results are extended to the case of Bianchi type I.

Future stability of homogeneous cosmological models

Ernesto Nungesser
Trinity College Dublin, Ireland
ernesto@maths.tcd.ie

There are several recent deep results concerning future stability of solutions to the Einstein-Vlasov and Einstein-Euler-system with a cosmological constant. In this talk we will present some results concerning the case of a vanishing cos- mological constant and assuming that the spacetime is homogeneous. We will motivate the interest of kinetic models in cosmology with an outlook of how they can be generalized in both directions, past and future.

Global Classical Solutions of the Relativistic Vlasov-Maxwell-Fokker-Planck System

Stephen Pankavich
Colorado School of Mines, USA
pankavic@mines.edu

The Vlasov-Maxwell system is a fundamental kinetic model of plasma dynamics. When one considers relativistic velocities and includes effects due to collisions with a fixed background of particles, the result is the relativistic Vlasov-Maxwell-Fokker-Planck system. The first Lorentz-invariant model of this type was recently derived by Calogero and Alcantara in 2010. Here, we shall discuss the first well-posedness results for global-in-time classical solutions of this system posed in a lower-dimensional setting. Our methods utilize a gain in regularity stemming from the diffusive term to arrive at smooth solutions stemming from initial data which fail to be even weakly differentiable.

A blow up result for fourth order differential equations

Petronela Radu
University of Nebraska-Lincoln, USA
pradu@math.unl.edu
Daniel Toundykov and Jeremy Trageser

The study of fourth order differential equations has recently intensified in the context of studying the behavior of traveling waves for nonlinear suspension bridges. I will present a blow-up result for the equation

$$u^{(4)} + ku'' + f(u) = 0$$

where f is super linear with $f(u)u > 0$ and when $k > 0$. Previous work by Gazzola and his collaborators solved the case $k \leq 0$. The case $k > 0$ is physically significant as it corresponds to $k = c^2$ with c being the speed of propagation of the traveling wave.

Blow-Up of Positive Solutions to Wave Equations in High Space Dimensions

Mohammad Rammaha
University of Nebraska-Lincoln, USA
mrammaha1@math.unl.edu
Hiroyuki Takamura, Hiroshi Uesaka, Kyouhei Wakasa

This talk is concerned with the Cauchy problem for the semilinear wave equation:

$$u_{tt} - \Delta u = F(u) \quad \text{in} \quad \mathbb{R}^n \times [0, \infty),$$

where the space dimension $n \geq 2$, $F(u) = |u|^p$ or $F(u) = |u|^{p-1}u$, with $p > 1$. Here, the Cauchy data are non-zero and non-compactly supported. Our results on the blow-up of positive radial solutions (not necessarily radial in low dimensions $n = 2, 3$) generalize and extend the results of Takamura-1995, and Takamura, Uesaka and Wakasa-2011. The main technical difficulty in this work lies in obtaining appropriate lower bounds for the free solution when both initial position and initial velocity are non-identically zero, especially in even space dimensions.

Decay rates for solutions of conservation laws with diffusion-type terms of regularity-gain and regularity-loss

Belkacem Said Houari
Alhosn University, United Arab Emirates
saidhouarib@yahoo.fr

We consider the Cauchy problem of a nonlocal scalar conservation law with a diffusion type source term. Using the energy method together with Fourier analysis, we show a faster decay rate of the solution. Our studies includes, (as particular cases) the convection diffusion equation, the hyperbolic elliptic system and many other examples. We show that by taking initial data in some weighted spaces, we where able to improve the decay rates of the solution. This method can be also extended to many other systems.

Estimates for nonlocal analogues of the wave equation

John Stalker
Trinity College Dublin, Ireland
stalker@maths.tcd.ie

I will discuss some integrodifferential equations of which the simplest is

$$u_{tt} + \mu u = J \star u$$

where $J \in L^1(\mathbf{R}^{1+n})$ is nonnegative and $\mu = \int J$. This equation is very far from satisfying Huygens' principle. In fact, the support of any nonzero solution is all of $\mathbf{R}^{1+n}$. It can, nonetheless, be thought of as an analogue of the wave equation and satisfies some of the same estimates. It even exhibits a sort of "almost finite speed of propagation."

The Dirac point electron in zero-gravity Kerr–Newman spacetime

A. Shadi Tahvildar-Zadeh
Rutgers University, USA
shadi@math.rutgers.edu
Michael Kiessling

Dirac's wave equation for a point electron in the electromagnetic Kerr–Newman spacetime is studied in the zero-gravity limit; here, "zero-gravity" means $G \to 0$, where G is Newton's constant of universal gravitation. The zero-G limit eliminates the troublesome Cauchy horizon of the Kerr–Newman spacetime and also its physically problematic acausal region of closed timelike loops. While the gravitational features of the Kerr–Newman manifold vanish as well when $G \to 0$, this limit retains the nontrivial topology associated with its ring singularity, and all its electromagnetic features. We first show that the formal Dirac Hamiltonian on a static spacelike slice of the maximal analytically extended zero-G Kerr–Newman spacetime is essentially self-adjoint, and that the spectrum of its self-adjoint extension is symmetric about zero. It is next shown that the Dirac operator on the zero-G Kerr–Newman spacetime has a continuous spectrum with both positive and negative energies, separated by a gap about zero that contains a pure point spectrum. The pure point spectrum is associated with time-periodic normalizable solutions, representing bound states of Dirac's point electron in the electromagnetic field of the ring singularity of the zero-G Kerr–Newman spacetime.

Local Hadamard well - posedness and blow -up for reaction - diffusion equations with non-linear dynamical boundary conditions

Enzo Vitillaro
Universita degli Studi di Perugia, Italy
enzo.vitillaro@unipg.it
Alessio Fiscella

The talk will deals with local well–posedness, global existence and blow–up results for reaction–diffusion equations coupled with nonlinear dynamical boundary conditions. The typical problem studied is

$$u_t - \Delta u = |u|^{p-2}\, u \qquad \text{in} \quad (0,\infty)\times\Omega,$$

$$u = 0 \qquad \text{on} \quad [0,\infty)\times\Gamma_0,$$

$$\frac{\partial u}{\partial \nu} = -\,|u_t|^{m-2}\, u_t \qquad \text{on} \quad [0,\infty)\times\Gamma_1,$$

$$u\,(0,x) = u_0\,(x) \qquad \text{in} \quad \Omega.$$

where Ω is a bounded open regular domain of $I\!R^n$ $(n \geq 1)$, $\partial\Omega = \Gamma_0 \cup \Gamma_1$, $2 \leq p \leq 1 + 2^*/2$, $m > 1$ and $u_0 \in H^1(\Omega)$, $u_{0|\Gamma_0} = 0$.After showing local well–posedness in the Hadamard sense we give global existence and blow–up results when Γ_0 has positivesurface measure. Moreover we discuss the generalization of the above mentioned results to more general problems where the terms $|u|^{p-2}u$ and $|u_t|^{m-2}u_t$ are respectively replaced by $f\,(x,u)$ and $Q(t,x,u_t)$ under suitable assumptions on them.

On the Long Time Behavior of Solutions to the Thermoelastic Plate Equations

Borislav Yordanov
Institute of Mathematics and Informatics, Bulgaria
byordanov@math.bas.bg

We study the long time behavior of solutions to the linear thermoelastic plate in abstract setting:

$$\begin{cases} u''(t) + A^2u(t) - A\theta(t) &= 0, \qquad t \in (0,\infty), \\ \theta'(t) - A\theta(t) + Au'(t) &= 0, \qquad t \in (0,\infty), \end{cases}$$

with the initial condition

$$u(0) = u_0, \quad u'(0) = v_0, \quad \theta(0) = \theta_0.$$

Here $A : \mathcal{D}(A) \subset H \to H$ is a non-negative self-adjoint operator in $(H, \|\cdot\|)$ with a dense domain $\mathcal{D}(A)$. The total energy of $(u(t), \theta(t))$ is given by

$$E(t) = \|u'(t)\|^2 + \|Au(t)\|^2 + \|\theta(t)\|^2, \quad t \in (0,\infty).$$

We show that the behavior of $E(t)$ is determined by the abstract diffusion semigroup e^{-tA} as $t \to \infty$. This is yet another example of the diffusion phenomenon in dissipative wave equations.

Special Session 53: Infinite Dimensional Stochastic Systems and Applications

Wilfried Grecksch, Martin-Luther-University Halle-Wittenberg, Germany

This special session should give a general overview as well about new tendencies in the field of infinite dimensional stochastic systems as applications to sciences, economic sciences and problems of optimal control. The invited speakers will discuss especially issues related to stochastic partial differential equations, stochastic differential equations with memory, stochastic integral equations, stochastic integrodifferential equations and stochastic difference equations in infinite dimensional spaces.

Dynamic Programming Equation for Portfolio Optimization under Partial Information with Expert Opinions

Gabih Abdelali
Marrakech University, Morocco
a.gabih@uca.ma
Rüdiger Frey and Ralf Wunderlich

This paper investigates optimal portfolio strategies in a market where the drift is driven by an unobserved Markov chain. Information on the state of this chain is obtained from stock prices and expert opinions in the form of signals at random discrete time points. As in Frey et al. (2012), Int. J. Theor. Appl. Finance, 15, No. 1, we use stochastic filtering to transform the original problem into an optimization problem under full information where the state variable is the filter for the Markov chain. The dynamic programming equation for this problem is studied with viscosity-solution techniques and with regularization arguments.

Continuous Data Assimilation with Stochastically Noisy Data

Hakima Bessaih
University of Wyoming, USA
bessaih@uwyo.edu
Eric Olson and Edriss S. Titi

We analyze the performance of a data-assimilation method based on a linear feedback control when used with observational data that contains measurement errors. Our model problem consists of dynamics governed by the two-dimension incompressible Navier–Stokes equations, observational measurements given by finite volume elements or nodal points of the velocity field and measurement errors which are represented by stochastic noise. Under these assumptions, the data-assimilation algorithm consists of a system of stochastically forced Navier–Stokes equations. The main result of this paper gives conditions on the observation density which guarantee that the expected value of the approximating solution will converge to the actual solution to within a factor related to the variance of the noise in the measurements.

Amplitude equations for stochastic Swift Hohenberg equation

Luigi Amedeo Bianchi
Universität Augsburg, Germany
luigi.bianchi@math.uni-augsburg.de
Dirk Blömker

In this talk we will consider the stochastic Swift Hohenberg equation

$$\frac{\partial u}{\partial t} = -(1+\partial_x^2)^2 u + \epsilon^2 \nu u - u^3 + \epsilon^{3/2}\xi(t,x)$$

in an unbounded domain, where the noise ξ is a space-time white noise. Close to the change of stability at $\nu = 0$ we can describe the solution u of the equation as a modulated wave

$$u(t,x) = \epsilon\mathcal{A}(\epsilon^2 t, \epsilon x)e^{ix} + \text{c.c.}$$

We derive this so called amplitude equation, which is a Ginzburg Landau equation. As a first case we limit ourselves to the linear problem.

Center Manifolds for infinite dimensional random dynamical systems

Xiaopeng Chen
Peking University, Peoples Rep of China
chenxiao002214336@outlook.com
Anthony J. Roberts, Jinqiao Duan

Stochastic center manifolds theory are crucial in modelling the dynamical behavior of complex systems under stochastic influences. The multiplicative ergodic theorem is an important concept for random dynamical systems. A multiplicative ergodic theorem on Hilbert space is proved to be satisfied to the exponential trichotomy condition. Then the existence of stochastic center manifolds for infinite dimensional random dynamical systems is shown under the assumption of exponential trichotomy. The theory provides a support for the discretisations of nonlinear stochastic partial differential equations with space-time white noise.

Random attractors for retarded SPDEs with time smooth diffusion coefficients

Maria Garrido-Atienza
University of Seville, Spain
mgarrido@us.es
Hakima Bessaih, Bjoern Schmalfuss

The aim of this talk is to study the long-time dynamics of mild solutions to retarded stochastic evolution systems driven by a Hilbert-valued Brownian motion. We do not assume that the noise is given in additive form or that it is a non-trivial multiplicative noise, but the price we have to pay for is to assume that the diffusion coefficient is smooth in some sense. In that way, and by using an integration by parts, the stochastic integral can be expressed in terms of non-stochastic integrals and the noisy path. As we will show, this latter term causes that in a first moment only a local mild solution can be obtained. Nevertheless, by using appropriate stopping times, we will be able to derive the existence and uniqueness of a global mild solution. Furthermore, we shall show that the global mild solution generates a random dynamical system that, under an appropriate smallness condition for the time lag, have associated a random attractor.

On a class of Sobolev spaces and applications to PDE and SPDE

Michael Hinz
Bielefeld University, Germany
mhinz@math.uni-bielefeld.de

In this talk we discuss a class of Sobolev spaces related to bilinear forms (e.g. Dirichlet forms) that generalize the classical spaces introduced by Mazja. They allow the immediate extension of numerous results formerly known for PDE and SPDE on Euclidean spaces or domains to abstract or highly singular spaces. Some examples in terms of SPDE will be provided. Parts of these results are based on joint work with Michael Roeckner and Alexander Teplyaev.

A mild Ito formula for SPDEs

Arnulf Jentzen
ETH Zurich, Switzerland
arnulf.jentzen@sam.math.ethz.ch
Giuseppe Da Prato and Michael Roeckner

This talk introduces a certain class of stochastic processes, which we suggest to call mild Itô processes, and a new - somehow mild - Itô type formula for such processes. Examples of mild Itô processes are mild solutions of SPDEs and their numerical approximations. This talk is based on a joint work with Giuseppe Da Prato and Michael Röckner.

A Schrödinger Problem with Kerr-Nonlinearity and Multiplicative Noise

Diana Keller
Martin Luther University Halle-Wittenberg, Germany
diana.keller@mathematik.uni-halle.de

A nonlinear Schrödinger problem perturbed by multiplicative Gaussian noise will be investigated over a finite time horizon and a bounded one-dimensional domain. The appearing nonlinearity is the cubic Kerr-nonlinearity $f(z) = |z|^2 z$ for $z \in \mathbb{C}$, which has many applications in mathematical physics. Being interested in the existence and uniqueness of a variational solution, a further process will be introduced which allows to transfer the stochastic Schrödinger problem into a pathwise one. Exploiting the absence of noise and using Galerkin approximations and compact embedding results, one considers first a priori estimates, existence and uniqueness of a variational solution of the pathwise Schrödinger problem. Thereafter, it is possible to extend these results to the variational solution of the nonlinear Schrödinger problem with multiplicative noise.

Homogenization of the evolution Stokes equation in a perforated domain with a stochastic Fourier boundary condition

Razvan Maris
King Abdullah University of Science and Technology, Saudi Arabia
florinmaris@yahoo.com
Hakima Bessaih, Yalchin Efendiev

We consider the evolution Stokes equation in a perforated domain subject to the Fourier boundary condition on the boundaries of the holes is considered. We assume that the dynamics are driven by a stochastic perturbation on the interior of the domain and another stochastic perturbation on the boundaries of the holes. The macroscopic (homogenized) equation is derived as a stochastic partial differential equation, defined in the whole non-perforated domain, and the initial stochastic perturbation on the boundary becomes part of the homogenized equation as an additional stochastic force.

Contributions to random operator equations

Antje Mugler
BTU Cottbus, Germany
mugler@tu-cottbus.de

In recent years the topic of uncertainty quantification has become of great interest. Complex systems in physical, biological, social, engineering, and technology sciences can be formulated as random equations of various classes. For example the subsurface flow with uncertain permeability can be modelled as stationary diffusion problem with random coefficient. In this talk we consider random operator equations with which we are able to capture a broader class of such random equations than only random elliptic partial differential equations. We investigate in this context the existence of a unique weak solution and, furthermore, we study the stochastic Galerkin method using generalized polynomial chaos in order to obtain an approximate solution.

Random dynamical systems in Banach Spaces

Alexandra Neamtu
Friedrich-Schiller-University Jena, Germany
alexandra.neamtu

We consider linear and nonlinear nonautonomous random evolution equations given by

$$u'(t) = A(\theta_t)u + F(\theta_t, u), \quad u(0) := x \in X,$$

where X is a separable Banach space and $(\theta_t)_{t\in\mathbb{R}}$ is a metric dynamical system. We give conditions under which such equations generate random dynamical systems and present important applications. Finally, the long-time behaviour of the corresponding random dynamical systems is studied by means of a multiplicative ergodic theorem proved in [**?**]. Stable and unstable manifolds are also discussed.

Model Order Reduction for Infinite Dimensional Stochastic Systems

Martin Redmann
Max Planck Institute Magdeburg, Germany
redmann@mpi-magdeburg.mpg.de

In this talk, we consider controlled linear stochastic evolution equations with Levy noise. To solve such systems numerically, finite dimensional approximations are needed. So, we apply a Galerkin scheme leading to a sequence of ordinary linear stochastic differential equations. In order to obtain a good approximation the Galerkin solution can be of high dimension. To reduce the high dimension for practical computations we consider model order reduction techniques. In this talk, we describe a particular model order reduction technique, provide an error bound for the estimation, and show some numerical results for a particular example.

Stochastic integration with respect to cylindrical Lévy processes in Hilbert spaces

Markus Riedle
King's College London, England
markus.riedle@kcl.ac.uk
A. Jakubowski

The objective of this talk is the introduction of cylindrical Lévy processes and their stochastic integrals in Hilbert spaces. The degree of freedom of models in infinite dimensions is often reflected by the request that each mode along a dimension is independently perturbed by the noise. In the Gaussian setting, this leads to the *cylindrical Wiener process* including from a model point of view the very important possibility to model a Gaussian noise in both time and space in a great flexibility (space-time white noise). Up to very recently, there has been no analogue for Lévy processes. Based on the classical theory of cylindrical processes and cylindrical measures we introduce *cylindrical Lévy processes* as a natural generalisation of cylindrical Wiener processes. In Hilbert spaces we introduce a stochastic integral for operator-valued stochastic processes with respect to cylindrical Lévy processes.

An operatorial approach to stochastic partial differential equations driven by linear multiplicative noise

Michael Roeckner
University of Bielefeld, Germany
roeckner@math.uni-bielefeld.de
Viorel Barbu

In this talk, we develop a new general approach to the existence and uniqueness theory of infinite dimensional stochastic equations of the form $dX(t) + A(t, X(t))dt = X(t)dW(t) in (0, T) \times H$, where A is a time-dependent nonlinear monotone and demicontinuous operator from V to V', coercive and with polynomial growth. Here, V is a reflexive Banach space continuously and densely embedded in a Hilbert space H of (generalized) functions on a domain $\mathcal{O} \subset \mathbb{R}^d$ and V' is the dual of V in the duality induced by H as pivot space. Furthermore, W is a Wiener process in H. The new approach is based on an operatorial reformulation of the stochastic equation which is quite robust under perturbation of A. This leads to new existence and uniqueness results for a larger class of equations with linear multiplicative noise than the one treatable by the known approaches. In addition, we obtain regularity results for the solutions with respect to both the time and spatial variable which are sharper than the classical ones. New applications include stochastic partial differential equations, as e.g. stochastic transport equations.

Degenerate semigroups and stochastic flows of mappings in foliated manifolds

Paulo Ruffino
University of Campinas, Brazil
ruffino@ime.unicamp.br
Paulo Henrique P. da Costa

Let $(M, \mathcal{F})$ be a compact Riemannian foliated manifold. We consider a family of compatible Feller semigroups in $C(M^n)$ associated to laws of the n-point motion. Under some assumptions (Le Jan and Raimond, Annals of Prob. 2004) there exists a stochastic flow of measurable mappings in M. We study the degeneracy of these semigroups such that the flow of mappings is foliated, i.e. each trajectory lays in a single leaf of the foliation a.s, hence creating a geometrical obstruction for coalescence of trajectories in different leaves. As an application, an averaging principle is proved for a first order perturbation transversal to the leaves. Estimates for the rate of convergence are calculated.

Fractional Riesz-Bessel motion on the ball

M.D. Ruiz-Medina
University of Granada, Spain
mruiz@ugr.es
V.V. Anh, N.N. Leonenko

The fractional Riesz-Bessel equation with random innovations was introduced in Angulo, Ruiz-Medina, Anh and Grechsch (2000), and Angulo, Anh, McVinish and Ruiz-Medina (2005), for the bounded (rectangle) and unbounded domain cases. In the case of random initial conditions, a fractional version in time and in space was derived in Anh and Leonenko (2001, 2002) in the case of unbounded domain (see also the references therein). In this paper we consider fractional Riesz Bessel equation on the ball. Indeed, we assume that the time evolution of the propagator in an open bounded connected domain is governed by a fractional subdiffusion equation, whose fractional-orders of differentiation in time and space lie in suitable intervals. Specifically, Caputo-Djrbashian fractional derivative is used in time, and the inverses of Riesz and Bessel potentials are considered in space. The symmetry of the domain is exploited to deriving the explicit solution to the associated eigenvalue problem, and hence, the corresponding random solution.

EnKF and PCE-KF and uncertainty quantification for pdes with random parameters

Hans-Jörg Starkloff
University of Applied Sciences Zwickau, Germany
hans.joerg.starkloff@fh-zwickau.de

Partial differential equations with random parameters play a great role in uncertainty quantification. As the random parameters are often not known exactly and not directly observable, one has to solve a stochastic inverse problem in an infinite dimensional space. One recent approach to tackle this problem uses Bayesian techniques, which result in complicated and expensive calculations. Therefore in the literature some approximate methods for the Bayesian inversion problem, like Ensemble Kalman Filter (EnKF) or Polynomial Chaos Expansion Kalman Filter (PCE-KF) were proposed, but without sound mathematical foundation. In the talk the stochastic model underlying these methods is given and it is shown, that they usually cannot solve the problem of full Bayesian inversion.

Dudley's representation theorem in infinite dimensions and weak characterizations of stochastistic integrability

Mark Veraar
TU Delft, Netherlands
m.c.veraar@tudelft.nl
Martin Ondrejat

In this talk we present an infinite dimensional version of Dudley's representation theorem for random variables and an extension of Doob's representation for martingales. We will present certain related weak characterizations of stochastic integrability. The limitations of weak characterizations will be demonstrated with a nontrivial counterexample.

Stochastic Delay Equation with Lévy Noise

Frank Wusterhausen
Martin Luther University Halle-Wittenberg, Germany
frank.wusterhausen@mathematik.uni-halle.de

We consider an abstract time-delayed stochastic evolution equation in an infinite dimensional separable Hilbert space driven by additive or multiplicative Lévy noise. The setting can handle both finite and infinite delay. Furthermore we include Lipschitz nonlinearities. We present a transformation approach, which transforms the original problem into a classic stochastic abstract Cauchy problem. We show the equivalence of mild solutions for the two problems. Since the solution neither has stochastic differentials nor need to be càdlàg we present approximations, where each member of the approximating sequence has those properties. Finally we give an outlook how these approximations can be used to handle problems involving stochastic delay equations.

Special Session 54: Nonlocal Fractional Problems and Related Topics

Raffaella Servadei, Università della Calabria, Italy

Fractional and nonlocal operators appear in concrete applications in many fields such as, among the others, optimization, finance, phase transitions, stratified materials, anomalous diffusion, crystal dislocation, soft thin films, semipermeable membranes, flame propagation, conservation laws, ultra-relativistic limits of quantum mechanics, quasi-geostrophic flows, multiple scattering, minimal surfaces, materials science and water waves. This is one of the reason why, recently, nonlocal fractional problems are widely studied in the literature. The aim of this session is to get together experts in the field of nonlocal fractional equations in order to discuss recent progress, their related issues and applications and future trends.

A Widder's type theorem for the heat equation with nonlocal diffusion.

Begoña Barrios
Universidad Autonoma de Madrid, Spain
bego.barrios@uam.es
F. Soria, I. Peral, E. Valdinoci

The main goal of this talk is to prove that every non-negative strong solution $u(x,t)$ to the nonlocal heat equation, can be written as

$$u(x,t) = \int_{\mathbb{R}^n} p(x-y,t)u(y,0)\,dy,$$

where $p(x,t)$ is the fundamental solution. This result shows uniqueness in the setting of non-negative solutions and extends some classical results for the heat equation by D. V. Widder to the nonlocal diffusion framework. Work done in collaboration with I. Peral , F. Soria and E. Valdinoci.

A Liouville-type result for a nonlinear and nonlocal equation in the Heisenberg group

Eleonora Cinti
Universitá di Bologna, Italy
eleonora.cinti5@unibo.it
J. Tan

In this talk I will present a Liouville-type Theorem for a nonlinear problem involving a fractional operator in the Heisenberg group. Very recently Frank, Gonzalez, Monticelli and Tan have established an extension result, analogue to the one of Caffarelli-Silvestre, in the context of CR manifold. Thanks to this result, we can study the nonlocal problem in $\mathbb{H}^n$, by studying a local Neumann problem in $\mathbb{H}^n \times \mathbb{R}^+$. The main tool of the proof of our Liouville-type result relies on the CR-inversion and the moving plane method, adapted to a subriemannian context. This is a joint work with J. Tan.

Sharp Essentially Self-adjointness of Relativistic Schrödinger Operators

Mouhamed Moustapha Fall
African Institute for Mathematical Scineces in Senegal, Senegal
mouhamed.m.fall@gmail.com
Veronica Felli

Essential self-adjointness of an operators H has immediate applications in probability and physics. Indeed, in general an operator H has several self-adjoint extensions H'. This yields Markov processes with transition semigroups $p_t = e^{-tH'}$. The essential self-adjointness of H implies that there is only one extension H_F: the Frederich extension. We therefore have a unique such semigroup and thus a unique Markov process with generator H_F. On the other hand, the essential self-adjointness of H, has the analytic consequence of the uniqueness of the quantum dynamics defined by H. We will talk about necessary and sufficient conditions for Essentially Self-adjointness of a class of Relativistic Schrödinger Operators with, as a particular case, the Coulomb potential.

Unique continuation properties and local asymptotics of solutions to fractional elliptic equations

Veronica Felli
Milano-Bicocca University, Italy
veronica.felli@unimib.it
M. M. Fall

The asymptotic behavior of solutions to fractional elliptic equations with Hardy type potentials is discussed. By an Almgren type monotonicity formula, separation of variables, and blow-up arguments, we describe the exact behavior near the singularity of solutions to linear and semilinear fractional elliptic equations with a homogeneous singular potential related to the fractional Hardy inequality. As a consequence, we obtain some unique continuation properties for fractional elliptic equations.

The CR fractional Laplacian

Maria del Mar Gonzalez
Universitat Politecnica de Catalunya, Spain
mar.gonzalez@upc.edu

We consider the (CR) conformally covariant fractional powers of the sub-Laplacian on the Heisenberg group. As in the real case, they are given in terms of the Dirichlet-to-Neumann problem for an extension problem to the complex hyperbolic space in relation to the scattering operator. We look at some energy inequalities and applications to the CR fractional Yamabe problem.

Lower semicontinuity of functionals via the (fractional) concentration-compactness principle

Giovanni Molica Bisci
University of Reggio Calabria, Italy
gmolica@unirc.it

In this talk we discuss the weak lower semicontinuity of some classes of functionals associated to fractional elliptic problems, using a suitable concentration-compactness principle. The semicontinuity result allows us to prove existence results for quasilinear elliptic equations. These contributions are obtained in collaboration with R. Servadei.

Boundary regularity for integro-differential equations

Xavier Ros-Oton
Universitat Politecnica de Catalunya, Spain
xavier.ros.oton@upc.edu
Joaquim Serra

We study the boundary regularity of solutions to elliptic integro-differential equations. First we show that, for the fractional Laplacian $(-\Delta)^s$, solutions u are comparable to d^s, where $d(x) = dist(x, \partial\Omega)$, in the sense that u/d^s is Hölder continuous up to the boundary. Moreover, we show that $u/d^s|_{\partial\Omega}$ plays the role that the normal derivative plays in second order elliptic equations. We also present new boundary regularity results for fully nonlinear integro-differential equations. Namely, we show that the class of kernels of Caffarelli-Silvestre is “too big” to obtain this type of results, and we obtain the regularity up to the boundary of u/d^s for a class of anisotropic kernels that correspond to infinitesimal generators of α-stable Lévy processes.

A nonlocal two phase Stefan problem

Silvia Sastre-Gomez
Universidad Complutense de Madrid, Spain
silviasastre@mat.ucm.es
Emmanuel Chasseigne

We study a nonlocal version of the two-phase Stefan problem, which models a phase transition problem between two distinct phases evolving to distinct heat equations. Mathematically speaking, this consists in deriving a theory for sign-changing solutions of the equation, $u_t = J * v - v$, $v = \Gamma(u)$, where the monotone graph is given by $\Gamma(s) = \text{sign}(s)(|s| - 1)_+$. We give general results of existence, uniqueness and comparison, in the spirit of [2]. Then we focus on the study of the asymptotic behaviour for sign-changing solutions, which present challenging difficulties due to the non-monotone evolution of each phase.

References

[1] Baiocchi, C.,Su un problema di frontiera libera conneso a questioni di idraulica. Ann. Mat. Pura. Appl. (4) 92 (1972), 107-127.

[2] Brändle, C.; Chasseigne, E.; Quirós, F., Phase transition with mid-range interactions: a nonlocal one-phase Stefan model, SIAM J. Math. Anal., Vol. 44, No. 4, (2012) 3071–3100.

[3] Meirmanov, A. M. “The Stefan problem”. Walter de Gruyter, Berlin, 1992.

[4] Stefan, J., Über einige Probleme der Theorie der Wärmeleitung. Sitzungsber, Wien, Akad. Mat. Natur. 98 (1889), 473-484; see also pp. 614-634; 965-983; 1418-1442.

Asymptotic Behavior of Solutions to Nonlinear Nonlocal Fractional Functional Differential Equations

Madhukant Sharma
Indian Institute of Technology, Madras, India
sharmamk003@gmail.com
Shruti Dubey

In this paper, we discuss the asymptotic behavior of solution to nonlocal initial value problems of nonlinear fractional order functional differential equations in a Banach space. We prove our results with the assumption that $\{-A(t) : t \geq 0\}$ generates a resolvent operator family and nonlinear part is a Lipschitz continuous function. Also, we assume that $-A(t)$ generates the analytic semigroup for each $t \geq 0$. At the end a fractional order partial differential equation is given to illustrate the obtained abstract results.

Multiplicity of solutions for non-local elliptic equations driven by the fractional Laplacian

Xifeng Su
Beijing Normal University and Universite de Bordeaux (IMB), Peoples Rep of China
xfsu@bnu.edu.cn
Yuanhong Wei

We consider the semi-linear elliptic PDE driven by the fractional Laplacian:

$$\begin{cases} (-\Delta)^s u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^n \backslash \Omega. \end{cases}$$

An L^∞ regularity result is given, using De Giorgi-Stampacchia iteration method. By the Mountain Pass Theorem and some other nonlinear analysis methods, the existence and multiplicity of non-trivial solutions for the above equation are established. The validity of the Palais-Smale condition without Ambrosetti-Rabinowitz condition for non-local elliptic equations is proved. Two non-trivial solutions are given under some weak hypotheses. Non-local elliptic equations with concave-convex nonlinearities are also studied, and existence of at least six solutions are obtained. Moreover, a global result of Ambrosetti-Brezis-Cerami type is given, which shows that the effect of the parameter λ in the nonlinear term changes considerably the nonexistence, existence and multiplicity of solutions.

Standing waves for the Chern-Simons-Schrodinger systems without (AR) condition

Jinggang Tan
Universidad Tecnica Federico Santa Maria, Chile
jinggang.tan@usm.cl
Y. Wan

We study the nonlinear Chern-Simons-Schrodinger systems (non local) with an external potential. We show the existence, non-existence, and multiplicity of standing waves to this problem with asymptotically linear nonlinearities, which do not hold the Ambrosetti-Rabinowitz condition.

Multiple solutions for non-local fractional problems via Morse theory

Binlin Zhang
Heilongjiang Institute of Technology, Peoples Rep of China
zhbinlin@gmail.com
Giovanni Molica Bisci

In this talk we focus on the existence of non-trivial solutions for equations driven by a non-local integrodifferential operator with homogeneous Dirichlet boundary conditions introduced by Servadei and Valdinoci. By using Three Critical Point Theorem and Morse theory, multiplicity results for the above-mentioned equations are obtained. As a particular case, we present some multiplicity results for the fractional Laplacian. Our results extend classical results for semilinear elliptic boundary value problems to the non-local fractional context.

Special Session 55: Microlocal Analysis and the Inverse Conductivity Problem

Raluca Felea, Rochester Institute of Technology, USA
Romina Gaburro, University of Limerick, Ireland

The subject of this special session is focused on the new developments in the fields of microlocal analysis and inverse conductivity problems. We aim to bring together both leading experts in these fields, as well as young researchers. Microlocal analysis has a great impact on inverse scattering theory, where its techniques can be used to recover images. Inverse problems to which microlocal analysis is particularly well-suited include those of seismology, Synthetic Aperture Radar (SAR) and Single Photon Emitted Computed Tomography (SPECT). In these problems, the forward operator which maps the image to the data is typically known and the goal is to invert it by applying to it the backprojection operator. In doing so, artifacts appear and the focus is to describe these artifacts, understand their strength and diminish their strength. The inverse conductivity problem is the problem of recovering the conductivity of the interior of a body from knowledge of currents and voltages applied to its surface. Such problem appears in geophysics, where the method is used in prospecting, in archaeology, in industrial process tomography and in medical imaging, where it is also known as Electrical Impedance Tomography (EIT). The mathematical problem consists in finding the conductivity function on a domain D (the body under inspection), from the knowledge of the so-called Dirichlet-to-Neumann map which describes measurements of boundary voltages and currents densities. In the context of EIT, different tissues inside the human body present different conductivities so that a map of the conductivity in the human body provides unique information about tissues and organs that no other medical imaging technique would provide.

From the conductivity equation to frequency-dependent PDEs: using multiple frequencies to satisfy non-zero constraints

Giovanni Alberti
University of Oxford, England
giovanni.alberti@maths.ox.ac.uk

In many hybrid imaging techniques the coefficients of a PDE have to be reconstructed from the knowledge of some internal measurements. In order to reconstruct the parameters, we need suitable boundary conditions (BCs) so that the corresponding solutions satisfy certain non-zero local constraints inside the domain. It is often easy to satisfy these constraints with the conductivity equation, but hard with other PDEs. The focus of this talk is to present an alternative strategy to the use of complex geometric optics solutions to find suitable BCs for frequency-dependent PDEs (conductivity equation with complex admittivity, Helmholtz equation or Maxwell's equations). In particular, I will show that if the non-zero constraints are satisfied for the conductivity equation with some BCs, then they are satisfied also for the other equations with the same BCs, provided that a finite number of frequencies, given a priori, are chosen in a fixed range.

Microlocal analysis of an elliptical Radon transform in circular geometry of data acquisition

Gaik Ambartsoumian
University of Texas at Arlington, USA
gambarts@uta.edu
Jan Boman, Venkateswaran P. Krishnan, Eric Todd Quinto

The talk discusses a generalized Radon transform that appears in ultrasound reflection tomography. In our model, the ultrasound emitter and receiver move along a circle with a fixed distance between them. The measured data corresponds to the integrals of the unknown image function along ellipses with foci at the emitter and receiver locations. We analyze the microlocal properties of the transform R that arises from this model. As a consequence, we show that, for distributions with support contained in a disc D_b sufficiently inside the circle, R^*R is an elliptic pseudo-differential operator. We provide a local filtered back projection algorithm $L = R^*DR$, where D is a well-chosen differential operator. Finally, we discuss an extension with some modifications of our result outside of D_b.

Solving the inverse problem for the 2D Schrödinger equation with L^p-potential

Eemeli Blåsten
University of Helsinki, Finland
eemeli.blasten@helsinki.fi
Oleg Imanuvilov, Masahiro Yamamoto

We will show that if the potential of a Schrödinger equation is in L^p, $p > 2$, then the boundary data consisting of traces and normal derivatives of all H^1-solutions determines the potential. If it is in a Sobolev space with a positive smoothness parameter, then there is conditional stability. This is made possible by proving a Carleman estimate with stronger decay rate than before. This fills a gap from Bukhgeim's paper of 2008. The result is based on a joint work with Imanuvilov and Yamamoto.

Stability of the unique continuation for the wave operator and applications

Roberta Bosi
University of Helsinki, Finland
roberta.bosi@helsinki.fi
Y. Kurylev, M. Lassas

We study the stability of the unique continuation for the anisotropic wave operator, with coefficients independent of time. Using a Carleman-type estimate by Tataru and other tools of microlocal analysis and subharmonic functions, we prove a logarithmic inequality in a ball whose

radius has an explicit dependence on the C^1-norm of the coefficients and on the other geometric properties of the operator. As possible application we consider the stability estimate for the inverse conductivity problem on a Riemannian manifold, in the hyperbolic case.

Stability estimates for the Radon transform with restricted data

Pedro Caro
University of Helsinki, Finland
pedro.caro@helsinki.fi
David Dos Santos Ferreira, Alberto Ruiz

In this talk, we will show a stability estimate going from the Radon transform of a function with limited angle-distance data to the L^p norm of the function itself, under some conditions on the support of the function. This estimate was originally motivated by the study of the stability for the inverse Calderón problem with partial data. The Segal-Bargmann transform, often called FBI transform, is one of the main tools to obtain our estimate.

Inverse scattering at fixed energy on asymptotically hyperbolic Liouville surfaces

Thierry Daude
Universite de Cergy-Pontoise, France
thierry.daude@u-cergy.fr
Niky Kamran, Francois Nicoleau

We study an inverse scattering problem on asymptotically hyperbolic Liouville surfaces and prove uniqueness of such surfaces (up to isometries consistent with the Liouville structure) from the knowledge of the scattering matrix at a fixed energy associated with scalar waves. The main ingredients to prove this result are the separability of the wave equation into a system of ODEs, the Complex Angular Momentum method and a reinterpretation of the partial scattering coefficients as generalized Weyl-Titchmarsh functions for a certain Sturm-Liouville equation having the complex angular momentum as spectral parameter.

Problems in Microlocal Analysis Motivated by Seismic Imaging

Allan Greenleaf
University of Rochester, USA
allan@math.rochester.edu
Raluca Felea

High frequency linearized sesimic imaging deals with the map that sends a small, singular perturbation of a known smooth sound speed to the resulting perturbation of the acoustic pressure field measured at the surface. Under fairly weak assumptions, this map is an FIO, but its detailed structure, and the possibility for approximate inversion, depend on an interaction between the ray geometry for the background sound speed and the geometry of the data set. I will discuss some of what is known and suggest open problems.

Microlocal analysis of the geodesic X-ray transform

Sean Holman
University of Manchester, England
sean.holman@manchester.ac.uk

Given a Riemannian manifold with boundary (M, g) the geodesic X-ray transform is the mapping $\mathcal{X}$ which takes a function on M to it's integrals along the geodesics of g. A standard approach to begin the analysis of this transform is to construct a parametrix for the so-called normal operator $N = \mathcal{X}^t \circ \mathcal{X}$. Such a construction shows that the problem of inverting N is Fredholm and therefore only has a finite dimensional kernel and the inversion is stable on a complement of that kernel. In the absence of conjugate points N is known to be an elliptic pseudodifferential operator of order -1 and so the parametrix construction is standard, but when there are conjugate points the situation is more complicated. This talk will present new results showing that under certain hypotheses the operator N is equal to a pseudodifferential operator plus some Fourier integral operators whose canonical relations can be given in terms of the geometry of the conjugate points in M.

Spectral Properties of Schroedinger Operators on Perturbed Lattices

Hiroshi Isozaki
Tsukuba University, Japan
isozakih@math.tsukuba.ac.jp
K. Nado and H. Morioka

We talk about the spectral properties of Schroedinger operators on perturbed lattices. We shall prove non-existence of embedded eigenvalues, limiting absorption principle for the resolvent, construct spectral representation, and define the S-matrix. Based on these forward problem results, we will discuss the inverse scattering, i.e. reconstruction of the potential term or the perturbation of the local part of the lattice from the knowledge of the S-matrix of one fixed energy. Our theory covers square, triangular, hexagonal, Kagome, diamond, subdivision lattices, as well as ladder and graphite.

Singular Fourier integral operators in radar imaging

Venky Krishnan
TIFR Centre for Applicable Mathematics, India
pkvn@live.com
Gaik Ambartsoumian, Raluca Felea, Clifford Nolan and Eric Todd Quinto

We consider the microlocal properties of the linearized scattering operator and the associated imaging operator in synthetic aperture radar imaging for several interesting data acquisition geometries. The imaging operator for all these acquisition geometries introduces artifacts in addition to recovering the true singularities. We analyze the structure and strength of the artifacts for these cases.

Monostatic SAR with fold-cusp singularities.

Clifford Nolan
University of Limerick, Ireland
clifford.nolan@ul.ie
Raluca Felea

We analyse the back-projected image obtained from synthetic aperture RADAR data collected on a flight-track with inflection points. Artefacts that are of equal strength as the bona-fide part of the image are predicted by this analysis. We also give a weak normal form for operators associated to a fold-cusp canonical relation, which we hope will be of use in other inverse problems where such a structure may arise.

Microlocal analysis and bistatic data acquisition in seismic imaging.

Eric Todd Quinto
Tufts University, USA
todd.quinto@tufts.edu
Gaik Ambartsoumian, Raluca Felea, Venkateswaran P. Krishnan, and Clifford Nolan

We will describe a simple model of bistatic seismic imaging in which the sound source and receiver are at different locations. We analyze the microlocal properties of the reconstruction operator under the assumption of constant wave speed and without multiple reflections. We show that one common data acquisition method is better than another one commonly used. This suggests that the same data acquisition could be better for models that include multiple reflections and variable sound speed.

Optimal measurements for elliptic boundary value problems

Luca Rondi
Università di Trieste, Italy
rondi@units.it
Michele Di Cristo

We consider the problem of determining defects, such as inhomogeneities or cavities, inside a conducting body through electrostatic measurements at the boundary. We discuss the issue of which measurements should be used, if we may perform only a fixed and finite number of them, to obtain the most stable reconstruction. In the particular case when the conducting body is a ball with a constant background conductivity, the optimal measurements are related to the spherical harmonics. We extend this result to different geometries or background conductivities and we find that the optimal measurements are related to suitable Steklov eigenfunctions.

The anisotropic Calderon problem

Mikko Salo
University of Jyvaskyla, Finland
mikko.j.salo@jyu.fi

We discuss recent progress in inverse problems in anisotropic media, mostly concentrating on Calderon's inverse conductivity problem. We mention connections between the anisotropic Calderon problem, microlocal analysis, integral geometry problems, and control theory methods for the wave equation.

Non-local boundary conditions and domain truncation in electrical impedance tomography

Erkki Somersalo
Case Western Reserve University, USA
ejs49@case.edu

In electrical impedance tomography, the electric current is injected to the body through contact electrodes that usually cover only a part of the body. The electrode positions are chosen so that the current probes mostly the part of the body that we are interested in. However, when the conductivity reconstruction is done in only a part of the body, artificial boundaries of the computational domain may introduce significant and non-local artifacts. In this talk, the truncation boundary conditions are discussed and it is shown that by proper modeling using non-local boundary conditions, the artifacts can be significantly reduced. Results with both simulated and real data are shown.

The geodesic X-ray transform on Riemannian surfaces

Plamen Stefanov
Purdue University, USA
stefanov@math.purdue.edu
F. Monard and G. Uhlmann

We study the geodesic X-ray transform X on compact Riemannian surfaces with conjugate points. Regardless of the type of the conjugate points, we show that we cannot recover the singularities and therefore, this transform is always unstable. We describe the microlocal kernel of X and relate it to the conjugate locus. We present numerical examples illustrating the cancellation of singularities.

Special Session 56: IsoDifferential Calculus, IsoDynamical Systems and Their Applications

Svetlin Georgiev, Sorbonne University, France

The session aims to present the latest advances in the field of iso-, geno- and hyper- mathematics and their applications in mathematics, physics, mechanics, chemistry, biology, engineering and other applied sciences. The proposed session intends to gather experts working on various actually important aspects of iso-, geno-, hyper-mathematics and to explore new connections between different research areas. We expect a fruitful exchange of new ideas and collaboration regarding the research development of these disciplines among the participants.

Iso-even and iso-odd iso-functions and their properties

Gerard Berthier
University of Paris 13, France
gerard.berthier2010@yahoo.com

Professor Svetlin Georgiev is defined two new kinds iso-functions different than Santilli iso-functions (http://www.cliffordanalysis.com/ Announcement). Using them we give definitions of odd and even iso-functions and give conditions for the isotopic element so that when we have odd or even real function the corresponding iso-lift to be odd and even iso-functions and the inverse.

Introduction to Santilli Iso-Numbers and Iso-Mathematics

Christian Corda
Istituto Universitario di Ricerca Scientifica Santa Rita, Firenze, Italy
cordac.galilei@gmail.com

Santilli iso-numbers and iso- mathematics, which represent the fundamental mathematical framework of Hadronic Mechanics and Hadronic Chemistry are introduced and reviewed from both the historical and pedagogical points of view.

Extremal properties of the iso-functions of several iso-variables

Svetlin Georgiev
Sorbonne University, France
svetlingeorgiev1@gmail.com

In my previous lecture, given at ICRAM 2014, India, they were defined all classes iso-functions of several isovariables. They were deducted partial iso-deriavtives. Here we use these main definitions so that to be found the main conditions for the isotopic element under which we have local and global maximum and minimum..

Periodic iso-functions

Carmen Ivone
National University of Madrid, Spain
carmen23456carmen@yahoo.com

In this lecture we use given fourth kind iso-functions in the book by Professor Svetlin Georgiev (http://www.cliffordanalysis.com/ Announcement) to give and classify so-called iso-periodic iso-functions. In the lecture will be given the connections between the periodic and iso-periodic iso-functions in accordance with the main properties of the isotopic element.

Advances in the Lie-Santilli IsoTheory

Jerdsay Kadeisvilli
Institute for basic research, Palm Harbor, Florida, USA, USA
jvkadeisvilli@gmail.com

We review the physical origin of the isotopies of the various branches of Lie's theory formulated in the late 1970s by Prof. R. M. Santilli when at the Department of Mathematics of Harvard University and present various advances.

Isodual functions

Nicolas Laurence
Imperial College of London, England
nicolas_laurence111@yahoo.com

Basing on the main definitions of isofunctions in the book by Professor Svetlin Georgiev (http://www.cliffordanalysis.com/ -Announcement) "Foundations of IsoDifferential Calculus" we introduce the main classes isodual functions. From them we deduct the main definitions for some special iso-functions which we can consider as iso-lift of the classical special functions. It will be given their properties and some of them applications..

Iso-homogeneous iso-functions

Peter Maers
Public University of Uruguay, Uruguay
peter_maers@yahoo.com

In this lecture we give definition for iso-homogeneous iso-functions basing on the main definitions given in the book by Professor Svetlin Georgiev for isofunctions of first, second, third and fourth kind. We deduct their main properties using the properties of the isotopic element.

Category Formulation of Santilli's Isotopies

Stepan Moskaliuk
Bogoliubov Institute for Theoretical Physics of NAS of Ukraine, Ukraine
mss@bitp.kiev.ua

This talk consists from 5 Parts and presents a reformulation of the basic results of Santilli Iso-Mathematics in terms of modern mathematical framework called category theory. Part 1 introduces the basic notions of category theory, functors and natural transformation: Categories, monoids and groupoids; Functors and natural transformations; Representations of categories; Multiplicative structures on categories; Method of additional structures on objects of a category. Examples of categories of basic isostructures are presented in Part 2: Category of Isogroups; Category of Isorings; Category of Isofields; Category of Linear Isospaces and Category of Isoalgebras. In Part 3 there are formulated the basic results of categories of topological, incidence and metric isostructures: Category of topological isospaces; Category of Affine isospaces; Category of projective isospaces; Category of Euclidean isospaces; Category of pseudo-Euclidean isospaces; Category of conformal isospaces; Category of pseudoconformal isospaces. In Part 4 it is shown how Lie-Santilli Isogroups and Isoalgebras. are represented in category theory. In Part 5 it is described the categorical concept of infinite-dimensional isospaces: Category of infinite-dimensional linear isospaces; Category of Hilbert isospaces; Category of Banach isospaces and Categories of Hilbert and Banach isoalgebras.

Iso-Galois Fields and Isotopically Isomorphic Realization of a Group

Arun Muktibodh
Mohota College of Science, India
amukti2000@yahoo.com

Rugero Maria Santilli defined iso-fields of characteristic zero. In this paper we extend this definition to define Iso-Galois fields which are essentially of non-zero characteristic. Iso-Galois fields of first and second kind are also defined with specific examples. We have also proved some results regarding the structure of these constructs. Our main purpose is to apply Santilli's ideas to the fields of non-zero characteristic and seek for further development in this direction. We have answered the open problems 'Can we construct finite isofields of first kind?' and 'Can we construct finite isofields of second kind?' in the affirmative. If G is a group then the permutation of the group elements of G by any element $a \in G$ is an isogroup $\widehat{G}$ with isomultiplication $\widehat{\times}$ defined on it. We say that $\widehat{G}$ is the **isotopically isomorphic realization** of the group G. Thus, G itself is the isotopically isomorphic realization of the group G where $a = e$. Isotopically isomorphic realizations of a group defines a clear cut distinction between automorphic groups and isotopic groups.

Isodifferential of first, second, third and forth kind

Laurent Peterson
Sorbonne University, France
laurent.peterson234@yahoo.com

In this talk, using the main definitions given by Professor Svetlin Georgiev for fourth kinds iso-functions (http://www.cliffordanalysis.com/ see Announcement), we deduct the main properties of the iso-differentiable iso-functions and we give conditions for the iso-topic element so that when we have increasing or decreasing of some real functions to have increasing and decreasing of the corresponding iso-lift.

On the main properties of the iso-integrals

John Ruan
Government University of Liberia, Liberia
john_ruan@yahoo.com

In this lecture, in accordance with given definitions for the iso-integrals by Professor Svetlin Georgiev in his book(http://www.cliffordanalysis.com/-Announcement) we investigate all cases for the isotopic element when the considered iso-integral is convergent and divergent..

Origin and applications of the isodifferential calculus and isodynamical systems

Ruggero Maria Santilli
Institut for Basic Research, Palm Harbor, Florida, USA, USA
research@thunder-fusion.com

We review the physical and chemical origin of the isotopies of the differential calculus, today known as the Santilli-Georgiev IsoDifferential Calculus (IDC); we review its use for the isotopies of dynamical systems; and outline various novel applications in physics and chemistry http://www.santilli-foundation.org/Madrid-2014-Santilli.pdf

Extremal properties of the iso-functions

John Smidth
London University of Art Sciences, England
john11112222@abv.bg

In this talk it is followed the main definitions for iso-functions of first, second, third and fourth kinds given by Professor Svetlin Georgiev given in the books (http://www.cliffordanalysis.com/ in Announcements). Using them we give definitions for local and global maximum of the iso-functions of first, second, third and fourth kinds. It is given some physical applications of the obtained results.

Special Session 57: Inverse Problems in PDE and Geometry

Daniel Faraco, Universidad Autonoma de Madrid/ICMAT, Spain
Mikko Salo, University of Jyvaskyla, Finland

This minisymposium will cover several fundamental inverse problems in PDE and geometry broadly interpreted. Topics include uniqueness, stability and computational aspects of inverse boundary value problems, inverse scattering problems, multi-wave methods and various tomography problems. This field has experienced remarkable growth in recent years, and the aim of the minisymposium is to bring together experts working on different aspects of the field and to highlight the power of analytic and geometric methods in solving inverse problems.

Stable determination of coefficients in the dynamical Schrödinger equation in a magnetic field

Mourad Bellassoued
Faculty of Sciences of Bizerte, Tunisia
bellassoued@yahoo.fr

This talk is devoted to the study of the following inverse boundary value problem: given a Riemannian manifold with boundary determine the magnetic potential in a dynamical Schrödinger equation in a magnetic field from the observations made at the boundary.

A scattering map in two dimensions

Russell Brown
University of Kentucky, USA
russell.brown@uky.edu
Katharine Ott

We consider the scattering map introduced by Beals and Coifman and Fokas and Ablowitz that may be used to transform one of the Davey-Stewartson equations to a linear evolution. We give mapping properties of this map on weighted L^2 Sobolev spaces that mimic well-known properties of the Fourier transform.

Stability of the Calderón problem in admissible geometries

Pedro Caro
University of Helsinki, Finland
pedro.caro@helsinki.fi
Mikko Salo

In this talk we will show a *log log* type stability estimates for inverse boundary value problems (IBVPs) on admissible Riemannian manifolds of dimension $n \geq 3$. These inverse problems arise naturally when studying the anisotropic Calderón problem. The basic approach to prove these estimates bases on the use of complex geometrical optics, which restricts our study to suitable Riemannian manifold denoted as admissible. In the way to prove the stability for these IBVPs, we will need to get stability estimates for a mixed Fourier/attenuated geodesic ray transform.

L^p-Carleman estimates for the Schrödinger equation and applications to unique continuation

David Dos Santos Ferreira
Institut Élie Cartan, Université de Lorraine, France
ddsf@math.cnrs.fr
Camille Laurent

The aim of this work is to establish L^p Carleman estimates for the (dynamical) Schrödinger equation on Riemannian manifolds which are products of an Euclidean factor and a compact RIemannian manifold, and to underline the relations with resolvent estimates and Strichartz estimates for the Schrödinger. Our interest lies primarily in applications to unique continuation and control theory for nonlinear equations but this type of estimates have been useful for inverse problems in the context of elliptic problems, so we hope it might be of some interest. This is joint work with Camille Laurent.

Stable determination of interfaces using boundary measurements

Elisa Francini
Università di Firenze, Italy
elisa.francini@unifi.it
E. Beretta, M. DeHoop, S. Vessella

We discuss the inverse boundary value problem for the equation

$$\Delta u + q(x)u = 0$$

with piecewise constant potential $q(x)$. We show a stability result concerning the reconstruction of the polyhedral regions in which the potential is constant.

Reconstruction from boundary measurements for less regular conductivities

Andoni Garcia
University of Jyväskylä, Finland
andoni.a.garcia@jyu.fi
Guo Zhang

Following Nachman's reconstruction scheme and using Haberman and Tataru's idea for the uniqueness for Calderón's problem with low regular coefficients, we reconstruct C^1 conductivity γ or Lipchitz conductivity γ with small enough value of $|\nabla log\gamma|$ in a Lipschitz domain Ω from the Dirichlet-to-Neumann map Λ_γ.

Sparse Reconstruction in Partial Data Electrical Impedance Tomography

Henrik Garde
Technical University of Denmark, Denmark
hgar@dtu.dk
Kim Knudsen

We consider the reconstruction problem in electrical impedance tomography for partial boundary data in 2D and 3D. That is from current and voltage measurements on part of the boundary to reconstruct inclusions with a conductivity different from the background. This is known to be a highly ill-posed problem. We describe a numerical algorithm for the solution based on a regularised least squares formulation of the problem. This formulation is flexible and allows us to consider partial data and apply prior information in terms of estimates of the support of the inclusions. We show by numerical examples that this approach improves both the resolution and the contrast in the reconstruction.

An Integral Geometry Problem Arising in Travel Time Tomography

Fikret Golgeleyen
Bulent Ecevit University, Turkey
golgeleyen@yahoo.com

We discuss the uniqueness of the solution of an integral geometry problem which arises in the problem of finding a Riemannian metric by the distance between the boundary points of a convex domain. First, the problem is reduced to an inverse source problem for a kinetic equation on a Riemannian manifold and then the uniqueness theorem is proved in semi-geodesic coordinates by using the tools of Fourier analysis.

On the Solution of an Inverse Problem for an Integro-differential Transport Equation

Ismet Golgeleyen
Bulent Ecevit University, Turkey
ismetoncu717@yahoo.com

In this work, the solvability conditions for an inverse problem for an integro-differential transport equation are obtained and a numerical approximation method based on the finite difference method is developed. A comparison between the numerical solution and the exact solution of the problem is presented. Experimental results show that proposed method is robust to data noises.

Superdimensional resonators modeled by Helmholtz-type equations

Allan Greenleaf
University of Rochester, USA
allan@math.rochester.edu
Henrik Kettunen, Yaroslav Kurylev, Matti Lassas, Gunther Uhlmann

Weyl's well-known law describes the asymptotic distribution of eigenfrequencies. In the physical setting of resonators for polarized electromagnetic or acoustic waves, modeled by Helmholtz-type equations, we describe inhomogeneous, anisotropic media that, over certain intervals, support anomalously large numbers of eigenvalues, an effect that we call superdimensionality.

Passive measurements in an inverse problem on Lorentzian manifolds

Yaroslav Kurylev
University College London, England
y.kurylev@math.ucl.ac.uk
M. Lassas, G. Uhlmann

We consider an inverse problem of the recovering of a part of a Lorentzian manifold (M, g) from passive measurements. Namely, let $\mu([0,1])$ be a time-like curve and U is its neighbourhood. Let V be a relatively compact region in $J^-(\mu(1)) \setminus I^-(\mu(0))$, where $J^-(p)$ is the causal past of p. We assume that, for any $q \in V$ we know the set of the light observations $U \cap L^+(q)$, where $L^+(q)$ is the future light cone from q. Note that this type of data corresponds to the observation of supernovas, quasars, etc in astronomy. Assuming that (M, g) is strongly hyperbolic, we prove that this data uniquely determine V and the conformal class of $g|_V$.

Carleman estimates for high-order elliptic operators

Jérôme Le Rousseau
Université d'Orléans, France
jlr@univ-orleans.fr
Mourad Bellassoued

We consider elliptic operators of even order with complex coefficients and we derive microlocal and local Carleman estimates near a boundary, under sub-ellipticity and strong Lopatinskii conditions or near an interface under sub-ellipticity and proper transmission conditions. Carleman estimates are weighted a priori estimates for the solutions of the associated elliptic problem. The weight is of exponential form, $\exp(\tau\varphi)$, where τ is meant to be taken as large as desired. Such estimates have numerous applications in unique continuation, inverse problems, and control theory.

Unique continuation property of solutions to anomalous diffusion equations

Gen Nakamura
Inha University, Korea
nakamuragenn@gmail.com
Ching-Lung Lin

A Carleman estimate and the unique continuation of solutions for an anomalous diffusion equation with fractional time derivative of order $0 < \alpha < 1$ are given. The estimate is derived via some subelliptic estimate for an operator associated to the anomalous diffusion equation using calculus of pseudo-differential operators.

Cloaking using complementary media in finite frequency regime

Loc Nguyen
Ecole Polytechnique Federale de Lausanne, Switzerland
loc.nguyen@epfl.ch
Hoai-Minh Nguyen

The proof for cloaking using complementary media in quasistatic regime was given by Hoai-Minh Nguyen (see arXiv:1310.5483). In this talk, we will confirm this phenomenon when the frequency is nonzero. A strong unique continuation property will be discussed as well.

Inverse problem for the Riemannian wave equation with Dirichlet data and Neumann data on disjoint sets

Lauri Oksanen
University College London, England
l.oksanen@ucl.ac.uk
Matti Lassas

We consider the wave equation on a smooth compact Riemannian manifold with boundary and show that acoustic measurements with sources and receivers on disjoints sets on the boundary determine the manifold uniquely assuming that the wave equation is exactly controllable from the set of sources. The exact controllability can be characterized in terms of the billiard flow of the manifold. We will also consider the problem to determine lower order terms in the wave equation.

Nonlinear Fourier analysis for discontinuous conductivities: computational results

Juan Manuel Reyes
Cardiff University, Wales
reyes.juanmanuel@gmail.com
Kari Astala, Lassi Päivärinta and Samuli Siltanen

Some new computational results are presented aimed at comparing two reconstruction methods of Electrical Impedance Tomography (EIT) for nonsmooth conductivities in the plane based on the use of complex geometrical optics (CGO) solutions to D-bar equations involving the constructive global uniqueness proofs for Calderón problem exposed in [Nachman; Annals of Mathematics **143** (1996)] and [Astala and Päivärinta; Annals of Mathematics **163** (2006)]. This is a joined work with Kari Astala, Lassi Päivärinta and Samuli Siltanen from the University of Helsinki.

Rough Potential Recovery in the Plane

Keith Rogers
ICMAT-CSIC, Spain
keith.rogers@icmat.es
Kari Astala and Daniel Faraco

We will consider the inverse scattering problem at a fixed energy in the plane. That is to recover a potential from the scattered plane waves with a fixed frequency, incoming from all directions and then measured in all directions. Following the pioneering work of Bukhgeim, we will provide explicit formulae with which one can recover compactly supported potentials with half a derivative in L^2, and show that below this level of regularity the recovery process can fail.

A stability analysis for direct and inverse scattering problems in the high frequency asymptotics

Luca Rondi
Università di Trieste, Italy
rondi@units.it
Mourad Sini

We discuss the stability issue for the determination of a scattered wave from its far-field pattern, in the high frequency asymptotics. Applications include stability results for the determination by far-field data of solutions of direct scattering problems with sound-soft obstacles and an instability analysis for the corresponding inverse obstacle problem.

G-convergence and Dirichlet to Neumann Map

Alberto Ruiz
Universidad Autonoma de Madrid, Spain
alberto.ruiz@uam.es
Daniel Faraco and Slava Kurylev

G-convergence is a tool used to check Stability in the inverse conductivity problem (Calderon Inverse Problem). I present a sufficient condition on a sequence of conductivities so that G-convergence implies convergence of the corresponding Dirichlet to Neumann maps in the norm topology. An example proves the necessity of such a condition. I will mention some applications.

Elastic wave propagation by many small rigid obstacles of arbitrary shapes and applications.

Mourad Sini
RICAM, Austrian Academy of Sciences, Austria
mourad.sini@oeaw.ac.at
Durga Prasad Challa

We are concerned with the linearized, isotropic and homogeneous elastic scattering problem by many small rigid obstacles of arbitrary, Lipschitz regular, shapes in 3D. We derive and justify the first order approximation of the scattered fields taking into account the whole denseness of the obstacles, i.e. the number of the obstacles M, their maximum radius a and the minimum distance between them d. We give two applications to this approximation: 1. In case the number of obstacles is moderate, $M = O(1)$, we localize the centers of mass of the obstacles and estimate their sizes using the far fields corresponding to a finite number of incident plane waves. 2. In case the number of obstacles is large, precisely $M := M(a) = O(a^{-1})$ and $d := d(a) = O(a^{1/3})$ as $a \to 0$, we derive the corresponding effective medium modeled by a density defined by the capacitances of the collection of obstacles. We will then discuss the corresponding inverse medium scattering problem.

The inverse Robin boundary value problem in a half-space

Miren Zubeldia
University of Helsinki, Finland
miren.zubeldia@helsinki.fi
Lassi Päivärinta

We study the inverse Robin problem for the Schrödinger equation in a half-space. The potential is assumed to be compactly supported. We first solve the direct problem for dimensions two and three. We then show that the Robin-to-Robin map uniquely determines the potential q.

Special Session 58: Dynamics in Systems With Interfaces

Tatiana Savina,Ohio University, USA
Alexander Nepomnyashchy, Technion, Israel

Modeling dynamics of systems with interfaces, which are widespread in nature and important in engineering, for many years serves as a stimulus for development of different branches of mathematics, including multidimensional complex analysis, integrability theory, nonlinear stability theory etc. The goal of the present session is to bring together mathematicians and physicists studying different phenomena related to the surface dynamics, and present recent achievements in that field. The subjects of the talks include the analysis of free boundary problems, Laplacian growth, viscous film dynamics, elastic interfaces and other aspects of that versatile problem.

Hydrodynamics in the quantum Hall regime

Eldad Bettelheim
Physics, Hebrew University of Jerusalem, Israel
eldadb@phys.huji.ac.il

I will discuss hydrodynamics in the quantum Hall regime and how these are connected to hidden quantum symmetries.

Stability and Evolution of Bilayer Interfaces in Amphiphilic Systems

Gurgen Hayrapetyan
Carnegie Mellon University, USA
ghayrap@andrew.cmu.edu

Functionalized energies, such as the Functionalized Cahn-Hilliard, model phase separation in amphiphilic systems, in which interface production is energetically favorable, but is limited by competition for the surfactant phase, which wets the interface. This is in contrast to classical phase-separating energies, such as the Cahn-Hilliard, in which interfacial area is energetically penalized. In binary amphiphilic mixtures interfaces are characterized by bilayers, which divide the domain of the dominant phase, A, via thin layers of phase B formed by homoclinic connections. We discuss the stability and evolution of bilayer interfaces, including the onset of pearling bifurcations which lead to development of pore dominated network morphologies.

Universality of singularity in Hele-Shaw flow

Seung-Yeop Lee
University of South Florida, USA
lees3@usf.edu
Nikolai Makarov, Sha Chang

It has been argued in physics literature that the asymptotic dynamics near singularities of Hele-Shaw flow is scaling-invariant. We give a rigorous proof of this statement when the boundary of Hele-Shaw droplet is described in terms of a rational conformal map.

Large-time Hele-Shaw solutions in terms of conserved moments

Yu-Lin Lin
Royal Institute of Technology, Sweden
ylli@kth.se
Sam Howison

In this talk, zero-surface-tension Hele-Shaw flow driven by injection at a single point is considered. We show that for a special set of solutions, we can construct their large-time solutions from their moments which are conserved quantities.

Algebraicity of higher-dimensional quadrature domains

Erik Lundberg
Purdue University, USA
elundberg9@gmail.com
Alexandre Eremenko

B. Gustafsson showed (1983) that planar quadrature domains are always algebraic (they have boundary contained in the zero set of a polynomial). H. S. Shapiro asked (1992) whether this is true in higher dimensions. We answer this question in the negative by constructing explicit four-dimensional examples that have transcendental boundary. In terms of exterior potential, our examples behave like a point charge superimposed on a dipole source. The construction involves elliptic integrals of the third kind and uses the relation between the Schwarz potential (in four dimensions) and the Schwarz function of the generating curve in the plane. This is joint work with Alexandre Eremenko.

Dynamics of a pore in a lipid membrane

Alexander Nepomnyashchy
Technion, Israel
nepom@fermat.technion.ac.il
Vladimir Volpert

It is known that vesicles formed by lipid bilayer membranes, which are permeable for water and small molecules but impermeable for large molecules, can be used for transportation of a toxic drug to a target, where the drug is released through created pores. A lipid membrane can be considered as a two-dimensional liquid medium surrounded by a three-dimensional medium of the ambient liquid. The dynamics of a liquid membrane includes a viscous or viscoelastic two-dimensional flow inside the membrane and an elastic deformation of the membrane itself. The pores in membrane show a rather nontrivial dynamics, which thus far has been studied by means of simplified models. In the present talk,

we describe the dynamics of a pore in a stretched membrane by means of a self-consistent mathematical modeling. The viscous dissipation in the membrane and surrounding liquids are accounted for. The effects of the membrane rheology are considered. The work has been partially supported by the US-Israel Binational Science Foundation (grant No. 2008122).

On the uniqueness of quadrature surfaces

Michiaki Onodera
Kyushu University, Japan
onodera@imi.kyushu-u.ac.jp

A new geometric flow describing the motion of quadrature surfaces is introduced. This characterization enables us to study quadrature surfaces through the investigation of the flow. It is proved that the flow is uniquely solvable under the geometric condition that the initial surface has positive mean curvature. As a consequence, a bifurcation criterion for quadrature surfaces is obtained.

Creation of localized drop train in a liquid film by traveling thermal waves

Alex Oron
Technion-Israel Inst. of Technology, Israel
meroron@tx.technion.ac.il
V. Frumkin, Wenbin Mao and A. Alexeev

Using long-wave theory and direct numerical simulations (DNS) of the Navier-Stokes and energy equations, we investigate the nonlinear dynamics of a bilayer system consisting of a thin liquid film and an overlying gas layer driven by the Marangoni instability. The bottom solid substrate is heated in the form of periodical thermal waves propagating along the substrate with a constant frequency ω.In the case of a stationary thermal wave, $\omega = 0$,the liquid film rupture takes place with a flattish wide trough when both Marangoni number and the amplitude of the thermal wave are sufficiently large. Regimes in which the film forms a train of localized drops traveling along the substrate arise for sufficiently small, but non-zero ω, In this case, localized traveling drops are interconnected by thin liquid bridges with negligible small flow velocities.With an increase of ω the interfacial profiles represent traveling waves with less localized shapes. We show that the liquid is trapped inside the drops during the drop train motion, and the total flow rate is linearly proportional to ω.When the minimal thickness of the bridges between consecutive drops is increased, a faster flow rate can be achieved than in the localized drop-train configuration. In this case, however, fluid in the bridges has the negative (backward) velocity and can migrate from one drop to another, thereby leading to a gradual exchange of the content of drops in the train.

Hele-Shaw flow with a time-dependent gap

Tatiana Savina
Ohio University, USA
savin@ohio.edu
K. Malaikah, A.A. Nepomnyashchy

We consider the interior Hele-Shaw problem when the upper plate is lifted uniformly changing the gap between the plates. We discuss the exact solutions when the surface tension is negligible and asymptotic solutions when the surface tension is taken into account.

Vibration influence on a longwave Marangoni convection in a liquid layer

Sergey Shklyaev
Institute of Continuous Media Mechanics, Russia
shklyaev@yandex.ru
A.A. Alabuzhev, M. Khenner

We consider a thin liquid film atop a heated substrate. The substrate is subject to vertical oscillations of amplitude b and frequency ω. Two particular cases are analyzed: (i) the isothermal substrate and (ii) the substrate thermally insulated for perturbations. In both cases we consider a wide interval of frequencies ω, from ultra-high (the Stokes layer is thin with respect to mean film thickness) to ultra-low (the vibration period is compared to the typical time of the layer thickness relaxation τ). Two frequency regimes are analyzed in detail: a high frequency ($\omega\tau \gg 1$), when the averaging approach is applied; and an ultra-low frequency ($\omega\tau = O(1)$), when the vibration results in the gravity modulation only. At high frequencies, in case (i) the conventional Kopbosynov–Pukhnachev equation [Oron et al., Rev. Mod. Phys. (1997)] holds with additional vibro-generated averaged terms. In case (ii) the analysis is more involved because the temperature oscillations cannot be neglected; thus additional terms enter the set of amplitude equations derived in [Shklyaev et al., PRE (2012)]. Intermediate asymptotics is developed to match the amplitude equations at high-frequency and ultra-low frequency. S.Sh. is supported by RFBR (grant N13-01-96010).

Deformation quantization of Laplacian growth and weak resolution of singularities

Razvan Teodorescu
University of South Florida, USA
razvan@usf.edu

The zero surface tension limit of Hele-Shaw flows (Laplacian growth) is known for many important features, including the finite-time singularities that limit the evolution (in the Cauchy-problem sense) of a large family of classical (strong) solutions. Finding an appropriate generalization of the classical problem has been the focus of many recent developments in this field. We reformulate the deformation quantization of the Poisson structure of Laplacian growth (first introduced a decade ago) and discuss the function-theoretic aspect of the associated weak solutions. Finally, using the family of deformed solutions, we derive and interpret the weak resolution of conical singularities and their geometric characteristics.

Dynamic unbinding transitions in dragged film flow

Uwe Thiele
University Muenster, Germany
u.thiele@lboro.ac.uk
M. Galvagno, D. Tseluiko, H. Lopez

We study a liquid film that is deposited onto a flat plate that is extracted from a liquid bath. We analyse steady-state solutions of a long-wave evolution equation for the film thickness. Using centre manifold theory, we first obtain an asymptotic expansion of solutions in the bath region. The presence of an additional temperature gradient significantly changes these expansions and leads to the presence of logarithmic terms that are absent otherwise. Next, we obtain numerical solutions of the steady-state equation and analyse the behaviour of the solutions as the plate velocity is changed. We observe that the bifurcation curve exhibits snaking behaviour when the plate inclination angle is beyond a certain critical value. Otherwise, the bifurcation curve is monotonic. The solutions along these curves are characterised by a foot-like structure that is formed close to the meniscus and is preceded by a thin precursor film further up the plate. Finally, we explain that the snaking behaviour of the bifurcation curves is caused by the existence of an infinite number of heteroclinic orbits close to a heteroclinic chain that connects in an appropriate three-dimensional phase space the fixed point corresponding to the precursor film with the fixed point corresponding to the foot and then with the fixed point corresponding to the bath.

Interface dynamics in multiply connected domains: Exact solutions for multiple Hele-Shaw bubbles and the selection problem

Giovani Vasconcelos
Imperial College London and UFPE, Brazil, England
g.vasconcelos@imperial.ac.uk

Many free boundary problems naturally arise from the consideration of different types of Hele-Shaw systems, where two viscous fluids (typically with one fluid much less viscous than the other) are sandwiched between two closely spaced parallel plates. If the flow domain is simply (or at most doubly) connected, a plethora of analytical solutions can be found for both steady and time-dependent Hele-Shaw flows by using conformal mapping techniques. In the case of higher connectivity, however, the situation is much more complicated because conformal mappings for such domains are notoriously difficult to obtain. In this work, I will describe a large class of conformal mappings from a bounded circular domain to radial and circular slit domains. The slit maps are written in terms of functions appropriately chosen from a family of Schottky-Klein prime functions associated with the circular domain which we have recently obtained. Using these slit maps, exact solutions for an arbitrary number (finite or infinite) of steady bubbles in a Hele-Shaw cell can be constructed in closed form. Nonsingular time-dependent solutions for multiple Hele-Shaw bubbles can also be obtained. In particular, it will be shown that the steady solutions for which the bubbles move twice as fast as the fluid at infinity are the only attractor for the nonsingular solutions, thus showing that the velocity selection is inherently determined by the zero surface tension dynamics.

Special Session 59: Central Configurations, Periodic Solutions, Variational Method and Beyond in Celestial Mechanics

Tiancheng Ouyang, Brigham Young University, United States
Zhifu Xie, Virginia State University, USA
Duokui Yan, Beihang University, Peoples Rep of China

This special session will concentrate on the latest developments in the field of celestial mechanics. In classical celestial mechanics, the motion of the planets and stars is governed by universal gravity and it is mathematically formulated as a second-order system of differential equations, known as the N-body problem. One of the most important types of solutions to the N-body problem is periodic in nature. Among this class of solutions, analyzing the structure and stability of simple, rigidly rotating orbits known as relative equilibria (or central configurations) leads to a greater understanding of the complexities in the full problem. The study of the N-body problem continues to attract researchers in a wide range of fields including dynamical systems, topology, variational methods, algebraic geometry and numerical methods. Much progress has been made recently in the study of the finiteness and stability questions in the N-body problem. This special session provides a marketplace for ideas, and helps identify trends and areas of new opportunity in the field. The session brings established researchers and recent Ph. D graduates together, some of whom are women or from groups underrepresented in mathematics. Some specific topics to be covered include central configurations, discovery of new periodic solutions, variational methods, regularization of collisions, absence of collisions, stability of periodic solutions, spacecraft orbital design and applications of Morse index and Maslov index to the N-body problem. If the schedule permits, we anticipate ending the session with a discussion on open problems.

Lower and upper bounds on the number of planar central configurations

Alain Albouy
Observatoire de Paris, CNRS, France
albouy@imcce.fr

The planar central configurations are configurations of a finite number of gravitating points in a relative equilibrium. They are obtained by solving a system of algebraic equations. This is one of the most natural systems having configurations as roots. By looking at the set of solutions we observe very simple facts, which we are unable to prove. For example, whatever be the masses, there are at most 50 planar central configurations of 4 bodies. We will analyse results by Marshall Hampton, Rick Moeckel, Carles Simó and Zhihong Xia which point toward an interesting conjecture about a general lower bound.

Mountain Pass Lemma and New Periodic Solutions of the Singular Second Order Hamiltonian System

Li Bingyu
Si Chuan University, Peoples Rep of China
sbtgvpgf@163.com
Li Fengying and Zhang Shiqing

We generalize the classical Ambrosetti-Rabinowitz's Mountain Pass Lemma with Palais-Smale condition for C1 functional to some singular case with Cerami- Palais-Smale condition and then to study the existence of new periodic solutions with a fixed period for the singular second order Hamiltonian systems with Strong Force Potential.

Quantifying the bifurcation of stable symmetric periodic solutions from the center mass in the Sitnikov Problem

Daniel Nunez
Pontificia Universidad Javeriana de Cali, Colombia
denunez@javerianacali.edu.co

The Sitnikov problem is a restricted three body problem where the primaries bodies are moving in a symmetric way on elliptic orbits and such that the infinitesimal body is moving in the orthogonal direction of them and passing by the center of mass O. The position $z(t)$ of the infinitesimal body relative to O satisfies the following differential equation in appropriate units,

$$\ddot{z} + \frac{z}{(z^2 + r(t,e)^2)^{3/2}} = 0, \qquad (1)$$

where $e \in [0,1[$ is the eccentricity of the primaries and r(t,e) measures the distance between any them and O. This problem became important in 1960 when Stnikov used it to show, for the first time, the possibility of the existence of oscillatory motion in the 3-body problem. Many contributions have been made since then like for instance a paper series of Alekseev in 1968-1969, where he proved that all of the possible combinations of final motions in the sense of Chazy were realized in the Sitnikov problem. To the last decade of the past century and at the present one the families of symmetric periodic orbits of the Sitnikov problem (depending of the eccentricity) have been studied by several authors like Belbruno and Llibre [1], Jimenez-Lara and Escalona Buen-Dia [2], Ortega and Llibre [3], Ortega and Rivera [6], and many others. Worth mentioning the results of Ortega-Llibre where the authors have obtained analytically and by topological methods the continuation of some periodic orbits for $e = 0$ to all values of $e \in [0,1[$. However it seems that nothing has been said about the stability properties of some part of these branches. In this talk we present some strategies for quantifying the periodic even elliptic segment of the branch emerging from the center of mass of the circular problem ($e = 0$). These techniques can be also exploited to estimate the size of a stable segment of symmetric and periodic solutions in oscillators like forced pendulum, obtaining similar results in the line of [7]. For this purpose the results in [5] could be useful.

References

[1] E. Belbruno, J. Llibre, M. Ollé. *On the families of the periodic orbits which bifurcate from the circular Sitnikov motions*, Celestial Mech. Dynam. Astronm., 60 (1994), 99-129.

[2] L. Jiménez-Lara, A. Escalona-Buendía. *Symmetries and bifurcations in the Sitnikov problem*, Celestial Mech. Dynam. Astronom., 79 (2001) 97-117.

[3] J. Llibre, R. Ortega. *On the families of periodic orbits of the Sitnikov problem*, Siam J. Applied Dynamical Systems Vol. 7, (2008) 561-576.

[4] J. Moser, *Stable and Random Motions in Dynamical Systems*, Annals of Math. Studies 77, Princeton University Press, Princeton, NJ, 1973.

[5] D. Núñez, The Method of Lower and Upper Solutions and the Stability of Periodic Oscillations, Nonlinear Analysis, Theory, Methods Applications, **51** (2002), pp. 1207-1222.

[6] R. Ortega, Andrés Rivera. *Global bifurcations from the center of mass in the Sitnikov problem*, Discrete and Continuos Dynamical Systems series B 14 (2010), 719,732.

[7] J. Lei, X. Li, P. Yan, M. Zhang, Twist character of the least amplitude periodic solution of the forced pendulum, SIAM J. Math. Anal., 35(2003), pp. 844–867.

A variational principle for elliptic stable orbits in low dimensional Hamiltonian systems

Daniel Offin
Queen's University, Canada
offind@mast.queensu.ca

We describe a simple variational principle for periodic solutions in Hamiltonian systems whose solutions correspond to elliptic stable periodic solutions. We use a variant of the Maslov index together with mountain pass geometry to characterize such periodic solutions. Applications in the the theory of exact symplectic period mappings for planar Hamiltonian systems are discussed. We also place this discussion in the context of periodic solutions of the N-body problem.

Variational method on N-body problem of Celestial Mechanics

Tiancheng Ouyang
Brigham Young University, USA
ouyang@math.byu.edu

In this talk, I will give a survey of the new development of the variational methods of N-body problem in our collarate resaearch group. It combines numerical computation and theoretical analysis to find many new stable and semi-stable orbits of three body as well as more nodies of two and tree dimensions.

Symmetries and reductions in many–body problems

Gabriella Pinzari
Università di Napoli Federico II, Italy
pinzari@mat.uniroma3.it

The planetary many body problem is not integrable; however, it possesses symmetries by rotation and reflection invariance. We shall talk about the relation between symmetries and the symplectic structure of the problem and, in particular, on their relation with bounded and stable motions.

Bifurcations of periodic solutions from the center of mass in the Generalized Sitnikov (N+1)-body problem

Andres Mauricio Rivera Acevedo
Pontificia Universidad Javeriana Cali, Colombia
amrivera@javerianacali.edu.co

The Sitnikov problem is a restricted three body problem where two point masses $m_1 = m_2$ (primaries) moves around the center of mass in elliptic orbits lying on the plane x, y as solutions of the 2 body problem and a infinitesimal mass m_3 moves on the z axis. In appropiate units the equation of motion of the infinitesimal mass is given by

$$\ddot{z} + \frac{z}{(z^2 + r(t,e)^2)^{3/2}} = 0,$$

where $e \in [0, 1[$ is the eccentricity of the ellipses described by the primaries and $r(t, e)$ is the distance from the primaries to the center of mass. In this talk, we consider the Sitnikov problem with N primaries bodies with equal mass moving around the origin in the plane x, y as generalized solutions of the Lagrange problem. We call this problem the Generalized Sitnikov Problem. This special restricted N+1-body problem can be reduced to the Sitnikov problem with an appropriate positive parameter λ. More precisely we study the differential equation

$$\ddot{z} + \frac{\lambda z}{(z^2 + r(t,e)^2)^{3/2}} = 0,$$

where the parameter λ is given by

$$\lambda = \frac{N}{2\sum_{k=1}^{N-1} \csc(\frac{k\pi}{N})}.$$

According to the number of bodies we proved the existence (or non-existence) of a finite number (or infinite number) of symmetric families of periodic solutions that bifurcate from the equilibrium $z = 0$ (center of mass of the system) at certain values of the eccentricity. Worth mentioning that the type of bifurcation from the equilibrium that one expects is the pitchfork bifurcation, but the standard transversality conditions are not easy to check, therefore we present a slight variant of the theorem on pitchfork bifurcation adapted to our problem. See [5,6]

References

[1] E. Belbruno, J. Llibre, M. Ollé. *On the families of the periodic orbits which bifurcate from the circular Sitnikov motions*, Celestial Mech. Dynam. Astronm., 60 (1994), 99-129.

[2] L. Jiménez-Lara, A. Escalona-Buendía. *Symmetries and bifurcations in the Sitnikov problem*, Celestial Mech. Dynam. Astronom., 79 (2001) 97-117.

[3] J. Llibre, R. Ortega. *On the families of periodic orbits of the Sitnikov problem*, Siam J. Applied Dynamical Systems Vol. 7, (2008) 561-576.

[4] J. Moser, *Stable and Random Motions in Dynamical Systems*, Annals of Math. Studies 77, Princeton University Press, Princeton, NJ, 1973.

[5] R. Ortega, Andrés Rivera. *Global bifurcations from the center of mass in the Sitnikov problem*, Discrete and Continuos Dynamical Systems series B 14 (2010), 719-732.

[6] Andrés Rivera, *Periodic solutions in the Generalized Sitnikov (N+1)-body problem*, Siam J. Applied Dynamical Systems Vol. 12, No. (2013) 1515-1540.

On the Uniqueness of the Regular N-gon Central Configuration

Gareth Roberts
College of the Holy Cross, USA
groberts@holycross.edu
Josep Cors, Glen Hall

Due to symmetry, the equal mass regular n-gon is a central configuration for the n-body problem, the n-vortex problem, and similarly constructed systems. In an article on perverse choreographies, Alain Chenciner asked if the regular n-gon was the only central configuration lying on a circle, with the additional property that the center of the circle coincides with the center of mass. Using a topological approach, we show that for any choice of positive masses (or circulations), if such a central configuration exists, then it is unique. It quickly follows that if the masses are all equal, then the only solution is the regular n-gon. For the planar n-vortex problem and any choice of the vorticities, we show that the only possible co-circular central configuration with center of vorticity at the center of the circle is the regular n-gon with equal vorticities.

Non-integrability criterion for homogeneous Hamiltonian systems via blowing-up technique of singularities

Mitsuru Shibayama
Osaka University, Japan
shibayama@sigmath.es.osaka-u.ac.jp

It is a big problem to distinguish between integrable and non-integrable Hamiltonian systems. I will provide a new approach to prove the non-integrability of homogeneous Hamiltonian systems with two degrees of freedom. The homogeneous degree can be taken from real values (not necessarily integer). The proof is based on the blowing-up theory which McGehee established in the collinear three-body problem. I will also compare our result with Molares-Ramis theory which is the strongest theory in this field.

A continuum of periodic solutions to the planar four-body problem with various choices of masses

Zhifu Xie
Virginia State University, USA
zxie@vsu.edu
Tiancheng Ouyang

In this paper, we apply the variational method with the Structural Prescribed Boundary Conditions (SPBC) to prove the existence of periodic and quasi-periodic solutions for planar four-body problem with $m_1 = m_3$ and $m_2 = m_4$. A path $q(t)$ in $[0, T]$ satisfies SPBC if the boundaries $q(0) \in \mathbf{A}$ and $q(T) \in \mathbf{B}$, where $\mathbf{A}$ and $\mathbf{B}$ are two structural configuration spaces in $(\mathbf{R}^2)^4$ and they depend on a rotation angle $\theta \in (0, 2\pi)$ and the mass ratio $\mu = \frac{m_2}{m_1} \in \mathbf{R}^+$. We show that there is a region $\Omega \subseteq (0, 2\pi) \times R^+$ such that there exists at least one local minimizer of the Lagrangian action functional on the path space satisfying SPBC $\{q(t) \in H^1([0, T], (\mathbf{R}^2)^4) | \, q(0) \in \mathbf{A}, q(T) \in \mathbf{B}\}$ for any $(\theta, \mu) \in \Omega$. The corresponding minimizing path of the minimizer can be extended to a non-homographic periodic solution if θ is commensurable with π or a quasi-periodic solution if θ is not commensurable with π. In the variational method with SPBC, we only impose constraints on boundary and we do not impose any symmetry constraint on solutions. Instead, we prove that our solutions extended from the initial minimizing pathes have the symmetries. The periodic solutions can be further classified as simple choreographic solutions, double choreographic solutions and non-choreographic solutions. Among the many stable simple choreographic orbits, the most extraordinary one is the stable star pentagon choreographic solution when $(\theta, \mu) = (\frac{4\pi}{5}, 1)$. Remarkably the unequal-mass variants of the stable star pentagon are just as stable as the basic equal mass choreography.

Classification of periodic orbits in the planar four-body problem

Duokui Yan
Beihang University, Peoples Rep of China
duokuiyan@gmail.com
Tiancheng Ouyang and Zhifu Xie

Many periodic orbits were discovered as local action minimizers in the N-body problem. A natural question is how to classify these periodic orbits. In this talk, we present our variational method and apply it to classify them in the planar equal-mass four-body problem. Specific planar configurations are considered: collinear shape, rectangle, diamond, trapezoid, double isosceles, kite, etc. The classification of these periodic orbits is based on pairs of different special configurations an orbit passing. It only includes 8 categories and each category corresponds to two different special configurations. Furthermore, it helps find several new sets of periodic orbits.

A family of orbits in the equal mass 3-body problem

Elizabeth Zolllinger
St. Joseph's College, USA
ezollinger@sjcny.edu

In this talk we give the topology of a family of periodic orbits in the planar Newtonian three body problem, known as "comet" orbits, where one body (the comet) orbits the binary system formed by the other two. The original proof of existence uses the Implicit Function Theorem with the distance between the bodies of the binary pair as the small parameter and hence applies only when the distance is small. By using the Principle of Least Action and variational techniques, we can extend the existence proof of orbits where the comet comes close to the binary pair. All of these orbits have the same topology and they can be deformed into each other without passing through collision.

Special Session 60: Recent Advances in Evolutionary Equations

Ryo Ikehata,Hiroshima University, Japan
Tuoc Phan, University of Tennesse, USA
Grozdena Todorova, University of Tennessee Knoxville, USA

The special session focuses on evolutionary partial differential equations and systems which arise from both theoretical and applied sciences. The main theme is to gather experts to discuss some recent results on well-posedness, regularity, formation of singularities, stability/instability and asymptotic behavior of solutions. New theories and methods in analysis of evolutionary equations and their applications will be proposed and discussed.

Asymptotic stability for the Klein Gordon equation on non compact Riemannian manifolds - A sharp result

Marcelo Cavalcanti
State University of Maringa, Brazil
mmcavalcanti@uem.br
Valeria Cavalcanti, Cesar Bortot, Paolo Piccione

The Klein Gordon equation subject to a nonlinear and locally distributed damping, posed in a complete and non compact n dimensional Riemannian manifold $(\mathcal{M}^n, \mathbf{g})$ without boundary is considered. Let us assume that the dissipative effects are effective in $(\mathcal{M}\backslash\overline{\Omega}) \cup (\Omega\backslash V)$, where Ω is an arbitrary open bounded set with smooth boundary. In the present article we introduce a new class of non compact Riemannian manifolds, namely, manifolds which admit a smooth function f, such that the Hessian of f satisfies the *pinching conditions* (locally in Ω), for those ones, there exist a finite number of disjoint open subsets V_k free of dissipative effects such that $\bigcup_k V_k \subset V$ and for all $\varepsilon > 0$, $meas(V) \geq meas(\Omega) - \varepsilon$, or, in other words, the dissipative effect inside Ω possesses measure arbitrarily small. It is important to be mentioned that if the function f satisfies the pinching conditions everywhere, then it is not necessary to consider dissipative effects inside Ω.

Weak solutions to the incompressible Euler equations

Antoine Choffrut
University of Edinburgh, Scotland
antoine.choffrut@ed.ac.uk
Laszlo Szekelyhidi Jr.

We consider weak stationary solutions to the incompressible Euler equations and show that the analogue of the h-principle obtained by the second author in joint work with Camillo De Lellis for time-dependent weak solutions continues to hold. The key difference arises in dimension $d = 2$, where it turns out that the relaxation is strictly smaller than what one obtains in the time-dependent case. This is joint work with Laszlo Szekelyhidi Jr.

On New Decay Rates for the Plate and Wave Equations with Fractional Damping

Ruy Coimbra Charao
Federal University of Santa Catarina, Brazil
ruycharao@gmail.com
Ryo Ikehata and Cleverson R. da Luz

In this report we show recent (almost) optimal decay rates for the total energy and the L^2 norm of solutions associated to the linear dissipative wave equation and plate equation with effects of rotational inertia. The damping is given by a fractional damping term depending on a parameter $\theta \in [0,1]$. We observe that the dissipative structure of the plate equation with $\theta = 0$ is of the regularity-loss type. This decay structure still remains true for the plate equation with a power of fractional damping $\theta \in [0,1)$. This means that we can have an optimal decay estimate of solutions under an additional regularity assumption on the initial data. The structure of regularity-loss becomes more weak when θ increase and does not occur when θ arrive in $\theta = 1$. The wave equation does not have a such structure of regularity loss and due to this fact it is not necessary to assume additional regularity on the initial data when get estimates in the region of high frequencies in Fourier space. In fact, in that region the energy decays exponentially. Our results generalized previous recent. We use a special method based on the energy method in the Fourier space combined with the Haraux-Komornik lemma. This method shows to be very effective to study decay properties for several initial problems in $\mathbf{R}^n$ [R. C. Charão, C. R. da Luz and R. Ikehata, Sharp decay rates for wave equations with a fractional damping via new method in the Fourier space, JMAA 408 (2013), 247-255; R. C. Charão, C. R. da Luz and R. Ikehata, New decay rates for a problem of plate dynamics with fractional damping, JHDE 10 (2013), 563-575].

Decay Estimates for Abstract Evolution Equations of Second Order

Cleverson Roberto da Luz
Federal University of Santa Catarina, Brazil
cleverson.luz@ufsc.br
Ruy Coimbra Charao and Ryo Ikehata

In this work we study decay estimates for the energy and the L^2-norm of solutions for several models associated to the following abstract second order evolution equation

$$A_1 u_{tt}(t,x) + A_2 u_t(t,x) + A_3 u(t,x) = 0$$

with initial conditions

$$u(0,x) = u_0(x) \quad \text{and} \quad u_t(0,x) = u_1(x);$$

where $t \in R^+$, $x \in R^n$ and A_i $(i = 1, 2, 3)$ are positive self adjoint differential operators with symbols given by functions $P_i(y)$ $(i = 1, 2, 3)$. To obtain decay estimates to the above problem the idea is to work with the associated problem in the Fourier space. To this end we take the Fourier transform of the above problem to obtain the following initial problem

$$P_1(y)v_{tt}(t,y) + P_2(y)v_t(t,y) + P_3(y)v(t,y) = 0;$$
$$v(0,y) = v_0(y) \quad \text{and} \quad v_t(0,y) = v_1(1)$$

where v is the Fourier transform of u. This problem is an initial value one to a linear ordinary differential equation of second order with coefficients depending on a parameter $y \in R^n$. Thus, we obtain estimates for an abstract initial value problem in Fourier space with different conditions in $P_1(y)$, $P_2(y)$, $P_3(y)$ and the initial data. Then, the decay rates of the L^2-norm and of the energy associated to the abstract problem will be consequences of these results. In particular, we can use such results to get several estimates for initial problems associated to the wave equation, plate equation, IBq equation and more. The framework used in this work is supported on the energy method in the Fourier space combined with the integrability of functions of type $|y|^{-\alpha} f(y)$ for suitable $f(y)$ depending on the symbols $P_i(y)$ $(i = 1, 2, 3)$ and the Haraux-Komornik Lemma.

Critical exponent for damped waves with nonlinear memory

Marcello D'Abbicco
University of Sao Paulo, FFCLRP - FAPESP JP, Brazil
m.dabbicco@gmail.com

In this talk we study the influence of a nonlinear memory

$$F(t,u) = \int_0^t (t-s)^{-\gamma} |u(s,x)|^p ds, \quad \gamma \in (0,1),$$

with $p > 1$, on the global existence of small data solutions to

$$\begin{cases} u_{tt} - \triangle u + \mu u_t = F(t,u), & t \geq 0, \quad x \in \mathbb{R}^n, \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x), \end{cases}$$

where $\mu > 0$, in space dimension $1 \leq n \leq 5$. We prove the global existence for $p > \overline{p}(n,\gamma)$, where $\overline{p}(n,\gamma)$ is the critical exponent, i.e. no global weak solution exists for $1 < p \leq \overline{p}(n,\gamma)$ for suitable, arbitrarily small, data in some Sobolev space. We also show how the critical exponent changes if we consider a problem with structural damping

$$\begin{cases} u_{tt} - \triangle u + \mu(-\triangle)^{\frac{1}{2}} u_t = F(t,u), & t \geq 0, \quad x \in \mathbb{R}^n, \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x), \end{cases}$$

and how the stronger dissipation allow us to obtain a global existence result in any space dimension $n \geq 2$.

References
[1] M. D'Abbicco, The influence of a nonlinear memory on the damped wave equation, Nonlinear Analysis 95 (2014), 130–145, doi:10.1016/j.na.2013.09.006.
[2] M. D'Abbicco, A wave equation with structural damping and nonlinear memory, Nonlinear Differential Equations and Applications, to appear, doi:10.1007/s00030-014-0265-2.

Asymptotic behavior for the solutions to damped Navier-Stokes equations

Hermenegildo de Oliveira
FCT - Universidade do Algarve, Portugal
holivei@ualg.pt

In this talk we will consider different Navier-Stokes equations modified by the presence of a damping term. We show in what conditions the solutions of the associated initial and boundary-value problem extinct in a finite time and we study the large time behavior of the solutions as well. We also shall consider the case when anisotropy is present whether in the diffusion or in the damping term.

References
[1] S.N. Antontsev and H.B. de Oliveira, Asymptotic behavior of trembling fluids, To appear in Nolinear Analysis: Real World Applications (2014).
[2] H.B. de Oliveira, Existence of weak solutions for the generalized Navier-Stokes equations with damping. NoDEA Nonlinear Differential Equations Appl. 20 (2013) no. 3, 797-824.

Exponential decay and resonances for Systems of Schrodinger equations

Claudio Fernandez
P.U.C. Chile, Chile
cfernand@mat.puc.cl
M.A.Astaburuaga, R. Coimbra Charao, G.Perla Menzala

Let k be a resonance (scattering frequency) for the operator $H = -\Delta + V(x)$. This means that $\Im k$ is negative and the problem

$$(-\Delta + V(x))\varphi = k^2 \varphi,$$

has a nontrivial outgoing solution. Since the outgoing condition rules out square integrability, it is reasonable to truncate the resonant solution φ in a compact region. One then expects that the evolution of the truncated solution under the Schrodinger equation, will exhibit an exponential behavior, when the time t approaches infinity. We shall review a method, due to R. Lavine, that proves this fact on the half line $[0, \infty)$ and then extend it to systems of Schrödinger equations,

$$-\Delta u + V(x)u + g(x)v = k^2 u,$$
$$-\Delta v + W(x)v + g(x)u = k^2 v.$$

Qualitative properties of maximal entropy solutions for 2D Euler's equation

Juraj Foldes
IMA, University of Minnesota, USA
foldes@ima.umn.edu
Vladimir Sverak

Two dimensional turbulent flows for large Reynold's numbers can be approximated by solutions of incompressible Euler's equation. As time increases, the solutions of Euler's equation are increasing their disorder; however, at the same time, they are limited by the existence of infinitely many invariants. Hence, it is natural to assume that the limit profiles are functions which maximize an entropy given the values of conserved quantities. Such solutions are described by methods of Statistical Mechanics and are called maximal entropy solutions. Nevertheless, there is no general agreement in the literature on what is the right notion of the entropy. We will show that on symmetric domains, independently of the choice of entropy, the maximal entropy solutions with small energy respect the geometry of the domain.

Estimates in $W^{1,\infty}$ for generalized Forchheimer equations in porous media

Luan Hoang
Texas Tech Univeristy, USA
luan.hoang@ttu.edu
Thinh Kieu and Tuoc Phan

We consider generalized Forchheimer (non-Darcy) flows of slightly compressible fluids in porous media. The resulting degenerate parabolic equation for pressure is studied in a bounded domain with time-dependent boundary data. We obtain long-time estimates for the L^∞-norm of the pressure, its gradient and time derivative. Exploiting the special structure of the equation, we combine the De Giorgi and Ladyzhenskaya-Uraltseva iteration techniques with uniform Gronwall-type inequalities. This is joint work with Thinh Kieu and Tuoc Phan.

Asymptotic profiles for strongly damped wave equations

Ryo Ikehata
Hiroshima University, Japan
ikehatar@hiroshima-u.ac.jp

I will talk about a simple method to derive asymptotic profiles for strongly damped wave equations as time goes to infinity. In this case we impose rather stronger conditions only on the initial velocity. The method is classical and self-contained based on the Fourier analysis only. This result will become a kind of partial preciseness of that introduced by Y.Shibata in 2000. The method will be widely applicable to the other evolutionary equations.

Stability of traveling waves for time-delayed reaction-diffusion equations: (II) Critical oscillating waves

Ming Mei
Champlain College & McGill University, Canada
ming.mei8@gmail.com
Xiongfeng Yang and Qifeng Zhang

This talk is devoted to the study on stability of oscillatory traveling waves to a kind of time-delayed reaction-diffusion equations including the Nicholson's blowflies equation. The equation under consideration losses its monotonicity, and its traveling waves are oscillatory when the time-delay r or the wave speed c is large. This causes the study of stability of these non-monotone traveling waves not easy. For all non-critical traveling waves $\phi(x+ct)$ with $c > c_* > 0$, where $c_* > 0$ is the minimum wave speed, these oscillating waves are proved to be exponentially stable by Lin-Lin-Lin-Mei. But the stability on the critical oscillating waves $\phi(x + c_*t)$ are more challenging and still remain open due to some technical reasons. Here, we give a positive answer, that is, all critical traveling waves $\phi(x + c_*t)$ including the critical oscillating wavefronts are time-asymptotically stable, when the initial perturbations around the wavefronts in a certain weighted Sobolev space are small. Furthermore, when the initial perturbations exponentially decay much fast, we derive the algebraic convergence rate $O(t^{-1/2})$ for the solutions to the corresponding critical oscillating waves. Such a rate is optimal in L^∞-sense. The adopted method is the technical weighted energy method, but with some new development so then we can treat the case of the critical oscillating waves. Finally, numerical simulations are also carried out, which further confirm and support our theoretical results.

Resolvent estimates and space-time estimates for the wave equations

Kiyoshi Mochizukii
Emeritus, Tokyo Metropolitan University, Japan
mochizuk@math.chuo-u.ac.jp

The wave equations with small potentials are considered in exterior domain of star-shard obstacles.Based on our resolvent estimates we shall show space time weighted estimates of solutions. Moreover, we treat behaviors of solutions when some dissipative terms are added. In two space-dimensional problem, the resolvent estimates are obtained as a joint work with H. Nakazawa (cf. next talk).

Sharp uniform resolvent estimate for Helmholtz and Schrödinger equations in an exterior domain in $\mathbb{R}^2$ and their applications.

Hideo Nakazawa
Nippon Medical School, Japan
nakazawa-hideo@nms.ac.jp
Kiyoshi Mochizuki

Sharp uniform resolvent estimate for Helmholtz and Schrödinger equations in an exterior domain in $\mathbb{R}^2$ is presented. Our inequality is obtained by noting the weight function which appears in Hardy inequality in an exterior domain in $\mathbb{R}^2$, and the duality in the both sides of our inequality is kept. As applications, some smoothing estimates for time dependent Schrödinger or wave equations are obtained. This is a joint work with Prof. Kiyoshi Mochizuki (Emeritus, Tokyo Metropolitan University).

On the Cauchy problem for weakly coupled system of damped wave equations

Kenji Nishihara
Waseda University, Japan
kenji@waseda.jp

In this talk we consider the Cauchy problem for the weakly coupled system of damped wave equations

$$(P) \quad \begin{cases} u_{tt} - \Delta u + b_1 \cdot u_t = |v|^p, \\ v_{tt} - \Delta v + b_2 \cdot v_t = |u|^q, \end{cases} \qquad (t,x) \in \mathbf{R}_+ \times \mathbf{R}^N.$$

Our main interest is concerning to the critical exponent of p, q. Note that, when $b_1 = b_2 \equiv 1$, the critical exponent is given by

$$A := \max\left(\frac{p+1}{pq-1}, \frac{q+1}{pq-1}\right) \begin{cases} < N/2 & : \text{supercritical}, \\ = N/2 & : \text{critical}, \\ > N/2 & : \text{subcritical}, \end{cases}$$

in the sense that, in the supercritical case (P) has a globa in time solution (u, v) for a small data, while the local solution (u, v) with some data blows up within a finite time in the critical and subcritical cases.

Semi-linear structurally damped evolution models

Michael Reissig
Technical University Freiberg, Germany
reissig@math.tu-freiberg.de
Pham Trieu Duong, Marcello D'Abbicco

The goal of the lecture is to study the Cauchy problem for structurally damped σ-evolution models

$$\begin{aligned} u_{tt} + (-\Delta)^{\sigma} u + (-\Delta)^{\delta} u_t &= f(u, u_t, |D|^{\alpha} u), \\ u(0, x) &= u_0(x), \\ u_t(0, x) &= u_1(x), \end{aligned}$$

where $\sigma \geq 1$, $\delta \in (0, \sigma]$ and $\alpha \in [0, \sigma]$. Our main issue is the global existence (in time) of small data solutions. It is related to determine the critical exponent (of Fujita type). The considerations base on a very precise linear theory. The main tool are Strichartz decay estimates not necessarily on the conjugate line.

Damping by heat conduction in the Timoshenko system: Fourier and Cattaneo are the same

Belkacem Said Houari
Alhosn University, United Arab Emirates
saidhouarib@yahoo.fr
Aslan Kasimov

We consider the Cauchy problem for the one-dimensional Timoshenko system coupled with heat conduction, wherein the latter is described by either the Cattaneo law or the Fourier law. We prove that heat dissipation alone is sufficient to stabilize the system in both cases, so that additional mechanical damping is unnecessary. However, the decay of solutions without the mechanical damping is found to be slower than that with mechanical damping. Furthermore, in contrast to earlier results, we find that the Timoshenko Fourier and the Timoshenko Cattaneo systems have the same decay rate. The rate depends on a certain number, which is a function of the parameters of the system.

Dynamic Boundary Conditions with Memory: Well-Posedness of the Coleman-Gurtin Equation

Joseph Shomberg
Providence College, USA
jshomber@providence.edu
Ciprian Gal

We report some recent advances concerning the well-posedness of the Coleman-Gurtin equation equipped with dynamic boundary conditions that also possess memory relaxation. Weak solutions are obtained from a Galerkin procedure whereby nonlinear terms with arbitrary polynomial growth are defined on the interior of the domain and on the boundary, subject to either classical dissipation assumptions, or to a generalized balance condition. In general, we do not assume the interior and the boundary share the same memory response. Under additional assumptions, we arrive at a formulation for strong solutions, and a hybrid-solution situated between the strong and weak formulations.

Global existence for semilinear wave equations including the blow-up term in four space dimensions

Hiroyuki Takamura
Future University Hakodate, Japan
takamura@fun.ac.jp
Kyouhei Wakasa

We are interested in the fact that the lifespan $T(\varepsilon)$, the maximal existence time, of a classical solution of $\begin{cases} u_{tt} - \Delta u = u^2 \quad \text{in} \quad \mathbf{R}^4 \times [0, \infty), \\ u(x, 0) = \varepsilon f(x), u_t(x, 0) = \varepsilon g(x) \end{cases}$ with a small parameter $\varepsilon > 0$, compactly supported smooth functions f and g, has an estimate $\exp\left(c\varepsilon^{-2}\right) \leq T(\varepsilon) \leq \exp\left(C\varepsilon^{-2}\right)$, where c and C are positive constants depending only on f and g. This result is due to Li & Zhou in 1995 for the lower bound and our previous work, Takamura & Wakasa in 2011 for the upper bound. We note that its importance is quite huge as the problem is the final open part of Strauss' conjecture on semilinear wave equations as well as one of the last open optimality of the general theory for nonlinear wave equations. In this talk, I would like to show you a special equation with u^2+some integral terms admits a global solution. Such integral terms are deeply related to "derivative loss due to high dimensions". This fact may help us to make a criterion to get the global existence in four space dimensions for nonlinear term even with u^2.

Global existence and asymptotic behavior of solutions to the nonlinear beam equation with weak damping

Hiroshi Takeda
Fukuoka Institute of Technology, Japan
h-takeda@fit.ac.jp

In this talk, we consider the Cauchy problem for a nonlinear beam equation with weak damping. First we show the the existence of a global solution and its decay estimates under small initial data condition. Next we derive asymptotic behavior of the global solution, which shows that the estimates obtained here are optimal. Our strategy for the proof is an application of the estimates for the linear solution of the damped beam equation.

Diffusion phenomenon for dissipative wave equations with two non-commuting operators

Grozdena Todorova
University of Tennessee Knoxville, USA
todorova@math.utk.edu
Petronela Radu, Borislav Yordanov

We consider the asymptotic behavior of solutions to dissipative wave equations involving two non-commuting self-adjoint operators in Hilbert space. The main result is that the abstract diffusion phenomenon takes place. We obtain precise estimates for consecutive diffusion approximations and remainder. In this process we use an abstract version of weighted estimates. When the diffusion semigroup has the Markov property, relying on the maximal regularity, we get sharp results. We apply the abstract results to derive optimal decay estimates for dissipative hyperbolic equations with variable coefficients in an exterior domain. Applications to nonlocal settings provide new decay rates for solutions of nonlocal dissipative wave equations.

On diffusion phenomena for the linear wave equation with space-dependent damping

Yuta Wakasugi
Osaka University, Japan
y-wakasugi@cr.math.sci.osaka-u.ac.jp

We consider the diffusion phenomena for the linear wave equation with space-dependent damping. It is well known that if the damping term has a constant coefficient, then the solution behaves like that of the corresponding heat equation as time goes to infinity. On the other hand, it is also known that when the coefficient of the damping term decays sufficiently fast near infinity, the solution behaves like that of the free wave equation. In this talk, we consider the intermediate case, that is, the case where the coefficient of the damping term decays slowly near infinity, and we show that the asymptotic profile of the solution is given by a solution of the corresponding heat equation. This result corresponds to that of J. Wirth, who treated the time-dependent damping cases. The key point of the proof is weighted energy estimates for higher order derivatives, which are based on some recent results by G. Todorova, B. Yordanov, R. Ikehata and K. Nishihara.

Transient growth phenomenon in a parabolic-elliptic chemotaxis system

Michael Winkler
University of Paderborn, Germany
michael.winkler@math.upb.de

We consider variants of the Keller-Segel system of chemotaxis which contain logistic-type source terms and thereby account for proliferation and death of cells. We briefly review results and open problems with regard to the fundamental question whether solutions exist globally in time or blow up. The primary focus will then be on the prototypical parabolic-elliptic system

$$u_t = \varepsilon u_{xx} - (uv_x)_x + ru - \mu u^2, 0 = v_{xx} - v + u,$$

in bounded real intervals. The corresponding Neumann initial-boundary value problem, though known to possess global bounded solutions for any reasonably smooth initial data, is shown to have the property that the so-called *carrying capacity* $\frac{r}{\mu}$ can be exceeded dynamically to an arbitrary extent during evolution in an appropriate sense, provided that μ_0 is sufficiently small. This is in stark contrast to the case of the corresponding Fisher-type equation obtained upon dropping the term $-(uv_x)_x$, and hence reflects a drastic peculiarity of destabilizing action due to chemotactic cross-diffusion, observable even in the simple spatially one-dimensional setting. Numerical simulations underline the challenge in the analytical derivation of this result by indicating that the phenomenon in question occurs at intermediate time scales only, and disappears in the large time asymptotics.

Diffusion phenomena for partially dissipative hyperbolic systems

Jens Wirth
University of Stuttgart, Germany
jens.wirth@mathematik.uni-stuttgart.de

We consider a partially dissipative hyperbolic system with possibly time-dependent coefficients and show that under natural assumptions its solutions behave like solutions to an associated parabolic problem modulo faster decaying terms. This generalizes the well-known diffusion phenomenon for damped waves and gives some further insight into the structure of dissipative hyperbolic systems.

Special Session 61: Enhanced Sampling Techniques in Simulation of Complex Systems

Elena Akhmatskaya, Basque Center for Applied Mathematics - BCAM, Basque Country, Spain
Jesus Maria Sanz-Serna, Universidad de Valladolid, Spain

Effective sampling is the key element for successfully simulating the properties and behavior of complex systems. Large sizes, slow processes, and sophisticated structures typical for such systems make achieving efficient sampling of their possible realizations a challenging task. Faster computer technologies, allowing for longer length simulations, greatly contributed to recent progress in understanding complex systems and processes. However, they alone are not sufficient to tackle these great computational challenges. Recently, many interesting approaches have been developed in an attempt to improve the sampling techniques in complex systems simulation. These include Monte Carlo approaches (e.g. simulated annealing, importance sampling, hybrid Monte Carlo, parallel tempering), harmonic approximations, coarse-graining formulations, multi-scale integrators, transition path sampling, Markov Chain models, new parallel algorithms, and specialized hardware systems. The most successful methods rely on adaptation of novel mathematical ideas to specific applications, on latest developments in computer sciences and engineering innovations. It has become clear that any future progress in the study of complex systems and processes is impossible without the use of novel multidisciplinary approaches, which allow for enhanced sampling and transcend the current limits on imposed spatial and temporal scales. The purpose of this session is to bring together experts in mathematics, computer and natural sciences, statistics, and engineering for exchanging experiences and ideas, in order to combine efforts in developing new methodologies for efficient simulation of complex systems.

Mathematical modeling of latex polymerization processes

Elena Akhmatskaya
Basque Center for Applied Mathematics, Spain
akhmatskaya@bcamath.org
J.M. Asua, S. Rusconi, D. Sokolovski

Modeling the morphology development in latex particles as well as the branching of polymers, are of great interest to practitioners, since they determine the resulting performance of synthetic latex polymers. We present two novel modeling methodologies. While using different levels of detail, both of them are able to reproduce experimental observations. Our high resolution approach for modeling the dynamics of particle morphology development in the composite waterborne systems is based on stochastic dynamics. It utilizes Langevin dynamics for predicting non-equilibrium morphologies for various technological conditions, and uses the generalized shadow hybrid Monte Carlo technique for faster identification of equilibrium morphologies. The model takes into account the effects of phase compatibility and internal viscosity of the particles, and provides a detailed description of the morphology of a single particle. For modeling the branching mechanism in control radical polymerization, we developed the delayed stochastic simulation algorithm, which operates with molecular numbers rather than with individual molecules, as does the high resolution approach. The algorithm has been successfully tested on simple exactly solvable models. The predictions obtained with both approaches closely reproduce experimental results. The possibilities for combining two approaches in the multiscale stochastic modeling method are discussed.

Establishing some order amongst exact approximations of MCMCs

C. Andrieu
University of Bristol, England
c.andrieu@bristol.ac.uk
Matti Vihola

Exact approximation Markov chain Monte Carlo (MCMC) algorithms are a general class of sampling algorithms particularly well suited to Bayesian inference in complex models or computations in statistical physics where some of the quantities involved are intractable. One of the main ideas behind exact approximations consists of replacing intractable quantities required to run standard algorithms, such as the target probability density in a Metropolis-Hastings algorithm, with estimators. Perhaps surprisingly, suitable and implementable approximations turn out to lead to exact algorithms in the sense that they are guaranteed to target the probability distribution of interest without introducing any approximation. In this presentation we present a general framework which allows one to compare, or order, performance measures of two such approximate implementations. We focus in particular on the mean acceptance probability, the first order autocorrelation coefficient, the so-called asymptotic variance and the right spectral gap. The key notion we identify as relevant to our purpose is that of the convex order between random variables, in our case two approximations of the aforementioned quantities required to implement standard algorithms. An important point is that we show that using the variance of such approximations as a means to compare performance is not sufficient whereas the convex order turns out to be natural and powerful. Indeed the literature concerned with the convex order is vast and we detail some examples by identifying extremal distributions within given classes of approximations, showing that averaging replicas improves performance in a monotonic fashion and that stratification may improve performance–it is in fact that case in all situations for the standard implementation of the Approximate Bayesian Computation (ABC) method. We also point to other applications and future developments.

Accurate actions for Path Integral Monte Carlo simulations

Jordi Boronat
Technical University of Catalonia, Spain
jordi.boronat@upc.edu
G. Ferrer, F. Casas, J. Casulleras

The Path Integral Monte Carlo (PIMC) method allows for calculation of many-particle properties of quantum systems at low temperatures. By mapping the action to a problem of classical interacting polymers one is able to calculate energetic and structural properties of the system using Metropolis Monte Carlo. However, the calculation becomes more and more difficult when the temperature is progressively decreased. To reduce this drawback it is very important to work out actions of higher order, beyond the primitive approximation. We will present results obtained using higher order expansions and show the efficiency achieved using different approximate schemes. Finally, we will comment on the symmetrization of the action when dealing with boson particles, which implies sampling in the permutation space.

On the performance of explicit sampling methods with variable mass matrices

Mari Paz Calvo
IMUVA. Universidad de Valladolid, Spain
maripaz@mac.uva.es
J. M. Sanz-Serna

Girolami and Calderhead suggested the use of variable masses to improve the performance of Monte Carlo algorithms to sample from a target density. Recently explicit methods for simulating diffusions whose generator is self-adjoint with respect to a known density have been proposed in the context of Molecular Dynamics. Furthermore, these methods do not require the evaluation of the divergence of the mass matrix. In this talk we investigate the applicability and efficiency of these methods when dealing with sampling problems arising in Bayesian Statistics.

Integrators for the hybrid Monte Carlo method (HMC)

Cédric M. Campos
ImUVA (Inst. Matemáticas UVA), Spain
cedricmc@icmat.es
J M Sanz-Serna

I shall present numerical comparisons between several HMC numerical integrators recently suggested in the literature. The problems used for benchmarking include a wide range of degrees of freedom.

Multiple-time-stepping generalized hybrid Monte Carlo methods

Bruno Escribano
Basque Center for Applied Mathematics, Spain
bescribano@bcamath.org
Elena Akhmatskaya, Sebastian Reich, Jon M. Azpiroz

The simulation of large molecular systems can take advantage of their inherent multi-scalar nature. The different components of the interaction potentials used in molecular dynamics vary greatly both in the spatial and temporal range. Force splitting or multiple-time-stepping methods improve the computational efficiency by using different integration step-sizes for updating each component, i.e. longer step-sizes for the slower, long-range interactions. This allows for longer simulated times at the expense of introducing numerical instabilities due to resonances in the slow forces. Here we present a multi-time-stepping method adapted to hybrid Monte Carlo, MTS-GSHMC. The new method combines the high sampling efficiency and stability of GSHMC with a force splitting method that includes the mollification of the slow forces. For benchmarking it has been implemented on the open-source software ProtoMol and tested on a water droplet and protein systems. Initial results show improved performance compared to the similar best performed multi-step method through achieving larger time-steps and enhanced sampling efficiency without compromising the accuracy of dynamical measurements as well as the numerical stability.

Geometric sampling: an efficient method to analyze uncertainty in very complex systems

Juan Luis Fernandez Martinez
Universidad de Oviedo, Spain
jlfm@uniovi.es

In this talk I will present several methodologies that serve to analyze uncertainty in the solution of very high dimensional nonlinear discrete inverse problems, and specially the case where the forward problem has a very high computational cost. These different methodologies involve model reduction, sparse deterministic sampling, stochastic sampling and local optimization, and have protected under 2 different USA patents.

Simulation based on Lagrangian and Hamiltonian Dynamics on Manifolds

Mark Girolami
University of Warwick, England
m.girolami@warwick.ac.uk

Simulation from complex systems described by intractable probability distributions is at the core of computational Bayesian statistics. This talk will consider the construction of Markov chain based simulation techniques to target a desired measure by exploiting stochastic Lagrangian and Hamiltonian dynamics defined on manifolds with metric G and Levi-Civita connection. This approach to stochastic simulation is illustrated for a number of challenging Bayesian inference problems and the strengths and weaknesses of the methodology are discussed.

Using Langevin dynamics splitting algorithms to reduce discretization bias

Charles Matthews
University of Edinburgh, Scotland
c.matthews@ed.ac.uk
Ben Leimkuhler

Many molecular dynamics sampling algorithms rely on efficiently generating trajectories that sample the canonical (NVT) ensemble. A popular choice for creating suitable trajectories is to use Langevin dynamics to provide a thermostatted dynamics whose solutions are consistent with the desired Boltzmann-Gibbs distribution. These dynamics then require numerical discretization in order to approximate the solution in time, which in turn introduces a bias in the sampling. In this talk, we will discuss some results on the invariant distributions sampled by Langevin dynamics discretization schemes defined through exactly solving parts of the SDE vector field in sequence. We focus on the case where purely configurational sampling of the ensemble is of interest.

Enhanced sampling techniques for spin and biological systems

Yuko Okamoto
Nagoya University, Japan
okamoto@phys.nagoya-u.ac.jp

Conventional Monte Calro and molecular dynamics simulations of spin and biological systems are greatly hampered by the multiple-minima problem, where the simulations tend to get trapped in some of a huge number of local-minimum-energy states. In order to overcome this difficulty, we have been advocating the uses of generalized-ensemble algorithms which are based on non-Boltzmann weight factors. With these algorithms we can explore a wide range of the configurational space. The advantage of generalized-ensemble algorithms such as multicanonical algorithm, simulated tempering, and replica-exchange method lies in the fact that from only one simulation run, one can obtain various thermodynamic quantities as functions of temperature and other parameters of the system. In this talk, I will present the results of our recent applications of generalized-ensemble algorithms to spin and biomolecular systems.

Computing Free Energy Landscapes of Small Molecules Interacting with DNA

Garegin Papoian
University of Maryland, USA
gpapoian@umd.edu
Ignacia Echeverria

DNA molecules are highly charged semi-flexible polymers that are involved in a wide variety of highly dynamical processes such transcription, replication and packing into organized structures that have different degrees of compaction. Characterizing the free energy landscape around DNA molecules is essential to understanding the energetics and kinetics of such processes. We developed a novel three-dimensional umbrella sampling technique for computing the potential of mean force for ligands interacting with DNA. Using this approach we performed molecular dynamics simulations to quantitatively elucidate the thermodynamic basis for 1D-sliding motions of small charge ligands along the DNAs minor groove. We also determined the three-dimensional potential of mean force (3D-PMF) for a Na+ cation and for the arginine analog methyl guanidinium. The computed PMFs show that, even for small ligands, the free energy landscapes are far from smooth.

Improving on the Verlet integrator

J Sanz-Serna
Universidad de Valladolid, Spain
sanzsern@mac.uva.es
F Casas, S Blanes

We construct numerical integrators for Hamiltonian problems that may advantageously replace the standard Verlet time-stepper within Hybrid Monte Carlo and related simulations. Past attempts have often aimed at boosting the order of accuracy of the integrator and/or reducing the size of its error constants; order and error constant are relevant concepts in the limit of vanishing step-length. We propose an alternative methodology based on the performance of the integrator when sampling from Gaussian distributions with not necessarily small step-lengths. We construct new splitting formulae that require two, three or four force evaluations per time-step. Numerical experiments suggest that the new integrators may provide an improvement on the efficiency of the standard Verlet method, especially in problems with high dimensionality.

Compressible Generalized Hybrid Monte Carlo

Robert Skeel
Purdue University, USA
rskeel@purdue.edu
Y. Fang and J.M. Sanz-Serna

One of the most demanding calculations is to generate random samples from a specified probability distribution (usually with an unknown normalizing prefactor) in a high-dimensional configuration space. One often has to resort to using a Markov chain Monte Carlo method, which converges only in the limit to the prescribed distribution. Such methods typically inch through configuration space step by step, with acceptance of a step based on a Metropolis(-Hastings) criterion. An acceptance rate of 100% is possible in principle by embedding configuration space in a higher-dimensional phase space and using ordinary differential equations. In practice, numerical integrators must be used, lowering the acceptance rate. This is the essence of *hybrid Monte Carlo* methods. Presented is a general framework for constructing such methods under relaxed conditions: the only geometric property needed is (weakened) reversibility; volume preservation is not needed. The possibilities are illustrated by deriving a couple of explicit hybrid Monte Carlo methods, one based on barrier-lowering variable-metric dynamics and another based on isokinetic dynamics.

Simultaneous computation of dynamical and equilibrium information using a weighted ensemble of trajectories

Ernesto Suarez
University of Pittsburgh, USA
esuarez@pitt.edu
Steven Lettieri, Matthew C. Zwier, Carsen A. Stringer, Sundar Raman Subramanian, Lillian T. Chong and Daniel M. Zuckerman

Equilibrium formally can be represented as an ensemble of uncoupled systems undergoing unbiased dynamics in which detailed balance is maintained. Many non-equilibrium processes can be described by suitable subsets of the equilibrium ensemble. Here, we employ the "weighted ensemble" (WE) simulation protocol [Huber and Kim, *Biophys. J.*, 1996] to generate equilibrium trajectory ensembles and extract non-equilibrium subsets for computing kinetic quantities. States do not need to be chosen in advance. The procedure formally allows estimation of kinetic rates between arbitrary states chosen after the simulation, along with their equilibrium populations. We also describe a related history-dependent matrix procedure for estimating equilibrium and non-equilibrium observables when phase space has been divided into arbitrary non-Markovian regions, whether in WE or ordinary simulation. In this proof-of-principle study, these methods are successfully applied and validated on two molecular systems: explicitly solvated methane association and the implicitly solvated Ala4 peptide.

Weighted-ensemble Brownian dynamics simulation: Sampling of rare events in non-equilibrium systems

Raul Toral
IFISC (CSIC-UIB), Spain
raul@ifisc.uib-csic.es
Justus A. Kromer, Lutz Schimansky-Geier

Rare events are ubiquitous in many biological, chemical and physical processes. Whereas the density of states is known in systems at thermal equilibrium, interesting phenomena often occur in non-equilibrium systems. Unfortunately, many such problems are inaccessible to analytic methods. Therefore computer simulations are a widely used tool to estimate the density of states or transition rates between them. Since standard Brownian dynamic simulation provides computational costs that are inversely proportional to the state's probability, specialized methods have to be used to adequately sample rare events, i.e. states with low probability or low transition rates. We have developed an algorithm [J . A. Kromer, L. Schimansky-Geier, R. Toral Physical Review **E87**, 063311 (2013)], based on the previously developed weighted-ensemble (WE) Brownian dynamics simulations that allows one to calculate the stationary probability density function (SPDF) as well as transition rates between particular states. Like in WE simulations the space of interest is divided into several subregions and the probability for finding the system in them is calculated by generating equally weighted walkers in each region. By moving to the underlying dynamics, the walkers transport probability between the subregions. Thus, WE methods are usually applied to systems of Brownian particles moving in a potential landscape. Our algorithm is based on WE Brownian dynamic simulations, but uses a uniform distribution of walkers within each subregion. Our method outperforms Brownian dynamics simulation by several orders of magnitude and its efficiency is comparable to weighted-ensemble Brownian dynamic simulations in all studied systems and lead to impressive results in regions of low probability and small rates. As an example, we show in the first figure the pdf computed for the classic double well potential $U(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$ as well as, in the second figure, the verification of Kramers law for the probability current. Note that the vertical scale in both cases shows the high efficiency of the method in sampling low-probability events, as we are able to sample correctly events with probability of the order of 10^{-300} and currents of the order of 10^{-300}.

Efficient non-adiabatic excited state dynamics simulations in extended molecular systems

Sergei Tretiak
Los Alamos National Laboratory, USA
serg@lanl.gov

Modelling of non-adiabatic dynamics in extended molecular systems and solids is a next frontier of atomistic electronic structure theory. The underlying numerical algorithms should operate only with a few quantities (that can be efficiently obtained from quantum chemistry), provide a controlled approximation (which can be systematically improved) and capture important phenomena such as branching (multiple products), detailed balance and evolution of electronic coherences. This talk will overview recently developed theoretical methodologies applicable for simulating large molecules underlying an efficient Non-Adiabatic Excited State Molecular Dynamics (NA-ESMD) framework incorporating non-adiabatic quantum transitions. Our calculations of several molecular systems show intricate details of photoinduced vibronic relaxation and identify the conformational degrees of freedom leading to ultrafast dynamics and energy transfer. This theoretical modeling allows us to understand and to potentially manipulate energy transfer pathways in molecular materials suitable for solar energy conversion.

Using sketch-map coordinates to bias molecular dynamics simulations

Gareth Tribello
Queen's University Belfast, No Ireland
g.tribello@qub.ac.uk

Atomistic simulation is now routinely used to shed light on the atomic scale mechanisms that underlie experimentally-observed phenomena. This approach is useful because statistical mechanics tells us that there is a relationship between the probability of adopting a particular microscopic configuration and the free energy of the corresponding state. This is both a blessing and a curse - on the one hand the ability to calculate free energies is what makes these sorts of simulations useful. At the same time, however, because it is unlikely that the system will adopt the high-energy configuration at the top of an energetic barriers it is unlikely that the system will cross such barriers during a short simulation. A large number of algorithms have been put forward to solve this problem many of which are based on the adding of a bias that forces the system to adopt particular configurations. There are many ways in which this bias can be generated but for the most part the bias is a function of a small number of collective variables (CVs). In the majority of the applications of these methods CVs are selected based on what the simulator knows about the chemical reaction/physical process under study, which works well in many cases but rather poorly in others. As such we have, over the past few years, tried to develop an alternative approach in which one tries to build a low dimensional representation that describes the spatial relationships between frames taken from a short trajectory. This representation is constructed using an algorithm called sketch-map that is itself based on the multidimensional scaling algorithm. In this talk I will explain how we adaptively construct a bias as a function of such collective coordinates and how such biases can be used to enhance sampling. I will then provide examples where this technique has been used to enhance the sampling on a model potential energy surface and to bias the sampling of a short protein model.

Special Session 62: Mathematical Models of Cell Migration, Tumor Growth and Cancer Dynamics

Yangjin Kim, Konkuk University, Korea

Cancer is a complex, multi-scale process, in which genetic mutations occurring at a sub-cellular level manifest themselves as functional changes at the cellular and tissue scale. The main aim of this session is to discuss current stages and challenges in modeling tumor growth and developing therapeutic strategies. Specific goals of the session include: (i) to analyze both computational and analytical solutions to mathematical models from tumor modeling (ii) to discuss creative ways of laboratory experimentation for better clinical diagnosis (iii) to improve our biochemical/biomechanical understanding of fundamental mechanism of tumor growth such as analysis of signaling pathways in relative balances between oncogenes and suppressors. Both the immediate microenvironment (cell-cell or cell-matrix interactions) and the extended microenvironment (e.g. vascular bed) are considered to play crucial roles in tumour progression as well as suppression. Microenvironment is known to control tumor growth and cancer cell invasion to surrounding stromal tissue. However, it also prohibits therapeutics from accessing the tumor cells, thus causing drug resistance. Therefore, a thorough understanding of the microenvironment would provide a foundation to generate new strategies in therapeutic drug development. At the cellular level, cancer cell migration is a main step for metastasis and further progression of cancer and metastasis in a given microenvironment. Thus, understanding of cell motility under the control of signal transduction pathways would improve technical and specific advances in cancer therapy by targeting the specific pathways that are associated with the diseases. Analysis of mathematical models would identify fundamental (abstract) structure of the model system and shed a light on our understanding of tumor growth in the specific host tissue environment and biochemical and biomechanical interactions between players in cancer progression. More comprehensive multi-scale (hybrid) models can be used to meet the needs of designing patient-specific agents. The focus of this session is threefold: (a) to present mathematical models of tumor growth and analysis of the models (b) to discuss therapeutic strategies for curing the disease and to showcase mathematical models incorporating mechanical aspects of cancer cell movement and clinical implications (c) see recent development of cell-mechanical aspect of cell-ECM interactions.

Emergence of microstructural patterns and morphological changes in skin cancers

Martine Ben Amar
Ecole Normale Superieure,Paris, France
benamar@lps.ens.fr

Current diagnostic methods for skin cancers are based on some morphological characteristics of the pigmented skin lesions, including the geometry of their contour. We develop a model for the early growth of melanoma accounting for the biomechanical characteristics of the tumour micro-environment as well as the spatial distribution of tumour cells and diffusing molecules in a three-dimensional multiphase model. It incorporates general cell-to-cell mechanical interactions, a dependence of cell proliferation on contact inhibition, as well as a local diffusion of nutrients and inhibiting molecules. We derive a 2D model in a lubrication limit accounting for the thin geometry of the epidermis. Comparing the theoretical results with a large amount of clinical data we show that our predictions describe accurately both the morphology of melanoma observed in vivo and its variations with the tumour growth rate. We also explain the presence of microstructures (dots, nests) sparsely distributed within the tumour lesion. Finally, taking into account the skin structure in glabrous parts allows to recover the patterns in acral melanoma or naevi and I will discuss the biological implications that we can deduce from modelling such as cancer stem cells localization and senescence.

Tumor-Immune cells competition under the action of immunoboosting

Marcello Delitala
Politecnico di Torino, Italy
marcello.delitala@polito.it
Tommaso Lorenzi, Matteo Melensi

How immunotherapies affects the evolutionary dynamics of cancer cells? Can we slow down cancer evolution by using immune boosters? Bearing these questions in mind, we present a mathematical model of cancer-immune competition under immunotherapies. The model consists of a system of structured equations for the dynamics of cancer cells and activated T-cells. Numerical results suggest that the selection of proper infusion schedules may play a key role in the success of anti-cancer therapies. In particular, we highlight how cancer evolution can be effectively slowed down by immunotherapeutic protocols relying on successive infusions of agents that boost the proliferation of activated T-cells and agents that enhance immune memory.

Interacting scales in modeling cancer and disease dynamics

Marisa Eisenberg
University of Michigan, USA
marisae@umich.edu

Cancer often involves processes at multiple scales, which can affect both how we build models of these systems and the data sets needed to estimate model parameters. In this talk I will discuss some examples using identifiability and parameter estimation to understand cancer cell dynamics, including examining the interactions of human papillomavirus (HPV) transmission dynamics with carcinogenesis and population cancer trends.

Models of Solid Cacner Chemotherapy

Harsh Jain
Florida State University, USA
hjain@fsu.edu

In this talk, we will present delayed and ordinary differential equation models of periodic chemotherapy targeting solid tumors. Necessary and sufficient conditions for stability of the cancer free equilibrium are derived. The existence of numerically (and experimentally) observed periodic solutions is investigated using tools from functional analysis and floquet theory. A number of open problems regarding these results in a more general setting will be presented. We will also discuss the problem of parameter identifiability for some of our models. Finally, we will present a clinical application of the model, by applying it to the treatment of ovarian cancers with combination chemotherapy. The drugs considered include: platinum based compounds that induce cell death by inflicting DNA damage; taxols that target cells undergoing mitosis; and small molecule cell death inducers. The model is calibrated versus in vitro experimental results, and is then used to predict optimal doses and administration time scheduling for the treatment of a tumor growing in vivo.

A mathematical model for microRNA in lung cancer

Hye-won Kang
University of Maryland, Baltimore County, USA
hwkang@umbc.edu
Melissa Crawford, Muller Fabbri, Gerard Nuovo, Michela Garofalo, S. Patrick Nana-Sinkam, Avner Friedman

Lung cancer is the leading cause of cancer-related deaths. Lack of early detection and the limited options for targeted therapies are the main factors to contribute to these statistics. MicroRNAs represent a class of non-coding RNAs that regulate genes and may serve as both diagnostic and prognostic biomarkers in lung cancer. Based on the experimental data, two microRNAs, miR-9 and let-7, are dysregulated in non-small cell lung cancer (NSCLC) and this feature may be helpful to identify lung cancer. In this talk, I will suggest a key signaling pathway involving two microRNAs and introduce a mathematical model using a system of differential equations. Simulations of the model demonstrate that EGFR and Ras mutations in NSCLC result in miR-9 upregulation and let-7 suppression. By putting random perturbation on microRNAs using stochastic differential equations, I can conclude that the signaling pathway is somewhat robust against random input into miR-9 and more strongly robust against random input into let-7.

Mathematical modeling of breast cancer

Hyunji Kang
Konkuk University, Korea
gagul0618@gmail.com
Yangjin Kim

Ductal carcinoma in situ (DCIS) is an early stage noninvasive breast cancer that originates in the epithelial lining of the milk ducts, but it can evolve into comedo DCIS and ultimately, into the most common type of breast cancer, invasive ductal carcinoma. Understanding the progression and how to effectively intervene in it presents a major scientific challenge. The extracellular matrix (ECM) surrounding a duct contains several types of cells and several types of growth factors that are known to individually affect tumor growth, but at present the complex biochemical and mechanical interactions of these stromal cells and growth factors with tumor cells is poorly understood. Here we develop a mathematical model that incorporates the cross-talk between stromal and tumor cells and focus on the EGF and TGF-beta signaling pathways. We show how up- or down-regulation of components in these pathways affects cell growth and cancer invasion.

How to eradicate glioma cells: can a radical idea lead to a new therapeutic strategy?

Yangjin Kim
Konkuk University, Korea
ahyouhappy@konkuk.ac.kr
Gibin Powathil, Dumitru Trucu, Sean Lawler, Mark Chaplain

Glioblastoma is the most aggressive type of brain cancer with the median survival time of one year. The therapeutic control of glioblastomas depends critically on the spatial location within and outside the tumour, and intracellular responses of the individual tumour cells such as cell cycle and activating motility module. These responses are influenced by external biophysical signals from other cells as well biochemical signals such as oxygen and glucose. A particular microRNA, miR-451 and its downstream signaling molecules, AMPK complex and mTOR, are known to determine a balance between rapid proliferation and aggressive invasion in response to metabolic stress in the harmful microenvironment. We propose a multi-scale mathematical model of glioblastoma invasion. We study the effects of the microenvironment on regulation of the miR-451-AMPK-mTOR system and cell cycle, and, more importantly, tumour treatments. The model identifies a key mechanism behind the molecular switches between proliferative phase and migratory phase in response to metabolic stress and biophysical interaction between cells. We show how up- or down-regulation of components in these pathways affects the key cell fate, proliferation and migration, and cell cycle.

Modeling effects of viral burst size and immunostimulatory capability of anti-cancer virus treatment

Peter Kim
University of Sydney, Australia
pkim@maths.usyd.edu.au
Joseph Crivelli, Joanna Wares, Il-Kyu Choi, Chae-Ok Yun

Recent experiments with engineered oncolytic adenovirus have caused substantial reduction in growth rates of tumors in mice. We develop mathematical models based on data from four different treatments: (Ad) oncolytic adenovirus, (Ad/4-1BBL) Ad virus co-expressing the molecule 4-1BBL, (Ad/IL-12) Ad virus co-expressing the cytokine IL-12, and (Ad/4-1BBL/IL-12) Ad virus co-expressing both 4-1BBL and IL-12. By fitting time series data of tumor growth to mathematical models, we attempt to elucidate the underlying cancer-virus and cancer-immune dynamics to clarify the strengths and limitations of oncolytic virotherapy and suggest improved methods of treatment.

Using modeling, we consider how existing oncolytic adenoviruses can be used to (1) rapidly kill the tumor with a goal of complete elimination or (2) maintain the tumor long-term at low levels. We also consider the possibility of using two specialist viruses, Ad/4-1BBL and Ad/IL-12, instead of the more general virus, Ad/4-1BBL/IL-12. We also show how increases in viral burst size or immunostimulatory capability could affect treatment outcomes.

Tumor Heterogeneity and its Effects on Optimal Drug Administration

Urszula Ledzewicz
Southern Illinois University, USA
uledzew@siue.edu
Heinz Schaettler

Tumor cells typically are genetically highly unstable and as a response to mutations, they frequently consist of heterogeneous agglomerations of various cell populations that exhibit a wide range of sensitivities towards particular chemotherapeutic agents. However, in response to different growth and apoptosis rates as well as increasing tumor cell densities, specific traits become dominant. We consider a mathematical model for cancer chemotherapy with a single chemotherapeutic agent for three distinctly separate levels of drug sensitivity and analyze the dynamic properties of the system under metronomic (continuous low-dose) chemotherapy. More generally, the optimal control problem of minimizing the tumor burden over a prescribed therapy interval is considered. Interestingly, when several levels of drug sensitivity are taken into account in the model, lower time-varying dose rates become a viable option. For simpler models that only distinguish between sensitive and resistant subpopulations, this only holds once a significant residuum of resistant cells has developed. For heterogeneous tumor populations, a more modulated approach that varies the dose rates of the drugs may be more beneficial than the classical maximum tolerated dose approach pursued in medical practice.

Stochastic Computations for Reaction Systems

Chang Hyeong Lee
Ulsan National Institute of Science and Technology, Korea
chlee@unist.ac.kr

In this talk we consider general reaction systems in which various reactions occur between biological or chemical species. To investigate the dynamics of the reaction systems such as genetic networks, enzymatic reaction systems and epidemic compartment models, researchers utilize mathematical models based on the principle of mass-action kinetics, Michaelis-Menten kinetics or other kinetics. If a reaction system is characterized by the interactions of a small number of species, a stochastic modeling should be used to describe the dynamics of the system due to the intrinsic fluctuations. We first introduce mathematical background for describing the reaction systems and then we present recent stochastic computational methods for time-evolution of the reaction networks.

A mathematical model of glioma invasion based on the role of myosin II

Wanho Lee
National Institute for Mathmatical Sciences, Korea
wlee@nims.re.kr
Sookkyung Lim, Yangjin Kim

In this talk, we present a numerical simulation of glioma invasion using a two-dimensional hybrid method under physical constraints in the given microenvironment via the regulation of myosin II in response to chemoattractants. Gliomas are malignant tumors that are commonly observed in primary brain cancer. Experimentally it was known that glioma cells can use myosin II to migrate and invade surrounding brain tissue when injected into mouse brain. We investigated a mathematical model based on cell mechanics which predicts how glioma cells infiltrate the brain tissue under the complex biochemical and biomechanical signals. The model also predicts the myosin II plays an essential role in regulating the cell migration through fixed glial normal cells in the brain.

Choindroitinase ABC I-mediated enhancement of oncolytic virus spread and anti tumor efficacy: Mathematical model and numerical simulation

Hyun Geun Lee
Institute of Mathematical Sciences, Ewha Womans University, Korea
leeh@korea.ac.kr
Yangjin Kim, Nina Dmitrieva, Junseok Kim, E. Antonio Chiocca, Balveen Kaur, Avner Friedman

Oncolytic viruses are genetically engineered viruses that are designed to kill cancer cells while doing minimal damage to normal healthy tissue. After being injected into a tumor, they infect cancer cells, multiply inside them, and when a cancer cell is killed they move on to spread and infect other cancer cells. Chondroitinase ABC (ChaseABC) is a bacterial enzyme that can remove a major glioma ECM component, chondroitin sulfate glycosoamino glycans from proteoglycans without any deleterious effects in vivo. It has been shown that ChaseABC treatment is able to promote the spread of the viruses, increasing the efficacy of the viral treatment. In this talk we develop a mathematical model to investigate the effect of the Chase ABC on the treatment of glioma by oncolytic viruses (OV). We show that the model's predictions agree with experimental results for a spherical glioma. We then use the model to test various treatment options in the heterogeneous microenvironment of the brain. In particular the model predicts that separate injections of OV, one into the center of the tumor and another outside the tumor will result in better outcome than if the total injection is outside the tumor.

The role of CD200-CD200R in cancer immunoediting

Kang-Ling Liao
Mathematical Biosciences Institute, The Ohio State Univeristy, USA
liao.92@mbi.osu.edu
Xue-Feng Bai, Avner Friedman

CD200 is a cell membrane protein that interacts with CD200 receptor (CD200R) of myeloid lineage cells. During tumor initiation and progression, CD200-positive tumor cells can interact with M1 and M2 macrophages through CD200-CD200R-compex to silence macrophages. However, the functions of M1 and M2 cells in tumor growth are different, so CD200-CD200R has been shown to have apparently two contradictory experimental results in tumor growth: inhibition and promotion. We used a system approach to determine the combined effect of CD200-CD200R interaction on tumor proliferation by developing a mathematical model. We explained why these two opposite experimental results can both take place depending on the "affinity" of M1 and M2 macrophages to form the complex CD200-CD200R with tumor. If M1 cell has more affinity than M2 cell, then CD200-CD200R promotes tumor growth and block CD200-CD200R could be a good therapy. However, if M2 cell has more affinity than M1 cell, then we have opposite result that block CD200-CD200R will only increase tumor growth. Our results help understanding the complexities of tumor microenvironment.

Biomechanical models of the developing colorectal crypt

Martin Nelson
Nottingham Trent University, England
martin.nelson@ntu.ac.uk

The colon's epithelial lining exhibits a number of invaginations into the underlying tissue, called the crypts of Lieberkühn. Housing stem cells at their bases, these crypts play an essential role in the maintenance of the epithelium, facilitating a complete renewal every 5–6 days. Disruptions to the healthy crypt shape and cellular dynamics are strongly linked to the onset of colorectal cancer. Formation of the crypts is known to occur approximately seven days after birth in mice; prior to this the intestinal wall is smooth. However, the processes that underlie colorectal crypt formation are not conclusively understood. This study investigates one potential mechanism for crypt formation: that in the developing intestine, proliferation and growth of epithelial cells generate compressive stresses within the epithelium, resulting in buckling instabilities, which initiate crypt formation. We present a simple cell culture experiment to validate the proposed mechanism, and describe associated biomechanical models for epithelial buckling based upon Euler–Bernoulli beam theory (in 1D) or von Kármán plate theory (in 2D). We demonstrate the range of attainable buckling patterns in each model, and show how these are affected by varying adhesion to underlying tissue, the mechanical properties of the epithelium, and patterning of cellular growth.

Mathematical modeling and simulation of tumor immune response

Shinji Nakaoka
RIKEN, Japan
snakaoka@rcai.riken.jp

The immune system plays a major role in protecting our body against invasion of pathogens (antigens) such as bacteria and viruses. A T cell population is a major type of immune cell which is educated in the thymus to discriminate self and non-self antigens. Antigen specific elimination of pathogens is known as the adaptive immune response. Tumor cells are initially given risen from a normal tissue that is not recognized as an enemy by immune cells. Moreover, immune responses are maintained by complex interactions among immune cells via secretion of cytokines. For these reasons, immune responses against tumor at the initial phase are genetically ineffective. *Ex vivo* boosting of a particular type of immune cells is therefore an essential therapeutic manipulation to initiate effective and sustained immune responses against tumor cells. We formulate mathematical models that describe complex positive and negative interactions among immune cells against tumor cells. We investigate effects of *ex vivo* boosting of immune cells by mathematical analysis and numerical simulations. In the symposium, we will show our theoretical classification for the effects of existing immunotherapies such as adoptive T cell transfer and NKT cell adjuvant-based tumor vaccine to tumor killing and growth suppression.

Optimizing delivery of therapeutic agents in pancreatic tumors: a multiscale model

Katarzyna Rejniak
Moffitt Cancer Center & Research Institute, USA
Kasia.Rejniak@moffitt.org

Pancreatic cancers contain significant amounts of stromal tissue that contributes to poor penetration of both metabolites and drug particles. As a result these tumors are hypoxic and standard therapeutic compounds show much lower efficacy in vivo then in laboratory experiments. To test optimal administration schedules of a novel class of drugs that are activated in the tumor hypoxic regions (HAPs, hypoxia-activated pro-drugs), as well as strategies to make the tumors more sensitive to these drugs, we developed a mechanistic computational model carefully parameterized with experimental data. This interdisciplinary approach has been used to investigate complex interplay between HAPs and tumor microenvironmental heterogeneity. We will present a current state of the computational model integrated with experimental data and calibrated to pancreatic tumor xenografts that aim on providing an analytical tool in designing drug properties and drug administration schedules that will optimize drug penetration into the tumor tissue and enhance their therapeutic efficacy.

Understanding the role of cell-substrate interaction in cell and focal adhesion shapes

Magdalena Stolarska
University of St. Thomas, USA
mastolarska@stthomas.edu
Aravind R. Rammohan

Cell shape and substrate properties have important implications in many contexts, and are particularly important in determining stem cell fate. We present a two-dimensional mathematical model and finite element simulations of a biological cell interacting with a deformable substrate and use this model to gain a better understanding of how the mechanical interaction between the cell and substrate affects cell shape and focal adhesion (FA) dynamics during cell spreading. The cell is treated as a hypoelastic actively-deforming continuum and the substrate is modeled as a linearly elastic continuum. The active deformation, captured by the addition of an active rate of deformation tensor, models local cytoskeletal reorganization. FAs connecting the cell and the substrate are modeled as a collection of discrete elastic springs, which can be dynamically added and removed. This model of mechanics is coupled to a model describing the intracellular stress-dependent evolution of the volume fraction of a single FA molecule representing all FA associated proteins. We investigate how cell mechanics and FA evolution should be coupled to obtain observable cell shapes. We also investigate how cell and substrate material properties affect cell and FA shapes, and how FA strength plays a role in FA dynamics.

A hybrid model for cell motility in angiogenesis

Nicoleta Tarfulea
Purdue University Calumet, USA
ntarfule@purduecal.edu

The process of angiogenesis is regulated by the interactions between various cell types such as endothelial cells and macrophages, and by biochemical factors. In this talk, we present a hybrid mathematical model in which cells are treated as discrete units in a continuum field of a chemoattractant that evolves according to a system of reaction-diffusion equations, whereas the discrete cells serve as sources/sinks in this continuum field. It incorporates a realistic model for signal transduction and VEGF production and release, and gives insights into the aggregation patterns and the factors that influence stream formation. In particular, it serves as a tool for investigating tumor-vessel signaling and the role of mechano-chemical interactions of the cells with the substratum.

A minimal physical model for a crawling cell

Adriano Tiribocchi
University of Edinburgh, Scotland
atiriboc@ph.ed.ac.uk
E. Tjhung, D. Marenduzzo, M. E. Cates

Cell motility is of enormous importance to biological functions such as wound healing [1], immune response [2], and cancer [3]. Experiments in vitro [4] shed a significant light into motility mechanisms, in particular on cells crawling on hard surfaces. They focus on the role of actomyosin and support the idea that crawling relies on a combination of actin polymerization, pushing the front of a cell forward, and myosin-induced contractile stress, retracting the rear [2]. We present a simplified physical model of a crawling cell, consisting of a droplet of active polar fluid with contractility throughout, but actin polymerization and polarization confined to a thin layer near the surface. The model shows several distinct shapes and motility regimes for spreading and crawling cells such as fried-egg and lamellipodia. Our work supports the view that cellular motility exploits autonomous physical mechanisms whose functioning does not require continuous regulations via biochemical networks.

References

[1] Basan M. et al.,Proc. Natl. Acad. Sci. USA 110, 2452 (2013).

[2] Bray D., Cell Movements: From Molecules to Motility, 2nd Ed., Garland, New York (2000).

[3] Poincloux R. et al., Proc. Natl. Acad. Sci. USA 108, 1943 (2011).

[4] Kohler S. et al., Nat. Mat. 10, 462 (2011).

Multiscale modelling and analysis of cancer growth and spread: exploring the role of matrix-degrading enzymes and adhesion in invasion

Dumitru Trucu
University of Dundee, Scotland
trucu@maths.dundee.ac.uk
Mark A. J. Chaplain

Most processes in biology and medicine occur over different but inter-connected spatial and temporal scales - from genes to cells to tissues to organs to populations. Known as one of the "hallmarks of cancer", cancer invasion is a complex "multi-scale" phenomenon involving many inter-related genetic, biochemical, and cellular processes at many different spatial and temporal scales that play a crucial role in the overall cancer development. The process of invasion consists of cancer cells secreting various matrix-degrading enzymes, which degrade the surrounding tissue or the extracellular matrix. Combining abnormal proliferation with favourable migratory conditions enabled by altered cell-cell and cell-matrix adhesion characteristics, the cancer cells actively spread locally into the surrounding tissue. As these multiscale phenomena lead naturally to a question concerning the establishment of a fundamental framework that would enable a rigorous analysis and modelling of cancer invasion, in this talk we will first present a novel multiscale modelling framework involving two scales - cell and tissue. This will be accompanied by a multiscale analysis framework, which is based on our new concept of "three-scale convergence". Finally, we will present computational simulations of our multiscale moving boundary model and discuss a number of important fundamental properties that follows.

Special Session 63: Advanced High Order Geometric Numerical Integration Methods for Differential Equations

Sergio Blanes, Universitat Politecnica de Valencia, Spain
Fernando Casas, Universitat Jaume I, Spain

The aim of the proposed special session is to bring together experts in the development, analysis and applications of methods for the numerical integration of differential equations arising in different fields of science and technology. The framework of these schemes is the field of Geometric Numerical Integration, its main goal being to reproduce the qualitative features of the solution of the differential equation which is being discretised, in particular its geometric properties. The motivation for developing such structure-preserving algorithms arises independently in areas of research as diverse as celestial mechanics, molecular dynamics, control theory, particle accelerators physics, and numerical analysis. In this special session we will focus on the analysis and applications of numerical methods within this class that, in addition, provide high accurate approximations.

Geometric integration methods of high order for gradient systems

Miguel Atencia
University of Malaga (Spain), Spain
matencia@ctima.uma.es
Yadira Hernandez-Solano

We propose the higher order extension of numerical methods that preserve the energy-diminishing feature of a system with Lyapunov function. Therefore we focus on ODEs with a Lyapunov function, so that they can be rewritten in the form of a linear gradient, i.e, the right hand side of the ODE consists of the product of a negative-definite matrix and the gradient of the Lyapunov function. Then, the formal construction of discrete gradient methods is straightforward, since they amount to replacing the negative definite matrix with an approximation, and the gradient with a discrete gradient. There is considerable freedom in the choice of these parameters. The technique used in this contribution results from composing a first order discrete gradient method together with its adjoint, yielding a second order method. The basis first order method is based upon the component-wise discrete gradient, whereas the adjoint method is computed in a similar vein, but the order of components is reversed. Finally the proposed method is validated by numerical experiments, showing the main features of the method: it preserves the Lyapunov function, it approximates the continuous system up to second order, and it can be computed explicitly in particular examples.

Splitting methods for Gross-Pitaevskii equations in real and imaginary time

Philipp Bader
Universidad Politecnica de Valencia, Spain
aims@philippbader.de

We study the numerical solution of the cubic nonlinear Schrödinger equation in real and imaginary time. In this work, we introduce two techniques to achieve higher order splittings for this nonlinear equation: modified nonlinear potentials and complex coefficients.

Structure preseving methods for port-Hamiltonian systems

Elena Celledoni
Department of Mathematical Sciences, NTNU, Norway
elenac@math.ntnu.no
Eirik H. Høiseth and Nataliya Ramzina

In this talk we will discuss port-Hamiltonian systems and their numerical approximation with structure preserving numerical methods. We will present a class of methods preserving a discrete form of passivity. We will consider also splitting methods and other geometric integrators and discuss their performance compared to the passivity preserving methods. If time permits, we will present a concrete application where the methods are used.

Superconvergence of Strang splitting for NLS in the d-dimensional torus

Philippe Chartier
INRIA-ENS-IRMAR, France
philippe.chartier@inria.fr
Florian Mehats, Mechthild Thalhammer and Yong Zhang

In this talk we investigate the convergence properties of semi-discretized approximations by Strang splitting method applied to fast-oscillating nonlinear Schrödinger equations. In a first step and for further use, we briefly adapt a known convergence result for Strang method in the context of NLS on T^d for a large class of nonlinearities. In a second step, we examine how errors depend on the length of the period ε, the solutions being considered on intervals of fixed length (independent of the period). Our main contribution is to show that Strang splitting with constant step-sizes is unexpectedly more accurate by a factor ε as compared to established results when the step-size is chosen as an integer fraction of the period, owing to an averaging effect.

Positive geometric integrators for population dynamics

Fasma Diele
IAC-CNR, Bari, Italy
f.diele@ba.iac.cnr.it
Carmela Marangi

Positive Poisson integrators for approximating Lotka-Volterra predator-prey model will be described. They are based on the trasformation of the systems into the field of positive solutions by means of a log trasformation. Splitting and composition schemes for positive approximation of population dynamics with prey logistic growth and Holling II type functional response will be also introduced.

Symplectic Splitting Methods for Dynamical Astronomy

Ariadna Farrés
Universitat de Barcelona, Spain
ariadna.farres@maia.ub.es
S. Blanes, F. Casas, J. Laskar and A. Murua

In this talk we will present new families of splitting methods designed for the numerical integration of near-integrable Hamiltonian systems, with special interest on the planetary N-body problems. We use a Newtonian model for the motion of the planetary system, which can be written as $H_A + \varepsilon H_B$ with an appropriate set of coordinates. We will perform an extensive comparison between different high-order symplectic splitting schemes, in order to determine the best scheme for the particular case of the solar system.

A high-order semiclassical splitting for the Schroedinger Equation for nuclei

Vasile Gradinaru
ETH Zurich, Switzerland
vasile.gradinaru@math.ethz.ch
Sergio Blanes

We note the existence of a systematic way of improving the convergence of the newly proposed semiclassical splitting for solving the semiclassical time-dependent Schrödinger equation. We propose and test algorithms that appear to be up to order three in the semiclassical parameter and order four in the time step.

Symmetric multistep cosine methods for second-order partial differential systems

M. Jesus Moreta
Universidad Complutense de Madrid, Spain
mjesusmoreta@ccee.ucm.es
B. Cano

In this talk we present the main results we have obtained in the study of symmetric and explicit multistep cosine methods, which are a subtype of what are called 'exponential integrators'. They integrate the linear and stiff part of a problem in an exact way, which makes it possible to obtain methods which are explicit and stable at the same time for linearly stiff problems. These methods are efficient numerical solvers when integrating second-order in time partial differential problems subject to periodic boundary conditions, like the nonlinear wave or the beam equation, which have a hamiltonian character. The condition of symmetry allows them to show a good long term behaviour. Besides, we have obtained a systematic way to get high accuracy avoiding instability and resonances.

A new look at geometric integrators for nonautonomous linear systems

Brynjulf Owren
Norwegian University of Science and Technology, Norway
bryn@math.ntnu.no
Håkon Marthinsen

The geometry of nonautonomous Hamiltonian systems are well studied in the literature, essentially they can be understood by adding an extra variable to obtain an autonomous Hamiltonian system. In the last few decades, some very efficient integrators for nonautonomous linear system have been devised by authors like Iserles, Nørsett, Blanes, Casas and others. In this talk we take a closer look at some of these integrators and consider their companion in extended phase space. Thereby we are able to say something about how well the integrators can approximate a time varying energy. We also derive general criteria for this type of methods to be canonical. Numerical experiments will confirm our findings.

Geometric Integration of Polynomial ODEs

Reinout Quispel
La Trobe University, Australia
r.quispel@latrobe.edu.au
Celledoni, McLachlan, McLaren, Owren

We discuss the preservation of geometric properties (e.g. energy and volume) in the discretization of quadratic ODEs using Kahan's method. Time permitting, we will also present a generalization to cubic and higher-order polynomial ODEs.

Adaptive multi-revolution Runge-Kutta methods

Luis Randez
University of Zaragoza, Spain
randez@unizar.es
M. Calvo, J.I. Montijano

Multi-revolution methods are meant to numerically solve highly oscillatory problems or almost-periodic systems. In this talk we present an embedded pair of Dormand and Prince type with orders 5 and 4 of multi-revolution RK formulae that allows us to implement it in an adaptive way. A technique for automatic correction of the period is also proposed. Numerical experiments show the efficiency of these schemes when compared with standard integrators.

Taylor method as a geometric integrator

Marcos Rodriguez
Centro Universitario de la Defensa de Zaragoza, Spain
marcos@unizar.es
Alberto Abad, Roberto Barrio and Fernando Blesa

In this talk we briefly introduce the basics of the Taylor series method, to jump into its capabilities of Computing in Extended precision, compute variational equations, and how to achieve behavior of symplectic methods, preserving integrals of motion such as Energy. We also present several tests comparing to well established codes of Runge Kuta schemes.

Consistent higher order integration of nonsmooth mechanical systems

Thorsten Schindler
Technische Universitaet Muenchen, Germany
thorsten.schindler@mytum.de

We focus on the numerical integration of nonsmooth mechanical systems with impacts and friction. Thereby basically, we split non-impulsive and impulsive force propagation. As a consequence, we are able to consistently represent impulsive periods by using the velocity as independent variable and the impact as respective Lagrange multiplier.Further, we can benefit from higher order integration rules in non-impulsive periods using the contact force as Lagrange multiplier and acceleration, velocity or position as independent variable. We bring together these two types of motion in the concept of time-discontinuous Galerkin methods. Thereby, we assume that trial and test functions for the velocity may have jumps across discretization intervals, and that they are of higher order inside discretization intervals. In this talk, we introduce the idea of this general concept resulting in two families of Runge-Kutta collocation methods and present their theoretical behavior. Then, we focus on half-explicit methods, e.g. containing the explicit trapezoidal rule. We present their numerical behavior within exemplary mechanical systems with impacts and Coulomb friction, e.g. the bouncing ball example or an imperfect slider-crank type mechanism. Thereby, we compare the evaluation of non-impulsive periods on acceleration-force and velocity-force level.

Efficient exponentiation of perturbed matrices by splitting methods

Muaz Seydaoglu
Ege Universty, Turkey
muasey@upvnet.upv.es
P. Bader, S. Blanes

We consider the computation of the flow of partial differential equations that can be discretized as $\partial_t u = (A + \epsilon B)u$. The flow corresponds to the matrix exponential $\exp(A + \epsilon B)$, where A is cheaply diagonalizable (for example by Fourier transforms) and the perturbation B is a dense matrix with $|\varepsilon| \ll 1$. After proper design and application, higher order splitting methods have been found to be superior to standard methods for certain classes of perturbed matrices.

Convergence analysis of high-order commutator-free exponential integrators for non-autonomous linear evolution equations

Mechthild Thalhammer
University of Innsbruck, Austria
mechthild.thalhammer@uibk.ac.at
Sergio Blanes, Fernando Casas

In this talk a stability and convergence analysis of high-order commutator-free exponential integrators for the numerical solution of non-autonomous linear evolution equations is presented. Employing a general framework of self-adjoint operators in Hilbert spaces and sectorial operators generating analytic semigroups, respectively, linear Schrödinger equations involving a time-dependent Hamiltonian operator as well as parabolic initial-boundary value problems are included in the scope of applications. Stability bounds and error estimates with respect to the norms of the underlying function spaces are deduced, and it is shown that commutator-free exponential integrators retain their nonstiff orders of convergence for the considered classes of problems, provided that the problem data satisfy suitable regularity and compatibility requirements. Numerical illustrations for initial-boundary value problems of Schrödinger and parabolic type confirm the theoretical convergence results.

Special Session 64: Traveling Waves and Patterns

Anna Ghazaryan, Miami University, USA
Vahagn Manukian, Miami University, USA

Traveling waves and spatial or spatiotemporal patterns are ubiquitous in natural and man-made systems. In mathematics, these phenomena are encoded as solutions of nonlinear partial differential equations. In this special session, we bring together researchers who study various aspects of nonlinear waves and patterns using analytical and numerical techniques, in particular addressing the existence and the stability issues.

On the Existence of Breathers in Periodic Media: An Approach Via Inverse Spectral Theory

Martina Chirilus-Bruckner
School of Mathematics and Statistics, University of Sydney, Australia
martinac@maths.usyd.edu.au
Clarence Eugen Wayne

Breathers are considered a rare phenomenon for constant coefficient nonlinear wave equations. Recently, a nonlinear wave equation with spatially periodic step potentials has been found to support breathers by using a combination of spatial dynamics, center manifold reduction and bifurcation theory. Via inverse spectral theory for weighted Sturm-Liouville equations, we characterize a larger class of potentials that allow breathers. The research is motivated by the quest of using photonic crystals as optical storage.

Orbital stability of solitary waves of moderate amplitude in shallow water

Anna Geyer
Universitat Autonoma de Barcelona, Austria
annageyer@mat.uab.cat
Nilay Duruk-Mutlubas

We study the orbital stability of solitary traveling wave solutions of an equation for surface water waves of moderate amplitude in the shallow water regime. Our approach is based on a method proposed by Grillakis, Shatah and Strauss (1987), and relies on a reformulation of the evolution equation in Hamiltonian form. We deduce stability of solitary waves by proving the convexity of a scalar function, which is based on two nonlinear functionals that are preserved under the flow.

Coherent Structures in a Population Model for Mussel-Algae Interaction

Anna Ghazaryan
Miami University, USA
ghazarar@miamioh.edu
Vahagn Manukian

We consider a model that describes the interaction of mussel biomass with algae in the water layer overlying the mussel bed. The model consists of a system of two coupled pdes where both the diffusion and the advection matrices in are singular. We use the Geometric Singular Perturbation Theory to capture nonlinear mechanisms of pattern and wave formation in this system.

Spectral stability of two-dimensional gravity-capillary periodic water waves

Mariana Haragus
University of Franche-Comté, France
mharagus@univ-fcomte.fr

We study the stability of two-dimensional gravity-capillary periodic water waves in the case of large surface tension. In this parameter regime, predictions based on model equations suggest that periodic traveling waves are stable with respect to two-dimensional perturbations, and unstable with respect to three-dimensional perturbations which are periodic in the direction transverse to the direction of propagation. In this talk we focus on the first prediction and show that these traveling periodic waves are spectrally stable with respect to co-periodic two-dimensional perturbations.

Anomalous spreading in a system of coupled Fisher-KPP equations

Matt Holzer
George Mason University, USA
mholzer@gmu.edu

We study anomalous spreading in a system of coupled Fisher-KPP equations. When a single parameter is set to zero, the system consists of two uncoupled Fisher-KPP equations which give rise to traveling fronts propagating with the unique, minimal KPP speed. When the coupling parameter is nonzero various behaviors can be observed. Anomalous spreading occurs when one component of the system spreads at a speed significantly faster in the coupled system than it does in isolation, while the speed of the second component remains unchanged. We study these anomalous spreading speeds and show that they arise due to poles of the pointwise Green's function corresponding to the linearization about the unstable homogeneous state. These poles lead to anomalous spreading in the linearized system and come in two varieties – one that persists and leads to anomalous spreading for the nonlinear system and one that does not. We describe mechanisms leading to these two behaviors and prove that one class of poles are irrelevant as far as nonlinear wavespeed selection is concerned. Finally, we show that the same mechanism can give rise to anomalous spreading even when the slower component does not spread.

Grain boundaries of the regularized Cross-Newell equation

Joceline Lega
University of Arizona, USA
lega@math.arizona.edu
Nick Ercolani, Nikola Kamburov

Grain boundaries in extended two-dimensional pattern-forming systems are curves separating regions of slanted rolls. When the angle between the rolls in each of the two regions exceeds a certain threshold, it is known [1,2] that the core of the grain boundary transforms into a chain of convex-concave disclinations. Even though the regularized Cross-Newell (RCN) phase diffusion equation cannot describe this transition all the way to the appearance of defects, it can nevertheless be used to address the question of whether the transition results from an instability of the grain boundary core, and if so, to describe this instability. To this end, we will take full advantage of the existence of an exact grain-boundary solution of RCN and of the variational nature of this equation. I will also show numerical simulations and connect our results to those of Haragus and Scheel [3] on grain boundaries of the Swift-Hohenberg equation.

References

[1] N.M. Ercolani, R. Indik, A.C. Newell, and T. Passot, J. Nonlinear Sci. 10, 223-274 (2000).

[2] N.M. Ercolani and S.C. Venkataramani, J. Nonlinear Sci. 19, 267-300 (2009).

[3] M. Haragus and A. Scheel, European Journal of Applied Mathematics 23, 737-759 (2012).

Universal structure of blowup in hyperbolic systems of conservation laws

Alexei Mailybaev
IMPA, Brazil
a.mailybaev@gmail.com

We develop the renormalization approach for describing the asymptotic form of blowup solutions in systems of conservation laws. This approach is aimed to reveal the universal structure of renormalized solutions in a neighborhood of the blowup point. Here, the time evolution is substituted by the equivalent evolution with increasing scaling parameter, similarly to the renormalization-group (RG) method. A stationary state of the RG equations describes self-similar blowup solutions. We argue that this stationary state is an attractor of the RG equations in the space of analytic functions. The asymptotic stability of this stationary state is what implies universality of the blowup for generic initial conditions. We prove the asymptotic stability for a linearized system and make steps toward the proof of the non-linear stability.

Traveling Waves in Holling-Tanner Model with Diffusion

Vahagn Manukian
Miami University, USA
manukive@miamioh.edu
Anna Ghazaryan

We study diffusive Holling -Tanner model for wide range of parameters from the point of view of existence of traveling waves. We geometrically construct qualitatively different traveling waves that include fronts, periodic wave-trains and solutions that asymptotically connect wave-trains to a constant state.

Open problems on traveling waves in combustion in porous media

Dan Marchesin
Institute for Pure and Applied Mathematics, Brazil
marchesi@impa.br
Lucas Furtado, Grigori Chapiro, Stephen Schecter

We use a very simplified model in which air enters a porous media and burns part of the medium. Analysis indicates the existence of combustion waves. Numerical experiments show that some of them are stable, some are not. Oscillating reactions also occur. Some of these phenomena have been studied before in more realistic models that do not easily allow rigorous analysis. This is not the case for the current model, which is very simple despite considering oxygen, fuel and temperature as possible controlling quantities for the reactions. The phenomena are illustrated by a number of computer simulations; we hope to entice someone in the audience to further rigorous analysis.

Localized Traveling Waves in Heterogeneous Media

Yasumasa Nishiura
Advanced Institute for Materials Research, Tohoku University, Japan
nishiura@wpi-aimr.tohoku.ac.jp
M.Yadome, T.Teramoto, Yuan Xiaohui and Zhijun Gao

Localized traveling wave is the main careers of information and the effect of heterogeneity in the media is of great importance for the understanding of signaling processes in biological and physiological systems. The role of heterogeneity in the media does not remain a perturbative effect, in fact it is well-known that it causes propagation failure such as pinning-depinning, and annihilation. There is, however, another aspect of heterogeneity-induced dynamics, namely it creates a spontaneous generator of pulses without any triggers and external forces. We try to clarify the underlying mechanism for such pulse generators from dynamical system viewpoint. Secondly, we discuss 2D spot dynamics in heterogeneous media. The simplest case is a jump-discontinuity along the line. Unlike the refraction of light, the Snell's law does not hold for dissipative traveling spots, but slightly different formula holds. We also discuss about the difference between monotone tail case and oscillatory tail case.

Stability of multi-bump, blowup solutions of the Ginzburg-Landau equation

Vivi Rottschäfer
Leiden University, Netherlands
vivi@math.leidenuniv.nl
Arjen Doelman, Martin van der Schans

In this talk we study, the stability of radially symmetric blowup solutions of the Ginzburg-Landau equation with respect to radially symmetric and non-radially symmetric perturbations. Upon writing the Ginzburg-Landau equation as a small perturbation of the nonlinear Schrödinger equation, the existence of multi-bump blowup solutions, especially of ring-like solutions, has already been established. So far, the stability of these blowup solutions had only been examined numerically. We use Evans function techniques developed for perturbations of Hamiltonian systems to study the stability of the ring-type solutions depending on the parameters in the system. The application of these methods is not straightforward and they have to be modified to suit our system.

Concatenated traveling waves

Stephen Schecter
North Carolina State University, USA
schecter@ncsu.edu
Xiao-Biao Lin

We consider concatenated traveling wave solutions of reaction-diffusion systems. These are solutions that look like a sequence of traveling waves with increasing velocity, with the right state of each wave equal to the left state of the next. I will present an approach to the stability theory of such solutions that does not rely on treating them as a sum of traveling waves. It is based instead on exponential dichotomies and Laplace transform.

Linear stability of traveling standing waves for the Klein-Gordon equation

Milena Stanislavova
University of Kansas, USA
stanis@ku.edu

We consider the traveling standing waves of the Klein-Gordon equation $u_{tt} - u_{xx} + u - |u|^{p-1}u = 0$ in the whole line case as well as the periodic case. In both situations we study the stability of the waves via recently developed abstract stability criteria. The exact ranges of the wave speeds and the frequencies needed for stability are derived for all solitary waves and for some integer values of p in the periodic case.

Hamilton-Krein instability index with applications to KdV like eigenvalue problems

Atanas Stefanov
University of Kansas, USA
stefanov@ku.edu
Todd Kapitula

We develop a Hamilton-Krein instability index, which allows one to treat KdV like eigenvalue problems. Note that the classical indices, including the Grillakis-Shatah-Strauss theory require a gap between the zero eigenvalue and the continuous spectrum, which we do not. As an application, we completely characterize the stability of travelling waves of certain fractional KdV and BBM versions of the classical models.

Periodic patterns in singularly perturbed reaction-diffusion systems

Frits Veerman
University of Oxford, England
veerman@maths.ox.ac.uk
Arjen Doelman, Björn de Rijk

We prove the existence of periodic solutions in a very general class of singularly perturbed reaction-diffusion systems using geometric singular perturbation theory. The construction method, which allows for a large variety of periodic solutions, is applied to an example system. The linear stability of the constructed solutions is assessed, both in the general and applied context. A rich bifurcation structure emerges; also, the homoclinic limit will be addressed.

Special Session 65: Kinetic Equations: Theory and Applications

Francesco Salvarani, University of Pavia, Italy

The kinetic description plays an increasing role in modelling systems composed by many elementary entities, that exchange between themselves and/or with a host medium some quantities carried by them. It is a crucial tool for the multiscale approach to many applied problems and it provides the correct framework for studying such systems in far-from-equilibrium situations. There is a twofold purpose of this special session. On one hand, it aims to spread the recent advances on the modeling, the analysis and the numerical simulations of kinetic systems. On the other hand, this special session encourages the intedisciplinary interactions in order to find new problems and applications.

The Cauchy problem for the Vlasov-Dirac-Benney equation and related issues in Fluid Mechanics and Semi-classical Limits

Claude Bardos
Laboratoire Jacques Louis Lions, France
claude.bardos@gmail.com
Nicolas Besse

This contribution concerns a one dimensional version of the Vlasov equation dubbed Vlasov Dirac Benney in short VDB where the self interacting potential is replaced by a Dirac mass. Emphasis is put on the relation between the linearized version, the full non linear problem and equations of fluids. In particular the connection with the so called Benney equation leads to new stability results. Eventually the VDB equation appears to be at the cross road of several problems of mathematical physic which have as far as stability is concerned very similar properties.

A hydrodynamic model for electron transport in graphene obtained by entropy maximization

Luigi Barletti
Università degli Studi di Firenze, Italy
luigi.barletti@unifi.it

By using the maximum entropy principle, we formally derive semiclassical, isothermal, Euler-like equations describing fermions (electrons and holes) in a graphene sheet. Some general properties of such equations as well as their asymptotic forms corresponding to particular physical regimes (low and high temperatures, diffusive limit, collimation limit) will be discussed.

On the convergence to equilibrium for degenerate transport problems

Etienne Bernard
IGN / LAREG, France
esteve.bernard@gmail.com
Francesco Salvarani

The long-time behavior of the kinetic linear equations is well-known in the case where the cross-section is bounded from below by a strictly positive constant. In the case where the cross-section vanishes in a portion of a domain – such an equation is called "degenerate" – , the results mentioned above have no obvious extension. The presentation is based on a series of three papers written by the authors on the subject. We show that the geometry of the portion of the domain where the cross-section vanishes is the key feature of the problem. More precisely, we first exhibit an example where the convergence to equilibrium can not be exponential. Then we give a necessary and sufficient condition on the geometry of the vanishing zone so that we have uniform convergence to equilibrium. Eventually, we will discuss the issue of finding an explicit rate of convergence.

A kinetic model of BGK-type for polyatomic gas mixtures

Marzia Bisi
Parma University, Italy
marzia.bisi@unipr.it
Maria José Caceres

In kinetic theory, polyatomic gases may be described as a mixture of monoatomic components, each one characterized by a different energy level. Besides classical elastic collisions, particles are subject to inelastic encounters changing their internal energy state (transforming part of kinetic energy into internal energy or vice versa) and/or to chemical reactions implying also transfer of mass. We derive consistent BGK approximations of the cumbersome Boltzmann equations for polyatomic gas mixtures, where parameters of the Maxwellian attractors are uniquely determined as functions of the main macroscopic fields (densities, velocities, and temperatures) of the interacting gases, by imposing preservation of exact equilibria and collision invariants of the original kinetic approach. Numerical tests of the proposed model are also shown for illustrative purposes.

Mathematical Models for Opinion Dynamics

Laurent Boudin
UPMC & Inria, France
laurent.boudin@upmc.fr

About a decade ago, mathematicians started to study opinion dynamics models. Between various viewpoints, we shall mostly focus on kinetic models for opinion formation and discuss various phenomena affecting the evolution of opinions inside a closed community.

Instantaneous filling of the vacuum for the Boltzmann equation

Marc Briant
University of Cambridge, England
m.j.briant@maths.cam.ac.uk

Imagine a box divided into two compartments thanks to a wall, one compartment being empty and the other one containing gas. What would happen if you take the wall out? A mathematical answer to that is the fact that immediately the gas will fill the entire box. In this talk we derive this result for the full Boltzmann equation in the torus and C^2 convex bounded domains in general, and also give an explicit exponential lower bound for the solution.

Duality estimates and applications to reaction-diffusion equations

José A. Cañizo
University of Birmingham, British Virgin Islands
j.a.canizo@bham.ac.uk
Laurent Desvillettes, Klemens Fellner

We present a refined duality estimate for parabolic equations. This estimate entails new results for systems of reaction-diffusion equations, including smoothness and exponential convergence towards equilibrium for equations with quadratic right-hand sides in two dimensions. For general systems in any space dimension, we obtain smooth solutions of reaction-diffusion systems coming out of reversible chemistry under an assumption that the diffusion coefficients are sufficiently close one to another.

Steady States for a Class of Kinetic Models Arising on Population Biology

Maria Carvalho
University of Lisbon, Portugal
mcarvalh@cii.fc.ul.pt

We study a Boltzmann type equation for densities on the unit circle arising from a binary interaction model coming from population dynamics. The uniform density is always invariant, but in certain parameter ranges, it is not stable. We construct non-uniform invariant densities and study their stability. This is joint work with E. Carlen, P. Degond and B. Wennberg.

Kinetic modelling and study of a dusty rarefied gas.

Frederique Charles
UPMC Paris 6, France
charles@ann.jussieu.fr

The evolution of dust particles in a rarefied gas can be described through the coupling of two Boltzmann-type equations. We propose here a modelling of the collisional operators describing the gas-particles interactions and a mathematic study of the system in a spacially homogeneous context.

The viscous quantum hydrodynamic model for semiconductors

Michael Dreher
Heriot-Watt University, Edinburgh, Scotland
M.Dreher@hw.ac.uk
Li Chen

We consider the transport of charged particles in a semiconductor. Starting from a Wigner equation with a collision term of Fokker-Planck type, a moment method can be employed to derive partial differential equations for several macroscopic quantities. Assuming that the density function is near the thermal equilibrium, a closure of the system becomes possible, and the viscous quantum hydrodynamic model is obtained. For this model, I present several results on local correctness, semigroup properties, stability, boundary layers, numerical simulations. This is joint work with Li Chen.

Two-Scale Numerical Methods for Kinetic Equations

Frenod Emmanuel
Universite de Bretagne-Sud, France
emmanuel.frenod@univ-ubs.fr

In this talk, I will introduce the concept of Two-Scale convergence on which Two-Scale Numerical methods are based. Then, I will show on examples related to charged particle beam and of tokamak plasma how such methods can be built. I will also illustrate their accuracy and robustness.

On coagulation-fragmentation models

Klemens Fellner
University of Graz, Austria
Klemens.Fellner@uni-graz.at
J. A. Canizo, J. A. Carrillo, L. Desvillettes

We consider existence, large-time behaviour and fast-reaction limits of coagulation-fragmentation models with spatial diffusion. As continuous-in-size model, we study Smoluchowski's equation with constant coefficients. Discrete-in-size models are considered with more general coefficients. The diffusion coefficients are allowed to degenerate in size. The main techniques include a-priori estimates based on the dissipation of an entropy functional, entropy entropy-dissipation methods, moment bounds and duality methods.

Selfsimilar solutions to Smoluchowski's coagulation equation in singular and non singular cases

Marco Fontelos
ICMAT, Spain
marco.fontelos@icmat.es
Giancarlo Breschi

Smoluchowski's coagulation equation is a mean field model describing cluster growth that has been used in a very wide set of applications, ranging from physical chemistry to astrophysics and population dynamics. For a good introductory survey, see [?] and the references therein. Many dynamical properties depend on the integration kernel $K(x, y)$, which determines the reactivity between couples of masses. It is known that, for certain kernels such as $K_* = xy$, a singularity in finite time occurs: the solution develops a heavy tail in finite time and the total mass is no longer conserved. This phenomenon is called gelation and represents the formation of a cluster with infinite density that drains mass from the coagulating system. In this talk we will consider homogeneous kernels $K(x, y) = (xy)^\lambda$ with $\lambda \leq 1$ and present some results about selfsimilar solutions both in singular and non singular cases. Such self-similar solutions depend on a free exponent that cannot be determined from dimensional considerations -self-similar solution of the second kind, in the notation of Barenblatt [?]-; it can be fixed imposing the behaviour at the origin and infinity. This is joint work with Marco A. Fontelos.

From Kinetic Theory to the Dynamics of Sprays

Francois Golse
Ecole polytechnique, Paris, France
golse@math.polytechnique.fr
E. Bernard, L. Desvillettes, V. Ricci

Sprays are multiphase flows in which a dispersed phase such as droplets or solid particle is mixed with a gaseous phase. An important class of models for such flows involve a Vlasov type kinetic equation for the dispersed phase, coupled to various fluid dynamic models for the gaseous phase. The coupling involves the friction force between the droplets or solid particles and the surrounding gaseous phase. The talk will present various models of this type and explain how they can be derived from the system of coupled Boltzmann equations for the binary particle system consisting of the particles in the dispersed phase and the gas molecules. The various fluid models coupled to the Vlasov equation for the dispersed phase are obtained under different scaling assumptions on the physical parameters involved.

Global well-posedness for a relaxed Landau equation.

Maria Pia Gualdani
George Washington University, USA
gualdani@gwu.edu
Nestor Guillen

We present global well-posedness results for a relaxed version of the Landau equation with Coulomb potential. The initial distribution is only assumed to be bounded and decaying sufficiently fast at infinity. Despite lack of a comparison principle for the equation, the proof of existence relies on barrier arguments and parabolic regularity theory. The Landau equation arises in kinetic theory of plasma physics. It was derived by Landau and serves as a formal approximation to the Boltzmann equation when grazing collisions are predominant.

Exponential trend to equlibrium for Becker-Doring equation

Bertrand Lods
Università di Torino, Italy
lodsbe@gmail.com
José A. Cañizo

We prove that any subcritical solution to the Becker-Döring equations converges exponentially fast to the unique steady state with same mass. Our convergence result is quantitative and we show that the rate of exponential decay is governed by the spectral gap for the linearized equation, for which several bounds are provided. This improves the known convergence result by Jabin & Niethammer (2003). Our approach is based on a careful spectral analysis of the linearized Becker-Döring equation (which is new to our knowledge) in both a Hilbert setting and in certain weighted ℓ^1 spaces. This spectral analysis is then combined with uniform exponential moment bounds of solutions in order to obtain a convergence result for the nonlinear equation.

Kinetic models for aerosols and numerical simualtions

Alexander Lorz
UPMC, France
alex.lorz@gmail.com
L. Boudin, C. Grandmont, A. Moussa

We write a model for an aerosol (air/particle mixture) in the respiratory system for a fixed or moving domain. It consists of the incompressible Navier-Stokes equations for the air and the Vlasov equation for the particles, coupled through a drag force. We propose a discretisation of the model and investigate fundamential properties of the numerical scheme.

Fractional-Stokes limit for kinetic equations

Sara Merino Aceituno
University of Cambridge, England
sm851@cam.ac.uk
Sabine Hittmeir

Fractional diffusion limits have been derived for collisional kinetic models conserving only the total mass (0-th moment). Their derivation is due, mainly, to the presence of a heavily tailed equilibrium distribution function in the collisional operator (instead of a Maxwellian) and a particular rescaling in time. In the present work, we extend the previous results to a linear kinetic model conserving the first three moments. Our approach is based on the 'moments methods' introduced by Antoine Mellet. In the limit we obtain the Stokes equation with fractional laplacian, under some conditions. This is a joint work with Sabine Hittmeir from Technische Universitat of Vienna.

Some measure convolution operators in transport theory

Mustapha Mokhtar-Kharroubi
University of Franche-Comte Besancon, France
mmokhtar@univ-fcomte.fr

We show how some fundamental spectral properties of neutron transport semigroups such as stability of essential spectra or critical spectra and related results can be infered from the study of two measure convolution operators

A fluid / kinetic coupling in a moving domain

Ayman Moussa
LJLL, UPMC, France
moussa@ann.jussieu.fr
L. Boudin and C. Grandmont

We present a fluid / kinetic coupling in a moving domain. Such a model has potential applications for the description of particle transport in the humang lung. We will explain the main obstructions in the mathematical analysis of this system.

A dispersive model for the unzipping of double-stranded DNA molecules

Juanjo Nieto
University of Granada, Spain
jjmnieto@ugr.es
Juan Calvo, Juan Soler and M. Ofelia Vásquez

This work deals with the analysis of a nonlinear Fokker-Planck equation modeling the mechanical unzipping of double-stranded DNA under the influence of an applied force. The dependent variable is the probability density of unzipping m base pairs. The non-linear Fokker-Planck equation we propose here is obtained when we couple a model proposed by Lubensky and Nelson in PRE (2000), with a transcendental equation for the applied force. The resulting model incorporates non-linear effects in a different way than the usual models in kinetic theory. We treat the well-posedness of this model. For that we require a combination of techniques coming from second order kinetic equations and compensated compactness arguments in conservation laws.

Mathematical and physical problems of chemically reactive mixtures

Ana Jacinta Soares
University of Minho, Portugal
ajsoares@math.uminho.pt

Chemically reactive mixtures can be modeled in the frame of the kinetic theory of gases. Some interesting problems can be formulated concerning the mathematical structure of the governing equations, consistency of the macroscopic analogue derived in the hydrodynamic limit, properties of the linearized reacting system, extension of the model to a moderately dense gas regime, existence and hydrodynamic stability of detonation wave solutions. In this talk we will discuss some of these problems with reference to a particular kinetic model. Some numerical simulations and representative computational results are also presented and discussed.

A toy model study of grazing collision effect of non-cutoff potentials

Shigeru Takata
Kyoto University, Japan
takata.shigeru.4a@kyoto-u.ac.jp

Consider the behavior of a rarefied gas whose initial state is such that the velocity distribution function (VDF) has a discontinuity. Though decaying, the discontinuity propagates in time for $t > 0$. A similar propagation of the discontinuity on the boundary into a gas occurs in steady problems around convex bodies. These phenomena are well understood both theoretically and numerically. On the other hand, according to recent mathematical studies, the infinite range intermolecular potential has a regularizing effect on the solution of the Boltzmann equation. Thus, starting from the initial data with a discontinuity, the velocity distribution function has no discontinuity immediately after the initial time, $t > 0$. The difference of the above two pictures is due to whether the collision frequency is finite or infinite. In the former case the gain and loss terms of the collision integral can be treated separately, while, in the latter case, they diverge individually and should be treated all together. Hence, in practical applications, one often introduces the cutoff model for the latter case, expecting little influence of the grazing collision effect on main physical picture. In the present work, we would like to discuss the effect of grazing collision via comparisons between the cutoff and non-cutoff models. To this aim, we introduce a toy model, a two-dimensional Lorentz gas model, and study its spatially homogeneous initial-value problem with a discontinuous initial data.

Special Session 66: Deterministic and Stochastic Models in Biology and Medicine

Mostafa Adimy, Inria, Dracula Team, France
Oscar Angulo, Departamento de Matemática Applicada, ETSIT, Universidad de Valladolid, Spain
Fabien Crauste, CNRS, University Lyon 1 & Inria Dracula, France
Laurent Pujo-Menjouet, University Lyon 1 & Inria Dracula,

This session is dedicated to the most recent deterministic and stochastic models in biology and medicine. Some of the biological fields covered are the following: mutation and cancer (DNA damage response, cancer dynamics), blood cell formation (dialysis, erythropoiesis, leukemia), epidemiology (infections, epidemiological models, salmonella), population dynamics (hermaphrodite population, adaptive dynamics)... Some of the mathematic tools used are the following: splitting particle methods, structured partial differential equations, pulse waves, individual based models...

Estimation of Distributions of Parameters Using Aggregate Data: Material Reflectivity

H. Thomas Banks
N C State University, USA
htbanks@ncsu.edu
Jared Catenacci, Shuhua Hu

In this presentation we investigate the properties of materials through the reflectivity, where the permittivity is described by the Lorentz model in which an unknown probability measure is placed on the model parameters. We summarize the computational and theoretical methodology (consistency of the probability measure estimator, the bias due to the approximation and the pointwise asymptotic normality of the approximated probability measure estimator) developed by our group in the past two decades for nonparametric estimation of probability measures using a least-squares method under the *Prohorov Metric Framework*. We demonstrate the feasibility of our proposed methods by numerical results.

References

[1] H.T. Banks, J. Catenacci, S. Hu, and Z.R. Kenz, Decomposition of permittivity contributions from reflectance using mechanism models, Tech. Report CRSC-TR13-11, N. C. State University, Raleigh, NC, September, 2013; *Proceedings 2014 American Control Conference*, Portland, OR.

[2] H. T. Banks, S. Hu, and W. Clayton Thompson, *Modeling and Inverse Problems in the Presence of Uncertainty*, CRC Press/Taylor and Frances, Boca Raton, April, 2014.

Understanding hermaphrodite species through game theory

Slimane Ben Miled
University of Tunis el Manar, Tunisia
slimane@ipeit.rnu.tn
Amira Kebir

Classical size advantage model (SAH), has been successful in assessing the adaptive significance of sex change and predicting within-species patterns, but fail to explain adaptive significant of sex-reversal strategy. In fact, experiments have confirmed an increasing list of sequential hermaphrodite species in which multiple sex reversals can occur: sex allocation is sensitive to individuals size relative to others in the social group and the sex-ratio of the social group. Our main objective is to study how the distribution by size and gender can affect the sexual behavior of a hermaphrodite. So we assume that among hermaphrodite there are different kind of sexual strategies depending on their environment like: sexual preference, gender partner distribution, size distribution .... In this work we are interested in the mathematical analysis of a general game theoretical model for a hermaphrodite population structured by size and sexual strategy. Our main objective is to study how the distribution by size and gender can affect the sexual behavior of a hermaphrodite from evolutionary point of view. So we assume that among hermaphrodite there are different kind of sexual strategies depending on their environment like: sexual preference, gender partner distribution, size distribution ....

Dynamics of slow cell renewal in humans

Samuel Bernard
University of Lyon, France
bernard@math.univ-lyon1.fr

Human tissues constantly replace dying cells with newborn cells. The pace at which they are replaced, however, varies by orders of magnitudes between blood cells, which are renewed every day and neurons, for which renewal is non-existent or limited to specific regions of the brain. Between those extreme are many tissues that turnover on a time scale of years, although no direct measurements have been done. We present here a mathematical method to estimate cell turnover in slowly renewing biological systems. Age distribution of DNA can be estimated from the integration of radiocarbon derived from nuclear bomb-testing during the cold war (1955-1963). For slowly renewing tissues, this method provides a better estimate of the average age of the tissue than direct estimates from the bomb-curve. Moreover, death, birth and turnover rates can be estimated. We highlight this method with data from hippocampal neurons and cardiomyocytes.

A model of healthy and cancer cells dynamics in Acute Myeloid Leukemia

Catherine Bonnet
Inria, France
Catherine.Bonnet@inria.fr
J.L. Avila Alonso, H. Ozbay, J. Clairambault and S. Niculescu

We present a coupled model for healthy and cancer cell dynamics in Acute Myeloid Leukemia consisting of several stages of maturation (for both healthy and cancer cells). The cell dynamics are modelled by nonlinear partial differential equations which can be reduced with classical methods to nonlinear systems with distributed delays. The interconnection phenomenon between the healthy and cancer cells is modeled through the re-introduction function which, in the cell cycle model, takes cells from the resting compartment into the proliferating compartment. For a particular healthy equilibrium point, a stability analysis is performed.

Population Model with an Age-weight Structured Two-stage Life History

Rafael Bravo de la Parra
University of Alcala, Spain
rafael.bravo@uah.es
Eva Sanchez, Hassan Hbid

In this talk we present a model for the dynamics of a physiologically structured population of individuals whose life cycle is divided into two stages: the first stage is structured by the weight, while the second one is structured by the age, the exit from the first stage occurring when a threshold weight is attained. The model originates in a complex one dealing with a fish population, and covers a large class of situations encompassing two-stage life histories with a different structuring variable for each state, one of its key features being that the maturation process is determined in terms of a weight threshold to be reached by individuals in the first stage. Mathematically the model is based on the classical Lotka-MacKendrick linear model, which is reduced to a delayed renewal equation including a constant delay that can be viewed as the time spent by individuals in the first stage to reach the weight threshold. The influence of the growth rate and the maturation threshold on the long term behaviour of solutions is analyzed by using Laplace transform methods.

Positive equilibria of structured population dynamics and fixed points of set valued maps

Angel Calsina
Universitat Autonoma de Barcelona, Spain
acalsina@mat.uab.cat

We consider the existence of positive steady states of nonlinear evolution equations arising in structured population dynamics. Often these problems can be reduced to an eigenvector problem for a parameterized family of unbounded linear operators plus a finite dimensional fixed point problem for a (in general) set-valued map. When the vital rates are monotonous functions of the interaction variables this map is single-valued and the existence of equilibria can be established by standard procedures. In the general case, some results can be obtained in the case of two dimensional nonlinearities [1]. As an example, we will consider the case of a selection mutation equation for the density of individuals with respect to an evolutionary trait, namely the age at maturity, and with respect to physiological age ([2]).

References

[1] A. Calsina, J. Farkas, Positive steady states of structured population models with finite dimensional nonlinearities. Submitted.

[2] A. Calsina and J. M. Palmada, Steady states of a selection-mutation model for an age structured population, J. Math. Anal. Appl., 400 (2013), 386-395.

Adaptive dynamics and evolutionary branching in the limit of small mutations in an individual-based population model

Nicolas Champagnat
Inria Nancy - Grand Est, France
Nicolas.Champagnat@inria.fr

The mathematical study of adaptive dynamics, and more specifically of the phenomenon of evolutionary branching by which a population is driven by selective forces to subdivide into two interacting subpopulations with different phenotypes, has been done in the last years using either an approach based on an assumption of rare mutations and large population on a stochastic individual-based model, or an approach based on a limit of small mutations on a PDE model. Both approaches suffer from irrealistic features: the first one requires a very long time scale to observe evolutionary branching; in the second one, exponentially small population densities can have a very strong impact on the future evolutionary dynamics. The goal of this talk is to present an intermediate approach that may solve these two drawbacks, consisting in applying a combination of limits of small mutations and large population on the stochastic individual-based model.

Modelling the evolution of a reversible drug-tolerant phenotype in a cancer cell population, mediated by stochastic and drug-induced epimutations

Rebecca Chisholm
Inria, France
rebecca.chisholm@inria.fr
Jean Clairambault, Tommaso Lorenzi, Alexander Lorz, Benoit Perthame

Epigenetic mechanisms are becoming increasingly implicated in the evolution of drug-resistance in cancer cell populations. However, these mechanisms are not well understood. Recent experiments on NSCLC-derived (PC9) cell lines revealed a sub-population of cells that, in response to anti-cancer drugs, were able to acquire transient drug-resistance (Sharma et al., Cell, 2010). Interestingly, this reversible drug-tolerant state was shown to be the result of epigenetic modifications, rather than genetic mutation. Motivated by these results, we formulate a mathematical model of phenotype variation in a cancer cell population in order to understand the mechanisms responsible for the reversible, drug-resistant phenotype observed in NSCLC-derived (PC9) cell lines. We assume that both stochastic variation in cell phenotype due to biological noise and stress-induced adaption are functioning in parallel without genetic mutation. Our model recovers the evolution of the PC9 cells into the drug-tolerant cells in the presence of a cytotoxic drug therapy. It also captures the dynamics of drug-resensitisation that were observed after drug-washout.

Drug resistance in cancer cell populations: Mathematical and biological assessment

Jean Clairambault
INRIA & UPMC, Paris, France
jean.clairambault@inria.fr
Rebecca Chisholm, Alexandre Escargueil, Tommaso Lorenzi, Alexander Lorz, Benoit Perthame, Emmanuel Trelat

Considering cancer as an evolutionary disease, we aim at understanding the means by which cancer cell populations develop resistance mechanisms to drug therapies, in order to circumvent them by using optimised therapeutic combinations. Rather than focusing on molecular mechanisms such as overexpression of intracellular drug processing enzymes or ABC transporters that are responsible for resistance at the individual cell level, we propose to introduce abstract phenotypes (that nevertheless may be experimentally identified and controlled in cell cultures, according to the drug and to the cell line at stake) of resistance structuring cancer cell populations. The models we propose rely on continuous adaptive dynamics of cell populations, and are amenable to predict evolution of these populations with respect to the phenotypic traits of interest. According to the cell populations at stake and to the exerted drug pressure, is drug resistance in cancer a permanently acquired phenotypic trait or is it reversible? Can it be avoided or overcome by rationally (model-guided) designed combinations of drugs (to be optimised)? These are some of the questions we will try to answer in a collaboration between a team of mathematicians and another one of biologists, both dealing with cancer and Darwinian evolution of cell populations.

Emergence of Spatial Patterns in a Model for the Dynamics of Epithelial and Mesenchymal Cells

Marcello Delitala
Politecnico di Torino, Italy
marcello.delitala@polito.it
Tommaso Lorenzi

In this talk, we present a model for the evolution of a monolayer sample of epithelial and mesenchymal cells. The model consists of a system of integro-differential equations and includes the effects of chemotaxis, proliferation and interactions amongst cells. We provide an interesting sample of numerical results, which show the emergence of spots, stripes and honeycomb patterns, depending on parameters and initial data. Simulations also show how epithelial and mesenchymal cells can segregate when there is little competition for nutrients. Furthermore, computational results suggest that the interplay between epithelial cell adhesion and mesenchymal cell spreading can pave the way for the formation of ring-like structures, which resemble the fibrous capsules frequently observed in hepatic tumors.

Structured population equations modeling erythropoiesis and applications to dialysis treatment

Franz Kappel
University of Graz, Austria
franz.kappel@uni-graz.at
Doris H. Fuertinger

We present a model for erythropoiesis including a submodel for the iron dynamics in the body. The core of the model consists of structured population equations for the different cell stages which have to be considered. In order to link the model for the iron dynamics to the structured population equations in a physiologically correct way we have to consider structured population equations with up to three structuring attributes. We shall also consider approximation schemes based on the theory of abstract evolution equations which provide approximating finite dimensional dynamical systems which can advantageously be used for parameter estimation. Finally, we shall present some applications of the model to issues in the dialysis treatment.

References

[1] Fuertinger, D.H., Kappel, F., Thijssen, S., Levin, N., and Kotanko, P.: *A* model of erythropoiesis in adults with sufficient iron availability. J. Math. Biol. **66**(6), 1209 – 1240 (2013).

[2] Ito, K., and Kappel, F.: Evolution Equations and Approximations. World Scientific, Singapoore (2002).

[3] Kappel, F., and Zhang, K.: *A*pproximation of linear age-structured population models using Legendre polynomials. J. Math. Anal. Appl. **180**, 518 – 549 (1993).

Effects of density dependent sex allocation on the dynamics of a sequential hermaphrodite population: modeling and analysis

Amira Kebir
Tunis University, Tunisia
amira.kebir@gmail.com
Mostafa Adimy, Slimane Ben Miled, Moulay Lhassan Hbid

In this work, we introduce and analyze a model of sequential hermaphroditism in the framework of continuously structured population models with sexual reproduction. The mathematical tools used for this model are systems of two partial differential equations where each one describe immature and mature life stage of hermaphrodite population. We assume that birth function depends on female and male allocation functions, the competition between adults for reproduction will affect the mature mortality rate and that the recruitment of immature individuals didn't occur at a fixed size. Using the method of characteristics we convert the problem to an equivalent system of a non-linear integral equations involving the birth rate and then we transform it in a delay system. The steady states of the delay system are analyzed and illustrated for several cases. In particular, neglecting the competition for resources we have explicitly found a conditions of stability for the of the non-trivial equilibrium.

A mathematical model of adult stem cell regeneration with crosstalk between genetic and epigenetic regulation

Jinzhi Lei
Tsinghua University, Peoples Rep of China
jzlei@tsinghua.edu.cn
Simon A. Levin, Qing Nie

Adult stem cells, which exist throughout the body, multiply by cell division to replenish dying cells or to promote regeneration to repair damaged tissues. To perform these functions during the lifetime of organs or tissues, stem cells need to maintain their populations in a faithful distribution of their epigenetic states, which are susceptible to stochastic fluctuations during each cell division, unexpected injury, and potential genetic mutations that occur during many cell divisions. However, it remains unclear how the three processes of differentiation, proliferation and apoptosis in regulating stem cells collectively manage these challenging tasks. Here, without considering any molecular details, we propose a genetic optimal control model for adult stem-cell regeneration that includes the three fundamental processes, along with cell division and adaptation based on differential fitnesses of phenotypes. In the model, stem cells with a distribution of epigenetic states are required to maximize expected performance after each cell division. The overall modeling results highlight the importance of crosstalk between genetic and epigenetic regulation and the performance objectives during homeostasis in shaping a desirable heterogeneous distribution of stem cells in terms of epigenetic states.

Mathematical modelling of cell polarization

Thomas Lepoutre
Inria and Universite Lyon 1, France
thomas.lepoutre@inria.fr
Vincent Calvez, Nicolas Meunier and Nicolas Muller

We consider different models of spontaneous cell polarization. Firstly we consider a detailed study of the asymptotic behaviour of a one-dimensional toy model. Secondly, we consider the phenomenon of exchange between two cells. In both cases, a critical mass phenomenon appears. The models are close to Keller Segel type.

Approximating the survival probability in age-structured population models with finite life-span

Miguel Lopez-Marcos
University of Valladolid, Spain
malm@mac.uva.es
O. Angulo, J. C. Lopez-Marcos

Realistic age-structured population models should incorporate a finite life-spam of individuals. To this end, some models introduce mortality rates that are unbounded near the finite maximum age. However, such condition introduces difficulties in the numerical analysis of the model. During the last years, some numerical methods have been proposed for some specific mortality functions. In this work we propose an appropriate procedure for the approximation of these kinds of problems based on the numerical approach of the survival probability.

Intermediate dynamics of population models in fluctuating environments

Tommaso Lorenzi
Laboratoire J.-L. Lions, UPMC, France
tommaso.lorenzi@upmc.fr
Alexander Lorz

We consider a class of differential equations that describes the dynamics of phenotype-structured populations embedded in time-fluctuating environments. The solutions of these equations are known to concentrate as one or several Dirac masses, whose concentration points evolve in time. We study how the frequency of environmental oscillations affects the evolution of the concentration points. In particular, we find sufficient conditions that allow the coexistence of several Dirac masses, on intermediate time scales. Moreover, we provide numerical simulations that illustrate analytical results and show an interesting sample of solutions. Our motivation comes from structured equations that describe the evolution of cancer cell populations and the emergence of resistance under anti-cancer drugs.

Asymptotic properties for an infection-load structured epidemiological model with exponential growth

Antoine Perasso
Franche-Comte university, France
antoine.perasso@univ-fcomte.fr
Ulrich Razafison

We consider an infection load-structured epidemiological model with exponential growth that translates the dynamic of transmission of some prion pathologies. The incorporated infection load leads to the formulation of a nonlinear partial differential equation of transport type, coupled with an ordinary differential equation. We exhibit epidemiological thresholds of the model, such as the basic reproduction number R0, in order to study some asymptotic properties of the solution.

Stability analysis of a complex time-delay system for leukemia

Rodica Radulescu
University "POLITEHNICA" of Bucharest, Romania
nicola_rodica@yahoo.com
D. Candea, A. Halanay

In this work, we characterize the steady states and analyze their stability properties in a complex, strongly nonlinear model of delay-differential equations with multiple delays, for cell evolution in leukemia. The competition on space between healthy and leukemic cell populations is taken into consideration. Three types of division that a stem-like cell can undergo, asymmetric division, self-renew and differentiation are also considered. Numerical results and simulations are discussed in relation to clinical implications of the proposed model.

On a Discrete Selection-Mutation Model: Competitive Exclusion and Uniform Persistence

Paul Salceanu
University of Louisiana at Lafayette, USA
salceanu@louisiana.edu
Azmy S. Ackleh and Robert J. Sacker

A discrete-time population model in which individuals are distributed over a discrete phenotypic trait-space is studied. It is shown that, for an irreducible mutation matrix Γ, if mutation is small, then an interior equilibrium exists, that is globally asymptotically stable in $\mathbb{R}^n \setminus \{0\}$, while for arbitrary large mutation each trait persists uniformly. For the model reduced to only two traits, conditions for the global stability of the interior equilibrium are provided. When structure is introduced in the model, namely when mutation matrix Γ has block-diagonal form, with each diagonal block being irreducible, competitive exclusion among traits is analyzed and sufficient conditions are given for one trait to drive all the other traits to extinction.

A mean-field SAIS epidemic model with awareness decay

Joan Saldana
Universitat de Girona, Spain
joan.saldana@ima.udg.edu
Istvan Z. Kiss, David Juher

The existence of a die-out threshold (different from the classic disease-invasion one) defining a region of slow extinction of an epidemic has been proved elsewhere for susceptible-alert-infected models without awareness decay. By means of an equivalent mean-field model defined on regular random networks, we interpret the dynamics of the system in this region and prove that the existence of this second epidemic threshold is not a generic property of this class of models. We show that the continuum of equilibria that characterizes the slow die-out dynamics collapses into a unique equilibrium when a constant awareness decay is assumed, no matter how small, and that the resulting bifurcation from the disease-free equilibrium is equivalent to that of standard epidemic models.

Conditions for growth and extinction in stochastic matrix models

Luis Sanz
Universidad Politecnica de Madrid, Spain
lsanz@etsii.upm.es
Juan Antonio Alonso

In this work we study some features of the behavior of Leslie type matrix models for age structured populations subjected to environmental stochasticity. In these models, the population lives in an environment that changes randomly with time in each projection interval. These changes can account for the variability in resources, climatic conditions, etc., that reflect in the fertility and survival coefficients os the population. In this way, there is a collection of Leslie matrices, each one of them corresponding to an environmental condition, and in each time step of the model the environment to which the population is subjected is defined by a certain random variable that is usually chosen to be a Markov chain. The main parameter that controls the dynamics of these kind of models is the so called stochastic growth rate (s.g.r.). When the s.g.r is positive the population grows exponentially with probability one and when the s.g.r is negative the population goes extinct with probability one. However, even in very simple situations, it is not possible to calculate the s.g.r. analytically. As a result, there are not simple expressions that biologists can use to check the extinction-explosion of populations. In this work we build different upper and lower bounds for the s.g.r. that are tighter than the ones already existing in the literature. We analyze the conditions under which each bound works best. Finally, the different bounds are used to give necessary-sufficient conditions, based in simple expressions easy to check in practice, for the explosion and the extinction of the population. The general results are applied to the case of a population structured in juveniles and adults living in an ambient in which there two possible environments.

Modeling of Stem Cell Dynamics in Hematopoiesis and Leukemia: Clinical and Scientific Perspectives

Thomas Stiehl
Institute of Applied Mathematics, University of Heidelberg, Germany
thomas.stiehl@iwr.uni-heidelberg.de
Natalia Baran, Anthony Ho, Anna Marciniak-Czochra

The blood forming system is maintained by a population of hematopoietic (blood forming) stem cells (HSCs). The HSCs guarantee life-long blood production and regeneration after chemotherapies or bone marrow transplantation. In spite of its robustness to perturbations, hematopoiesis can be outcompeted by leukemias which lead to expansion of malignant cells. Leukemias are maintained by a small but heterogeneous population of leukemic stem cells (LSCs) that resist treatment and trigger relapse. We propose different multi-compartment models of cell differentiation. The models consist of nonlinear systems of ordinary differential equations describing time evolution of healthy and cancerous cell populations subjected to different possible feedbacks. Mathematical results and simulations are discussed based on patient data. The models will be applied to better understand outcompetition of hematopoiesis by leukemic cells, possible impacts of LSC properties on the course of the disease, changes of cell properties due to clonal competition, and interindividual differences in responses to treatment. Estimation of LSC properties and simulation of chemotherapy and bone marrow transplantation may help to discuss mechanisms of relapse and effective treatment strategies.

Cross-contamination of fresh produce and poultry during commercial wash procedures

Jianhong Wu
York University, Canada
wujh@yorku.ca
D. Munther, S. Tang, Y. Xiao, H. Shimozako

We look at the cross-contamination dynamics in the fresh produce commercial wash procedures using delay differential equations, and explore its applications to inform industrial guidelines to reduce food-borne disease spread in the population.

Special Session 67: Topological Methods for the Qualitative Analysis of Differential Equations and Inclusions

Pierluigi Benevieri, University of Sao Paulo, Brazil
Marco Spadini, University of Florence, Italy

The main topic of the session will be topological methods such as degree theory, fixed point index theory, Morse theory, Maslov index, spectral flow, and their applications to various problems in ordinary, functional and partial differential equations, differential-algebraic equations and differential inclusions. Particular emphasis will be given to existence, multiplicity and bifurcation of solutions. Contribution to the theoretical aspects of the above mentioned methods will be welcome as well.

A concept of degree for multivalued perturbations of Fredholm maps in Banach spaces and its application to differential inclusions

Pierluigi Benevieri
University of Sao Paulo, Brazil
pluigi@ime.usp.br
Pietro Zecca

We present an integer valued topological degree theory for locally compact multivalued perturbations of Fredholm maps of index zero in Banach spaces. The construction is based on an infinite dimensional nortion of orientation for nonlinear Fredholm maps and by an analogous concept of degree for Fredholm maps of index zero. This theory is applied to existence and bifurcation problems for differential inclusions.

Periodic perturbations with delay of coupled differential equations on manifolds with applications to sunflower-like equations

Luca Bisconti
University of Florence, Department of Mathematics and Informatics “U. Dini”, Italy
luca.bisconti@unifi.it
Marco Spadini

In this talk we present recent results jointly obtained with Marco Spadini (University of Florence). We investigate the structure of the set of T-periodic solutions of periodically perturbed coupled delay differential equations on differentiable manifolds. More precisely, given $M \subseteq \mathbb{R}^k$ and $N \subseteq \mathbb{R}^s$ boundaryless smooth manifolds, we consider the following system of delay differential equations for $\lambda \geq 0$:

$$\begin{cases} \dot{x}(t) = \lambda f\big(t, x(t), y(t), x(t-r), y(t-r)\big), \\ \dot{y}(t) = a(t)g(x(t), y(t)) \\ \qquad\qquad + \lambda h\big(t, x(t), y(t), x(t-r), y(t-r)\big), \end{cases}$$

where $r > 0$ is given, $a\colon \mathbb{R} \to \mathbb{R}$ is a continuous map with nonzero average, $f\colon \mathbb{R} \times M \times N \times M \times N \to \mathbb{R}^k$ is tangent to M and $g\colon M \times N \to \mathbb{R}^s$ and $h\colon \mathbb{R} \times M \times N \times M \times N \to \mathbb{R}^s$ are tangent to N. Also, we assume that f, h and a are T-periodic, $T > 0$ given, in the t-variable. By using fixed point index and degree-theoretic methods we provide sufficient conditions for the existence of branches of T-periodic solutions to the above system of equations. As main application of our results, we study generalized versions of the sunflower equation.

On the uniqueness of the topological degree for quasi-Fredholm maps

Alessandro Calamai
Università Politecnica delle Marche, Italy
calamai@dipmat.univpm.it
Pierluigi Benevieri and Massimo Furi

In 2006, Benevieri and Furi presented a quite simple construction of a topological degree for compact perturbations of Fredholm maps of index zero between Banach spaces, called quasi-Fredholm maps. This degree verifies the three fundamental properties of the classical degree theory: Normalization, Additivity and Homotopy invariance. We show here that this degree is unique. Precisely, by an axiomatic approach similar to the one due to Amann-Weiss, we prove that there exists at most one real function satisfying the above properties, and this function must be integer valued. In addition, we show that the degree for quasi-Fredholm maps provides in a natural way a generalization of the Leray-Schauder degree. This is a joint work with Pierluigi Benevieri and Massimo Furi.

A quasilinear bistable equation in cylinders and timelike heteroclinics in special relativity

Isabel Coelho
Université Libre de Bruxelles, Belgium
isabel.coelho@ulb.ac.be
Denis Bonheure, Manon Nys

We study the action functional

$$\int_{\mathbb{R}\times\omega} 1 - \sqrt{1 - |\nabla u|^2} + W(u)\; dx.$$

We assume that $\omega \subset \mathbb{R}^{N-1}$ is a bounded domain and that $W\colon \mathbb{R} \to [0, +\infty[$ is a double-well potential, i.e., W is of class C^1 and satisfies $W(-1) = W(1) = 0$ and $W(u) > 0$ if $u \neq \pm 1$. Using variational arguments and a rearrangement technique, we prove the existence, one-dimensionality and uniqueness (up to translations) of a smooth minimizing phase transition between the stable states -1 and 1. We also discuss the existence of minimizing heteroclinic connections for the non autonomous model

$$\int_{\mathbb{R}} 1 - \sqrt{1 - |u'|^2} + a(t)W(u)\; dt,$$

where $a \in C^1(\mathbb{R})$ is a bounded positive function that can have a constant asymptotic behaviour at infinity or can be periodic. The Euler-Lagrange equations associated with these functionals find significant applications in Riemannian geometry and in the theory of relativity from where timelike heteroclinic solutions take their name.

Subharmonic solutions of the prescribed curvature equation

Chiara Corsato
University of Trieste, Italy
chiara.corsato@phd.units.it
Pierpaolo Omari, Fabio Zanolin

We study the existence of subharmonic solutions of the capillarity equation

$$-\left(u'/\sqrt{1+u'^2}\right)' = f(t,u).$$

According to the specific structure of the curvature operator and to the behaviour of f at infinity and at zero, we get the existence of arbitrarily large bounded variation subharmonic solutions, or arbitrarily small classical subharmonic solutions. The method of proof relies on the use of non-smooth critical point theory in the former case and on the Poincaré-Birkhoff theorem in the latter one.

Nodal solutions for supercritical Laplace equations

Francesca Dalbono
Università degli Studi di Palermo, Italy
francesca.dalbono@unipa.it
Matteo Franca

We study asymptotic behaviour and nodal properties of the radial solutions to a superlinear Laplace equation. Our approach is based on the Fowler transformation combined with invariant manifold theory.

Semilinear Elliptic Systems involving Exponential Nonlinearities

Joao Do O
Federal University of Paraiba, Brazil
jmbo.pq@gmail.com

We establish a priori bounds for positive solutions of a class semi-linear elliptic systems defined in a bounded and smooth domain in the plane. We obtain results concerning such bounds when the nonlinearities have exponential growth.

Long time behavior of solutions to a non-linear non-homogenous heat equation

Matteo Franca
Universit'a Politecnica delle Marche, Italy
franca@dipmat.univpm.it
Luca Bisconti

In this talk we consider a non-linear heat equation of the form

$$u_t = \Delta u + f(u,|x|)$$

and we assume that the non-linearity f is supercritical with respect to $n/(n-2)$ in some weak sense. We prove local existence of regular and singular solutions and we use Fowler transformation in order to establish existence and separation properties of radial Ground States. Then we use these results to detect long time behavior of both radial and non-radial initial data. In particular we describe the threshold separating initial data in the basin of attraction of the null solution, from initial data which blow up in finite time. We show that such a border is made up by Ground States when f is supercritical with respect to $(n+2)/(n-2)$, thus extending results by Wang, and Deng, Li, Liu to potential including the Matukuma case $f(u,r) = \frac{u^q}{1+r^a}$. We also obtain a family of sub and super-solutions which are new even for the original problem $f(u) = u^q$ for any $q > n/(n-2)$.

Chaotic dynamics for switched nonlinear planar systems in presence of an equilibrium changing its nature

Maurizio Garrione
Università di Milano-Bicocca, Italy
maurizio85.g@gmail.com

We study the dynamics originated by the superposition of two autonomous systems in the plane for which the origin is a local center and a saddle, respectively. We will show that if we alternatively switch the two systems maintaining each one for a sufficiently large time, then the overall Poincaré map induces chaotic dynamics on two symbols near the origin. In particular, under the T-periodicity of the switching (for T large), there is existence of T-periodic solutions and subharmonics of any order. We will briefly provide an outline of the proof and an insight into the main topological tool used for its fulfillment, the so called "stretching along the paths" method. If time permits, we will show other configurations (different from the center-saddle one) giving possibly rise to chaotic dynamics in the coin-tossing sense.

Some analytical results about periodic orbits in the restricted three body problem with dissipation

Alessandro Margheri
CMAF and FCUL, Universidade de Lisboa, Portugal
margheri@ptmat.fc.ul.pt
Rafael Ortega, Carlota Rebelo

We present some analytical results about the existence of periodic orbits for the planar restricted three body problem with dissipation considered recently by Celletti et al. (2011). We show that, under fairly general conditions on the dissipation term, the circular orbits cannot be continued to the dissipative framework. Moreover, we give general conditions for the occurrence or not of a Hopf bifurcation around the libration points L_4 and L_5. Our results are consistent with the numerical findings of Celletti et al.(2011).

Invertibility of nonsmooth maps arising in optimal control

Laura Poggiolini
Università degli Studi di Firenze, Italy
laura.poggiolini@unifi.it
Marco Spadini

We consider second order sufficient conditions for state-local optimality of a Pontryagin extremal in the minimum time problem. When the extremal control being considere is bang-bang with simultaneous switches, non trivial problems appear in the invertibility of the projected maximised Hamiltonian flow. Such problems are solved by non smooth analysis and degree theory.

Second order impulsive retarded differential inclusions with nonlocal conditions

Marcos Rabelo
Federal University of Goias, Brazil
rabelo@dmat.ufpe.br
Hernán Henríquez and Luciana Vale

In this work we establish some existence results for abstract second order Cauchy problems modeled by a retarded differential inclusions involving nonlocal and impulsive conditions. Our results are obtained using fixed point theory for the measure of noncompactness.

Multiplicity of solutions of BVP associated to asymptotically linear second order equations

Carlota Rebelo
CMAF and FCUL, Universidade de Lisboa, Portugal
mcgoncalves@fc.ul.pt

In this talk we will discuss some multiplicity results associated to asymptotically linear second order equations in R^n. Dirichlet, Neumann and periodic boundary conditions will be considered. We will focus in the cases $n = 1$ and $n = 2$.

Existence, stability and regularity properties of periodic solutions of a capillarity-type equation

Sabrina Rivetti
University of Trieste, Italy
sabrina.rivetti@phd.units.it
Franco Obersnel, Pierpaolo Omari

We consider the periodic problem associated with the capillarity equation

$$-\left(u'/\sqrt{1+u'^2}\right)' = f(t,u).$$

In the setting of bounded variation functions we present some existence results in the presence of lower and upper solutions, both in the case where the lower solution α and the upper solution β satisfy $\alpha \leq \beta$ and in the case where $\alpha \not\leq \beta$. In the former case we find a solution as a local minimizer of the associated action functional without any additional assumption on the right-hand side f. In the latter case, we still prove the existence of at least one solution, but now a control on f is needed with respect to the first branch of the Dancer-Fučik spectrum of the T-periodic problem for the 1-Laplace operator. Some results concerning regularity and stability are also produced.

Patterns from Bifurcations: A Symmetry Analysis of Networks with Delayed coupling

Haibo Ruan
University of Hamburg, Germany
haibo.ruan@math.uni-hamburg.de
Fatihcan M. Atay

We study systems of coupled units in a general network configuration with a coupling delay. We show that the destabilizing bifurcations from an equilibrium are governed by the extreme eigenvalues of the coupling matrix of the network. Based on the equivariant degree method and its computational packages, we perform a symmetry classification of destabilizing bifurcations in bidirectional rings of coupled units. Both stationary and oscillatory bifurcations are discussed. We also introduce the concept of secondary dominating orbit types to capture bifurcating solutions of submaximal nature.

On nonhomogeneous elliptic problems involving exponential critical growth

Uberlandio Severo
UFPB, Brazil
uberlandio@mat.ufpb.br
Everaldo de Medeiros and Manasses de Souza

In this talk we will be present the existence of solutions for elliptic equations of the form $-\text{div}(|\nabla u|^{n-2}\nabla u) + V(x)|u|^{n-2}u = g(x,u) + \lambda h$ in $\mathbb{R}^n$ with $n \geq 2$. Here the potential $V(x)$ can change sign and the nonlinearity $g(x,u)$ is possibly discontinuous and may exhibit exponential growth. The proof relies on the application of a fixed point result and a version of the Trudinger-Moser inequality.

On periodic problems for periodically perturbed equations on manifolds

Marco Spadini
Università di Firenze, Italy
marco.spadini@unifi.it

In this talk we explore some applications of oriented intersection theory to 'special' (i.e. with partially assigned initial condition) periodic problems for periodically perturbed coupled ordinary differential equations on manifolds. Analogous situations for second order equations are analyzed too. Our remarks will be compared to known results obtained for (standard) periodic problems using fixed point index theory. Simple examples will be investigated numerically.

Smooth circle actions on the 7-sphere and nonlinearizable multicentres

Massimo Villarini
University Modena R.E., Italy
massimo.villarini@unimore.it

We will give an example of a smooth free action of $S^1 = U(1)$ on S^7 whose orbits have unbounded lenghts (equivalently: unbounded periods). As an application of this example we construct a C^∞ vector field X, defined in a neighbourhood U of $0 \in \mathbb{R}^8$, such that: $U - \{0\}$ is foliated by closed integral curves of X, the differential $DX(0)$ at 0 defines a 1-parameter group of nondegenerate rotations and X is *not* orbitally equivalent to its linearization. This proves in the C^∞ category that the classical Poincaré Centre Theorem, true for planar non degenerate centres, is not generalizable to multicentres.

Special Session 68: Entropy-Like Quantities and Applications

J.M. Amigo, Universidad Miguel Hernandez, Spain
K. Keller, University of Luebeck, Germany
B. Pompe, Ernst-Moritz-Arndt-University, Germany

Entropy is a general concept that appears in different settings with different meanings. Thus, it measures disorder in physics, uncertainty in information theory, minimum code length in coding theory, (pseudo-)randomness in measure-preserving dynamical systems, complexity in topological dynamics, and algorithmic complexity in computer science. As for its importance, let us remind that it enters the second axiom of thermodynamics, related to the direction of time in many-particle mechanical systems. In information theory it defines the very concept of information, beside lying at the core of the fundamental results. And, last but not least, in ergodic theory entropy is perhaps the most important invariant of metric and topological conjugacy, which are the equivalence concepts in measure-preserving and topological dynamics, respectively. In the last decades new versions of entropy has come to the fore. Sequence entropy, correlation entropy, permutation entropy, transfer entropy, approximate entropy, sample entropy, etc. are some of the entropy-like quantities proposed by researchers to cope with new challenges in ergodic theory, chaos, synchronization and control, information theory, time series analysis, etc. Along with these new developments, some traditional topics, like the computation (and even computability) of metric and topological entropy, still remain the subject of current research in applied mathematics. Also in physics, entropy is the objective of undiminished research activity, topics ranging from axiomatic aspects to its formulation in non-stationary processes. This Special Session is organized with the scope that researchers on the theoretical or practical aspects of entropy and akin quantities can share their interests and latest results in a multidisciplinary environment. Therefore, participants from all fields of science, mathematics and engineering are very much welcome.

The power of the min-max symbols

Jose Amigo
Miguel Hernandez University, Spain
jm.amigo@umh.es
A. Gimenez

Min-max symbols are a convenient generalization of the well-known kneading symbols in one-dimensional dynamics generated by multimodal mappings. From the computational point of view, min-max symbols require virtually the same effort as the kneading ones, with the plus that they provide additional information about the minimim/maximum character of the iterates of the critical values. We will present some theoretical and practical results which involve min-max symbols, including the computation of topological entropy.

Applications of symbolic dynamics in the context of cryptography

David Arroyo
Universidad Autónoma de Madrid, Spain
david.arroyo@uam.es

The relationship between the dynamical systems theory and the principles of cryptography was underlined by Shannon in his seminal work about perfect secrecy. Indeed, it is possible to interpret the main requirements of encryption systems by means of general concepts of dynamical systems theory, as the sensitivity to initial conditions/control parameters and the ergodicity property. However, this relationship could imply security flaws when an attacker can link partial information extracted from the encryption process to a subset of the secret keys of the related cryptosystem. In this work we highlight the outcomes of the applied theory of symbolic dynamics as a tool to detect such a vulnerability.

On non-autonomous systems through perturbations

Francisco Balibrea
Universidad de Murcia, Spain
balibrea@um.es

A great variety of problems on discrete dynamical systems and difference equations, both depending on parameters, can evolve to non-autonomous when such parameters are perturbed with a perturbation depending on n. We are dealing with this problem and give results proving that even when the perturbations has a small amplitude, the dynamical behavior of the system or equation can change drastically In particular, we will use trigonometric and general perturbations trough sinam or cosam Jacobi functions. We will give results on Lyapunov stability, entropy and other dynamics properties.

Order patterns: a diagnostic tool for time series

Christoph Bandt
Institute of Mathematics, University of Greifswald, Germany
bandt@uni-greifswald.de

Permutation entropy is widely used to evaluate the complexity of time series of moderate length. For long time series, multiscale analysis of all order patterns will give more information, as we shall demonstrate with examples from medicine, wheather, internet traffic, and music. A very interesting case is pink noise where frequencies of order patterns do not depend on the scale, We analyse several models and applications of pink noise and present a simple new model.

On the shape of isentropes for multimodal maps.

Henk Bruin
University of Vienna, Austria
henk.bruin@univie.ac.at
Sebastian van Strien

It is well-known that topological entropy. depends monotonically on the parameter in the family of quadratic maps. The same was proved by Milnor and Tresser for cubic maps, and recently monotonicity of entropy for polynomials of arbitrary degree was established by Bruin and Van Strien. This means that the isentropes (i.e., level sets of entropy) are connected subset of parameter, but I will demonstrate in this talk that the topological shape of such sets can be very complicated still, and many questions remain open.

Significance Test for Mutual Information

Ned Corron
RDECOM, USA
ned.corron@us.army.mil
Shawn Pethel, Dan Hahs

A common experimental goal is to detect dependency among multiple time series based on a limited number of observations. Mutual information is an information theoretic measure that is often used to detect dependency. Although the mutual information is theoretically zero for independent processes, estimates derived using finite data sets are imprecise and ambiguous. In this case, one must distinguish nonzero values to determine a likelihood of dependency. A rigorous approach is to use a statistical significance test to assess the null hypothesis of independent processes. In this talk, we present a significant test for mutual information that is accurate for finite (small) data sets. The key development is a method for generating and uniformly sampling surrogates from the set of all sequences that exactly match the n-th order properties of the observed data sets. Examples using coupled chaotic maps demonstrate the effectiveness of the test.

Interdependencies in climate networks constructed using information measures at different time scales

Juan Ignacio Deza
Universitat Politècnica de Catalunya, Spain
juan.ignacio.deza@upc.edu
Cristina Masoller, Marcelo Barreiro

We study global climate networks constructed by means of ordinal time series analysis. Climate interdependencies among the nodes are quantified by mutual information, computed from time series of surface air temperature anomalies, and from their symbolic ordinal representation (OP). This analysis allows identifying topological changes in the network when varying the time-interval of the ordinal pattern. We consider intra-season time-intervals (e.g., the patterns are formed by anomalies in consecutive months) and inter- annual time-intervals (e.g., the patterns are formed by anomalies in consecutive years). We discuss how the network density and topology change with these time scales, and provide evidence of correlations between geographically distant regions that occur at specific time scales. In particular, we find that an increase in the ordinal pattern spacing (i.e., an increase in the timescale of the ordinal analysis), results in climate networks with increased connectivity on the equatorial Pacific area. As the equatorial Pacific is known to be dominated by El Niño-Southern Oscillation (ENSO) on scales longer than several months, our methodology allows constructing climate networks where the effect of ENSO goes from mild (monthly OP) to intense (yearly OP), independently of the length of the ordinal pattern and of the thresholding method employed.

g-Entropies: its connections with Shannon and KS entropies and an invariant based on this concept

Fryderyk Falniowski
Cracow University of Economics, Poland
fryderyk.falniowski@uek.krakow.pl

Considering a concave function $g\colon [0,1] \mapsto \mathbb{R}$ vanishing at the origin instead of the Shannon entropy function $\eta(x) = -x \ln x$, $\eta(0) := 0$, leads to the generalizations of dynamical and measure-theoretic entropies . We will show the connections of these generalizations with the dynamical and Kolmogorov-Sinai entropies. We will also introduce the concept of types of g-entropy convergence rates introduced by Blume [1] and discuss some results concerning this quantity.

References
[1] F. Blume. Possible rates of entropy convergence *Ergodic Theory & Dynam. Systems* (1997), **17**, 45-70.

Finite-time entropy: a probabilistic approach for measuring nonlinear stretching

Gary Froyland
University of New South Wales, Australia
g.froyland@unsw.edu.au
Kathrin Padberg-Gehle

Transport and mixing processes in dynamical systems are often difficult to study analytically and therefore a variety of numerical methods have been developed. Finite-time Lyapunov exponents (FTLEs) or related stretching indicators are frequently used as a means to estimate transport barriers. Alternatively, eigenvectors, singular vectors, or Oseledets vectors of numerical transfer operators find almost-invariant sets, finite-time coherent sets, or time-asymptotic coherent sets, respectively, which are minimally dispersed under the dynamics. While these families of approaches (geometric FTLEs and the probabilistic transfer operator) often give compatible results, a formal link is still missing; here we present a small step toward providing a mathematical link. We propose a new entropy-based methodology for estimating finite-time expansive behaviour along trajectories in autonomous and nonautonomous dynamical systems. We introduce the finite-time entropy (FTE) field as a simple and flexible way to capture nonlinear stretching directly from the entropy growth experienced by a small localised density evolved by the transfer operator. The FTE construction elucidates in a straightforward way the connection between the evolution of probability densities and the local stretching experienced. We develop an extremely simple and numerically efficient method of constructing an estimate of the FTE field. The FTE field is instantaneously calculable from a numerical transfer operator – a transition matrix of conditional probabilities that describes a discretised version of the dynamical system; once one has such a transition matrix, the FTE field may be computed "for free". We also show (i) how to avoid long time integrations in autonomous and time-periodic systems, (ii) how to perform backward time computations by a fast matrix manipulation rather than backward time integration, and (iii) how to easily employ adaptive methods to focus on high-value FTE regions.

Entropy-like quantities in the assessment of cardiovascular regulation

Grzegorz Graff
Gdansk University of Technology, Poland
graff@mif.pg.gda.pl
Beata Graff, Roberto Monetti, Jose M. Amigo, Krzysztof Narkiewicz

The interaction between various regulatory mechanisms in healthy persons is still not fully understood. Entropy-like quantities are valuable methods which allow the assessment of the information flow between biological time series. We discuss their application to elucidate the effect of hyperoxia on cardiovascular regulation in healthy subjects.

Evolution of random Boolean network toward close to criticality based on local transfer entropy

Taichi Haruna
Kobe University, Japan
tharuna@penguin.kobe-u.ac.jp

Random Boolean network (RBN) is an abstract model of gene regulation networks. It is known that RBN exhibits a continuous order-disorder phase transition. In recent years, it has been shown that real-world gene regulation networks are working at close to criticality. It has been suggested that optimization of information transfer on gene regulation networks has certain evolutionary advantage because there are evidences that information processing ability of RBN is maximized at criticality. Some authors have been proposed adaptive network models based on simple local rewiring rules and showed that their models evolve toward close to criticality by numerical simulations. However, the role of information transfer in the course of evolution is still unclear in these models because the rewiring rules include no quantities related to information transfer. Here, we propose a new adaptive RBN model whose local rewiring rule involves local information transfer through a single node in the network. Local information transfer is quantified by the local transfer entropy which reflects the quality of information transfer over a single link. We show that our model can evolve toward close to criticality by both numerical simulation and master equation analysis of the in-degree distribution.

A theorem in queueing theory with applications to queueing networks

James Keesling
University of Florida, USA
kees@ufl.edu
Louis Block, Jo Ann Lee, Lenka Rucka

Let T^n be an $n-$torus and let $A : T^n \to T^n$ be a hyperbolic automorphism on T^n. We give a method with a theoretical basis for using A to generate a sequence of pseudorandom numbers on $[0, 1]^n$. Such a pseudo-random number generator would have useful applications.

On determining the Kolmogorov-Sinai entropy

Karsten Keller
University of Luebeck, Germany
keller@math.uni-luebeck.de

Recent results show that there is a close relation between the Kolmogorov-Sinai entropy and the relatively new concept of Permutation entropy. Here we discuss these results in a broader framework. In particular, we show that under certain separation conditions the distribution of ordinal patterns, which describe the up and down in a system, is sufficient for determining the Kolmogorov-Sinai entropy. Moreover, we adress methods of estimatiing Kolmogorov-Sinai entropy on the base of the given approach and compare them with known methods.

Symbolic Correlation Integral

Mariano Matilla-Garcia
UNED, Spain
mmatilla@cee.uned.es
M. Victoria Caballero, Manuel Ruiz

We introduce the symbolic correlation integral SC(m), which avoids the noisy parameter "distance" of the classical correlation integral defined by Grassberger-Procaccia. Moreover we provide the asymptotic distribution of SC(m) under the null of i.i.d.. With a MonteCarlo simulation we show the size and the power performance of the new test under linear and nonlinear processes.

Periodic points of latitudinal sphere maps

Michal Misiurewicz
Indiana University-Purdue University Indianapolis, USA
mmisiure@math.iupui.edu

For the maps of the two-dimensional sphere into itself that preserve the latitude foliation and are differentiable at the poles, lower estimates of the number of fixed points for the maps and their iterates are obtained. Those estimates give also show that the growth rate of the number of fixed points of the iterates is larger than or equal to the logarithm of the absolute value of the degree of the map.

Characterizing thalamocortical information flow using ordinal symbolic measures

Roberto Monetti
Fundacion Escuela Medicina Nuclear (FUESMEN-CNEA-UNC), Mendoza, Argentina
r.monetti@gmail.com
Nuria Tort-Colet, Jose M. Amigo, and Maria V. Sanchez-Vives

The thalamus and the cerebral cortex are reciprocally connected brain structures. In the physiological realm, the path of information flow during spontaneous activity is still being debated. Here we investigate information flow within the cortical-thalamo-cortical loop during slow oscillations (≤1Hz), i. e. spontaneous activity emerging during slow wave sleep and anaesthesia. We apply information directionality measures within the framework of the Ordinal Symbolic Dynamics to local field potential recordings simultaneously obtained from the visual thalamus and visual cortex layers 2/3 and 5. We compared across two types of waves, spontaneous and evoked by visual stimulation. In the case of spontaneous activity, we find evidence that supports models triggered by the cortical layer 5. Thus, in the absence of external stimuli cortical layer 5 leads the flow of information towards other cortical layers and thalamus. In the evoked waves, the thalamus plays a more important role, triggering activity on cortical layers 2/3 as expected from the anatomical connectivity. However, our information flow measurements reveal that cortical layer 5 has still a prominent role inducing activity on the remaining cortical layers. We will discuss how this information measurement matches and enhances the current understanding of the anatomical/physiological models of connectivity.

Computing the topological entropy of continuous maps using kneading sequences

María Muñoz Guillermo
Universidad Politécnica de Cartagena, Spain
maria.mg@upct.es
Jose S. Cánovas

Following the ideas of Block, Keesling, Li and Peterson we introduce an algorithm to compute the topological entropy, with prescribed accuracy, of maps with four pieces of monotonicity. We also show how to extend our algorithm for multimodal maps with at most three different kneading sequences. As an application, we compute the topological entropy of 3–periodic sequences of logistic maps.

Temporal symbolic transfer entropy: Measuring information transfer in real-time

Kohei Nakajima
ETH Zurich, Switzerland
jc_mc_datsu@yahoo.co.jp
Taichi Haruna

In nature, dynamical systems with many interacting elements, such as neural networks, often change their modality of couplings over time. Here, we propose an information theoretic measure that effectively monitors these dynamic changes for information transfer. Dealing with time series in real-time inevitably introduces two issues: the impossibility of predefining the range of the state space and of obtaining the probability distribution of occurrence of states, because, in principle, the entire time series are not provided beforehand. We here propose a measure called "temporal symbolic transfer entropy" to overcome these issues in assessing information transfers in real-time. The measure is based on transfer entropy, with two modifications in its setting; one is the use of the permutation partitioning, and second is an introduction of the time evolution scheme of the probability distributions. We will illustrate the power of this measure in a number of experiments using time series data from model systems, neuroscience, and soft robotics by comparison with conventional approaches.

Entropy, inverse limits and attractors

Piotr Oprocha
AGH University, Poland
oprocha@agh.edu.pl
Jan Boroński

Works of Barge and Martin provide a nice tool for construction of attractors by inverse limits of one dimensional maps. It was also discovered that topological structure of these inverse limits (attractors) is related to dynamical properties of bonding maps such as structure of periodic orbits or topological entropy. In this talk we will present some more recent result related to that topic, in particular rigidity of topological entropy when the attractor is a hereditarily indecomposable continuum.

Identifying dynamical features using ordinal patterns

Ulrich Parlitz
Max Planck Institute for Dynamics and Self-Organization, Germany
ulrich.parlitz@ds.mpg.de

Ordinal pattern statistics is a versatile method for analyzing time series from dynamical systems. In this contribution we shall present and discuss applications such as signal classification, parameter estimation, detection of structural changes, and synchronization analysis.

Detecting order in an ordinal pattern

Bernd Pompe
University of Greifswald, Germany
pompe@uni-greifswald.de

In practical time series analysis we are dealing with samples of finite size T, often $10 \ll T < 1000$. On an ordinal level, such a sample forms an ordinal pattern (OP). If there are no tied ranks, the OP is a permutation of $(1, 2, \dots, T)$. If the series is a finite realization of a continuous iid process, each possible OP occurs with the same probability $1/T!$. This is called completely unordered behavior. However, a given process might not generate all these permutations. This holds especially for chaotic time series. In the talk some proposals are made to detect order by a recurrence analysis within an OP leading to entropy–like quantities.

Maximum Entropy Complex Networks, and how to measure them

Michael Small
University of Western Australia, Australia
michael.small@uwa.edu.au
Kevin Judd and Thomas Stemler

We use an edge-switching algorithm to produce random realisations of a particular network degree distribution. These networks, we claim are maximum entropy realisations. That is, they are random realisations of a given class of networks. When we apply this to the particular case of scale-free complex networks and we find a richer variety of complex networks than typified by (for example) preferential attachment. For weighted (and unweighted) complex networks we define a path-length depedent measure of variability in the structure of the network: this quantity we call the network entropy - as it is defined in an entropy-like manner based on the homogeneity of path-lengths between random nodes. We study the variation in this property across network classes and apply it to the special case of networks generated from time series via an ordinal partitioning of the quantised scalar signal.

Ordinal time-series analysis applied to the characterization of a forced excitable system

Taciano Sorrentino
Universitat Politecnica de Catalunya, Spain
taciano@ufersa.edu.br
A. Aragoneses, M. C. Torrent and C. Masoller

Semiconductor lasers have been used in the last few decades to experimentally explore a variety of dynamical phenomena, among them regular and extreme pulses, multistability, intermittency, quasiperiodicity and chaos. When a semiconductor laser is operated near the threshold current with a moderate level of optical feedback, the output intensity shows sudden, apparently random, dropouts known as low frequency fluctuations (LFF). In the LFF regime semiconductor lasers exhibit excitable behavior and, through the modulation of the injection current, the system can be easily periodically forced. In this work we study the dynamics of such a system using ordinal time-series analysis. This method takes into account the order of consecutive inter-dropout time intervals and allows us to uncover serial correlations in the sequence of intensity dropouts exhibited by the laser. We transform the sequence of inter-dropout intervals into a sequence of ordinal patterns and we analyze the statistics of the patterns and of the transitions between them. We unveil correlations among several consecutive dropouts and we identify clear changes in the dynamics as the modulation amplitude and frequency vary.

Conditional entropy of ordinal patterns and its possible applications

Anton Unakafov
University of Luebeck, Germany
anton@math.uni-luebeck.de
Karsten Keller

In this talk we discuss a quantity called conditional entropy of ordinal patterns (CEofOP). It is similar to the celebrated permutation entropy: the latter characterizes the diversity of ordinal patterns themselves, whereas the CEofOP – the average diversity of the ordinal patterns succeeding a given ordinal pattern. We observe that in several relatively simple cases including systems with regular dynamics and Markov shifts over the binary alphabet, the CEofOP for a finite order d coincides with the Kolmogorov-Sinai entropy, while the permutation entropy only asymptotically approaches it. Moreover, we demonstrate that under certain assumptions CEofOP provides a better estimation of the KS entropy than the permutation entropy. Finally we discuss possible applications of the CEofOP to the segmentation of time series.

Relationship between permutation entropy and Kolmogorov-Sinai entropy with examples from interval maps

Valentina Unakafova
University of Luebeck, Germany
unakafov@math.uni-luebeck.de
Karsten Keller

In this talk we discuss the relationship between permutation entropy and Kolmogorov-Sinai entropy. The considerations are based on comparing two special partitions of the state space induced by ordinal patterns. An ordinal pattern of order d describes the order relations between the components of a $(d+1)$-dimensional vector. We consider, on the one hand, the partition $\mathcal{P}(d+n-1)$ provided by ordinal patterns of order $d+n-1$ and, on the other hand, the partition $\mathcal{P}(d)_n$ provided by n successive ordinal patterns of order d. Due to recent results, the answer to the question of how much more information longer ordinal patterns of order $d+n-1$ provide than n successive shorter ordinal patterns of order d gives an approach to comparing the entropies. We present some extreme examples shedding some new light on the problem of coincidence of the entropies. We also discuss some combinatorial properties regarding the partitions $\mathcal{P}(d+n-1)$ and $\mathcal{P}(d)_n$, which provide a better understanding of the considered problem.

Generalized entropies, Large Deviations, and Multifractal Analysis

Evgeny Verbitskiy
Leiden University, Netherlands
evgeny@math.leidenuniv.nl

Generalized entropies appear naturally in problems of multifractal analysis, but also in the so-called large deviations rate functions of dynamical systems. This link will be discussed, using standard, as well, as some and novel examples of multifractal spectra.

Special Session 69: Lie Symmetries, Conservation Laws and Other Approaches in Solving Nonlinear Differential Equations

Chaudry Masood Khalique, North-West University, So Africa
Maria Gandarias, University of Cadiz, Spain
Mufid Adudiab, Texas A&M University-Corpus Christi, USA

This session is devoted to all research areas that are related to nonlinear differential equations and their applications in science and engineering. The main focus of this special session is on the Lie symmetry analysis, conservation laws and their applications to ordinary and partial differential equations. Other approaches in finding exact solutions to nonlinear differential equations will also be discussed. This includes, but not limited to, asymptotic analysis methodologies, the simplest equation method, the multiple exp-function method, the (G'/G) expansion method, inverse scattering transform techniques, the upper-lower solutions method, the Hirota method, the Adomian decomposition method, and others.

Traveling wave solutions and conservation laws for a nonlinear generalized Benjamin Equation

Mufid Abudiab
Texas A&M University-Corpus Christi, USA
mufid.abudiab@tamucc.edu
Chaudry Masood Khalique

In this talk we study a nonlinear generalized Benjamin Equation, which is used in the analysis of long waves in shallow water. Traveling wave solutions are obtained by direct integration and simplest equation method. Moreover, conservation laws are derived by employing the multiplier method

Lie symmetry analysis and conservation laws of a (2+1) dimensional Haragus-Courcelle-Ilichev model

Abdullahi Adem
North-West University, So Africa
Abdullahi.R.Adem@gmail.com
Chaudry Masood Khalique

In this talk, we investigate the exact solutions and conservation laws of a (2+1) dimensional 6th-order model proposed recently by Haragus-Courcelle and Ilichev that models the wave propagation on an ocean or water surface in the presence of sea ice and also surface tension. The Lie group method in conjunction with a new extended simplest equation method is employed to seek exact solutions. Furthermore, the conservation laws are constructed for the underlying equation.

Symmetry analysis of a generalized two-dimensional nonlinear Kadomtsev-Petviashivilli-modified equal width equation

Khadijo Adem
North-West University, So Africa
Khadijo.R.Adem@gmail.com
Chaudry Masood Khalique

In this talk Lie symmetry analysis is performed on the generalized two-dimensional nonlinear Kadomtsev-Petviashivilli-modified equal width (KP-MEW) equation. The symmetries and adjoint representations for this equation are given, an optimal system of one-dimensional subalgebras is derived. The similarity reductions and exact solutions with the aid of (G'/G)-expansion method are obtained based on the optimal systems of one-dimensional subalgebras.

Mathematical Treatment of Thomas-Fermi Equation by Trigonometric and Hyperbolic Method

Iftikhar Ahmad
University of Gujrat, Pakistan
dr.iftikhar@uog.edu.pk
Siraj-ul-Islam Ahmad, Muhammad Bilal

We propose a new Mathematical technique for solving the highly nonlinear ordinary differential equation which is called Thomas-Fermi equation for finite interval with the help of Trigonometric and Hyperbolic functions. This approach is based on Active Set (AS), Sequential Quadratic Programming (SQP) and hybrid method with combination of Mathematica that is indeed a good combination in present as well as for future. Our method reduces the error between reference solution and numerical solution. Comparison of our numerical results with those obtained by other approaches shows high accuracy of our method which provide evidence that the present solution is highly accurate.

A certain diffusive epidemic model with spreading fronts of an infective environment

Inkyung Ahn
Korea University, Korea
ahnik@korea.ac.kr

A certain diffusive epidemic model is investigated to understand infective environment. The free boundary is introduced to describe the expanding front of an infective environment induced by transmitted disease. The basic reproduction number for the free boundary problem is introduced, and the behaviour of positive solutions to the reaction-diffusion system is discussed. Sufficient conditions for the bacteria to vanish or spread are given.

Generalization of Noether's method to conservation laws of non-Lagrangian PDEs

Stephen Anco
Brock University, Canada
sanco@brocku.ca

I will review some old and new work on algorithmic methods (Anco & Bluman, Ibragimov) for finding conservation laws of PDEs without a Lagrangian.

A spatial-temporal model of macrophage-tumor interaction with chemotaxis

Seunghyeon Baek
Korea University, Korea
seunghyeon9@gmail.com
Inkyung Ahn

In this talk, we consider a spatio-temporal mathematical model which describes the spatial interaction of macrophages, tumor cells, normal tissue cells, and chemoattractants. The ODE model[1] is extended to a diffusive model with the directed movement of macrophage to represent the chemotaxis between macrophage and chemoattractant. We examine the effects of chemotaxis of macrophage by chemicals such as MCP-1 secreted by tumor cells. In particular, the change of density of tumor and macrophage is investigated by using various coefficients of the chemotaxis by means of chemoattractant. Numerical simulations for tumor and macrophage in two dimension are provided under various scenarios on the coefficient of directed movement.

References

[1] Macrophage-Tumor interactions: In vivo dynamics, H.M.Byrne, S.M.Cox, C.E.Kelly, DCDS-B 4(1):81-98 2004

Classical symmetries and exact solutions for a Generalized Benjamin equation

Maria Bruzon
University of Cadiz, Spain
m.bruzon@uca.es
Jose C. Camacho

We apply the Lie-group formalism to deduce symmetries of the generalized Benjamin equation,

$$u_{tt} + a(u^p u_x)_x + \beta u_{xxxx} = 0 \tag{1}$$

where a, β and p are arbitrary constants different to zero. This kind of equation is one of the most important nonlinear partial differential equations, used in the analysis of long wave in shallow water. We make an analysis of the symmetry reductions of equation (1). In order to obtain travelling wave solutions we apply an indirect F-function method. We obtain some periodic wave solutions expressed by various single and combined nondegenerative Jacobi elliptic function solutions and their degenerative solutions.

Similarity Reductions of a nonlinear model for vibrations of beams

Jose Carlos Camacho
Universidad de Cadiz, Spain
josecarlos.camacho@uca.es
Maria de los Santos Bruzon

In this paper we make an analysis of the symmetry reductions of the nonlinear beam equation of Kirchoff type given by,

$$\Delta \equiv u_{tt} + u_{xxxx} - M\left(\int |u_x|^2 dx\right) u_{xx} + \nu u_t = 0.$$

where $u = u(x,t)$ is the transverse deflection of beam which changes its configuration at each instants of time, increasing its deformation and hence increasing its tension. We use the classical Lie method of infinitesimals. We consider travelling wave reductions depending on the form of an arbitrary function. We present some reductions.

Strict self-adjointness and shallow water models

Priscila Leal da Silva
UFABC, Brazil
pri.leal.silva@gmail.com
Igor Leite Freire

In this work we establish conditions for a class of third order partial differential equations to be strictly self-adjoint. Then, from a strictly self-adjoint subclass we consider those equations that admit a certain scaling symmetry. Thus we obtain a strictly self-adjoint and scaling invariant family of equations which includes the Benjamin-Bona-Mahony, Camassa-Holm and Novikov equations, and finally construct conservation laws for them using Ibragimov's conservation theorem.

Lie symmetries and reductions of a simplified model for a benign tumour growth

Rafael de la Rosa
University of Cadiz, Spain
rafael.delarosa@uca.es
Maria Luz Gandarias and Maria de los Santos Bruzon

The purpose of this work is to find local symmetries and exact solutions of the following model given by Perumpanani et al.:

$$\frac{\partial u}{\partial t} = f(u) + u_{xx}$$
$$\frac{\partial v}{\partial t} = k(v_x u_x + v u_{xx})$$

This model described benign tumour growth where *u(x,t)* represents the concentrations of the tumour cells, *v(x,t)* the connective tissue, and x and t are the space and time coordinates. The model studies the averaged behaviour of the tumour cells in the direction of expansion only, disregarding all the variations in the plane perpendicular to the direction growth. Symmetry groups have several different applications in the context of nonlinear differential equations. In particular, they are used to obtain exact solutions of partial differential equations. We calculate the symmetries of the model applying Lie's classical method. Then, we construct the optimal system, to determine those solutions that cannot be derived from others under the action of a symmetry group. Using the optimal system, we obtain the variables and similarity solutions. This, we allow us to transform our system of partial differential equations into a system of ordinary differential equations. From these reductions we find exact solutions of the model.

On the symmetries of a high order class of ordinary differential equations

Igor Freire
UFABC, Brazil
igor.leite.freire@gmail.com
Priscila Leal da Silva

In this talk we discuss about a class of autonomous, even-order ordinary differential equations from the point of view of Lie symmetries. It is shown that for a certain power nonlinearity, the Noether symmetry group coincides with the Lie point symmetry group. First integrals are established and exact solutions are found. Furthermore, these results complements, for the one-dimensional case, some results in the literature of Lie group analysis of poliharmonic equations and Noether symmetries obtained in the last twenty years.

Symmetries, solutions and conservation laws of a class of nonlinear dispersive wave equation

Maria Gandarias
University of Cadiz, Spain
marialuz.gandarias@uca.es

In a previous work we have considered a forced Korteweg-de Vries which serves as an analytical model of tsunami generation by submarine landslides. In this work we consider a damped externally excited Korteweg-de Vries equation with a forcing term. We derive the classical Lie symmetries admitted by the equation. Looking for travelling waves solutions we find that the damped externally excited KdV equation has some exact solutions which are periodic waves and solitary waves. These solutions are derived from the solutions of a simple nonlinear ordinary differential equation. By using a general theorem on conservation laws and the multiplier method, we find some conservation laws for some of these partial differential equations.

Lie point symmetries for a generalized Drinfel'd-Sokolov equations

Tamara Garrido
Cadiz University, Spain
tamara.garrido@uca.es
Maria S. Bruzon

We consider the system of generalized Drinfeld–Sokolov equations which models one-dimensional nonlinear wave processes in two-component media

$$u_t + \alpha_1 u u_x + \beta_1 u_{xxx} + \gamma(v^\delta)_x = 0,$$
$$v_t + \alpha_2 u v_x + \beta_2 v_{xxx} = 0.$$

We obtain the Lie group classification depending on the parameters. We derive the reduction to systems of ordinary differential equations from the optimal system of subalgebras. We present some particular solutions.

Symmetry analysis and conservation laws for a coupled (2+1)-dimensional hyperbolic system

Chaudry Masood Khalique
North-West University, So Africa
Masood.Khalique@nwu.ac.za

In this talk we perform Lie symmetry analysis and Noether symmetry classification of a coupled (2+1)-dimensional hyperbolic system. Moreover, conservation laws for several cases which admit Noether point symmetries are established for the underlying system

On Diffusive Predator-prey Models With Stage Structure

Seong Lee
Korea University, Korea
imp57@korea.ac.kr
Inkyung Ahn

In this talk, we study the asymptotic property of a diffusive delayed predator-prey model with Beddington-DeAngelis type functional response under homogeneous Neumann boundary conditions, where the discrete time delay covers the period from the birth of immature preys to their maturity. We establish the threshold dynamics of their permanence and the extinction of the predator. We also give sufficient conditions for the global attractiveness of the semi-trivial and coexistence equilibria. Furthermore, we provide sufficient conditions for the local asymptotical stability at these equilibria under some assumptions. Biologically, the results imply that the variation of prey stage structure can affect the permanence of the system and drive the predator into extinction by having a large handling time or a low capture rate. Our results show that the predator coexists with prey if the system has sufficiently large mutual interference of predators or low death rate of it. The methods used in this paper are comparison argument and upper-lower solution method to examine the existence of unique coexistence solution. Furthermore, the linearized stability theory and Lyapunov function theory for the asymptotical stability of equilibria are employed.

Characterizing Derivatives of Solutions to Nonlocal Boundary Value Problems with Parameter

Jeffrey Lyons
Nova Southeastern University, USA
jlyons@nova.edu

In this talk, we will introduce a nonlocal boundary value problem with parameter and its associated variational equation. Next, we show that if certain conditions are placed upon the boundary value problem, a solution to

the may be differentiated with respect to the boundary data and parameter. The results either solve the previously introduced variational equation or another related boundary value problem each with their own specific nonlocal boundary conditions.

Lagrangian formulation of a generalized (2+1)-dimensional hyperbolic system

Ben Muatjetjeja
North-West University, So Africa
Ben.Muatjetjeja@nwu.ac.za

In this talk we perform Noether symmetry classification of a generalized (2+1)-dimensional hyperbolic system and compute the Noether operators corresponding to its first-order Lagrangian. In addition conservation laws of the various cases which admit Noether point symmetries are constructed for the underlying system.

Comparison of different approaches to construct first integrals for ordinary differential equations

Imran Naeem
Lahore University of Management Sciences (LUMS), Pakistan
imran.naeem@lums.edu.pk
Rehana Naz, Igor Leite Freire

Different approaches to construct first integrals for ordinary differential equations and systems of ordinary differential equations are studied here. We apply these different approaches to derive the first integrals of harmonic oscillator equation. We also study first integrals for some physical models. The first integrals for nonlinear jerk equation and the free oscillations of a two degrees of freedom gyroscopic system with quadratic nonlinearities are derived. Moreover solutions via first integrals are also constructed.

A Hamiltonian Approach to Equations of Economics growth theory

Rehana Naz
Centre for Mathematics and Statistical Sciences, Lahore School of Economics, Pakistan
rehananaz_qau@yahoo.com
Fazal M Mahomed, Azam Chaudhry

We develop a partial Hamiltonian framework to obtain reductions and closed-form solutions via first integrals of current value Hamiltonian systems of ordinary differential equations (ODEs). The approach is algorithmic and applies to many state and costate variables of the current value Hamiltonian. However, we apply the method to models with one control, one state and one costate variable to illustrate its effectiveness. The current value Hamiltonian systems arise in economic growth theory and other economic models. We explain our approach with the help of a simple illustrative example and then apply it to two widely used economic growth models: the Ramsey model with a constant relative risk aversion utility function and Cobb Douglas technology and a one-sector AK model of endogenous growth are considered.

Determination of First Integrals and Symmetries of the Second Order Ordinary Differential Equations Using Linearization Methods

Ozlem Orhan
Istanbul Technical University, Turkey
orhanozlem@itu.edu.tr
Teoman Ozer

We study first integrals, λ-symmetries and integrating factor of the second order ordinary differential equation. We characterize the second order ordinary differential equations by the coefficients of the equation and we obtain first integrals, λ-symmetries and integrating factor in the specific form. Then using linearization methods, we determine first integrals, λ-symmetries and integrating factor of nonlinear fin equation in which the thermal conductivity and heat transfer coefficient are assumed to be functions of the temperature. And we examine these finding results for different types of thermal conductivity and the heat transfer coefficient functions. Finally, we analyze symmetry classification for conservation forms.

Classical symmetries for a μ-Hunter-Saxton equation.

Elena Recio
Cadiz University, Spain
elena.recio@uca.es
María de los Santos Bruzón, María Luz Gandarias

Recently, the μ-Hunter-Saxton (μHS) equation has attracted a lot of attention from researchers. The μHS equation was originally derived and studied by Khesin *et al.*, where the authors showed that this equation admits both periodic one-peakon solutions and multipeakons. In this work, we write this equation in terms of μ and we study classical Lie symmetries and reductions.

A model of malignant gliomas through symmetry reductions

María Rosa
Cádiz University, Spain
maria.rosa@uca.es
María Luz Gandarias , María de los Santos Bruzón

A glioma is a kind of tumor that starts in the brain or spine. The most common site of gliomas is in the brain. Most of the mathematical models in use for malignant gliomas are based on a simple reaction-diffusion equation: the Fisher equation. A nonlinear wave model describing the fundamental features of these tumors has been introduced by V.M. Pérez and collaborators. In this work, we study this model from the point of view of the theory of symmetry reductions in partial differential equations. We obtain the classical symmetries admitted by the system, then, we use the transformations groups to reduce the equations to ordinary differential equations. Some exact solutions are derived from the solutions of a simple nonlinear ordinary differential equation.

On symmetries of a Klein-Gordon equation

Lucia Vazquez
Cadiz University, Spain
lucy_86torre@hotmail.com
Maria L. Gandarias, Maria S. Bruzon

The Klein-Gordon equation which plays an important role in mathematical physics. The equation has attracted much attention in studying solitons and condensed matter physics, in investigating the interaction of solitons in collisionless plasma and the recurrence of initial states. In this work we consider the nonlinear Klein-Gordon equation from the point of view of Lie symmetries. We derive Lie classical generators and we obtain similarity solutions, reductions and some exact solutions.

On the structure of the solution set to Killing-type equations and LRL conservation

Gaetano Zampieri
Università di Verona, Italy
gaetano.zampieri@univr.it
Gianluca Gorni

Infinitesimal gauge-invariance of the action functional in Lagrangian ODEs is equivalent to Killing-type equations, which are related to first integrals through Noether theorem. We discuss the structure of the solution set of two interpretations of Killing-type equations. In particular, we give an explicit solution formula for the “on-flow” interpretation. By means of these results, we study LRL (Laplace-Runge-Lenz) vector conservation for the Kepler problem, showing new solutions, in particular without gauge terms.

Special Session 70: Nonlinear Phenomena: Theory and Applications

Francesco Demontis, Department of Mathematics and Computer Science, University of Cagliari, Italy
Sara Lombardo, Northumbria University, Department of Mathematics and Information Sciences, England
Giovanni Ortenzi, Department of Mathematics and Applications, University of Milano-Bicocca, Italy
Matteo Sommacal, Department of Mathematics and Information Sciences, Northumbria University (UK), England

Integrable nonlinear evolution equations arise in many applications, such as propagation of signals in optical fibers, water waves, Bose-Einstein condensation, two-dimensional gravity, interface motions, propagation of deformations along the DNA chain. The importance of this topic makes clear why it is important to get information on the behavior of the solutions of these equations. To achieve this aim, many methods and different techniques have been developed over the years, including the Inverse Scattering Transform (IST), the Darboux transformation, the Hirota and d-bar dressing method. In this session we intend to analyze the recent advancements in this field considering different points of view. The session broadly focuses on nonlinear phenomena amenable of being described by means of tools derived within the theory of integrable systems, such as rogue waves and Peregrine-like solitons, dispersive shock waves, wave breaking in multidimensions, viscous conservation laws, stratified fluids, magnetic droplet solitons, isochronicity and chaos in many-body problems. In particular, the possibility to get explicit solutions by using the Inverse Scattering Transform and the use of the associated Hamiltonian structure for finding the symmetries of the system will be discussed.

The modulational instability revisited

Gino Biondini
State Univ. of New York at Buffalo, USA
biondini@buffalo.edu
Emily Fagerstrom

The modulational instability (MI), also known as Benjamin-Feir instability in the context of water waves, is one of the most widespread phenomena in nonlinear science. In many cases, the underlying dynamics is governed by the nonlinear Schrodinger (NLS) equation. The initial stage of MI can therefore be described by linearizing the NLS equation around a constant background. Once the perturbations have grown, however, the linearization ceases to be valid. On the other hand, the NLS equation is a completely integrable infinite-dimensional Hamiltonian system, and the initial-value problem is therefore amenable to solution via the inverse scattering transform (IST). In this talk I will describe how the recently-developed IST for the focusing NLS equation with non-zero boundary conditions can be used to elucidate the nonlinear stage of the MI.

On conservation laws for stratified incompressible Euler fluids in 2D channels

Gregorio Falqui
Milano-Bicocca University, Italy
gregorio.falqui@unimib.it
R. Camassa and S. Chen (UNC at Chapel Hill)
G. Ortenzi and M. Pedroni (UniBG).

We shall discuss aspects of the theory of incompressible stratified Euler fluids in a $2D$ channel. In particular, our focus will be on conserved quantities, both for continuous and sharp (two-layer) stratification. We shall analytically show the existence of classes of initial data for which total horizontal momentum is not conserved, and briefly compare these results with long-wave asymptotic models. Hamiltonian pictures (both in the full $2D$ case and in the $1D$ long wave limit(s)) will then be discussed in order to illustrate these results.

Exceptional Hermite polynomials and rational solutions of NLS

David Gomez-Ullate
ICMAT and Universidad Complutense, Spain
dgomezu@ucm.es

Exceptional Hermite polynomials are complete systems of orthogonal polynomials that satisfy a Sturm-Liouville problem in the real line. They differ from Hermite polynomials in that there are a finite number of gaps in their degree sequence. We will show a complete classification of such polynomial systems by using Darboux-Crum transformations and showing that the associated potentials are monodromy free. A convenient representation of exceptional Hermite polynomials is through Wronskian determinants of certain sequences of classical Hermite polynomials, and we will discuss some properties of their real and complex zeros. Particular cases of exceptional Hermite polynomials appear in the construction of rational solutions to Nonlinear Schrödinger's Equation, and we will try to illustrate this connection.

Dark soliton dynamics in the presence of higher order and perturbative effects

Theodoros Horikis
University of Ioannina, Greece
horikis@uoi.gr

Dark solitons are intensity dips on a constant background with a phase jump across their intensity minimum. In physically relevant settings, linear/nonlinear dissipative effects are naturally important, and usually lead to the decay of the background; this way, the composite object composed by the background and the soliton is destroyed. In this context, a particularly interesting problem concerns the stabilization of dark solitons. Unlike their bright counterparts, i.e., solitons that decay at infinity, dark solitons -under the influence of perturbations-develop a shelf that propagates with speed determined by the background intensity. Using a newly developed perturbation theory we show that the perturbing effects give rise to a shelf that accompanies the soliton in its propagation. Although, the stable solitons are not affected by the shelf it remains an integral part of the dynamics otherwise missed so far in the literature. Furthermore, it is found that for certain values of the parameters a stable evolution can exist for both the soliton and the relative continuous wave background.

Dispersive deformations of integrable 2+1 dimensional hydrodynamic-type systems

Benoit Huard
Northumbria University, England
benoit.huard@northumbria.ac.uk
V. Novikov

It has long been recognised that waves and their interactions play a fundamental role in the theory of integrability of partial differential equations. On the one hand, the presence of solitonic interactions and Lax pair representations characterise integrability of dispersive equations. On the other hand, the presence of sufficiently many interacting Riemann waves can be used as a criterion for detecting integrability of dispersionless equations, leading to solutions in implicit form through the method of generalized hodograph. Remarkably, integrable dispersionless equations also arise as the compatibility condition of nonlinear pseudo-potentials. In this contribution, we propose a new approach to the classification of multidimensional integrable dispersive systems of PDEs based on the requirement that the quasi-classical limit of such a system should be generated by a polynomial pseudo-potential. An integrability-preserving perturbative procedure based on the method of hydrodynamic reductions then allows to reconstruct the corresponding dispersive systems along with associated Lax pairs. We illustrate this approach by providing a classification of systems of the Davey-Stewartson type and new integrable systems within this class are presented.

Using IST in evaluating the capacity of optical fiber channel

Morteza Kamalian
Scuola Superiore Sant'Anna, Italy
kamalian_morteza@yahoo.com
Marco Secondini, Enrico Forestieri, Ali Akbar Tadaion

The evolution of the optical signal in fiber is described by Nonlinear Schrödinger Equation (NLSE) which can be solved by means of Inverse Scattering Transformation formalism. In the eigenvalue communication paradigm, where data are mapped on the spectral parameters of the signal, the fiber channel capacity has to be evaluated using the statistical characteristics of the spectrum of the Zakharov-Shabat operator, the underlying linear operator of the NLSE. Approximating the operator by a large matrix, we can use the Green's function to calculate the spectral density i.e. the density of eigenvalues of the operator. Since the operator in not self-adjoint (non-Hermitian matrix) the conventional diagramatic methods of finding the Green's function can not be applied. Here we try to use some recently proposed methods of dealing with non-hermitian matrices such as Generalized Green's function for the problem of evaluating the capacity of optical fiber in the presence of ASE noise (non-distributed additive noise) considering the noise as the random part of the operator.

Lauricella functions, critical points and hydrodynamic systems of Whitham type

Boris Konopelchenko
Department of Mathematics and Physics, University of Salento, Lecce, Italy
boris.konopeltchenko@unisalento.it
Y. Kodama, W.K. Schief

Semi-Hamiltonian integrable hydrodynamic type systems which describe critical points of functions obeying the linear Darboux systems are discussed. It is shown that a wide class of such systems can be constructed starting with the Lauricella type solutions of the Euler-Poisson-Darboux equations. Classical multi-phase Whitham equations for the Korteweg-de Vries equation are particular examples of such systems.

On integrable conservation laws

Paolo Lorenzoni
University of Milano-Bicocca, Italy
paolo.lorenzoni@unimib.it
Alessandro Arsie, Antonio Moro

We study normal forms of scalar integrable dispersive (non necessarily Hamiltonian) conservation laws via the Dubrovin-Zhang perturbative scheme. Our computations support the conjecture that such normal forms are parametrised by infinitely many arbitrary functions that can be identified with the coefficients of the quasilinear part of the equation. More in general, we conjecture that two scalar integrable evolutionary PDEs having the same quasilinear part are Miura equivalent. This conjecture is also consistent with the tensorial behaviour of these coefficients under general Miura transformations.

Topological selection in stratified fluids: an example from air-water systems

Marco Pedroni
University of Bergamo, Italy
marco.pedroni@unibg.it
R. Camassa, S. Chen, G. Falqui, and G. Ortenzi

In this talk I will consider a 2-dimensional air-water system (i.e., a two-fluid system with the density of one of the fluids limiting to zero) in a channel. Both fluids are supposed to be inviscid, incompressible, and homogeneous. I will show that the connection properties of the air domain affect total horizontal momentum conservation, despite the apparent translational invariance of the system.

Full-vectorial finite element mode solver for lossy and non-reciprocal ring resonators

Paolo Pintus
Scuola Superiore Sant'Anna, Italy
paolo.pintus@sssup.it

Ring resonators are important building blocks used in integrated optical circuits to implement many different functions. Such devices are characterize by a cylindrical symmetry and their main benefits are compactness and possibility of dense integration. When a signal is injected into a ring resonator, its intensity is built up to a higher value and this can lead to an enhanced nonlinear response. In this work, we present a new mode solver for non-reciprocal and lossy ring resonators. Starting from Maxwell's equations, we derive the variational formulation in the cylindrical coordinate systems. The node-based finite element method is used to solve the problem and the penalty function is added to remove the spurious solutions. Finally, the bending loss are simulated by assuming perfectly matched layer (PML) as boundary condition. Unlike the earlier developed mode solver, our approach allows for the precise computation of both clockwise (CW) and counterclockwise (CCW) modes in the non-reciprocal case, ensuring high accuracy and computational efficiency. Although the preliminary results shown in this talk refer to the linear condition, the model can be easily extended for analyzing nonlinear optical regime. This work is partially supported by the European Commission FP7 grant IRIS, project no. 619194 FP7-ICT.

Inverse Scattering Transform for the focusing NLS equation with fully asymmetric boundary conditions

Barbara Prinari
University of Colorado Colorado Springs, USA
bprinari@uccs.edu
F. Demontis, C. van der Mee, F. Vitale

We present the inverse scattering transform (IST) for the focusing nonlinear Schrödinger equation: $iq_t = q_{xx} + 2|q|^2 q$, with non-zero boundary conditions $q(x,t) \sim q_{l/r}(t) = A_{/r} e^{i\theta_{l/r}(t)}$ as $x \to \mp\infty$ in the fully asymmetric case. The direct problem is shown to be well-posed for NLS solutions $q(x,t)$ such that $q(x,t) - q_{l/r}(t) \in L^{1,1}(\mathbb{R}^{\mp})$ with respect to x for all $t \geq 0$, for which analyticity properties of eigenfunctions and scattering data are established. The inverse scattering problem is formulated both via (left and right) Marchenko integral equations, and as a Riemann-Hilbert problem on a single sheet of the scattering variables $\lambda_{l/r} = \sqrt{k^2 + A_{l/r}^2}$, where k is the usual complex scattering parameter in the IST. The time evolution of the scattering coefficients is then derived, showing that, unlike the case of solutions with the same amplitude as $x \to \pm\infty$, here both reflection and transmission coefficients have a nontrivial time dependence.

Efficient numerical methods for the solution of the NLS in fiber-optic systems

Marco Secondini
Scuola Superiore Sant'Anna, Italy
marco.secondini@sssup.it
Domenico Marsella, Enrico Forestieri

Propagation of light through optical fibers is governed by the nonlinear Schrödinger (NLS) equation. The availability of efficient numerical methods for computing the NLS solution given an arbitrary initial condition is, therefore, crucial for the design of fiber-optic communication systems. Several methods are currently available: numerical integration (e.g., the split-step Fourier method—SSFM), analytical approximations (e.g., perturbation methods), the inverse scattering transform (when attenuation can be neglected). In the first part of this talk, we investigate the problem of accuracy versus complexity for the main available methods when applied to fiber-optic communications. In particular, we show how and why, despite decades of research and significant advances in the field of integrable equations, the SSFM is still the most widely used method. In the second part of the talk, we introduce a novel numerical integration method for the NLS and show that, according to some preliminary results, it can provide a better trade-off between accuracy and complexity with respect to the SSFM.

On Nonlocal Reductions of S-integrable Nonlinear Equations

Tihomir Valchev
Dublin Institute of Technology, Ireland
Tihomir.Valchev@dit.ie

Recently an integrable nonlocal version of nonlinear Schrödinger equation that could be of physical interest in the context of PT-symmetric optics has been proposed by Ablowitz and Musslimani. This motivated us to present here an adaptation of the gauge covariant formalism suited for local S-integrable equations. That will allow us to similarly treat nonlinear equations whose Lax pairs are subject to nonlocal reductions. A few examples to illustrate the general ideas shall be discussed in more detail.

Time-evolution-proof characterization of Zakharov-Shabat scattering data

Cornelis van der Mee
University of Cagliari, Italy
cornelis110553@gmail.com

In this talk we discuss the characterization of the scattering data of the Zakharov-Shabat system and, more generally, the AKNS system. More precisely, we provide a $1,1$-correspondence between (a) potentials having their entries in $L^1(\mathbb{R})$ and not leading to singularities in the transmission coefficients, and (b) Marchenko integral kernels. These kernels faithfully encode the "classical" scattering data consisting of a reflection coefficient, the discrete eigenvalues, and the norming constants. We consider the defocusing and focusing cases as well as potentials not having any adjoint symmetry. Unfortunately, the class of L^1 potentials is not invariant under time evolution according to integrable nonlinear evolution equations such as the nonlinear Schrödinger (NLS), modified Korteweg-de Vries (mKdV), and sine-Gordon equations. The main reason is that the conservation laws regard potentials belonging to Sobolev spaces, whereas the characterization regards L^1-potentials. We present sufficient conditions on the Marchenko kernels in order that the potentials are L^2. We explore sufficient conditions on L^2 potentials in order that the Marchenko kernels remain in the same class upon time evolution. At present there is no full time-evolution-proof characterization of scattering data.

Special Session 71: Recent Progress in Spintronics: Experiment, Theory and Simulation

Jingrun Chen, University of California, Santa Barbara, USA
Carlos J. Garcia-Cervera, University of California, Santa Barbara, USA
Xu Yang, University of California, Santa Barbara, USA
Sookyung Joo, Old Dominion University, USA

In solid-state physics, the active control and manipulation of spin degrees of freedom, known as Spintronics, can lead to technological advances and the design of devices with new functionality. It involves a number of processes at different time and length scales: polarization of the spin, transport and diffusion of the polarized spin between the ferromagnetic layers, and interaction with the underlying magnetization. The objective of this minisymposium is to bring together specialists in the field from different backgrounds, and facilitate the exchange of ideas and recent developments. Topics on experiment, theory and simulation will be discussed.

Energy-Minimizing Nematic Elastomers

Patricia Bauman
Purdue University, USA
bauman@math.purdue.edu
Andrea Rubiano

We prove weak lower semi-continuity and existence of energy minimizers for a free energy describing stable deformations and the corresponding director configuration of an incompressible nematic liquid-crystal elastomer subject to physically realistic boundary conditions. The energy is a sum of the trace formula developed by Warner, Terentjev and Bladon (coupling the deformation gradient and the director field) and the bulk term for the director with coefficients depending on temperature. A key step in our analysis is to prove that the energy density has a convex extension to non-unit length director fields. Our results apply to the setting of physical experiments in which a thin incompressible elastomer in R^3 is clamped on its sides and stretched perpendicular to its initial director field, resulting in shape-changes and director reorientation.

Spin-polarized transport in ferromagnetic multilayers

Jingrun Chen
University of California, Santa Barbara, USA
cjr@math.ucsb.edu
Carlos J. Garcia-Cervera, Xu Yang

Magnetic storage devices rely on the fact that ferromagnetic materials are typically bistable, and that it is possible to switch between different states by applying a magnetic field. The discovery of the Giant Magneto-Resistance effect has enabled the use of layered ferromagnetic materials in magnetic devices, such as magnetic memories (MRAMs). Even in the absence of thermal effects, there are limitations in the storage capacity of such devices due to the fact that as the size is decreased, the magnitude of the switching field increases, due to an increase in shape anisotropy. Given that magnetic fields have long range interactions, the density of such devices is limited. A new mechanism for magnetization reversal in multilayers was proposed by Slonczweski and Berger. In this new mechanism, an electric current flows perpendicular to the layers. The current is polarized in the first layer, and the polarization travels with the current to the second layer, where it interacts with the underlying magnetization. Since currents are localized in each cell, long range effects can be reduced. In this talk we will discuss the connection between several models for the description of the spin transfer torque at different physical scales. Specifically, we connect the quantum and kinetic descriptions with the help of the Wigner transform, and the kinetic and diffusion models by a specific parabolic scaling. Numerical examples will presented to illustrate the applicability and limit of the different models.

Magnonics: current status and outlook

Andrii Chumak
TU Kaiserslautern, Germany
chumak@physik.uni-kl.de

With conventional CMOS technology data is carried by flows of electrons that generate heat which is responsible for the device's power consumption. An alternative to this principle is the employment of other particles or quasi-particles as information carriers which are subject to dissipation to a much lesser degree than electrons. I will show that eigen excitations of magnetic media - magnons can be used for this role.

In my talk, after an introduction on spin waves and their quanta magnons, I will concentrate on the key information processing elements realized by means of magnons: data buffering element, time reverser of complex signals, and magnon transistor. These proof of concept devices are made out of an insulator in order to exclude any motion of free electrons and are based on a magnonic crystal (also known as a magnetic meta-material): a man-made medium with properties derived from an engineered structuring. Data buffering has been realized in a magnonic crystal by the excitation of standing quasi-normal modes. Data read-out has been implemented by means of phase-sensitive parametric amplification of the stored mode. The second device, a time reverser, is based on a dynamic magnonic crystal: a crystal with properties that can be varied using external controls on a very fast time scale. We have shown that a wave packet, while being reflected by the dynamic crystal, reverses its time profile. The time-reversal mechanism is purely linear and is applicable to waves of any nature. Finally, I will present the most recent breakthrough in the field - the realization of a magnon transistor. The device uses control of magnons by magnons, which is realized through an enhancement of nonlinear magnon interactions by the magnonic crystal. We have shown that the magnon transistor allows for the design of all-magnon logic gates as well as for enhancement of magnonic signals.

Numerical advances in Self-Consistenf Field Theory simulations of block copolymers: Applications to polymer lithography and evaporation-driven pattern selection.

Carlos Garcia-Cervera
UC Santa Barbara, USA
cgarcia@math.ucsb.edu
S. Hur, S.P. Paradiso, K.T. Delaney, H.D. Ceniceros, and G.H. Fredrickson

I will discuss some recent developments in the numerical simulation of self-consistent field theory (SCFT) for block copolymers. I will focus on the following applications:

- SCFT simulations of block copolymers laterally confined in a square well: Here we explore the conditions for which self-assembly in laterally confined thin block copolymer films results in tetragonal square arrays of standing up cylinders. More specifically, we study the equilibrium phase behavior of thin films composed of a blend of AB block copolymer and A homopolymer laterally confined in square wells. By using suitable homopolymer additives and appropriately sized wells, we observed square lattices of upright B cylinders that are not stable in pure AB block copolymer systems. Considering the potential application of such films in block copolymer lithography, we also conducted numerical SCFT simulations of the role of line edge roughness at the periphery of the square well on feature defect populations. Our results indicate that the tetragonal ordering observed under square confinement is robust to a wide range of boundary perturbations.

- SCFT simulations of block copolymers on the surface of a sphere: In this model, we assume that the composition of the thin block copolymer film is independent of the radial direction. Using this approach we were able to study the phase separation process, and specifically the formation of defects in the lamellar and cylindrical phases, and its dependence on the radius of the sphere.

- Evaporation-Induced Ordering in Block Copolymer Thin Films.

Analysis of a de Gennes-Landau model for bent-core liquid crystals

Tiziana Giorgi
New Mexico State University, USA
tgiorgi@math.nmsu.edu
Yousef Feras

We analyze a de Gennes-Landau energy functional, which Vaupotiĉ and Ĉopiĉ introduced to describe polarization modulation instabilities in bent-core molecules liquid crystals. We consider existence of energy minimizers for the full model, and use Gamma-convergence to study the switching mechanisms in the large limit domain in a simplified model.

Field response of smectic A liquid crystals

Sookyung Joo
Old Dominion University, USA
sjoo@odu.edu

We study the Landau-de Gennes free energy to describe the undulatory instability in smectic A liquid crystals subjected to magnetic fields. We prove this phenomena by the bifurcation theory to the nonlinear system of Landau-de Gennes model for smectic liquid crystals. We find the critical field and oscillatory description of the undulations which are consistent with experimental results. When the applied field is sufficiently large, the smectic states are maintained, with the director parallel to the field. We perform numerical simulations to illustrate the results of our analysis and demonstrate the tilted layers and directors near the boundary at the equilibrium state. The three dimensional study for smectic liquid crystals under the magnetic field will be also discussed.

Electric Field Control of Magnetism in Multiferroic Tunnel Junctions

Nicholas Kioussis
Caifornia State University Nortrhidge, USA
nick.kioussis@csun.edu
P. V. Ong, Artur Useinov, Alan Kalitsov, and Julian Velev

I will present electronic structure calculations of the effect of electric field and heavy metal (Ta, Hf, Pd) cap on the magnetocrystalline anisotropy (MCA) of magnetic tunnel junctions. I will discuss predictions of giant sensitivity of the interfacial magnetic anisotropy energy to applied electric field which can be tuned via epitaxial strain. The underlying mechanism lies on the electric field- or strain-induced re-distribution of d-orbital character of the electronic states. I will show how the interfacial MCA can be selectively tuned by the heavy metal cap thickness through the spin-orbit coupling of the spin-polarized quantum well states. Finally, I will present predictions of the ferroelectric control of spin transfer torque (STT) in multiferroic tunnel junctions including a novel polarization-assisted STT reversal. The underlying mechanism is the interplay of the bias-induced and polarization-induced spin-dependent screening. These results open a new avenue for applications in multiferroic devices.

Liquid crystal enabled electrophoresis and electroosmosis

Oleg Lavrentovich
Liquid Crystal Institute, USA
olavrent@kent.edu

The review presents recent progress in exploration of dynamic phenomena enabled by liquid crystals and topological defects in them namely, (i) Nonlinear electrophoresis of colloidal particles and (b) Nonlinear electroosmosis of a liquid crystal fluid around immobilized spheres. The main object of study represents a colloidal sphere with perpendicular anchoring of the director embedded into an otherwise uniform nematic bulk and subject to an external electric field. The sphere distorts the director field around itself, introducing a topological point defect, the so-called hyperbolic hedgehog. In the applied electric field, a free sphere experiences electrophoretic motion with a velocity that grows as a square of the applied field. By combining a classic linear electrophoretic dynamics with the non-linear liquid crystal-enabled electrophoresis, one can control three dimensional trajectories of the particles. A sphere immobilized at a substrate serves as an electroosmotic pump. The research establishes the patterns of the liquid crystal flows around the particles. The work is supported by NSF DMR 1104850.

Magneto-Seebeck effect and THz spintronics

Markus Muenzenberg
Institute of Physics, Greifswald University, Germany
mmuenze@gwdg.de

Creating temperature gradients has resulted in a new research direction in spinelectronics – spincaloritronics. By optical heating using a small laser focus on a magnetic tunnel junction, I show that a large Seebeck voltages can be generated. The tunnel junctions' Seebeck coefficients can be changed by the magnetic configuration. New techniques for the detection of ultrafast spin currents have to be developed to bring spin-based information technology to ultrafast time scales. These can be quanitatively determined by THz emission. The original transient spincurrents can be calculated by the theory of super diffusion.

Properties of minimizers to the Maier-Saupe energy for liquid crystals.

Daniel Phillips
Purdue University, USA
phillips@math.purdue.edu
Patricia Bauman

We prove regularity properties and determine bounds for local minimizers to an energy derived from Maier-Saupe theory that is used to characterize order in nematic liquid crystal materials.

Giant thermal spin torque assisted magnetic tunnel junction switching

Aakash Pushp
IBM Almaden Research Center, USA
apushp@us.ibm.com
Timothy Phung, Charles Rettner, Brian P. Hughes, See-Hun Yang, Stuart S.P. Parkin

It is well established that spin-polarized charge currents can induce magnetic tunnel junction (MTJ) switching by virtue of spin transfer torque (STT). Recently, by taking advantage of the spin-dependent thermoelectric properties of magnetic materials, novel means of generating spin currents from temperature gradients, and their associated thermal spin torques (TSTs) have been proposed, but so far these TSTs have not been large enough to influence MTJ switching. In this talk, I will show evidence of significant TSTs in MTJs by generating large temperature gradients across ultrathin MgO tunnel barriers that considerably affect the switching fields of the MTJ. Furthermore, I will show that the TST strongly depends on the relative orientation of the free and the reference layers of the MTJ, consistent with recent theoretical work, and then discuss how the origin of the TST can be attributed to an asymmetry of the tunnelling conductance across the zero bias voltage of the MTJ. Remarkably, the estimates of the charge currents that are inevitably generated through Magneto-Seebeck voltages give rise to STT that are a thousand times too small to account for the changes in switching fields observed experimentally.

From spin pumping to spin Seebeck effect

Eiji Saitoh
Tohoku University, Japan
satou.yumiko@imr.tohoku.ac.jp

Spin current, a flow of electrons' spins in a solid, is the key concept in spintronics that will allow the achievement of efficient computing devices, and energy converters. I here review phenomena which allow us to use spin currents in insulators [1]: inverse spin-Hall effect [2,3], spin pumping, and spin Seebeck effect [3,4]. We found that spin pumping and spin torque effects appear at an interface between an insulator YIG and Pt. Using this effect, we can connect a spin current carried by conduction electrons and a spin-wave spin current flowing in insulators. Seebeck effect (SSE) is the thermal spin pumping [4]. The SSE allows us to generate spin voltage, potential for driving nonequilibrium spin currents, by placing a ferromagnet in a temperature gradient. Using the inverse spin-Hall effect in Pt films, we measured the spin voltage generated from a temperature gradient in various ferromagnetic insulators.

References

[1] Y. Kajiwara & E. Saitoh et al. Nature 464 (2010) 262.
[2] E. Saitoh et al., Appl. Phys. Lett. 88 (2006) 182509.
[3] K. Uchida & E. Saitoh et al., Nature 455 (2008)778.
[4] K. Uchida & E. Saitoh et al., Nature materials 9 (2010) 894.

Nonhomogeneous spin-orbit interaction in nanostructures

David Sanchez
University of the Balearic Islands, Spain
david.sanchez@uib.es

We consider spin-orbit interaction of the Rashba type due to interfacial electric fields in asymmetric semiconductor heterostructures. In spin transistors, the Rashba interaction is localized in a one-dimensional channel, giving rise to a spin precession of injected electrons from attached ferromagnetic leads. Interestingly, we find strongly modulated transmission lineshapes when the leads are normal metals. The effect is due to asymmetric scattering with Rashba induced quasi-bound states in the localized region, as shown in our numerical calculations based on the quantum transmitting boundary algorithm. Furthermore, we discuss the role of external magnetic fields, multimode effects in quantum wires and thermoelectric properties of spin-orbit graphene monolayers.

Switching probability of spintronics devices

Hideo Sato
Center for Spintronics Integrated Systems, Tohoku University, Japan
hsato@csis.tohoku.ac.jp
S. Kubota, M. Yamanouchi, S. Fukami, S. Ikeda, F. Matsukura, and H. Ohno

Magnetic tunnel junction (MTJ) that compose of two ferromagnetic electrodes (recording and reference layers) sandwiching barrier layer is one of attractive systems for future electronics. In MTJs, information is stored as magnetization direction of the recording layer. For writing information, spin-transfer-torque (STT) is utilized by flowing current through MTJs. From application viewpoint, understanding of the switching by STT in ns regime is very important. The magnetization switching by STT is affected by relative magnetization angle between reference and recording layers. Because magnetization is fluctuated by thermal energy, there is distribution of initial relative magnetization angle, resulting in statistical switching behavior [1,2]. The statistical switching was also observed in domain wall motion by STT [3]. We will discuss the switching probability in ns regime for MTJ using CoFeB-MgO with perpendicular easy axis in which high performances were reported [4].

References

[1] D. Bedau et al., Appl. Phy. Lett. 97, 262502 (2010).

[2] J. He et al., J. Appl. Phys. 101, 09A501 (2007).

[3] S. Fukami et al., Nature. Communications, 4, 2293 (2013).

[4] S. Ikeda et al., Nature Mat. 9, 721 (2010).

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Quantum computing with defects

Joel Varley
Lawrence Livermore National Laboratory, USA
varley2@llnl.gov
J.R. Weber, L. Gordon, A. Janotti, and C.G. Van de Walle

The nitrogen-vacancy (NV^{-1}) center in diamond has emerged as a promising qubit candidate as it is an individually-addressable quantum system that can be controlled at room temperature. The success of the NV^{-1} stems from its electronic structure as a point defect, yet only recently has their been a systematic effort to identify other defects that might behave in a similar way.[1] We present a list of physical criteria that these centers and their hosts should meet and explain how these requirements can be paired with electronic structure theory to intelligently sort through candidate systems. To illustrate, we compare calculations of the NV^{-1} with those of several prospects in 4H silicon carbide. Using hybrid functionals, we report formation energies and configuration-coordinate diagrams to compare and contrast the properties of these defects. We also discuss how the criteria can serve as guidelines for discovering NV analogs in other tetrahedrally coordinated materials. [1] J.R. Weber, W.F. Koehl, J.B. Varley, A. Janotti, B.B. Buckley, C.G. Van de Walle, and D.D. Awschalom, Proc. Nat. Acad. Sci. **107**, 8513 (2010). This work was supported by ARO, AFOSR, and NSF.

Dynamics of domain wall in thin film driven by spin current

Xiao-Ping Wang
Hong Kong University of Science and Technology, Peoples Rep of China
mawang@ust.hk
Lei Yang

The dynamics of magnetization under the applied spin current is modeled by the generalized Landau-Lifshitz-Gilbert equation with a spin transfer torque term. Using matched asymptotic expansion with the domain wall thickness ϵ as the small parameter, we derive analytically the dynamic law for the domain wall motion induced by the spin current. We show that the domain wall driven by adiabatic current spin-transfer torque moves with a decreasing velocity and eventually stops. With a pinning potential, the domain wall motion is a damped oscillation around the pinning site with an intrinsic frequency which is independent of the strength of the current. When the AC current is applied, the dynamic law shows that the frequency of the applied current can be turned to maximize the amplitude of the oscillation. The results obtained are consistent with the recent experimental and numerical results.

Computational spin caloritronics

Ke Xia
Beijing Normal University, Peoples Rep of China
kexia@bnu.edu.cn
ShiZhuo Wang

An efficient first-principles method was developed to calculate spin-transfer torques in layered system with noncollinear magnetization. The complete scattering wave function is determined by matching the wave function in the scattering region with the Bloch states in the leads. The spin-transfer torques are obtained with the aid of the scattering wave function. With this method, we studied the thermoelectric coefficients (Seebeck and spin-transfer torques) of FeCo|MgO|FeCo(001) magnetic tunnel junctions (MTJs). We find FeCo|MgO|FeCo(001) MTJs usually yield smaller thermoelectric effects compared with epitaxial Fe|MgO|Fe(001) MTJs. The magneto-Seebeck signal is sensitive to the details of the FeCo|MgO interfaces. Additionally, we compute angular dependent Seebeck coefficients that provide more information about the transport process. We report large deviations from the Wiedemann-Franz law at room temperature.

Onsager relations for the thermal transport through FI—NM interface

Jiang Xiao
Fudan University, Peoples Rep of China
xiaojiang@fudan.edu.cn
Gerrit E. W. Bauer

For an ferromagnetic insulator (FI) and normal metal (NM) bilayer structure, we predict theoretically that due to magnetic fluctuations, a longitudinal (parallel to equilibrium magnetization) spin current can be injected into a normal metal by applying i) a magnetic field, ii) a spin accumulation in the NM, both parallel to the magnetization, or iii) a temperature difference across the interface. In addition to the previously studied spin Seebeck effect, we find several other previously unknown effects including the recently discovered spin Peltier effect, in which a longitudinal spin accumulation heats up or cools down the magnetization. An Onsager matrix revealing all effects has been identified.

Angular and Linear Momentum of Excited Ferromagnets

Peng Yan
Delft University of Technology, Netherlands
p.yan@tudelft.nl
Akash Kamra, Yunshan Cao, and Gerrit Bauer

The angular momentum vector of a Heisenberg ferromagnet with isotropic exchange interaction is conserved, while under uniaxial crystalline anisotropy the projection of the total spin along the easy axis is a constant of motion. Using Noether's theorem, we prove that these conservation laws persist in the presence of dipole-dipole interactions. However, spin and orbital angular momentum are not conserved separately anymore. We also define the linear momentum of ferromagnetic textures. We illustrate the general principles with special reference to spin transfer torques and identify the emergence of a non-adiabatic effective field acting on domain walls in ferromagnetic insulators.

Special Session 72: Kinetic Models - Analysis, Computation, and Applications

Stephen Pankavich, Colorado School of Mines, USA
Ricardo Alonso, Pontifical Catholic University of Rio de Janeiro, 0

In the large context of Kinetic Theory, the session will focus on analytical and computational issues related to the mathematical treatment of kinetic models and their applications. Such models appear in a variety of areas that represent interacting particle systems as an intermediate level of description between microscopic (molecular dynamics) and macroscopic (fluid) models. Numerous applications arise within the study of rarefied gases, plasmas, radiative transport, semiconductors, wave propagation in disordered media, inverse problems, and materials science, as well as, flocking and social dynamics. Additionally, a significant interest in applications of kinetic models to biological systems and soft matter has recently emerged. A secondary goal of the session is to bring together specialists who, owing to their respective fields of study, might not otherwise have occasion to meet and exchange ideas and perspectives.

Spatially homogeneous solutions of the Vlasov-Nordström-Fokker Planck system

Jose Antonio Alcantara Felix
University of Granada, Mexico
jaaf@correo.ugr.es
Simone Calogero, Stephen Pankavich

The purpose of this talk is to study the existence and uniqueness of solutions of the Vlasov-Nordström-Fokker-Planck system for the corresponding initial value problem in three momentum dimensions by stochastic methods. Additionally, we show that there exists a non-trivial asymptotic profile in the absence of a friction term and we obtain an explicit formula of the long time behavior for spherically symmetric initial data.

Self-similar solutions for a kinetic annihilation model

Veronique Bagland
Blaise Pascal University, France
bagland@math.univ-bpclermont.fr
Bertrand Lods

We consider a modified Boltzmann equation describing probabilistic ballistic annihilation in a spatially homogeneous setting. Such a model describes a system of hard-spheres such that, whenever two particles meet, they either annihilate with probability $\alpha \in (0,1)$ or they undergo an elastic collision with probability $1-\alpha$. For such a model, the number of particles, the linear momentum and the kinetic energy are not conserved. Physicists expect that any solution to the Boltzmann equation for ballistic annihilation should approach for large times a self-similar solution. We shall investigate here the existence of such self-similar solutions.

Effects of emotion on a flocking model

Alethea Barbaro
Case Western Reserve University, USA
alethea.barbaro@case.edu
Jesus Rosado, Andrea Bertozzi

The effects of emotion on a crowd, such as fear or excitement, is not yet fully understood. However, many models have been proposed to simulate the dynamics of socially interacting organisms. The Cucker-Smale model and the model by Vicsek et al. are the most well-studied, particularly in the kinetic community. In this talk, we explore how incorporating a variable representing a contagious emotion can affect the dynamics of flocking models.

A uniform ergodic theorem for flows on weighted spaces

Jonathan Ben-Artzi
University of Cambridge, England
J.Ben-Artzi@damtp.cam.ac.uk

Von Neumann's original proof of the ergodic theorem for one-parameter families of unitary operators relies on a delicate analysis of the spectral measure of the associated flow operator and the observation that over long times only functions that are invariant under the flow make a contribution to the ergodic integral. In this talk I shall show that for a specific class of generators – namely vector fields – the spectral measure is rather simple to understand. For some nicely behaved flows this allows us to obtain a uniform ergodic theorem, while for other flows we show that the spectral measure can be purely singularly continuous. These results are closely related to recent results on the 2D Euler equations, and have potential applications for other conservative flows, such as those governed by the Vlasov equation.

Propagation of Chaos in some Thermostatted Kinetic Models

Eric Carlen
Rutgers University, USA
carlen@math.rutgers.edu

We present an approach to quantitatively controlling propagation of chaos in several kinetic models in which particles interact not only through collisions, but also through the effects of a thermostat that is responsible for maintaining an eventual non-equilibrium steady state. The results presented have been contained in several joint works with collaborators: Bonetto, Esposito, Lebowitz, Mara, Mustafa and Wennberg.

Entropy production inequalities for the linear Boltzmann equation

José A. Cañizo
University of Birmingham, British Virgin Islands
j.a.canizo@bham.ac.uk
Marzia Bisi, Bertrand Lods

We prove an inequality between the entropy and the production of entropy for the linear Boltzmann equation $\partial_t f = Q(f, M)$, where M is a Maxwellian. This is an interesting kind of inequality, similar to a log-Sobolev inequality, and in fact we show that in the limit of grazing collisions one can obtain from it the log-Sobolev inequality for a particular Fokker-Planck equation. We show that this inequality is related to other known ones involving the bilinear Boltzmann operator, and give some applications.

On the analysis of a coupled kinetic-fluid model

Young-Pil Choi
Imperial College London, England
young-pil.choi@imperial.ac.uk
José A. Carrillo and Trygve K. Karper

In this talk, we study global existence, hydrodynamic limit, and large-time behavior of weak solutions to a kinetic flocking model coupled to the incompressible Navier-Stokes equations. The model describes the motion of particles immersed in a Navier-Stokes fluid interacting through local alignment.

Numerical computation of the Boltzmann collision operator with angularly dependent cross section.

Jeff Haack
University of Texas, USA
haack@math.utexas.edu
Irene M. Gamba

I will present an extension of the conservative spectral method to the case of angular dependent scattering mechanisms arising from potential interactions between particles. In particular, we test this method by computing the Boltzmann equation with screened Coulomb potentials and numerically study the rate of convergence of the Boltzmann collision operator in the grazing collisions limit to the limiting Landau collision operator, in Fourier space. We show that the decay rate to equilibrium depends on the parameters associated with the collision cross section, and specifically study the differences between the classical Rutherford scattering angular cross section, which has logarithmic error, and an artificial one with a linear error.

Stability of steady states in the Einstein-Vlasov system

Mahir Hadzic
King's College London, England
mahir.hadzic@kcl.ac.uk

Einstein-Vlasov system describes a fully relativistic description of galaxy evolution. I will discuss some recent progress on the stability and instability theory for spherically symmetric steady states of the Einstein-Vlasov system.

Quantitative stability inequalities for Vlasov-Poisson and 2d Euler systems

Mohammed Lemou
CNRS and University of Rennes 1, IRMAR, France
mohammed.lemou@univ-rennes1.fr

In this work, we prove a new functional inequality of Hardy-Littlewood type for generalized rearrangements of functions. We then show how this inequality yields a quantitative stability result for dynamical systems that essentially have the important property to preserve the rearrangement and the Hamiltonian. In particular we derive a quantitative stability result for a large class of steady state solutions to Vlasov-Poisson systems by providing a quantitative control of the L^1 norm of the perturbation (uniformly in time) by the perturbed Hamiltonian and the L^1 norm of the perturbation at initial time . In fact, such non linear stability has already been recently obtained, but our proof was based in a crucial way on compactness arguments which by construction provide no quantitative control of the perturbation. We finally investigate the application of our inequality to other contexts such as the relativistic Vlasov-Poisson and 2D-Euler systems.

Unstable manifolds of Vlasov-Poisson systems

Zhiwu Lin
Georgia Institute of Technology, USA
zlin@math.gatech.edu
Chongchun Zeng

We consider the homogeneous equilibrium of 1D Vlasov-Poisson system modeling electrostatic plasmas. A sharp linear instability criterion was obtained by Penrose in 1960. With C. Zeng, we constructed stable and unstable invariant manifolds near linearly unstable homogeneous states. This implies the nonlinear instability in the macroscopic sense. There is no restriction on the homogeneous profiles. In particular, we can treat the cases with vanishing or compactly supported steady distribution functions, which were excluded in the previous study of nonlinear instability.

Maxwellian intermediate asymptotics for visco-elastic hard spheres

Bertrand Lods
Università di Torino, Italy
lodsbe@gmail.com
Ricardo J. Alonso

We investigate the long time behavior of a system of viscoelastic particles modeled with the homogeneous Boltzmann equation. In this case, the coefficient of normal restitution, which measures the inelasticity of collisions, is not constant and depends on the impact velocity. For such a model, we prove the existence of a universal Maxwellian intermediate asymptotic state and explicit the rate of convergence towards it.

Weak coercivity and convergence to equilibrium of some kinetic equations

Tran Minh Binh
Basque Center for Applied Mathematics, Spain
tranmbinh85@gmail.com

The convergence to equilibrium of kinetic equations was first systematically studied by L. Desvillettes and C. Villani. A few different approaches have been developed to study the problem: Lyapunov functional technique, the pseudodifferential calculus, the macroscopic and microscopic decomposition. We introduce in this talk a new constructive approach for the problem of the convergence to equilibrium for a large class of kinetic equations. The approach consists of two types of weak coercive inequalities, which imply exponential or polynomial convergence rate. Our methods work very well not only for hypocoercive systems in which the coercive parts are degenerate but also for the linearized classical Boltzmann equation and the linearized quantum Boltzmann equation.

Kinetic theory for collective displacements

Sebastien Motsch
Arizona State University, USA
smotsch@asu.edu

Collective displacements such as a flock of birds or a school of fish are one of the most astonishing examples of emergent behavior in nature. To model these phenomena, we can either use “microscopic models” describing the motion of each individual or “macroscopic models” describing the evolution of the density of individuals. In this talk, we discuss how we can link the two approaches using “kinetic theory”. The difficulty is that, in contrast with particle systems in physics, models of collective displacements do not conserve momentum or energy. This lack of conservation requires to introduce new tools to derive and analyze their macroscopic limits.

A numerical study of a model system of kinetic wave turbulence

Vladislav Panferov
California State University, Northridge, USA
vladislav.panferov@csun.edu

Kinetic wave turbulence equations have been proposed to describe the wavenumber spectrum for a system of weakly interacting phase incoherent waves. In certain cases, explicit steady solutions to these equations, the so-called Kolmogorov-Zakharov spectra are available. I will present results of numerical work on a model system of gravity waves on a surface of incompressible fluid, studying the regime of applicability of the known steady solutions.

A Spectral Study of the Diffusively Excited Granular Gases Equation

Thomas Rey
University of Maryland, USA
trey@cscamm.umd.edu

We will present a work concerning the quasi-elastic hydrodynamic limit of the diffusively excited granular gases equation. The granular gases equation is a Boltzmann-like kinetic equation arising when one wants to give a statistical description of a rarefied gas composed of macroscopic particles, interacting via energy-dissipative binary collisions (pollen flow in a fluid, or planetary rings for example). The purpose of an hydrodynamic limit is to give a reduced description of this equation, using a fluid approximation. As a first step to prove mathematically the validity of the quasi-elastic hydrodynamic limit of this equation, we will present results inspired from the seminal paper of Ellis and Pinsky about the spectrum of the linearized collision operator. The main differences in our case concern the lack of energy conservation and the use of an exponentially weighted Banach L^1 setting instead of the classical Hilbertian L^2 one with Gaussian weights. We will give a precise localization of the spectrum, and an expansion of the branches of eigenvalues of this operator, for small Fourier (in space) frequencies and small inelasticity, allowing to explain some of the classical features of this equation and its hydrodynamic limit, such as the clustering instability.

Existence and non-existence of steady-states for a two species segregation model

Nancy Rodriguez
UNC Chapell Hill, USA
nrodriguez@math.stanford.edu

The focus of this talk will be on the analysis of steady-state solutions in $\mathcal{R}^d$ to a coupled system of PDEs, which was originally introduced as a model for social segregation. The existence of steady-state solutions are important as they provide some insight into long-time behavior of solutions. We explore the existence and non-existence of steady states via a free energy functional.

On the relativistic Vlasov-Maxwell System

Robert Strain
University of Pennsylvania, USA
strain@math.upenn.edu
Jonathan Luk

We consider the relativistic Vlasov-Maxwell (VM) system with initial data of unrestricted size. In the 3D case, since the work of Glassey-Strauss in 1986, it has been known that as long as the 3D momentum support remains bounded then solutions can be continued and they will remain regular. We prove that as long as there exists a plane upon which the momentum support remains bounded then solutions can be continued and they will remain regular. We will also discuss other related recent results. These are joint works with Jonathan Luk.

A Spectral Method for Linear Half-Space Kinetic Equations

Weiran Sun
Simon Fraser University, Canada
weirans@sfu.ca
Qin Li, Jianfeng Lu

In this talk we will present a spectral method to solve a family of linear half-space kinetic equations. This type of equations naturally arise in boundary layer analysis of kinetic equations. In order to achieve a quasi-optimality for the numerical scheme, we modify the half-space equation first to make the collision operator coercive and then reconstruct the solution to the original kinetic equation. The quasi-optimality is proved by verifying that the bilinear operator involved in the variational form of the half-space equation satisfies an inf-sup condition. A numerical example is shown for the linearized BGK equation. This is joint work with Qin Li and Jianfeng Lu.

Numerical methods on kinetic flocking models

Changhui Tan
University of Maryland, USA
ctan@cscamm.umd.edu
Thomas Rey

In this talk, I will discuss kinetic representation of flocking systems, like Cucker-Smale model and Motsch-Tadmor model. Flocking and clustering behaviors are observed, where we prove that flocking is guaranteed with strong nonlocal interaction. It leads to an infinite time concentration in velocity variable. Such δ-singularity brings difficulties in numerical implementation. We use a discontinuous Galerkin method to construct high order positive preserving scheme to solve kinetic flocking systems. In the case of flocking, a method based on a scaling argument of the velocity variable is also introduced to solve the system.

A Finite Difference Approximation of the Spatially Homogeneous Fokker-Planck-Landau Equation

Stephen Wollman
College of Staten Island - CUNY, USA
wollman@math.csi.cuny.edu

The Fokker-Planck-Landau(FPL) equation is a kinetic model for the time evolution of a plasma. Our focus at present is on the numerical approximation of the Landau collision operator hence we consider the spatially homogeneous equation. This equation is

$$\frac{\partial f}{\partial t} = q\Phi(f, f), \quad f(v, 0) = f_0(v),$$

where $v = (v_1, v_2, v_3) \in R^3$, $f(v, t)$ is the distribution function for a single charged species, and $\Phi(f, f)$ is the Landau collision operator for Coulomb collisions. The equation (1) is put into a form so that finite difference methods for parabolic type PDE's can be applied. Through a change of independent variable the equation is reformulated so that finite differencing in velocity space is done on a bounded domain with well defined boundary conditions. The reformulate FPL equation is approximated by a semi-implicit difference equation on a uniform grid and is solved by the SOR algorithm. The coefficients in the difference equation are obtained as derivatives of a Rosenbluth potential. By taking advantage of regularity properties of the solution (which are assumed to exist although a formal proof is lacking) the time needed to compute the Rosenbluth potential is reduced and accuracy is maintained by doing numerical integrations on courser subgrids of the finite difference grid. Also parallel programming is easily applied to further reduce computation time. Some analysis is carried out on the stability and accuracy of the numerical method. A number of computational examples are given showing the convergence of the solution to the steady state.

Special Session 73: Entropy and Statistical Properties for Smooth Dynamics

Todd Fisher, Brigham Young University, USA
Amie Wilkinson, University of Chicago, USA

The main ingredient in studying the complexity of the orbit structure of a dynamical system is the idea of entropy. The related notion of statistical properties of a dynamical system refers to the statistical behavior of typical trajectories of the system. This special session will bring researchers from around the world to examine entropy and statistical properties for smooth systems.

Tower constructions and specification properties

Vaughn Climenhaga
University of Houston, USA
climenha@math.uh.edu

Given a dynamical system with some hyperbolicity, the equilibrium states associated to sufficiently regular potentials often display stochastic behaviour. Two important tools for studying these equilibrium states are specification properties and tower constructions. I will describe how both uniform and non-uniform specification properties can be used to deduce existence of a tower with exponential tails, and hence to establish various statistical properties.

A criterium for the existence of nonhyperbolic ergodic measures with positive entropy

Lorenzo Diaz
PUC-Rio, Brazil
lodiaz@mat.puc-rio.br
J. Bochi and Ch. Bonatti

We prove that there is a C^1-open and dense subset of the C^1-open set of diffeomorphisms with robust cycles consisting of diffeomorpphisms with non-hyperbolic ergodic measures with positive entropy. Examples of diffeomorphisms with C^1-robust cycles are those having a chain recurrence class or a homoclinic class containing hyperbolic saddles of different index (dimension of the stable bundle). As a consequence we get that there is a C^1-open and dense subset of the set of non-Anosov robustly transitive diffeomorphisms of systems with non-hyperbolic ergodic measures with positive entropy. The main technical ingredient of our results is a semi-local mechanism called flip-flop families that allows us to construct compact sets with controlled Birkhoff averages. We use this mechanism to control the average of the central Jacobian of partially hyperbolic sets with one-dimensional center direction.

Multiple phase transitions in non-hyperbolic dynamics

Katrin Gelfert
Federal University of Rio de Janeiro, Brazil
gelfert@im.ufrj.br
L. J. Diaz, M. Rams

We study Lyapunov exponents for a family of partially hyperbolic and topologically transitive local diffeomorphisms that are step skew-products over a horseshoe map. These maps are genuinely non-hyperbolic and the central Lyapunov spec trum contains negative and positive values. We show that in a first model, besides one gap, this spectrum is complete. The principal ingredients of our proofs are minimality of the underlying iterated function system and shadowing- like arguments. In another model we study multiple phase transitions for the topological pre ssure of geometric potentials. We prove that for every k there is a diffeomorphism with a transitive set as above such that the pressure map of the parametrized geometric potential has k rich phase transitions. This means that there are k parameters where pressure is not differentiable and this lack of differentiability is due to the coexistence of two equilibrium states with positive entropy and different Birkhoff averages. Each phase transition is associated to a gap in the central Lyapunov spectrum

On Sums of Dynamically Defined Cantor Sets

Anton Gorodetski
University of California Irvine, USA
asgor@math.uci.edu

Palis' Conjecture on sums of Cantor sets claims that typically a sum of two dynamically defined Cantor sets either has zero measure or contains an interval. In 2001 Moreira and Yoccoz proved that generically a sum of two dynamically defined Cantor sets with sums of Hausdorff dimensions larger than one contains an interval, therefore confirming Palis' Conjecture for generic nonlinear Cantor sets. The genericity assumptions there do not allow to apply the result to a specific one- or finite- dimensional family of Cantor sets, which is the setting encountered in many applications. In particular, Palis' Conjecture for affine Cantor sets is still open. Based on the recent results on convolutions of singular measures (joint with D.Damanik and B.Solomyak), we show that generically (for almost all values of parameters) the sum of two affine Cantor sets has positive Lebesgue measure provided the sum of their Hausdorff dimensions is greater than one (this is a joint result with S.Northrup). Moreover, in the current work in progress (joint with D.Damanik and Y.Takahashi) we show that the sum of a fixed compact set and a Cantor set from a one-parameter family has positive Lebesgue measure for almost all parameters under the assumption that the Hausdorff dimension of the Cantor set changes monotonically with the parameter (once again, provided the sum of Hausdorff dimensions of the compact set and the Cantor set is greater than one). Some applications of the these results to spectral theory will be also given.

Iterated function systems on compact manifolds; minimality and synchronization

Ale Jan Homburg
University of Amsterdam, Netherlands
a.j.homburg@uva.nl

I'll consider iterated function systems generated by finitely many diffeomorphisms on compact manifolds. These give skew product systems over shift operators. I'll discuss aspects of their dynamics, in particular minimality and synchronization.

Stochastic entropy for compact manifolds: properties, rigidity, regularity

Francois Ledrappier
University of Notre Dame, USA
Francois.M.Ledrappier.1@nd.edu
Lin Shu

We consider the stochastic (Kaimanovich) entropy of the Brownian motion on the universal cover of a compact Riemannian manifold M. We discuss the relation with other global invariants, rigidity results if M has no focal points and the regularity under (conformal) changes if M has negative curvature.

Rigorous computation of invariant measures for homeomorphisms of the circle

Maurizio Monge
Universidade Federal do Rio de Janeiro, Brazil
maurizio.monge@im.ufrj.br
S. Galatolo and I. Nisoli

We describe an algorithm which is able to approximate invariant measures of dynamical systems up to small errors in the Wasserstein distance, and its practical implementation. The use of Wasserstein distance, allows to replace some difficult a priori estimations on the regularity of the invariant measure and exploit as much as possible some a posteriori estimates which are made by the computer. We will show a variation of the algorithm for computing the invariant measure of general homeomorphisms of the circle with irrational rotation number, where we can rigorously compute the invariant measure up to small errors. Such maps form a family of examples which are not mixing but just ergodic, and for which we are able to compute the invariant measure.

Persistence of global spiral attractors

Maria Jose Pacifico
Universidade Federal do Rio de Janeiro, Brazil
pacifico@im.ufrj.br

We prove the existence of flows in 3-dimensional manifolds displaying global "strange" attractors with spiral geometry. Moreover, the attractors are measure-theoretically persistent (positive Lebesgue measure set in parameter space) in parametrized families of flows going through saddle-focus homoclinic bifurcations.

Smooth deformations of the Kuperberg plug

Ana Rechtman
IRMA, Universite de Strasbourg, France
rechtman@math.unistra.fr
Steven Hurder

In 1993 K. Kuperberg constructed examples of smooth flows without periodic orbits on any closed 3-manifold. These examples continue to be the only known examples with such properties and are constructed using plugs. A. Katok's theorem implies that such flow have topological entropy equal to zero. In an earlier work, S. Hurder and I studied the topology of the minimal set of these plugs. After reviewing K. Kuperberg's construction, I will present a C^∞-small pertubartion of the construction whose flow has positive entropy.

Uniqueness of equilibrium states for a family of partially hyperbolic systems

Isabel Rios
Universidade Federal Fluminense, Brazil
rios@impa.br
Jaqueline S. Rocha

In this work we study a family of partially hyperbolic horseshoes introduced by L. Diaz *et al* [Destroying horseshoes via heterodimensional cycles: generating bifurcations inside homoclinic classes, *Ergod. Th. and Dynamical Systems* **29**, 2009]. In [*Equilibrium States for partially hyperbolic horseshoes.* Ergod. Th. and Dynamical Systems. **31** 2011], R. Leplaideur *et al* proved the existence of equilibrium states for the diffeomorphisms F in this family, associated to continuous potentials. They also proved the existence of a spectral gap associated to the central Lyapunov exponents, and that, for a positive value of t_0, the smooth potencial $t_0 \log ||DF|_{E^c}||$ admits at least two equilibrium states. Here we prove the uniqueness of equilibrium states for the class of Holder-continuous potentials with small variation, wich includes $t_0 \log ||DF|_{E^c}||$, for small values of t.

Continuity of entropy for partially hyperbolic diffeomorphisms

Radu Saghin
PUCV, Chile
rsaghin@gmail.com

I will discuss different results about computing the topological entropy for partially hyperbolic diffeomorphisms, and about the continuity of the topological entropy for these diffeomorphisms.

Minimal yet measurable foliations

Ali Tahzibi
University of Sao Paulo- Campus Sao Carlos, Brazil
tahzibi@icmc.usp.br

We study the disintegration of Lebesgue measure along central leaves of partially hyperbolic diffeomorphisms. Considering an special open class of derived from Anosov diffeomorphisms, we obtain minimal yet measurable foliation of 3-Torus.

Entropy for generalised beta-transformations.

Daniel Thompson
Ohio State University, USA
thompson@math.osu.edu

Generalised β-transformations are the class of piecewise continuous interval maps given by taking the β-transformation $x \mapsto \beta x \pmod 1$, where $\beta > 1$, and replacing some of the branches with branches of constant negative slope. If the orbit of 1 is finite, then the map is Markov, and we call the map a PCF (post-critically finite) generalised β-transformation. We would like to describe the set of β for which these maps can be PCF. We know that β (which is the exponential of the entropy of the map) must be an algebraic number. Our main result is that the Galois conjugates of such β have modulus less than 2. This extends an analysis of Solomyak for the case of β-transformations, who obtained a sharp bound of the golden mean in that setting.

Transience and multifractal analysis

Mike Todd
University of St Andrews, Scotland
mjt20@st-andrews.ac.uk
Godofredo Iommi and Thomas Jordan

Pressure, entropy, Lyapunov exponents and local dimension are connected concepts which help describe the statistical properties of a dynamical system. These ideas can be extremely effective in the case that the system being studied is recurrent. In this talk, I'll discuss how this analysis might be extended to systems which exhibit both transience and recurrence.

How robust are the intermingled basin for partially hyperbolic diffeomorphisms?

Carlos Vasquez
PUCV, Chile
cvasqueze@gmail.com
Raul Ures

It is well-known that it is possible to construct a partially hyperbolic diffeomorphism on the 3-torus like Kam example. It has two hyperbolic physical measures with intermingled basins on two embedded tori with Anosov dynamics. A natural question is how robust are the intermingled basin phenomena for diffeomorphisms defined on boundaryless manifolds? In this talk we will show that in the 3-torus the only partially hyperbolic examples having hyperbolic physical measures with intermingled basins are not robust.

Equilibrium states for certain partially hyperbolic systems

Daniel Visscher
University of Michigan, USA
davissch@umich.edu

I will discuss uniqueness of equilibrium measures for some examples of partially hyperbolic systems, including time one maps of Anosov flows.

Special Session 74: Collective Behaviour in Biological and Social Aggregations

Joep Evers, Eindhoven University of Technology, The Netherlands
Razvan Fetecau, Simon Fraser University, Canada

Describing and predicting the behaviour of interacting social individuals in a group is a challenging task, since there is no obvious system of physical laws that should be obeyed. Various modelling approaches have been employed by researchers in the field: microscopic/ODE and macroscopic/PDE models, measure-valued formulations, cellular automata, local or nonlocal descriptions, kinetic formulations, etc. This session aims at bringing together various views on modelling these systems and their dynamics. Specific applications (bacteria, animals, people), as well models that incorporate universal features of aggregations, will be presented.

Stochastic clustering and aggregation in biological systems

Maria D'Orsogna
California State University at Northridge, USA
dorsogna@csun.edu

The binding of individual components to form composite structures is a ubiquitous phenomenon within the sciences. Nucleation and growth have been extensively studied in the past decades, often assuming infinitely large numbers of building blocks and unbounded cluster sizes. These assumptions led to the use of mass-action, mean field descriptions such as the well known Becker Döering equations. In cellular biology, however, nucleation events often take place in confined spaces, with a finite number of components, so that discrete and stochastic effects must be taken into account. We examine finite sized nucleation by considering a fully stochastic master equation, solved via Monte-Carlo simulations and via analytical insight. We find striking differences between the mean cluster sizes obtained from our discrete, stochastic treatment and those predicted by mean field treatments, even in the limits of large system sizes. We discuss homogeneous and heterogeneous nucleation, coagulation and fragmentation events, and first assembly times.

Nonlocal Hyperbolic and Parabolic Models for Self-Organised Biological Aggregations

Raluca Eftimie
University of Dundee, Scotland
reftimie@maths.dundee.ac.uk
Pietro-Luciano Buono

Parabolic and hyperbolic models have been used intensively over the past years to describe the formation and movement of self-organised biological aggregations. Here we use symmetry and bifurcation theory to investigate a class on nonlocal hyperbolic models and their parabolic counterparts. We show that the parabolic models exhibit a loss of bifurcation dynamics (i.e., loss of Hopf bifurcations) compared to the hyperbolic models (which can exhibit both codimension-1 and codimension-2 bifurcations: Hopf, steady-state, Hopf/Hopf, Hopf/steady-state and steady-state/steady-state bifurcations). This explains the less rich patterns exhibited by the parabolic equations for self-organised aggregations.

Mass evolution: application to pedestrians

Joep Evers
Eindhoven University of Technology, Netherlands
j.h.m.evers@tue.nl
Adrian Muntean, Razvan C Fetecau, Fons van de Ven

We study the behaviour of groups of pedestrians, in which macroscopic effects arise from the microscopic, social interactions between individuals. In this talk I introduce and discuss the kind of models I use. I will focus on one of the ingredients that make these models different from systems of non-living particles: mutual interactions are anisotropic since they are vision-based and thus direction-dependent. My main interest is to investigate how anisotropy influences the model, both in a mathematical sense as in view of the interpretation given to the solution. This influence turns out to be rather large, and affects e.g. pattern-formation and (stability of) equilibria.

Continuum models of cohesive stochastic swarms

Klemens Fellner
University of Graz, Austria
Klemens.Fellner@uni-graz.at
B. Hughes

Mathematical models of swarms of moving agents with non-local interactions have many applications and have been the subject of considerable recent interest. For modest numbers of agents, cellular automata or related algorithms can be used to study such systems, but in the present work, instead of considering discrete agents, we discuss a class of one-dimensional continuum models, in which the agents possess a density $\rho(x, t)$ at location x at time t. The agents are subject to a stochastic motility mechanism and to a global cohesive inter-agent force. The motility mechanisms covered include classical diffusion, nonlinear diffusion (which may be used to model, in a phenomenological way, volume exclusion or other short-range local interactions), and a family of linear redistribution operators related to fractional diffusion equations. A variety of exact analytic results are discussed, including equilibrium solutions and criteria for unimodality of equilibrium distributions, full time-dependent solutions, and transitions between asymptotic collapse and asymptotic escape. We address the behaviour of the system for diffusive motility in the low-diffusivity limit for both smooth and singular interaction potentials and show how this elucidates puzzling behaviour in fully deterministic non-local particle interaction models. We conclude with speculative remarks about extensions and applications of the models.

Aggregation Models with Nonlinear Local Repulsion

Razvan Fetecau
Simon Fraser University, Canada
van@math.sfu.ca
Martin Burger, Yanghong Huang

We consider a continuum aggregation model with nonlinear local repulsion given by a degenerate power-law diffusion with general exponent. The steady states and their properties in one dimension are studied both analytically and numerically, suggesting that the quadratic diffusion is a critical case. The focus is on finite-size, monotone and compactly supported equilibria. We also investigate numerically the long time asymptotics of the model by simulations of the evolution equation. Issues such as metastability and local/ global stability are studied in connection to the gradient flow formulation of the model.

Predator-swarm interactions

Theodore Kolokolnikov
Dalhousie, Canada
tkolokol@gmail.com
Yuxin Chen

We propose a minimal model of predator-swarm interactions which captures many of the essential dynamics observed in nature. Different outcomes are observed depending on the predator strength. For a "weak" predator, the swarm is able to escape the predator completely. As the strength is increased, the predator is able to catch up with the swarm as a whole, but the individual prey are able to escape by "confusing" the predator: the prey forms a ring with the predator at the center. For higher predator strength, complex chasing dynamics are observed which can become chaotic. For even higher strength, the predator is able to successfully capture the prey. Our model is simple enough to be amenable to a full mathematical analysis which is used to predict the shape of the swarm as well as the resulting predator-prey dynamics as a function of model parameters. We show that as the predator strength is increased, there is a transition (due to a Hopf bifurcation) from confusion state to chasing dynamics, and we compute the threshold analytically. Our analysis indicates that the swarming behaviour is not helpful in avoiding the predator, suggesting that there are other reasons why the species may swarm. The complex shape of the swarm in our model during the chasing dynamics is similar to the shape of a flock of sheep avoiding a shepherd.

Stability analysis and existence results for particular solutions of self-propelled interacting particle systems

Stephan Martin
Imperial College London, England
stephan.martin@imperial.ac.uk
J.A. Carrillo, Y. Huang

We consider a second-order model of self-propelled interacting agents, which has been frequently used to model complex behavior of swarms such as fish schools or birds flocks. Particular solutions such as aligned flocks and rotating mills can be found on the particle level and the associated kinetic PDEs. We present some recent advances in the analysis of such solutions: First, we consider the particle system and show that nonlinear stability of flock solutions is inherited from the first-order aggregation equation to the second-order model. Flocks are shown to be stable as a family of particular solutions. Second, we turn to the mean-field equations and present existence and uniqueness results for the so-called Quasi-Morse potential, which is a special interaction potential allowing for an explicit expression of density profiles of particular solutions in terms of special functions (in 2D and 3D).

References

[1] J.A. Carrillo, Y. Huang, S. Martin. *Nonlinear stability of flock solutions in second-order swarming models*, Nonlinear Analysis: Real World Applications, vol. 17 (2014), pp. 332 - 343

[2] J.A. Carrillo, Y. Huang, S. Martin. *Explicit Flock Solutions for Quasi-Morse potentials*, preprint: http://arxiv.org/abs/1308.2883

[3] J. A. Carrillo, S. Martin, V. Panferov: http://dx.doi.org/10.1016/j.physd.2013.02.004,*A new interaction potential for swarming models*, Physica D - Nonlinear Phenomena, vol. 260 (2013), pp. 112-126

Collective behavior in stochastic populations

Daniela Morale
University of Milano, Italy
daniela.morale@unimi.it

Over the past couple of decades, a large amount of literature has been devoted to the mathematical modelling of self-organizing populations, based on forces, external and/or internal at short-range and/or long-range, acting at the individual level. The main interest has been in catching the main features of the interaction at the lower scale of single individuals, showing evident stochasticity, that are responsible, at a larger scale, for a more complex behaviour that leads to the formation of aggregating patterns. Here we discuss the modelling of the dynamics of some self-organization populations at different scales and the role of stochasticty, via some examples in biology, medicine and evolution theory.

Jamology - from mathematical modeling and analysis to engineering applications

Akiyasu Tomoeda
Musashino University / JST CREST, Japan
atom@isc.meiji.ac.jp

Various kinds of the collective motion of Self-Driven Particles (SDP), such as vehicles, pedestrians and social insects, have attracted a great deal of attention in a wide range of fields during the last few decades. Most of these complex systems are interesting not only from the point of view of natural sciences for fundamental understanding of how nature works but also from the points of view of applied sciences and engineering for the potential practical use of the results of the investigations. Especially, interdisciplinary research for the dynamics of jamming phenomena in SDP systems, so-called "Jamology", has progressed by developing sophisticated mathematical models considered as a system of interacting particles driven far from equilibrium with a central focus on the jamming phenomena in traffic flow. In this talk, starting from the definition of the "jamming state" in 1-D transportation system, several mathematical models and their relations are shown in the case of traffic flow. Mathematical modeling and its analysis suggest the mechanism for forming the spontaneous traffic jam. Then our jam absorbing driving technique is introduced as a fundamental theoretical framework.

The basic principles of hollow spherical self-assembly

David Uminsky
University of San Francisco, USA
duminsky@usfca.edu
James H. von Brecht

Despite their elegance, nature seldom adopts an empty shell - or sphere - as an energetically favorable state. Exceptions to this rule do exist at the nanoscale, with the self-assembly processes of viral capsids and the more recently discovered spontaneous assembly of polyoxometalate (POM) macroions into supramolecular spherical structures called "blackberries" as examples. The underlying physical mechanisms that govern these assemblies remain largely unknown, and models for these processes have grown increasingly complex and problem-dependent. We exploit recent mathematical advances to present a simpler approach. Specifically, using only pairwise and isotropic interactions we derive the basic principles on the interaction that predict when hollow shells will form. We then use these principles to find, for the first time, a purely isotropic physical model that will produce such assemblies. Time permitting we will also consider the effects on spherical self assembly associated to non-isotropic forces.

Parameter Estimation of Social Forces in Crowd Dynamics Models via a Probabilistic Method

Kiamars Vafayi
TU Eindhoven, Netherlands
k.vafayi@tue.nl
Alessandro Corbetta, Adrian Muntean, Federico Toschi

Crowd models are powerful tools aiming at exploring complex dynamics of pedestrians walking in a crowd. In order for these models to have some predictive power, and hence being of use in civil engineering (e.g. when pedestrian evacuation is concerned), the parameters on which they depend must be estimated. It is not hard to imagine, though, that the behavior of pedestrians, even in simple scenarios, features a wide heterogeneity, determined by the so called *inter-subject* and *intra-subject* variabilities. Such heterogeneities make the task of estimating free-parameters by means of experimental data (e.g. by detailed pedestrian trajectories) both *unclear* and *unconventional*. We present a Bayesian probabilistic framework, which, on the basis of given experimental data, estimates the values and quantifies uncertainties (in the form of probability density functions) in the parameters of chosen models. In this framework, we also introduce a fitness measure for the models to classify several model structures (forces) according to their fitness to the experimental data, preparing the stage for a more general model-selection and validation strategy inspired by probabilistic data analysis. The results are based on a large amount of experimental pedestrian data obtained after a long time measurement at Eindhoven University of Technology.

Special Session 75: Differential and Difference Equations on Graphs and Their Applications

Sergei Avdonin, University of Alaska Fairbanks, USA
Serge Nicaise, Université de Valenciennes et du Hainaut Cambrésis, France

There are many problems in science and engineering where dynamics on networks, particularly dynamics on the graph edges, plays a fundamental role. For example, applications from vibration theory include bridges, antennas, and space structures, while from diffusion theory applications include neuronal dendritic trees, river delta systems, and the circulatory system. From a mathematical viewpoint, these applications pose outstanding inverse and forward problems concerning differential and difference equations on graph domains. This session will touch on a number of applications in wave propagation theory, nanotechnology, biomathematics and methodologies, with the intent to foster collaboration to further developments in this research area.

Energy flow of dispersive waves above the threshold of tunnel effect

Felix Ali Mehmeti
University of Valenciennes, France
felix.ali-mehmeti@univ-valenciennes.fr
R. Haller-Dintelmann (Darmstadt) and V. Régnier (Valenciennes)

We consider Klein-Gordon Equations with piecewise constant coefficients on a star-shaped network with semi-infinite branches. Using the spectral theory of the associated spatial operator developed in and a version of the stationary phase method, we calculate the leading term of an asymptotic expansion for large times including an error estimate for solutions in frequency bands. This yields the exact time decay of the solution inside the cone of group lines issued by the frequency band on the outgoing half axis. The special case of two branches can be interpreted in physical terms for example as a simplified model for two connected semi infinite electromagnetic waveguides with different dielectric constants. In this case we show that the ratio of the energy on the outgoing half axis and inside the cone of group lines is bounded by a constant which is essentially independent of the height of the potential step but is determined by the frequency band. This suggests that the particle character of a wave packet is quite independent of the potentials with which it interacts, but depends mainly on its frequency band.

Source Identification for the Wave Equation on Graphs

Sergei Avdonin
University of Alaska Fairbanks, USA
s.avdonin@alaska.edu
Serge Nicaise

In this talk we consider source identification problems for the wave equation on trees and graphs with cycles. The main advantage of our approach is its locality: to recover unknown coefficients on a part of the graph we use an information relevant only to this sub-graph. This feature of our method allows us to propose a very efficient identification algorithm which is new even for an interval and much more simple than known algorithms.

Neuronal cable theory on graphs

Jon Bell
University of Maryland Baltimore County (UMBC), USA
jbell@umbc.edu

Dendritic tree morphology is basically a finite metric tree graph. For some reduced cable nonlinear dynamics we investigate solution behavior on such domains, particularly threshold characteristics, solution propagation, bounds on propagation speed, and conduction block. We also outline an inverse problem concerning recovering a spatially distributed conductance parameter from boundary measurements.

Myopic Population Models on Infinite Networks

Robert Carlson
CU Colorado Springs, USA
rcarlson@uccs.edu

Reaction-diffusion equations with spatial difference operators are treated on infinite networks using semigroup methods. The main function space B is a Banach algebra generated by functions which are flat at infinity. There is an associated compactification of the network which promotes the discussion of spatial asymptotics. For certain evolution problems in the space B, diffusive effects vanish at infinity, greatly simplifying the remote dynamics. Some related eigenfunction expansions are also considered.

Gaussian packets on decorated graphs

Vsevolod Chernyshev
HSE, Russia
vchern@gmail.com
Anton Tolchennikov

We study a topological space obtained from a graph by replacing vertices with smooth Riemannian manifolds, i.e. a decorated graph. We study the problem of propagation of Gaussian packet on such space. We obtain asymptotical formulae for number of packets at time T for some decorated graphs. We use formula for the number of states of Bose-Maslov gas.

Neuronal Model Reduction: Cells, Junctions and Circuits

Steven Cox
Rice University, USA
cox@rice.edu
Kathryn Hedrick

The spatial component of input signals often carries information crucial to a neuron's function, but models which map synaptic inputs to transmembrane potential can be computationally expensive. Existing reduced models of the neuron either merge compartments, thereby sacrificing the spatial specificity of inputs, or apply model reduction techniques which sacrifice the biological interpretation of the model. We use Krylov subspace projection methods to construct reduced models of the quasi-active neurons which preserve both the spatial specificity of inputs and the biological interpretation as an RLC circuit, respectively. Each reduced model accurately computes the potential at the spike initiation zone given a much smaller dimension and simulation time, as we show numerically and theoretically. The structure is preserved through the similarity in the circuit representations, for which we provide circuit diagrams and mathematical expressions for the circuit elements. Furthermore, the transformation from the full to the reduced system is straightforward and depends on the intrinsic properties of the dendrite. As each reduced model is accurate and has a clear biological interpretation, the reduced models can be used not only to simulate morphologically accurate neurons but also to examine the underlying functions performed in dendrites.

Topographic open periodic waveguides

Patrick Joly
INRIA, France
patrick.joly@inria.fr
Berangere Delourme, Sonia Fliss, Elizaveta Vasilevskaya

Open periodic waveguides play an important role in applications, in particular in optics. We investigate how a purely geometric perturbation along a line of a periodic domain occupied by a homogeneous medium can create guided waves. This amounts to solving a self-adjoint eigenvalue problem in an unbounded medium. The guided modes correspond to eigenvalues in the spectral gaps of the unperturbed operator. Using asymptotic analysis, we shall show how to derive existence results of such eigenvalues for thin propagation domains that degenerate into perturbed periodic graphs at the small thickness limit. We shall present a numerical method for computing such guided waves and provide various numerical results.

Quantum network models of nanomaterials

Peter Kuchment
Texas A&M UNiversity, USA
kuchment@math.tamu.edu
Ngoc T. Do

Applications of quantum graph models to spectral analysis of a variety of carbon nano-materials are described.

Exact controllability of networks of nonlinear strings and beams

Guenter Leugering
FAU Erlangen-Nuremberg, Germany
leugering@math.fau.de

We consider fully nonlinear elastic strings, some nonlinear Timoshenko beams and Cosserat rods on networks. We discuss equilibrium solutions and analyze the dynamic problems in a neighbourhood of these equilibria. We show well-posendes of the corresponding systems and discuss their controllability properties when acting on the boundary of the treelike network.

Two velocity inverse problem on graphs

Victor Mikhaylov
Senior Researcher/St. Petersburg Department of V.A.Steklov Institute of Mathematics of the Russian Academy of Sciences, Russia
ftvsm78@gmail.com

On a tree-like graph we investigate the inverse boundary problem for a two velocity wave equation which holds on each edge for a two component vector displacement. Physical parameters of the graph: the densities and lengths of the edges, and also the topology of the tree as well as the angles between branching edges are recovered from the Weyl matrix function. We extend the approach and results of the paper: (S. Avdonin, G. Leugering and V. Mikhaylov, *On an inverse problem for tree-like networks of elastic strings*, Zeit. Angew. Math. Mech., **90** (2010), 136–150) to the case of variable velocities. It is shown that the inverse problem can be uniquely solved by applying measurements at all, or at all but one, boundary vertices. This talk is based on a joint work with S. Avdonin, A. Choque Rivero and G. Leugering.

Dynamical systems and general graph symmetries

Delio Mugnolo
University of Ulm, Germany
delio.mugnolo@uni-ulm.de

By definition, permutations associated with graph automorphisms commute with the graph's adjacency matrix, as well as with its discrete Laplacian. This property is important when one is looking for non-simple eigenvalues. We will review other notions of graph symmetries that generalize this property of automorphisms and present some applications to dynamical systems.

Inverse problem and Lipschitz stability for the heat equation with a discontinuous diffusion coefficient

Julie Valein
University of Lorraine, France
julie.valein@univ-lorraine.fr
Emmanuelle Crépeau and Lionel Rosier

We consider a star-shaped network $\mathcal{R}$ of $n+1$ edges e_j, of length $l_j > 0$, $j \in \{0, .., n\}$, connected at one vertex that we assume to be the origin 0 of all the edges. For any function $f : \mathcal{R} \to \mathbb{R}$ we set

$f_j = f\,|_{e_j}$ the restriction of f to the edge e_j,

$[f]_0 = \sum_{j=0}^{n} f_j(0)$ the transmission bracket at the vertex 0.

We consider on this plane 1-D network a heat equation with a different diffusion coefficient on each string, given by the following system

$$\begin{cases} u_{j,t}(x,t) - (c_j(x)u_{j,x}(x,t))_x = g_j(x,t), \\ \quad \forall j \in \{0,\cdots,n\}, (x,t) \in (0,l_j) \times (0,T), \\ u_j(l_j,t) = h_j(t), \\ \quad \forall j \in \{0,\cdots,n\}, t \in (0,T), \\ u(x,0) = u^0(x), \quad x \in \mathcal{R}, \end{cases} \tag{1}$$

under the assumptions of continuity and of Kirschoff law at the vertex 0, given by

$$u_j(0,t) = u_i(0,t) =: u(0,t), \quad \forall i,j \in \{0,...,n\}, \; 0 < t < T, \tag{2}$$

$$[u_x(t)]_0 := \sum_{j=0}^{n} u_{j,x}(0,t) = 0, \quad 0 < t < T. \tag{3}$$

The diffusion coefficient c is assumed to be piecewise regular such that $c_j \in \mathcal{C}^1([0,l_j])$, $j \in \{0,\cdots,n\}$ and

$$0 < c_{min} \le c \le c_{max}.$$

If we change the diffusion coefficient c into $\tilde{c}$ we let $\tilde{u}$ be the solution of (1) associated to $\tilde{c}$ and $\tilde{u}_0$ for initial condition.

First we are interested in the inverse problem of the determination of the diffusion coefficient c on each of the $n+1$ strings of the network from only n boundary measurements $(u_{j,x}(l_j,\cdot))_{j=1..n}$ on $(0,T)$ and from measurements at time T' on the whole network $(u_{j,xx}(\cdot,T')|_{\mathcal{R}})_{j=1..n}$ (where $T' \in (0,T)$). We will prove the well-posedness of this problem, giving the appropriate stability estimate. The proof will mainly rely on a global Carleman estimate for this network of heat equations (see for instance [1] for an example of a Carleman estimate on a $1D$ network for the wave equation), and is based on [2].

Statement of the inverse problem: Is it possible to retrieve the diffusion coefficient $c = c(x)$, for $x \in \mathcal{R}$ from the n measurements $(u_{j,x}(l_j,\cdot))_{j=1..n}$ on $(0,T)$ and from measurements at time T' on the whole network $(u_{j,xx}(\cdot,T')|_{\mathcal{R}})_{j=1..n}$ where u is the solution to (1)-(3) and $T' \in (0,T)$?

We will actually give a positive answer to this question, assuming that the diffusion coefficients are constant on each edge e_j. Under some suitable assumptions we prove that

$$\sum_{j=0}^{n} |c_j(0) - \tilde{c}_j(0)|^2 \le C \int_{\mathcal{R}} \left|u_{j,xx}(x,T') - \tilde{u}_{j,xx}(x,T')\right|^2 dx$$
$$+ C \sum_{j=1}^{n} \int_0^T |u_{j,tx}(l_j,t) - \tilde{u}_{j,tx}(l_j,t)|^2 \, dt.$$

Second we are interested in the inverse problem of the determination of the diffusion coefficient c on each of the $n+1$ strings of the network from only n interior measurements $(u_j|_{\omega_j \times (0,T)})_{j=1..n}$ and from measurements at time T' on the whole network $(u_j(\cdot,T')|_{\mathcal{R}})|_{j=0..n}$ (where $T' \in (0,T)$). Here $\omega_j \subset (0,l_j)$ ($j \in \{1,\cdots,n\}$) are n non empty subsets. In this case, we prove this stability result without the assumption that the diffusion coefficients are constant on each edge e_j, but assuming that the diffusion coefficients c and $\tilde{c}$ are equal at each vertex. Our method is based on the methods of [4] and [3], proving a Carleman estimate in $L^2(\mathcal{R})$, then a Carleman estimate in $H^{-1}(\mathcal{R})$ and a stability result for a stationnary problem.

This is joint work with Emmanuelle Crépeau and Lionel Rosier.

References

[1] L. Baudouin, E. Crépeau, and J. Valein, *Global Carleman estimate on a network for the wave equation and application to an inverse problem*, Math. Control Relat. Fields, 1 (2011), pp. 307–330.

[2] A. Benabdallah, P. Gaitan, and J. Le Rousseau, *Stability of discontinuous diffusion coefficients and initial conditions in an inverse problem for the heat equation*, SIAM J. Control Optim., 46 (2007), pp. 1849–1881.

[3] M. Cristofol, P. Gaitan, K. Niinimäki, and O. Poisson, *Inverse problem for a coupled parabolic system with discontinuous conductivities: One-dimensional case*, Inverse Problems and Imaging, 7 (2013), pp. 159–182.

[4] O. Poisson, *Uniqueness and Hölder stability of discontinuous diffusion coefficients in three related inverse problems for the heat equation*, Inverse Problems, 24 (2008), p. 025012.

Inverse Problems for Differential Operators of Variable Orders on Graphs

Viacheslav Yurko
Saratov University, Russia
yurkova@info.sgu.ru

We study inverse spectral problems for ordinary differential equations on compact star-type graphs when differential equations have different orders on diferent edges. As the main spectral characteristics we introduce and study the so-called Weyl-type matrices which are generalizations of the Weyl function (m-function) for the classical Sturm-Liouville operator. We provide a procedure for constructing the solution of the inverse problem and prove its uniqueness.

Special Session 76: Viscosity, Nonlinearity and Maximum Principle

Isabeau Birindelli, University of Rome 1, Italy
Italo Capuzzo Dolcetta, University of Rome 1, Italy
Fabiana Leoni, University of Rome 1, Italy
Antonio Vitolo, University of Salerno, Italy

The session is mainly, but non exclusively, devoted to actual results about maximum principles for viscosity solutions of fully nonlinear elliptic PDEs and related inequalities such as ABP estimate, Harnack inequality and gradient estimates with applications to eigenvalue problems, degenerate elliptic operators, existence and uniqueness of positive, entire and blow-up solutions, removable singularities, overdetermined problems as well as with attention to motivations from geometry, physics, engeenering and/or economics.

Regularity of solutions for fully nonlinear equations

Isabeau Birindelli
Sapienza Universitá di Roma, Italy
isabeau@mat.uniroma1.it

Results concerning regularity of solutions have an intrinsic interest which doesn't need to be explained. We will consider cases concerning both global Holder regularity for a large class of domains and Holder regularity of the gradient when the domains are bounded and smooth but the operator is either degenerate or singular.

Existence of boundary blow up solutions for singular or degenerate fully nonlinear equations

Francoise Demengel
University of Cergy Pontoise, France
demengel@math.u-cergy.fr
Olivier Goubet

We prove here the existence of boundary blow up solutions for fully nonlinear equations in general domains, for a nonlinearity satisfying Keller-Osserman type condition. If moreover the nonlinearity is non decreasing , we prove uniqueness for boundary blow up solutions on balls for operators related to Pucci's operators.

Asymptotic Behavior of Degenerate Parabolic System

Zhaosheng Feng
University of Texas-Pan American, USA
zsfeng@utpa.edu

In this talk, we are concerned with approximate solutions to a degenerate parabolic system. We provide a connection between the Abel equation of the first kind, an ordinary differential equation that is cubic in the unknown function, and the degenerate parabolic system, a partial differential equation that is the dispersion model of biological populations with both density-dependent diffusion and nonlinear rate of growth. We present the integral forms of the Abel equation with the initial condition. By virtue of the integral forms and the Banach Contraction Mapping Principle we derive the asymptotic expansion of bounded solutions in the Banach space, and use the asymptotic formula to construct approximate solutions to the degenerate parabolic system.

The extended maximum principle and removable singularities of fully nonlinear second-order elliptic operator

Giulio Galise
University of Salerno, Italy
ggalise@unisa.it
M.E. Amendola, A. Vitolo

In the spirit of Potential Theory we present an Extended Maximum Principle (EMP) result for viscosity solutions of fully nonlinear second-order elliptic equations $F(D^2u) = 0$ by supposing that no boundary condition is given on a set of null Riesz capacity. Next we consider the wide class of uniformly elliptic equations depending on the gradient variable and we analyse in what extent the lower-order terms influence (EMP) with respect to principal part D^2u. As application we show that (EMP) is a powerful tool to treat removable singularities. Finally we extend such results for a class of degenerate elliptic operator which are partial sum of eigenvalues.

On quasilinear fractional PDEs and their local limits. Viscosity solution theory.

Espen Jakobsen
NTNU, Norway
erj@math.ntnu.no
Emmanuel Chasseigne

We introduce a class of fully non-linear quasilinear nonlocal equations of the form

$$F\big(x, u, Du, L[u, Du]\big) = 0 \qquad (1)$$

in whole space where L is a nonlocal quasilinear Levy type operator. We show well-posedness of bounded viscosity solutions under assumptions that for the first time includes i) a gradient-dendence in L and ii) cover the generators of SDEs driven by general pure jump Levy process. Next we study limit problems where the non-local operators converge to local ones:

$$L_\epsilon[u, Du] \to \mathrm{tr}\big(\sigma(x, Du))\sigma(x, Du)D^2u\big)$$

as $\epsilon \to 0$. We prove that the solution u_ϵ of the (1) with $L = L_\epsilon$ converges locally uniformly to the solution of the local problem

$$F\Big(x, u, Du, \mathrm{tr}\big(\sigma(x, Du)\sigma(x, Du)D^2u\big)\Big) = 0 \text{ in } \mathbb{R}^N.$$

We identify general conditions for this convergence, and are able to obtain in the local limit essentially any nonsingular gradient-depending quasi-linear equation. Finally, we present non-local p-Laplace, infinity-Laplace and mean curvature of graph operators.

Fully nonlinear elliptic inequalities in unbounded domains

Fabiana Leoni
Sapienza Università di Roma, Italy
leoni@mat.uniroma1.it

We present a collection of recent results concerning existence or non existence of viscosity solutions in unbounded domains of fully nonlinear second order differential inequalities, with possibly degenerate elliptic principal parts and "absorbing" or "reaction" zero order terms.

Liouville type theorems for nonlinear Choquard equations

Vitaly Moroz
Swansea University, Wales
v.moroz@swansea.ac.uk
Jean Van Schaftingen

The Choquard equation, also known as Hartree equation or nonlinear Schrödinger-Newton equation is a stationary nonlinear Schrödinger type equation where the nonlinearity is coupled with a nonlocal convolution term associated with an attractive gravitational potential. We present a survey of recent results on Choquard type equations, focusing on Liouville type nonexistence theorems and sharp a priori decay estimates of the solutions and super-solutions. The techniques of proofs include an integral form of Phragmen-Lindelöf type comparison estimates and a nonlocal nonlinear extension of the Agmon-Allegretto-Piepenbrink positivity principle which relates the existence of a positive super-solutions to an integral inequality.

Generalized principal eigenvalues for elliptic operators in non-compact scenarios

Luca Rossi
Università di Padova, Italy
lucar@math.unipd.it

What is the analogue of the principal eigenvalue for elliptic operators with non-compact resolvents? Focusing on the case where the lack of compactness is due to the unboundedness of the domain, we show that the answer depends on the property one is looking for: existence of a positive eigenfunction, simplicity, lower bound of the spectrum, characterization of the maximum principle. Indeed, there is not a unique notion fulfilling all such properties in general. In the last part of the talk we present some recent results concerning degenerate elliptic operators.

Viscosity solutions for closed curves driven by a singular weighted mean curvature flow

Piotr Rybka
The University of Warsaw, Poland
rybka@mimuw.edu.pl
Y.Giga, P.Górka

Earlier, we constructed variational solutions for closed curves driven by a singular wmc flow. Actually, we considered curves which are small perturbations of a scaled Wulff shape, i.e. a ball in the norm specified by the anisotropy function. Recently, M.-H.Giga and Y.Giga have developed the viscosity theory for singular parabolic problems in one-dimension. In order to make this theory work, we treat our evolving curve as graph over a suitable reference manifold and we rewrite the wmc flow as a singular parabolic pde for an evolving graph. Using the methods of the viscosity theory we study issues, which were not tractable with the tools we used earlier. In particular, we show uniqueness of solutions and address the corner persistence problem.

Combination and mean width rearrangements of viscosity solutions of elliptic equations in convex domains

Paolo Salani
Università di Firenze, Italy
paolo.salani@unifi.it

I present a refinement of a method by Alvarez-Lasry-Lions to prove convexity of viscosity solutions of elliptic equations, which permits to compare solutions of different equations in different domains. As a consequence, it is possible to define a new kind of rearrangement, which applies to viscosity solution of fully nonlinear equations $F(x, u, Du, D^2u) = 0$ (not necessarily in divergence form), and to obtain Talenti's type results for this kind of rearrangement.

From constant mean curvature surfaces to overdetermined elliptic problems

Pieralberto Sicbaldi
Aix-Marseille Universite, France
pieralberto.sicbaldi@univ-amu.fr

Analysis tools (PDE's Theory, Functional Analysis, Measure Theory, Harmonic Analysis,...) are often used in Differential Geometry to solve natural problems concerning minimal or constant mean curvature surfaces. Conversely, the use of geometry as a tool to solve analytical problems is less frequent. In this talk I would like to show that minimal and constant mean curvature surfaces can be used in order to find a classification of solutions to some overdetermined elliptic problems in domains of the Euclidean space, i.e. solutions to some elliptic differential equations with two boundary conditions.

Boundary regularity and overdetermined problems for fully nonlinear elliptic operators

Boyan Sirakov
PUC-Rio de Janeiro, Brazil
bsirakov@yahoo.com
Luis Silvestre

We prove that the existence of a solution to a fully nonlinear elliptic equation in a bounded domain Ω with an overdetermined boundary condition prescribing both Dirichlet and Neumann constant data forces the domain Ω to be a ball, under any of the following conditions: (a) the operator is C^1 in the second-order derivative, (b) the space dimension is 2, (c) the domain is strictly convex. This is a generalization of Serrin's classical result from 1971.

Representation formulas for solutions of Isaacs integro-PDE

Andrzej Swiech
Georgia Tech, USA
swiech@math.gatech.edu
Shigeaki Koike

We will present sub- and super-optimality inequalities of dynamic programming for viscosity solutions of Isaacs integro-PDE associated with two-player, zero-sum stochastic differential game driven by a Lévy type noise. This implies that the lower and upper value functions of the game satisfy the dynamic programming principle and they are the unique viscosity solutions of the lower and upper Isaacs integro-PDE. The method uses PDE techniques and is based on regularization of viscosity sub- and super-solutions of Isaacs equations to smooth sub- and super-solutions of slightly perturbed equations, and approximate optimal synthesis. It is constructive and provides a fairly explicit way to produce almost optimal controls and strategies.

Characterization of ellipsoids through an overdetermined boundary value problem of Monge-Ampére type

Cristina Trombetti
University of Naples, Italy
cristina@unina.it
B. Brandolini, N. Gavitone, C. Nitsch

The study of the optimal constant in an Hessian-type Sobolev inequality leads to a fully nonlinear boundary value problem, overdetermined with non standard boundary conditions. We show that all the solutions have ellipsoidal symmetry. In the proof we use the maximum principle applied to a suitable auxiliary function in conjunction with an entropy estimate from affine curvature flow.

Gradient estimates and symmetry results in anisotropic media

Enrico Valdinoci
Weierstrass Institute, Berlin, Germany
enrico@math.utexas.edu

We consider a Wulff-type energy functional in an anisotropic setting. The critical points of this functional satisfy a possibly singular or degenerate, quasilinear equation in an anisotropic medium. We prove that the gradient of the solution is bounded at any point by the potential and we deduce several rigidity and symmetry properties. The results presented were obtained in collaboration with M. Cozzi and A. Farina.

Maximum principles for degenerate elliptic equations

Antonio Vitolo
University of Salerno, Italy
vitolo@unisa.it

In this talk we consider some degenerate elliptic operators, including partial sums of eigenvalues of the Hessian matrix as well as of second derivatives plus lower order terms, which go back to different papers of Harvey-Lawson and Caffarelli-Li-Nirenberg. We focus the attention on maximum principles and discuss some applications.

Special Session 77: Theoretical, Technical, and Experimental Challenges in Closed-Loop Approaches in Biology

David Arroyo, Universidad Autónoma de Madrid, Spain
Pablo Varona, Universidad Autónoma de Madrid, Spain

Characterization and control of nonlinear and non-stationary processes is an active topic in the general field of the applied theory of dynamical systems, and in the specific scope of complex biological systems. In this context classical control techniques cannot be applied straight away, and thus observation and actuation should be properly incorporated into a real-time feedback (or closed-loop) methodology. This being the case, modern activity-dependent stimulation protocols should be used to reveal dynamics (otherwise hidden under traditional stimulation techniques), achieve control of natural and pathological states, induce learning, bridge between disparate levels of analysis and for a further automation of experiments. Furthermore, closed-loop interaction calls for novel real time analysis, prediction and control tools and a new perspective for designing stimulus-response experiments, which can have a large impact in biological research. In this special session the closed-loop methodology is discussed through recent contributions in both the theoretical and experimental study of biological systems.

Automatic event detection and characterization in the context of real-time control of complex time-varying dynamical systems

David Arroyo
Universidad Autónoma de Madrid, Spain
david.arroyo@uam.es
Pablo Varona and Francisco B. Rodriguez

The general framework of control theory is very dependent on the outcomes of previous and/or simultaneous system identification. Given a set of standard inputs, we can proceed by deriving a describing function for the input-output functional correlation. However, in an experimental setup this need is very difficult to be satisfied. First, from a general point of view we do not have total access to the inner state of the system to be controlled. Certainly, our data is mainly constructed by partial observations of the underlying system dynamics. Second, partial observations should be modeled in a fast and accurate way to achieve controlability and observability. In fact, if the considered systems are nonlinear and time-varying then the describing function must be adapted as the system evolves. Furthermore, the input-output relationship could be history dependent, which results in an adequate system identification in case we apply classical identification techniques. As a possible alternative we can construct data-driven control procedures guided by efficient and precise tools for detecting and characterizing events automatically. In this communication we discuss this possibility by means of our recent results on the application of symbolic dynamics and time-frequency signal processing to such a goal.

A biophysical observation model for neural field effects

Peter Beim Graben
Humboldt-Universität zu Berlin, Germany
peter.beim.graben@hu-berlin.de
Serafim Rodrigues

Neural field effects, such as ephaptic interactions, can be described by a closed-loop between neural activity and the surrounding electromagnetic field: active neurons generate their characteristic electric dipole fields which superimpose to the local field potential (LFP) of a coherent neural population. This endogenous electric activity could feed back on the activation threshold of the individual neurons. Sofar, this mechanism has been described by phenomenological mean field models, disregarding the biophysical details of electric dipole field generation. We suggest to close this gap by a spatially extended model of a simple leaky integrate-and-fire (LIF) neuron. Introducing three compartments for the apical dendritic tree, for the perisomatic dendritic tree and for the axon hillock, we derive the standard LIF evolution equation augmented by an observation model of the dendritic field potential that allows estimation of the LFP of a neural population. We present simulation results of the forward model and indicate possible ways for the closed-loop approach toward ephaptic field effects.

References

[1] beim Graben, P. & Rodrigues, S. (2013). A biophysical observation model for field potentials of networks of leaky integrate-and-fire neurons. Frontiers in Computational Neuroscience, 6.

The role of sensorimotor feedback in cognition

Christopher Buckley
University of Sussex, England
c.l.buckley@sussex.ac.uk
Taro Toyoizumi

Recently there has been a significant movement in the cognitive sciences that emphasises the centrality of sensorimotor feedback for accounts of cognition. Despite this success the migration of this focus to mainstream systems neuroscience has been slow. Recent experimental innovations mean that this state of affairs is beginning to radically change. Closed-loop experimental paradigms that utilise virtual reality in mice and fish and well circumscribed sensory-motor systems are becoming more widespread. Consequently, in vivo electrophysiology and optogenetics of behaving animals is quickly becoming an achievable gold standard. This work places the sensorimotor loop at the heart of neural processing and promises to give sensorimotor accounts renewed relevance for mainstream neuroscience. Here we utilise these technologies to examine the role of sensorimotor feedback for accounts of neural dynamics and brain function.

Ready to close the loop: modulation of cortical oscillations through DC electric fields

Mattia D'Andola
IDIBAPS, Spain
mattia.dandola@gmail.com
Julia F. Weinert, Maria V. Sanchez-Vives

Brain stimulation techniques such as tDCS or TMS play a growing role in the treatment of neurological disorders. However the thorough understanding of the cellular and network mechanisms recruited by these strategies is missing. We use the slow oscillatory activity from the cortical network (Steriade et al (1993) J Neurosci 13, 3252 and Sanchez-Vives, M. V. & McCormick, D. A. (2000) Nat Neurosci 3, 1027) as a model of a relatively regular activity amenable to be modulated by electric fields (Frohlich, F. & McCormick, D. A. (2010) Neuron 67, 129). To investigate the underlying mechanisms that lead to changes in network activity we applied DC electric fields in vitro to active brain slices. We find that the DC fields effects are limited to specific aspects that characterize slow oscillations and they are highly dependent on the initial activity. Considering the high complexity and non-linear dynamics of the involved networks, the linearity of the changes that we observe in some specific parameters provide an operational window that could lead to an online control through a closed-loop system, a solution with wide clinical implications. Supported by Ministerio de Economia y Competitividad (BFU2011-27094) EU PF7 FET CORTICONIC contract 600806.

Experience-guided combination of principal and independent component analyses to rescue pathway-specific electrical fields in the brain

Oscar Herreras
Cajal Institute, Spain
herreras@cajal.csic.es
V. Makarov, G. Martín Vázquez, and J. Makarova

The unpredictable discontinuous activation of electrical sources in the brain plus their mixing in the volume constitute major handicaps for analysis based on averaging or frequency-decomposition. Indeed, none captures the natural temporal dynamics of current sources contained in local field potentials (LFPs). Since sources are static, they are amenable to disentanglement through blind-source separation techniques. We earlier combined spatial independent component analysis (sICA) and hierarchical clustering to segregate intracerebral sources into spatially coherent groups that represent pathway-specific activations of target populations. The spatial and temporal mixings influence the efficiency of the separation, which can be strongly optimized by a priori knowledge of the sources features. Thus, we used known mixtures of sources from real experiments to reproduce spatiotemporal fluctuations of LFPs in silico through a realistic multineuronal multicompartmental model. Virtual LFPs were essayed to check the advantages of using PCA prior to ICA. The flexibility of this approach allows the repeated modification of the initial conditions for data choice and pretreatment in an experience-guided cyclic process. We also present hints to increase the relative variance of weak sources and to reduce cross-contamination.

Weakly electric fish information processing analyzed through close-loop code-driven stimulation

Angel Lareo
Universidad Autonoma de Madrid, Spain
angel.lareo@estudiante.uam.es
Pablo Varona, Francisco B. Rodriguez

Activity-dependent stimulation techniques in Neuroscience have been implemented through the concept of dynamic clamp in electro-physiological experiments. The same generic principles underlying the dynamic-clamp technique can be used to develop novel protocols to study information processing in the context of electrocommunication of weakly electric fish. Specifically, here we address the use of these protocols in the elephant fish, Gnathonemus petersii, an animal that uses a weak electric field to locate obstacles or food while navigating, as well as for electro-communication with other fishes. To investigate the electrical coding in this fish, we use an adaptive electrical stimulation as a function of the animal's electrical activity. To map this electrical activity to a time series of events, we represent the recorded electrical signal as a binary string. Then we take data words of predetermined length to define information events by considering the correlation between consecutive pulses. From the information analysis, we chose a representative word to trigger the stimulation delivered in the close-loop. We compare the electrical activity generated by Gnathonemus petersii during the closed-loop stimulation protocol versus a random stimulation. Finally, we discuss how this comparison can serve to understand the underlying information processing.

Close-loop stimulation in real-time functional magnetic resonance imaging

Norberto Malpica
Universidad Rey Juan Carlos, Spain
norberto.malpica@urjc.es
Pablo Garcia-Polo, Juan Antonio Hernandez-Tamames

Real time analysis of Functional Magnetic Resonance Imaging allows quality control and analysis of the data while the subject is in the scanner. In experimental settings, very fast processing times have been achieved, and recent studies have shown that RT fMRI feedback is feasible (Weiskopf et al. 2012). RT fMRI, in comparison to all other human brain mapping techniques, represents the only non-invasive method allowing feedback regulation of deep subcortical brain regions. In current fMRI experiments, activity is regulated by the user as a result of the feedback. Closing the loop in real-time fMRI setups will allow the implementation of online activity-dependent stimulation protocols. The close-loop will take a major role in the control of brain activity, facilitating and improving subject self-regulation. The combination of different sources of stimuli (olfactory, visual and auditory) with a certain temporal structure will expose brain pathways and connectivity. By controlling all these stimuli, and modulating them online as a function of activation, we will unveil sequential activations associated to the encoding of sensorial input including cognitive responses. Weiskopf et al. Real-time fMRI and its application to neurofeedback. Neuroimage 62(2): 682-92, 2012.

Long range dependence and the dynamics of exploited fish populations

Rui Mendes
CMAF, University of Lisbon, Portugal
rvilela.mendes@gmail.com
Hugo C. Mendes, Alberto Murta

Long range dependence or long memory is an important notion in many processes in the natural world. Sometimes considered a nuisance in the study of these processes, the existence of long memory is in fact a bonus, in the sense that it provides a window on the underlying mechanisms that generate the observed data. Long range dependence may only be found in the higher order characteristics of the process. A process that looks short range when looked at through second-order properties, may in fact have an underlying long range dependence of higher order properties. This mechanism is illustrated by analyzing some data related to the abundance of exploited fish populations in the North Atlantic.

Inhibitory synapses control anticipation in neuronal circuits

Claudio Mirasso
Universitat de les Illes Balears, Spain
claudio@ifisc.uib-csic.es
Fernanda S. Matias, Leonardo L. Gollo, Pedro V. Carelli, Mauro Copelli

When the connectivity between two brain regions is such that one them (the sender) strongly influences the other (the receiver), a positive phase lag is often expected. The assumption is that the time difference implicit in the relative phase reflects the transmission time of neuronal activity. However, experiments performed in monkeys engaged in processing a cognitive task, a dominant directional influence from one area of sensorimotor cortex to another may be accompanied by either a negative or a positive time delay. Here we present a model of two brain regions, coupled with a well-defined directional influence, that displays similar features to those observed in the experimental data. By reproducing experimental delay times and coherence spectra, our results provide a theoretical basis for the underlying mechanisms of the observed dynamics, and suggest that inhibitory neurons might play a crucial role in the response time of neuronal populations.

Neuro-DYVERSE: building hybrid systems neuroscience

Eva Navarro-Lopez
The University of Manchester/School of Computer Science, England
eva.navarro@cs.man.ac.uk

Tools from control engineering, formal methods of computer science and network science hold the promise of transforming the course of computational neuroscience. In this talk, we will explore how these tools can be combined in a framework called Neuro-DYVERSE. Neuro-DYVERSE is a work in progress and aims towards a further understanding of the adaptive dynamical processes involved in the formation and consolidation of memory in the human brain. In neuroscience, this is known as neuroplasticity: the brainś ability to change due to experience or damage. The dynamical behaviour of networks of billions of neurons is still poorly understood, as is its relationship to the emergence of learning and memory. Preexisting models are still fairly limited. The multi-scale complexity of the problem requires the combination of paradigms from different fields, mainly: hybrid systems, control engineering, automated verification, dynamical systems and network science. Neuro-DYVERSE is built upon the computational-mathematical framework DYVERSE. DYVERSE stands for the DYnamically-driven VERification of Systems with Energy considerations, and focuses on hybrid systems models and tools capturing the mixture of continuous dynamics with discontinuities - that is, abrupt changes or transitions. This work leads towards a new branch of computational neuroscience: hybrid systems neuroscience, to coin a term.

Dynamic Observer: Can we use closed loop observation to characterize individual neurons?

Thomas Nowotny
University of Sussex, England
t.nowotny@sussex.ac.uk

The standard method for characterising ion channels in neurons is voltage clamp in patch clamp recordings [1]. However, in the classical procedure measurements are performed with constant voltage steps and chemical channel blockers are used to isolate individual ion channel types. Because chemical blockers can be irreversible, different ion channels of the same neuron type have to be measured in different individual cells, potentially even in separate preparations from different individual animals. This can be highly problematic and it has been observed that so combining measurements from many different cells does not allow to build accurate whole cell models [2]. Here we introduce a proposal to go beyond the classical constant steps and try to design optimised stimulation patterns to isolate the effect of different ion channels without blockers. Furthermore, we propose to use closed-loop online parameter estimation methods with simulated "dynamic clamp" coupling to then build a model of all ionic currents in an individual neuron simultaneously. If successful this new closed-loop experimentation technology could have deep impacts on our understanding of how individual neurons vary in their ion channel content.

References

[1] Dunlop J, Bowlby M, Peri R, Vasilyev D, Arias R. (2008) Nature Reviews Drug Discovery 7:358-368.

[2] Golowasch J, Goldman MS, Abbott LF, Marder E. (2002) J Neurophysiol. 87(2):1129-1131.

Towards Model-Based Closed-Loop Control of Neuronal Networks

Steven Schiff
Penn State University, USA
sschiff@psu.edu

Since the 1950s, there have been a steadily maturing theory for model-based control of systems, as well as a steadily improving computational neuroscience demonstrating models with increasing fidelity to the biophysics and dynamics of neuronal systems. We have recently demonstrated that there is an emergent unification between seizures and spreading depression (the physiology of migraine auras), derived by improving the biophysical modeling and introducing conservation principles into an extension of the Hodgkin-Huxley equations. Such models represent nodes within neuronal graph directed networks. The topology of how such nodes are connected determines the structural observability and controllability of neuronal systems, which is highly dependent upon the presence of symmetries in such connectivity. We discuss the implications of nodal dynamics and topology on strategies for closed-loop control of neuronal systems.

Closed-loop aproaches to characterize transient neural dynamics

Pablo Varona
Universidad Autonoma de Madrid, Spain
pablo.varona@uam.es
David Arroyo, Francisco B. Rodriguez

During the last 30 years there has been a large development in experimental and theoretical approaches in neuroscience research. However, the assessment of neural dynamics, both in experimental and in modeling efforts, continues to be based to a large extent on a classical, but at the same time very limited, stimulus-response paradigm. In this paradigm a stimulus is deliver to a neural system, and the characteristics of the response are then studied offline in relationship to the given stimulus. In addition, the study of information processing in the nervous system is also often based on the assumption that neural mechanisms are well approximated by steady-state measurements of neural activity. However, transient states may, in many cases, better describe neural network activity. In this talk we will emphasize the need of closed-loop approaches to deal with transient neural dynamics. Our goal is to link new theoretical descriptions of transient neural dynamics with experiments that implement goal-driven closed-loop activity-dependent stimulation. Real-time stimulation feedback enables a wide range of innovative studies to characterize information processing with transient dynamics in neural networks, and to exploit the robustness, flexibility and capacity of this type of dynamics.

Special Session 78: the Navier-Stokes Equations and Related Problems

S. Necasova, Institute of Mathematics, Academy of Sciences, Czech Republic
R. Rautmann, University of Paderborn, Germany
W. Varnhorn, University of Kassel, Germany

The Navier-Stokes equations represent the fundamental equations of fluid dynamics. Recently researchers from many countries around the world have found important new results to the theory of these equations and their applications: Existence proofs for weak, or strong, or even analytical solutions to time-dependent as well as for stationary boundary value problems of the fully nonlinear Navier- Stokes equations or for the linear Stokes equations, considered with a wide variety of initial- and boundary conditions, uniqueness classes in the frame of suitably adapted abstract spaces, questions of asymptotic behavior and stability of solutions, of maximum regularity, new proofs of the maximum modulus theorem, and convergence results with vanishing viscosity. A further important part of research is the interaction of fluid flow with moving bodies or particles, or with additional heat flow, leading to enlarged dynamical systems which combine the Navier-Stokes equations with another equation of evolution. The strong progress in the theory opens the way to efficient numerical schemes for problems in technology and medicine. The aim of our special session will be to bring together leading researchers from all parts of the world and from the different working directions mentioned above, and to initiate exchange of ideas as well as future cooperation.

On estimates for the Stokes flow in a space of bounded functions

Ken Abe
Nagoya University, Japan
kabe@ms.u-tokyo.ac.jp

We consider a composition operator for the Stokes semigroup subject to the Dirichlet boundary condition and the Helmholtz projection on a space of bounded functions. It is known that some regularizing estimate for this composition on L^p plays an important role for studying the Navier-Stokes equations. We show some a priori estimate for this composition operator on a space of bounded functions.

Semi-group theory for the Stokes operator with Navier-type boundary conditions on $\boldsymbol{L}^p$-spaces

Cherif Amrouche
Universite de Pau et des Pays de l'Adour, France
cherif.amrouche@univ-pau.fr
H. Al Baba, M. Escobedo, N. Seloula

The aim of this work is to study the analyticity of the Stokes operator with Navier-type boundary conditions on $\boldsymbol{L}^p$-spaces in order to get strong, weak and very weak solutions to the following evolution Stokes problem:

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} - \Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f},\ \operatorname{div} \boldsymbol{u} = 0 \ \text{ in } \Omega \times (0,T), \\ \boldsymbol{u} \cdot \boldsymbol{n} = 0,\ \mathbf{curl}\, \boldsymbol{u} \times \boldsymbol{n} = \mathbf{0} \ \text{ on } \Gamma \times (0,T), \\ \boldsymbol{u}(0) = \boldsymbol{u}_0 \ \text{ in } \Omega. \end{cases}$$

In this work we prove that the Stokes operator with Navier-type boundary conditions generates a bounded analytic semi-group on the space

$$\boldsymbol{L}^p_{\sigma,T}(\Omega) = \big\{ \boldsymbol{v} \in \boldsymbol{L}^p(\Omega) : \operatorname{div} \boldsymbol{v} = 0 \text{ in } \Omega \\ \text{and } \boldsymbol{v} \cdot \boldsymbol{n} = 0 \text{ on } \Gamma \big\}.$$

The idea is to study the resolvent of the Stokes operator:

$$\begin{cases} \lambda \boldsymbol{u} - \Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f},\ \operatorname{div} \boldsymbol{u} = 0 \text{ in } \Omega, \\ \boldsymbol{u} \cdot \boldsymbol{n} = 0,\ \mathbf{curl}\, \boldsymbol{u} \times \boldsymbol{n} = \mathbf{0} \text{ on } \Gamma, \end{cases} \tag{1}$$

where $\lambda \in \mathbb{C}^*$ with $\operatorname{Re} \lambda \geq 0$. We prove the existence of weak, strong and very weak solutions to Problem (1) satisfying the following resolvent estimate

$$\|\boldsymbol{u}\|_{\boldsymbol{L}^p(\Omega)} \leq \frac{C(\Omega, p)}{|\lambda|} \|\boldsymbol{f}\|_{\boldsymbol{L}^p(\Omega)}.$$

Asymptotic flocking dynamics of Cucker-Smale particles immersed in compressible fluids

Hyeong-Ohk Bae
Ajou University, Korea
hobae@ajou.ac.kr
Young-Pil Choi, Seung-Yeal Ha and Moon-Jin Kang

We propose a coupled system for the interaction between Cucker-Smale flocking particles and viscous compressible fluids, and present a global existence theory and time-asymptotic behavior for the proposed model in the spatial periodic domain. Our model consists of the kinetic Cucker-Smale model for flocking particles and the isentropic compressible Navier-Stokes equations for fluids, and these two models are coupled through a drag force, which is responsible for the asymptotic alignment between particles and fluid. For the asymptotic flocking behavior, we explicitly construct a Lyapunov functional measuring the deviation from the asymptotic flocking states. For a large viscosity and small initial data, we show that the velocities of Cucker-Smale particles and fluids are asymptotically aligned to the common velocity.

Higher order variational space-time approximation of elastic wave propagation

Markus Bause
Helmut Schmidt University Hamburg, Germany
bause@hsu-hh.de

Advanced composites such as carbon fibre reinforced plastics have become one of the most promising materials to build light-weight structures, for instance in aerospace engineering. In the field, these structures are subject to fluid-structure interaction with material fatigue and damage. Material inspection by piezoelectric induced ultrasonic waves is a relatively new and an intelligent technique

for non-destructive evaluation of such structures. For the phenomenological understanding of wave propagation in composite materials and the design of structural health monitoring systems the ability to solve numerically the elastic vector-valued wave equation in three space dimensions,

$$\rho\,\partial_t^2 \boldsymbol{u} - \nabla\cdot\boldsymbol{\sigma}(\boldsymbol{u}) = \boldsymbol{f}\,,$$
$$\boldsymbol{\sigma} = \boldsymbol{C}(\boldsymbol{x})\boldsymbol{\epsilon}\,,$$
$$\boldsymbol{\epsilon} = \left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^\top\right)/2\,,$$

for $\boldsymbol{u} : \Omega\times[0,T]\mapsto\mathbb{R}^3$, with $\Omega\subset\mathbb{R}^3$, is particularly important from the point of view of physical realism. In this contribution we propose and analyze higher order variational space-time Galerkin methods for the numerical approximation of solutions to (1). Continuous and discontinuous variational time discretization schemes that are at least A-stable are studied. Discontinuous Galerkin finite element methods are used for the spatial approximation. The numerical analysis of these methods as well as implementational issues are addressed. Their stability properties and their potential to predict complex wave propagation phenomena are illustrated by numerical experiments. Realistic external loads are computed from solving a fluid-structure interaction problem that is considered further.

Vanishing theorems for the discretely self-similar solutions to the Euler equations

Dongho Chae
Chung-Ang University, Korea
dchae@cau.ac.kr

Discretely self-similar solution is a generalized notion of the self-similar solution, which is equivalent to the time-periodic solution to the self-similar form of the Euler equations. We deduce sufficient conditions to guarantee that the solution is identically zero. More specifically, conditions for the decays of the velocity at spatial infinity implies that the solution is zero (Liouville type theorems). Also, conditions of the velocity at the origin implies that the solution vanishes on the whole of $\mathbb{R}^n$ without decay condition at spatial infinity(unique continuation type theorems).

On the triviality of attractor of viscous Burgers equation on line with periodic boundary conditions and nonautonomous forcing

Jacek Cyranka
Warsaw University, Poland
jacek.cyranka@ii.uj.edu.pl
P. Zgliczynski

This is our current work in progress on proving a conjecture related to the viscous Burgers equation

$$u_t + u\cdot u_x - \nu u_{xx} = f(t,x),$$

where the forcing $f(t,x)$ is a bounded and continuous with respect to time function. The conjecture is that the attractor of the viscous Burgers equation on the line with periodic boundary conditions and nonautonomous forcing is composed of an unique solution u_∞ for any forcing bounded, continuous in time, and periodic in space. Moreover, the convergence towards u_∞ is exponential, and $||u_\infty|| \approx O(1/c)$, where $c = \int_\Omega u_0$. The conjecture is supported by previous works by H.R. Jauslin, H.O. Kreiss, J. Moser (1999), Y. Sinai (1991), and our recent computer assisted proof of existence of globally attracting solutions of the viscous Burgers equation on the line with periodic boundary conditions and nonautonomous forcing. Methods we apply are general and, if successful, shall be applied to other similar partial differential equations including the Navier-Stokes equations.

The existence of a global attractor for the forced critical surface quasi-geostrophic equation in L^2

Mimi Dai
University of Illinois Chicago, USA
mdai@uic.edu
Alexey Cheskidov

We prove that the critical surface quasi-geostrophic equation driven by a force f possesses a compact global attractor in $L^2(\mathbb{T}^2)$ provided $f\in L^p(\mathbb{T}^2)$ for some $p>2$.

A Linearized Model for Compressible Flow past a Rotating Obstacle

Reinhard Farwig
TU Darmstadt, Germany
farwig@mathematik.tu-darmstadt.de
Milan Pokorny

Consider the flow of a compressible Newtonian fluid around/past a rotating obstacle in $\mathbb{R}^3$. After a coordinate transform to get a problem in a time-independent domain we assume the new system to be stationary, then linearize and use Fourier transform to find an explicit solution and estimates in L^q-spaces. However, in contrast to the incompressible case with multipliers based on the heat kernel the new multiplier functions are of a very different type...

Lq regularity of generalized Newtonian flows

Petr Kaplicky
Charles University, Prague, Czech Rep
kaplicky@karlin.mff.cuni.cz
L. Diening

We will present results about regularity of weak solutions to the system describing a flow of a generalized Newtonian fluid. We focus on optimal local estimates in Lebesgue spaces.

Stability of strong solutions of the Navier-Stokes equations in L^3-norm

Petr Kucera
Czech Technical University, Czech Rep
kucera@mat.fsv.cvut.cz

We solve a system of the Navier-Stokes equations with Navier's boundary condition or with Navier-type boundary conditions. We deal with perturbations of initial conditions of strong solutions of our system. We prove that if these perturbations are sufficiently small in L^3 - norm then corresponding solutions are strong too.

Self-propelled motion in a viscous compressible fluid

Vaclav Macha
The Academy of Sciences of the Czech Republic, Czech Rep
macha@math.cas.cz
Sarka Necasova

We focus on an existence of a weak solution to a system describing a self-propelled motion of a single deformable body in a viscous compressible fluid which occupies a bounded domain in the 3 dimensional Euclidean space. The considered governing system for the fluid is the isentropic compressible Navier-Stokes equation. We present a proof an existence of a weak solution up to a collision.

Stability in PDEs and fluid-dynamics: simmetry, optimal Lyapunov functions and norms

Giuseppe Mulone
University of Catania, Italy
mulone@dmi.unict.it
Andrea Giacobbe

The stability of a basic motion plays a central role in fluid dynamics. There are several methods for investigating stability. The spectral methods study the real parts of the eigenvalues of the linear operator of the system and give sufficient conditions of instability. The Lyapunov second method introduces suitable energy-norms (or Lyapunov functions) and gives sufficient conditions for nonlinear stability of the flow. If the linear operator is symmetric with respect to an energy-norm, the linear instability threshold coincides with the energy nonlinear stability. Many fluid systems show stabilizing effects mainly due to forces which give skew-symmetric contributions to linear operator (rotation, chemical concentration, magnetic fields). The energy norm is not sensitive to such effects. Here we prove the coincidence of linear and nonlinear thresholds when the stabilizing effects are present. The coincidence is obtained with the introduction of optimal Lyapunov functions or norms. To this end, we use and improve the reduction method introduced in JMAA, 342, pp.461–476. We show, in particular cases (motion of a protoplanetary disk and some Bénard problems) that the definition of optimal Lyapunov functions does not give coincidence for all physical parameters, and needs to be weakened to prove coincidence and asymptotic stability.

Regularity of a weak solution to the Navier-Stokes equations via a spectral projection of vorticity

Jiri Neustupa
Academy of Sciences of the Czech Republic, Czech Rep
neustupa@math.cas.cz

We formulate sufficient conditions for regularity of a suitable weak solution v to the Navier-Stokes system in R^3 by means of Serrin-type integrability conditions imposed on certain spectral projection of the vorticity $\mathrm{curl}\, v$. The projection is defined by means of the spectral resolution of identity, associated with the self-adjoint operator curl.

Recent results about elliptic problems in the half-space and bounded domains

Huy Hoang Nguyen
Federal University of Rio de Janeiro, Brazil
nguyen@im.ufrj.br
Cherif Amrouche

In this talk, I will present some recent results about div-grad operators, vector potentials, the Laplace equation and the Stokes problem in the half-space with data in L^1-data or in weighted Sobolev spaces. I also talk about very weak solution for some elliptic problems with non regular data in bounded domains.

Asymptotic stability of mild solutions to Navier-Stokes system

Dominika Pilarczyk
University of Wrocław, Institute of Mathematics, Poland
dominika.pilarczyk@math.uni.wroc.pl
Maria Elena Schonbek, Grzegorz Karch

We consider the following initial value problem for the Navier-Stokes system for an incompressible fluid in the whole three dimensional space

$$\begin{aligned} u_t - \Delta u + \nabla \cdot (u \otimes u) + \nabla p &= F,\ (x,t) \in \mathbf{R}^3 \times (0,\infty) \\ \mathrm{div}\, u &= 0, \\ u(x,0) &= u_0(x). \end{aligned}$$

It is well-known that this problem has a unique global-in-time mild solution for a sufficiently small initial condition u_0 and for a small external force F in suitable scaling invariant spaces. We show that these global-in-time mild solutions are asymptotically stable under every (arbitrary large) L^2-perturbation of their initial conditions.

On the existence of weak solutions to the equations of steady flow of heat-conducting fluids with dissipative heating

Milan Pokorny
Charles University in Prague, Czech Rep
pokorny@karlin.mff.cuni.cz
Joachim Naumann, Joerg Wolf

We study a system of partial differential equations describing the steady flow of a heat conducting incompressible fluid in a bounded three dimensional domain, where the right-hand side of the momentum equation includes the buoyancy force. In the present work we prove the existence of a weak solution under both the smallness and a sign condition on physical parameters α_0 and α_1 which appear on the right hand side. The presentation is based on the paper: *On the existence of weak solutions to the equations of steady flow of heat-conducting fluids with dissipative heating*, Nonlinear Analysis, Series B: Real World Applications **13** (2012), No. 4, 1600–1620.

Navier-Stokes Equations with Navier Boundary Condition

Ahmed Rejaiba
University of Pau, France
ahmed.rejaiba@univ-pau.fr
Cherif Amrouche

To study Stokes and Navier-Stokes equations in bounded domain in 3 dimensional, it is necessary to add some boundary conditions. Note that, these equations are mostly studied with no-slip Dirichlet's boundary condition, which is applicable in the case where the boundary of the flow is solid. However, in the physical applications, we are encounter the situations where this condition is not quite feasible. In this case, it is really important to introduce another type of boundary condition, such as Navier boundary condition introduced by Navier in 1827. In this talk, we present some results on existence, uniqueness and regularity for stationary Stokes and Navier-Stokes equations with Navier boundary condition in Hilbert case and in Lp-theory.

Exponential Stability of Electrokinetic Flows

Jürgen Saal
Heinrich-Heine-Universität Düsseldorf, Germany
Juergen.Saal@hhu.de

We prove existence and exponential stability of steady states to the Navier-Stokes-Nernst- Planck-Poisson equations. This system describes the dynamics of charged particles dispersed in an incompressible fluid. The results are based on a suitable Lyapunov functional and conservation of mass.

Stability of the linearized MHD-Maxwell free interface problem

Paolo Secchi
University of Brescia, Italy
paolo.secchi@unibs.it
Davide Catania, Marcello D'Abbicco

We consider the free boundary problem for the plasma-vacuum interface in ideal compressible magnetohydrodynamics (MHD). In the plasma region, the flow is governed by the usual compressible MHD equations, while in the vacuum region we consider the Maxwell system for the electric and the magnetic fields, in order to investigate the well-posedness of the problem, in particular in relation with the electric field in vacuum. At the free interface, driven by the plasma velocity, the total pressure is continuous and the magnetic field on both sides is tangent to the boundary. Under suitable stability conditions satisfied at each point of the plasma-vacuum interface, we derive a basic a priori estimate for solutions to the linearized problem in the Sobolev space $H^1_{\tan}$ with conormal regularity. The proof follows by a suitable secondary symmetrization of the Maxwell equations in vacuum and the energy method. An interesting novelty is represented by the fact that the interface is characteristic with variable multiplicity, so that the problem requires a different number of boundary conditions, depending on the direction of the front velocity (plasma expansion into vacuum or viceversa). To overcome this difficulty, we recast the vacuum equations in terms of a new variable which makes the interface characteristic of constant multiplicity. In particular we don't assume that plasma expands into vacuum.

L^p theory for the Navier-stokes equations with non standard boundary conditions

Nour Seloula
Caen University, France
nour-elhouda.seloula@unicaen.fr
Cherif Amrouche

In a three dimensional bounded eventually multiply-connected domain of class $\mathcal{C}^{1,1}$, we prove existence and uniqueness of vector potentials associated with a divergence-free function and satisfying some boundary conditions in L^p theory. We also present some results concerning various Sobolev's inequalities. Next, we consider the stationary Stokes equations with nonstandard boundary conditions of the form $\boldsymbol{u}\cdot\boldsymbol{n} = g$ and $\mathbf{curl}\,\boldsymbol{u}\times\boldsymbol{n} = \boldsymbol{h}\times\boldsymbol{n}$ or $\boldsymbol{u}\times\boldsymbol{n} = \boldsymbol{g}\times\boldsymbol{n}$ and $\pi = \pi_0$ on the boundary Γ. We prove the existence and uniqueness of weak, strong and very weak solutions corresponding to each boundary condition in L^p theory. To prove the solvability, we study the well-posdeness of some elliptic systems. For this end, it is necessary to establish $Inf - Sup$ conditions which play an essential role in our proofs. Furthermore, two Helmoholtz decompositions which consist of two kinds of boundary conditions such as $\boldsymbol{u}\cdot\boldsymbol{n}$ and $\boldsymbol{u}\times\boldsymbol{n}$ on Γ are given. Finally, we give an application to the Navier-Stokes equations where the proof of solutions is obtained by applying a fixed point theorem over the Oseen equations.

Coupling multiscale fluid-structure interaction models for blood flow simulations

Adélia Sequeira
Lisbon University, Portugal
adelia.sequeira@math.ist.utl.pt

Blood flow in arteries is characterized by pulse pressure waves due to the interaction with the vessel walls. To avoid spurious reactions due to the truncation of the domain for numerical simulations, we consider the coupling between 3D and reduced 1D fluid-structure interaction (FSI) models to describe blood flow in compliant vessels and derive energy estimates for the fully 3D-1D FSI coupling. Numerical tests are presented to illustrate the coupling procedure. This is a joint work with J. Janela and A. Moura.

On the existence and uniqueness of solutions for the Kevin-Voigt equations

Ana Silvestre
Instituto Superior Técnico, Universidade de Lisboa, Portugal
ana.silvestre@math.ist.utl.pt
P. Damázio and P. Manholi

We present some results on the existence and uniqueness of L^q-solutions for the Kevin-Voigt equations.

The time periodic Stokes system in a layer: asymptotic behavior of the solutions at infinity

Maria Specovius-Neugebauer
University of Kassel, Germany
specovi@mathematik.uni-kassel.de
Konstantin Pileckas

We consider solutions to the time periodic Stokes problem in a layer $\Omega = \mathbb{R}^2 \times (0,1) \ni x = (y,z)$:

$$u_t - \Delta u + \nabla p = f, \quad \operatorname{div} u = g \text{ in } \Omega$$
$$u|_{z=1} = 0, \quad u|_{z=0}, \quad u|_{t=0} = u|_{t=2\pi},$$

where the data f, g are also time periodic and smooth with bounded support for simplicity. Starting from solutions with $u \in L^2(L^2_\beta)$, $p \in L^2(L^2_\beta)$, where $L^2_\beta(\Omega)$ is a weighted L^2-space with polynomial weight at infinity, we derive the main asymptotic terms of u, p as $|y|$ tends to infinity.

Long Time Behavior of the Forced Critical Surface Quasi-geostrophic Equation

Andrei Tarfulea
Princeton University, USA
tarfulea@math.princeton.edu
Peter Constantin, Vlad Vicol

We prove the existence of a compact global attractor in $H^1(\mathbb{T}^2)$ for the dynamics of the forced critical surface quasi-geostrophic equation (SQG). After a transient time, the solution is bounded in C^α and H^1 by higher regularity norms of the forcing term f and independently of the initial data. The attractor also has finite fractal (box-counting) dimension.

Stabilization in a two-dimensional chemotaxis-Navier-Stokes system

Michael Winkler
University of Paderborn, Germany
michael.winkler@math.upb.de

We consider a model which was proposed to describe processes of spontaneous pattern formation in populations of swimming aerobic bacteria. This model couples the incompressible Navier-Stokes equations to two evolutionary PDEs which, besides reaction, diffusion and convection, involve nonlinear cross-diffusion of bacteria towards oxygen. The presentation briefly addresses topics of existence theory by reporting on known well-posedness results, as well as on remaining challenges in the analysis. The main focus will then be on the question in how far the considered system is indeed able to generate structures. A partially negative answer in this direction is presented, which asserts stabilization towards spatially homogeneous equilibria in a corresponding two-dimensional initial-bounday value problem.

Local regularity for weak solutions to the equations of unsteady motion of power law fluids $q > 2$

Joerg Wolf
Humboldt University of Berlin, Germany
jwolf@math.hu-berlin.de

In our talk we discuss the local regularity properties of weak solutions to the equations of unsteady motions of a non-Newtonian fluid with shear rate dependent viscosity $\nu \sim |\boldsymbol{D}(\boldsymbol{u})|^{q-2}$ $(q > 0)$. We consider weak solutions $\boldsymbol{u} \in C_w([0,T]; \boldsymbol{L}^2_\sigma(\Omega))$ with $\boldsymbol{D}(\boldsymbol{u}) \in \boldsymbol{L}^q(Q_T)$ satisfying the corresponding integral identity for all smooth solenoidal test functions. For such weak solutions we present a sufficient condition on $\boldsymbol{f}$ which implies that $\boldsymbol{u}$ is continuous in Q. The proof of this result is divided into three steps. First, based on the local pressure decomposition we prove the weak differentiability of $\boldsymbol{V} = |\boldsymbol{D}(\boldsymbol{u})|^{(q-2)/2}\boldsymbol{D}(\boldsymbol{u})$. Secondly, by using the local pressure decomposition $p = \frac{\partial p_H}{\partial t} + p_0$ we verify the time regularity $\frac{\partial}{\partial t}(\boldsymbol{u} + \nabla p_H) \in L^2(0,T; \boldsymbol{L}^2_{\mathrm{loc}}(\Omega))$. Finally, with help of the method of differences with respect to time together with a standard scaling argument we obtain the required regularity property of $\boldsymbol{u}$.

The Oberbeck-Boussinesq approximation in R^3 as a limit of the compressible Naver-Stokes-Fourier system with a low Mach number

Aneta Wroblewska-Kaminska
Polish Academy of Sciences, Poland
a.wroblewska@impan.pl

We will present the asymptotic analysis of solutions to the compressible Navier-Stokes-Fourier system, when the Mach number is small proportional to ϵ, Froud number is proportional to $\sqrt{\epsilon}$ and $\epsilon \to 0$ and the domain containing the fluid varies with changing parameter ϵ. In particular, the fluid is driven by a gravitation generated by object(s) placed in the fluid of diameter converging to zero. As $\epsilon \to 0$, we will show that the fluid velocity converges to a solenoidal vector field satisfying the Oberbeck-Boussinesq approximation on R^3 space with a concentric gravitation force. The proof is based on the spectral analysis of the associated wave propagator (Neumann Laplacian) governing the motion of acoustic waves.

References

[1] A. Wróblewska-Kamińska. Asymptotic analysis of complete fluid system on varying domain: from compressible to incompressible flow. Preprint on http://mmns.mimuw.edu.pl/preprints.html.

[2] E. Feireisl, T. Karper, O. Kreml, J. Stebel. Stability with respect to domain of the low Mach number limit of compressible viscous fluids. *M3AS*, 23(13):2465-2493, 2013.

[3] E. Feireisl, M. Schonbek. On the Oberbeck-Boussinesq approximation on unbounded domains. *Nonlinear partial differential equations*, edited by: H.Holden, K.H.Karlsen, Abel Symposial, vol. 7, Springer, Berlin, 2012.

[4] E. Feireisl. Local decay of acoustic waves in the low mach number limits on general unbounded domains under slip boundary conditions. *Commun. Partial Differential Equations* 36,1778-1796, 2011.

Special Session 79: Modeling and Computation in Cell Biology, Stem Cells and Development

Ching-Shan Chou, Ohio State University, USA,
Qing Nie, University of California at Irvine, USA

Cell and developmental biology has been major areas in the mathematical modeling world because of their fundamental importance. More recently, stems cells have attracted a lot of attention, and it provides a new venue to understand development and other areas of biology. This minisymposium aims to focus on these areas and brings together researchers working on modeling complex biological systems using differential equations and computational approaches. Areas represented by the speakers include computational modeling of morphogenesis, cell signaling, gene regulatory network, stem cell, cancer and early embryonic development, etc. Future advances in interaction of computational methods and modeling for various biological problems will be discussed.

Mathematical Methods for Multiscale Modelling in Cell Biology

Radek Erban
University of Oxford, England
erban@maths.ox.ac.uk

I will discuss methods for spatio-temporal modelling in cell biology. Three classes of models will be considered:

(i) microscopic (molecular-based) models which are based on the simulation of trajectories of molecules and their localized interactions (for example, reactions);

(ii) mesoscopic (lattice-based) models which divide the computational domain into a finite number of compartments and simulate the time evolution of the numbers of molecules in each compartment; and

(iii) macroscopic (deterministic) models which are written in terms of mean-field reaction-diffusion-advection partial differential equations (PDEs) for spatially varying concentrations.

In the first part of my talk, I will discuss connections between the modelling frameworks (i)-(iii). I will consider chemical reactions both at a surface and in the bulk. In the second part of my talk, I will present hybrid (multiscale) algorithms which usemodels with a different level of detail in different parts of the computational domain. The main goal of this multiscale methodology is to use a detailed modelling approach in localized regions of particular interest (in which accuracy and microscopic detail is important) and a less detailed model in other regions in which accuracy may be traded for simulation efficiency.

Rule-based kinetic modelling of phosphorylation-dephosphorylation systems

Vahid Shahrezaei
Imperial College London, England
v.shahrezaei@imperial.ac.uk
Omer Dushek

One of the challenges in modelling biochemical networks is their combinatorial complexity that gives rise to large number of molecular species even in relatively simple systems that involve multiple modifications. Rule-based modelling has been proposed as a framework to effectively deal with this problem. Here, I will present two example of applications of rule-based modelling to the study of signalling in cellular systems. The first example is on the study of ultra-sensitivity in multi-site phosphorylation systems. We show that multi-site phosphorylation can produce local saturation of enzymes, providing a mechanism for ultra-sensitivity even if globally enzymes are not saturated. In the second example, we will model oligomerization in FRET-based biosensors and illustrate that the read-out of biosensors can be highly non-linear. We propose optimal design and working regimes for biosensors to produce reliable reporting.

An Enzymatic Model of Prion Aggregate Dynamics in Yeast

Suzanne Sindi
UC Merced, USA
ssindi@ucmerced.edu
Jason K. Davis

Prion proteins are responsible for a variety of diseases in mammals such as Creutzfeldt-Jakob disease in humans and mad-cow diease in cattle. According to the prion hypothesis, misfolded versions of a protein appear and form prion aggregates, complexes of multiple misfolded proteins ranging in size from tens to hundreds of proteins. The prion state is infectious and spreads to healthy proteins by conversion of the healthy confirmation to the misfolded state (which increases the size of the aggregate). Prion aggregates also increase in number by fragmentation, thus increasing the number of templates which act to convert healthy proteins. The dynamics of prion aggregates have been investigated with a number of mathematical models. Most mathematical models assume that the fragmentation rate is proportional to the size of the aggregate; we present yeast data to demonstrate the inadequacy of this assumption, then extend the model to include the effects of an enzymatic limitation. Experiments have shown that changing a separate protein's expression levels has measurable effects on the aggregate size distribution, suggesting its role as a molecular chaperone in the fragmentation process. We perform general analyses of our more complete model, then compare it with experimental data.

Mathematical model of stochastic-like but deterministic and stable dynamics in cell biology

Yevhen Suprunenko
Lancaster University, England
y.suprunenko@lancaster.ac.uk
P. T. Clemson, and A. Stefanovska

In cell biology, certain time-dependent processes or characteristics can have time-dependent and stochastic-like dynamics. However, simultaneously this dynamics is stable and it resists external perturbations. We now ask, what is a mechanism of stability that can account for such time-dependent dynamics? We present an answer to this question and discuss it in detail using the non-excitable cell membrane potential dynamics as an example. The membrane potential in such cells is typically considered to be fluctuating stochastically around a constant value. However, under metabolic or oxidative stress a membrane potential can have strong and stable deterministic oscillations which resist external perturbations. Considering a detailed model, we show that the considered example belongs to a special class of nonautonomous deterministic oscillatory systems which have a time-dependent point attractor (driven steady state). Such systems were recently introduced and named chronotaxic (from chronos - time and taxis - order) because they possess a time-dependent point attractor which makes the dynamics ordered in time. Thus, chronotaxic systems are able to model complex, time-dependent dynamics which may look stochastic.

A New Approach to Feedback for Robust Signaling Gradients(1)

Frederic Wan
University of California, Irvine, USA
fwan@uci.edu
Taisa Kushner(2), Aghavni Simonyan(3) and Frederic Y. M. Wan(3)

The patterning of many developing tissues is orchestrated by gradients of morphogens through a variety of elaborate regulatory interactions. Such interactions are thought to make gradients robust–i.e. resistant to change in the face of genetic or environmental perturbations–but just how this might be done is a major unanswered question. Past numerical simulations and analytical studies suggest that robustness of signaling gradients cannot be attained by negative feedback (of the Hill's function type) on signaling receptors but can be achieved through morphogen degradation by non-signaling receptor molecules (or non-receptors for short) such as heparan sulfate proteoglycans. However, evidence of feedback regulating signaling gradients has been reported in the literature. The present paper undertakes a different approach to the role of feedback in robust signaling gradients. The overall goal of the project is to investigate the effective-ness of feedback on ligand synthesis, receptor-mediated degradation, non-receptor synthesis and other regulatory processes in morphogen gradient systems. As a first step, we present in this talk a proof of concept examination of a new spatially uniform feedback process that is distinctly different from the conventional spatially non-uniform Hill function approach.

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(2) Undergraduate at St. Olaf College supported by the NSF REU awarded to MBI of Ohio State University to participate in the 2013 MCBU Program at U.C. Irvine.

(3) Department of Mathematics, University of California, Irvine

Computation of Transition State and its Applications in System Biology

Lei Zhang
Peking University, Peoples Rep of China
zhangl@math.pku.edu.cn
Qing Nie

The dynamics of complex biological systems is often driven by multiscale, rare but important events. In this talk, I will introduce the numerical methods for computing transition states, and then give two examples in distinct biological systems: one is a multiscale stochastic model to investigate a novel noise attenuation mechanism that relies on more noises in different cellular processes to coordinate cellular decisions during embryonic development; the other is a phase field model to study the neuroblast delamination in Drosophila.

Special Session 80: Theory, Numerical Methods, and Applications of Stochastic Systems and SDEs/SPDEs

Wanyang Dai, Nanjing University, China

In this special session, theory and numerical methods and related analysis of stochastic systems and SDEs/SPDEs will be addressed. These SDEs/SPDEs and systems include forward/backward stochastic ordinary/partial differential equations. Their interactions and applications with the areas of diffusion approximation, optimal control, queueing system, information theory and technology, wireless and wireline communication systems, finance, etc. will be highlighted.

A relaxed stochastic maximum principle for singular optimal control of forward-backward doubly stochastic differential equations with jumps

Abdulrahman Al-Hussein
Qassim University, Saudi Arabia
alhusseinqu@hotmail.com
Boulakhras Gherbal

In this talk we derive the relaxed stochastic maximum principle for singular control problems with jumps. The problem is governed by a fully coupled forward-backward doubly stochastic differential equations (FBDSDEs) with Poisson jumps and have both regular controls, which are absolutely continuous, and singular controls. All coefficients appearing in these equations depend on control variables. We establish sufficient conditions for optimality for this problem under either full or partial information.

Optimal stopping for diffusions via sieve empirical minimization: convergence and complexity

Denis Belomestny
Duisburg-Essen University, Germany
denis.belomestny@uni-due.de

In this talk I present a new approach towards solving optimal stopping problems for multidimensional diffusion with Monte Carlo. The approach is based on the dual representation for the solution of optimal stopping problems and uses martingale sieves to numerically solve the dual optimization problem. I analyze the complexity of the proposed algorithm and prove its convergence.

High order approximations for the filtering problem

Dan Crisan
Imperial College London, England
d.crisan@imperial.ac.uk

The solution of the continuous time filtering problem can be represented as a ratio of two expectations of certain functionals of the signal process. These functionals are parametrized by the observation path. We discuss a new class of discretization methods for these functionals and their convergence rate as a function of the corresponding partition mesh.

Adapted Solution, Numerical Methods and Analysis via Malliavin Calculus for A Unified B-SPDE and Their Applications in Finance

Wanyang Dai
Nanjing University, Peoples Rep of China
nan5lu8@netra.nju.edu.cn

The aim of this research is to study the adapted solution, numerical methods, and related convergence analysis for a unified backward stochastic partial differential equation (B-SPDE). The equation is vector-valued, whose drift and diffusion coefficients may involve nonlinear and high-order partial differential operators. Under certain generalized Lipschitz and linear growth conditions, the existence and uniqueness of adapted solution to the B-SPDE are justified. The methods are based on completely discrete schemes in terms of both time and space. The analysis concerning error estimation or rate of convergence of the methods is conducted. The key of the analysis is to develop new theory for random field based Malliavin calculus to prove the existence and uniqueness of adapted solutions to the first-order and second-order Malliavin derivative based B-SPDEs under random environments. Furthermore, we will also address the related issues of our unified B-SPDE involving jumps. In addition, we will present the applications of our unified B-SPDE in finance, particularly, in the fields of optimal portfolio decision-making and mean-variance hedging with external random environmental risk factors.

Nonequilibrium steady-states for particle systems

Yao Li
New York University, USA
yaoli@cims.nyu.edu
Lai-Sang Young

In this talk I will present our new results on the nonequilibrium steady states (NESS) for a class of (stochastic) particle systems coupled to unequal heat baths. These stochastic models are derived from the mechanical chains studied by Eckmann and Young by randomizing certain quantities while retaining other features of the model. Our results include the existence and uniqueness of nonequilibrium steady states, their relation to Lebesgue measure, tail bounds on total energy and number of particles in the system, and exponential convergence to steady states from suitable initial conditions.

Some Results on HJB equations with Quadratic and Superquadratic Hamiltonian

Federica Masiero
Universita‘ di Milano Bicocca, Italy
federica.masiero@unimib.it
Adrien Richou

We present some results on Hamilton Jacobi Bellman equations in an infinite dimensional Hilbert space, where the Hamiltonian has quadratic and superquadratic growth with respect to the derivative of the value function, and the coefficients can have polynomial growth with respect to the state variable. The results allow to study stochastic optimal control problems for suitable controlled state equations with unbounded control processes. In the case of quadratic hamiltonian, we show some situations where it is possible to deal with final datum only continuous. The talk is partially based on a joint work with A. Richou.

Numerical approximation schemes for fractional diffusions

David Nualart
The University of Kansas, USA
nualart@math.ku.edu
Yaozhong Hu and Yanghui Liu

The purpose of this talk is to present a new modified Euler scheme for stochastic differential equations driven by a fractional Brownian motion with Hurst parameter $H > \frac{1}{2}$. The rate of convergence of this numerical scheme with step size $1/n$ turns out to be $n^{\frac{1}{2}-H}$ if $H\frac{3}{4}$. These results have been obtained applying techniques of Malliavin calculus. We will also discuss the corresponding weak approximation results and central limit theorems for the fluctuations of the approximation error.

Non-standard Skorokhod convergence of Lévy-driven convolution integrals in Hilbert spaces

Markus Riedle
King's College London, England
markus.riedle@kcl.ac.uk
Ilya Pavlyukevich

In many problems of engineering, physics or finance, the evolution of a stochastic system can be described by stochastic convolution integrals of a deterministic kernel with respect to a Lévy process in a Hilbert space. In this talk we study the convergence in probability of a stochastic convolution integral process for kernels depending on a parameter. For many examples, the appropriate topology is not the standard but another Skorokhod topology (M_1), which is much less often studied. It turns out that in infinite dimensional Hilbert spaces there are three different kinds of this non-standard Skorokhod topology, i.e in a strong, weak and product sense. We establish a general characterisation for a sequence of stochastic processes in a Hilbert space to converge in these non-standard Skorokhod topologies in terms of a corresponding oscillation function. The results are applied to the infinite dimensional integrated Ornstein-Uhlenbeck process with a diagonalisable generator.

The Stochastic Porous Media Equations on $\mathbb{R}^d$

Michael Roeckner
University of Bielefeld, Germany
roeckner@math.uni-bielefeld.de
Viorel Barbu, Francesco Russo

This talk is on existence and uniqueness of solutions to the stochastic porous media equation $dX - \Delta\psi(X)dt = XdW$ on $\mathbb{R}^d$. Here, W is a Wiener process and ψ is a maximal monotone graph in $\mathbb{R} \times \mathbb{R}$ such that $\psi(r) \leq C(|r|^m + 1), \forall r \in \mathbb{R}$. In this general case the dimension is restricted to $d \geq 3$. When ψ is Lipschitz, the well-posedness, however, holds for all dimensions on the classical Sobolev space $H^{-1}(\mathbb{R}^d)$. If $\psi(r)r \geq \rho|r|^{m+1}$ and $m := \frac{d-2}{d+2}$, we prove finite time extinction of the solutions with strictly positive probability.

A linear-quadratic optimal control problem of forward-backward stochastic differential equations with partial information

Jie Xiong
University of Macau and University of Tennessee, Macau
jiexiong@umac.mo
Guangchen Wang, Zhen Wu

We study a linear-quadratic optimal control problem derived by forward-backward stochastic differential equations, where drift coefficient of observation equation is linear with respect to state, and observation noise is correlated with state noise, in the sense that the cross-variation of state and observation is nonzero. A backward separation approach is introduced. Combining it with variational method and stochastic filtering, two optimality conditions and a feedback representation of optimal control are derived. Closed-form optimal solutions are obtained in some particular cases. As an application of the optimality conditions, a generalized recursive utility problem from financial markets is solved explicitly.

Special Session 81: Improving Climate and Weather Prediction Through Data-Driven Statistical Modeling

Dimitris Giannakis, Courant Institute, New York University, USA
John Harlim, The Pennsylvania State University, USA

Advancing weather and climate prediction on time scales from several days to years is limited by the capability of operational and research prediction systems to represent coupled processes in the Earth system involving precipitating convection, low-frequency modes in the ocean, and interactions between the atmosphere, ocean, and cryosphere. A major challenge in contemporary applied science is to create efficient yet faithful models of the subgrid-scale processes responsible for these organized phenomena, as well as to assimilate sparse noisy observations to minimize initial-condition and parametric uncertainties in forecast models. Data-driven statistical modeling is a promising interdisciplinary approach to address these issues combining ideas from dynamical systems theory, stochastic processes, and data analysis algorithms. This special session aims to bring together researchers from across the spectrum of disciplines related to statistical-stochastic modeling of climate to discuss the development and application of emerging ideas and techniques for these important and difficult practical issues.

Multi model mixture density estimators & information theory for stochastic filtering and prediction

Michal Branicki
Department of Mathematics, University of Edinburgh, Scotland
m.branicki@ed.ac.uk

Multi Model Ensemble (MME) predictions are a popular ad-hoc technique for improving imperfect predictions of high-dimensional, multi-scale dynamical systems. The heuristic idea behind MME prediction framework is simple: given a collection of imperfect models, one considers predictions obtained through the convex superposition of the individual probabilistic forecasts in the hope of mitigating model error. However, it is not obvious which models - and with what weights - should be included in the MME forecast in order to achieve the best predictive performance. I will show that an information-theoretic approach to this problem allows for deriving a sufficient condition for improving dynamical predictions within the MME framework; moreover, this formulation gives rise to practical conditions for optimising data assimilation techniques based on multi model ensembles.

Sequential Monte Carlo parameter estimation for dynamical systems

Daniela Calvetti
CWRU, USA
dxc57@case.edu
Andrea Arnold, Erkki Somersalo

In sequential Monte Carlo methods, the posterior distribution of an unknown of interest is explored in a sequential manner, by updating the Monte Carlo sample as new data arrive. In a similar fashion, particle filtering encompasses different sampling techniques to track the time course of a probability density that evolves in time based on partial observations of it. Methods that combine particle filters and sequential Monte Carlo have been developed for some time, mostly in connection with estimating unknown parameters in stochastic differential equations. In this talk, we present some new ideas suitable for treating large scale, non-stochastic, severely stiff systems of differential equations combining sequential Monte Carlo methods with classical numerical analysis concepts.

Modeling of unresolved scales with data-inferred stochastic processes

Daan Crommelin
CWI Amsterdam, Netherlands
Daan.Crommelin@cwi.nl

I will discuss a data-driven stochastic approach to modeling unresolved scales, in which feedback from micro-scale processes is represented by a network of Markov processes. The Markov processes are conditioned on macro-scale model variables, and their properties are inferred from pre-computed high-resolution (micro-scale resolving) simulations. These processes are designed to emulate, in a statistical sense, the feedback observed in the high-resolution simulations, thereby providing a statistical-dynamical coupling between micro- and macro-scale models. This work is primarily aimed at applications in atmosphere-ocean science (stochastic parameterizations of atmospheric convection and of mesoscale oceanic eddies).

Extracting spatiotemporal patterns from data with dynamics-adapted kernels

Dimitrios Giannakis
New York University, USA
dimitris@cims.nyu.edu

Kernel methods provide an attractive way of extracting features from data by biasing their geometry in a controlled manner. In this talk, we discuss a family of kernels for dynamical systems featuring an explicit dependence on the dynamical vector field operating in the phase-space manifold, estimated empirically through finite differences of time-ordered data samples. The associated diffusion operator is adapted to the dynamics in that it generates diffusions along the integral curves of the dynamical vector field. We present applications to toy dynamical systems and comprehensive climate models.

Linear theory for filtering nonlinear multiscale systems with model error

John Harlim
The Pennsylvania State University, USA
jharlim@psu.edu
Tyrus Berry

We will discuss filtering of multiscale dynamical systems with model error arising from limitations in resolving the smaller scale processes. From the mathematical analysis, we learn that for a continuous time linear model with Gaussian noise, there exists a unique choice of parameters in a linear reduced model for the slow variables which gives the optimal filtering when only the slow variables are observed. Moreover, these parameters simultaneously gives the best equilibrium statistical estimates, and as a consequence they can be estimated offline from the equilibrium statistics. By examining a nonlinear test model, we show that the linear theory extends in this non-Gaussian, nonlinear configuration as long as we know the optimal stochastic parameterization and the correct observation model. However, when the stochastic parameterization model is inappropriate, parameters chosen for good filter performance may give poor equilibrium statistical estimates and vice versa; this finding is based on analytical and numerical results on our nonlinear test model and the two-layer Lorenz-96 model. Finally, even when the correct stochastic ansatz is given, it is imperative to estimate the parameters simultaneously and to account for the nonlinear feedback of the stochastic parameters into the reduced filter estimates. In numerical experiments on the two-layer Lorenz-96 model, we find that the parameters estimated *online*, as part of a filtering procedure, simultaneously produces accurate filtering and equilibrium statistical prediction. In contrast, a linear regression based offline method, which fits the parameters to a given training data set independently from the filter, yields filter estimates which are worse than the observations or even divergent when the slow variables are not fully observed. This finding does not imply that all offline methods are inherently inferior to the online method in nonlinear estimation problems, it only suggests that an ideal estimation technique should estimate all parameters simultaneously whether it is online or offline.

On extensions of Markov models towards multiscale setting: unresolved scales and data-driven models

Illia Horenko
University of Lugano, Switzerland
horenkoi@usi.ch
J. de Wiljes

Accumulation of large amounts of measurements and simulation data give strong impulses to development of new data-driven approaches. One of the central challenges for the statistical analysis and data-driven long-time modelling methods in this area is posed by the intrinsically multiscale and multiphysics nature of the underlying phenomena. One of the significant manifestations of this issue is that such approaches should be confronted with a problem of systematically missing information from unresolved or unmeasured scales. However, this missing information might be crucial for better understanding of the corse-grained dynamics. As demonstrated recently in general mathematical context, such systematically missing data/information may induce non-stationarity of the resulting data-driven model for the observed/analyzed quantities and can lead to biased and distorted results when applying standard stationary data analysis methods to such problems. In this talk some new theoretical results for the case of spatio-temporal Markov process subject to impact of resolved and systematically-missing external impacts will be presented. Limitations of standard data-analysis tools from machine learning (e.g., artificial neuronal networks and support vector machines) in this generic setting will be illustrated on a toy model system and application of the new framework to analysis and understanding of satellite ice measurement data will be demonstrated.

Deterministic filtering of discretely observed SDE

Kody Law
KAUST, Saudi Arabia
kody.law@kaust.edu.sa
Hamidou Tembine, Raul Tempone

I will talk about filtering a continuous-time stochastic process which is observed at discrete observation times. A mean-field ensemble Kalman filter (EnKF) is introduced, and a deterministic discretization of its density is proven to asymptotically recover the filtering distribution more accurately than the traditional finite-ensemble Monte Carlo version, for sufficiently low dimensional state-space. This improvement is lost when the underlying distribution deviates from Gaussian, due to the intrinsic assumptions in the EnKF methodology. However, the analogous deterministic discretization of the true filtering density is proven to be both consistent and asymptotically more accurate than the analogous particle methods. The prospect of extension to higher-dimensional state-spaces will be discussed.

Optimal model-free prediction of multivariate time series with applications to climate

Jakob Runge
Humboldt University, Germany
jakobrunge@gmail.com

We address the problem of predicting future measurements of a single time series from a set of multivariate predictor time series in an information theoretic framework. We investigate in how far this can be done optimally given the available information. Such an optimality criterion has to balance the effect of including too few or the wrong predictors (model-misspecification) and too many predictors (practical problem of overfitting) and develop a practical prediction algorithm to address these issues. The performance and challenges are demonstrated on multivariate nonlinear stochastic delay processes as well as on real climate data.

Diffusion map methods in attractor reconstruction

Tim Sauer
George Mason University, USA
tsauer@gmu.edu
Tyrus Berry

We survey recent progress in the application of spectral dimension reduction methods to reconstruct attractors from time series data. In 2012, Giannakis and Majda showed how to combine diffusion map methodology with delay coordinates to more faithfully represent dynamics. We discuss this technique and variations that can reveal desired intrinsic aspects of the topology and geometry of the underlying dynamical system.

Multi-scale properties of idealized Walker Cell

Joanna Slawinska
New York University Abu Dhabi, United Arab Emirates
js6206@nyu.edu
Olivier Pauluis, Andrew Majda, Wojciech Grabowski

Two-dimensional Walker circulation over a planetary scale domain is simulated with cloud resolving model for an extended period of time. The simulated flow is characterized by complex multi-scale interactions that are difficult to disentangle. In particular, impact of convection on the dynamics of other scales, although established to be crucial, lacks detailed understanding. Thus, novel approaches have to be applied. Here, we test these approaches in an idealized setting of planetary scale circulation, with tens of convective systems sampled over several hundred days. The circulation exhibits intra-seasonal variability on a time-scale of about 20 days with quasi-periodic intensification of the circulation and broadening of the convective regime. The low frequency oscillation has four main stages: a suppressed stage with strengthened mid-level circulation intensification phase, active phase with strong upper level circulation and a weakening phase. Various physical processes driving low-frequency variability are discussed, with the particular emphasis on the moisture impact. Low-frequency variability and the associated expansion and contraction of the Walker circulation are closely tied to various kinds of organized convective systems that propagate throughout the domain. Associated flow properties across scales, convective, to mesoscale, to synoptic and global, are diagnosed. For that, individual scales are decomposed by wavelet decomposition applied to recently introduced isentropic stream function. Also, prevailing regimes of convection and convective organization are revealed and characterized by clustering of isentropic streamfunction and by applying spatio-temporal analysis.

Asymmetry in the Signals of Madden-Julian Oscillation Deep Convection

Wen-wen Tung
Purdue University, USA
wwtung@purdue.edu
Dimitrios Giannakis, Andrew J. Majda

Equatorial heating has a profound impact on the general circulation of the atmosphere. We examine north-south asymmetry in convection associated with the 20–90-day Madden-Julian oscillation (MJO) propagating across the Indo-Pacific warm-pool region, using satellite IR brightness temperatures and the ECMWF Interim Reanalysis. We show the predominantly antisymmetric MJO convection has a distinctly different origin over the Indian Ocean than its symmetric counterpart, as well as a drastically distinct propagation over the Maritime Continent. In particular, the asymmetric signal remains intact and strong while the symmetric signal becomes significantly diminished upon passage over the Maritime Continent. Implications of energy conversion associated with both signals will be discussed.

Special Session 82: Celestial Mechanics

Marian Gidea, Yeshiva University, USA
Tere Seara, Universitat Politecnica de Catalunya, Spain

Historically speaking, celestial mechanics laid the foundations for the birth of dynamical systems. Numerous problems and methods in modern dynamics are rooted in the study of the motion of satellites, planets, stars, galaxies, the stability of the solar system, etc. Regular and singular perturbation methods, exponentially small phenomena, KAM theory, normally hyperbolic invariant manifolds, shadowing lemmas, instability and diffusion, are just a few of the directions that witnessed recent advances in conjunction with fundamental research on celestial mechanics. This special session on Celestial Mechanics is devoted to survey some of the latest mathematical progress in these areas and its application to concrete models of celestial mechanics.

Shilnikov lemma for a nondegenerate critical manifold and second species solutions of the 3 body problem

Sergey Bolotin
Department of Mathematics, University of Wisconsin, USA
bolotin@math.wisc.edu
Piero Negrini

We consider a Hamiltonian system possessing a normally hyperbolic symplectic manifold M consisting of equilibria and prove an analog of Shilnikov lemma (or strong λ-lemma). We use it to show that certain chains of heteroclinic orbits to M can be shadowed by a trajectory with energy H close to $H|_M$. This is an generalization of a theorem of Shilnikov and Turayev. Applications to the Poincaré second species solutions of the 3 body problem will be given. The talk is based on [1,2].

References

[1] S. Bolotin, P. Negrini, Variational approach to second species periodic solutions of Poincare of the three-body problem. Discrete Contin. Dyn. Syst., 33 (2013), 1009–1032.

[2] S. Bolotin, P. Negrini, Shilnikov lemma for a nondegenerate critical manifold of a Hamiltonian system. Regular and Chaotic Dynamics, 18 (2013), 779–805.

Few bodies problems. Models to understand the dynamics of the solar system.

Jaime Burgos
Yeshiva University, USA
america1587@hotmail.com

Few bodies problems have been studied for long time in Celestial Mechanics in order to understand the complex dynamics of our solar system. Two, three and more recently, four body problems have been studied as simplified models of planetary systems or as benchmark models where new mathematical theories can be tested. In this talk we will show some new results regarding to these problems, such results are related to invariant objects of the dynamics: periodic orbits, equilibrium points and their invariant manifolds.

Arnold diffusion in the planar elliptic restricted three-body problem

Maciej Capinski
AGH University of Science and Technology, Poland
mcapinsk@agh.edu.pl
Marian Gidea, Rafael de la Llave

We present a diffusion mechanism for time dependent perturbations of autonomous, integrable Hamiltonian systems. This mechanism is based on shadowing of pseudo-orbits generated by two dynamics; unlike other approaches, it does not rely on the KAM theory and/or Aubry-Mather theory. As an application, we show, in the case of the planar elliptic restricted three-body problem, the existence of diffusing trajectories whose energy changes between some given levels, while moving across a perturbed family of Lyapunov orbits. Some of the assumptions for this result have only been verified through high precision numerical computations.

Dynamics around the collinear points in the circular restricted three body problem, including Solar Radiation Pressure and Oblateness.

Marta Ceccaroni
Universitá degli studi di Roma Tor Vergata, Italy
ceccaron@mat.uniroma2.it
S. Bucciarelli, A. Celletti, G. Pucacco

The bifurcation thresholds of the Halo orbits is investigated for the Circular Restricted Three Body Problem (CR3BP) with the inclusion of Solar Radiation Pressure (SRP) and oblateness. This is carried on both numerically and analytically. After reducing the system to the central manifold, which eliminates the hyperbolic directions of the motion, the Bifurcation thresholds are thus numerically evaluated using Fast Lyapunov Indicators (FLI) and Frequency analysis techniques. The accordance between the numerical results and the analytical estimations is shown. Finally the variation of the bifurcation values as the SRP varies into a feasible range, accounting near term technology advances, is investigated.

Hunting for resonances within space debris

Alessandra Celletti
University of Roma Tor Vergata, Italy
celletti@mat.uniroma2.it

Since the beginning of space exploration a large number of space debris accumulated in the neighborhood of the Earth, from the near atmosphere to the geosynchronous region. Indeed, the fragmentation of operative spacecraft and satellites could result in the dramatic formation of dangerous space debris. Understanding the overall orbital evolution of these objects is essential for maintenance and control strategies, as well as for space debris mitigation. In this talk, I present a description of the dynamics of space debris in the main resonances by using the Hamiltonian formalism and simple mathematical tools. The structure of the resonances is analyzed in detail, either using analytical methods or implementing numerical techniques, like the computation of the Fast Lyapunov Indicators (FLIs). In particular, the FLIs provide a cartographic study of the resonances, showing regular and chaotic behaviors. The results are validated by a comparison with a model developed in Cartesian coordinates, including the geopotential, the lunar and solar gravitational attractions and the solar radiation pressure. This work is in collaboration with C. Gales.

Global Instability in the ERTBP

Amadeu Delshams
Universitat Politecnica de Catalunya, Spain
Amadeu.Delshams@upc.edu
Vadim Kaloshin, Abraham de la Rosa and Tere M. Seara

The goal of this talk is to show the existence of global instability in the elliptic restricted three body problem (ERTBP). The main tool is to compute two different scattering maps associated to the infinity manifold, which behaves topologically as a normally hyperbolic invariant manifold (NHIM). The combination of both scattering maps allows us to build trajectories whose angular momentum increases arbitrarily. The computation of such scattering maps relies heavily on the computation of the Melnikov potential associated to the separatrix of the NHIM.

Stability in The Sitnikov Problem with restriction to a circle

Luis Franco Perez
Universidad Autonoma Metropolitana - Cuajimalpa, Mexico
lfranco@correo.cua.uam.mx
Ernesto Perez-Chavela, Marian Gidea

We introduce a new problem: the Sitnikov Problem with restriction to a circle, that is two bodies with positive masses (primaries) follow elliptic Keplerian orbits on a fixed plane and a third massless body (secondary) restricted to move on a circle, passing through the center of mass of the primaries orthogonally. In this talk we assume eccentricity zero for the primaries and we take as a parameter the ratio between the radii of the circle followed by the primaries and the circle of the secondary. We begin showing that the introduced statement is the link between another two problems already studied: the classical Sitnikov problem (when primaries follow circular paths) and the 2-body problem on the circle. After we determine two equilibria: the origin and the opposite in the circle. We analyze the stability of each equilibrium. Particularly, we will show the alternation of the stability for the latter equilibrium as the parameter varies.

Exponentially small splitting of separatrices to whiskered tori with frequencies of constant type in the RTBP

Marina Gonchenko
Technische Universitaet Berlin, Germany
gonchenk@math.tu-berlin.de
Amadeu Delshams, Pere Gutiérrez

We study the splitting of invariant manifolds of whiskered tori with two frequencies in nearly-integrable Hamiltonian systems which come from the three-dimensional RTBP. It is known that in a neighborhood of the elliptic equilibrium L_4 there are KAM tori as well as single resonance zones where lower dimensional (2D) hyperbolic tori appear. We consider a 2-dimensional torus with a fast frequency vector $\omega/\sqrt{\epsilon}$, with $\omega = (1, \Omega)$, where Ω is an irrational number of constant type, i.e. a number whose continued fraction has bounded entries. Applying the Poincaré-Melnikov method, we find exponentially small lower bounds for the maximal splitting distance between the stable and unstable invariant manifolds associated to the invariant torus, and we show that these bounds depend strongly on the arithmetic properties of the frequencies. Such lower bounds imply the existence of splitting between the invariant manifolds, which provides a strong indication of the non-integrability of the system near the given torus, and opens the door to the application of topological methods for the study of Arnold diffusion in such systems.

The scattering map in a piecewise-smooth mechanical system

Albert Granados
Inria, France
albert.granados@inria.fr
John Hogan and Tere Seara

In this talk we consider a piecewise-smooth Hamiltonian system with two degrees of freedom submitted to a periodic perturbation. The system consists of a generalization of a mechanical system with impacts, which occur when trajectories collide with two manifolds where the Hamiltonian is not differentiable. The unperturbed system possesses two invariant manifolds with C^0 stable and unstable manifolds. After showing their persistence and transversal intersection after the perturbation, we study the scattering map, in which is based a common geometric approach for the study of Arnol'd diffusion in Hamiltonian systems relevant in celestial mechanics. It allows us to find trajectories that accumulate energy from an external forcing when following certain heteroclinic connections.

Annulus-like global surfaces of section for tight Reeb flows on SO(3)

Umberto Hryniewicz
Universidade Federal do Rio de Janeiro, Brazil
umbertolh@gmail.com
Pedro A. S. Salomao and Kris Wysocki

The existence question of global surfaces of section on energy levels of Hamiltonian systems with two degress of freedom goes back to Poincaré and his studies on the three body problem. In the planar restricted circular case with small mass ratio and energy, Poincaré considers an annular global surface of section bounded by the direct and retrograde orbits. The study of the associated first return map motivated him to state what is known today as the Poincaré-Birkhoff Theorem, a result that has been a strong driving force in the field of Dynamical Systems for the last century. The above example is only one of many other important systems given by the Hamiltonian dynamics on contact-type energy levels contactomorphic to $SO(3)$ with its standard contact structure. Other such important systems are: geodesic flows of Riemannian or Finsler metrics on the two sphere, magnetic flows on the two sphere for high energy levels, two vortices moving on the two sphere etc. Thus, it becomes of relevance to investigate under which conditions one can find annulus-like global surfaces of section of Reeb flows for the standard contact structure on $SO(3)$. The old theorem of Birkhoff on the existence of such annuli for positively curved Riemannian metrics on the two sphere serves a guide. Our main result discussed in this talk, which is joint work with Pedro Salomão and Kris Wysocki, states that if such a Reeb flow on $SO(3)$ admits two closed orbits forming a Hopf link L, then these orbits bound an annular global surface of section provided that: (1) all periodic trajectories have positive Conley-Zehnder indices, and (2) all periodic trajectories in the complement of L are positively linked with L. These conditions are easily verified for positively curved geodesic flows on the two sphere.

A finite information KAM theorem

Piotr Kamieński
Jagiellonian University, Poland
piotr.kamienski@ii.uj.edu.pl

We present a KAM type theorem for two-dimensional tori which does not assume the classical diophantine condition, instead it is based on finite approximation of the rotation vector. Specifically, we prove that for all $p > 0$ a KAM theorem holds with a perturbation threshold ε_*, which depends only on $N(p)$ first "digits" of the continued fraction expansion of the rotation frequency ω provided that ω belongs to a set of probabilistic measure $1 - p$. The quantities are explicitly computed, which makes the method amenable for computer assisted proofs. Our reasoning relies on a careful analysis of the small divisors - the ones which are "truly small" are related to the continued fraction expansion of ω, which can be controlled statistically in spirit of the theorem on Khintchine's constant.

Some dynamical consequences of tidal effects.

Mark Levi
Penn State, USA
levi@math.psu.edu

I will describe some effects of tidal dissipation on the motion of (an idealized version of) celestial bodies.

Oscillatory motions in the restricted and non restricted three body problem

Pau Martín
Universitat Politècnica de Catalunya, Spain
martin@ma4.upc.edu
Marcel Guàrdia, Tere M. Seara

Oscillatory motions in the three body problem or its simplified versions, the different instances of the restricted three body problem, R3BP, are solutions such that two of the bodies, the primaries, evolve describing bounded orbits while the third one moves closer and closer to infinity, but always returning to a fixed neighborhood of the primaries. They are one of the seven types of possible orbits in the three body problem, according to Chazy (four in the case of the R3BP), who knew examples of orbits of the other six types, but not of oscillatory ones. The existence of oscillatory orbits has been proved in several instances of the R3BP by several authors, Sitnikov, Alexeev, Moser, Llibre and Simó, Moeckel, among others. In this work we first address the existence of oscillatory motions in the restricted planar circular three body problem, and we prove that they exist for all values of the masses of the primaries, thus closing the problem in this case. Next, in the planar three body problem with arbitrary masses, we consider the case when two of the masses evolve in bounded orbits close to elliptic motions with small eccentricity while the third one performs oscillatory motion.

Transport in the Solar System using dynamical systems tools

Merce Olle
Universitat Politecnica de Catalunya, Spain
merce.olle@upc.edu

Our main goal in this work is to give an explanation of transport in the solar system based in dynamical systems theory. More concretely we consider (as a first approximation) different bicircular problems (i.e. Sun, Jupiter, a planet and an infinitesimal mass), we take *natural* periodic orbits which are unstable and we study their invariant manifolds as well as the existence of possible heteroclinic connections. The role that these particular trajectories play in relation with transport from exterior planets to the inner ones is discussed. Finally, some comments concerning a more realistic model of the Solar System are given and dynamical substitutes of invariant objects (from simpler models) are obtained. This is a joint work with E. Barrabés (U. de Girona), G. Gómez (U. de Barcelona) and J.M. Mondelo (U. Autònoma de Barcelona)

The Kolmogorov set of the planetary (1+n)–body problem

Gabriella Pinzari
Università di Napoli Federico II, Italy
pinzari@mat.uniroma3.it

In 1963 V. A. Arnold stated that, in the planetary problem, a positive measure set of bounded motions may be found, provided the masses of the planets, eccentricities and inclinations are sufficiently small. The complete proof of this statement took more than fifty years to come to the light. After the first proof given by M. Herman and J. Féjoz, the theorem was directly (and quantitatively) reproved in 2011 by L. Chierchia and myself. In particular, we proved that the "density" of bounded motions tends to one as soon as eccentricities and inclinations go to zero. In this talk, we shall discuss the possibility of rendering this density independent of eccentricities.

Analytical investigation of the dynamics around the collinear points

Giuseppe Pucacco
Tor Vergata University, Italy
pucacco@roma2.infn.it
Marta Ceccaroni, Alessandra Celletti

We study the main families of periodic orbits around the collinear points of the restricted spacial 3 body problem. We construct a resonant normal form on the central manifold and exploit it to compute bifurcation thresholds of halo orbits in the whole range of mass parameter $0 \leq \mu \leq 1/2$ recovering, as a limit case, the Hill problem.

Transition chains of invariant tori around L_1 in the Sun-Earth system

Pablo Roldan
University of Maryland, USA
pablo.roldan@upc.edu
Marian Gidea and Amadeu Delshams

We consider the spatial Restricted Three-Body Problem modelling the Sun-Earth system. We focus on the center manifold $W^c(L_1)$ of the collinear equilibrium point L_1, with linear type center $\times$ center $\times$ saddle. We present a systematic numerical exploration of heteroclinic connections between different invariant tori in the center manifold $W^c(L_1)$. The results show that, as energy increases, there exist longer transition chains of tori. For high enough energy, there exist transition chains linking almost all tori in the energy manifold.

Contact homology and CPR3BP

Pedro A. Salomao
University of Sao Paulo, Brazil
psalomao@gmail.com
U. Hryniewicz and A. Momin

I will talk about Reeb flows on the universally tight RP^3. If it admits a pair of periodic orbits forming a Hopf link L and satisfying a certain non-resonance condition then there exist infinitely many periodic orbits with prescribed linking numbers with the components of L. The proof of this result makes use of contact homology in the complement of L. No global surface of section is assumed to exist. This generalizes Poincaré-Birkhoff, so it has a natural application to the CPR3BP. This is joint work with U. Hryniewicz and A. Momin.

Global surfaces of section and holomorphic curves

Otto Van Koert
Seoul National University, Korea
okoert@snu.ac.kr

We discuss how holomorphic curve methods from symplectic geometry can be used to construct global surfaces of section for the restricted three-body problem. These modern methods work for very general Hamiltonian systems and can, in addition, be used to find finite energy foliations, a notion that generalizes global surfaces of section. We indicate how one can find the necessary holomorphic curves numerically, and we present some numerical results inspired by these methods.

On the bifurcation creating 3:1 resonant islands for PCR3BP

Piotr Zgliczynski
Jagiellonian University, Poland
umzglicz@cyf-kr.edu.pl

We consider the planar rescricted three body problem with very small mass μ of the second body. The limit problem is the Kepler problem in rotating frame. It is integarble. We study how the 3:1 resonant periodic orbits are created when $\mu \neq 0$. We will present some pictures and some proofs (computer assisted)

Special Session 83: Fluid Flows in Unbounded Domains

Reinhard Farwig, Technical University Darmstadt, Germany
Jiri Neustupa, Czech Academy of Sciences, Institute of Mathematics, Czech Republic

The lectures in the session will be focused on qualitative analysis of mathematical models of flows of viscous or inviscid fluids in unbounded domains. The discussed topics will involve questions of existence, uniqueness, regularity, decay (in time and space) and other properties of solutions in domains with fixed as well as moving boundaries.

Stokes and Oseen equations in an exterior domain

Cherif Amrouche
Universite de Pau et des Pays de l'Adour, France
cherif.amrouche@univ-pau.fr
M. Meslameni, S. Necasova

In a first part of this work, we will study the following Stokes problem in exterior domain of $\mathbb{R}^3$:

$$(\mathcal{S}) \quad -\Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f} \quad \text{and} \quad \operatorname{div} \boldsymbol{u} = h \quad \text{in } \Omega,$$
$$\boldsymbol{u} = \boldsymbol{g} \quad \text{on } \Gamma,$$

where $\boldsymbol{u}$ denote the velocity and π the pressure and both are unknown. We are interested in the existence and the uniqueness of very weak solutions. Here, we extend a result proved by Farwig et al [**?**] and we prove the existence and the uniqueness of a second type of very weak solution. In a second part, we will study the linearized Navier-Stokes equations in an exterior domain of $\mathbb{R}^3$ at the steady state, that is, the Oseen equations:

$$(\mathcal{O}) \quad -\Delta\, \boldsymbol{u} + \operatorname{div}(\boldsymbol{v} \otimes \boldsymbol{u}) + \nabla \pi = \boldsymbol{f}$$
$$\text{and} \quad \operatorname{div} \boldsymbol{u} = h \quad \text{in} \quad \Omega, \quad \boldsymbol{u} = \boldsymbol{g} \quad \text{on} \quad \Gamma.$$

We are interested in the existence and the uniqueness of weak, strong and very weak solutions. Our analysis is based on the principle that linear exterior problems can be solved by combining their properties in the whole space $\mathbb{R}^3$ and the properties in bounded domains.

Remark on the Helmholtz decomposition in unbounded domain with noncompact boundary

Yasunori Maekawa
Tohoku University, Japan
maekawa@m.tohoku.ac.jp
Hideyuki Miura

In this talk we study the Helmholtz decomposition of vector fields in an unbounded domain whose boundary is given as a Lipschitz graph. Since the boundary is noncompact and nonsmooth the choice of function spaces for vector fields is a critical issue in this problem. It is classical that the standard L^2 space admits the Helmholtz decomposition, while the problem becomes complicated if the function space includes vector fields with infinite energy. In this talk we show that the Helmholtz decomposition is valid in certain anisotropic spaces which include some infinite energy vector fields.

On uniqueness of symmetric Navier-Stokes flows around a body in the plane

Tomoyuki Nakatsuka
Nagoya University, Japan
m09033b@math.nagoya-u.ac.jp

In this talk, we investigate the uniqueness of symmetric weak solutions to the stationary Navier-Stokes equation in a two-dimensional exterior domain Ω. It is known that, under suitable symmetry condition on the domain and the data, the problem admits at least one symmetric weak solution tending to zero at infinity. Given two symmetric weak solutions u and v, we show that if u satisfies the energy inequality $\|\nabla u\|^2_{L^2(\Omega)} \leq (f, u)$ and $\sup_{x\in\Omega}(|x|+1)|v(x)|$ is sufficiently small, then $u = v$. The proof relies upon a density property for the solenoidal vector field and the Hardy inequality for symmetric functions.

Linearized stationary incompressible flow around rotating and translating- Leray solution, asymptotic profile

Sarka Necasova
Institute of Mathematics, Academy of Sciences, Czech Rep
matus@math.cas.cz
Paul Deuring, Stanislav Kracmar

We consider a stationary viscous incompressible flow around a translating and rotating body. We are interested to show that our solution which was derived on the basis of a representation formula involving a fundamental solution constructed by R.B.Guenther and E.A.Thomann is also Leray solution and we proved the asymptotic profile.

On the rotating Navier-Stokes problem in weighted spaces

Penel Patrick
Toulon University, France
patrick.penel@gmail.com

In the talk I will review some recent results on the study of the time-periodic incompressible $3D$-Navier–Stokes system describing the flow around a rotating rigid body. The existence in weighted $\mathbf{L}^q$-spaces of very weak solutions will be discussed and treated, for the three modified steady models, Stokes, Oseen and Navier–Stokes. In the nonlinear case, we need weighted variants of the embedding of homogeneous Sobolev spaces into Lebesgue spaces. My talk is based on joint works with S. Necasova. and S. Kracmar.

On Decomposition of the Hilbert Space $W^{1,2}$ with Respect to the Quadratic Form $\langle$curl u, curl v$\rangle$ and Applications.

Reimund Rautmann
Institute of Mathematics, University of Paderborn, Germany
rautmann@uni-paderborn.de

The vorticity curl v of a viscous incompressible fluid flow with velocity v plays a prominent role in regularity theory as well as for approximation methods to the Navier-Stokes equations. Of special interest is the creation of vorticity for product formula approaches based on transport-diffusion splitting schemes, which require the alternating change from slip- to no-slip boundary condition. In order to get bounds to the resulting change of vorticity, recently we had introduced a suitable orthogonal decomposition of the Hilbert space $W^{1,2}(\Omega)$ with respect to the quadratic form $\langle$curl u, curl v$\rangle$ on 3-dimensional domains Ω with C^2-smooth boundaries. The decomposition leads immediately to a lower bound for the considered change of vorticity. In my talk, this result will be extended to domains having less regular boundaries. In addition, on domains with C^2-regular boundaries, we present a transport-diffusion splitting scheme which is consistent with the full Navier-Stokes equations under no-slip condition, and we get also an upper bound to the change of vorticity by transport-diffusion stepping.

Fluid Flow in Wedge Type Domains

Jürgen Saal
Heinrich-Heine-Universität Düsseldorf, Germany
Juergen.Saal@hhu.de

We prove maximal regularity for the Stokes equations with partial slip type boundary conditions on wedge type domains. The approach is based on operator theoretical methods, such as the operator sum method and H^∞-calculus, and the reduced Stokes system. As an application we present well-posedness of related Navier-Stokes problems.

Regularity criteria for the solutions of the Navier-Stokes equations based on the velocity gradient

Zdenek Skalak
Czech Technical University, Czech Rep
skalak@mat.fsv.cvut.cz

We study regularity criteria for the nonstationary solutions of the Navier-Stokes equations in the whole three-dimensional space or with periodic boundary conditions based on the velocity gradient. We pay a special attention to the criteria based on an additional regularity of 1) one directional derivative of the velocity field, 2) the gradient of one velocity component, 3) only one of the nine entries of the velocity gradient and 4) two or more entries of the velocity gradient possibly combined with an additional information on the regularity of one velocity component.

Long time solvability for the 3D rotating Euler equations

Ryo Takada
Mathematical Institute, Tohoku University, Japan
ryo@m.tohoku.ac.jp

In this talk, we consider the initial value problem of the 3D incompressible rotating Euler equations. We prove the long time existence of classical solutions for initial data in $H^s(\mathbb{R}^3)$ with $s > 5/2$ provided the speed of rotation is sufficiently high. Also, we give an upper bound of the minimum rotating speed for the long time existence when initial data belong to $H^{\frac{7}{2}}(\mathbb{R}^3)$.

Uniqueness of solutions on the whole time axis to the Navier-Stokes equations in unbounded domains

Yasushi Taniuchi
Shinshu University, Japan
taniuchi@math.shinshu-u.ac.jp
Reinhard Farwig and Tomoyuki Nakatsuka

We consider the uniqueness of bounded continuous L^3_w-solutions on the whole time axis to the Navier-Stokes equations in 3-dimensional unbounded domains. Thus far, uniqueness of such solutions to the Navier-Stokes equations in unbounded domain, roughly speaking, is known only for a small solution in $BC(R; L^3_w)$ within the class of solutions which have sufficiently small $L^\infty(L^3_w)$-norm. In this talk, we discuss another type of uniqueness theorem for solutions in $BC(R; L^3_w)$ using a smallness condition for one solution and a precompact range condition for the other one.

Necessary and sufficient conditions for the existence of Helmholtz decompositions in general domains

Werner Varnhorn
Kassel University, Germany
varnhorn@mathematik.uni-kassel.de
Christian G. Simader and Hermann Sohr

Consider a general domain $\Omega \subseteq \mathbb{R}^n$, $n \geq 2$, and let $1 < q < \infty$. Our first result is based on the estimate for the gradient $\nabla p \in G^q(\Omega)$ in the form

$$\|\nabla p\|_q \leq C \sup |\langle \nabla p, \nabla v\rangle_\Omega| / \|\nabla v\|_{q'},$$
$$\nabla v \in G^{q'}(\Omega), \quad q' = \frac{q}{q-1},$$

with some constant $C = C(\Omega, q) > 0$. This estimate was introduced by Simader and Sohr for smooth bounded and exterior domains. We show for general domains that the validity of this gradient estimate in $G^q(\Omega)$ and in $G^{q'}(\Omega)$ is necessary and sufficient for the validity of the Helmholtz decomposition in $L^q(\Omega)$ and in $L^{q'}(\Omega)$. Another new aspect concerns the estimate for divergence free functions $f_0 \in L^q_\sigma(\Omega)$ in the form

$$\|f_0\|_q \leq C \sup |\langle f_0, w\rangle_\Omega| / \|w\|_{q'}, \quad w \in L^{q'}_\sigma(\Omega),$$

for the second part of the Helmholtz decomposition. We show again for general domains that the validity of this estimate in$L^q_\sigma(\Omega)$ and in $L^{q'}_\sigma(\Omega)$ is necessary and sufficient for the validity of the Helmholtz decomposition in $L^q(\Omega)$ and in $L^{q'}(\Omega)$.

Time-dependent singularities in the Navier-Stokes system

Xiaoxin Zheng
Uniwersytet Wroclawski, Peoples Rep of China
xiaoxinyeah@163.com
Grzegorz Karch

We showed that, for a given Hölder continuous curve in $\{(\gamma(t), t) : t > 0\} \subset R^3 \times R^+$, there exists a solution to the Navier-Stokes system for an incompressible fluid in R^3 which is smooth outside this curve and singular on it. This is a pointwise solution of the system outside the curve, however, as a distributional solution on $R^3 \times R^+$, it solves an analogous Navier-Stokes system with a singular force concentrated on the curve.

Special Session 84: Dynamics and Games

Alberto Pinto, University of Porto, Portugal
Michel Benaïm, University of Neuchatel, Switzerland

The session aims to bring together world top researchers and practitioners from the fields of Dynamical Systems, Game Theory and applications to such areas as Biology, Economics, Engineering, Energy, Natural Resources and Social Sciences. This session is organized by the founders and editors-in-chief of the Journal of Dynamics and Games (JDG), published by the American Institute of Mathematical Sciences (AIMS).

Anosov diffeomorphisms and γ-tilings

João Almeida
LIAAD - INESC TEC and Polytechnic Institute of Bragança, Portugal
jpa@ipb.pt
Alberto A. Pinto

Inspired in the works of Y. Jiang and A. Pinto and D. Sullivan, A. Pinto et al. introduced the notion of golden tiling and proved the existence of a natural correspondence between golden tilings, smooth conjugacy classes of Anosov diffeomorphisms with invariant measure absolutely continuous with respect to Lebesgue measure and solenoid functions. Here we extend their result and introduce the notion of γ-tiling. Like the golden tilings, the γ-tilings record the infinitesimal geometric structure determined by the dynamics of an Anosov diffeomorphism G along the unstable leaf that is invariant under the action of G. The properties of γ-tilings are defined using a decomposition of natural numbers that we call γ-Fibonacci decomposition. The main contribution of this work consists in understanding the way how this γ-Fibonacci decomposition encodes the combinatorics determined by the Markov partition of G along the unstable leaf. We exhibit a natural correspondence between γ-tilings, smooth conjugacy classes of Anosov diffeomorphisms with invariant measure absolutely continuous with respect to Lebesgue measure and solenoid functions

What is the length of the land frontier between Portugal and Spain?

João Almeida
LIAAD - INESC TEC and Polytechnic Institute of Bragança, Portugal
jpa@ipb.pt
Alberto A. Pinto

The land frontier between Portugal and Spain is one of the oldest and longest in Europe. In 1961 L.F. Richardson published a paper entitled “The Problem of Contiguity: An Appendix to 'Statistics of Deadly Quarrels' ”, where he tackled the problem of determining the real length of coastlines or geographical borders between two countries. Noting that in many cases neighbour countries didn't agree about the length of their common frontier, he showed that the length of a coastline or a land frontier depended upon the yardstick or scale with which this length was measured. This prevalent phenomenon is commonly refered as the 'Richardson effect'. In his paper, Richardson also derived a log-linear relationship between length and scale and this log-log scatter plot of perimeter lengths versus scale intervals came to be known as a 'Richardson plot'. In late 60s, Mandelbrot related the Richardson effect with fractal geometry, this way unvealing the fractal dimension of coastlines and land frontiers.

Model for Wolf's sunspot numbers

Ricardo Cruz
University of Porto, Portugal
rpmcruz@fc.up.pt
Renato Fernandes and Alberto A. Pinto

In this work we expand a model by Gonçalves and Pinto characterizing the fluctuation of the monthly number of sunspots, based on the BHP distribution.

Testing the Universality of the BHP distribution

Ricardo Cruz
University of Porto, Portugal
rpmcruz@fc.up.pt
Renato Fernandes and Alberto A. Pinto

We expand on work by Gonçalves et al [2011] who have developed tools to find the BHP distribution as describing the daily flows of two data set: the Portuguese Paiva river and the S&P 100 index.

A game theoretical analysis of vaccination games

José Martins
Polytechnic Institute of Leiria and LIAAD-INESC TEC, Portugal
jmmartins@ipleiria.pt
A. Pinto

In the definition of the course of an epidemic, vaccination plays a key role. If enough individuals are vaccinated the extinction threshold of the disease can be exceeded and the epidemic disappears. In the case of diseases for which vaccination is voluntary, people have to decide if the benefit of vaccination overcomes the morbidity risks of vaccine. The decision of each individual is also influenced by the decisions of all other people. In this work, we make a game theoretical analysis of the vaccination game to better understand the people decisions with respect to vaccination. Previous analyses were made considering the basic SIR model with vaccination. Here, we consider the SIRI model that incorporates the possibility of reinfection and the results obtained leads to a much more diverse human behavior. For epidemic models with multiple pathogen strains this analysis is a subtle challenge.

An economic model to achieve monopoly without committing dumping

José Martins
Polytechnic Institute of Leiria and LIAAD-INESC TEC, Portugal
jmmartins@ipleiria.pt
A. Pinto

The phenomena of dumping, selling a good in the foreign market at a price lower than the domestic price, usually happens when a large company wants to expand and eliminate competition in the foreign market to consolidate as monopolies. In this work, we study this phenomena and we propose a simple mathematical model where a company that are competing in quantities with another one in a foreign market can expand without make dumping by changing its production quantity in the domestic market. We compare the profit in this strategy along several periods with the one obtained when dumping is committed and we observe when each strategy is preferred.

A transcritical bifurcation in an immune response model

Bruno Oliveira
University of Porto / INESC TEC, Portugal
bmpmo@fcna.up.pt
I.P. Figueiredo, A.A. Pinto, N.J. Burroughs

We study the immune response by T cells with the presence of regulatory T cells (Treg) using an ODE model. Tregs act mainly by inhibiting interleukine 2 secretion by T cells. We introduced an asymmetry reflecting that the difference between the growth and death rates can be higher forthe active T cells and the active Tregs than for the inactive T cells and inactive Tregs. We obtain an explicit formula that gives the equilibria of the model. Furthermore, when model the correlation between the antigenic stimuli, we find a transcritical bifurcation that might explain two alternative scenarios: in one case the appearance of autoimmune responses and in another the suppression of the immune response.

Convergence of Edgeworthian prices in a random matching economy

Bruno Oliveira
University of Porto / INESC TEC, Portugal
bmpmo@fcna.up.pt
Alberto A. Pinto, Athanassios Yannacopoulos, Barbel Finkenstadt

We study a random matching economy, where pairs of participants trade two goods. We show that under some fairly general and easy to check symmetry conditions, depending on the initial distribution of endowments and the agents preferences, the sequence of Edgeworthian prices in this economy converges to the Walrasian prices for this economy. Additionally, we associate a selfishness factor to each participant in this market. this brings up a game alike the prisoner's dilemma, where trade may occur in a different point in the core or may not even be allowed. We discuss how the selfishness affects the sequence of Edgeworthian prices.

Regularity for time dependent mean-field games in the subquadratic case

Edgard Pimentel
CAMGSD-IST-UTL, Portugal
edgard.pimentel@gmail.com

In this we consider time dependent mean-field games (MFG) with a local power-like dependence on the measure and Hamiltonians satisfying subquadratic growth conditions. We establish existence and uniqueness of smooth solutions under a certain set of conditions depending both on the growth of the Hamiltonian as well as on the dimension. It is done by combining a Gagliardo-Niremberg type of argument with a new class of polynomial estimates for solutions of the Fokker-Planck equation in terms of L^rL^p-norms of D_pH. Also, we consider MFG systems with initial-initial boundary conditions. In this setting, we establish existence of classical solutions under conditions certain conditions on the mean-field hypothesis. This is based on joint works with D. Gomes and H. Sánchez-Morgado

Regularity for time dependent mean-field games in the superquadratic case

Edgard Pimentel
CAMGSD-IST-UTL, Portugal
edgard.pimentel@gmail.com

In this talk we investigate time dependent mean-field games (MFG) with superquadratic Hamiltonians and a power-like dependence on the measure. Existence and uniqueness of smooth solutions is established under a set of conditions depending on the dimension as well as on the growth of the Hamiltonian. In particular, our results recover the quadratic case. This is done by recurring to a delicate argument that combines the non-linear adjoint method with polynomial estimates for solutions of the Fokker-Planck in terms of $L^\infty L^\infty$-norms of D_pH. To the best of our knowledge, superquadratic MFG have not been addressed in the literature yet. In fact, it is likely that our estimates may also add to the current understanding of Hamilton-Jacobi equations with superquadratic Hamiltonians. This is a based on a joint work with D. Gomes and H. Sánchez-Morgado.

Hoteling town with uncertainty

Alberto Pinto
University of Porto / INESC TEC, Portugal
aapinto1@gmail.com
T. Parreira

This paper develops a theoretical framework to study price competition in a Hotelling-type network game, extending the Hotelling model of price competition with linear transportation costs from a line to a network. Under explicit conditions on the production costs and road lengths we show the existence of a pure Nash price equilibrium. Furthermore, we introduce incomplete information in the production costs of the firms and we find the Bayesian-Nash price equilibrium.

Uncertainty effects in the stakelberg model

Alberto Pinto
University of Porto / INESC TEC, Portugal
aapinto1@gmail.com
Mohamad Choubdar

It is well known that in the stakelberg model without uncertainty the leader has always advantage over the following firms. We will study when the uncertainty effects reverse this phenomenon giving advantage to the follower.

Special Session 85: Transport Processes in Biology: Modelling and Analysis

Jozsef Farkas, University of Stirling, Scotland
Mariya Ptashnyk, University of Dundee, Scotland

The complex dynamical nature of biological systems gives rise to many challenging questions in mathematical modelling and analysis. Transport processes are key mechanisms ensuring functioning, development and growth of cell tissues and organs. Important questions include direction of the movement, and underlying communication between individuals leading to self-organisation of populations. Transport processes take place at different scales, ranging from the movement of molecules in cells, the transport through cell tissues and organs, to the spatial organisation in ecological systems. The interconnections between the processes at different spatial and temporal scales often requires multiscale modelling and analysis. For the more complete understanding of biological systems (often populations of individuals) not only spatial distribution but also changes in state and structural properties are to be analysed. Physiologically structured population models are widely used to study qualitative questions in population dynamics, where structuring may be based on age, internal cell cycle, body mass, volume, etc. The presentations in the session will demonstrate how the derivation of new models inspire the simultaneous development of novel mathematical techniques.

A Structured Model for the Spread of Mycobacterium marinum

Azmy Ackleh
University of Louisiana at Lafayette, USA
ackleh@louisiana.edu
Mark L. Delcambre, Karyn L. Sutton, Don G. Ennis

Mycobacterium marinum (Mm), a genetically similar bacterium to Mycobacterium tuberculosis, affects a number of fish industries (fisheries, aquaculture, aquariums and research stocks) on a comparable scale to tuberculosis (TB) in humans. Because of this, and the practical advantages of working with animal models as opposed to humans, Mm infections in recently established fish models provide a unique opportunity for the study of mycobacterial infections. We derive a structured model for the transmission dynamics of Mm in fish. The model consists of a system of nonlinear hyperbolic partial differential equations coupled with three nonlinear ordinary differential equations. First order and second order finite-difference schemes to approximate the solutions of this model are developed and convergence results are established. These numerical methods are used to conduct preliminary studies on the long-term behavior of the model.

A mathematical model of systemic inhibition of angiogenesis in metastatic development

Sebastien Benzekry
INRIA, France
benzekry@phare.normalesup.org
Alberto Gandolfi, Philip Hahnfeldt

We present a mathematical model describing the time development of a population of tumors subject to mutual angiogenic inhibitory signaling. Based on biophysical derivations, it describes organism-scale population dynamics under the influence of three processes: birth (dissemination of secondary tumors), growth and inhibition (through angiogenesis). The resulting model is a nonlinear partial differential transport equation with nonlocal boundary condition. The nonlinearity stands in the velocity through a nonlocal quantity of the model (the total metastatic volume). The asymptotic behavior of the model is numerically investigated and reveals interesting dynamics ranging from convergence to a steady state to bounded non-periodic or periodic behaviors, possibly with complex repeated patterns. Numerical simulations are performed with the intent to theoretically study the relative impact of potentiation or impairment of each process of the birth/growth/inhibition balance. Biological insights on possible implications for the phenomenon of "cancer without disease" are also discussed.

Singularly perturbed transport and diffusion systems on network and state lumping

Jacek Banasiak
University of KwaZulu-Natal, So Africa
banasiak@ukzn.ac.za

We consider asymptotic behaviour of diffusion and transport process on networks, when the parameters related to the transport speed/diffusion go to infinity, while the permeability coefficients at the nodes tend to zero. We show that, under certain conditions, such processes exhibit the so called *state lumping*; that is, they can be approximated by linear dynamical systems governed by the adjacency matrices of the line graph of the original network.

Spatial spread of intestinal bacteria and bacteriophage within a single host

Angel Calsina
Universitat Autonoma de Barcelona, Spain
acalsina@mat.uab.cat

We consider a hyperbolic system of semilinear partial differential equations for the bacteria and their bacteriophage populations interaction within animal intestine taking into account spatial spread and the existence of two different bacteria populations, the bacteria attached to the wall and the unattached ones ([1]). We will study the existence of free-infection and of infected equilibria as well as the equilibria distribution shapes, depending on the administered dose of phage and the amount of bacteria re-ingested. The formulation of the initial value problem can be dealt with by means of the sun dual framework

and the variation of constants formula and subsequently to prove the linearized stability principle ([2]). Local existence and uniqueness and positivity and global existence, as well as linear stability of the stationary solutions will be briefly discussed.

References
[1] Boldin, B. Persistence and spread of gastro-intestinal infections: the case of enterotoxigenic Escherichia coli in piglets. Bull. Math. Biol. 70 (2008) 2077-2101.
[2] Lichtner M., Variation of constants formula for hyperbolic systems, J. Appl. Anal. 15 (2009), no. 1, 79-100.

A spatially-extended model of kinase-receptor interaction

Elaine Crooks
Swansea University, Wales
e.c.m.crooks@swansea.ac.uk
Bogdan Kazmierczak, Tomasz Lipniacki

This talk is concerned with the analysis of a spatially-extended model describing mutual phosphorylation of cytosolic kinases and membrane receptors in immune cells. A prototype for the systems considered has the form

$$\frac{\partial K}{\partial t} = d\Delta K - A\frac{HK}{H+K}, \text{ in } \Omega \times (0,\infty),$$
$$\frac{\partial R}{\partial t} = (K^w + c_0)(P-R) - bR, \text{ in } \partial\Omega \times (0,\infty),$$

with appropriate initial conditions and a nonlinear Robin-type boundary condition

$$d\nabla K \cdot n = aR(1-K) \quad \text{in } \partial\Omega \times (0,\infty),$$

that couples the variables K and R on $\partial\Omega$. Here $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary, K represents a concentration of kinase molecules and R is a concentration of non-diffusing receptors that live on the cell membrane $\partial\Omega$. The given function $a : \partial\Omega \to [0,\infty)$ is typically zero on part of $\partial\Omega$, giving a zero Neumann condition, and positive on the remainder of $\partial\Omega$, with the other model parameters positive constants. We discuss the existence and stability of stationary spherically-symmetric solutions when Ω is a spherical shell via an auxiliary problem in which the Robin boundary condition is replaced by a uniform Dirichlet boundary condition.

On a cyclin-structured cell population model: equilibria, oscillations and delay formulation

Silvia Cuadrado
Universitat Autonoma de Barcelona, Spain
silvia@mat.uab.cat
Ricardo Borges, Angel Calsina, Odo Diekmann

We consider a nonlinear cyclin content structured model of a cell population divided into proliferative and quiescent cells. Under suitable hypotheses, we show existence and uniqueness of a steady state of this model by using positive linear semigroup theory. We also show, for particular values of the parameters, the existence of solutions that do not depend on the cyclin content. We make numerical simulations for the general case obtaining, for some values of the parameters convergence to the steady state, but for others oscillations of the population. Finally we use the delay equation formulation of structured population dynamics to write a different version of the cell population model for which we characterize steady states and establish the validity of the principle of linearized stability.

Bifurcation dynamics in nonlocal hyperbolic models for self-organised biological aggregations

Raluca Eftimie
University of Dundee, Scotland
reftimie@maths.dundee.ac.uk
Pietro-Luciano Buono

Nonlocal hyperbolic models that are used to describe self-organised biological aggregations (i.e., aggregations that form in the absence of any leader) can display a wide variety of spatial and spatio-temporal patterns: from classical stationary and rotating waves, to more exotic zigzags, ripples and modulated rotating waves. Here we use weakly nonlinear analysis to investigate the codimension-1 and codimension-2 bifurcations that give rise to some of these patterns, and discuss the mechanisms behind the transitions between the patterns.

The effect of visual perception on systems of interacting individuals

Joep Evers
Eindhoven University of Technology, Netherlands
j.h.m.evers@tue.nl
Razvan Fetecau, Lenya Ryzhik, Adrian Muntean

This talk treats an extension of an established model for biological aggregations: a first-order system of ODEs for the trajectories of interacting individuals. The original model lacks a crucial ingredient that makes social aggregations different from systems of non-living particles: mutual interactions are anisotropic since they are vision-based and thus direction-dependent. My main interest is to investigate how anisotropy influences the model. Uniqueness and continuity of velocity are the main mathematical issues I will address.

Size-structured population dynamics with diffusion

Nobuyuki Kato
Kanazawa University, Japan
nkato@se.kanazawa-u.ac.jp

We are concerned with size-structured population models with spacial diffusion. Consider a biological population living in a habitat $\Omega \subset \mathbb{R}^n$ with smooth boundary $\partial\Omega$. Let $p(s,t,x)$ be the population density of size $s \in [0,s_\dagger)$ and position $x \in \Omega$ at time $t \in [0,T]$, where $s_\dagger \in (0,\infty)$ is the maximal size, $T \in (0,\infty)$ is a given time. Let $\Omega_T := (0,T) \times \Omega$, $Q := (0,s_\dagger) \times \Omega$, $\mathcal{Q}_T := (0,s_\dagger) \times (0,T) \times \Omega$ and $\Sigma_T := (0,s_\dagger) \times (0,T) \times \partial\Omega$.

We first consider the following linear model:

$$\partial_t p + \partial_s(g(s,t)p) = \Delta p(s,t,x) - \mu(s,t,x)p(s,t,x) + f(s,t,x), \text{ in } \mathcal{Q}_T,$$

$$g(0,t)p(0,t,x) = C(t,x)+ \int_0^{s_\dagger} \beta(s,t,x)p(s,t,x)\,ds, \text{ in } \Omega_T,$$

$$\frac{\partial p}{\partial \nu}(s,t,x) = 0, \text{ on } \Sigma_T,$$
$$p(s,0,x) = p_0(s,x), \text{ in } Q.$$

Here $g(s,t)$ represents the growth rate depending on individual's size s and time t. The functions $\mu(s,t,x)$ and $\beta(s,t,x)$ stand for the mortality and fertility rates, respectively. The spacial diffusion is prescribed by Laplacian Δ and the Neumann boundary condition. The functions $f(s,t,x)$ and $C(t,x)$ represent certain inflows of s-size and zero-size populations, respectively, from outside. We introduce a notion of mild solution and establish the existence of a unique mild solution. Next, we develop a nonlinear problem in which mortality and fertility rates are supposed to depend on the population density $P(x,t) = \int_0^{s_\dagger} p(s,t,x)\,ds$ in position x at time t.

Epidemic spread on transportation networks with travel related infection

Diana Knipl
MTA-SZTE Analysis and Stochastics Research Group, University of Szeged, Hungary
knipl@math.u-szeged.hu
Gergely Rost, Jianhong Wu

We introduce an SEAIR-based, anti-gravity model to investigate the spread of an infectious disease in two regions which are connected by transportation. As a submodel, an age structured system is constructed to incorporate the possibility of disease transmission during travel, where age is the time elapsed since the start of the travel. The model is equivalent to a large system of differential equations with dynamically defined delayed feedback term. After describing fundamental, but biologically relevant properties of the system, we obtain disease transmission dynamics results in terms of the basic reproduction number. We parametrize our model for influenza and use real demographic and air travel data for the numerical simulations. The model is also fitted to the first wave of the A(H1N1) 2009 pandemic influenza.

Effects of space structure and combination therapies on phenotypic heterogeneity and drug resistance in solid tumors

Tommaso Lorenzi
Laboratoire J.-L. Lions, UPMC, France
tommaso.lorenzi@upmc.fr
J. Clairambault, A. Escargueil, A. Lorz, B. Perthame

Histopathological evidence supports the idea that the emergence of phenotypic heterogeneity and resistance to cytotoxic drugs can be considered as a process of adaptation, or evolution, in tumor cell populations. In this framework, can we explain intra-tumor heterogeneity in terms of cell adaptation to local conditions? How do anti-cancer therapies affect the outcome of cell competition for nutrients within solid tumors? Can we overcome the emergence of resistance and favor the eradication of cancer cells by using combination therapies? Bearing these questions in mind, we develop a model describing cell dynamics inside a tumor spheroid under the effects of cytotoxic and cytostatic drugs. Cancer cells are assumed to be structured as a population by two real variables standing for space position and the expression level of a cytotoxic resistant phenotype. The model takes explicitly into account the dynamics of resources and anti-cancer drugs as well as their interactions with the cell population under treatment. We analyze the effects of space structure and combination therapies on phenotypic heterogeneity and chemotherapeutic resistance. Furthermore, we study the efficacy of combined therapy protocols based on constant infusion and/or bang-bang delivery of cytotoxic and cytostatic drugs.

Analysis and simulation of a one-dimensional swimmer

Marco Morandotti
Instituto Superior Técnico, Portugal
marco.morandotti@ist.utl.pt
Gianni Dal Maso, Antonio DeSimone, Luca Heltai

We consider a one-dimensional swimmer in an infinite viscous fluid. An appealing mathematical framework is described to study the equations of motion. We prove that the motion of the swimmer is uniquely determined by the history of its shapes. Furthermore, we address the problem of controllability of the swimmer, by using elements of control theory, and we prove the existence of optimal (i.e., power minimizing) swimming strategies. We also present a numerical approach to the optimization problem. This is joint work with Gianni Dal Maso, Antonio DeSimone, and Luca Heltai.

A model for pattern formation in Dictyostelium discoideum slug migration: The double-negative feedback mechanism

Miguel Pineda
University of Dundee, Scotland
mpineda@maths.dundee.ac.uk

The life cycle of the cellular slime mold Dictyostelium discoideum represents a fascinating example of self organisation, and the molecular mechanisms underlying its development has presented a challenge for many years. In this talk, we introduced a hyperbolic system to address the role of the so-called double -negative feedback mechanism on the spatial patterning and cell-type proportioning in Dictyostelium discoideum slug migration. The model involves the dynamics of excitable cAMP signalling, differences in chemotactic and signalling behaviour between different cell populations and generation of local signals. We analyse how stable is this proportion mechanism under different sorting conditions.

Stochastic homogenization of the one-dimensional Keller-Segel chemotaxis system

Mariya Ptashnyk
University of Dundee, Scotland
mptashnyk@maths.dundee.ac.uk
Anastasios Matzavinos

In this talk the one-dimensional Keller-Segel chemotaxis system in a random heterogeneous domain will be considered. It is assumed that the diffusion and chemotaxis coefficients are given by stationary ergodic processes. Methods pertaining to stochastic two-scale convergence are applied to derive the homogenized macroscopic equations. Special attention is paid to developing numerical algorithms for approximating the homogenized asymptotic coefficients.

Numerical Bifurcation Analysis of PSPM

Julia Sanchez
BCAM, Spain
jsanchez@bcamath.org
A.M. de Roos, O. Diekmann, Ph. Getto

Physiologically structured population models are used in biology and ecology to study, from a mathematical point of view, the behaviour of populations, in which the individuals differ due to physiological characteristic, and the interactions of the populations with the environment. The models can be defined in terms of Delay Differential Equations and Volterra Functional equations. Due to the complexity of the models, it is not possible to handle the problem analytically, and so it is necessary to use numerical methods, even to obtain steady states solutions. We present the formulation for a general type of models, and numerical methods to compute equilibrium branches and bifurcation curves under one and two parameter variations. Using this type of formulation and the algorithms proposed, it is possible to obtain biological conclusions for different type of models, including consumer-resource, three trophic, or cannibalistic models.

Special Session 86: Nonlinear Evolution Equations and Related Topics

Mitsuharu Otani, Osaka University, Japan
Tohru Ozawa, Waseda University, Japan

This session will focus on the recent developments in the theory of Nonlinear Evolution Equations and Related Topics including the theory of abstract evolution equations in Banach spaces as well as the studies (the existence, regularity and asymptotic behaviour of solutions) of various types of Nonlinear Partial Differential Equations.

Some fully nonlinear parabolic equation and unidirectional evolution

Goro Akagi
Kobe University, Japan
akagi@port.kobe-u.ac.jp

In this talk, we discuss the local (in time) solvability and the finite time blow-up of positive solutions for the Cauchy-Neumann problem in a bounded domain Ω of $\mathbb{R}^N$ for a fully nonlinear parabolic equation involving a "positive part function" $(x)_+ := x \vee 0$ for $x \in \mathbb{R}$,

$$\partial_t u = g(u)\,(\lambda\Delta u + u)_+ \quad \text{in } \Omega \times (0, +\infty),$$

where $g(u)$ is a positive function and $\lambda > 0$.

Finite time blowup for the fourth-order NLS

Yonggeun Cho
Chonbuk National University, Korea
changocho@jbnu.ac.kr
Tohru Ozawa, Chengbo Wang

In this talk, we will consider the finite time blowup for the mass-critical focusing NLS of fourth order:

$$iu_t = \Delta^2 u - |x|^{-2}|u|^{\frac{4}{n}}u, \ \ n \geq 5.$$

The model of inhomogeneous term of nonlinearity is known as the laser beam in Kerr media affected by electrons. The equation is mass-critical in the scaling and satisfies the mass and energy conservation laws. The main ingredient of this talk is to show the fiite time blowup of this equation. We adopt Glassey's virial argument. For this purpose we assume the radial symmetry of solutions, finite fourth moment within the existence time and negative energy.

Periodic solutions for parabolic evolution equations on R^N

Aleksander Cwiszewski
Nicolaus Copernicus University, Poland
aleks@mat.umk.pl

The parabolic partial differential equation

$$u_t = \Delta u - V(t, x)u + f(t, x, u)$$

on $\mathbb{R}^N$ with a time T-periodic potential V and nonlinearity f shall be considered. The tail estimates method shall be applied to study the compactness properties of the translation along trajectories operator. By averaging arguments and fixed point techniques we prove that a topologically nontrivial stationary solution of

$$-\Delta u = \widehat{V}(x)u + \widehat{f}(x, u),$$

where $\widehat{V}$, $\widehat{f}$ are the time averages of V and f, respectively, is the source of a branch of periodic solutions. Finally, by a continuation argument, we derive criteria for the existence of T-periodic solutions for asymptotically linear f that interacts properly with the spectrum of $-\Delta + \widehat{V}$.

Abstract reaction-diffusion problems for protein networks

Jan Elias
Université Pierre et Marie Curie - Paris 6, France
jan.elias@inria.fr
Jean Clairambault, Benoît Perthame

Spatio-temporal dynamics of proteins in individual cells composed from one, two (or more) compartments can be mathematically described, recalling Michaelis-Menten and Hill kinetics, by a non-linear reaction-diffusion system(s) defined in each compartment coupled with Robin-like boundary conditions on common membranes simulating thus exchange of the species between the compartments. Motivated by spatio-temporal dynamics of the protein p53, we study abstract non-linear reaction-diffusion systems defined in one and more domains with suitable boundary conditions on common boundaries (that, we believe, can be applied to any other protein network).

Blow up and asymptotic behavior in a nondissipative nonlinear wave equation

Jorge Esquivel-Avila
Universidad Autonoma Metropolitana, Mexico
jaea72@gmail.com

A nonlinear wave equation without damping and with a superlinear source term is considered. Qualitative behavior of solutions is studied. In particular, the dynamics around the ground state is analyzed. Partial results for blow up, boundedness, convergence and rates of decay to the set of nonzero equilibria as $t \to \infty$ are proved. Several invariant and positive invariant sets are defined.

References

[1] Esquivel-Avila, J. A. Blow up and asymptotic behavior in a nondissipative nonlinear wave equation. Applicable Analysis. In press, 2014.

The well-posedness of the Cauchy problem for a semirelativistic system

Kazumasa Fujiwara
Waseda University, Japan
k-fujiwara@asagi.waseda.jp
Shuji Machihara, Tohru Ozawa

We study time local and global well-posedness of the Cauchy problem for a system of semirelativistic equations with quadratic nonlinearities. The existence of local solutions in H^s with $s > 1/2$ follows easily by the Sobolev embedding $H^s \hookrightarrow L^\infty$. In the case where $0 \le s \le 1/2$, the uniform control by H^s norm breaks down and Strichartz type estimates are not sufficient for a contraction argument unless the uniform control by H^s norm is available. In this talk, we introduce the Fourier restriction method to study the Cauchy problem of the semirelativistic system in H^s with $0 \le s \le 1/2$. We obtain the existence of local solutions in H^s with $s \ge 0$ by a contraction argument based on a Fourier restriction norm. In addition, under a constraint of nonlinearities, the charge conserves and the solutions in H^s with $s \ge 0$ are shown to extend globally.

Maximal regularity for parabolic equations.

Chiara Gallarati
Delft University of Technology, Netherlands
c.gallarati@tudelft.nl
Mark Veraar

Maximal regularity can often be used to obtain a priori estimates which give global existence results. For example, using maximal regularity it is possible to solve quasi-linear and fully nonlinear PDEs by elegant linearization techniques combined with the contraction mapping principle. In this talk we will prove a new mixed $L^p(0,T;L^q(\mathbb{R}^d))$ maximal regularity estimate for the PDE:

$$\begin{cases} u'(t,x) + A(t,x)u(t,x) = f(t,x), & x \in \mathbb{R}^d,\ t \in [0,T] \\ u(0,x) = u_0(x), & x \in \mathbb{R}^d. \end{cases}$$

Here $A(t,x)$ is a second order elliptic differential operator. The main novelty in our result is that the coefficients are merely measurable in time and we allow the full range $p, q \in (1,\infty)$

Analytic smoothing effect for a system of nonlinear Schrödinger equations.

Gaku Hoshino
Waseda University, Japan
gaku-hoshino@ruri.waseda.jp
Tohru Ozawa

We consider the Cauchy problem for a system of nonlinear Schrödinger equations of the form

$$\begin{cases} i\partial_t u + \frac{1}{2m}\Delta u = \lambda v\overline{u}, \\ i\partial_t v + \frac{1}{2M}\Delta v = \mu u^2. \end{cases}$$

In this study, we consider the analytic smoothing property for the above system under the mass resonance condition $M = 2m$ for suffciently small Cauchy data with exponential decay in space dimensions $n \ge 3$. We prove the global existence in framework of the critical Sobolev space $\dot{H}^{n/2-2}$ for $n \ge 4$ and $1/2$ order Sobolev type space defined by the generator of Galilei transform $x + i\frac{t}{m}\nabla$ for $n = 3$.

On the critical Hardy-type inequalities and related variational problems

Michinori Ishiwata
Osaka University, Japan
ishiwata@sss.fukushima-u.ac.jp
Norisuke Ioku

We consider Hardy inequalities of critical type and the solvability of the associated variational problems. We introduce two types of inequalities, one is the log-Hardy type and another is the mean-oscillation type. Both inequalities, in spite of the criticality, possess natural scale invariance and we prove the nonexistence of minimizers for these inequalities by using this invariance.

Solvability and Long time Behavior of Nonlinear Parabolic Equation under the Third Type Boundary Condition

Kerime Kalli
Hacettepe University, Turkey
kerime@hacettepe.edu.tr
Kamal Soltanov

We study some third type boundary value problems for a general semilinear parabolic equation in divergence form:

$$\frac{\partial u}{\partial t} + Lu + g(x,t,u) = h(x,t), \quad (x,t) \in Q_T \equiv \Omega \times (0,T] \tag{1}$$

$$u(x,0) = u_0(x),\ x \in \Omega \subset \mathbb{R}^n, n \ge 3 \tag{2}$$

$$\left(\frac{\partial u}{\partial \nu} + k(x,t)u\right)\Big|_{\Gamma_T} = \varphi(x,t), \quad \Gamma_T \equiv \partial\Omega \times [0,T], T > 0 \tag{3}$$

Here Ω is a bounded domain with sufficiently smooth boundary $\partial\Omega$; L denotes a second order linear elliptic operator in divergence form:

$$Lu := -\sum_{i,j=1}^{n} D_i(a_{ij}(x,t)\,D_j u) + \sum_{i=1}^{n} b_i(x,t)\,D_i u + c(x,t)u,$$

where a_{ij}, b_i and c are given coefficient functions $(i,j = 1,...,n)$; $g : Q_T \times \mathbb{R} \longrightarrow \mathbb{R}$ and $k : \Gamma_T \longrightarrow \mathbb{R}$ are given functions; h and φ are given generalized functions. For the existence and the uniqueness of the generalized solution of problem (1)-(3), we obtain sufficient conditions for L, g and k. Under these conditions we prove that problem (1)-(3) is uniquely solvable in corresponding spaces by applying a general existence theorem. For the long time behavior of solution, we obtained the existence of the absorbing sets in two different spaces for the autonomous case of the problem.

Nonlinear Evolution Equations and Application to Mathematical Models of Medicine

Akisato Kubo
Fijita Health University, Japan
akikubo@fujita-hu.ac.jp

In this talk we consider initial-Neumann boundary value problem of nonlinear evolution equations with strong dissipation and proliferation arising from mathematical biology and medicine formulated as

$$(NE)\begin{cases} u_{tt} = D\nabla^2 u_t + \nabla\cdot(\chi(u_t, e^{-u})e^{-u}\nabla u) \\ +\mu_1 u_t(1-u_t) \text{ in } (x,t)\in\mathbf{\Omega}\times(0,\infty)\ (1.1) \\ \dfrac{\partial}{\partial\nu}u|_{\partial\mathbf{\Omega}} = 0 \text{ on } \partial\mathbf{\Omega}\times(0,\infty)\ (1.2) \\ u(x,0)=u_0(x), u_t(x,0)=u_1(x) \text{ in } \mathbf{\Omega}\ (1.3) \end{cases}$$

where constants D, μ_1 are positive, $\mathbf{\Omega}$ is a bounded domain in R^n with a smooth boundary $\partial\mathbf{\Omega}$ and ν is the outer unit normal vector. We show the existence and asymptotic behavior of the solution. Under some conditions of the coefficient $\chi(u_t, e^{-u})$ of (1.1), we can derive the energy esitmate of (NE), which enables us to show the global existence in time of the soluton and asymptotic behaviour. We will deal with an extended case of our result in this talk and apply it to Chaplain type of mathematical models of biology and medicine (cf. [1]).

References
[1] Chaplain, M.A.J., and Lolas, G., Mathematical modeling of cancer invasion of tissue: Dynamic heterogeneity, Networks and Heterogeneous Media, vol 1, Issue 3, 399-439(2006).

Normal form reduction for unconditional well-posedness of canonical dispersive equations.

Soonsik Kwon
KAIST, Korea
soonsikk@kaist.edu

Normal form method is a classical ODE technique begun by H. Poincare. Via a suitable transformation one reduce a differential equation to a simpler form, where most of nonresonant terms are cancelled. We present how we apply the method to nonlinear dispersive equations such as KdV, NLS to obtain unconditional well-posedness for low regularity data in both periodic and nonperiodic setting.

The onset of Sobolev instability for analytical solutions to first order systems

Baptiste Morisse
Institut de Mathematiques de Jussieu - PRG, France
morisseb@math.jussieu.fr

I will talk about quasilinear first order systems, and specifically on the onset of instabilities for a hyperbolic to non hyperbolic transition. My work is based on the method developed by Guy Métivier in 2005, which proves the existence of a family of analytical solutions that grow exponentially in time. This proves the Hadamard instability of the system: even a small perturbation of an initial datum lead to a solution infinitely distant to the reference solution in a very short time.

Identification of the diffusion coefficient in evolution equations

Noboru Okazawa
Tokyo University of Science, Japan
okazawa@ma.kagu.tus.ac.jp
Gianluca Mola, Tomomi Yokota

This talk concerns the identification of the diffusion coefficient in linear evolution equations in a Hilbert space. The problem has already been solved by G. Mola (2012) in which Faedo-Galerkin methods was employed. The employed methods mean that a range of the application is restricted to the parabolic problems on *bounded* domains. The purpose of this talk is to replace Faedo-Galerkin methods with methods of Yosida approximation in operator-semigroup theory. The new methods enable us to widen the range of applicability up to the problems on unbouded domains such as the whole Euclidean space.

Attractors of the Hyperbolic Relaxation of Reaction Diffusion Equations with Dynamic Boundary Conditions

Joseph Shomberg
Providence College, USA
jshomber@providence.edu
Ciprian Gal

Under consideration is the hyperbolic relaxation of the semilinear reaction-diffusion equation,

$$\varepsilon u_{tt} + u_t - \Delta u + f(u) = 0,$$

$\varepsilon \in [0,1]$, with the prescribed dynamic boundary condition,

$$\partial_{\mathbf{n}} u + u + u_t = 0.$$

For all singular and nonsingular values of the perturbation parameter, we obtain global attractors with optimal regularity. After fitting both problems into a common framework, a proof of the upper-semicontinuity of the family of global attractors is given. The result is motivated by the seminal work of J. Hale and G. Raugel in *Upper semicontinuity of the attractor for a singularly perturbed hyperbolic equation*, J. Differential Equations 73 (1988). We also show that for all values of the perturbation parameter, the corresponding solution operator admits an exponential attractor.

Large time behavior of solutions for Double-diffusive convection systems based on Brinkman-Forchheimer equation

Shun Uchida
Waseda University, Japan
u-shun@suou.waseda.jp
Mitsuharu Ôtani

Double-diffusive convection is one of fluid phenomena reflecting the interaction between temperature and concentration of solute. In particular, the model of double-diffusive convection in a porous medium has a large area of application, for instance, the behavior of polluted water in the soil. In order to describe the behavior of the fluid velocity under these situations, it is appropriate to apply so called Brinkman-Forchheimer equation, which is derived from a modified Darcy's law. In previous researches, we have obtained some result for global solvability of double-diffusive convection systems. In this talk, we consider the large time behavior of those solutions.

Periodic solutions of completely resonant nonlinear wave equations

Tatsuya Watanabe
Kyoto Sangyo University, Japan
tatsuw@cc.kyoto-su.ac.jp

We consider the following nonlinear wave equation:

$$(1) \quad \begin{cases} \omega^2 u_{tt} - u_{xx} + \varepsilon f(x,u) = 0 \\ \qquad\qquad \text{in } (x,t) \in (0,\pi) \times \mathbb{R}, \\ u(x,t) = u(x,t+2\pi). \end{cases}$$

We are interested in the case $\omega \in \mathbb{R} \setminus \mathbb{Q}$, $f(x,0) = f_s(x,0) = 0$ and $f(x,s)$ has a superlinear growth at infinity. In this talk, we discuss the existence of nontrivial periodic solutions of (1) under Dirichlet boundary condition. We also study the existence of spatially nonconstant periodic solutions of the orresponding free vibration problem.

Life span of solutions for a reaction-diffusion system with non-decaying initial data

Yusuke Yamauchi
Hiroshima Institute of Technology, Japan
y.yamauchi.bm@cc.it-hiroshima.ac.jp
Satoshi Sasayama

We consider the blow up time of positive solutions for the reaction-diffusion system:

$$u_t = \Delta u + u^{p_1} v^{q_1},$$
$$v_t = \Delta v + u^{p_2} v^{q_2}.$$

Especially for non-decaying initial data, it is shown that the blow up time for the system is strongly related to that for the corresponding system of ordinary differential equations. The proof is divided into three cases by whether the nonlinearities are superlinear or not.

Special Session 87: Evolution Equations and Integrable Systems

Alex Himonas, University of Notre Dame, USA
Gerson Petronilho, Federal University of Sao Carlos, Brazil

The theme of this session is nonlinear evolution equations and integrable systems including the NLS equation, the KdV equation, the Camassa-Holm equation, and the Euler equations of hydrodynamics. Topics covered for these equations include, among others, local and global well-posedness, scattering and stability issues, integrability and solitary waves.

Integrable multi-component nonlinear wave equations

Stephen Anco
Brock University, Canada
sanco@brocku.ca

I will survey some recent work that uses a geometric approach to derive new integrable multi-component nonlinear wave equations which have a Lax pair, a bi-Hamiltonian structure, and a symmetry recursion operator.

A Lagrangian approach for Navier-Stokes equations in critical spaces

Raphaël Danchin
Université Paris-Est Créteil, France
danchin@univ-paris12.fr

We are concerned with the Cauchy problem for models describing the evolution of nonhomogeneous fluids in the whole space or with periodic boundary conditions. Both the compressible and the incompressible cases are considered. In Eulerian coordinates, the corresponding systems are of mixed hyperbolic/parabolic type so that uniqueness conditions are stronger than existence conditions. We here show that it is no longer the case if those systems are reformulated in Lagrangian coordinates. As a matter of fact, it turns out to be possible to solve the Cauchy problem with critical regularity data, by means of the standard Banach fixed point theorem. As a by-product, in this framework, the flow is Lipschitz continuous.

Local well-posedness for a class of nonlocal evolution equations of Whitham type

Mats Ehrnstrom
Norwegian University of Science and Technology, Norway
mats.ehrnstrom@math.ntnu.no

For a class of pseudodifferential evolution equations of the form

$$u_t + (n(u) + Lu)_x = 0,$$

we prove local well-posedness for initial data in the Sobolev space H^s, $s > 3/2$. Here L is a linear Fourier multiplier with a real, even and bounded symbol m, and n is a real measurable function with $n'' \in H^s_{\mathrm{loc}}(\mathbb{R})$, $s > 3/2$. The proof, which combines Kato's approach to quasilinear equations with recent results for Nemytskii operators on general function spaces, applies equally well to the Cauchy problem on the line, and to the initial-value problem with periodic boundary conditions. This is based on joint work with J. Escher, Leibniz University Hanover, and L. Pei, Norwegian University of Science and Technoogy.

Global well-posedness and blow-up for the L^2-supercritical and H^1-subcritical Inhomogeneous Nonlinear Schrödinger Equation

Luiz Gustavo Farah
UFMG, Brazil
lgfarah@gmail.com

We consider the supercritical inhomogeneous nonlinear Schrödinger equation (INLS)

$$i\partial_t u + \Delta u + |x|^{-b}|u|^{2\sigma}u = 0,$$

where $(2-b)/N < \sigma < (2-b)/(N-2)$ and $0 < b < \min\{2, N\}$. We prove a new Gagliardo-Nirenberg estimate and use it to establish sufficient conditions for global existence and blow-up in $H^1(\mathbb{R}^N)$.

Microlocal Analysis for Evolution Equation in the Einstein and de Sitter Spacetime

Anahit Galstyan
University of Texas-Pan American, USA
agalstyan@utpa.edu

The talk is concerned with the waves propagating in the universe modeled by the so-called Einstein and de Sitter cosmological model. We introduce the initial value problem for this equation and give the parametrixes in the terms of Fourier integral operators. The estimates for the energy of solutions will be presented as well.

On the integrability of a class of generalized Davey-Stewartson system

Burak Gurel
Bogazici University, Turkey
bgurel@boun.edu.tr
Alp Eden

We investigate the integrable cases of the elliptic-hyperbolic-hyperbolic (EHH) generalized Davey-Stewartson (GDS) system using the vertex method developed by Zakharov and Shulman. The system we analyze is called the GDS system because it was derived as a 3-component system so as to incorporate the effect of the second spatial coordinate, that was omitted in the derivation of the 2-component Davey-Stewartson (DS) system. Physically meaningful cases of the GDS system comprise the EHH version which we consider in this work. The method produces necessary conditions for the inverse scattering transform to be applied successfully. Implementing this method, we prove that the EHH-GDS system with physical parameters is integrable only when it can be reduced to an integrable, necessarily elliptic-hyperbolic, DS system.

Stability of periodic waves of the coupled Klein-Gordon equations

Sevdzhan Hakkaev
Shumen University and Istanbul Aydin University, Bulgaria
shakkaev@fmi.shu-bg.net

We consider linear stability for periodic wave solutions for the coupled Klein-Gordon equation. We find conditions on parameters of the waves which imply linear stability and instabilty of periodic waves.

The Cauchy problem for a generalized CH equation

Alex Himonas
University of Notre Dame, USA
himonas.1@nd.edu

We shall discuss the initial value problem for a generalized Camassa-Holm equation with higher order nonlinearities and containing as its members three integrable equations–the Camassa-Holm, the Degasperis-Procesi and the Novikov equations. For $s > 3/2$ we shall show that this equation is well-posed in Sobolev spaces H^s on both the circle and the line in the sense of Hadamard. That is, the data-to-solution map is continuous. However, it is not uniformly continuous. This is work in collaboration with Curtis Holliman.

Integrable and nonintegrable peakon equations

Andrew Hone
University of Kent, England
anwh@kent.ac.uk

We consider partial differential equations admitting peaked soltion solutions (peakons), and give a survey of results on the integrability of such equations. Some of the properties of peakons are related to their shape, which is inherited from the one-dimensional Helmholtz operator. Green's functions for other operators are considered, in particular in an economic model for urban growth due to Krugman, where integrability is absent, but exact results on the long-term evolution of the system are still possible.

Matrix Riemann-Hilbert problems with jumps across Carleson contours

Jonatan Lenells
Baylor University, USA
Jonatan_Lenells@baylor.edu

I will describe a theory of matrix Riemann-Hilbert problems for a class of jump contours of very low regularity. As an application, the long-time asymptotics for the Degasperis-Procesi equation on the half-line will be analyzed. Paper reference: arXiv:1401.2506.

Initial-boundary value problems for certain evolution equations

Dionyssios Mantzavinos
University of Notre Dame, USA
mantzavinos.1@nd.edu
Alex Himonas

For initial and boundary data in appropriate Sobolev spaces, we will use the novel solution representations obtained via Fokas' transform method in order to study well-posedness of certain evolution equations on the half-line.

Autonomous Ovsyannikov theorem and applications to nonlocal evolution equations and systems

Gerson Petronilho
Federal University of Sao Carlos, Brazil
gersonpetro@gmail.com
Rafael F. Barostichi, A. Alexandrou Himonas

This work presents an Ovsyannikov type theorem for an autonomous abstract Cauchy problem in a scale of decreasing Banach spaces, which in addition to existence and uniqueness of solution it provides an estimate about the analytic lifespan of the solution. Then it presents applications to the Cauchy problem for Camassa-Holm type equations and systems with initial data in spaces of analytic function on both the circle and the line. Finally, it studies the continuity of the data-to-solution map in spaces of analytic functions.

A functional-analytic technique for the study of analytic solutions of PDEs

Eugenia Petropoulou
University of Patras, Greece
jenpetr@upatras.gr
Panayiotis D. Siafarikas

A functional-analytic method is used to study the existence and the uniqueness of bounded, analytic and entire complex solutions of partial differential equations. The predicted solutions are in power series form. As a benchmark problem, this method is applied to the nonlinear Benjamin–Bona–Mahony equation and the associated to this, linear equation.

Vector nonlinear Schrödinger equation: Global existence and spectral stability

Natalie Sheils
University of Washington, USA
nsheils@amath.washington.edu

We consider the Cauchy problem for a Schrödinger system with power-type nonlinearities

$$i\frac{\partial \psi_j}{\partial t} + \triangle\psi_j + \sum_{k=1}^{m} \alpha_{jk}|\psi_k|^p|\psi_j|^{p-2}\psi_j = 0,$$

where $\psi_j : \mathbb{R}^N \times \mathbb{R} \to \mathbb{C}$ for $j = 1, 2, \ldots, m$ and α_{jk} are real. I will provide an overview of our recent results relating to this problem. This includes establishing global existence for a certain range of p with $\alpha_{jk} = \alpha_{kj}$ positive and examining spectral stability of solitary wave solutions (both bright and dark) analytically and numerically for the case $p = 2$, $N = 2$.

Closed solutions for the degenerate parametric oscillator and inhomogeneous paraxial wave equation

Erwin Suazo
Arizona State University/University of Puerto Rico, USA
esuazo@asu.edu
P. B. Acosta, A. Mahalov and S. K. Suslov

Using Ermakov and Riccati type systems, we present how we can construct explicit solutions for the propagator for a generalized harmonic oscillator. This has applications describing the process of degenerate parametric amplification in quantum optics as well as light propagation in a nonlinear anisotropic waveguide. We also present explicit solutions of the inhomogeneous paraxial wave equation in a linear and quadratic approximation, showing the existence of oscillating laser beams in a parabolic waveguide and spiral light beams in varying media. Finally, using a similar approach we show how we can construct soliton solutions for the nonautonomous nonlinear Schroedinger equation.

Colliding peakons and the formation of shocks in the Degasperis-Procesi Equation

Jacek Szmigielski
University of Saskatchewan, Canada
szmigiel@math.usask.ca
Lingjun Zhou

The Degasperis-Procesi equation (DP) is one of several equations known to model important nonlinear effects such as wave breaking and shock creation. What is quite unique about the DP equation is that these two effects can be studied in an explicit way with the help of the multipeakon ansatz. In essence, one is presented with the view of wave breaking as a collision of hypothetical particles (peakons and antipeakons). I will show that the system of ODEs describing DP multipeakons has the Painlevé property which leads to a universal wave breaking behaviour. Other interesting features are: (i) multipeakons can collide only in pairs; (ii) there are no multiple collisions other than, possibly simultaneous, collisions of peakon-antipeakon pairs at different locations; (iii) each peakon-antipeakon collision results in creation of a shock thus making possible a multi-shock phenomenon. This is joint work with Lingjun Zhou (Tongji University, Shanghai, P.R. China).

Weak solutions of an integrable evolution equation

Feride Tiglay
Ohio State University, USA
tiglay.1@osu.edu

The μHS equation was introduced by B. Khesin, J. Lenells and G. Misiolek as a geodesic equation on the group of circle diffeomorphisms. I will present some existence results for weak solutions of the Cauchy problem of this equation.

Biomagnetic fluid flow in an aneurism

Efstratios Tzirtzilakis
TEI of Western Greece, Greece
etzirtzilakis@teimes.gr

In this study, we numerically investigate the fundamental problem of biomagnetic fluid flow in an aneurismal geometry under the influence of a steady localized magnetic field. The mathematical model used for the formulation of the problem is consistent with the principles of ferrohydrodynamics (FHD). Blood is considered as a homogeneous non-isothermal Newtonian fluid and is treated as an electrically non-conducting magnetic fluid. For the numerical solution of the problem, which is described by a coupled, non-linear system of PDEs, with appropriate boundary conditions, the stream function vorticity formulation is adopted. The solution is obtained by the application of an efficient pseudotransient numerical methodology using finite differences. This methodology is based on the application of a semi-implicit numerical technique, transformations stretching of the grid and proper construction of the boundary conditions for the vorticity. Results concerning the velocity and temperature field, skin friction and rate of heat transfer indicate that the presence of the magnetic field influences the flow field considerably especially in the region of the aneurism.

Transverse instability of generalised solitary waves

Erik Wahlen
Lund University, Sweden
ewahlen@maths.lth.se
Mariana Haragus

A generalised solitary wave is a travelling wave consisting of a localised central part and periodic non-decaying oscillations extending to infinity. They arise in weakly nonlinear equations due to a resonance between a long wave and a short wave with finite wave number. An important example is the water wave problem with weak surface tension (Bond number less than 1/3). The localised part is then described to leading order by the KdV equation. Although the KdV soliton is stable, instabilities may arise from the periodic wave trains at infinity. I will discuss this question for a simpler model equation. The Kawahara equation is a fifth order version of the KdV equation, modelling waves with Bond number close to 1/3. It's known to have generalised solitary-wave solutions. There is also a 2D version called the 5th order KP equation. I will discuss the spectral instability for the generalised solitary waves (seen as generalised line solitary waves of the 5th order KP equation) to perturbations in the transverse direction.

The evolution equations in the curved spacetime

Karen Yagdjian
University of Texas-Pan American, USA
yagdjian@utpa.edu

The talk is concerned with the global in time solutions of the Cauchy problem for waves propagating in the curved spacetime, which can be, in particular, modeled by cosmological models. We examine the global in time solutions of some class of semililear hyperbolic equations in the de Sitter spacetime. The crucial tool for the obtaining those results is a new approach, which is based on the integral transform with the kernel containing the hypergeometric function. That integral transform allows to write solutions of the initial-value problem for one partial differential equation via solution of another one.

Numerical methods for higher-order water waves

Juan-Ming Yuan
Financial and computational Mathematics Department, Providence University, Taiwan
jmyuan@pu.edu.tw
Jiahong Wu

This talk shows some recent results on a new version of higher-order models for water waves derived by Bona et al. Preliminary analysis and implementation of numerical methods reveal significant mathematical differences among these model equations.

Special Session 88: Stochastic Processes and Spectral Theory for Partial Differential Equations and Boundary Value Problems

Francis Nier, Univesité de Rennes 1, France
Tony Lelievre, Ecole des Ponts ParisTech, France

The purpose of this session is to gather people interested in partial differential equations associated with stochastic processes either from the point of view of theoretical analysis or from the point of view of numerical and stochastic simulations. This covers spectral theory, hypoelliptic techniques, kinetic theory, entropy methods, stochastic analysis, numerical algorithms and molecular dynamics. We have especially in mind boundary value problems, low temperature asymptotics and longtime behaviors.

Mathematical foundations of Temperature Accelerated Dynamics (TAD)

David Aristoff
University of Minnesota, USA
daristof@umn.edu
T. Lelievre

We give a mathematical framework for Temperature Accelerated Dynamics (TAD), a popular algorithm due to M.R. Sorensen and A.F. Voter for efficiently generating molecular dynamics in the presence of metastability. Using the notion of quasistationary distributions, we show that, under certain idealizing assumptions and with some small modifications to the algorithm, TAD becomes exact. We hope our framework will allow for a rigorous analysis of the error in TAD.

Entropy method for hypocoercive Fokker-Planck equations with linear drift

Anton Arnold
Vienna University of Technology, Austria
anton.arnold@tuwien.ac.at
Jan Erb

For degenerate parabolic equations, the entropy dissipation may vanish for states other than the equilibrium. Hence, the aproach due to Bakry-Emery does not carry over. In the hypocoercive case, we first establish a condition that is equivalent to the existence of a unique normalised steady state. By introducing an auxiliary functional we prove the exponential decay of the solution towards the steady state in relative entropy. Finally, we show that the obtained rate is indeed sharp (both for the logarithmic and quadratic entropy).

On the well-posedness of Stochastic Lagrangian Models

Mireille Bossy
INRIA, France
mireille.bossy@inria.fr
Jean Francois Jabir

In this talk, I will present some recent results on the well-posedness of a Stochastic Lagrangian Model (SLM). Our interest in such nonlinear model is strongly connected with the modeling of turbulent flows by the so-called PDF methods or fluid-particle methods. These models are intensively used in the context of computational fluid dynamics for reactive turbulent flows. We will focus on the following particular result: when the SLM is endowed with a specular boundary condition, we show that for such stochastic Kinetic equation, a mean-no-permeability boundary condition is satisfied. This is a joint work with Jean Francois Jabir (University of Valparaiso)

Sharp spectral asymptotics for discrete metastable diffusions

Giacomo Di Gesu
Roma La Sapienza, Italy
digesu@mat.uniroma1.it

We consider a metastable diffusion moving in a multiwell potential on the rescaled n-dimensional integer lattice. From a spectral point of view metastability effects correspond to the presence of nearly degenerate small eigenvalues of the generator, each one linked to a well of the potential. In this talk a result providing complete asymptotic expansions of these small eigenvalues is presented. The proof, inspired by previous work of B. Helffer, M. Klein and F. Nier in continuous space setting, is based on tools of semiclassical analysis (Harmonic approximation, WKB expansions) and on a supersymmetric extension à la Witten of the generator on the level of discrete 1-forms.

Long time behaviour of SDE : from coupling to functional inequalities

Arnaud Guillin
CNRS - Blaise Pascal University and IUF, France
guillin@math.univ-bpclermont.fr
P. Cattiaux, A. Eberle

We will review here various methods to study long time behaviour of Markov processes (coupling, Lyapunov conditions, functional inequalities) and will illustrate them on the Ornstein-Uhlenbeck pinball, namely an Ornstein Uhlenbeck with reflections on a non convex domain.

A probabilistic trajectorial interpretation of the dissipations of entropy and Fisher information for SDEs

Benjamin Jourdain
Université Paris-Est Cermics, France
jourdain@cermics.enpc.fr
Joaquin Fontbona

The dissipation of general convex entropies for continuous time Markov processes can be described in terms of backward martingales. The relative entropy is the expected value of a backward submartingale. In the case of (non necessarily reversible) Markov diffusion processes, we use

Girsanov theory to explicit its Doob-Meyer decomposition and provide thereby a stochastic analogue of the well known entropy dissipation formula, which is valid for general convex entropies, including total variation distance. Under additional regularity assumptions, and using Itô's calculus and ideas of Arnold, Carlen and Ju 2008, we obtain a new Bakry Emery criterion which ensures exponential convergence of the entropy to 0. This criterion is non-intrisic since it depends on the square root of the diffusion matrix, and cannot be written only in terms of the diffusion matrix itself.

Conditioned Brownian motions

Martin Kolb
University of Paderborn, Germany
kolb@reading.ac.uk
Mladen Savov

In this talk we present joint work with Mladen Savov (Reading) concerning several conditioned limit theorems, which are inspired by random polymer models. We consider Brownian motion conditioned on the behaviour of the associated local time as well as Brownian motion with constant drift conditioned on its range process and derive certain conditional limit theorems. Our work was motivated by previous results of Berestycki and Benjamini as well as Povel.

Analysis in large dimension of the first non zero eigenvalue of some Witten Laplacian

Dorian Le Peutrec
Paris-Sud University, France
dorian.lepeutrec@math.u-psud.fr
Giacomo Di Gesu

This talk will be about the precise computation of the first non zero eigenvalue of some semi-classical positive Schroedinger operator, the Witten Laplacian, at the semi-classical limit $h \to 0$, in large dimension. These operators are distorted semi-classical Laplacians by means of a Morse function f. The study of their low spectrum at the semi-classical limit is closely related to the study of the metastability for the overdamped Langevin processes $dX_t = -\nabla f(X_t) + \sqrt{2h}dW_t$. In this talk, we will look at the Witten Laplacian associated with a particular Morse function corresponding to some coupled bistable system, in large dimension.

W-entropy formula and optimal transport on Riemannian manifolds with weighted measure

Xiangdong Li
Academy of Mathematics and Systems Science, Peoples Rep of China
xdli@amt.ac.cn

In this talk, we present some recent results on the W-entropy formula and the optimal transport problem on Riemannian manifolds with weighted measure. Inspired by the seminal work of G. Perelman, we prove a W-entropy formula for the heat equation of the Witten Laplacian on complete Riemannian manifolds. Under the CD(0, m) condition, we derive the monotonicity of the W-entropy and prove a rigidity theorem. We also present a W-entropy formula for the geodesic flow on the Wasserstein space over compact Riemannian manifolds. We observe that these two W-entropy formulas have the same spirit. This leads us to prove a convexity theorem of the Boltzmann entropy along a familly of geometric flows which interpolate the geodesic flow on the Wasserstein space and the heat flow on the underlying Riemannian manifolds with CD(0, m) condition.

The gluing formula of the Ray-Singer analytic torsion

Xiaonan Ma
Institut de Mathematiques de Jussieu, France
ma@math.jussieu.fr

Real analytic torsion is a spectral invariant of a compact Riemannian manifold equipped with a flat Hermitian vector bundle, that was introduced by Ray-Singer in 1971. Ray and Singer conjectured that for unitarily flat vector bundles, this invariant coincides with the Reidemeister torsion, a topological invariant. This conjecture was established by Cheeger and Mueller, and extended by Bismut-Zhang to arbitrary flat vector bundles. We derive the Bismut-Zhang theorem for manifolds with boundary and the gluing formula for the analytic torsion of flat vector bundles in full generality, i.e., we do not assume that the Hermitian metric on the flat vector bundle is flat nor that the Riemannian metric has product structure near the boundary.

Quasi-tensorization of the relative entropy for discrete spin systems.

Georg Menz
Stanford University, USA
gmenz@stanford.edu
Pietro Caputo, Prasad Tetali

In this talk we discuss the quasi-tensorization of the relative entropy for Gibbs measures. By transferring Katalyn Marton's argument from continuous to discrete state-spaces, we derive a criterion for the quasi-tensorization in discrete state-spaces. The criterion seems to be the first one of its kind and is optimal for product measures. The criterion can be interpreted as a generalization of the Otto-Reznikoff criterion for the logarithmic Sobolev inequality (LSI) to discrete state spaces. Applying the criterion to the Curie-Weiss and Ising modell, one can deduce uniform estimates on the modified LSI for the Gibbs sampler in the regime of small interaction.

Spectral analysis of hypo elliptic random walk

Laurent Michel
Universite de Nice, France
lmichel@unice.fr
G. Lebeau

We study the spectral theory of a reversible Markov chain associated to a hypoelliptic random walk on a manifold M. This random walk depends on a parameter $h \in]0, h_0]$ which is roughly the size of each step of the walk. Under a Hormander type assumption, we prove uniform bounds with respect to h on the rate of convergence to equilibrium.

Diffusion Limit for Random Walk Metropolis algorithm started out of stationarity

Michela Ottobre
Imperial College London, England
m.ottobre08@imperial.ac.uk
A. M. Stuart

We study a diffusion limit for the Random Walk Metropolis algorithm for target measures in non-product form, when the chain is started out of stationarity. Random Walk Metropolis is a popular Metropolis-Hastings algorithm which samples from a given probability distribution by creating a reversible Markov chain which has the target distribution as unique invariant measure. When the target measure is in product form, a diffusion limit for the resulting Markov chain has been first studied in the seminal paper by G.O. Roberts et al (1997), assuming the chain is started in stationarity, as a way of understanding the efficiency of the algorithm in high dimensions. Since then, the overwhelming majority of the literature on the subject has been concerned with the stationary phase of the chain. Recently Jourdain at al (2012) have analysed the transient phase as well, in the case in which the target measure is in product form. This talk will present a method to extend the results in the literature by considering measures in non-product form, when the chain is started out of stationarity.

Exponential return to equilibrium for hypoelliptic quadratic systems and applications

Grigorios Pavliotis
Imperial College London, England
g.pavliotis@imperial.ac.uk
M. Ottobre, K. Pravda-Starov

We study the problem of convergence to equilibrium for evolution equations associated to general quadratic operators. Quadratic operators are non-selfadjoint differential operators with complex-valued quadratic symbols. Under appropriate assumptions, a complete description of the spectrum of such operators is given and the exponential return to equilibrium with sharp estimates on the rate of convergence is proven. Some applications to the study of chains of oscillators, to the generalized Langevin equation are given and to the analysis of Markov Chain Monte Carlo methods and of stochastic thermostats for molecular dynamics simulations are given.

Pseudospectral landscape around a doubly characteristic point

Karel Pravda-Starov
Université de Cergy-Pontoise, France
karel.pravda-starov@u-cergy.fr

We discuss the spectral and pseudospectral properties of doubly characteristic non-selfadjoint operators of the Kramers-Fokker-Planck type. We shall explain how the structure of the doubly characteristic set allows to describe the pseudospectral properties of these accretive operators near the bottom their spectra.

Tanaka's quantitative dissipation, Markov coupling, and Kac's conservative N-particle systems.

Mathias Rousset
INRIA, France
mathias.rousset@inria.fr

An explicit Markov coupling of collisional conservative Kac's N-particle system with Maxwell collisions is constructed. Parallel geometric coupling of simultaneous collisions is used. In agreement with Tanaka's dissipation of the space homogenous Maxwell-Boltzmann in Wasserstein distance, the resulting coupling is almost surely decreasing, and the L2-coupling creation is computed explicitly. Some quasi-contractive and uniform in N coupling / coupling creation inequalities are then proved, relying on $2 + a$-moments ($a > 0$) of velocity distributions; upon N-uniform propagation of moments of the particle system, it yields a N-scalable a-power law trend to equilibrium. The case of order 4 moment yields a simple solution of Kac's program for convergence to equilibrium of collisional N-particle systems.

Relative Entropy Preconditioning for Markov Chain Monte Carlo

Gideon Simpson
Drexel University, USA
simpson@math.drexel.edu
F.J. Pinski, A.M. Stuart, H. Weber

One of the challenges in using Markov Chain Monte Carlo methods to sample from a target distribution is finding a good prior distribution. An ideal prior distribution would both be easy to sample from and have a high acceptance rate in the Metropolis step of the algorithm. This latter property ensures that the Markov chain will rapidly explore the configuration space under the target distribution. In this talk, we present work to use functionalized Gaussian priors which are preconditioned to minimize the distance, with respect to relative entropy, to the target measure. This will then be seen to give much more favorable sampling properties than the naive prior.

Inequalities for Markov Operators and Applications to Forward and Backward PDEs.

Holger Stephan
Weierstrass Institute Berlin, Germany
stephan@wias-berlin.de

Typical examples of Markov operators are solution operators to linear PDEs of second order, describing the evolution of probability densities for classical particles. A general mathematical framework, applicable for general classical physical systems, can be developed to describe such equations and many others in a unique setting. This allows to understand better the underlying structure of such equations. In this setting, Markov operators provide some general inequalities defining a natural order. This order is well known as order in majorization theory in linear algebra or order of rearrangement of functions in integration theory. In addition, the inequalities show the irreversibility of time for both forward as well as backward running time, depending on the physical meaning of the variables in the underlying equations.

Langevin dynamics with space-time periodic nonequilibrium forcing

Gabriel Stoltz
Ecole des Ponts, France
stoltz@cermics.enpc.fr
G. Pavliotis, R. Joubaud

I present results on the ballistic and diffusive behavior of Langevin dynamics under a space-time periodic driving force. In the hyperbolic scaling, a non-trivial average velocity can be observed even if the external forcing vanishes. More surprisingly, an average velocity in the direction opposite to the forcing may develop at the linear response level – a phenomenon called negative mobility. The diffusive limit of (possibly strongly) forced systems is studied using appropriate solutions of Poisson equations, extending recent works on pointwise estimates of the resolvent for the generator associated with Langevin dynamics.

Special Session 89: Applications of Topological and Variational Methods to Boundary Value Problems

John R. Graef, University of Tennessee at Chattanooga, USA

Topological methods have proved to be an important technique in the study of boundary value problems and related topics for ordinary and partial differential equations. There has been a rapidly growing interest in applying variational methods and critical point theory to such problems in recent years. This session is devoted to the use of these methods in the study of boundary value problems including singular problems and those with multi-point conditions.

Existence and multiplicity results for nonlinear differential problems

Gabriele Bonanno
Messina University, Italy
bonanno@unime.it

In this talk existence results for nonlinear differential problems are presented. Precisely, under a suitable behaviour of the nonlinearity, the existence of a non zero solution is proved for wide classes of differential problems. The main tool is a local minimum theorem. However, in some case, also a coincidence point theorem for weakly sequentially continuous maps, obtained by set-valued arguments, is applied. Finally, multiplicity results are established by using suitable remarks on the classical mountain pass theorem.

Higher-order BVP's defined on unbounded domains

Hugo Carrasco
University of Evora, Portugal
hugcarrasco@gmail.com
Feliz Minhos

In this work is presented sufficient conditions for the existence of unbounded solutions of a Sturm-Liouville boundary value problem on the half-line. One-sided Nagumo condition plays a special role because allows an asymmetric unbounded behavior on the nonlinearity. The arguments are based on the Schauder fixed point theorem and lower and upper solutions method. An example is given to show the applicability of our results.

A Fixed Point Theorem for Weakly Inward A-proper Maps

Casey Cremins
University of Maryland, USA
ctc@math.umd.edu

A fixed point theorem for weakly inward A-proper maps defined on cones in Banach spaces is established using a fixed point index for such maps introduced by Lan and Webb.

Positive solutions of a three-point boundary value problem for p-Laplacian dynamic equation on time scales

Abdulkadir Dogan
Abdullah Gul University, Turkey
abdulkadir.dogan@agu.edu.tr

We consider a three-point boundary value problem for p-Laplacian dynamic equation on time scales. We prove the existence at least three positive solutions of the boundary value problem by using the Avery and Peterson fixed point theorem. The interesting point is that the non-linear term f involves a first-order derivative explicitly. Our results are new for the special cases of difference equations and differential equations as well as in the general time scale setting.

Existence of Positive Solutions for a Periodic Functional Differential Equation System

Wenying Feng
Trent University, Canada
wfeng@trentu.ca
G. Zhang

We consider the existence, multiplicity and nonexistence of ω-periodic positive solutions for an n-dimensional functional differential equation system involving one parameter. Applying a fixed point theorem and the monotone techniques, we obtain some existence and nonexistence results for the system depending on the parameter λ.

Higher order periodic impulsive problems

Joao Fialho
College of the Bahamas, Bahamas
jfzero@gmail.com
Feliz Minhos

The theory of impulsive problem is experiencing a rapid development in the last few years. Mainly because they have been used to describe some phenomena, arising from different disciplines like physics or biology, subject to instantaneous change at some time instants called moments. Second order periodic impulsive problems were studied to some extent, however very few papers were dedicated to the study of third and higher order impulsive problems. One can refer for instance and the references therein. The high order impulsive problem considered is composed by the fully nonlinear equation

$$u^{(n)}(x) = f\left(x, u(x), u'(x), ..., u^{(n-1)}(x)\right)$$

for a. e. $x \in I := [0,1] \setminus \{x_1, ..., x_m\}$ where $f : [0,1] \times \mathbb{R}^n \to \mathbb{R}$ is L^1-Carathéodory function, along with the periodic boundary conditions

$$u^{(i)}(0) = u^{(i)}(1), \quad i = 0, ..., n-1,$$

and the impulsive conditions

$$u^{(i)}\left(x_j^+\right) = g_j^i\left(u\left(x_j\right)\right), \quad i = 0, ..., n-1,$$

where g_j^i, for $j = 1, ..., m$,are given real valued functions satisfying some adequate conditions, and $x_j \in (0,1)$, such that $0 = x_0 < x_1 < ... < x_m < x_{m+1} = 1$. The arguments applied make use of the lower and upper solution method combined with an iterative technique, which is not necessarily monotone, together with classical results such as Lebesgue Dominated Convergence Theorem, Ascoli-Arzela Theorem and fixed point theory.

Stabilization of unidimensional discrete population models via conditional strategies

Daniel Franco
UNED (Universidad Nacional de Educación a Distancia), Spain
dfranco@ind.uned.es

We will describe different harvesting/thinning control strategies in the framework of one dimensional discrete population models. These strategies have the common feature of considering a threshold population size, commonly called Biomass at the limit, under which the population is not altered, and they differ in how the harvesting/thinning is applied when that threshold is surpassed. We will discuss the basic properties of both strategies and compare them with other simpler control methods that have been studied in the literature. Particularly, we will focus on the possibility of applying these strategies to control the chaotic behaviour predicted by some one dimensional discrete population models.

Multiplicity results for systems of first order differential inclusions

Marlene Frigon
University of Montreal, Canada
frigon@dms.umontreal.ca
M. Lotfipour

Abstract: Multiplicity results are obtained for systems of first order differential inclusions with periodic boundary or initial value conditions. To this aim, we introduce notions of strict solution-tubes. The cases where the nonlinearity satisfies an upper or lower semi-continuity condition are considered. Our results are new even in the particular cases where the nonlinearity is single valued. Keywords: System of differential inclusions; first order differential inclusion; multiple solutions; solution-tube; strict solution-tube

Existence of homoclinic solutions for second order difference equations with p-laplacian

John Graef
University of Tennessee at Chattanooga, USA
john-graef@utc.edu
Lingju Kong and Min Wang

Using the variational method and critical point theory, the authors study the existence of infinitely many homoclinic solutions to the difference equation

$$-\Delta\big(a(k)\phi_p(\Delta u(k-1))\big) + b(k)\phi_p(u(k)) = \eta f(k, u(k))),$$

where $k \in \mathbb{Z}$, $p > 1$ is a real number, $\phi_p(t) = |t|^{p-2}t$ for $t \in \mathbb{R}$, $\eta > 0$ is a parameter, $a, b : \mathbb{Z} \to (0, \infty)$, and $f : \mathbb{Z} \times \mathbb{R} \to \mathbb{R}$ is continuous in the second variable. Related results in the literature are extended.

Infinitely many periodic solutions for a class of damped vibration problems via variational methods

Shapour Heidarkhani
Razi University, Iran
sh.heidarkhani@yahoo.com

Using variational methods and critical point theory, we established the existence of infinitely many periodic solutions for a class of damped vibration problems, under suitable assumptions on the nonlinear terms. We illustrate the results by presenting convenient examples.

On the second order equations with nonlinear impulses - Fredholm alternative type results

Martina Langerova
NTIS, University of West Bohemia, Czech Rep
mlanger@ntis.zcu.cz
Pavel Drabek

We consider the semilinear homogeneous Dirichlet boundary value problem for the second order equation on a finite interval with nonlinear impulses in the derivative at prescribed points. We introduce Landesman-Lazer type necessary and sufficient conditions for resonance problems and generalize the Fredholm alternative results for linear operators. An interaction between nonlinear restoring force and nonlinear impulses is presented.

An Application of an Avery Type Fixed Point Theorem to a Second Order Antiperiodic Boundary Value Problem

Jeffrey Lyons
Nova Southeastern University, USA
jlyons@nova.edu

In this presentation, an application is made of an Avery type fixed point theorem to a second order boundary value problem with antiperiodic boundary conditions. The fixed point theorem is an extension of the traditional Leggett-Williams fixed point theorem. Under certain conditions and with a nice concavity result, an antisymmetric solution is shown to exist.

On higher order nonlinear impulsive boundary value problems

Feliz Minhos
University of Evora, Portugal
fminhos@uevora.pt
R. Carapinha

This work studies some three point impulsive boundary value problems composed by a fully differential equation, which higher order contains an increasing homeomorphism, by three point boundary conditions and impulsive effects. We point out that the impulsive functions are given via multivariate generalized functions, including impulses on the referred homeomorphism. The method used apply lower and upper solutions technique together with fixed point theory. Therefore we have not only the existence of solutions but also the localization and qualitative data on their behavior. Moreover a Nagumo condition will play a key role in the arguments.

Existence of a Smallest Eigenvalue for a Fractional Boundary Value Problem

Jeffrey Neugebauer
Eastern Kentucky University, USA
jeffrey.neugebauer@eku.edu
Paul W. Eloe

We show the existence of a smallest eigenvalue for the fractional linear differential equation $D^{\alpha}_{0+}u + \lambda p(t)u = 0$, $0 < t < 1$, satisfying the boundary conditions $u(0) = u(1) = 0$. This is accomplished by showing the operator $Mu(t) = \int_0^1 G(t,s)p(s)u(s)ds$, where $G(t,s)$ is the appropriate Green's function, is a u_0-positive operator. Some consequences of this existence will be explored.

Solvability of BVPs with state-dependent impulses via fixed point theorem

Irena Rachunkova
Palacky University Olomouc, Czech Rep
irena.rachunkova@upol.cz
Jan Tomecek

We investigate the solvability of a differential system with general linear boundary conditions and state-dependent impulse conditions on a compact interval. This is the case when impulse moments satisfy a predetermined relation between state and time variables. The boundary conditions are expressed by a linear bounded operator on the space of left-continuous regulated vector-functions. Such operators are uniquely represented by a constant matrix and by the Kurzweil-Stieltjes integral of a matrix-function whose elements have finite variation. Impulse points are determined as intersection points of a solution with barriers stated in a formulation of the boundary value problem. We provide transversality conditions which guarantee that each possible solution of the problem crosses each barrier at a unique point. Further, we construct a Banach space as a product space and an operator having a fixed point which consists from a finite number of functions. This number corresponds to a number of the barriers. The fixed point can be used to a construction of a solution of the boundary value problem under consideration.

Recent advances on mathematical models involving singular nonlinearities

Pedro Torres
Universidad de Granada, Spain
ptorres@ugr.es

A nonlinearity is said to be *singular* if it becomes infinite when the state variable approaches a certain point. Beyond the classical gravitational and electrostatic forces, singular nonlinearities arise in a wide variety of mathematical models in the applied sciences, like Celestial Mechanics, molecular dynamics, matter-state Physics, Fluid Mechanics, vortex dynamics, Mechanical Engineering and more. The purpose of this talk is to present a general review of some recent advances on the study of this relevant family of mathematical models, with a particular emphasis on open problems. Our interest is focused on the existence and stability of periodic oscillations.

Positive solutions for a system of singular higher-order multi-point boundary value problems

Rodica Luca Tudorache
Technical University of Iasi, Romania
rlucatudor@yahoo.com
Johnny Henderson

We investigate the existence of positive solutions of a system of higher-order nonlinear differential equations subject to multi-point boundary conditions, where the nonlinearities do not possess any sublinear or superlinear growth conditions and may be singular. In the proof of our main results, we use the Guo-Krasnosel'skii fixed point theorem.

Positive periodic solutions of a singular problem modeling valveless pumping with friction

Milan Tvrdy
Institute of Mathematics ASCR, Czech Rep
tvrdy@math.cas.cz
José Ángel Cid and Georg Propst

The talk is based on the joint paper with José Ángel Cid and Georg Propst accepted for Physica D. We consider the periodic problem

$$\text{(P)}\quad u'' + a\,u' = \frac{1}{u}\left(e(t) - b\,(u')^2\right) + c, \qquad u(0) = u(T),\ u'(0) = u'(T)\,,$$

where $T, a, c > 0$, $b > 1$ and the forcing term e is essentially bounded on $[0, T]$. As shown by G. Propst, this problem describes valveless pumping in the one pipe - one tank system. In general, valveless pumping assists in fluid transport in various biomedical and engineering systems and it describes e.g. blood circulation in the cardiovascular system when the heart's valves fail or when the embryonic vertebrate heart begins pumping blood long before the development of discernable chambers and valves. Despite the formal simplicity of the differential equation in (P), the singularity on the right-hand side makes it more difficult to analyze and, in fact, there is a lack of general existence results for (P). From the physical point of view we are interested in the search of positive solutions. It is easy to see that (P) has a positive solution $u(t) > 0$ only if the mean value $\bar{e}$ of e is positive. The aim of the contribution is to deliver conditions sufficient for the existence of a positive solution to (P).

On second-order non-local boundary value problem with singularities of space variables

Miroslawa Zima
University of Rzeszow, Poland
mzima@ur.edu.pl

The aim of the talk is to discuss the existence of positive solutions for a non-local boundary value problem of second-order, where the involved nonlinearity depends on the derivative and may be singular. The boundary conditions are given by Riemann-Stieltjes integrals. Our approach is based on the Krasnoselskii-Guo fixed point theorem on cone expansion and compression.

Special Session 90: Analysis of Hyperbolic PDEs

Anahit Galstyan, University of Texas-Pan American, United States
Fumihiko Hirosawa, Yamaguchi University, Japan
Jens Wirth, University of Stuttgart, Germany

This session is devoted to recent developments in the theory of linear and nonlinear hyperbolic equations and related topics including applied models appearing in physics and engineering. The main focus of the session will be related to questions concerning representations of solutions, regularity properties, propagation of singularities, local and global existence and asymptotic behavior.

Low regularity solutions for nonlinear Dirac equations

Nikolaos Bournaveas
University of Edinburgh, Scotland
n.bournaveas@ed.ac.uk

We look at various nonlinear systems involving the Dirac equation and examine whether local or global well-posedness can be established for rough data (joint works with Timothy Candy and Shuji Machihara).

Local existence of solutions of self gravitating relativistic perfect fluids

Uwe Brauer
Complutense University Madrid, Spain
oub@mat.ucm.es
Lavi Karp

This is a joint work together with Lavi Karp in which we consider the Einstein–Euler system in asymptotically flat spacetimes and therefore use the condition that the energy density might vanish or tend to zero at infinity, and that the pressure is a fractional power of the energy density. In this setting we prove local in time existence, uniqueness and well-posedness of classical solutions. The zero order term of our system contains an expression which might not be a $C^?$ function and therefore causes an additional technical difficulty. In order to achieve our goals we use a certain type of weighted Sobolev space of fractional order. In a previous work we constructed an initial data set for these of systems in the same type of weighted Sobolev spaces. We obtain the same lower bound for the regularity as in the case of the vacuum Einstein equations. However, due to the presence of an equation of state with fractional power, the regularity is bounded from above.

Global existence of equivariant wave maps on a curved background

Piero Antonio D'Ancona
Sapienza - University of Roma, Italy
dancona@mat.uniroma1.it
Qidi Zhang

In a joint work with Qidi Zhang, we study the global existence of small solutions in critical spaces for the equivariant wave maps equation bewteen two manifolds with rotational symmetry. We prove that global existence holds for several classes of base manifolds, and in particular we can allow for base manifolds which are not flat at infinity. The main tools are sharp (non-endpoint) Strichartz estimates for the linear wave and Klein-Gordon equation on a curved background.

General limits for systems of fluid dynamics

Michael Dreher
Heriot-Watt University, Edinburgh, Scotland
M.Dreher@hw.ac.uk

We consider various systems of fluid dynamics and present a generalized functional analytic framework for limit procedures as the incompressible limit of the Euler equations.

Global solutions for semilinear Klein-Gordon equation in FLRW spacetimes

Anahit Galstyan
University of Texas-Pan American, USA
agalstyan@utpa.edu

We consider waves, which obey the semilinear Klein-Gordon equation, propagating in the Friedmann-Lemaitre-Roberson-Walker spacetimes. The equations in the de Sitter and Einstein-de Sitter spacetimes are the important particular cases. We show the global in time existence n the energy class of solutions of the Cauchy problem.

On a class of iso-quasilinear symmetric hyperbolic systems

Svetlin Georgiev
Sorbonne University, France
svetlingeorgiev1@gmail.com

In this talk will be considered the Cauchy problem for a class quasilinear symmetric hyperbolic systems in the language of iso-mathematics. Will be given conditions for the isotopic element so that when the initial datum is a smooth with compact support we have local and global existence and uniqueness of the classical solutions. Also, it will be considered several cases for the isotopic element under which can be given some energy estimates of the solutions of the considered system.

A class of non-analytic functions for the global solvability of Kirchhoff equation

Fumihiko Hirosawa
Yamaguchi University, Japan
hirosawa@yamaguchi-u.ac.jp

We consider the global solvability for the Cauchy problem of a non-linear wave equation of Kirchhoff type with non-analytic data. The class of functions which we introduce in this paper is defined by a non-standard weight function in the Fourier space, and it is an extension of so-called Manfrin's class.

Semilinear wave equations with variable coefficients

Sandra Lucente
Bari University, Italy
sandra.lucente@uniba.it
M. D'Abbicco, M. Reissig

In the present talk we discuss the global existence theory for some wave equation of kind

$$u_{tt}(t,x) - a(t)\Delta u(t,x) + b(t)u_t(t,x) = \Gamma(t,x)f(u(t,x))$$

for $t > 0$, $x \in \mathbb{R}^n$ with positive a, b and $|f(u)| \simeq |u|^p$ with $p > 1$. This kind of results depends in on the size of the initial data, on the growth of the nonlinear term, on the growth or on the zero order of the time-coefficients, a, b, Γ After a brief review of the literature on this problem, we shall present in detail recent results on this subject.

Remarks on a dispersive equation in de Sitter spacetime

Makoto Nakamura
Yamagata University, Japan
nakamura@sci.kj.yamagata-u.ac.jp

Nonrelativistic limit of a nonlinear Klein-Gordon equation is considered in de Sitter spacetime. Local and global solutions of the Cauchy problem are shown in Sobolev spaces.

Linear structurally damped evolution models

Michael Reissig
Technical University Freiberg, Germany
reissig@math.tu-freiberg.de
Lu Xiaojun, Kainane Mohamed Mezadek, Takashi Narazaki

The goal of the lecture is an analysis of qualitative properties of solutions to the Cauchy problem for structurally damped evolution models $u_{tt} + (-\Delta)^\sigma u + b(t)(-\Delta)^\delta u_t = 0$, $u(0,x) = u_0(x)$, $u_t(0,x) = u_1(x)$. The main concern are estimates of energies of higher order not necessarily on the conjugate line. Here $L^1 - L^1$ estimates are of special interest. Moreover, we present Gevrey smoothing. By studying scale-invariant models we prove optimality of our results. The main tools are a refined WKB-analysis of hyperbolic-elliptic coupled type, theory of Fourier multipliers and modified Bessel functions. Finally, parameter dependent Cauchy problems are discussed.

Some well-posedness and analytic regularity results for weakly hyperbolic problems

Giovanni Taglialatela
University of Bari, Italy
giovanni.taglialatela@uniba.it

According to Jannelli (Comm. P.D.E. 1989) and D'Ancona Spagnolo (Boll. U.M.I. 1998), we say that an $N \times N$ matrix A admits a quasisymmetrizer if there exists a family $\{Q_\varepsilon\}_{0<\varepsilon\le 1}$ of positive definite matrices such that

$$(1) \qquad (Q_\varepsilon u, u) \ge {}^\exists C\, \varepsilon^{N-1}\, \|u\|^2$$

$$(2) \qquad \big|(R_\varepsilon u, v)\big| \le {}^\exists C\, \varepsilon\, (Q_\varepsilon u, u)^{1/2}\, (Q_\varepsilon v, v)^{1/2}$$

where $R_\varepsilon := Q_\varepsilon A - (Q_\varepsilon A)^*$. In this talk, we recall how to construct a quasisymmetrizer for some classes of matrices and we illustrate how to use it in order to derive apriori estimates for hyperbolic systems.

Constrained Hyperbolic Systems and Applications

Nicolae Tarfulea
Purdue University Calumet, USA
tarfulea@purduecal.edu

Important mathematical models involve hyperbolic systems of differential equations supplemented by constraint equations on infinite domains. In general, for the pure Cauchy problem one can prove that the constraints are preserved by the evolution. That is, the solution satisfies the constraints for all time whenever the initial data does (e.g., Maxwell's equations and Einstein's field equations in various hyperbolic formulations). Frequently, the numerical solutions to such evolution problems are computed on artificial space cutoffs because of the necessary boundedness of computational domains. Therefore, well-posed boundary conditions are needed at the artificial boundaries. Moreover, these boundary conditions have to be chosen in such a way that the numerical solution of the cutoff system approximates as best as possible the solution of the original problem on infinite domain, and this includes the preservation of constraints. In this talk, I will present a few ideas and techniques for finding constraint preserving boundary conditions for a large class of constrained hyperbolic systems. Then, I will talk about applications of the theoretical framework to certain models.

On the Cauchy prooblem for a class of hyperbolic operators whose coefficients depends only on the time variable

Seiichiro Wakabayashi
University of Tsukuba, Japan
wkbysh@math.tsukuba.ac.jp

We consider the Cauchy problem for a class of hyperbolic operators which satisfy the following conditions:

(1) The coefficients depends only on the time variable.

(2) The coefficients of the principal parts are real analytic.

(3) The multiplicities of the characteristic roots are at most two unless the operators are of third order.

Then we show that the Cauchy problem for the operators is C^∞ well-posed under Levi type conditions. Namely, for C^∞ well-posedness we impose some conditions on the subprincipal symbols, and, in addition, on so-called "sub-sub-principal symbols" if the operators are of third order.

Blow-up of solutions to semilinear wave equations with non-zero initial data

Kyouhei Wakasa
Hokkaido University, Japan
wakasa@math.sci.hokudai.ac.jp
Mohammad A. Rammaha, Hiroyuki Takamura, Hiroshi Uesaka

In this talk, we consider the initial value problem for semilinear wave equations with non-compactly supported data. With the initial data of zero initial position, the solution blows up for any power nonlinearity. This was first shown by Asakura (1986) under the assumption that the spatial decay is weak at infinity. On the other hand, Takamura & Uesaka & Wakasa (2010) have obtained the blow up result for non-zero initial position by making use of "time-derivative reduction". Our aim in this talk is to show the blow up result when both the initial position and the initial velocity do not identically vanish.

Wave equations with mass and dissipation

Jens Wirth
University of Stuttgart, Germany
jens.wirth@mathematik.uni-stuttgart.de
Wanderley Nunes do Nascimento

In this talk we consider wave equations with t-dependent lower order terms under optimal assumptions and study the interaction of them and their influence on large-time energy and L^p-L^q estimates. The approach is based on diagonalization for large frequencies and an asymptotic integration argument for small frequencies. Results are characterized in terms of a 'large-time principal symbol' associated to the lower order terms.

Integral transform approach to the initial-value problem for the evolution equations

Karen Yagdjian
University of Texas-Pan American, USA
yagdjian@utpa.edu

In this talk we describe some integral transform that allows to write solutions of the Cauchy problem for one partial differential equation via solution of another one. By application of this integral transform we give some new results on the electromagnetic wave equation in the de Sitter spacetime and on the weighted energy estimate for magnetic Klein-Gordon equation in the de Sitter spacetime.

Scattering for a quasilinear hyperbolic equation of Kirchhoff type with dissipation term integrable in time

Taeko Yamazaki
Tokyo University of Science, Japan
yamazaki_taeko@ma.noda.tus.ac.jp

We are concerned with a abstract quasilinear hyperbolic equations of Kirchhoff type with dissipation term integrable in time, and shows the existence of the wave operators and the scattering operator for small data. These operators are shown to be homeomorphic with respect to a suitable metric.

Structure preserving finite difference schemes for some thermoelastic systems

Shuji Yoshikawa
Ehime University, Japan
yoshikawa@ehime-u.ac.jp

We introduce new finite difference schemes for some thermoelastic systems. These schemes preserve physical structures: both the energy conservation law and the Clausius-Duhem inequality (the second principle of thermodynamics). Moreover, we show the existence of solution to the scheme and an error estimate between approximate solution and rigorous solution.

Special Session 91: Variational Methods for Evolution Equations

Ulisse Stefanelli, University of Vienna, Austria
Goro Akagi, Kobe University, Japan

Evolution models are of a paramount relevance in applications. As such they are constantly attracting the strongest attention. From the mathematical standpoint, the formulation of evolution systems in variational terms and the corresponding variational treatment have been a leading paradigm in modeling and analysis in the last decades. The field of variational methods for evolution equations has recently witnessed an even stronger development as a number of interesting new ideas and tools are getting to the stage. Among these, we shall mention global variational principles on trajectories, gradient flows in probability spaces, and evolutionary Gamma-convergence and relaxation techniques, just a to mention a few hot topics. The aim of the Special Session is that of bringing together the leading experts in the field in order to promote a stimulating enviroment for both the dissemination of the latest developments and the discussion of new ideas.

Error control for elastoplastic evolution problems

Soeren Bartels
University of Freiburg, Germany
bartels@mathematik.uni-freiburg.de

Quasistationary elastoplasticity provides a simple mathematical model problem for general rate-independent evolution problems. The numerical approximation is typically based on an implicit discretization in time and an appropriate finite element discretization in space. We discuss in this talk various a priori and a posteriori error estimates that control the temporal and spatial discretization errors. A new contribution is a quasi-optimal error estimate for the temporal discretization error under minimal regularity assumptions which follows from a reformulation of the model problem as a subdifferential flow.

Singular limit of Allen-Cahn equation with constraint and its Lagrange multiplier

Takeshi Fukao
Kyoto University of Education, Japan
fukao@kyokyo-u.ac.jp
M. H. Farshbaf-Shaker and N. Yamazaki

A problem of singular limit of Allen-Cahn equation with constraint is considered. This problem is modeled by Blowey and Elliott in the 1990s and Chen and Elliott studied the asymptotics, where the constraint of the problem is given by the subdifferential of the indicator function on the closed interval. Bronsard and Kohn (1991) introduced some method for the singular limit of prototype Allen-Cahn equation. The motivation of the research is to apply this method of Bronsard and Kohn to the Allen-Cahn equation with constraint under the Neumann boundary condition, same as Chen and Elliott (1994). This method has an advantage for the numerical analysis. For example, it replaces the large time behavior of the original problem with the sharp interface problem on a short time length. By the characterization of the subdifferential like the Lagrange multiplier, the same kind of result of Bronsard and Kohn (1991) is obtained not only on the Neumann but also the dynamic boundary condition.

A variational approach to curvature dependent interfacial accelaration

Elliott Ginder
Hokkaido University, Japan
eginder@gmail.com
Karel Svadlenka

We introduce a method for computing interfacial motions governed by curvature dependent acceleration. Our method is a thresholding algorithm of the BMO-type which, instead of utilizing a diffusion process, thresholds evolution by the wave equation to obtain the desired interfacial dynamics. We also develop the numerical method and present the results of its application, including an investigation of the volume preserving motions. Curvature dependent accelerations (oscillating interfaces) contrast mean curvature flow, and it is natural to search for a thresholding algorithm to approximate their dynamics. The target of the current study is to construct such a method. Moreover, from the point of view of applications, since interfaces in nature are often observed to oscillate (e.g., elastic membranes, soap bubbles, and liquid droplets), we remark this class of motions includes interesting physical phenomena.

Structure-preserving finite difference scheme for the Landau-Lifshitz equation

Tetsuya Ishiwata
Shibaura Institute of Technology, Japan
tisiwata@shibaura-it.ac.jp
K. Kumazaki

In this talk, we treat some typical sphere-valued partial differential equations, for examples, the Heisenberg equation and the Landau-Lifshitz equation. These equations describe the evolution of spin fields in continuum ferromagnetism and have the following properties: (1) length preserving, (2) energy conservation or dissipation property. We propose a finite difference scheme for these equations which inherits the above properties and show some theoretical results on the scheme. And we also demonstrate numerical examples in order to show the effectiveness of our scheme.

An irreversible diffusion equation and a phase field model of crack propagation

Masato Kimura
Kanazawa University, Japan
mkimura@se.kanazawa-u.ac.jp
Goro Akagi, Takeshi Takaishi

We study a nonlinear diffusion equation with irreversibility condition: $u_t = (\Delta u + f)_+$ in a bounded domain of $\mathbf{R}^n$ with Dirichlet or mixed boundary condition. Under some suitable conditions, we prove the unique existence of a strong solution and show its gradient structure, comparison principle, and long time behaviour of the solution. The construction of the strong solution is done through the backward Euler time discretization by using a regularity estimate of the solution of the classical obstacle problem. An application to a phase field model of crack propagation phenomena is also presented with some numerical examples.

Thermodynamically-consistent mesoscopic model of the ferro/paramagnetic transition

Martin Kruzik
Academy of Sciences of the Czech Republic, Czech Rep
kruzik@utia.cas.cz

A continuum evolutionary model for micromagnetics is presented that, beside the standard magnetic balance laws, includes thermo-magnetic coupling. To allow conceptually efficient computer implementation, inspired by relaxation method of static minimization problems, our model is mesoscopic in the sense that possible fine spatial oscillations of the magnetization are modeled by means of Young measures. Existence of weak solutions is proved by backward Euler time discretization. We will also present a few numerical examples. This is a joint work with Tomas Roubicek (Prague) and Barbora Benesova (Aachen).

Variational Solutions to Nonlinear Diffusion Equations with Singular Diffusivity

Gabriela Marinoschi
Romanian Academy, Romania
gmarino@acad.ro

We provide existence results for nonlinear diffusion equations with multivalued time-dependent nonlinearities derived from convex l.s.c. potentials, under minimal growth and coercivity conditions. Following a variational principle, we prove that a generalized solution of the nonlinear equation can be retrieved as a solution of an appropriate minimization problem for a convex functional involving the potential and its conjugate. In some cases, under further assumptions the null minimizer in the minimization problem is found to coincide with a weak solution to the nonlinear equation. Applications to various physical models (e.g., self-organized criticality) are discussed.

New variational principles of symmetric boundary value problems

Abbas Moameni
Assistant Professor, Canada
abbas.momeni@uleth.ca

We study the concept and the calculus of *Non-convex self-dual (Nc-SD)* Lagrangians and their derived vector fields which are associated to many partial differential equations and evolution systems. They indeed provide new representations and formulations for the superposition of convex functions and symmetric operators. They yield new variational resolutions for large class of hamiltonian partial differential equations with variety of linear and nonlinear boundary conditions including many of the standard ones. This approach seems to offer several useful advantages: It associates to a boundary value problem several potential functions which can often be used with relative ease compared to other methods such as the use of Euler-Lagrange functions. These potential functions are quite flexible, and can be adapted to easily deal with both nonlinear and homogeneous boundary value problems.

Semilinear and linear approximations to nonlinear diffusion problems

Hideki Murakawa
Kyushu University, Japan
murakawa@math.kyushu-u.ac.jp

This talk deals with nonlinear diffusion problems. The framework is so general as to include the Stefan problem, porous medium equation and several kinds of cross-diffusion systems. We show that the solutions of the nonlinear diffusion problems can be approximated by those of semilinear reaction-diffusion systems. This indicates that the mechanism of nonlinear diffusion might be captured by reaction-diffusion interaction. The reaction-diffusion systems include only simple reactions and linear diffusions. Resolving semilinear problems is typically easier than dealing with nonlinear problems. Therefore, our ideas are expected to reveal effective approaches to the study of nonlinear problems. Applying the similinear aproixmation to numerical analysis, we constructed and analyzed a linear numerical scheme for the nonlinear diffusion systems. The linear algorithm is a very easy to implement scheme. We derive optimal rates of convergence of the linear scheme by means of the reaction-diffusion system approximation.

Large deviations, gradient flows, and taking limits

Mark A. Peletier
TU Eindhoven, The Netherlands
m.a.peletier@tue.nl

It is now well understood that there is a strong connection between gradient flows on one hand and large-deviation principles on the other hand. In a sense, this connection takes the form of a single functional that characterizes both the large deviations and the gradient-flow behaviour.

In this talk, which is work with Giovanni Bonaschi and Giuseppe Savaré, I will show how this insight produces a unification of both structure and method. I will focus on a very simple stochastic system, and show how the large-deviation rate functional is related to a generalized gradient flow - with a parameter. By taking various limits in this parameter we recover both linear gradient-flow behaviour and rate-independent behaviour.

The unification lies in the fact that we can base our entire discussion on this one functional. It characterizes the structure and is also the main actor in the limit-taking, leading both to compactness and to characterization of the limit. This work shows how the connection between large deviations and gradient flows is not only philosophically interesting but also provides tools for analysis.

Variational resolution for some general classes of nonlinear evolutions

Arkady Poliakovsky
Ben Gurion University of the Negev, Be'er Sheva, Israel, Israel
poliakov@math.bgu.ac.il

We develop a variational technique for some wide classes of nonlinear evolutions. The novelty here is that we derive the main information directly from the corresponding Euler-Lagrange equations. In particular, we prove that not only the minimizer of the appropriate energy functional but also any critical point must be a solution of the corresponding evolutional system.

Comparison of rate independent evolution problems in BV

Vincenzo Recupero
Politecnico di Torino, Italy
vincenzo.recupero@polito.it

We consider some classes of rate independent evolution processes in the space of functions of bounded variation. We compare the related notions of solutions and we study the continuity properties of their solution operators. In the finite dimensional case we provide a geometrical characterization of the cases when these notions coincide. We briefly discuss the infinite dimensional case.

References

[1] V. Recupero, BV solutions of rate independent variational inequalities, Ann. Sc. Norm. Super. Pisa Cl. Sc. (5), 10 (2011), 269–315.

[2] V. Recupero: A continuity method for sweeping processes, J. Differential Equations, 251 (2011), 2125–2142.

[3] P. Krejčí, V. Recupero: Comparing BV solutions of rate independent processes, to appear in J. Convex Anal.

[4] P. Krejčí, V. Recupero: BV solutions of rate independent differential inclusions, preprint.

Singular perturbations of infinite-dimensional gradient flows

Riccarda Rossi
Università degli studi di Brescia, Italy
riccarda.rossi@unibs.it
Virginia Agostiniani, Giuseppe Savaré

We address the asymptotic behavior, as $\varepsilon \downarrow 0$, of the solutions to the (Cauhy problem for the) gradient flow equation

$$\varepsilon u'(t) + \mathrm{D}\mathcal{E}(t, u(t)) \ni 0 \qquad \text{in } \mathcal{H}, \quad t \in (0, T),$$

where $\mathcal{H}$ is a (separable) Hilbert space and $\mathcal{E} : (0, T) \times \mathcal{H} \to (-\infty, +\infty]$ is a time-dependent energy functional with $u \mapsto \mathcal{E}(t, u)$ possibly nonconvex. The main difficulty attached to the analysis as $\varepsilon \downarrow 0$ for a family of solutions $(u_\varepsilon)_\varepsilon$ resides in the lack of estimates for u'_ε. We develop a variational approach to this problem, based on the study of the limit of the energy identity

$$\frac{\varepsilon}{2}\int_s^t |u'_\varepsilon(r)|^2\,\mathrm{d}r + \frac{1}{2\varepsilon}\int_s^t |\mathrm{D}\mathcal{E}(r, u_\varepsilon(r))|^2\,\mathrm{d}r + \mathcal{E}(t, u_\varepsilon(t)) = \mathcal{E}(s, u_\varepsilon(s)) + \int_s^t \partial_t \mathcal{E}(r, u_\varepsilon(r)\,\mathrm{d}r$$

for all $0 \le s \le t \le T$, and on a fine analysis of the asymptotic properties of the quantity

$$\int_s^t |u'_\varepsilon(r)||\mathrm{D}\mathcal{E}(r, u_\varepsilon(r))|\,\mathrm{d}r.$$

In this context, the crucial hypothesis is that for every $t \in (0, T)$ the critical points of $\mathcal{E}(t, \cdot)$ are isolated, a condition of which we discuss the genericity.

Some results on a variational model for nematic shells-Part A

Antonio Segatti
University of Pavia, Italy
antonio.segatti@unipv.it
M. Snarski, M. Veneroni

In this talk and in the subsequent Part B, we discuss some features of a newly proposed variational model for nematic shells, which consist in a thin film of liquid crystals coating a particle. In particular, we consider a surface energy, recently proposed by G. Napoli and L. Vergori, which exhibits a combination of effects related both to intrinsic and extrinsic geometric quantities. The aim of this talk is to present some results about the existence of equilibrium configurations for substrates with genus 1. Moreover, we will discuss also the evolution of the energy along a gradient flow. Particular emphasis will be given in explaining how the topology and the geometry of the surface influence the existence and the behaviour of the equilibrium configurations.

A stress-driven local-solution approach to quasistatic brittle delamination

Marita Thomas
WIAS Berlin, Germany
Marita.Thomas@wias-berlin.de
Tomáš Roubíček, Christos Panagiotopoulos

This contribution addresses several models describing the rate-independent fracture of a material compound along a prescribed interface. This unidirectional process is modeled in the framework of Generalized Standard Materials with the aid of an internal delamination parameter. In the context of the energetic formulation it has become a well-established procedure to obtain solutions of a so-called brittle delamination model via an adhesive-contact approximation based on tools from Gamma-convergence of rate-independent systems. This means that the non-smooth, local brittle constraint, confining displacement jumps to the null set of the delamination parameter, is approximated by a smooth, non-local surface energy term. Based on this idea we present a procedure to find local solutions for the brittle model. The behavior of local and energetic solutions is compared in a one-dimensional example.

Some results on a variational model for nematic shells - Part B

Marco Veneroni
University of Pavia, Italy
marco.veneroni@unipv.it
Antonio Segatti and Michael Snarski

In this talk and in the preceding Part A, we discuss some features of a newly proposed variational model for nematic shells, which consist in a thin film of liquid crystals coating a particle. In particular, we consider a surface energy, recently proposed by G. Napoli and L. Vergori, which exhibits a combination of effects related both to intrinsic and extrinsic geometric quantities. In this talk we focus on the case of toroidal shells: We present finer results on the qualitative behaviour of the equilibrium solutions and we study the well-posedness of the gradient flow of the energy, which we use to produce numerical approximations of the minimizers. Particular emphasis will be given in explaining how the topology and the geometry of the surface influence the existence and the behaviour of the equilibrium configurations.

Special Session 92: Analysis and Computation of Nonlinear Systems of the Mixed Type

Zhaosheng Feng, University of Texas-Pan American, USA

Many problems of an applied nature reduce to finding specific solutions and properties of equations of mixed type; in particular, problems of plane transonic flow of a compressible medium, and problems in the theory of envelopes. This session will discuss various parts of the theory of mixed type differential equations with/without boundary conditions such as: classical dynamical equation of mixed type, the theory of regularity of solutions, quasi-regularity of solutions in the classical sense etc.

Some interpolating techniques and non-parametric regression methods for geophysical and financial data analysis

Kanadpriya Basu
The University of Texas at El Paso, USA
kbasu@utep.edu
M.C.Mariani

In this work we applied several interpolation techniques including locally weighted scatterplot smoothing techniques (Lowess/Loess) to geophysical and high frequency financial data. The application of these methods to the two different data sets demonstrate that the overall methods are accurate and efficient. In addition, these methods are highly localized and data dependent so that results are dependent on the data trends. Unlike the previous modeling implementations, this modeling technique deals with the spatial analysis of the data although for the high frequency financial data we used the modified version of the non-parametric regression method to find out the curve of best fit. Overall this modeling approach proves out to be very reliable and useful for handling spatial data and time dependent financial data.

Dynamics Classification of Boolean Networks

Fangyue Chen
Hangzhou Dianzi University, Peoples Rep of China
fychen@hdu.edu.cn
Gancang Zhao, Qinbin He, Weifeng Jin

An effective scheme for coding Boolean networks is proposed, by which one can uniquely designate a distinguished integer for any given n-node Boolean network. More importantly, by analyzing the characteristic polynomial of the linearized matrix of any given Boolean network, the connection between the dynamics of the network and the solution of a linear Diophantine equation can be established. Based on the calculation of the number of nonnegative integer solutions of the Diophantine equation, all n-node Boolean networks can be classified into several classes and the members in the same one have the same limit dynamical behaviors, i.e., same topological structures of invariant sets such as attractors and isles of Eden.

Boundary Value Problems for Fractional p-Laplacian Equation

Taiyong Chen
China University of Mining and Technology, Peoples Rep of China
taiyongchen@cumt.edu.cn
Wenbin Liu

We consider the existence of solutions for some nonresonance and resonance boundary value problems for the fractional p-Laplacian equation with the following form

$$D^{\beta}_{0^+}\phi_p(D^{\alpha}_{0^+}x(t)) = f(t, x(t), D^{\alpha}_{0^+}x(t)),$$

where $\alpha, \beta \in (0, 1]$, $\phi_p(s) = |s|^{p-2}s, p > 1$, and $D^{\alpha}_{0^+}$ is a Caputo fractional derivative. By using Schaefer's fixed point theorem and Ge-Mawhin's continuation theorem, some new existence results are obtained under the certain nonlinear growth conditions of the nonlinearity.

Dynamics of a Cournot investment game with bounded rationality

Zhanwen Ding
Faculty of Science, Jiangsu University, Peoples Rep of China
dgzw@ujs.edu.cn
Shumin Jiang

This talk is concerned with a dynamic system of investment game played by two firms with bounded rationality. It is assumed that each firm in any period makes a strategy for investment and uses local knowledge to make investment strategy according to the marginal profit observed in the previous period. Theoretic work is done on the existence of equilibrium solutions, the instability of the boundary equilibriums and the stability conditions of the interior equilibrium. Numerical simulations are used to provide experimented evidence for the complicated behaviors of the system evolution. It is observed that the equilibrium of the system can loose stability via flip bifurcation or Neimark-Sacher bifurcation and time-delayed feedback control can be used to stabilize the chaotic behaviors of the system.

Existence and asymptotic behavior of traveling waves in a modified vector-disease model

Zengji Du
Jiangsu Normal University, Peoples Rep of China
duzengji@163.com
Ying Xu

This talk deals with the existence and asymptotic behavior of traveling wave fronts in a modified vector-disease model. Vector-borne diseases have become major public health problems throughout the world. The spatial spread of newly introduced diseases is a subject of continuing interest to both theoreticians and empiricists. We first establish the existence of traveling wave solutions for the modified vector-disease model without delay, then the existence of traveling fronts for the model with a special local delay convolution kernel are obtained by employing geometric singular perturbation theory and the linear chain trick.

Iterative methods for PDE constrained optimization problems

Jinyan Fan
Shanghai Jiao Tong University, Peoples Rep of China
jyfan@sjtu.edu.cn
Jianyu Pan

Optimization problems with constraints, which require the solution of a partial differential equation, arise widely in many areas of the sciences and engineering. The solution of such PDE-constrained optimization problems is usually a major computational task. In this talk, we consider the iterative methods for such optimization problems with convection-diffusion equation constraints. We employ optimize-then-discretize approach to discretize the problem. The effectiveness of our proposed methods is illustrated by numerical examples.

Quadratic Reversible Lotka-Volterra Systems with Two Centers

Zhaosheng Feng
University of Texas-Pan American, USA
zsfeng@utpa.edu
Linping Peng

This talk is concerned with the bifurcation of limit cycles from a quadratic reversible Lotka-Volterra system with two centers of genus one under small quadratic perturbations. It shows that the cyclicities of each period annulus and two period annuli of the considered system under small quadratic perturbations are two, respectively. This not only gives at least partially a positive answer to an open conjecture, but also improves the corresponding results in the literature. In addition, we present the configurations of limit cycles of the perturbed system as $(2, 0)$, $(1, 1)$, $(1, 0)$, $(0, 2)$, $(0, 1)$ and $(0, 0)$, where (i, j) indicates that the perturbed system has i limit cycles surrounding the positive singularity while it has j limit cycles surrounding the negative one.

Analysis on the dynamics of a Cournot investment game with bounded rationality

Shumin Jiang
Faculty of Science, Jiangsu University, Peoples Rep of China
jsm@ujs.edu.cn
Zhanwen Ding

In this work, a dynamic system of investment game played by two firms with bounded rationality is proposed. It is assumed that each firm in any period makes a strategy for investment and uses local knowledge to make investment strategy according to the marginal profit observed in the previous period. Theoretic work is done on the existence of equilibrium solutions, the instability of the boundary equilibriums and the stability conditions of the interior equilibrium. Numerical simulations are used to provide experimented evidence for the complicated behaviors of the system evolution. It is observed that the equilibrium of the system can loose stability via flip bifurcation or Neimark-Sacher bifurcation and time-delayed feedback control can be used to stabilize the chaotic behaviors of the system.

Iterative methods for PDE constrained optimization problems

Jianyu Pan
East China Normal University, Peoples Rep of China
jypan@math.ecnu.edu.cn
Jinyan Fan

Optimization problems with constraints, which require the solution of a partial differential equation, arise widely in many areas of the sciences and engineering. The solution of such PDE-constrained optimization problems is usually a major computational task. In this talk, we consider the iterative methods for such optimization problems with convection-diffusion equation constraints. We employ optimize-then-discretize approach to discretize the problem. The effectiveness of our proposed methods is illustrated by numerical examples.

New technology diffusion model considering two process stages

Mei Sun
Jiangsu University, Peoples Rep of China
sunm@ujs.edu.cn
Changsheng Jia, Dun Han, Cuixia Gao, Dandan Li

In this paper, a diffusion model is proposed to describe the diffusion process of technology. This model is suitable for modeling diffusion processes of all those technologies that require great initial investments and public subsidies, such as renewable energy technology. We divided the whole individuals into three groups, and the diffusion process is divided into two stages, each stage has internal and external factors affect the communication process, and the external factors are different in each stage which is media and policy respectively. The adoption process is described by a system of ordinary differential equations. Numerical simulations are carried out, and the model analysis shows that both mass media and incentive policies will accelerate the adapter of technology reach its equilibrium with much faster rate, and increase the density of adopters. It further shows that any one of the mass media and in-

centive policies is blocked, it will hinder another diffusion stage. Therefore, it needs to control the external factors reasonably when we popularize the technology, and make the technical diffusion more saving and efficient. Finally, the renewable energy technology diffusion stage is empirical analyzed.

Distributed controllability and asymptotic stabilization of the two-component Camassa-Holm equation

Lixin Tian
Nanjing Normal University, Peoples Rep of China
tianlixin@njnu.edu.cn
Yufeng Zhang, Jiangbo Zhou, Xinghua Fan

This paper is concerned with the problems of the controllability and asymptotic stabilization of the two-component Camassa-Holm equation on the circle by means of a distributed control.

Bifurcation Controlling for a Predator-Prey Model with Stage-Structure

Xuedi Wang
Jiangsu University, Peoples Rep of China
wxd959@ujs.edu.cn
Honglin Yang,Yang Zhou

In this paper, a predator-prey model with stage-structure is investigated. Firstly, the existence of Hopf bifurcation at the positive equilibrium point and the stability of the limit cycle are discussed. Then concerning the sustainable development need of the ecosystem, a self-designed linear feedback controller is used to realize a new balance at desired point. Numerical simulations are provided to verify our results.

Traveling-wave solutions of the modified Buckley-Leverett equation

Ying Wang
University of Oklahoma, USA
wang@ou.edu

Buckley-Leverett (MBL) equation describes two-phase flow in porous media. The MBL equation differs from the classical Buckley-Leverett (BL) equation by including a balanced diffusive-dispersive combination. The dispersive term is a third order mixed derivatives term, which models the dynamic effects in the pressure difference between the two phases. The classical BL equation gives a monotone water saturation profile for any Riemann problem; on the contrast, when the dispersive parameter is large enough, the MBL equation delivers non-monotone water saturation profile for certain Riemann problems as suggested by the experimental observations. In this talk, we show that the solution of the finite interval $[0, L]$ boundary value problem converges to that of the half-line $[0, +\infty)$ boundary value problem expoentially fast for the MBL equation as $L \to +\infty$. In this talk, I will discuss both the analytical and numerical results for the MBL equation. (This is a joint work with Chiu-Yen Kao.)

Bounded traveling wave solutions and peakon-antipeakon interaction for the generalized two-component Hunter-Saxton equation

Liqin Yu
Jangsu University, Peoples Rep of China
yulq@ujs.edu.cn
Honglin Yang

In this paper, the bifurcation method of planar dynamic systems is used to investigate bounded traveling wave solutions for the generalized two–component Hunter–Saxton equation, and then the peakon–antipeakon interaction in this equation is discussed. Employing the phase portrait bifurcation of traveling wave system, bounded traveling wave solutions are obtained under different parameter conditions. Moreover, the wave dynamics for multiply peaked solitons and the peakon–antipeakon interaction are analyzed.

Special Session 93: Partial Differential Equations Arising From Biology and Physics

Shangbin Cui, Sun Yat-Sen University, P. R. China
Jianhua Wu, Shannxi Normal University, Peoples Rep of China
Bei Hu, University of Notre Dame, U. S. A.
Joachim Escher, Leibniz University of Hannover, Germany

This special session focuses on PDE problems arising from biology and physics, including (but not restricted to) reaction-diffusion equations in ecology and epidemic of diseases, chemotactic equations, age, stage and size-structured population equations, tumor models, Nonlinear Schrodinger related partial differential equations, KdV related partial differential equations, Navier-Stokes related partial differential equations, etc. Special interest is paid on global behavior of solutions, including global existence of time-dependent solutions, asymptotic behavior of solutions as time goes to infinity, existence and multiplicity of steady-state solutions and their asymptotic stability.

Linearized eigenvalues for a free boundary problem modeling two-phase tumor growth

Shangbin Cui
Sun Yat-Sen University, Peoples Rep of China
cuishb@mail.sysu.edu.cn

In this talk we consder a free boundary problem modeling the growth of a tumor containing two species of cells: proliferating cells and quiescent cells. By using Fourier expansion via a basis of spherical harmonic functions and some techniques for solving singular differential integral equations developed in some previous literature, we prove that there exists a null sequence $\{\gamma_k\}_{k=2}^{\infty}$ for the surface tension coefficient γ, with each of them being an eigenvalue of the linearized problem, i.e., if $\gamma = \gamma_k$ for some $k \geq 2$ then the linearized problem has extra nontrivial solutions besides the standard nontrivial solutions, and if $\gamma \neq \gamma_k$ for all $k \geq 2$ then the linearized problem does not have other nontrivial solutions than the standard nontrivial solutions.

Asymptotic Stability of Steady-States for A Three-Dimensional Network of Ferromagnetic Ellipsoidal Samples

Sharad Dwivedi
Indian Institute of Technology Madras, Chennai, India
sharadiitm@gmail.com
Shruti Dubey

In this article, we present a mathematical study of stability properties of steady states for a three-dimensional network of ferromagnetic particles of ellipsoidal shapes. The dynamics of magnetization inside the material is governed by the Landau-Lifschitz equation of micromagnetism which is non-linear and parabolic in nature. We prove that under certain condition on the shape of the samples and on the network geometry, the stability property for the relevant configurations can be achieved. More precisely, we establish a sufficient condition on the volume of the ellipsoidal samples and on the distance between the samples for particular configuration to be locally asymptotically stable using variational estimate technique. In this talk, we will also discuss about the controllability of these relevant configurations in which the control is the magnetic field generated by a dipole whose position and amplitude can be selected.

An efficient numerical solution of Hsu model involving size variation in soybean hydration

Seda Gülen
Ege University, Turkey
abhishek.dutta@kuleuven.be
Zehra Pinar, Abhishek Dutta, Turgut Ozis

An important step in the production of soybean-derived products such as soya milk is the soaking process. Even for fermentation of Soybean Meal (SBM), soaking is a necessary preconditioning step. Moisture hydration during soaking depends mainly on the time-temperature binomial. The amount of absorbed water increases as soaking time and temperature increase until it reaches a saturation limit. Both emprical and phenomenological models that represent hydration have been developed to predict the necessary time to obtain the desired moisture content at a certain temperature, representing the dynamic behavior of the soaking process. A distributed parameter phenomenological model knows as the Hsu model, which has effective diffusivity as its most important parameter, is used in this study. As the model involves nonlinear Partial Differential Equations (PDEs) which are inherently "stiff", it is usually difficult to solve them numerically. Hence the solution strategy often becomes a key factor in solving these PDEs. In this study, the Hsu model is solved using the Optimal Homotopy Analysis Method (OHAM) by considering both the variation in concentration and size. It is found that the approximations given by OHAM, with a couple of convergence-controlling parameters, converges fast in general. The variation in volume is solved using an appropriate numerical method. The analytical approximate solutions are then compared with the experimental data obtained from FTBE Department, Jadavpur University, Kolkata (India) and a good agreement between the experiment results and the simulation model was obtained.

Positive Steady States for the Generalized Gause-type Predator-prey System with Cross-diffusion

Yunfeng Jia
Shaanxi Normal University, Peoples Rep of China
jiayf@snnu.edu.cn

The paper deals with the generalized Gause-type predator-prey system with cross-diffusion and homogeneous Neumann boundary condition, where the cross-diffusion is included in such a way that the prey runs away

from the predator. We first give a priori estimate for the positive steady states. Then the non-existence of the non-constant positive steady states is given by employing the energy integral method. Finally, we investigate the existence of the non-constant positive steady states by using the Leray-Schauder degree theory.

Multiple coexistence solutions to the unstirred chemostat model with plasmid and toxin

Hua Nie
Shaanxi Normal University, Peoples Rep of China
niehua@snnu.edu.cn
Wu Jianhua

We investigate the effects of toxins on the multiple coexistence solutions to an unstirred chemostat model of competition between plasmid-bearing and plasmid-free organisms when the plasmid-bearing organism produces toxins. It turns out that coexistence solutions to this model are governed by two limiting systems. Based on the analysis of uniqueness and stability of positive solutions to two limiting systems, the exact multiplicity and stability of coexistence solutions to this model are established by means of the combination of the fixed-point index theory, bifurcation theory and perturbation theory.

Periodic traveling waves in a periodic Lotka-Volterra competition-diffusion system

Zhi-Cheng Wang
Lanzhou University, Peoples Rep of China
wangzhch@lzu.edu.cn
Xiongxiong Bao

This talk is concerned with the time periodic Lotka-Volterra competition-diffusion system. We show that the system admits a periodic traveling wave $(u(x,t),v(x,t)) = (U(x+ct,t),V(x+ct,t))$ connecting two periodic solutions $(p(t),0)$ and $(0,q(t))$ as $x \to \pm\infty$. By using a dynamical method, we also show that the time periodic traveling wave solution $(U(x+ct,t),V(x+ct,t))$ is asymptotically stable and unique modulo translation for front-like initial values.

Existence and Uniqueness of Global Solution For A Model Of Immune Cells Inhibiting Tumor Immune Evasion

Xuemei Wei
Guangdong University of Technology, Peoples Rep of China
wxm_gdut@163.com
Baili Cong

In this paper, we study a mathematical model of immune cells inhibiting tumor immune evasion. The model consists of stronglycoupled parabolic PDEs. Applying the L^p-theory, Shauder-estimate and Banach fixed point theorem, we prove that this problem has a unique local solution. Then by extension method, we prove that the local solution is global.

The effect of interaction ratio in a chemical reaction

Jianhua Wu
Shaanxi Normal University, Peoples Rep of China
jianhuaw@snnu.edu.cn
Cui Ma and Gaihui Guo

This paper is concerned with the effect of interaction ratio in a chemical reaction with zero-flux boundary condition. Treating the interaction ratio as a parameter, the existence of non-constant positive steady states is discussed by the bifurcation theory. Especially, the steady-state bifurcation from the double eigenvalue is derived. The Hopf bifurcation analysis to both ordinary differential equations and partial differential equations systems is investigated in detail. Examples of numerical simulations are shown to support and complement the analytical conclusions.

Stability and bifurcation of a free boundary problem modeling the growth of multi-layer tumors with Gibbs-Thomson relation

Fujun Zhou
South China University of Technology, Peoples Rep of China
fujunht@scut.edu.cn
Junde Wu

Of concern is a free boundary problem modeling the growth of multi-layer tumors. The new feature of this problem lies in that nutrient concentration at the tumor boundary is given by the Gibbs-Thomson Relation, where surface tension effect of the free boundary is included. We investigate well-posedness, stability and bifurcation analysis of this free boundary problem and give the interesting biological implication.

Special Session 94: Homogenization Based Numerical Methods

Emmanuel Frenod, LMBA - Universite de Bretagne-Sud, France

The aim of this special session is to take stock of numerical methods for solving partial differential equations that manage multi-scale phenomena, oscillations and heterogeneities by incorporating concepts coming from Homogenization Theory. At the present time, there are several research program exploiting this kind of ideas. They concern hyperbolic, elliptic and parabolic PDEs. The application fields are Environmental Sciences, Fluid Dynamics, Elasticity, Tokamak Physic, among others. One of the goals of this special session is to gather people working in different teams of different countries and having in mind different applications in order to exhibit and synthesize what is common between the different fields.

The localized reduced basis multi-scale method with online enrichment

Felix Schindler
Applied Mathematics Muenster, Germany
felix.schindler@wwu.de
Mario Ohlberger

We are interested in the efficient and reliable numerical approximation of elliptic parametric multi-scale problems which consist of finding $p_h(\mu) \in V_h$, such that $b_h(p_h(\mu), q_h; \varepsilon, \mu) = l(q_h)$ for all $q_h \in V_h$ for an a-priori given multi-scale parameter $\varepsilon > 0$ either in a multi-query context, where we want to solve for many parameters μ, or in a real-time context, where we have to solve for some parameters μ as fast as possible. Model reduction using reduced basis (RB) methods is a well established and reliable technique to reduce the computational complexity of parametric problems with respect to μ. In the context of multi-scale problems, however, standard RB methods may become computationally too expensive. The localized RB multi-scale (LRBMS) method was introduced in [F. Albrecht and B. Haasdonk and S. Kaulmann and M. Ohlberger, "The localized reduced basis multiscale method", *Proceedings of Algoritmy 2012, Conference on Scientific Computing, Vysoke Tatry, Podbanske, September 9-14, 2012*, 393–403, (2012).] as a combination of model reduction and numerical multi-scale methods to overcome the shortcomings of classical RB methods. We will present recent advances in the context of the LRBMS based on a recently derived efficient a-posteriori error estimator.

Multilevel Finite Element Methods with applications to Multiscale Fluid-Structure Interaction

Donald Brown
King Abdullah University of Science and Technology, Saudi Arabia
donaldbrowdr@gmail.com
Yalchin Efendiev, Viet Ha Hoang, Peter Popov, Anastasia Protasov.

Simulating Fluid-Structure Interaction (FSI) in porous media is complex due to complex geometries and many scales involved. Homogenization and numerical techniques need to be developed to simplify the models and accelerate computation time. Assuming a medium is initially periodic, a primary feature of FSI in a porous media is that is exhibits a break in the periodic structure in a slowly varying way. In this talk, we present some recent developments in the theory of homogenization to handle these slowly varying domains by constructing correctors and estimates. In addition, we present a multilevel scheme for computing effective properties in slowly varying domains. By building a hierarchy of maco-grids and a nested collection of finite element spaces, we are able to calculate the effective properties at a reduced computational cost. Current research includes the integration of these techniques with model reduction methods such as the Proper Orthogonal Decomposition.

Age-structured Models of Articular Cartilage Lesion Formation

Bruce Ayati
University of Iowa, USA
bruce-ayati@uiowa.edu
Xiayi Wang, Marc J. Brouilette, James A. Martin

We present age- and space-structured models of articular cartilage lesion formation. We discuss the model derivation and parameterization, the manner in which we include multiple biological "scales" (intracellular, intercellular and multicellular), the reason to use age structure over time delay terms, and the numerical methods used to solve the system.

Asymptotic-Preserving all speed numerical methods for compressible gas dynamics and multi-phase flows

Pierre Degond
Imperial College London, England
pdegond@imperial.ac.uk
F. Cordier, A. Kumbaro

In this talk, we will present Asymptotic-Preserving methods for compressible gas dynamics equations in the small Mach number limit and for multi-phase flows in the limit of a vanishing phase. They are based on a suitable semi-implicit discretization of the equations and on the use of so-called "polynomial schemes", i.e. schemes which allow the computation of an approximate Roe matrix without computing its eigenvector basis.

An exponential integrator for a highly oscillatory Vlasov-Poisson system

Sever Hirstoaga
Inria, France
hirstoaga@math.unistra.fr
E. Frénod, M. Lutz, E. Sonnendrücker

We aim at studying the dynamics of long and thin beams of charged particles focused by external electric fields. Under the axisymmetric assumption on the beam, we use the paraxial approximation of the full Vlasov-Maxwell system. We therefore have to solve some small parameter dependent Vlasov-Poisson system in a two dimensional phase space. In a Particle-In-Cell framework, we propose a time-stepping method which works uniformly when the parameter vanishes. Based on an exponential time differencing approach, the scheme is able to use large time steps with respect to the typical size of the fast oscillations of the solution. Both short and long time simulations will be presented.

Numerical homogenization methods for nonlinear monotone parabolic multiscale PDEs

Martin Huber
École Polytechnique Fédérale de Lausanne, Switzerland
martin.huber@epfl.ch
A. Abdulle and G. Vilmart

In this talk we introduce numerical homogenization methods for a class of nonlinear monotone parabolic multiscale problems with data oscillating rapidly in space. We first introduce a method combining the implicit Euler method in time with a finite element heterogeneous multiscale method in space (coupling macro and micro finite element methods) for which we present optimal fully discrete a priori error estimates in both time and space. The upscaling procedure of the method however relies on nonlinear elliptic cell problems at the microscopic level. As this is computationally costly for practical simulations, we briefly discuss a new linearized scheme that is much more efficient as it only involves linear micro problems.

A MsFEM approach à la Crouzeix-Raviart for problems on perforated domains

Frederic Legoll
Ecole des Ponts, France
legoll@lami.enpc.fr
C. Le Bris and A. Lozinski

The Multiscale Finite Element Method (MsFEM) is a Finite Element type approximation method for multiscale PDEs, where the basis functions used to generate the approximation space are precomputed and are specifically adapted to the problem at hand. Many ways to define these basis functions have been proposed. Here, we introduce and analyze a specific MsFEM variant, in the spirit of Crouzeix-Raviart elements, where the continuity of the solution accross the mesh edges is enforced only in a weak sense. Our motivation stems from our wish to address multiscale problems for which implementing flexible boundary conditions on mesh elements is of particular interest. A prototypical situation is that of perforated media, where the accuracy of the numerical solution is generically very sensitive to the choice of values of the basis functions on the boundaries of elements. The Crouzeix-Raviart type elements we construct then provide an advantageous flexibility, as shown by our numerical results.

References
[1] C. Le Bris, F. Legoll and A. Lozinski, MsFEM à la Crouzeix-Raviart for Highly Oscillatory Elliptic Problems, Chin. Ann. Math. Series B, 34B (2013), 1-26 and arXiv 1307.0876.

Long time behaviour of an exponential integrator for a Vlasov-Poisson system with strong magnetic field

Mathieu Lutz
University of Strasbourg, France
mathieu.lutz@unistra.fr
E. Frenod, S. A. Hirstoaga, E. Sonnendrucker

In the framework of Particle-In-Cell methods for a 4D phase space Vlasov-Poisson system depending on a small parameter, we propose a time-stepping method which works uniformly when the parameter vanishes. As an exponential integrator the scheme is able to use large time steps with respect to the typical size of the solution's fast oscillations. In addition, the method has accurate long time behaviour since it follows the slow motion of the center of the rapid periodic rotations.

Uniformly accurate numerical method for highly oscillatory kinetic equations

Florian Mehats
IRMAR, University of Rennes 1, France
florian.mehats@univ-rennes1.fr
Nicolas Crouseilles, Mohammed Lemou

This work is devoted to the numerical simulation of a Vlasov-Poisson model describing a charged particle beam under the action of a rapidly oscillating external field. We construct a uniformly accurate numerical scheme for this kinetic equation in the highly oscillatory limit. This scheme enables to simulate the problem without using any time step refinement technique. Our method is based on a "two scale" reformulation of the initial equation, with the introduction of an additional periodic variable.

The IDSA for core-collapse neutrino transport and fuzzy domain decomposition tech- niques

Jerome Michaud
University of Geneva, Switzerland
jerome.michaud@unige.ch

In this talk we present the Isotropic Diffusion Source Approximation (IDSA) of the $\mathcal{O}(v/c)$ relativistic Boltzmann equation in the comoving frame that has been developed for the computation of neutrino radiative transfer in core-collapse supernovae. This approximation is based on homogenization techniques used to derive the relevant approximations. They are then coupled together in volume.

This kind of heterogeneous domain decomposition methods lead us to develop the formalism of fuzzy domain decomposition methods based on Zadeh's fuzzy set theory. We present this formalism and some results about the approximation quality of this method.

Expansion of a singularly perturbed equation with a two-scale converging convection term

Alexandre Mouton
CNRS - Univ. Lille 1, France
alexandre.mouton@math.univ-lille1.fr

In many physical contexts, evolution convection equations may present some very large amplitude convective terms. As an example, in the context of magnetic confinement fusion, the distribution function which describes the plasma satisfies the Vlasvov equation in which some terms are of the same order as ϵ^{-1}, $\epsilon \ll 1$ being the characteristic gyrokinetic period of the particles around the magnetic lines. In this paper, we aim to present a model hierarchy for modeling the distribution function for any value of ϵ by using some two-scale convergence tools. Following Frénod & Sonnendrücker's recent work, we choose the framework of a singularly perturbed convection equation where the convective terms admits a high amplitude part which periodically oscillates in time with high frequency $\epsilon^{-1} \gg 1$. In this abstract framework, we derive an expansion with respect to the small parameter ϵ and we recursively identify each term of this expansion. Finally, we apply this new model hierarchy to the context of a linear Vlasov equation in the presence of a high amplitude external magnetic field.

Multi-dimensional coupled systems in river hydraulics

Antoine Rousseau
Inria, France
antoine.rousseau@inria.fr
Eric Blayo, Mehdi-Pierre Daou, Manel Tayachi

Fluid dynamics, either for industrial or geophysical flows, become more and more complex, multidisciplinary and requires numerical modeling tools which must be efficient and interoperable. For numerous applications (impact studies or flood prevention for instance), it may be necessary to design a modeling system, coupling processes representing the different parts of the physical system. These models may differ in several ways, related either to the physics and/or to the numerics. The present work addresses more specifically the problem of coupling models with different spatial dimensions. We show that this problem can be tackled quite efficiently using the so-called Schwarz method, borrowed from domain decomposition theory. This iterative algorithm is non intrusive, i.e. allows the use of existing numerical models with very few modifications. The method is illustrated on an academic testcase, namely a free surface flow in a bend shaped canal.

On the Shallow Water Equation with a long term dynamics of sand dunes

Diaraf Seck
Universite Cheikh Anta Diop, Senegal
diaraf.seck@ucad.edu.sn
M. A. M.T. Baldé, E. Frénod

In this work we intend to couple a long term dynamic equation of dunes of sand in the Works due to I. Faye, E. Frénod and D. Seck (Long term behavior of singularity perturbed parabolic degenerated equation. may 12 2010.) with a shallow water equation. This coupling models the movement of sand dunes over a long period subject to the effect of the movement of the sea near the coast. In the literature some authors have worked on the coupling of the Saint-Venant equation with equations dynamics of sediment and important results ensued. Our work is quite specific to the type of sediment (sand), we take into account the different results of these authors. Thus we study the evolution of sand dunes over long periods in the marine environment near the coast.

Special Session 95: Modeling the Spread and Control of Infectious Diseases

Tufail Malik, Khalifa University of Science, Technology and Research, Abu Dhabi, UAE
Abba Gumel, University of Manitoba, Canada

The special session aims to address some recent developments and current challenges in mathematical modeling and analysis of dynamical systems associated with the spread and control of infectious diseases in populations.

Evaluating the Cost-Effectiveness of Vaccination Programs

Elamin Elbasha
Merck Research Laboratories, USA
elamin_elbasha@merck.com
Erik J. Dasbach

Cost-effectiveness analysis (CEA) is used routinely to inform vaccination policy decisions. An incremental cost-effectiveness ratio (ICER) of a vaccination program aimed at controlling the spread of infections in a population represents a measure of how efficient that program may be in improving the health of a population. When estimating these ICERs, however, the traditional static approach used in CEA methods does not account for the potential herd-immunity/protection effects of vaccination. To account for these indirect effects of vaccination when estimating ICERs, we borrow from the field of mathematical infectious disease modeling to extend the traditional cost-effectiveness methods using a dynamic approach. We characterize the difference between the estimates of ICER of vaccination programs under the static and the dynamic approaches. We use a general SIRS (susceptible-infected-removed-susceptible) model featuring a vaccine with several properties. The special case of an SIR model with an all-or-nothing vaccine is studied analytically. We also numerically simulate the general model, trace the transient dynamics, and conduct a probabilistic sensitivity analysis. Measures of vaccine effects differ across the static and dynamic models. We find that the static model generates ICERs showing vaccination programs are less valuable (i.e., less efficient) in improving health than the dynamic model. This gap in efficiency is biggest for diseases with low basic reproduction numbers and grows with increases in vaccine cost, waning immunity, and decrement in quality of life from disease. Analytic results from this study suggest it may be possible to reduce this bias by adjusting ICERs generated by a static model to better approximate what the ICER may be for the dynamic model counterpart. We recommend, however, use of a dynamic model within a CEA when the vaccine is likely to have important effects on transmission.

Dynamics of Bovine Tuberculosis in African Buffalo Population

Salisu Garba
University of Pretoria, So Africa
Salisu.Garba@up.ac.za
A.S. Hassan, A.B. Gumel and J.M-S. Lubuma

Bovine tuberculosis (BTB), a contagious disease caused by mycobacterium bovis (M. bovis), affects a wide range of hosts, including domestic livestock, wildlife and humans. The socio-economic impact of the disease is estimated at US $3 billion per year globally (50 million number of cases and over 100,000 BTB-related mortality are recorded annually). In this talk, a deterministic model for the transmission dynamics of BTB in African Buffalo population will be discussed. Theoretical and numerical simulation results, using data from South Africa's Kruger National Park, will be presented.

The Influence of Sexual Orientation on the Spread of HIV: A Five Population Mathematical Model

Katharine Gurski
Howard University, USA
kgurski@howard.edu
Kathleen Hoffman and Evelyn Thomas

We use dynamical systems techniques to study the effect bisexual males have on the spread of HIV to the heterosexual female population, within a system consisting of sexually active homosexual and bisexual males, both self-identified and non-identified, in addition to heterosexual females and males. We present an alternative method to describing the reproductive number and endemic equilibria by deconstructing the larger system of five populations into smaller subsystems and capturing the interactions between the smaller systems as external forces using an approximate model. Our approach is to study decoupled systems of the sexually active populations and determine the influence of other populations on the basic reproductive number and endemic equilibria of these subpopulations. We utilize a sliding scale to manipulate the percentage of bisexuals who are self-identified and non-identified. This tool, with added emphasis on prevention and control, allows us to measure the impact the non-identifying bisexual male population has on the spread of the disease to heterosexual females. We present analytics and numerics along with a statistical study on the effect of the parameters.

Mathematics of HPV Transmission Dynamics and Associated Dysplasia

Ali Javame
Manitoba University, Canada
javamea@cc.umanitoba.ca

Human papillomavirus (HPV) is a major sexually-transmitted disease that causes various cancers in females and males (including cervical cancer). Each year, 500,000 women develop cervical cancer (and about 50 percent of these women succumb to the cancer). The talk is based on the design and rigorous qualitative analysis of a new deterministic model for the transmission dynamics of HPV (and related dysplasia and cancers in females and males) in a community, in the presence of Pap cytology screening of pre-cancerous lesions for females.

A Model for the Transmission Dynamics and Control of Hep C

Adnan Khan
Lahore University of Management Sciences, Pakistan
adnan.khan@lums.edu.pk

Hepatitis C virus (HCV) is a common cause of liver diseases worldwide and a major public health problem. We present a mathematical model for the transmission dynamics of Hepatitis C. We consider two modes of transmission of the virus, these are, unsafe blood transfusions and intravenous drug use. The susceptible population is divided into two distinct compartments, the intravenous drug users and individuals undergoing unsafe blood transfusions. Individuals belonging to each compartment may develop acute and then possibly chronic infections. Chronically infected individuals may be quarantined. The analysis indicates that the eradication and persistence of the disease is completely determined by the magnitude of basic reproduction number R_c. It is shown that for the basic reproduction number $R_c 1$, an endemic equilibrium exists and the disease is uniformly persistent. When the infected population persists, we present a time dependent optimal quarantine strategy to minimize it. We compare numerical results for the optimal control against the constant controls and discuss their efficiency.

Timing and Effectiveness of Disease Intervention Mechanisms for Vector-Borne Diseases in the Presence of Human Population Movement in Africa

Sehjeong Kim
United Arab Emirates University, United Arab Emirates
sehjung.kim@uaeu.ac.ae
Dong Eui Chang, Abdessamad Tridane

In Africa, the spread of vector-borne diseases has been accelerated via human population movements due to poverty and geopolitical conflicts. Hence control of the vector-borne diseases has been strongly demanded. As one of traditional disease intervention mechanisms in the presence of human population movement, border screening has been used in many African countries. However, its effectiveness is still under debate. It is due to focusing only on the functionality of border screening without considering the timing to use. In this paper, we attempt to qualitatively answer whether the use of border screening is a desirable action in an epidemic. A novel mathematical model with border screening and a transition probability of status change during travelling is developed. In fact, these two features have not been explicitly considered in many previous studies. A condition to check timing of the border screening is established in terms of a lower bound of the basic reproduction number. If the lower bound is greater than one, which indicates a pandemic, then the border screening may not be effective and the disease persists. In this case, a local level control strategy should be conducted. Moreover, vector population control is a crucial factor to eradicate the disease.

Rich bifurcation structure of an epidemic model with vaccination and transportation between regions

Diana Knipl
MTA-SZTE Analysis and Stochastics Research Group, University of Szeged, Hungary
knipl@math.u-szeged.hu
Pawel Pilarczyk, Gergely Rost

Classical disease transmission models typically have a unique locally asymptotically stable equilibrium whenever a certain threshold, known as the basic reproduction number ($\mathcal{R}_0$) is less than unity. However the situation when the model undergoes a backward bifurcation at $\mathcal{R}_0 = 1$ is different, since in this case for values of $\mathcal{R}_0$ less than one the stable disease free fixed point coexists with one stable positive and one unstable positive equilibrium. We consider an SIVS (susceptible – infected – vaccinated – susceptible) model to describe the spread of an infectious disease in a population of individuals in two regions. Assuming the disease dynamics exhibits the phenomenon of backward bifurcation in both sub-populations, we incorporate the possibility of travel between the regions and give condition for the existence of backward bifurcation in the full system. The mathematical analysis reveals an unusually rich dynamical behavior: from triple transcritical and double saddle-node bifurcation points eight possible endemic equilibria bifurcate besides the disease free steady state. We investigate the global dynamics and the stability of fixed points with analytical tools and rigorous numerical computations.

Optimal quarantine and vaccination strategies in the control of the Middle East Respiratory Syndrome coronavirus

Tufail Malik
Khalifa University of Science, Technology and Research, United Arab Emirates
tufail.malik@kustar.ac.ae
Aliya A. Alsaleh, Mohammad A. Safi, Abba B. Gumel

The talk will address the transmission dynamics of Middle East Respiratory Syndrome (MERS) in two human populations with immigration. A deterministic model will be used to assess the combined impact of quarantine and a vaccine on the disease dynamics. Optimal control for MERS via the Pontryagin's Maximum Principle will be discussed. Rigorous qualitative analysis will be presented. Simulation results, using a realistic set of parameter values, will also be discussed.

Mathematical Modeling of the HIV/AIDS Epidemic in Cuba

Antonio Mastroberardino
Penn State Erie, USA
axm62@psu.edu
Yuanji Cheng, Ahmed Abdelrazec, Hao Liu

In this talk, we present a nonlinear mathematical model for the transmission dynamics of HIV/Aids in Cuba. Due to Cuba's highly successful national prevention program, we assume that the only mode of transmission is through contact with people who do not know that they are HIV positive. We find the equilibria of the governing nonlinear system, perform a linear stability analysis, and then determine the threshold for global stability. We conclude with an application of optimal control as a demonstration of the effectiveness of the Cuban prevention program.

Bifurcations in HCV Transmission Dynamics

Fereshteh Nazari
University of Manitoba, Canada
nazarif@myumanitoba.ca
Abba Gumel

Hepatitis C virus (HCV) is a major blood-born disease that continues to pose significant public health and socioeconomic burden globally. The talk is focused on modeling the transmission dynamics of HCV within an IDU (injecting drug user) population. Results for the existence and stability of equilibria, as well as bifurcation types, will be reported.

Age-Based Stochastic Model of HPV

Omayra Ortega
Arizona State University, USA
omayra.ortega@asu.edu

Approximately 630 million people are infected with Human Papillomavirus (HPV) worldwide with six million new cases every year [4]. With over 200 different types identified, HPV is the most common sexually transmitted infection contributing to multiple adverse health outcomes including cervical cancer. In our study, we develop a stochastic model which describes the transmission of HPV infection in both men and women. We separate the women into the age groups of 12-19, 20-29, 30-65, 65+ and observe the rates of progression to disease, pre-cancerous cells, cancer and death in women. Our studies show that the 12-19 age group progresses to infection with HPV at the lowest rate, while the number of woman presenting with Low-Grade Squamous Intraepithelial Lesion (LSIL) peaks during the ages of 30-65 and women progressing to oncogenic HPV types peak at the age of 65+.

Estimation of transmission rates of dengue fever in Puerto Rico for the years 2010-2012 via a mathematical epidemiological model with seasonality

Karen Ríos-Soto
University of Puerto Rico Mayaguez, USA
karen.rios3@upr.edu
Gerardo López

Dengue fever is an infectious disease cause by one of four serotypes indentify by DEN-1, DEN-2, DEN-3 and DEN-4, which are antigenically related. The virus is transmitted to humans by a bite of the female mosquito Aedes Aegypti (main vector of the disease). In the tropical island of Puerto Rico dengue is an endemic infectious disease thus, the aim of this work is to study a mathematical epidemiological model of non-linear ordinary differential equations with seasonality to estimate the transmission rate and degree of seasonality in the island. Monthly incidence data of confirmed dengue cases are used for the period of years between 2010 and 2012. The parameter estimation is performed using inverse problems theory for differential equations and the available data. Specifically, we use the minimum square method and a statistical model to measure the error in the estimation via residual plots. Finally, the type of variance of the errors (constant or non-constant) is analyzed and the results are subject to interpretation.

Spatio-temporal modeling of health information in social media and application in control of influenza

Haiyan Wang
Arizona State University, USA
haiyan.wang@asu.edu

Social media such as Twitter has gained tremendous popularity in information dissemination. Modeling information spreading in online social networks has become a challenging problem. Most of dynamical models arising from social media only involve ordinary differential equations which describe static or collective social processes over time. Building on intuitive friendship hops in social media, we recently propose to use partial differential equation models to describe the temporal and spatial characteristics of information diffusion in online social networks. In this talk, I will examine a diffusive logistic equation based on cyber-distance and geocoded data in Twitter to model diffusion of health related information in Twitter. We demonstrate that it can be used to real-timely monitor spread of flu related information in social media, and help control spread of influenza.

Special Session 96: Geometric Variational Problems With Associated Stability Estimates

Mouhamed Moustapha Fall, African Institute for Mathematical Scineces in Senegal, Senegal

Geometric variational problems arise in various applications e.g. in physics and engineering. Classical examples are the shape of solid bodies minimizing air resistance, area minimizing surfaces with prescribed boundary properties or the shape of membranes giving rise to the lowest fundamental mode. These and other geometric variational problems can be formulated as optimization problems for domain dependent functionals on given classes of subdomains of Riemannian manifolds. Isochoric (volume preserving) or isoperimetric (perimeter preserving) constraints play a fundamental role, and they used to define the relevant classes of domains among which the optimization is performed. Stability estimates (estimating the deviation from the optimal set) have shown several applications, for instance, in the study of variational problems in Riemannian manifolds. In this session, we will also consider related PDEs arising from the Euler-Lagrange equations from the above shape optimization problems (e.g. overdetermined problems, prescibed mean curvature problems) as well as those which share similar mathematical features (e.g. singularly perturbed problems, Allen-Cahn equations).

Stable hypersurfaces in Euclidean convex cones with homogeneous densities

Antonio Cañete
Universidad de Sevilla, Spain
antonioc@us.es

Given a manifold M, we can consider a density, which is just a positive function weighting volume and perimeter functionals on M. In this talk we shall focus on convex Euclidean solid cones endowed with homogeneous densities (these densities have a nice behavior with dilations), classifying the compact stable hypersurfaces in this setting. This is part of a joint work with César Rosales (Universidad de Granada).

Concentration phenomena for a nonlocal Schroedinger equation

Serena Dipierro
University of Edinburgh, Scotland
serydipierro@yahoo.it

We consider a nonlocal Schroedinger equation in a smooth domain with Dirichlet datum and we construct solutions that concentrate at interior points of the domain. The proof uses a scaling blow up of the space variable and an appropriate Lyapunov-Schmidt bifurcation argument. The leading of the reduced energy functional in this case is not exponential, but polynomial with respect to the distance from the boundary. The exponent of such asymptotics is also quite unexpected and the competition among different scalings causes of course some technical difficulties, that can be overcome by a careful analysis of the nonlocal Robin function. The results presented were obtained in collaboration with J. Dávila, M. del Pino and E. Valdinoci.

Domain optimizers for the first non-zero Neuman eigenvalue problem on the cylinder

Mouhamed Moustapha Fall
African Institute for Mathematical Scineces in Senegal, Senegal
mouhamed.m.fall@gmail.com
Tobias Weth

In this talk, we will discuss the characterization of domains maximizing the first non-zero Neuman eivgenvalue on a cylinder. We will also present our study of overdetermined problem which arises from the Hadamard formula for this eigenvalue.

Stable area-stationary surfaces in the sub-Riemannian Sol manifold

Matteo Galli
University of Bologna, Italy
galli@ugr.es

In this talk we will study the classification of area-stationary and stable C^2 regular surfaces in the space of the rigid motions of the Minkowski plane equipped with its sub-Riemannian structure. We construct examples of area-stationary surfaces that are not foliated by sub-Riemannian geodesics. We also prove that there exist an infinite number of C^2 area-stationary surfaces with a singular curve. Finally we show the stability of C^2 area-stationary surfaces foliated by sub-Riemannian geodesics.

Solutions for singularly perturbed Klein Gordon Maxwell Proca systems on a Riemannian manifold

Marco Ghimenti
University of Pisa, Italy
marco.ghimenti@dma.unipi.it
Monica Clapp, Anna Maria Micheletti, Angela Pistoia

Given a 3 or a 4 dimensional Riemannian manifold $(M; g)$, we investigate the existence of positive solutions of a singularly perturbed Klein-Gordon-Maxwell system in M. When the nonlinearity is subcritical, both the topology

and the geometry of the Riemannian manifold M influence the number of solution. In the case of critical or supercritical nonlinearity, if the manifold M exhibits some special type of symmetry, it is possible to find solutions of the KGM equation concentrating on a submanifold of M.

Comparison results for capacity

Ana Hurtado
University of Granada, Spain
ahurtado@ugr.es
Vicente Palmer and Manuel Ritoré

We obtain in this paper bounds for the capacity of a compact set K with smooth boundary. If K is contained in an $(n+1)$-dimensional Cartan-Hadamard manifold and the principal curvatures of ∂K are larger than or equal to $H_0 > 0$, then $Cap(K) \geqslant (n-1)\, H_0\, vol(\partial K)$. When K is contained in an $(n+1)$-dimensional manifold with non-negative Ricci curvature and the mean curvature of ∂K is smaller than or equal to H_0, we prove the inequality $Cap(K) \leqslant (n-1)\, H_0\, vol(\partial K)$. In both cases we are able to characterize the equality case. Finally, if K is a convex set in Euclidean space $\mathbb{R}^{n+1}$ which admits a supporting sphere of radius H_0^{-1} at any boundary point, then we prove $Cap(K) \geqslant (n-1)\, H_0\, vol(\partial K)$ and that equality holds for the round sphere of radius H_0^{-1}.

Overdetermined problems with fractional Laplacian

Sven Jarohs
Goethe-University Frankfurt, Germany
jarohs@math.uni-frankfurt.de
Mouhamed Moustapha Fall

We characterize bounded open sets Ω with C^2 boundary (not necessarily connected) for which the following overdetermined problem

$$(-\Delta)^s u = f(u) \text{ in } \Omega$$
$$u = 0 \text{ in } \mathbb{R}^N \setminus \Omega$$
$$(\partial_\eta)_s u = \text{Const. on } \partial\Omega$$

has a nonnegative and nontrivial solution, where η is the outer unit normal vectorfield along $\partial\Omega$ and for $x_0 \in \partial\Omega$

$$(\partial_\eta)_s\, u(x_0) = -\lim_{t\to 0} \frac{u(x_0 - t\eta(x_0))}{t^s}.$$

Under mild assumptions on f, we prove that Ω must be a ball. In the special case $f \equiv 1$, we obtain an extension of Serrin's result in 1971. The main ingredients in our proof are maximum principles and the method of moving planes.

Constant mean surfaces with boundary

Paul Laurain
Institut Mathématique de Jussieu, Paris 7, France
laurainp@math.jussieu.fr
O. Druet

Surfaces with constant mean curvature arise in one of the fundamental problem in differential geometry: the isoperimetric problem. When you try to minimize the area of a surface which enclose a given volume, your solution must have constant mean curvature. When considering surface with boundary it becomes the classical soap bubble problem that every one has already tested. But mathematically, less is known on the space of solutions. For instance, from the point of view of uniqueness, if the boundary is a circle we even don't know if the only two solutions are the spherical caps. However, from the point of view of existence, since the 80's and the work of Brezis and Coron, we know that for a given curve there exist at least two solutions up to assume that the mean curvature is small enough. In this talk I will describe more precisely the space of solutions, we will notably focus on their asymptotic behaviour when the mean curvature goes to zero. Under suitable condition we are able to avoid bubbling phenomena. The main tool is asymptotic analysis for the CMC equation in conformal coordinates which is know to be of critical growth.

Isoperimetric properties of complete non-compact Riemannian manifolds

Stefano Nardulli
Dep. de Matematica-IM-UFRJ, Brazil
nardulli@im.ufrj.br

In this talk we present some recents results obtained by the author among others about the isoperimetric problem in the setting of complete non-compact Riemannian manifolds.

Existence of solutions to supercritical problems on manifolds

Angela Pisotia
La Sapienza, Università di Roma, Italy
pistoia@dmmm.uniroma1.it

We study the problem

$$-\Delta_g u + h(x)u = u^p,\ u > 0,\ in\ (M, g)$$

where (M, g) is a $n-$dimensional Riemannian manifold without boundary and $p > 1$. We present some recent results about existence of solutions concentrating along $k-$dimensional minimal submanifolds of M for any integer k between 0 and $n-1$, provided the exponent p is close the $(k+1) - st$ critical exponent.

Isoperimetric inequalities for extremal Sobolev functions

Jesse Ratzkin
University of Cape Town, So Africa
jesse.ratzkin@uct.ac.za
Tom Carroll

Let $n \geq 2$, let $\Omega \subset \mathbf{R}^n$ be a bounded domain with a smooth boundary, and let $1 \leq p \leq \frac{2n}{n-2}$. The Sobolev embedding theorem and Rellich compactness together imply that

$$\mathcal{C}_p(\Omega) = \inf\left\{ \frac{\int_\Omega |\nabla u|^2 d\mu}{\left(\int_\Omega |u|^p d\mu\right)^{2/p}} : u \in W_0^{1,2}(\Omega) \right\}$$

is a finite, positive number, and it is realized by a nontrivial function ϕ. I will discuss some new results linking the number $\mathcal{C}_p(\Omega)$ and its associated extremal function ϕ to other, more classical quantities, such as volume, perimeter, principal frequency, and torsional rigidity. The inequalities we can prove generalize some theorems of Polya, Szego, Payne, Rayner, Kohler-Jobin, Chiti, and others. Moreover, we can classify which realize equality in our bounds. This is all joint work with Tom Carroll of the University of Cork, in Ireland.

Large isoperimetric regions in the product of a compact manifold with Euclidean space

Manuel Ritoré
Universidad de Granada, Spain
ritore@ugr.es
Efstratios Vernadakis

Given a compact Riemannian manifold M without boundary, we show that large isoperimetric regions in $M \times \mathbb{R}^k$ are tubular neighborhoods of $M \times \{x\}$, with $x \in \mathbb{R}^k$.

Sharp isoperimetric inequalities with densities via the ABP method

Xavier Ros-Oton
Universitat Politecnica de Catalunya, Spain
xavi.ros@gmail.com
Xavier Cabre, Joaquim Serra

We prove some old and new isoperimetric inequalities with the best constant using the ABP method applied to an appropriate linear Neumann problem. More precisely, we obtain a new family of sharp isoperimetric inequalities with weights (also called densities) in open convex cones of $\mathbb{R}^n$. Our result applies to all nonnegative homogeneous weights satisfying a concavity condition in the cone. Remarkably, Euclidean balls centered at the origin (intersected with the cone) minimize the weighted isoperimetric quotient, even if all our weights are nonradial –except for the constant ones. We also study the anisotropic isoperimetric problem in convex cones for the same class of weights. We prove that the Wulff shape (intersected with the cone) minimizes the anisotropic weighted perimeter under the weighted volume constraint. As a particular case of our results, we give new proofs of two classical results: the Wulff inequality and the isoperimetric inequality in convex cones of Lions and Pacella.

Isoperimetric and stable regions for some log-concave perturbations of the Gaussian measure

César Rosales
Universidad de Granada, Spain
crosales@ugr.es

In this talk we describe global minimizers and second order minima of the weighted perimeter for fixed weighted volume inside an open half-space or slab Ω in $\mathbb{R}^{n+1}$ endowed with a perturbation of the Gaussian density of the form $f(p) := \exp(\omega(p) - c|p|^2)$, where $c > 0$ and ω is a concave function only depending on the signed distance from the linear hyperplane parallel to $\partial\Omega$.

Geometrical Properties in Shape optimization

Diaraf Seck
Universite Cheikh Anta Diop, Senegal
diaraf.seck@ucad.edu.sn
M. Ould Badahi, I. Ly, B. Mb. Ndiaye

We present some works about the characterization of the shape of a domain or the reconstruction of a domain in $\mathbb{R}^n$ by considering for instance the 2-Laplace operator or p-Laplace operator in some problems : the first eigenvalue problem, the Bernoulli's free boundary problem. Some symmetry properties will be presented by the Serrin's approach.

Continuous solutions of a balance law and sub-Riemannian geometry

Francesco Serra Cassano
University of Trento, Italy
cassano@science.unitn.it
Francesco Bigolin, Laura Caravenna

We will discuss the equivalence about different notions of continuous weak solutions to the equation $\partial_t u + \partial_x[u^2/2] = g$, where g is a bounded measurable function. Moreover we will point out some relationships between this problem and the intrinsic regular graphs in the framework of the simplest sub-Riemannian structure, namely the Heisenberg group .

Extremal domains for the first eigenvalue in a general Riemannian manifold

Pieralberto Sicbaldi
Aix-Marseille Universite, France
pieralberto.sicbaldi@univ-amu.fr

In this talk we will give some examples of domains that are critical points for the first eigenvalue of the Laplacian in a general compact Riemannian manifold, without any hypothesis on the curvature of the manifold. This is a joint work with E. Delay.

Liouville-type theorems and 1-dimensional symmetry for solutions of competitive systems with several components

Nicola Soave
Justus Liebig University of Giessen, Italy
nicola.soave@gmail.com
Susanna Terracini

In this talk we consider solutions of the competitive elliptic system

$$\begin{cases} -\Delta u_i = -\sum_{j\neq i} u_i u_j^2 & \text{in } \mathbb{R}^N \\ u_i > 0 & \text{in } \mathbb{R}^N \end{cases} \qquad i = 1, \dots, k,$$

which appears in the analysis of phase separation phenomena for Bose-Einstein condensates with multiple states. The prototype of our main results is the following: for every $d > 0$ there exists $h = h(d, N) \in \mathbb{N}$ such that if $(u_1, \dots, u_k)$ is a solution of the considered system and

$$u_1(x) + \cdots + u_k(x) \leq C(1 + |x|^d) \qquad \text{for every } x \in \mathbb{R}^N,$$

then $k \leq h(d, N)$. This means that a bound on the growth of a positive solution imposes a bound on the number of components k of the solution itself. If $N = 2$, the expression of $h(d, N)$ is explicit and optimal, while in higher dimension it can be characterized in terms of an optimal partition problem. We discuss the sharpness of our results and, as a further step, for every $N \geq 2$ we can prove the 1-dimensional symmetry of the solutions satisfying suitable assumptions, extending known results which are available for $k = 2$. The proofs rest upon a blow-down analysis and on some monotonicity formulae.

A Geometric Uncertainty Principle and Applications

Stefan Steinerberger
Yale University, USA
stefan.steinerberger@gmail.com

Trivially, the Euclidean plane cannot be decomposed into disks of equal radii. Thus any partition requires either the sets to be not all disks (i.e. a hexagonal partition) or that they be of different size (Apollonian packings). We quantify the phenomenon and use it to improve Pleijel's bound on the number of nodal domains of a Laplacian eigenfunction (a numerically more explicit improvement has been given by Bourgain).

Large mass boundary condensation patterns in the stationary Keller-Segel system

Giusi Vaira
Sapienza, University of Rome, Italy
giusi.vaira@gmail.com
Manuel del Pino, Angela Pisotia

We present a recent result for the stationary Keller-Segel problem in a bounded domain of the plane. We establish the existence of a solution of the problem which exhibits a sharp boundary layer along the entire boundary of the domain.

Nonlocal minimal surfaces

Enrico Valdinoci
Weierstrass Institute, Berlin, Germany
enrico@math.utexas.edu

We present some rigidity and regularity results for the minimizers of a nonlocal perimeter functional. In particular, these minimizers are smooth in the plane and when the fractional parameter is sufficiently close to the integer value, and a Bernstein-type theorem holds true. The results were obtained in joint works with B. Barrios, L. Caffarelli, A. Figalli and O. Savin.

Special Session 97: Analysis and Control of Nonlinear Partial Differential Equation Evolution Systems

George Avalos, University of Nebraska-Lincoln, USA
Lorena Bociu, NC State University, USA
Francesca Bucci, Università degli Studi di Firenze, Italy

It is intended that this Special Session will provide a platform from which renowned specialists in Partial Differential Equations (PDE) and/or Mathematical Control Theory will present their latest research on nonlinear and evolutionary PDE. We anticipate that our speakers will have expertise in a wide-ranging array of topics, possibly including: (i) qualitative and quantitative properties enjoyed by solutions to nonlinear partial differential equations of hyperbolic, parabolic, or of mixed type. Such properties might include global existence and uniqueness; availability of so-called "hidden", or extra boundary trace regularity; for local well-posedness of solutions, with associated finite time blow-up. In addition, the topic of longtime behavior of solutions for given dissipative PDE could conceivably be broached by one or other of our Speakers, including the existence of attracting sets of finite dimension. (ii) Shape and sensitivity analysis of PDE, particularly with a view toward treating moving boundary phenomena. (iii) Optimization and control problems for nonlinear PDE processes, including the longstanding issue of globally controlling nonlinear hyperbolic PDE. Feedback control schemes to stabilize nonlinear and unstable PDE might also be under discussion in our Special Session, particularly if such schemes are amenable to numerical implementation. The Organizers of this Special Session are hopeful that this bringing together of the various Participants, from various parts of the globe, and each with his or her unique expertise, will spark fruitful discussions and possible future research work in nonlinear PDE control analysis.

Large time behavior of degenerate parabolic equations in domains with non-compact boundary

Daniele Andreucci
Sapienza Università di Roma, Italy
daniele.andreucci@sbai.uniroma1.it
Anatoli F. Tedeev

We investigate the asymptotic profile as time goes to infinity of solutions to porous-media like equations set in an unbounded open region of R^N, with zero Neumann data prescribed on the boundary. The boundary itself is assumed to be non-compact. We identify the asymptotic profile which in fact is anisotropic due to the shape of the domain itself. We also give an explicit rate of convergence to such a profile, depending again on the shape of the domain. The latter is assumed to satisfy suitable isoperimetric inequalities.

A Mixed Variational Formulation for the Wellposedness and Numerical Analysis of a Certain Fluid-Structure PDE System

George Avalos
University of Nebraska-Lincoln, USA
gavalos@math.unl.edu

We will present qualitative and numerical results on a partial differential equation (PDE) system which models a certain fluid-structure dynamics. The wellposedness of this PDE model is established by means of constructing for it a nonstandard semigroup generator representation; this representation is essentially accomplished by an appropriate elimination of the pressure. Wellposedness of this fluid-structure dynamics is attained through a certain nonstandard variational (inf-sup) formulation. Subsequently we show how our constructive proof of wellposedness naturally gives rise to a certain mixed finite element method for numerically approximating solutions of this fluid-structure dynamics.

Asymptotic stability for the Schrodinger equation on non compact Riemannian manifolds and exterior domains

Marcelo Cavalcanti
State University of Maringa, Brazil
mmcavalcanti@uem.br
Cesar A. Bortot

The Schrödinger equation subject to a nonlinear and locally distributed damping, posed in a connected, complete and non compact n dimensional Riemannian manifold $(M, \mathbf{g})$ is considered. Assuming that $(M, \mathbf{g})$ is non-trapping and, in addition, that the damping term is effective in $M\backslash\Omega$, where $\Omega \subset\subset M$ is an open bounded and connected subset with smooth boundary $\partial\Omega$, such that $\overline{\Omega}$ is a compact set, exponential and uniform decay rates of the L^2-level energy are established. The main ingredients in the proof of the exponential stability are: (A) an unique continuation property for the linear problem (as in Triggiani and Xu (Contemporary Math./2007)); and (B) a local smoothing effect for the linear and non-homogeneous associated problem (as in Burq, Gerard and Tzvetkov (AIHP/2004))

Control of fourth-order parabolic control systems

Eduardo Cerpa
Universidad Santa Maria, Chile
eduardo.cerpa@usm.cl

In this talk we address the boundary and internal controllability of some fourth-order parabolic control systems posed on a finite interval. We deal with the null controllability of single and coupled equations by applying the moment theory and a Carleman estimate approach.

Intrinsic decay rate estimates for the wave equation with competing viscoelastic and frictional dissipative effects

Valeria Domingos Cavalcanti
State University of Maringa, Brazil
vndcavalcanti@uem.br
Irena Lasiecka, Marcelo Cavalcanti , Flavio Falcao Nascimento

Wave equation defined on a compact Riemannian manifold $(M, \mathfrak{g})$ subject to a combination of locally distributed viscoelastic and frictional dissipations is discussed. The viscoelastic dissipation is active on the support of $a(x)$ while the frictional damping affects portion of the manifold quantified by the support of $b(x)$ where both $a(x)$ and $b(x)$ are smooth functions. Assuming that $a(x) + b(x) \geq \delta > 0$ for all $x \in M$ and that the relaxation function satisfies certain nonlinear differential inequality, it is shown that the solutions decay according to the law dictated by the decay rates corresponding to the slowest damping. In the special case when the viscoelastic effect is active on the entire domain and the frictional dissipation is differentiable at the origin, then the overall decay rates are dictated by the viscoelasticity. The obtained decay estimates are intrinsic without any prior quantification of decay rates of both viscoelastic and frictional dissipative effects. This particular topic has been motivated by influential paper of Fabrizio and Polidoro where it was shown that viscoelasticity with poorly behaving relaxation kernel destroys exponential decay rates generated by the linear frictional dissipation. In this paper we extend these considerations to: (i) nonlinear dissipation with unquantified growth at the origin (frictional) and infinity (viscoelastic) , (ii) more general geometric settings that accommodate competing nature of frictional and viscoelastic damping. To this end we use an intrinsic method for describing decay rates of the energy via solutions to an appropriate nonlinear ODE system .

Hyperbolic boundary problems with non-traditional boundary conditions

Matthias Eller
Georgetown University, USA
mme4@georgetown.edu

Sobolev regularity of the solutions to hyperbolic boundary value problems is a persistent issue in many questions concerning the control and stabilization of evolution equations. In the context of shape optimization non-traditional boundary conditions may arise. These boundary conditions satisfy in most cases the Lopatinskii condition but not the Kreiss-Sakamoto (uniform Lopatinskii) condition. Hence, the hyperbolic boundary problem is not well-posed in the usual L_2 or Sobolev spaces and a loss of derivatives occurs. We present a survey of the existing literature and show that in most cases the loss of derivatives occurs only in the boundary terms.

Long-time behavior of reaction-diffusion equations with nonlocal boundary conditions on rough domains

Ciprian Gal
Florida International University, USA
cgal@fiu.edu
Mahamadi Warma

We investigate the long term behavior in terms of finite dimensional global and exponential attractors, as time goes to infinity of solutions to semilinear reaction-diffusion equations on non-smooth domains subject to nonlocal Robin boundary conditions, characterized by the presence of fractional diffusion on the boundary. Our results are of general character and apply to a large class of irregular domains, including domains with a fractal-like geometry. We recover the most of the existing results on existence, regularity, uniqueness, stability, attractor existence, and finite dimension, which are known for the reaction-diffusion equation in smooth domains. The framework we develop also makes possible a number of new results for all diffusion models in other non-smooth settings.

The modified phase field crystal equation

Maurizio Grasselli
Politecnico di Milano, Italy
maurizio.grasselli@polimi.it

In recent years, the so-called phase-field crystal (PFC) approach has been recently employed to model and simulate the dynamics of crystalline materials, including, e.g., crystal growth in a supercooled liquid, dendritic and eutectic solidification, epitaxial growth. We consider a modification of the so-called (sixth order) PFC equation introduced by K.R. Elder et al. This variant has recently been proposed by P. Stefanovic et al. to distinguish between elastic relaxation and diffusion time scales. It consists of adding an inertial term (i.e. a second-order time derivative) into the PFC equation. We present some results regarding well-posedness, regularity and longtime behavior obtained in collaboration with Hao Wu (Fudan University).

Existence and Stability for Free Liquid Fibers and Films

Thomas Hagen
University of Memphis, USA
thagen@memphis.edu

In this talk we present recent results on the Matovich-Pearson equations of free liquid fibers and related equations for free liquid films. The governing equations are given by a nonlinear transport equation together with stationary constraints. We will give existence, regularity and stability results for the governing equations, both in the linear and nonlinear regimes, and motivate some open problems.

A new two-component system modelling shallow-water waves

Delia Ionescu-Kruse
Institute of Mathematics of the Romanian Academy, Romania
delia.ionescu@imar.ro

For propagation of surface shallow-water waves on irrotational flows, we derive a new two-component system of nonlinear partial differential equations. The system is obtained by a variational approach in the Lagrangian formalism. We show that the system has a non-canonical Hamiltonian formulation. We also find its exact solitary-wave solutions.

Strong stability to multiple equilibria in flow structure interactions.

Irena Lasiecka
University of Memphis, USA
lasiecka@memphis.edu
Justin Webster

It is well known that flutter is an endemic phenomenon in aeroelasticity. It occurs in high speed flying jets, suspension bridges, wind mills etc. Eliminating or controlling flutter is one of the fundamental issues arising in applications. From the mathematical point of view, the problem can be modeled by an evolutionary system of coupled PDE's with an interface. It involves a perturbed wave equation coupled - in a hybrid way- with a nonlinear system of elasticity. It will be shown that the resulting evolutionary system (i) generates a nonlinear semigroup and (ii) the semigroup is strongly stabilizable in the subsonic case. As a consequence, flutter can be eliminated all together in the subsonic regimes. For supersonic velocities it will be shown that the long time behavior of structural solutions is reducible to a finite dimensional attracting set. The above results extend the theory previously known only for the "regularized" models which account for the rotational inertia or thermal effects. The proof of this result relies on a newly developed method for studying strong stabilizability in non-dissipative evolutions with a non-compact resolvent.

A control problem for a non convex conservation law

Andrea Marson
University of Padova, Italy
marson@math.unipd.it
B. Andreianov and C. Donadello

We consider a scalar conservation law in one space dimension

$$\partial_t u + \partial_x f(u) = 0\,, \qquad t \geq 0\,,\ x \in \mathbb{R}\,, \tag{1}$$

with a smooth flux function f that suffers a single inflection point. We augment (1) with an initial datum

$$u(0,x) = \begin{cases} 0 & \text{if } x \notin [a,b] \\ z(x) & \text{if } x \in [a,b] \end{cases} \tag{2}$$

where a and b are given real numbers, and $z = z(x)$ is a bounded measurable function that it is regarded as a control. We are interested in studying the set of attainable profiles at a fixed time $T > 0$, i.e. the set

$$A(T) = \Big\{ v \in L^1(\mathbb{R}) : \exists z \in L^\infty(a,b) : v(x) = u(T,x) \text{ a.e.}, \\ u = u(t,x) \text{ is the solution to (1)-(2)} \Big\}.$$

In literature several control problems for conservation laws were studied, but in most cases only strictly convex flux functions were considered. Indeed, the presence of an inflection point changes the structure of the waves in a solution, allowing the presence of one-side contact discontinuities. The problem may be of interest for applications to traffic flow.

Null controllability of degenerate parabolic equations

Patrick Martinez
IMT, University Toulouse 3, France
patrick.martinez@math.univ-toulouse.fr
Piermarco Cannarsa, Judith Vancostenoble

We study the null controllability properties of degenerate parabolic equations, when degeneracy occurs at the boundary of the domain. In space dimension 1 and 2, we obtain null controllability results under some necessary and sufficient condition on the degeneracy parameter, providing new global Carleman estimates. In space dimension 1, we are also able to study the behaviour of the controllability cost with respect of the degeneracy parameter, using properties of Bessel functions.

Asymptotic stability for a class of semilinear evolution equations with time delay

Cristina Pignotti
L'Aquila, Italy
pignotti@univaq.it
Serge Nicaise

We study the asymptotic behaviour of solutions to an abstract semilinear evolution equation in presence of a time delay in the feedback. It is well-known that a time delay, even arbitrarily small, may destroy the stability of an exponentially stable model. On the other hand, time delay effects are often present in many applications and practical problems. We give an exponential stability result under suitable conditions. Some concrete examples that enter into our abstract framework are also illustrated.

Null controllability for parabolic equations with dynamic boundary conditions of reactive-diffusive type

Roland Schnaubelt
Karlsruhe Institute of Technology, Germany
schnaubelt@kit.edu
Lahcen Maniar, Martin Meyries

We prove null controllability for linear and semilinear heat equations with dynamic boundary conditions of surface diffusion type. The results are based on a new Carleman estimate for this type of boundary conditions.

On the spectral stability of kinks in $\mathcal{PT}$-symmetric Klein-Gordon type models

Milena Stanislavova
University of Kansas, USA
stanis@ku.edu
A. Demirkaya, T. Kapitula, P.Kevrekidis, A. Stefanov

We consider the introduction of $\mathcal{PT}$-symmetric terms in the context of classical Klein-Gordon field theories. We explore the implication of such terms on the spectral stability of coherent structures, namely kinks. We find that the conclusion critically depends on the location of the kink center relative to the center of the $\mathcal{PT}$-symmetric term. The main result is that if these two points coincide, the kink's spectrum remains on the imaginary axis and the wave is spectrally stable. If the kink is centered on the "lossy side" of the medium, then it becomes stabilized. On the other hand, if it becomes centered on the "gain side" of the medium, then it is destabilized. The consequences of these two possibilities on the linearization (point and essential) spectrum are discussed in some detail.

Blow-up for the wave equation with nonlinear source and boundary damping terms

Enzo Vitillaro
Universita degli Studi di Perugia, Italy
enzo.vitillaro@unipg.it
Alessio Fiscella

The talk will deals with blow–up for the solutions of an evolution problem consisting on a semilinear wave equation posed in a bounded $C^{1,1}$ open subset of $\mathbb{R}^n$, supplied with a Neumann boundary condition involving a nonlinear dissipation. The typical problem studied is

$$\begin{aligned}
&u_{tt} - \Delta u = |u|^{p-2}u \quad \text{in } [0,\infty) \times \Omega,\\
&u = 0 \quad \text{on } (0,\infty) \times \Gamma_0,\\
&\partial_\nu u = -\alpha(x)\left(|u_t|^{m-2}u_t + \beta|u_t|^{\mu-2}u_t\right) \text{ on } (0,\infty) \times \Gamma_1,\\
&u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x) \text{ in } \Omega,
\end{aligned}$$

where $\partial\Omega = \Gamma_0 \cup \Gamma_1$, $\Gamma_0 \cap \Gamma_1 = \emptyset$, $\sigma(\Gamma_0) > 0$, 21, $\alpha \in L^\infty(\Gamma_1)$, $\alpha \geq 0$, $\beta \geq 0$. The initial data are posed in the energy space. The aim of the talk is to present some recent improvements to previous blow–up results concerning the problem.

Special Session 98: Boundary-Value Problems for Linear and Nonlinear Integrable Problems

Bernard Deconinck, University of Washington, Department of Applied Mathematics, USA
Beatrice Pelloni, University of Reading, England
Natalie Sheils, University of Washington, Department of Applied Mathematics, USA

In the last decade the use of a new method for solving boundary value problems due to Fokas has been expanded by him, his collaborators, and others. This method contains the classical solution methods as special cases and allows for the explicit solution of problems which could not previously be solved. This session will bring together practitioners of the Fokas Method and expose interested parties to the many applications of this technique.

Nonlinear Schrodinger systems with non-zero boundary conditions

Gino Biondini
State Univ. of New York at Buffalo, USA
biondini@buffalo.edu
Daniel Kraus

Nonlinear Schrodinger (NLS) equations are universal models for the evolution of weakly nonlinear dispersive wave trains which are also completely integrable, infinite-dimensional Hamiltonian systems. Despite having been intensely investigated over the last forty years, these systems still offer a number of challenges, some of these involve the study problems in which non-zero boundary conditions (NZBC) are given. This talk with discuss a number of recent results in this area. In particular, I will discuss the solution of both focusing and defocusing, scalar and vector NLS equations with NZBC. A number of explicit soliton solutions will be discussed, as well as spectral problems for special classes of initial conditions.

Boundary-value problems for systems of nonlinear dispersive wave equations

Jerry Bona
University of Illinois at Chicago, USA
jbona@uic.edu

The lecture will focus upon both the motivation and some of the recent analysis of boundary-value problems for nonlinear, dispersive wave equations.

Explicit solutions of Interface Problems II

Bernard Deconinck
University of Washington, USA
bern4rd@gmail.com
Natalie Sheils

Interface problems for PDEs are initial boundary value problems for which equations in one domain prescribe the boundary conditions for equations in adjacent domains. In applications, these interface conditions follow from conservations laws. Few interface problems allow for a fully explicit closed-form solution, using classical solution methods. Using the Fokas method, we present such solutions for both dissipative and dispersive linear interface problems. Specific problems are heat conduction and optical transmission in composite media of both finite and infinite extent. The ideas are extended to problems with moving interfaces.

Asymptotics of the Neumann Value for the NLS on the Half-Line with Periodic Dirichlet Data

Thanasis Fokas
University of Cambridge, England
T.Fokas@damtp.cam.ac.uk

The most challenging problem in the implementation of the so-called unified transform to the NLS on the half line, is the characterisation of the unknown boundary value in terms of the given data. First, an effective perturbative method will be presented for the asymptotic computation of the Neumann boundary value for given t-periodic Dirichlet data. Then, certain analytical results will be discussed, extending the relevant pioneering results of A. Boutet de Monvel and collaborators.

On well-posedness of evolution equations

Alex Himonas
University of Notre Dame, USA
himonas.1@nd.edu
Dionyssios Mantzavinos

We shall begin with a presentation of well-posedness results for the initial value problem of a class of integrable evolution equations with initial data in Sobolev spaces. Then, we shall continue by discussing corresponding results for their initial-boundary value problem.

The Wavemaker Problem

Jonatan Lenells
Baylor University, USA
Jonatan_Lenells@baylor.edu
J. L. Bona, A. S. Fokas

I will discuss the problem of determining the wave profile generated by a periodically moving wavemaker mounted at one end of a laboratory wave tank. Mathematically, the problem can be expressed as a boundary value problem for a nonlinear evolution equation with time-periodic forcing. I will present a new technique for analyzing problems of this type and describe how it can be used to partially answer some questions raised by experiments 35 years ago.

On the Cauchy problem for a CH-type equation with cubic nonlinearity

Dionyssios Mantzavinos
University of Notre Dame, USA
mantzavinos.1@nd.edu
Alex Himonas

We will discuss the derivation and integrability properties of a certain Camassa-Holm type nonlinear integrable equation with cubic nonlinearity. Next, we will establish well-posedness for the Cauchy problem of this equation in Sobolev spaces, and also investigate the dependence of the solution on the initial data. Finally, we will derive peakon travelling wave solutions for this equation on the line and on the circle.

Explicit solutions of Interface Problems I

Natalie Sheils
University of Washington, USA
nsheils@amath.washington.edu
Bernard Deconinck

Interface problems for PDEs are initial boundary value problems for which equations in one domain prescribe the boundary conditions for equations in adjacent domains. In applications, these interface conditions follow from conservations laws. Few interface problems allow for a fully explicit closed-form solution, using classical solution methods. Using the Fokas method, we present such solutions for both dissipative and dispersive linear interface problems. Specific problems are heat conduction and optical transmission in composite media of both finite and infinite extent. The ideas are extended to problems with moving interfaces.

Special Session 99: Asymptotic Expansion for Nonoscillatory Solutions of Differential and Difference Equations

Serena Matucci, University of Florence, Dep. of Mathematics and Informatics, Italy
Pavel Rehak, Academy of Sciences of the Czech Republic, Czech Republic

The study of the asymptotic expansion for nonoscillatory solutions both for differential and for difference equations has attracted an increasing interest in the last years, also due to its applications in various physical, biological or chemical problems. The aim of this special session is to outline the recent advances in this field, with a particular emphasis to the application of powerful methods, such as, for instance, the theory of regularly varying functions and some of its ramifications.

On asymptotic behavior of blow-up and Knezer solutions to nonlinear higher order equations

Irina Astashova
Lomonosov Moscow State University, Russia
ast@diffiety.ac.ru

For the equation $y^{(n)} + p(x, y, y', ...y^{(n-1)})\ |y|^k \mathrm{sgn} y = 0$ with $n \geq 1$, real $k > 1$ and a continuous function p some results about the asymptotic behavior of blow-up and Kneser solutions are obtained. The existence of such solutions with non-power asymptotic behavior is proved for constant negative p and $n = 12, 13, 14$. It is also proved that the equation can have an oscillatory solution with special asymptotic behavior. This yields the existence of solutions with arbitrary number of zeros. The problem of asymptotic equivalence of solutions is investigated for the above equation and a slightly modified one.

Explicit criteria for the existence of positive solutions to the linear advanced equation $\dot{x}(t) = c(t)x(t + \tau)$

Josef Diblik
Brno University of Technology, Czech Rep
diblik@feec.vutbr.cz

In the talk will be investigated a linear differential equation with advanced argument $\dot{x}(t) = c(t)x(t + \tau)$ where $\tau > 0$ and the function c: $[t_0, \infty) \to (0, \infty)$, $t_0 \in R$ is bounded and locally Lipschitz continuous. New explicit criteria for the existence of a positive solution in terms of c and τ will be presented. An overview of known relevant criteria will be given together with relevant comparisons.

An asymptotic analysis of monotone solutions of the equation $(p(t)x')' = q(t)\phi(x)$

Vojislav Maric
Serbian Academy of Sci. and Arts, Yugoslavia
vojam@uns.ac.rs
T. Kusano and J. Manojlović

Let p, q, ϕ be positive, continuous functions on a half-axis $[a, \infty)$. In the recent years a number of papers appeared dealing with the asymptotic behavior for $t \to \infty$ of positive monotone solutions $x(t)$ of the mentioned equation in the framework of regular variation in the sense of Karamata. We present the precise asymptotic for $t \to \infty$ of increasing solutions for the cases

$$\text{a)}\quad \phi(x) = x^\gamma,\ \gamma > 1, \qquad \text{b)}\quad p(t) = 1.$$

Some of these result overlap with the ones of V.M. Evtukhov and A.M. Samoilenko for the n-th order equation $x^{(n)} = q(t)\phi(x)$ studied by a different approach.

Regular variation of extreme solutions to a system of n nonlinear differential equations

Serena Matucci
University of Florence, Dep. of Mathematics and Informatics, Italy
serena.matucci@unifi.it
Pavel Rehak

Consider the nonlinear differential system

$$x_i' = \delta a_i(t) F_i(x_{i+1}), \qquad i = 1, \ldots, n,\ t \in [a, \infty)$$

where $\delta \in \{-1, 1\}$, $a_i : [a, \infty) \to [0, \infty)$ and $F_i : \mathbb{R} \to \mathbb{R}$, are continuous functions, with $uF_i(u) > 0$ for every $u \neq 0$ and $i = 1, \ldots, n$. Here $x_{n+1} = x_1$. The existence and the asymptotic expansion of strongly increasing and strongly decreasing solutions will be analyzed under the assumptions that the coefficients a_i are regularly varying at infinity with index $\sigma_i \in \mathbb{R}$, and the nonlinear terms F_i are regularly varying (at zero or at infinity) with index $\alpha_i > 0$; a subhomogeneity condition is also assumed. In particular, if $F_i(u) = |u|^{\alpha_i} \mathrm{sgn}(u)$, we find conditions under which all extreme solutions are regularly varying.

Asymptotic properties of solutions of Volterra difference equations

Malgorzata Migda
Poznan University of Technology, Poland
malgorzata.migda@put.poznan.pl

Volterra difference equations are widely used in modeling of processes in many branches of natural sciences, economics and engineering. They arise also by applying numerical methods to Volterra integral equations. We consider the Volterra difference equation

$$x(n+1) = a(n) + b(n)x(n) + \sum_{i=0}^{n} K(n, i) f(x(i)),\ n \geq 0,$$

and its special cases. We present sufficient conditions, under which, for every real constant, there exists a solution of the studied equation convergent to this constant. We present also sufficient conditions under which all solutions are asymptotically constant. Some boundedness, periodicity and stability results are also given.

Asymptotic behavior of solutions of difference equations

Janusz Migda
A. Mickiewicz University, Poland
migda@amu.edu.pl

Asymptotic properties of solutions of difference equations of the form

$$\Delta^m x_n = a_n f(n, x_{\sigma(n)}) + b_n$$

are studied. Using fixed point theorems we obtain sufficient conditions under which for any solution y_n of the equation $\Delta^m y_n = b_n$ and for any real $s \leq 0$ there exists a solution x_n of the above equation such that $x_n = y_n + o(n^s)$. Using a discrete variant of the Bihari lemma and a certain new technique we give also sufficient conditions under which for a given real $s \leq m-1$ all solutions x_n of the equation satisfy the condition $x_n = y_n + o(n^s)$ where y_n is a solution of the equation $\Delta^m y_n = b_n$. In particular, taking $b_n = 0$ we obtain asymptotically polynomial solutions. Similarly, if b_n is a periodic sequence, we can obtain asymptotically periodic solutions.

On asymptotics of the Mittag-Leffler function and its discrete counterpart

Ludek Nechvatal
Brno University of Technology, Czech Rep
nechvatal@fme.vutbr.cz

A modified two-parametric Mittag-Leffler function plays a key role in solving the so-called fractional differential equations. Its asymptotic behaviour is known (at least for a subset of the domain and special choices of the parameters). The contribution discusses a discrete analogue of this function as a solution of a certain two-term linear fractional difference equation (involving both the Riemann-Liouville as well as the Caputo fractional h-difference operators) and describes its asymptotics. Some of our recent results on stability and asymptotics of solutions to the mentioned equation are employed here.

On the dynamics of the solutions of a biological model

Garyfalos Papaschinopoulos
Democritus University of Thrace, Greece
gpapas@env.duth.gr
C. J. Schinas and G. Ellina

In this paper we study the boundedness and the persistence of the positive solutions, the existence, the attractivity and the global asymptotic stability of the unique positive equilibrium and the existence of periodic solutions concerning the biological model given by

$$x_{n+1} = \frac{ax_n^2}{x_n + b} + c\frac{e^{k-dx_n}}{1 + e^{k-dx_n}}$$

where $0 < a < 1$, b, c, d, k are positive constants and x_0 is a positive real number.

Conditions for preserving convergence rates to equilibrium in perturbed differential equations with regularly varying nonlinearity of unit index

Denis Patterson
Dublin City University, Ireland
denis.patterson2@mail.dcu.ie
John Appleby

We first consider the ODE

$$x'(t) = -f(x(t)) + g(t),$$

which is a perturbed version of the autonomous ODE $y'(t) = -f(y(t))$. We assume the unperturbed equation has a unique globally stable equilibrium at zero and under the assumption that $f \in RV_0(1)$ we construct, essentially, necessary and sufficient conditions on g under which the rate of convergence to zero is preserved. Analogously, we define the perturbed SDE

$$dX(t) = -f(X(t))dt + \sigma(t)dW(t),$$

where W is a one dimensional Brownian motion. Once more, under the assumption that f is regularly varying with unit index, our goal is to characterise conditions under which we preserve almost sure rates of decay to zero. The constructions of the above results are greatly abetted by the study of the auxiliary ODE

$$z'(t) = -f(z(t) + \gamma(t)),$$

for a suitable forcing function γ. We refer to this as the internally perturbed equation and will discuss the utility and novelty of its study in advancing the solutions of our main problems.

This talk will be based on research that was funded by the Irish Research Council under the project GOIPG/2013/402.

De Haan type solutions of half-linear differential equations

Pavel Rehak
Academy of Sciences of the Czech Republic, Czech Rep
rehak@math.cas.cz

In the talk, we will consider the half-linear differential equation

$$(r(t)|y'|^{\alpha-2}y')' = p(t)|y|^{\alpha-2}y,$$

where r and p are positive continuous functions on $[a, \infty)$ and $\alpha \in (1, \infty)$. We will show how the Karamata theory of regularly varying functions and the de Haan theory can be utilized in the study of asymptotic behavior of positive solutions to the half-linear equation. In particular, we will give conditions guaranteeing that increasing solutions are in the de Haan class Γ and a similar type of statement will be obtained for decreasing solutions. Further, we will deal with a description of behavior of slowly varying solutions, where the theory of the de Haan class Π finds applications. We will also mention the concept of nearly (half-)linear equations.

Asymptotic properties of damped solutions of nonlinear singular ODE

Martin Rohleder
Palacky University in Olomouc, Czech Rep
mathin@seznam.cz

We investigate the singular second-order ordinary differential equation

$$(p(t)u'(t))' + q(t)f(u(t)) = 0 \tag{1}$$

on the half-line $[0,\infty)$. Here p, q are continuous on $[0,\infty)$ and positive on $(0,\infty)$. In addition, $p(0) = 0$ which yields the singularity at $t = 0$. Function f is continuous on $\mathbb{R}$ and has three zeros L_0, 0, L such that $L_0 < 0 < L$. We consider $u_0 \in [L_0, L]$ and describe the set of all solutions of equation (1) satisfying the initial condition

$$u(0) = u_0, \qquad u'(0) = 0.$$

This set consists of three disjunct classes: damped solutions, homoclinic solutions and escape solutions. It is proved that these classes are nonempty. We specify asymptotic properties, especially of damped solutions.

References

[1] M. Cecchi, M. Marini, G. Villari: *On some classes of continuable solutions of a nonlinear differential equation*, J. Differential Equations **118** (1995), No. 2, pp. 403–419.

[2] I. Rachůnková, J. Tomeček: *Bubble-type solutions of nonlinear singular problems*, Mathematical and Computer Modelling **51** (2010), 658–669.

[3] I. Rachůnková, J. Tomeček: *Existence of oscillatory solutions of singular nonlinear differential equations*, Abstract and Applied Analysis. Vol. 2011, Article ID 408525, 20 pages.

[4] I. Rachůnková, J. Tomeček, J. Stryja: *Oscillatory solutions of singular equations arising in hydrodynamics*, Advances in Difference Equations, Recent Trends in Differential and Difference Equations. Vol. 2010, Article ID 872160, 13 pages.

[5] I. Rachůnková, J. Tomeček: *Strictly increasing solutions of a nonlinear singular differential equation arising in hydrodynamics*, Nonlinear Analysis **72** (2010), pp. 2114–2118.

[6] M. Rohleder: *On the existence of oscillatory solutions of the second order nonlinear ODE*, Acta Univ. Palacki. Olomouc., Fac. rer. nat., Mathematica **51**, 2 (2012), 107–127.

On system of difference equations included negative exponential terms

Christos Schinas
Democritus University of Thrace, Greece
cschinas@ee.duth.gr
G. Papaschinopoulos, N. Fotiades

In this paper we study the asymptotic behavior of the positive solutions of the system of two difference equations

$$x_{n+1} = ay_n + bx_{n-1}e^{-y_n},$$
$$y_{n+1} = cx_n + dy_{n-1}e^{-x_n},$$

$n = 0, 1, \dots$ where a, b, c, d are positive constants and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive numbers.

Periodic Max-type Systems of Difference Equations

Stevo Stević
Mathematical Institute of the Serbian Academy of Sciences, Serbia
sstevic@ptt.rs

We present some systems of max-type difference equations whose all solutions are periodic, with a special attention to the general system of the following form

$$x_n^{(j)} = \max_{1\le i_j\le m_j}\left\{f_{ji_j}\left(x^{(1)}_{n-k^{(j)}_{i_j,1}},\dots,x^{(l)}_{n-k^{(j)}_{i_j,l}},n\right), x^{(\sigma(j))}_{n-t_js}\right\},$$

where $n \in \mathbb{N}_0$, $s, l, m_j, t_j, k^{(j)}_{i_j,h} \in \mathbb{N}$, $j, h \in \{1,\dots,l\}$, $(\sigma(1), \sigma(2), \dots, \sigma(l))$ is a permutation of $(1, 2, \dots, l)$, and $f_{ji_j} : (0,\infty)^l \times \mathbb{N}_0 \to (0,\infty)$, $j \in \{1,\dots,l\}$, $i_j \in \{1,\dots,m_j\}$. Some conditions are posed on functions f_{ji_j} which guarantee the periodicity of all positive solutions of the system. Our results extend those ones in papers [1,2,3].

References

[1] E. M. Elsayed, B. Iričanin and S. Stević, On the max-type equation $x_{n+1} = \max\{A_n/x_n, x_{n-1}\}$, *Ars. Combin.* 95 (2010), 187-192

[2] S. Stević, Periodicity of max difference equations, *Util. Math.* 83 (2010), 69-71

[3] S. Stević, Periodicity of a class of nonautonomous max-type difference equations, *Appl. Math. Comput.* 217 (2011), 9562-9566, and [4] S. Stević, On some periodic systems of max-type difference equations, *Appl. Math. Comput.* 218 (2012), 11483-11487.

Asymptotic Stability Criteria for Linear Difference Equations

Petr Tomasek
Brno University of Technology, Czech Rep
tomasek@fme.vutbr.cz

There exist several approaches to decide, whether or not some linear difference equation with constant coefficients is asymptotically stable. It turns out that for some special cases (linear difference equations with a few nonzero terms) it is possible to formulate some efficient asymptotic stability criteria. These special cases of difference equations are of use in numerical analysis for delay differential equations. In this talk we give a brief survey of criteria for asymptotic stability of linear difference equations and we discuss relations between them. Then we present some of recent results for linear difference equations with a few nonzero terms and we point out their advantages.

Asymptotic properties of Kneser solutions to nonlinear second order ODE with regularly varying coefficients

Jana Vampolova
Palacky University Olomouc, Czech Rep
jana.vampolova@upol.cz
Irena Rachunkova

In the recent join work with professor Irena Rachunkova (Palacky University Olomouc) we investigate one class of solutions to the second order differential equation

$$\big(p(t)u'(t)\big)' + q(t)f\left(u(t)\right) = 0,$$

on the interval $[a, \infty), a \geq 0$, where p and q are regularly varying functions. Our aim is to describe asymptotic behaviour of non-oscillatory solutions satisfying one of the following conditions

$$u(a) = u_0 \in (0, L),\ 0 \leq u(t) \leq L,\ t \in [a, \infty),$$

$$u(a) = u_0 \in (L_0, 0),\ L_0 \leq u(t) \leq 0,\ t \in [a, \infty),$$

where the interval $[L_0, L]$ is determined by function f. Asymptotic formulas are derived for solutions of the above problem and for their first derivatives. We prove the existence of Kneser solutions on $[a, \infty), a > 0$ and we also discuss the singular case $a = 0$ and $p(0) = 0$ provided $q \equiv p$.

Oscillation constants for second-order nonlinear difference equations related to Euler differential equations

Naoto Yamaoka
Osaka Prefecture University, Japan
yamaoka@ms.osakafu-u.ac.jp

This talk is concerned with the oscillatory behavior of difference equations corresponding to the second-order nonlinear differential equation $x'' + f(x)/t^2 = 0$, where f is continuous on $\mathbb{R}$ and satisfies $xf(x) > 0$ if $x \neq 0$. Obtained results are represented as a pair of oscillation theorem and non-oscillation theorem. These results are best possible in a certain sense. A discrete version of the Riemann-Weber generalization of the Euler differential equation and its extended equations play an important role to prove our results. The proofs of our results are based on Riccati technique and phase plane analysis of a system.

Special Session 100: Analysis of Free Boundary Problems

Patrick Guidotti, University of California, Irvine, USA
Joachim Escher, Leibniz Universität Hannover, Germany
Christoph Walker, Leibniz Universitaet Hannover, Germany

Free Boundary problems are ubiquitous in mathematics and many application fields. They often pose serious mathematical challenges and requires sophisticated techniques. This session brings together a wide cross section of experts in this active field of research.

On the solvability of the free boundary problems arising in combustion theory

Galina Bizhanova
Institute of mathematics and mathematical modeling, Kazakhstan
galina_math@mail.ru

Let $\Omega(t),\ t\in[0,T]$, be an unknown domain in $\mathbb{R}^n$, $n\geq 2$, with a boundary $\partial\Omega(t)=:\gamma(t)$, at the initial moment the domain $\Omega(0)$ and it's boundary $\partial\Omega(0)=\gamma(0)$ are known. There are studied multidimensional one-phase free boundary problems for the heat equation with unknowns $u(x,t)$ defined in $\Omega(t)$, and free boundary $\gamma(t)$, $t\in[0,T]$. On $\gamma(t)$, $t\in(0,t)$, we have the conditions

$$u=0,\ |\nabla u|=\varphi(x,t),\ \varphi(x,t)\geq d_0=\text{const}>0,$$

in the first problem, and

$$u=0,\ |\nabla u|=-V_N+\varphi(x,t)$$

in the second one, where V_N is the velocity of the free boundary on the direction of a vector $N(\xi)$, $\xi\in\gamma(0)$. In particular, the cases, when $\varphi(x,t)=c_0$, c_0 =const, are considered. The existence, uniqueness of the solutions of these problems are proved in the Hölder spaces locally in time, the estimates of the solutions are derived.

Absence of singularity formation for the Muskat problem

Francisco Gancedo
University of Seville, Spain
fgancedo@us.es

The purpose of this talk is to present global in time result for the Muskat problem. The Muskat contour evolution problem describes the dynamics of immiscible and incompressible fluids in porous media. We will rule out singularity blow-up scenarios in several situations.

Equilibria and Their Stability for a Quasi-Stationary Droplet Model

Patrick Guidotti
University of California, Irvine, USA
gpatrick@math.uci.edu

This talk will focus on the stability of the manifold of equilibria for the moving boundary problem

$$\begin{cases} -\Delta u=\lambda\,, & \text{in } \Omega(t) \text{ for } t>0\,,\\ u=0\,, & \text{on } \partial\Omega(t) \text{ for } t>0\,,\\ \int_{\Omega(t)}u(x)\,dx=V_0\,, & \text{for } t>0\\ V=F(|Du|)\,, & \text{on } \partial\Omega(t) \text{ for } t>0\,,\\ \Omega(0)=\Omega_0\,. \end{cases}$$

satisfied by a viscous droplet sitting on a homogeneous surface and driven by contact angle. The unknown function u describes the height of the droplet, $\Omega(t)$ the wetted region at time t, V_0 the droplet volume, V the front velocity in direction normal to $\partial\Omega(t)$, and λ is a Lagrange multiplier. After the system is recast into a nonlinear nonlocal evolution for the boundary of the moving domain, the analysis proceeds with the explicit computation of the linearization in the equilibria and the introduction of suitable coordinates for $\partial\Omega(t)$ in the space of shapes. In these coordinates the equation takes on a particularly convenient form which simplifies the stability analysis.

On a simplified model for the interactions between a viscous fluid and a dissipative beam

Matthieu Hillairet
Univ. Paris Dauphine, INRIA, France
hillairet@ceremade.dauphine.fr

In this talk, we consider a model introduce by A. Quarteroni, M. Tuveri and A. Veneziani for a simplified description of arteries. In this model, a viscous incompressible fluid, governed by the Navier Stokes equations, interacts with a flexible elastic beam located on one part of the container. The fluid domain evolvec according to the structure's displacement, itself resulting from the fluid force. We discuss the global-in-time well-posedness of this problem. A particular issue is the influence of the dissipation in the beam equation on the regularity of the fluid domain.

The Direct Mapping Approach to Free Boundary Problems in Fluid Dynamics

Matthias Köhne
Heinrich-Heine-Universität Düsseldorf, Germany
koehne@math.uni-duesseldorf.de

We study several well-known and new free boundary problems in fluid dynamics. The main emphasis will be placed on the powerful approach that exploits direct mapping techniques (i.e. Hanzawa transformations) in combination with maximal regularity in an L_p-setting. We discuss the advantages as well as the drawbacks of this approach and present some recent results concerning the well-posedness of the considered problems. The models studied here will range from dynamics driven by surface tension over viscous interfaces to elasticity driven dynamics as encountered for example in biological systems formed by membranes.

On the symmetry of steady water waves: an intrinsic characterization

Bogdan Matioc
Leibniz University Hanover, Germany
matioc@ifam.uni-hannover.de

Within the setting of periodic rotational water waves, we present a new intrinsic characterization of the symmetric gravity waves. Namely, we show that the symmetry property of the free water surface can be characterized in terms of the flow supporting the wave. To be more precise, we prove that a gravity wave - which may possess a priori arbitrary many crests and trough per period - is symmetric and has only one crest and trough per period if and only if there exists a vertical line within the fluid domain such that all the fluid particles located on that line minimize there simultaneously their distance to the fluid bed. This characterization is new even for Stokes waves.

Well-posedness for the two-phase Navier-Stokes equations with surface tension and surface viscosity

Stefan Meyer
University of Halle, Germany
stefan.meyer@mathematik.uni-halle.de

We consider a sharp-interface problem for the flow of two incompressible Newtonian fluids in a bounded domain. The motion of the moving interface is governed by a stress balance which includes both surface tension and surface viscosity according to the Boussinesq-Scriven law. Here the stress balance on the interface is of second order with respect to the tangential velocity but only of first order in the normal velocity. We prove that the problem is locally well-posed in an L_p-setting for a restricted class of initial data. By means of maximal regularity for linear problems, we obtain well-posedness by simply applying Banach's fixed-point theorem. We use a decomposition of the velocity and the stress balance into tangential and normal components and a suitable transformation to a fixed interface which respects this structure. Suitable function spaces are constructed by using the theory of parabolic mixed-order systems.

Analysis of Incompressible Two-Phase Flows with Marangoni Forces

Jan Pruess
University of Halle, Germany
jan.pruess@mathematik.uni-halle.de

We consider the basic model for incompressible two-phase flows with phase transitions consistent with thermodynamics in the case of temperature-dependent surface tension. We prove well-posedness in an L_p-setting, study the stability of the equilibria of the problem, and show that a solution which does not develop singularities exist globally, and if its limit set contains a stable equilibrium it converges to this equilibrium as time goes to infinity, in the natural state manifold for the problem.

Analyticity of the interface of a thermodynamically consistent two-phase Stefan problem

Yuanzhen Shao
Vanderbilt University, USA
yuanzhen.shao@vanderbilt.edu
Jan Pruss, Gieri Simonett

We study the regularity of solutions to a thermodynamically consistent two-phase Stefan problem with or without kinetic undercooling. It is shown that the free interface of the problem immediately becomes analytic jointly in time and space, provided the initial surface satisfies a mild regularity assumption. The proof is based on a combination of a family of parameter-dependent diffeomorphisms, L_p-maximal regularity theory, and the implicit function theorem.

Global existence, decay, and stability for the classical Stefan problem

Steve Shkoller
University of Oxford, USA
shkoller@gmail.com
Mahir Hadzic

I will discuss some recent results on global well-posedness and stability for the classical Stefan problem. The analysis involves a novel hybrid methodology, combining high-order energy methods together with quantitative Hopf-type inequalities. This is joint work with Mahir Hadzic.

On two-phase flows with phase transitions: the case of equal densities

Gieri Simonett
Vanderbilt University, USA
gieri.simonett@vanderbilt.edu

A thermodynamically consistent model for two-phase fluid flows with phase transitions driven by temperature is introduced and analyzed. Concentrating on the case of equal densities, we establish well-posedness and study the qualitative behavior of solutions. In particular, we characterize all equilibria and study their stability properties. The entropy turns out to be an important quantity in the stability analysis.

Special Session 101: Nonlinear Waves in Materials With Microstructure

Guillaume James, University of Grenoble, France
Dmitry Pelinovsky, McMaster University, Canada., Canada

Many challenging problems in the theory of nonlinear lattices and periodic partial differential equations arise from the modeling of nonlinear media with microstructure, such as photonic or phononic crystals, granular media and composite materials. Among many fundamental problems, this minisymposium will address the existence and qualitative properties of periodic or localized waves (solitary waves and breathers) in these systems. Topics will include dynamical stability, wave scattering by defects, trapping of propagating waves, and nontrivial modes of propagation such as direction-reversing waves. The important questions to be discussed are on how wave propagation can be influenced by the microstructure and can be potentially controlled. The minisymposium will emphasize a variety of analytical approaches, such as modulation equations, homogenization, reduced-order models, variational methods, and the rigorous derivation of lattice differential equations from periodic PDE. In the context of granular media, we will address new types of analytical difficulties linked with nonsmooth contact interactions and purely nonlinear energy propagation in the absence of phonon modes. Computational approaches and experimental results will be also presented.

From Newton's cradle to the discrete p-Schroedinger equation

Eric Dumas
Université de Grenoble, France
eric.dumas@ujf-grenoble.fr
Brigitte Bidegaray-Fesquet, Guillaume James

We investigate the dynamics of a chain of oscillators coupled by fully-nonlinear interaction potentials. This class of models includes Newton's cradle with Hertzian contact interactions between neighbors. By means of multiple-scale analysis, we give a rigorous asymptotic description of small amplitude solutions over large times. The envelope equation leading to approximate solutions is a discrete p-Schroedinger equation. Our results include the existence of long-lived breather solutions to the original model. For a large class of localized initial conditions, we also estimate the maximal decay of small amplitude solutions over long times.

Neoclassical field theory for electromagnetically interacting elementary charges

Alexander Figotin
University of California at Irvine, USA
afigotin@uci.edu
Anatoli Babin

Our motivations for introducing a neoclassical field theory for electromagnetically interacting elementary charges were as follows. First, we wanted to have a theory which completely accounts for the relativistic dynamics of a system of charges interacting electromagnetically. Second, we wanted to have a theory which accounts for particle properties as well as for wave phenomena in a single mathematically sound Lagrangian relativistic theory. When constructing such a theory we also wanted: (i) to stay on the solid ground of the Lagrangian mechanics and the relativity principle; (ii) to recover all well established experimental facts described by the classical electromagnetic theory; (iii) to have all particle properties to naturally follow from the field equations as approximations. We have constructed such a theory and demonstrated that it implies in the non-relativistic limit (i) the non-relativistic particle mechanics governed by the Newton equations with the Lorentz forces and (ii) the frequency spectrum for hydrogenic atoms. The relativistic version of our field theory implies, in particular, the relativistic point mechanics and the celebrated Einstein's energy-mass relation $E = mc^2$.

Phase-transition waves in FPU-type chains with double-well potential

Michael Herrmann
Saarland University, Germany
michael.herrmann@math.uni-sb.de
Karsten Matthies, Hartmut Schwetlick, Johannes Zimmer

Traveling waves in atomic chains with non convex potential energy play an important role in physics and materials science since they provide the kinetic relation for more complex phenomenological models. Unfortunately, very little is known about such waves and most of the available results are restricted to piecewise affine nonlinearities. In this talk we consider genuinely nonlinear perturbations of a bi-quadratic double-well potential and show that the corresponding three-parameter family of phase-transition waves persists provided that the perturbation is sufficiently small and localized with respect to the strain variable. Our perturbative methods is nonlocal and nonlinear but does not alter the asymptotic oscillations in front of the interface. As a consequence we can identify a unique one-parameter sub-family of waves satisfying the causality principle.

Interactions between ultrasonic waves and non-linear cracks with finite compressibility

Stephane Junca
Nice Sophia-Antipolis, France
junca@unice.fr
Bruno Lombard

The modelling of interactions between ultrasonic waves and cracks is of great interest in many fields of applied mechanics. When the wavelengths are much larger than the width of the cracks, the latter are usually replaced by zero-thickness interfaces and appropriate jump conditions. A realistic non-linear hyperbolic model accounting for the finite compressibility of crack faces under normal loading conditions has been presented for applications in mechanical engineering. Numerical experiments and the derivation of differential equations from the Partial Differential equations model will be first presented before the mathematical study. For N cracks, the model becomes a Neutral Delay Differential System with many mathematical challenges: non-linear stability, small divisors, homogenization.

Nonlinear Waves in Granular Crystals

Panayotis Kevrekidis
UMass, Amherst, USA
kevrekid@gmail.com

In this talk, we will provide an overview of results in the setting of granular crystals, consisting of beads interacting through Hertzian contacts. We will start from the simplest setting of one- dimensional, monoatomic chains where highly localized traveling waves exist and we will also examine states in the form of (dark) discrete breathers and shock waves therein. Wherever possible, we will corroborate these considerations with recent experimental results. We will then extend our considerations to the case of diatomic chains and examine how the properties of traveling waves and also of discrete breathers are modified in the latter setting. More highly heterogeneous chains will be briefly examined as well. In addition to considering the purely Hamiltonian case, select examples of the damped-driven variant of the system and its rich phenomenology, including chaotic response and bistability/hysteresis will also be shown. The results will be extended to two dimensions and some prototypical examples thereof in hexagonal, as well as square chains will be provided. Finally, some very recent developments involving acoustic transistors and logic gates, as well as a novel configuration involving a single column woodpile phononic crystal will be presented.

On the orbital stability of Gaussian solitary waves in the log-KdV equation

Dmitry Pelinovsky
McMaster University, Canada
dmpeli@math.mcmaster.ca
Remi Carles, Guillaume James

We consider the logarithmic Korteweg–de Vries (log–KdV) equation, which models solitary waves in anharmonic chains with Hertzian interaction forces. By using an approximating sequence of global solutions of the regularized generalized KdV equation in $H^1(\mathbb{R})$ with conserved L^2 norm and energy, we construct a weak global solution of the log–KdV equation in a subset of $H^1(\mathbb{R})$. This construction yields conditional orbital stability of Gaussian solitary waves of the log–KdV equation, provided uniqueness and continuous dependence of the constructed solution holds. Furthermore, we study the linearized log–KdV equation at the Gaussian solitary wave and prove that the associated linearized operator has a purely discrete spectrum consisting of simple purely imaginary eigenvalues in addition to the double zero eigenvalue. The eigenfunctions, however, do not decay like Gaussian functions but have algebraic decay. Nevertheless, using numerical approximations, we show that the Gaussian initial data do not spread out and preserve their spatial Gaussian decay in the time evolution of the linearized log–KdV equation.

Influence of Network Topology on Sound Propagation in Granular Materials

Mason Porter
University of Oxford, England
porterm@maths.ox.ac.uk

Granular media, whose features range from the particle scale to the force-chain scale and the bulk scale, are usually modeled as either particulate or continuum materials. In contrast with each of these approaches, network representations are natural for the simultaneous examination of microscopic, mesoscopic, and macroscopic features. In this talk, I treat granular materials as spatially embedded networks in which the nodes (particles) are connected by weighted edges obtained from contact forces. I use ideas from network science to provide insights into how sound propagates through granular materials. I test a variety of network measures to determine their utility in helping to describe sound propagation in granular networks and find that network diagnostics can be used to probe particle-, curve-, domain-, and system-scale structures in granular media. I also use network "community structure" — a mesoscopic property — to help characterize sound propagation, and I will illustrate how using different "null models" in community-detection methods can yield different insights into granular force networks.

Failure of amplitude equations

Guido Schneider
Universität Stuttgart, Germany
guido.schneider@mathematik.uni-stuttgart.de

We present a number of examples for homogeneous and periodic media where formally derived amplitude equations fail to make correct predictions about the dynamics in the original system.

Tempest in a Granular Alignment: solitary waves, quasi-equilibrium, rogue fluctuations and more

Surajit Sen
State Univ of New York Buffalo, USA
sen@buffalo.edu
Yoichi Takato, Edgar Avalos, Han Ding, Matthew Westley

We consider a simple alignment of elastic objects in mutual contact and held between two rigid walls. A simple velocity perturbation imparted across short enough times at an end of an unloaded alignment results in a solitary wave. A long lived perturbation results in a solitary wave train. These solitary waves break and reform upon collision, thereby in the absence of dissipation leading to the eventual emergence of a quasi-equilibrium phase. We will also show that there exists a critical compression where in addition to the quasi-equilibrium phase there are solitary waves that do not scatter as expected upon collision. The talk will discuss various properties of this quasi-equilibrium phase, in particular, the possibility of observing very large energy fluctuations and clusters of such fluctuations which we call rogue fluctuations in these systems at late times.

Resonant Attenuation, Redirection and Entrapment of Strongly Localized Wave-packets in Locally Resonant, Quasi-1D Chains Incorporating Internal Rotators

Yuli Starosvetsky
Technion - Israel Institute of Technology, Israel
staryuli@tx.technion.ac.il
K. Vorotnikov

As of date, there is a growing interest in the dynamics of higher dimensional granular crystals. One of the central objectives of the current research activity in the area of granular meta-materials is to achieve an efficient mechanism for attenuation, absorption and redirection of non-linear stress waves. Dynamics of locally resonant, granular meta-materials is a subject of intense study of the past few years and from the theoretical view point remains a relatively young and also hardly explored area. As of today there is a lack of a substantial theoretical understanding of the dynamics of locally resonant, non-linear lattices. Model under consideration comprises the chain of elements, incorporating internal rotators and subject to horizontal and vertical local potentials. In the present talk we present the recent results of the analytical and numerical study of the intrinsic mechanisms of complete, resonant energy redirection as well as the recurrent energy transport between the vertical and horizontal directions, spontaneous energy entrapment and the direction reversing localized wave packets. Based on the results of the theoretical studies we formulate the conceptual design of locally resonant, non-linear scalar model bringing to the desired response of the meta-material e.g. permanent wave redirection for the lower amplitude pulses, uni-directional wave transmission for the higher amplitude pulses, direction reversal of the moving breathers by the properly tuned internal rotators.

Traveling waves in a nonlinear lattice with competing interactions

Anna Vainchtein
University of Pittsburgh, USA
aav4@pitt.edu
Erik S. Van Vleck, Aijun Zhang

We consider an overdamped Frenkel-Kontorova lattice with piecewise linear bistable interaction force due to the onsite potential and harmonic first and second-neighbor interactions that may be either attractive or repulsive. We construct traveling wave solutions that connect various periodic equilibria to the undeformed equilibrium state. Such solutions may be interpreted as steadily propagating fronts of microstructural patterns. Existence, multiplicity and global structure of these solutions are investigated. We show that competing interactions bring interesting new features, such as a maximum speed of propagation and coexistence of multiple traveling waves.

Localized patterns and snaking in 2D reaction-diffusion systems

Daniel Wetzel
Unisersity of Oldenburg, Germany
daniel.wetzel@uni-oldenburg.de
Hannes Uecker, Jens Rademacher

We use our continuation and bifurcation software tool pde2path (see [1]) to determine solutions and solution branches numerically for elliptic PDEs over 1D and 2D domains. In my talk I present numerical and analytical results for localized patterns on patterned backgrounds and the snaking behavior of their branches for some reaction-diffusion systems over 2D domains.

References

[1] http://www.staff.uni-oldenburg.de/hannes.uecker/pde2path/

Special Session 102: Kinetic Models for Multi-Agent Systems Modeling Socio-Economic Behavior

Pierre Degond, Imperial College, UK
Sebastien Motsch, Arizona State University, USA
Christian Ringhofer, Arizona State University, USA

Social and economic interactions between an ensemble of agents can be modeled using kinetic equations based on binary interactions in a mean field limit. The advantage of this approach lies in the possibility to derive macroscopic equations for conserved quantities on large time scales, and therefore derive models for consensus, flocking, cooperation or competition on large time scales. Applications include c.f. the evolution of wealth in spatially distributed economies or the formation of low dimensional structures in swarms. This special session will bring together researchers in kinetic theory with backgrounds in different applications.

Swarms in bounded domains

Dieter Armbruster
Arizona State University, USA
armbruster@asu.edu
Andrea Thatcher, Sebastian Motsch, Christian Ringhofer

We numerically study the Vicsek swarm model in a circular, a rectangular and a channel domain as a function of the parameters governing the individual particles and their interaction and the geometry of the domain. Scale free parameters characterizing the observed swarm behavior are presented. We find different regimes and characterize them through experimental observables like the trajectory of the center of mass, the mean density profile of a swarming motion and the connectivity to the swarm. While individual particles are reflected at a boundary the swarm as a whole interacts in much more complicated ways with the boundary. We discuss how the interaction of the particles with the boundary creates boundary layers for the swarm flows and compare them to hydrodynamic boundary layers.

Optimal transport applied to potential games in economics

Adrien Blanchet
Toulouse School of Economics, France
adrien.blanchet@ut-capitole.fr
G. Carlier, P. Mossay, F. Santambrogio

We consider a non-cooperative non-atomic anonymous game with a continuum of players. Mathematically, let X be the space of types and Y the set of actions. If $\nu\mathcal{P}(X)$ is the distribution of actions of the players then an agent of type x who choose the action y pays the cost $\Gamma(x, y, \nu)$. We are interested in the Nash equilibrium of this game. Our results apply to potential games. Under general assumptions we prove that there is a unique equilibrium. Moreover this equilibrium is the minimiser of the following functional:

$$\mathcal{J}_\mu[\nu] := \mathcal{W}_c(\mu, \nu) + \mathcal{E}[\nu]$$

where μ is the distribution of types, $\mathcal{W}_c(\mu, \nu)$ is the Monge-Kantorovich distance between μ and ν for the cost c, and $\mathcal{E}$ is typically of the form

$$\mathcal{E}[\nu] := \int_C V[\nu(x)]dy + \int_C A(y)\lambda(y)dy + \frac{1}{2}\int_{C\times C} \phi(y-z)\lambda(y)\lambda(z)dydz \,.$$

This functional is not convex in the classical sense but it is convex along generalised geodesic in the Wasserstein metric. We characterise the equilibrium in term of a PDE and compute the taxes to restore the efficiency of this game.

Pedestrian macroscopic models: game-theoretical versus mechanistic viewpoints

Pierre Degond
Imperial College London, England
pdegond@imperial.ac.uk

Many Individual-Based Pedestrian models have been proposed in the literature. They can roughly be categorized in two classes: a first class considers pedestrians as rational agents able to optimize their trajectory in order to fulfil their goal. The second class of models rely on a mechanistic view of agents reacting to close encounters like physical particles. In both cases, we derive macroscopic models and discuss their similarities and differences.

Complex material flow problems: a multi-scale model hierarchy

Simone Göttlich
University of Mannheim, Germany
goettlich@uni-mannheim.de

A multi-scale model based on material flow systems is investigated that is divided into a microscopic and a macroscopic model scale. On the one hand macroscopic flow models are used for large scale simulations with a large number of parts. On the other hand microscopic models are needed to describe the details of the production process. We present a hierarchy of models for material flow problems ranging from detailed microscopic models to macroscopic models using scalar conservation laws with nonlocal interaction terms. Numerical simulations are presented on all levels of the hierarchy and validated against real-data test settings.

Mean field hierarchies for interacting particle systems: Numerical methods and applications

Axel Klar
TU Kaiserslautern, Germany
klar@mathematik.uni-kl.de

The talk will present numerical methods for mean field hierarchies derived from interacting particle systems. The hierarchy includes kinetic equations and their hydrodynamic and diffusive approximations. We discuss Semi-Lagrangian methods for the mean field equations and particle methods for the hydrodynamic approximations. Applications to swarming and to pedestrian flow simulations are shown.

A localized version of the Hughes model for pedestrian flow

Stephan Martin
Imperial College London, England
stephan.martin@imperial.ac.uk
J.A. Carrillo, M.T. Wolfram

In the classical model of Hughes for crowd motion pedestrians seek to minimize their travel time to a-priori known destinations/exits, but try to avoid regions of high density. One of the basic assumptions is that the overall density of the crowd is known to every agent at every time. We present results on a modification that includes localizing effects such as limited vision to a Hughes-type equation. The basic mechanism permits agents to perceive information on the current crowd density only in a local neighborhood, while taking assumptions on the density outside that region. We discuss the modelling aspects and consider both a microscopic and macroscopic perspective. As the suggested model leads to the problem of solving varying Hamilton-Jacobi equations depending on the agent's location, efficient numerical solvers that reduce computational costs are developed. Our main object of study is the reduction of the overall performance of the crowd. We quantify and illustrate with numerical experiments, how the ability of the crowd to evacuate effectively is reduced by limiting the information available to every agent.

Accurate crowd prediction system

Sebastien Motsch
Arizona State University, USA
smotsch@asu.edu

Optimizing pedestrian flow efficiency and crowd safety is a critical issue in contemporary urban societies. We advocate the need for "Crowd Weather" forecasting systems using similar concepts as those used in weather forecast. Such a system requires carefully validated models on high quality data. In this talk, we use real time tracking experiments based on automatic motion capture techniques to calibrate a macroscopic model for pedestrian dynamics. It relies on a Bi-directional Fundamental Diagram (BFD) which relates the pedestrian fluxes to both the co- and counter-moving pedestrian densities. This Bi-directional Macroscopic (BM) model using the BFD estimated from the data quantitatively reproduces the dynamical clustering patterns observed in the experiments. Finally, we use the BM model to examine the outcome of a corridor segregation strategy and demonstrate the need for a real-time crowd prediction system.

Kinetic description of control in multi-agent systems

Lorenzo Pareschi
University of Ferrara, Italy
lorenzo.pareschi@unife.it
Giacomo Albi, Micael Herty, Mattia Zanella

In this talk we introduce a general way to construct a Boltzmann description of optimal control problems for large systems of interacting agents. The approach is applied to a constrained microscopic model of opinion formation. The main feature of the method is that, thanks to a model predictive approximation, the control is explicitly embedded in the resulting binary interaction dynamic. In particular in the so-called quasi invariant opinion limit simplified Fokker-Planck models can be derived which admit explicit computations of the steady states. The robustness of the controlled dynamics is illustrated by several numerical examples which confirm the theoretical results. Different generalizations of the presented approach are possible, like the introduction of the same control dynamic through leaders or the application of this same control methodology to swarming and flocking models.

Averaged kinetic models for particle flows on complex networks

Christian Ringhofer
Arizona State University, USA
ringhofer@asu.edu
M. Herty

A kinetic model for flows on arbitrarily complex networks is presented and analyzed. Large time averages of the model equations give a drift - diffusion model on time scales, not reproducable by Monte Carlo simulations. The actual numerical implementation and applications to social and economic networks are discussed.

On multi-scale representations of flocking models

Changhui Tan
University of Maryland, USA
ctan@cscamm.umd.edu
Eitan Tadmor

Self-organized behavior is very common in nature and human societies. One widely discussed example is the flock formed by birds flying towards the same direction. Several models such as Cucker-Smale and Motsch-Tadmor are very successful in characterizing the flocking behavior. In this talk, we will discuss about the flocking models in different scales, describing the alignment phenomenon. In particular, the hydrodynamic representation of the flocking models yields a very interesting fluid system: compressible Eulerian dynamics with nonlocal alignment. We show a critical threshold phenomenon for the fluid system. Under suitable initial conditions, the system has global strong solution, and it converges to a flock. On the other hand, another set of initial conditions will lead to a finite time break down of the system.

Special Session 103: Periodic Solutions for Dynamical Systems

Adriana Buica, Universitatea Babes-Bolyai, Cluj-Napoca, Romania
Susanna Maza, Universitat de Lleida, Spain

The goal of this section is to present recent results in the theory of existence, bifurcations and stability of periodic solutions of dynamical systems. It includes both methods and applications. We are mainly interested in finite dimensional dynamical systems that are defined by differential equations, autonomous or nonautonomous, smooth or non-smooth. We hope to gather together established and young scientists actively working in these subjects.

Period Function for the sum of two homogeneous systems

Maria Jesus Alvarez
Universitat de les Illes Balears, Spain
chus.alvarez@uib.es
A. Gasull; R. Prohens

In this work we study the period function of the sum of two homogeneous Hamiltonian vector fields, non of them being linear. We get some bounds (upper and lower) for the number of critical periods of the period function, depending on the relationship of the degrees of the two homogeneous vector fields.

Hopf bifurcation in symmetric networks of coupled oscillators with hysteresis

Zalman Balanov
University of Texas at Dallas, USA
balanov@utdallas.edu
E. Hooton, W. Krawcewicz, D. Rachinskii, A. Zhezherun

The standard approach to study symmetric Hopf bifurcation phenomenon is based on the usage of the equivariant singularity theory developed by M. Golubitsky et al. In this talk, we present the equivariant degree theory based method which is complementary to the equivariant singularity approach. Our method allows systematic study of symmetric Hopf bifurcation problems in non-smooth/non-generic equivariant settings. The exposition is focused on a network of eight identical van der Pol oscillators with hysteresis memory, which are coupled in a cube-like configuration leading to S_4-equivariance. The hysteresis memory is the source of non-smoothness and of the presence of an infinite dimensional phase space without local linear structure. Symmetric properties and multiplicity of bifurcating branches of periodic solutions are discussed in the context showing a direct link between the physical properties and the equivariant topology underlying this problem. The global behavior of bifurcating branches is also discussed.

Stability of singular limit cycles for Abel equations

José Luis Bravo Trinidad
Universidad de Extremadura, Spain
trinidad@unex.es
M. Fernandez, A. Gasull

We obtain a criterion for determining the stability of singular limit cycles of Abel equations $x' = A(t)x^3 + B(t)x^2$. This stability controls the possible saddle-node bifurcations of limit cycles. Therefore, studying the Hopf-like bifurcations at $x = 0$, together with the bifurcations at infinity of a suitable compactification of the equations, we obtain upper bounds of their number of limit cycles. As an illustration of this approach, we prove that the family $x' = at(t - t_A)x^3 + b(t - t_B)x^2$, with $a, b > 0$, has at most two positive limit cycles for any t_A, t_B.

Center mechanisms for a 6-parameter subfamily of planar cubic vector fields

Magdalena Caubergh
Universitat Autònoma de Barcelona, Spain
leen@mat.uab.cat

Two of the main classical problems in the Qualitative Theory of real planar polynomial vector fields are Hilbert 16th Problem and the center-focus problem. It is well-known that the localized version of Hilbert 16th Problem is related with the center-focus problem. For instance, in case the singular point **s** is a non degenerate elliptic point, the maximum number of small amplitude limit cycles bifurcating from it, can be studied by calculating the Lyapunov quantities, which are also used to characterize the parameter for which **s** is a center. Analogously both the maximum number of large amplitude limit cycles as well as a center at infinity can be described in terms of Lyapunov quantities, that are obtained for the center at the origin after transformation $x = \frac{\cos\theta}{r}, y = \frac{\sin\theta}{r}$. Motivated by these problems we consider the cubic vector fields $\dot{x} = -y + ax^2 + bxy + cy^2 - y(x^2 + y^2)$, $\dot{x} = x + dx^2 + exy + fy^2 + x(x^2 + y^2)$, where $a, b, c, d, e, f \in \mathbb{R}$ are such that the phase portraits all have a center simultaneously at the origin and at infinity. Recently we have obtained a complete analytic classification of the global phase portraits for it. In this talk we focus on the separatrix bifurcations that appear as a key in the complete study.

TBC

Colin Christopher
Plymouth University, England
c.christopher@plymouth.ac.uk

TBA

Configurations of limit cycles in Liénard equations

Bartomeu Coll
University of Balearic Islands, Spain
tomeu.coll@uib.cat
F. Dumortier, R. Prohens

We show that every finite configuration of disjoint simple closed curves in the plane is topologically realizable as the set of limit cycles of a polynomial Liénard equation. The related vector field X is Morse-Smale. Moreover it has the minimum number of singularities required for realizing the configuration in a Liénard equation. We provide an explicit upper bound on the degree of X, which is lower than the results obtained before, obtained in the context of general polynomial vector fields.

Slow-fast Liénard systems and their limit cycles

Peter de Maesschalck
Hasselt University, Belgium
peter.demaesschalck@uhasselt.be

We give an overview of the results on the number of limit cycles of slow-fast Liénard systems, with a focus on recent results where existing known lower bounds are improved to a great extent. On top of that we extend the applicability of these results by pointing out that slow-fast Liénard systems are normal forms, up to exponentially small error, by general contact points appearing in planar slow-fast systems.

Average forcing and frequency locking in a fast-slow compartmental system

Jean-Pierre Francoise
Université Pierre-Marie Curie, Paris, France
Jean-Pierre.Francoise@upmc.fr

Average forcing and frequency locking in a fast-slow compartmental system In this talk, we consider, a fast-slow system modeling metabolic exchanges between different compartments. The time evolution is fast in one compartment and slow in others. There is a forcing term and a conidition is given on the average of the forcing to secure a frequency locking.

The 3-dimensional center problem for the analytic zero-Hopf singularity

Isaac Garcia
University of Lleida, Spain
garcia@matematica.udl.cat
Susanna Maza and Claudia Valls

In this work we extend well-known techniques for solving the Poincaré-Lyapunov nondegenerate analytic center problem in the plane to the 3-dimensional center problem at the analytic zero-Hopf singularity. We will see that both problems have much in common. Thus we characterize the existence of a neighborhood of the zero-Hopf singularity completely foliated by periodic orbits (including continua of equilibriums) via an analytic Poincaré return map. We also prove that the 3-dimensional center is characterized by the fact that the system is analytically completely integrable and also because the Poincaré-Dulac normal form is analytic and orbitally linearizable. Also we show that, when the system is polynomial and parametrized by its coefficients, the set of systems with 3-dimensional centers corresponds to an affine variety in the parameter space of coefficients.

Periodic Orbits and Noose Bifurcation in Piecewise Linear Systems

Elisabeth García-Medina
Universidad de Sevilla, Spain
egarme@us.es
V. Carmona, S. Fernández-García, F. Fernández-Sánchez, A. E. Teruel

We focus our attention on the existence of periodic orbits for the reversible two-zonal piecewise linear system

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z, \\ \dot{z} = 1 - y - \lambda(1+\lambda^2)|x|, \end{cases}$$

where the parameter λ is strictly positive. This system is formed from two linear systems separated by the plane $x = 0$ and can be considered as a continuous piecewise linear version of the Michelson system. Both systems present the well-known noose bifurcation. Indeed, a numerical study of the stability and bifurcations of the periodic orbits involved in the noose curve for the piecewise linear system allows us to assure that they exhibit the same configuration that the Michelson system has.

The orbits that take part in the noose bifurcation for the piecewise linear system have two and four points of intersection with the separation plane and they are arranged in two curves that are connected by a point where the periodic orbit has a crossing tangency with the separation plane. This periodic orbit plays an important rule because the crossing tangency forces a small loop that allows the appearance of the noose structure.

Vector fields with homogeneous nonlinearities and many limit cycles

Armengol Gasull
Universitat Autonoma de Barcelona, Spain
gasull@mat.uab.cat
Jiang Yu and Xiang Zhang

Consider planar polynomial differential equations of the form $\dot{\mathbf{x}} = A\mathbf{x} + X_n(\mathbf{x})$, where $\mathbf{x} = (x, y) \in \mathbb{R}^2$, A is a 2×2 real matrix and X_n is a homogeneous vector field of degree $n > 1$. Most known results above these equations, valid for arbitrary n, deal with the case where the origin is a focus or a node and give either non-existence of limit cycles or upper bounds of one or two limit cycles surrounding the origin. In this paper we improve some of these results and moreover we show that for $n \geq 3$ odd there are equations of this form having at least $(n+1)/2$ limit cycles surrounding the origin. Our results include cases where the origin is a focus, a node, a saddle or a nilpotent singularity.

Sufficient conditions to have a resonant complex center

Jaume Giné
Universitat de Lleida, Spain
gine@matematica.udl.cat

The notion of center is generalized to the case of a $p : -q$ resonant singular point of a polynomial vector field in $\mathbb{C}^2$. We give new methods to prove the sufficiency for a resonant singular point satisfying some necessary conditions and we provide a resume of the known techniques used to prove the sufficiency.

Essential perturbations of quadratic and cubic systems

Maite Grau
Universitat de Lleida, Spain
mtgrau@matematica.udl.cat
Adriana Buică, Jaume Giné

A way to study the limit cycles which bifurcate from the periodic orbits of a center of a planar polynomial differential system is to perturbate the system with the center in a certain parametric family of systems. In this context, it appears the notion of essential perturbation used for the first time by Iliev (1998) in a paper on quadratic systems. We will give its explicit definition. Given a perturbation of a particular family of centers of polynomial differential systems of arbitrary degree for which we explicitly know its Poincaré–Liapunov constants, we find its essential perturbations. As a consequence we give the structure of its k-th Melnikov function. This result generalizes the result obtained by Chicone and Jacobs (1991) for perturbations of degree at most two of any center of a quadratic polynomial system. Moreover we study the essential perturbations for all the centers of the differential systems

$$\dot{x} = -y + P_{\mathrm{d}}(x, y), \quad \dot{y} = x + Q_{\mathrm{d}}(x, y),$$

where P_{d} and Q_{d} are homogeneous polynomials of degree d, for d = 2 and d = 3.

Dynamics in unfoldings of HBT singularities

Santiago Ibáñez
University of Oviedo, Spain
mesa@uniovi.es
David Rivela

Let X be a 4-dimensional vector field with an equilibrium point where the 1-jet of X is linearly conjugated to

$$y\frac{\partial}{\partial x} - \omega v\frac{\partial}{\partial u} + \omega u\frac{\partial}{\partial v}.$$

with $\omega \neq 0$. We refer to X as a Hopf-Bogdanov-Takens (HBT in the sequel) singularity. We will discuss local bifurcations arising in generic unfoldings of HBT singularities of codimension 3. The catalogue includes several types of Hopf-Zero and Hopf-Hopf bifurcations. Some global aspects of the bifurcation diagram will also be treated. Our interest in HBT singularities is due to their role in applications. They arise in a natural way when planar systems displaying Hopf bifurcations are coupled by simple mechanisms. For instance, they are present in a model consisting of two brusselators linearly coupled by diffusion. They also play a crucial role in the understanding of the unfolding of a 4-dimensional nilpotent singularity of codimension four. Complex dynamical behaviour emerging from HBT singularities will be shown in the context of such applications.

Periodic solutions of Lipschitz differential systems via higher order averaging method

Douglas Novaes
Universidade Estadual de Campinas, Brazil
ddnovaes@gmail.com
Jaume Llibre and Marco A. Teixeira

We deal with nonlinear differential systems of the form

$$x'(t) = \sum_{i=0}^{k} \varepsilon^i F_i(t, x) + \varepsilon^{k+1} R(t, x, \varepsilon),$$

where $F_i : \mathbb{R} \times D \to \mathbb{R}^n$ for $i = 0, 1, \cdots, k$, and $R : \mathbb{R} \times D \times (-\varepsilon_0, \varepsilon_0) \to \mathbb{R}^n$ are continuous functions, T–periodic in the first variable and Lipschitz in the second variable, being D an open subset of $\mathbb{R}^n$, and ε a small parameter. For such differential systems, which do not need to be of class $\mathcal{C}^1$, under convenient assumptions we extend the averaging theory for computing their periodic solutions to k–th order in ε. The main tool used is the Brouwer degree theory for finite dimensional spaces.

Algebraic Geometry of the Center-Focus Problem for Polynomial Abel Equations

Fedor Pakovich
Ben Gurion University, Israel
pakovich@math.bgu.ac.il
M. Briskin and Y. Yomdin

Using recent solution of the Polynomial Moments vanishing problem, we show that the Center configurations for a polynomial Abel differential equation are very closely approximated by the composition ones.

Limit cycles bifurcated from a degenerate center

Chara Pantazi
Universitat Politecnica de Catalunya, Spain
chara.pantazi@upc.edu
J. Llibre

In this talk we concern about the maximum number of limit cycles that can bifurcated from a degenerate center of a cubic homogeneous polynomial differential system. Using the averaging method of second order and perturbing inside the class of all cubic polynomial differential systems we prove that at most three limit cycles can bifurcate from the degenerate center. This is the first time that such a complete study of second order is done for a degenerate center having neither a Hamiltonian first integral nor a rational one.

Vector fields defined by submersions of Euclidean spaces

Daniel Peralta-Salas
ICMAT, Spain
dperalta@icmat.es

A vector field is defined by the fibres of a submersion when it has a maximal number of independent first integrals. I will show that any link can be a set of periodic trajectories of such fields, and I will give some homological criteria for the existence of periodic orbits. This is based on joint work with Gilbert Hector.

On periodic orbits of piecewise linear systems with two zones

Enrique Ponce
University of Sevilla, Spain
eponcem@us.es
Emilio Freire and Francisco Torres

In discontinuous piecewise linear systems with two zones, it is shown that the existence of a focus in one zone is sufficient to get three nested limit cycles independently on the dynamics of the another linear zone. Starting from a situation with only one hyperbolic limit cycle, other two limit cycles are obtained by combining of a boundary focus bifurcation and a pseudo-Hopf bifurcation. After some generic assumptions, and taking $\gamma_L\gamma_R < 0$, $m_L = i$, $a_L \le 0$ (a left focus), we show our results for $a_R < 0$ and $m_R \in \{i, 0, 1\}$ in the family of systems

$$\dot{\mathbf{x}} = \begin{pmatrix} 2\gamma_{\{L,R\}} & -1 \\ \gamma^2_{\{L,R\}} - m^2_{\{L,R\}} & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} -b_{\{L,R\}} \\ a_{\{L,R\}} \end{pmatrix},$$

where the subscripts $\{L, R\}$ indicate the left/right half planes.

Alien Limit Cycles in Liénard Equations

Rafel Prohens
University of Balearic Islands, Spain
rafel.prohens@uib.cat
F. Dumortier

This paper aims at providing an example of a family of polynomial Liénard equations exhibiting an alien limit cycle. This limit cycle is perturbed from a 2-saddle cycle in the boundary of an annulus of periodic orbits given by a Hamiltonian vector field. The Hamiltonian represents a truncated pendulum of degree 4. In comparison to a former polynomial example, not only the equations are simpler but a lot of tedious calculations can be avoided, making the example also interesting with respect to simplicity in treatment.

On the multiplicity of algebraic limit cycles

Jesús S. Pérez del Río
Universidad de Oviedo, Spain
jspr@uniovi.es
Belén García, Jaume Giné, Maite Grau

In the present talk we tackle the problem of determining the multiplicity of periodic orbit as limit cycle of a planar differential system. We consider the particular case of a circumference as periodic orbit. We show that the conditions of multiplicity can be almost algebraically solvable. There are parameters in which these conditions depend transcendentally, as happens in the degenerate center-focus problem. Even though this difficulty, these transcendental dependence can be, in some sense, controlled because only a basis of fundamental functions appear. The appearance of this fundamental basis opens the path to approach these types of problems. We present several examples of families for which these conditions can be computed.

On the period function near the outer boundary.

David Rojas
Universitat Autònoma de Barcelona, Spain
rojas@mat.uab.cat
F. Mañosas, J. Villadelprat

Consider a planar family of planar differential systems with a center at p. The period function assigns to each periodic orbit in the period annulus its period. The problem of bifurcation of critical periods have been studied and there are three different situations to consider: bifurcations from the center, bifurcations from the interior of the period annulus and bifurcation from the outer boundary of the period annulus. The bifurcation of critical periods from the inner boundary is completely understood by using the so-called period constants. We study the bifurcation of critical periods from the outer boundary, which has an additional difficulty since the period function can not be analytically extended. We give some results to ensure that the period function extends to the outer boundary and identify the parameter regions where there are no bifurcations associated in the case of families of potential vector fields. Particularly, we found the bifurcation curves associated to the outer boundary of the bi-parametric family of potential system $X_{p,q} = -y\partial_x + V'_{p,q}(x)\partial_y$ with $V'_{p,q}(x) = (x+1)^p - (x+1)^q$.

Chaos in three dimensional Lotka-Volterra equations

Rizgar Salih
Plymouth University, England
rizgar.salih@postgrad.plymouth.ac.uk
Colin Christopher

We show that three dimensional Lotka-Volterra systems can exhibit chaos by demonstrating the existence of a horseshoe map. The proof uses a Shilnikov-type structure adapted to the geometry of the these systems.

Periodic solutions for a pair of coupled oscillators at resonance

Iulian Stoleriu
University "Al. I. Cuza" from Iasi, Romania
iulian.stoleriu@uaic.ro

We shall discuss here the bifurcation of small periodic solutions in a time-reversible and conservative four-dimensional nonlinear system that arises in Differential Geometry, in the study of biharmonic maps from torus into spheres. The linearized system has two pairs of pure imaginary eigenvalues, which are double (the case of 1:1 resonance) and non-semisimple. Using a normal form transformation, we reduce the original system to a three-dimensional system, which is analyzed qualitatively with regard to the unfolding parameters.

Polynomial solutions of Riccati Equations

Joan Torregrosa
Universitat Autònoma de Barcelona, Spain
torre@mat.uab.cat
Armengol Gasull, Xiang Zhang

We study the number of polynomial solutions in polynomial Riccati equations. We prove that any polynomial Riccati equation of degree n has either at most $n+1$ polynomial solutions if $n > 0$ or two if $n = 0$.

On the algebraic structure of the polynomial quadratic-like Hamiltonian isochrones

Jordi Villadelprat
Universitat Rovira i Virgili, Spain
jordi.villadelprat@urv.cat
C. Christopher and M. Grau

We deal with isochronous centers of the so-called quadratic-like Hamiltonian systems, i.e., planar differential systems of the form

$$\begin{cases} \dot{x} = -H_y(x,y), \\ \dot{y} = H_x(x,y), \end{cases}$$

where the Hamiltonian function is $H(x,y) = A(x) + B(x)y + C(x)y^2$. A necessary and sufficient condition for isochronicity in case that A, B and C are analytic was given in [A. Cima, F. Mañosas and J. Villadelprat, "Isochronicity for several classes of Hamiltonian systems", J. Differential Equations, 157 (1999) 373-413]. An involution σ inherited from the geometry of the periodic orbits surrounding the center plays a key role in this condition. In the present talk we will explain some recent results on the case that A, B and C are polynomials. Our results show that in order to study the polynomial isochrones one can reduce, essentially, to the case in which the involution σ is a Möbius transformation.

Hopf - Hopf bifurcation with 2:1 resonance. Periodic and chaotic solutions.

Dmitrii Volkov
Herzen University of Russia, Russia
dmitrivolkov@mail.ru
K. V. Galunova

A dissipative Hopf - Hopf bifurcation with 2:1 resonance are studied. A parameter dependent polynomial truncated normal form is derived. We study this truncated normal form (TNF system). This system displays a large variety of behaviour both regular and chaotic solution. Existence of the stationary and periodic solutions is proved. The stationary and periodic solutions of TNF will correspond with periodic solutions and two dimensional torus of the original system. The occurrence of chaos in the TNF is studied. It is shown that the chaotic dynamics of TNF can be described in terms of bimodal one dimensional map. Analogy between dissipative Hopf - Hopf bifurcation with 2:1 resonance, generations of second harmonics in non-linear optics and resonant interaction of waves in a plasma is presented.

Special Session 104: Instabilities and Bifurcations in Geophysical Fluid Dynamics

Ana Maria Mancho, Instituto de Ciencias Matematicas, CSIC, Spain
Jezabel Curbelo, Instituto de Ciencias Matemáticas-UAM, Spain

Instabilities and bifurcations are common in fluids of geophysical interest (ocean, atmosphere, lithosphere) and therefore relevant in the study and understanding of the natural features of the Earth and other planets. The aim of this special session is to present, discuss and exchange recent results, ideas and experiences about problems in the theoretical and computational aspects of stability and bifurcation theory related to geophysical fluid dynamics phenomena, such as shear and rotating flow instabilities, hydrodynamic stability, patterns of bifurcations, symmetry and symmetry-breaking, etc.

Quantitative thinking under the volcano: integrating rocks, numbers and experiments to advance the knowledge of sub-volcanic processes

Antonio Alvarez-Valero
Universidad de Salamanca, Spain
aav@usal.es
Francisco Pla

Modern Geology often utilizes numerical tools to elucidate and solve Earth Sciences questions that allow scientists to define different Earth properties and their variability in quantitative terms. In volcanology and petrology, numerical models of conduit dynamics are based on assumptions that need input from geologic studies on the petrologic and physicochemical evolution of magma systems; and vice versa, to fully interpret the petrological dataset requires key numerical parameters to constrain the entire information given by the rocks. We integrate classical petrology (and thermodynamic modelling), 2-D fluid dynamics simulations, and experiments (decompression rates and assimilation analogues) with the aim to advance in the knowledge of the country-rock's melting within a magma conduit before the eruption is already inevitable. Our approach confronts exhaustive field observations and chemical data (from Neogene silicic systems in SE Spain) with thermo-mechanical models of crust-magma interactions under the volcano. The ongoing results reveal fundamental information on the circulation and transport, from depth upward, of partly melted crustal restites (i.e., xenoliths) hosted by magma, as well as on the thermal interaction and assimilation processes between the crust and melt. They are based on the relationships of the microtextures and pressure-temperature conditions of crustal xenoliths to their position in a transient thermal regime in the wall-rock of the magma conduit, and to the time spent as a xenolith immersed in magma.

Onset of intermittent octahedral patterns in spherical Bénard convection

Philippe Beltrame
Université d'Avignon / UMR EmmaH, France
philippe.beltrame@univ-avignon.fr
Pascal Chossat

The onset of convection for spherically invariant Rayleigh-Bénard fluid flow is driven by marginal modes associated with spherical harmonics of a certain degree ℓ, which depends upon the aspect ratio of the spherical shell. At certain critical values of the aspect ratio, marginal modes of degrees ℓ and $\ell+1$ coexist. Initially motivated by an experiment of electrophoretic convection between two concentric spheres carried in the International Space Station (GeoFlow project), we analyze the occurrence of intermittent dynamics near bifurcation in the case when marginal modes with $\ell = 3, 4$ interact. The situation is by far more complex than in the well studied $\ell = 1, 2$ mode interaction, however we show that heteroclinic cycles connecting equilibria with octahedral as well as axial symmetry can exist near bifurcation under certain conditions. Numerical simulations and continuation (using the software AUTO) on the center manifold help understanding these scenarios and show that the dynamics in these cases exhibit intermittent behaviour, even though the heteroclinic cycles may not be asymptotically stable in the usual sense.

Slow dynamics in turbulence and their effect on geophysical flows

Javier Burguete
University of Navarra, Spain
javier@fisica.unav.es

The existence of slow scales in turbulence has an important effect in meteorology and in the geodynamo problem. In this work we will present the results of an experimental setup where these scales are investigated. Two different dynamics are observed: a multistability and random switching between different solutions of the average flow, and inverse cascades in the fully developed 3D experimental turbulent flow. In this last case we propose that the conserved magnitude is the angular momentum. The experimentl setup consists on a fluid in a closed cavity where two inhomogeneous and strongly turbulent flows collide in a thin region. The experimental volume is a closed cylinder (diameter of 20cm) where two impellers rotate in opposite directions.

Dynamic Patterns in an Asymptotically-Reduced Model of Upper Ocean Langmuir Circulation

Gregory Chini
University of New Hampshire, USA
greg.chini@unh.edu
Zhexuan Zhang, Keith Julien

Langmuir circulation (LC) is a convective flow in the ocean surface mixed layer characterized by an array of wind-aligned, counter-rotating vortices. The vortex system arises as an instability of a vertically-sheared current on which high-frequency surface gravity waves propagate. By filtering these waves, the Craik–Leibovich (CL) equations facilitate investigation of LC, but numerical simulations of the fully three-dimensional (3D) CL equations in spatially-extended domains nevertheless require immense

computing resources. Here, an asymptotically-reduced version of the CL equations – the rCL equations – is presented. The rCL equations render numerical simulations in very long (streamwise) domains more feasible by exploiting the strongly anisotropic structure of LC that emerges in the physically-relevant limit of strong surface-wave forcing. Linear and secondary stability analyses as well as pseudospectral nonlinear simulations of the rCL equations confirm that they capture dynamics reminiscent of LC. A novel 2:1 spatial resonance phenomenon, mediated by downwind variability of the convective flow, is shown to be a robust feature of the reduced dynamics. This 2:1 resonance is captured in its most pristine form in a numerically exact, steady 3D traveling wave solution of the rCL equations. The associated Lagrangian surface patterns reveal the presence of Y-junctions preferentially oriented downwind, as is commonly reported in field observations of LC.

Numerical evidence of inertia-gravity waves in a baroclinic cavity

Emilia Crespo del Arco
UNED, Spain
emi@fisfun.uned.es
Anthony Randriamampianina

Baroclinic flows can be studied in laboratory environments and in direct numerical simulations within a differentially heated rotating annulus, the so-called baroclinic cavity. In the present study a high resolution technique based on spectral collocation methods is used for the investigation of the complex flow regimes arising in a baroclinic cavity. The coupled Navier-Stokes and temperature equations are solved for a Boussinesq fluid. With a fluid with a high Prandtl number, Pr = 16, it is found that the stable stratified flow exhibits inertia-gravity waves. These waves have very small wavelength and period in comparison with the characteristic parameters of the baroclinic instability wave. In the atmosphere and in the oceans the thermal stratification is more important than the rotation rate, the representative Brunt Vaisälä frequency being typically about hundred times the Coriolis parameter f . In the present baroclinic cavity the local values of the Brunt Vaisälä frequency N are smaller than the rotation rate (of the order of one tenth). The inertia-gravity waves are first observed near the inner cold cylindrical wall, in the concave part of the baroclinic wave. In order to study the characteristic parameters of the inertia-gravity wave the numerical flow results have been filtered, the time dependent variables are decomposed into three parts: the time-averaged flow, the flow corresponding to the large-scale baroclinic wave and the flow representing the inertia-gravity wave. The baroclinic instability wave makes the effect of a background wind and the value of the intrinsic frequency for the small-scale wave is close to the Brunt Vaisälä frequency, thus in the low-frequency waves limit $f >> \omega \approx N$. In the results of the study, the phase velocity, the group velocity and the polarization of the inertia-gravity wave are discussed. An attempt to determine the generation mechanism of the spontaneous occurrence of these inertia-gravity waves simultaneously with the baroclinic instability is proposed.

Plate-like convection induced by symmetries in fluids with temperature-dependent viscosity

Jezabel Curbelo
Instituto de Ciencias Matemáticas- UAM, Spain
jezabel.curbelo@icmat.es
Ana M. Mancho

The study of instabilities in fluids in which viscosity experiences a transition at a certain temperature range is of great interest for the understanding of planetary interiors, since this phenomena is suitable for representing a lithosphere over a convecting mantle. To this end, we study a 2D convection problem with periodic boundary conditions along the horizontal coordinate, which introduce the O(2) symmetry, infinite Prandtl number and viscosity dependent on temperature. Notable solutions are found for a sharp transition viscosity law in which time-dependent solutions sporadically through abrupt bursts, alternate an upper stagnant lid with spontaneous plate-like behaviors that move either towards the right or towards the left.

Instabilities of the sidewall boundary layer in a rapidly rotating split cylinder

Paloma Gutierrez Castillo
Arizona State University, USA
paloma_gutierrez@hotmail.com
Juan M. Lopez

Understanding the behavior of shear layers and their instabilities is crucial because they are present in many rotating geophysical systems and they are responsible for important natural effects in the atmosphere and oceans. The flow in a completely filled rapidly rotating cylinder is studied numerically solving the three-dimensional Navier-Stokes equations using a spectral method. The cylinder is split in two with the top half rotating slightly faster than the bottom half. As the mean rotation is increased, the differential rotation drives thin boundary layers on the sidewall as well as on the top and bottom endwalls. In the absence of instabilities, the bulk is in solid-body rotation and the sidewall layer is of Stewartson-type. When the mean rotation increases, combined with an increase in the differential rotation, the sidewall boundary layer and the corner flow on the slower half-cylinder undergo a number of three-dimensional instabilities. These include slow low-azimuthal-wavenumber modes whose frequencies excite inertial waves in the interior as well as fast high-azimuthal-wavenumber modes whose impact is contained in the sidewall boundary layer region. Nonlinear competition due to Eckhaus instabilities and mode interactions abound.

Thermoconvective vortices in a cylindrical annulus varying inner radius

Henar Herrero
Universidad de Castilla-La Mancha, Spain
Henar.Herrero@uclm.es
Damián Castaño, María Cruz Navarro

The importance of thermoconvective processes in the formation and intensity of atmospheric phenomena such as dust devils, cyclones or hurricanes is well known. Dust devils are more likely to form in the presence of large horizontal temperature gradients, and the evolution of hurricane intensity depends, among other factors, on the heat exchange with the upper layer of the ocean under the core of the hurricane. All these atmosferic phenomena have a common vortical structure characterizes by a spiral up motion around an eye. No monotonic relationship is evident between eye diameter and intensity in tropical cyclones though the most intense ones all have small eye diameters. However, the change in the size of the eye can give useful information about the intensity trend. In previous work we reported that under particular thermal conditions (including vertical and horizontal temperature gradients) and geometrical conditions (aspect ratio) numerical vortices can be generated by a convective instability in a Rayleigh-Bénard problem in a cylindrical annulus with non-homogeneous heating from below and a lateral inflow/outflow permitted. In this talk we study numerically the generation of stable axisymmetric vortices by ther- moconvective mechanisms in a cylinder non-homogeneously heated from below compared to the case of an annulus. Moreover, we show the influence of the inner radius on the stability and intensity of those vortices. Little relation is found between the intensity of the vortex and the magnitude of the inner radius. Strong stable vortices can be found for both small and large values of the inner radius. The Rankine combined vortex structure, that characterizes dust devils, is clearly observed when small values of the inner radius are considered. A radial contraction on the radius of maximum azimuthal velocity is observed when the vortex is intensified. This radius becomes nearly stationary subsequently despite the vortex keeps intensifying. These results connect with the behavior of the radius of the maximum tangential wind associated with a hurricane. Finally we will relate these results with those obtained for the case where the domain is a cylinder.

Upscale energy transfer in three-dimensional rapidly rotating turbulent convection

Edgar Knobloch
University of California at Berkeley, USA
knobloch@berkeley.edu
A. Rubio, K. Julien and J.B. Weiss

Rotating Rayleigh-Bénard convection exhibits, in the limit of rapid rotation, a turbulent state known as geostrophic turbulence. This state is present for sufficiently large Rayleigh numbers representing the thermal forcing of the system, and is characterized by a leading order balance between the Coriolis force and pressure gradient. This turbulent state is itself unstable to the generation of depth-independent or barotropic vortex structures of ever larger scale through a process known as spectral condensation. This process involves an inverse cascade mechanism with a positive feedback loop whereby large-scale barotropic vortices organize small scale convective eddies. In turn, these eddies provide a dynamically evolving energy source for the large-scale barotropic component. Kinetic energy spectra for the barotropic dynamics are consistent with a k^{-3} downscale enstrophy cascade and an upscale cascade that steepens to k^{-3} as the box-scale condensate forms. At the same time the flow maintains a baroclinic convective component with an inertial range consistent with a $k^{-5/3}$ spectrum. The condensation process resembles a similar process in two dimensions but is fully three-dimensional.

Convection with semi-permeable boundary conditions

Stephane Labrosse
ENS de Lyon, France
stephane.labrosse@ens-lyon.fr
Fabien Dubuffet, Yanick Ricard

The large temperature excess in young planets leads to a large surface heat flow which is evacuated as much by intense volcanic activity as by thermal diffusion across a boundary layer. However, most fluid dynamic codes in Earth Science are implementing a zero vertical velocity at the surface of their models. This condition forbids any direct transport of material and therefore any advective extraction of heat from depth. We propose a new set of boundary conditions on the top surface for the momentum and energy equations. The vertical velocity is not imposed to zero but we compute the topography generated from this velocity. A diffusion term mimics the various processes that can redistribute the topography (mechanical and chemical erosion, gravity sliding, magma spreading on the surface...). The resulting topography affects the internal flow by imposing an equivalent vertical stress to the mantle. We show that with minimal approximations, the new condition can be implemented in numerical models. In the energy equation we impose a surface temperature only when the surface velocity is downward and a zero temperature gradient elsewhere. Our 2D and 3D numerical calculations of bottom and internally heated convection show that when increasing the Rayleigh number, the model evolves continuously from the typical pattern of convection and heat diffusion through a thermal boundary layer to a planform where all the heat is brought to the surface by large plumes. The extracted heat flux increases with the Rayleigh number from $Ra^{.3}$ at low Rayleigh number to $Ra^{.5}$ at high Rayleigh number where simultaneously, the temperature decreases from .5 to very low values. We discuss the implication of this model for the early evolution of the Earth and other solid planets, and the present state of Io.

Instabilities of plumes driven by localized heating in initially isothermal or stably stratified ambients

Juan Lopez
Arizona State University, USA
jmlopez@asu.edu
Francisco Marques

Plumes due to localized buoyancy sources are of wide interest due to their prevalence in many geophysical situations. This study investigates the transition from laminar to turbulent dynamics. Several experiments have reported that this transition is sensitive to external perturbations. As such, a well-controlled set-up has been chosen for our numerical study, consisting of a localized heat source at the bottom of an enclosed cylinder whose sidewall is maintained at either a fixed uniform temperature or a fixed temperature which varies linearly up the wall, and there is a localized heat source on the bottom. For uniform sidewall temperature, and a moderate heat source, the flow consists of a steady, axisymmetric purely poloidal plume. On the temperature of the hot spot, the flow undergoes a supercritical Hopf bifurcation to an axisymmetric "puffing" plume, where a vortex ring is periodically emitted from the localized heater. At higher Ra, this state becomes unstable to a sequence of symmetry-breaking bifurcations, going through a quasi-periodic "fluttering" stage where the axisymmetric rings are tilted, and other states in which the sequence of tilted rings interact with each other. The sequence of symmetry-breaking bifurcations in the transition to turbulence culminates in a torus breakup event in which all the spatial and spatio-temporal symmetries of the system are broken. With the linearly varying sidewall temperature, stratification effects come into play and the whole transition scenario changes. In particular, swirling flows states spontaneously appear. All of these various transition scenarios can be tied back to the possible ways that the symmetries of the system can be broken.

Inertial waves in a rapidly rotating cylinder flow

Francisco Marques
UPC, Spain
francisco.marques@upc.edu
Juan M. Lopez

Numerical simulations of a flow in a rapidly rotating cylinder subjected to a time-periodic forcing are presented. When the axial oscillation frequency is less than twice the rotation frequency, inertial waves in the form of shear layers are present. The driving mechanism is the oscillating Stokes layer on the sidewall and the corner discontinuities where the sidewall meets the top and bottom endwalls. A detailed numerical and theoretical analysis of the internal shear layers is presented. The system is physically realizable, and attractive because of the robustness of the Stokes layer that drives the inertial waves. The system losses stability to a complicated three-dimensional flow when the sidewall oscillation displacement amplitude is of the order of the cylinder radius, but this is far removed from the displacement amplitudes of interest, and there is a large range of governing parameters which are physically realizable in experiments in which the inertial waves are robust. We have computed the response diagram of the system for a large range of forcing frequencies and compared the results with inviscid eigenmodes and ray tracing techniques.

Subduction dynamics: influence of thermal heterogeneities in the upper plate.

Ana Negredo
Universidad Complutense Madrid, Spain
anegredo@fis.ucm.es
Juan Rodriguez-Gonzalez, Jorge M. Taramon

We present time-dependent three-dimensional (3D) fully-dynamic simulations of subduction to study the influence of a non-uniform overriding plate on the evolution of slab geometry and induced mantle flow. The equations of conservation of mass, momentum and energy are solved for a 3D incompressible fluid. Time-dependent subduction is driven by thermal density anomalies prescribed by the initial thermal structure. We find that along-strike variation in thermal thickness of the overriding plate causes increased hydrodynamic suction and shallower slab dip beneath the colder portion of the overriding plate; the variation in slab geometry drives strong trench-parallel flow beneath the slab and a complex flow pattern above the slab. This new mechanism for driving trench-parallel flow provides a good explanation for seismic anisotropy observations from the Middle and South America subduction zones, where both slab dip and overriding plate thermal state are strongly variable and correlated, and thus may be an important mechanism in other subduction zones. The development of hydrodynamic instabilities in the mantle wedge eventually leads to complete coupling between plates and subduction cessation. We further emphasize that the lithospheric structure of the overriding plate should be taken into account in analysis and modelling studies of subduction zones.

Influence of the composition on the onset of convection in rotating spherical shells

Marta Net
Universitat Politecnica de Catalunya, Spain
marta.net@upc.edu
Ferran Garcia, Juan Sanchez Umbria

The convection of a pure fluid in rotating spherical shells is well studied, but not much is known for binary fluids. The aim of this work is to study the influence of an externally forced concentration gradient on the onset of convection of a mixture of two components. To solve this problem two numerical methods are applied. The first computes the eigenvalues by evolving the linearized equations a selected time interval, and the second is based on a double complex shift. The results are compared with those obtained previously for a pure fluid of the same Prandtl number. For negative concentration gradients the critical thermal Rayleigh number, the precession frequency of the waves, and the azimuthal wave number, start to increase significantly when the compositional Rayleigh numbers are at least one order of magnitude larger than the thermal. On the contrary, for positive and sufficiently large concentration gradients, the thermal Rayleigh number decreases and suddenly changes sign indicating that the compositional gradients can trigger the convection even in presence of stabilizing temperature gradients. Then, the convection becomes almost stationary.

Localized convection in variable viscosity fluids: Implications for the dynamics of planetary interiors

Viatcheslav Solomatov
Washington University in St. Louis, USA
slava@wustl.edu

Theoretical and numerical studies of thermal convection on Earth and other planets have largely focused on convection above the critical Rayleigh number. However, convection in variable viscosity fluids can extend well below the critical Rayleigh number where it can exhibit new and interesting phenomena such as spatial localization. Two-dimensional and three-dimensional finite element calculations show that spatially localized convective cells can exist indefinitely at subcritical Rayleigh numbers. Among the factors which are responsible for the formation and the extent of the subcritical branch are the dependence of the viscosity on temperature and stress. Localized convection is shown to exist for several viscosity laws and in the parameter range applicable to rocky and icy planetary bodies in the Solar System. The interactions among spatially localized convective cells result in their attraction and cluster formation. Localized convection can contribute to the formation of localized surface features observed on Mercury, Mars, Venus and icy satellites of Jupiter.

Secondary bifurcations and connecting orbits in the thermal convection of rotating fluid spherical shells.

Juan Sánchez Umbría
Universitat Politécnica de Catalunya, Spain
juan.j.sanchez@upc.edu
F. Garcia, M. Net

The thermal convection of a pure fluid contained in a spherical shell rotating about a fixed axis is studied. The bifurcations from the conduction state give rise to azimuthal waves which have been computed by continuation methods, as steady solutions of a system for the waves, in the frame of reference of the spheres. Their stability has also been studied. We have found the coexistence of stable waves of different azimuthal wave number for the same value of the parameters of the problem due to the vicinity of a double-Hopf bfurcation. The eigenfunctions at the secondary bifurcation points reveal the nature of the modulation of the waves when they lose stability. If the waves are unstable, connecting orbits have been found by time integration, starting from the unstables states.

Stochastic slow-passage around resonance and saddle-node bifurcation in a forced Duffing oscillator

Bruno Welfert
Arizona State University, USA
welfert@asu.edu
Juan M. Lopez, Jason Yalim, Stephanie Taylor

Models of geophysical fluid flows typically depend on characteristic parameters such as a Reynolds number. At fixed critical values of the parameter(s) the solution may undergo bifurcations which change the nature of its behavior. However, in practice, shifts and uncertainties in physical or experimental conditions make it natural to investigate the effect of slow drifts and stochastic perturbations of such parameters on this behavior. In order to simplify the analysis and increase our understanding of these models we consider instead the more manageable case of a (normalized) damped Duffing oscillator

$$\ddot{x} + 2\gamma\dot{x} + k(x) = f(\theta),$$

where γ represents a (small) friction coefficient, $k(x)$ a (typically nonlinear) spring force, and $f(\theta)$ a 2π-periodic (possibly stochastic) forcing depending on an angle θ. The periodicity of f leads to sustained oscillations in x, whose amplitude depends on the specific choice of γ, $k(\cdot)$, and $f(\cdot)$. In the case $k(x) = x$ (linear spring) and $\theta = \omega t$ (static angular frequency ω), resonance effects typically occur (e.g. for $f(\theta) = \sin\theta$) when ω matches the natural frequency $(1-\gamma^2)^{1/2} \approx 1$ of the mass-spring system. In the case $k(x) = x - \eta x^3$ (softening nonlinear spring), the resonance amplitude undergoes a saddle-node bifurcation as η increases. In this work we investigate the effect of slow, stochastic, variations in the time evolution of θ, specifically

$$\dot{\theta} = \omega(\varepsilon t) + \sigma(t)\dot{W}_t,$$

where $\varepsilon \ll 1$, W_t is a Wiener process, and $\sigma^2(t)$ represents the variance of the (state-independent) stochastic contribution, on the system response. We focus in particular on how the resonance amplitude and the saddle-node bifurcation are modified from the static, deterministic, situation. This work is conducted in collaboration with Prof. Juan M. Lopez (Mathematics, ASU), Jason Yalim (graduate student, ASU), and Stephanie Taylor (undergraduate student, ASU).

Special Session 105: Geometric Mechanics

Hernan Cendra, Universidad Nacional del Sur, Argentina
Darryl Holm, Imperial College London, England
David Martin de Diego, ICMAT, Spain
Juan-Pablo Ortega, Centre National de la Recherche Scientifique (CNRS), France

In the 1960s new, powerful techniques from modern differential geometry and topology were introduced into the study of dynamical systems. This new field, now called Geometric Mechanics, has successfully reformulated quantum and classical analytic mechanics in geometric language and brought in new methods from topology and analysis. Geometric Mechanics has experienced a spectacular growth in the last years impacting all adjacent mathematical fields as well as mathematical physics and certain areas of engineering, particularly control and robotics. The guiding idea in Geometric Mechanics is the application of techniques and methods of differential geometry and Lie group theory to the formulation and analysis of dynamical systems (classical, field theoretical, or quantum). Symplectic structures and their natural generalizations (the Poisson and Dirac manifolds) comprise the natural framework in the description and study of various phenomena that appear in classical (quantum) mechanics, among which we mention the following: symmetry reduction (both for finite and infinite dimensional systems) in a classical and quantum setting, Hamilton-Jacobi theory, mechanical systems that are subjected to external (possibly non-holonomic) constraints, the modeling of friction, geometric methods for numerical integration . . . The aim of the session is to provide a forum to discuss recent significant research efforts in Geometric Mechanics that reveal the current challenges on this topic.

Double bracket vector fields and embedding problems

Petre Birtea
West University of Timisoara, Department of Mathematics, Romania
birtea@math.uvt.ro
Dan Comanescu

We will present a general mechanism for constructing gradient vector fields on regular submanifolds. This construction is related to the double bracket vector fields on semisimple Lie algebras. We will apply this geometrical setting to embedding problems in optimization algorithms.

Completeness for Sobolev Metrics on the Space of Plane Curves

Martins Bruveris
EPFL, Switzerland
martins.bruveris@epfl.ch
Peter W. Michor, David Mumford

Riemannian metrics on the space of curves are used in shape analysis to describe deformations that take one shape to another and to define a distance between shapes. The talk will focus on a particular class of metrics, metrics of Sobolev type. They arose from the need of strengthen the L^2-metric, which was found to have vanishing geodesic distance. I will describe recent work on the geodesic and metric completeness of Soblev metrics on the space of plane curves.

Higher Order Variational Integrators: the Galerkin method

Cédric M. Campos
ImUVA (Inst. Matemáticas UVA), Spain
cedricmc@icmat.es

It is well-known that a particular subset of partitioned Runge-Kutta methods are symplectic. In this talk, I will introduce a variational derivation of a family of higher order symplectic integrators and I will discuss their relation with respect to symplectic partitioned Runge-Kutta methods. Some numerical simulations will be also of interest for the discussion.

Cartan algorithm and contraints in field theory

Santiago Capriotti
Universidad Nacional del Sur, Argentina
santiago.capriotti@uns.edu.ar
Hernán Cendra

(Pre)multisymplectic manifolds provide a geometrical framework for the equations of motion of field theory, as (pre)symplectic manifolds does for dynamical systems. In the same way, there exists a procedure in the Hamiltonian formalism for field theories that allows us to deal with constraints, analogous to Gotay-Nester algorithm. The novel feature is that this algorithm has to deal with integrability issues, absent in the Gotay-Nester algorithm due to dimensional reasons; the natural outcome of this procedure is a set of decomposable multivectors, yielding to solutions of the Hamiltonian field equation whenever these integrability questions can be solved. On the other side, the Cartan algorithm is a procedure to ensure existence of integral manifold for linear Pfaffian exterior differential systems (EDS); namely, whether such algorithm can be applied to an EDS version of the Hamiltonian field equations, it is automatically ensured the question regarding integrability, and a set of constraints assuring integrability can be found. In the present talk we will show with some examples how to encode field theory and its equations in a linear Pfaffian EDS, and the way in which Cartan algorithm can be used in order to deal with them.

Geometric generalizations of the Virial Theorem

José Cariñena
Universidad de Zaragoza, Spain
jfc@unizar.es

The Virial's theorem is revisited from a modern geometric perspective by making use of the symplectic formalism as an approach, both in the case of Hamiltonian and Lagrangian systems. The particular case of Lagrangians of mechanical type will be examined and homothetic vector fields in the Riemann base manifold are related with some Virial-like theorems. It is also proved that the theory can be extended to the framework of mechanics on Lie algebroids. Several applications of the theory will be shown.

Structure of the Hamiltonian equations of gauge invariant problems

Marco Castrillon Lopez
ICMAT - Universidad Complutense de Madrid, Spain
mcastri@mat.ucm.es
Jaime Munoz Masque

Let $C \to M$ be the bundle of connections of a principal bundle on M. Solutions to Hamilton-Cartan equations for a first order gauge-invariant Lagrangian density on C are not equivalent to solutions of the Euler-Lagrange equations. In this talk it is proved that, under a weak condition of regularity, the set of Hamiltonian solutions admits the structure of an affine bundle over the set of solutions of the Lagrangian problem. This structure is also studied for the Jacobi fields and for the moduli space of extremals.

Dirac Systems and Applications

Hernan Cendra
Universidad Nacional del Sur, Argentina
hcendra@gmail.com
M. Barbero Liñán, M. Etchechoury and D. Martín de Diego

Dirac structures were introduced by A. Weinstein and T.J. Courant in 1987 as a geometric structure unifying Poisson and presymplectic structures, with a motivation from the Dirac theory of constraints. Since then, Dirac geometry has evolved in several directions. Part of its versatility, as compared with Poisson or symplectic geometry, comes from the transformation properties of Dirac structures, which leads to a unification of several fundamental physical models. This includes Lagrangian and Hamiltonian mechanics, nonholonomic mechanics, control, optimal control and interconnected systems. I will expose some recent developments in this direction.

A gravity-gradient intermediary in attitude dynamics. Relative equilibria and bifurcations

Francisco Crespo
Universidad de Murcia, Spain
francisco.crespo@um.es
S. Ferrer, F.J. Molero, J.C. van der Meer

Relative equilibria of an intermediary in attitude dynamics of a generic triaxial spacecraft in a circular orbit under gravity-gradient perturbation are discussed. Intermediary defines a Poisson flow over a large parameter space: three physical parameters (moments of intertia) and three distinguished parameters, the integrals M, G_3 and n. In the case of slow motion we identify conditions under which different bifurcations of the classic unstable trajectories occur, scenario of great interest in relation to stabilization and control purposes. Our study is based on the use of the invariants defining the full reduced $\mathbb{S}^2 \times \mathbb{S}^2$ space orbital space and the associated energy-momentum mapping. We identify bifurcation curves along which the system shows degeneracy, connected with the change of stability of the classic unstable equilibria of the second reduced space (Euler system). The role played by the triaxiality is illustrated with several examples.

Dissipative MHD equations via stochastic variational principles

Ana Bela Cruzeiro
Dep. Mathematics IST ULisboa, Portugal
abcruz@math.ist.utl.pt
Xin Chen, Tudor Ratiu

We derive Euler-Poincaré equations for stochastic processes defined on semi-direct product Lie algebras, that allow to consider dissipative systems. Specializing to the continuous case, we derive MHD equations for viscous compressible fluids.

Reduction of the Hamilton-Jacobi equation

Manuel de Leon
ICMAT, Spain
mdeleon@icmat.es

We study the Hamilton-Jacobi equation for hamiltonian systems with symmetry.

Generalized variational calculus for mechanical systems

Viviana Díaz
Universidad Nacional del Sur, Argentina
viviana.diaz@uns.edu.ar
David Martín de Diego

We will consider a formalism that generalizes the variational calculus and allows us to deal with different mechanical systems within the same framework. In particular, we will describe how the cases of Lagrangian and Hamiltonian mechanics, subjected to constraints or not, fit in this setting. The proposed formalism is based on the notions of the tangent lift of curves and the complete lift of a vector field. Hence it can be extended to the Lie algebroids theory, and thus it includes also the case of reduced mechanical systems.

Walking as a limit cycle through symmetry reduction

Jaap Eldering
Imperial College London, England
eldering@a-eskwadraat.nl
Henry Jacobs

We present a toy mass-spring model of bio-locomotion to identify core mathematical principles for modelling e.g. walking or crawling. The system is invariant under the isometry group of the (2D/3D) environment. We use a combination of symmetry, dissipation, and regularization of ground contact to prove the existence of a relative limit cycle, that is, a trajectory which is a limit cycle upon reduction by symmetry. After actuating the system and lifting a perturbed limit cycle to the original phase space, we find that the phase shift of each period can be viewed as the step size of a locomotive gait.

Bi-symplectic $\mathbb{N}Q$-algebras of weight 1

David Fernandez
ICMAT, Spain
david.fernandez@icmat.es

By a general principle in noncommutative algebraic geometry (as formulated by M. Kontsevich and A. Rosenberg), a property of an associative algebra A is 'geometric' if it induces standard geometric properties on its representation spaces $\mathrm{Rep}(A,V)$. Here, $\mathrm{Rep}(A,V)$ is the space of all representations of A in a finite-dimensional vector space V, which has a well-known affine-scheme structure. According to this principle, the family of affine schemes $\mathrm{Rep}(A,V)$ should be regarded as a substitute for a hypothetical affine noncommutative scheme 'Spec(A)'. This principle has been applied successfully to symplectic structures and Poisson structures on quiver algebras by W. Crawley-Boevey, P. Etingof, V. Ginzburg, and M. Van den Bergh, among others. In this talk we will consider noncommutative analogues of other standard geometric structures. More precisely, we will introduce bi-symplectic $\mathbb{N}Q$-algebras. These are noncommutative counterparts of symplectic $\mathbb{N}Q$-manifolds, which are basic ingredients encoding higher Lie algebroid structures in the Batalin-Vilkovisky formulation of Topological Quantum Field Theories. The natural context to study these structures is graded noncommutative algebraic geometry. In particular, we will explain how bi-symplectic $\mathbb{N}Q$-algebras of weight 1 are closely related to Van den Bergh's double Poisson algebras. To establish this result, we will explain an analogue of the odd Darboux Theorem in this setting. Joint work with Luis Álvarez-Cónsul

Relative equilibria for the n body problem in spaces of constant negative curvature

Luis Garcia Naranjo
UNAM, Mexico
luis.garcianaranjo@gmail.com
Juan Carlos Marrero, Ernesto Perez Chavela

We consider the n body problem in spaces of constant negative curvature. We study existence and dynamical properties of relative equilibria of the problem. These correspond to solutions in which the pairwise distance between the point masses is constant throughout the motion. Our results are given in terms of intrinsic Riemannian data and are therefore valid in any model of the hyperbolic space.

Hamiltonian reduction of the midpoint discretization for optimal control problems

Pedro Luis García
Universidad de Salamanca y Real Academia de Ciencias, Spain
pgarcia@usal.es

Given a regular optimal control problem with m control variables, we can construct m infinitesimal symmetries from its midpoint discretization. The Noether invariants associated to these symmetries are used to construct a Hamiltonian reduction and the corresponding variational integrator. The theory is illustrated with the elementary example of the Heisenberg problem. This is a joint work with Antonio Fernandez.

Lagrangian reduction and integrability in condensed matter

Francois Gay-Balmaz
Ecole Normale Superieure de Paris, France
gaybalma@lmd.ens.fr
M. Monastyrsky, T. Ratiu.

We consider a general approach for the process of Lagrangian and Hamiltonian reduction by symmetries in condensed matter. This approach is used to show the complete integrability of several one dimensional texture equations arising in liquid Helium phases and neutron stars. The key to the success of our geometric method is the fact that all physical systems under study have a natural Lagrangian and Hamiltonian formulation within the Lagrange-Poincaré and Hamilton-Poincaré theories, with the Lagrangian and Hamiltonian independent on a very special group of variables. This implies that these systems have an equivalent Euler-Poincaré and Lie-Poisson description which turns out to be considerably simpler and more appropriate to the study of the dynamics of the equations associated to the relevant phases. The possibility of using at once the four descriptions of the systems under consideration leads directly to the proof of complete integrability of the equations describing the system's behavior in different phases.

Stability of Hamiltonian relative equilibria in symmetric magnetically confined rigid bodies: orbitrons, levitrons, and generalizations

Lyudmila Grigoryeva
Université de Franche-Comté, France
lyudmyla.grygoryeva@univ-fcomte.fr
Juan-Pablo Ortega, Stanislav Zub

We study the symmetries, the associated momentum map, and relative equilibria of mechanical systems consisting of magnetically confined rigid bodies in axisymmetric external magnetic fields; some instances of such systems include the so called orbitrons, levitrons, and others. We study the nonlinear stability of branches of relative equilibria using the energy-momentum method and we provide sufficient conditions for the existence of G_μ-stability. These stability prescriptions are explicitly written down in terms of the field parameters, which can be used in the design of stable solutions. We propose new linear methods to determine instability regions in the context of relative equilibria that we use to conclude the sharpness of some of the nonlinear stability conditions obtained. The results that we present are proved to be generalizable to other specific symmetric systems of magnetically confined rigid bodies.

Momentum Maps, Shape Analysis and Solitons

Darryl Holm
Imperial College London, England
d.holm@ic.ac.uk
M Bruveris, CJ Cotter, F Gay-Balmaz, DM Meier, TS Ratiu

Much of this talk is based on work done with Jerry Marsden (1942 - 2010) on shared geometric properties in the analysis of fluid flow and shape transformations. The talk will discuss uses of geometric mechanics in the problem of registration of images, primarily in the example of planar closed curves. Many types of mathematics apply in this problem, including soliton theory and momentum maps. Some trade secrets will be revealed.

On a class of Kählerian Hamiltonian systems on infinite dimensional Kähler manifolds with holomorphic flows

Alberto Ibort
ICMAT, UCIIIM, Spain
albertoi@math.uc3m.es

Kato's representation theorem provides a necessary and sufficient condition for a densely defined positive Hermitean quadratic form on a Hilbert space to be representable by a (positive) self-adjoint operator, hence to define a strongly continuous one-parameter group of unitary operators and conversely. In this talk we will explore a non-linear extension of this correspondence: given an infinite dimensional Kähler manifold, we will introduce a class of functions (not necessarily continuous) that give rise to one-parameter groups of holomorphic transformations as in Kato's theorem, that is, they can be represented by a Hamiltonian dynamical system, whose flow is the desired one-parameter group of transformations. These functions will be called Kählerian and they will be characterized in terms of the analytical structure of the Kähler manifold.

Multisymplectic geometry and Lie groupoids

David Iglesias Ponte
University of La Laguna, Spain
diglesia@ull.edu.es

Multisymplectic groupoids, a higher-degree generalization of symplectic groupoids, are presented. Recalling that Poisson structures may be viewed as infinitesimal counterparts of symplectic groupoids, we describe "higher" versions of Poisson structures by identifying the infinitesimal counterparts of multisymplectic groupoids. Some basic examples and features are discussed, in particular, how they appear in the description Lie-Poisson reduction in field theories.

G-strand equations - some examples

Rossen Ivanov
Dublin Institute of Technology, Ireland
rossen.ivanov@dit.ie
Darryl Holm

The G-strand equations for a map $\mathbb{R}\times\mathbb{R}$ into a Lie group G are associated to a G-invariant Lagrangian. The Lie group manifold is also the configuration space for the Lagrangian. The G-strand itself is the map $g(t,s):\mathbb{R}\times\mathbb{R}\to G$, where t and s are the independent variables of the G-strand equations. The Euler-Poincaré reduction of the variational principle leads to a formulation where the dependent variables of the G-strand equations take values in the corresponding Lie algebra $\mathfrak{g}$ and its co-algebra, $\mathfrak{g}^*$ with respect to the pairing provided by the variational derivatives of the Lagrangian. We review examples of G-strand constructions, including matrix Lie groups of low ranks, and the Diffeomorphism group. In some cases the arising G-strand equations are completely integrable $1+1$ Hamiltonian systems that admit soliton solutions.Our presentation is aimed to illustrate the G-strand construction with several simple but instructive examples: (i) $SO(3)$ and $SO(4)$-strand integrable equations for Lax operators, quadratic in the spectral parameter; (ii) Diff$(\mathbb{R})$-strand equations. These equations are in general non-integrable; however they admit solutions in $2+1$ space-time with singular support (e.g., peakons). The one- and two-peakon equations obtained from the Diff$(\mathbb{R})$-strand equations can be solved analytically, and potentially they can be applied in the theory of image registration. Our example is with a system which is a $2+1$ generalization of the Hunter-Saxton equation.

A multiscale method for ideal fluid flow

Henry Jacobs
Imperial College London, USA
hoj201@gmail.com
Darryl D. Holm, Collin J. Cotter, David M. Meier

In this talk we present an infinite hierarchy of particle-like solutions to a regularized form of Euler's fluid equations. In these particle-like solutions each particle stores internal Lie group structures which correspond to higher order deformation gradients of the flow map. Collision experiments suggest that two particles at one level in the hierachy can asymptotically merge into a single particle at a higher-level in the hierarchy. We will display some of these collisions and provide a formal argument to explain this phenomena. These collision events are interpreted as a cascade to smaller scales.

Constrained Mechanics and Idealized Models for Propulsive Vortex Shedding in Fluids

Scott Kelly
University of North Carolina at Charlotte, USA
scott@kellyfish.net

Systems comprising free solid bodies interacting with singular distributions of vorticity in two- and three-dimensional ideal fluids exhibit noncanonical Hamiltonian structures. Velocity constraints like the classical Kutta condition can be imposed on such systems discretely or continuously in time to model localized boundary layer detachment and vortex shedding in an idealized way. This approach provides a basis for the reduced-order modeling of biological and robotic locomotion in viscous fluids. This talk will detail a selection of models obtained in this way, addressing the role played by symmetry breaking in enabling locomotion and the integrability of the constraints in question.

Towards a Geometric Mechanics of Acoustic Streaming

Jair Koiller
FGV - INMETRO, Brazil
jkoiller@fgv.br
Kurt Ehlers

Acoustic streaming (AS) is a phenomenon discovered by Faraday and later analyzed mathematically by Rayleigh. It is a somewhat undesired effect in medical ultrasound. In the 90's it was shown that it could be used in micro scale for pumping and mixing. This is now a mature technology. We speculated that certain algae have used AS 4.5 gyr ago. In this talk we review the phenomenology and the basic mathematical formulation as presented by Nyborg in the 50's. We discuss a geometry that we believe has good prospect. We are currently also preparing an experiment at the Brazilian Metrology Institute (INMETRO).

Space-Time Finite-Element Exterior Calculus and Variational Discretizations of Gauge Field Theories Presentation

Melvin Leok
University of California, San Diego, USA
mleok@math.ucsd.edu
Joe Salamon, John Moody

Many gauge field theories can be described using a multisymplectic Lagrangian formulation, where the Lagrangian density involves space-time differential forms. While there has been much work on finite-element exterior calculus for spatial and tensor product space-time domains, there has been less done for space-time simplicial complexes. One critical aspect is that the Hodge star is now taken with respect to a pseudo-Riemannian metric, and this is most naturally expressed in space-time adapted coordinates, as opposed to the barycentric coordinates that Whitney forms (and their higher-degree generalizations) are typically expressed in terms of. We introduce a novel characterization of Whitney forms and their Hodge dual with respect to a pseudo-Riemannian metric that is independent of the choice of coordinates, and then apply it to a variational discretization of the covariant formulation of Maxwell's equations. Since the Lagrangian density for this is expressed in terms of the exterior derivative of the four-potential, the use of finite-dimensional function spaces that respects the de Rham cohomology results in a discretization that inherits the gauge symmetries of the continuous problem. This then yields a variational discretization that exhibits a discrete Noether's theorem, which implies that an associated multi-momentum is automatically conserved by the discretization.

The exact discrete Lagrangian function on a Lie groupoid

J C Marrero
University of La Laguna, Spain
jcmarrer@ull.edu.es
D Martin de Diego and E Martinez

In this talk, I will present a definition of the exact discrete Lagrangian function associated with a continuous regular Lagrangian function on the Lie algebroid of a Lie groupoid. For this purpose, I will use some results on second order differential equations on the vertical bundle of a fibration. These results may be proved using classical theorems on second order differential equations. It is a work in collaboration with D Martín de Diego and E Martínez.

Optimal control of nonholonomic systems

David Martin de Diego
ICMAT, Spain
david.martin@icmat.es

We will describe a new geometric description for optimal control of nonholonomic systems.

Jacobi fields and the second variation formula

Eduardo Martinez
University of Zaragoza, Spain
emf@unizar.es

For a second order differential equation (sode) on a Lie algebroid, the analog of the Jacobi equation for a geodesic problem will be obtained. When the sode is defined by a regular Lagrangian, the relation with the second variation formula for the action will be analyzed.

Homogeneity and Lagrangian systems

Tom Mestdag
Ghent University, Belgium
tom.mestdag@ugent.be

In this talk we show that the unparametrized trajectories of a Lagrangian system, after a suitable choice of parameter, can be derived from a variational principle involving a 1-homogeneous function. Our approach is based on Routh's reduction procedure. We will also discuss the so-called Mane critical value in this context. This value of the total energy provides a lower bound for the 1-homogeneous function to be in fact a Finsler function.

Symmetries of b-manifolds and their generalizations

Eva Miranda
UPC, Spain
eva.miranda@upc.edu

The aim of this talk is to show some examples of simple Poisson manifolds which have some common features with symplectic manifolds (including the study of group actions). I will start presenting a Delzant theorem for toric b-symplectic manifolds (joint work with Victor Guillemin, Ana Rita Pires and Geoffrey Scott) taking the 2-dimensional case as starting point. Time permitting, I will report on an ongoing project with Geoffrey Scott on generalizing the notion of b-symplectic manifold. This notion includes other Poisson manifolds which share good properties with b-symplectic manifolds and seem to have less topological constraints.

Optimal information transport

Klas Modin
Chalmers University of Technology, Sweden
klas.modin@chalmers.se

In this talk I will present the framework of optimal information transport (OIT) and the connection to the Fisher-Rao information metric. I will focus on the underlying infinite dimensional Riemannian geometry, and compare it with the geometry of optimal mass transport (OMT) and the Wasserstein L^2 distance, as described by Otto (2001). There are similarities and dissimilarities between OMT and OIT: the "lifting equations" in both frameworks involve PDEs, it is non-linear in the former (the Monge-Ampere equation) and linear in the latter (the Poisson equation). I will discuss some applications currently employing OMT, where OIT might be an alternative. Since the lifting PDE for OIT is linear, one can expect significantly faster simulation algorithms.

Topology of Symmetric Loops Spaces

James Montaldi
University of Manchester, England
j.montaldi@manchester.ac.uk

Motivated by the search for symmetric periodic orbits of the n-body problem in the form of choreographies, I describe a method for determining the topology of the space of symmetric loops, for given spatio-temporal symmetry groups. In the example of choreographies, this is related to conjugacy in subgroups of the braid group and their cosets.

Reduction of symplectic Lie algebroids

Edith Padron
University of La Laguna, Spain
mepadron@ull.es

We will present in this talk several reduction procedures (with a momentum map or by a subalgebroid) of a symplectic Lie algebroid under certain conditions. We will present several examples and we will discuss the problem of extending our results for a general case.

A matrix-based framework to structure-preserving discretization of continuum theories

Dmitry Pavlov
Imperial College London, England
dmitry.pavlov@icloud.com

I will describe a framework for constructing discrete models of infinite-dimensional systems which preserve underlying geometry. These models lead to numerical methods that capture the dynamics of the system without energy or momenta loss and preserve momentum maps in discrete realm. This work starts with developing a matrix-based exterior calculus. This calculus extends the classical Discrete Exterior Calculus providing us with the notions of discrete Lie derivative, interior product etc., while preserving many properties of their continuous counterparts. This matrix-based exterior calculus has been used to create structure-preserving discretizations of various systems, such as fluid dynamics, magnetohydrodynamics, complex fluids etc. I will show how it can be used to construct a variational integrator for the Euler and EPDiff equations. I will also describe how these methods can be extended to create a new model of discrete differential geometry. This approach uses ideas of noncommutative geometry and can lead to a structure-preserving discretization of general relativity.

KAM theory and the 3D Euler equation

Daniel Peralta-Salas
ICMAT, Spain
dperalta@icmat.es

We will show that the dynamical system defined by the hydrodynamical Euler equation on any closed Riemannian 3-manifold is not ergodic. To prove this result we introduce a family of functionals on the space of divergence-free vector fields, which are integrals of motion of the 3D Euler equation. These functionals measure the part of the manifold foliated by ergodic invariant tori of fixed isotopy types, so KAM theory is the main tool to analyze some continuity properties of these new conservation laws. This is based on joint work with Boris Khesin and Sergei Kuksin.

Geometric theory of garden hose dynamics

Vakhtang Putkaradze
University of Alberta, Canada
putkarad@ualberta.ca
Francois Gay-Balmaz

A garden hose inevitably wiggles and twists when water is rushing through it. We derive a fully three-dimensional, geometrically exact theory for this phenomenon. The theory also incorporates the change of the cross-section available to the fluid motion during the dynamics. Our approach is based on the symmetry-reduced, exact geometric description for elastic rods, coupled with the fluid transport and subject to the volume conservation constraint for the fluid. We analyze the linear stability, and show that the change of cross-section plays an important role. We derive and analyze several analytical, fully nonlinear solutions of traveling wave type in two dimensions. Time permitting, we shall also discuss the effects of the boundary conditions and experimental results.

The Flaschka transformation

Tudor Ratiu
Ecole Polytechnique Federale de Lausanne, Switzerland
tudor.ratiu@epfl.ch
Anthony Bloch, Francois Gay-Balmaz

The Flaschka transformation has been introduced historically by Flaschka and Manakov in the early 70s in order to provide a Lax pair for the Toda lattice integrable system. It turns out that this is just an instance of a map that provides a symplectomorphism between certain coadjoint orbits and magnetic cotangent bundles. If the structure of the coadjoint orbit is simple enough, then this method provides global canonical coordinates; this is the case for the finite Toda systems associated to an arbitrary Dynkin diagram. We shall discuss the general geometric setup leading to this map, connect it to a momentum map and to Pukanszky's conditions. The example of the Toda lattice will be presented as well as other ones that, classically, do not fall in the same category of orbits.

Lagrangian reduction of quantum variational principles

Cesare Tronci
University of Surrey, England
c.tronci@surrey.ac.uk
Esther Bonet-Luz

Geometric quantum mechanics has various applications ranging from quantum tomography to quantum computation and control. In finite dimensional quantum systems, deep geometric concepts emerge naturally, such as Stiefel manifolds and Grassmannians, projective spaces and holonomy. While the first part of the talk will illustrate how these structures pop up, the second part will show how they are used in the Lagrangian reduction of the quantum (Dirac-Frenkel) variational principle. For example, the Schrödinger and Heisenberg pictures are shown to correspond respectively to the Eulerian and convective frameworks of Euler-Poincaré theory. Also, the geometric phase is shown to obey a Kelvin-Noether theorem arising from relabeling symmetry.

Variational Partitioned Runge-Kutta methods for Lagrangians linear in velocities

Tomasz Tyranowski
Imperial College, England
ttyranow@imperial.ac.uk
Mathieu Desbrun

In this talk we propose higher-order variational integrators for a class of degenerate systems described by Lagrangians that are linear in velocities. We analyze the geometry underlying such systems and develop the appropriate theory for variational integration. Our main observation is that the evolution takes place on the primary constraint and the 'Hamiltonian' equations of motion can be formulated as an index-1 differential-algebraic system. We also construct variational Runge-Kutta methods and analyze their properties. The general properties of Runge-Kutta methods depend on the 'velocity' part of the Lagrangian. If the 'velocity' part is also linear in the position coordinate, then we show that non-partitioned variational Runge-Kutta methods are equivalent to integration of the corresponding first-order Euler-Lagrange equations, which have the form of a Poisson system with a constant structure matrix, and the classical properties of the Runge-Kutta method are retained. If the 'velocity' part is nonlinear in the position coordinate, we observe a reduction of the order of convergence, which is typical of numerical integration of DAEs. We verify our results through numerical experiments for various dynamical systems.

k–symplectic Lie systems: theory and applications

Silvia Vilariño Fernandez
Centro Universitario de La Defensa- Zaragoza, Spain
silvia.vilarino@gmail.com
Javier de Lucas Araujo

A *Lie system* is a system of first–order ordinary differential equations describing the integral curves of a t-dependent vector field taking values in a finite-dimensional real Lie algebra of vector fields: a so-called *Vessiot–Guldberg Lie algebra*. We here suggest the definition of a particular class of Lie systems, the *k–symplectic Lie systems*, admitting a Vessiot–Guldberg Lie algebra of Hamiltonian vector fields with respect to the presymplectic forms of a k–symplectic structure. We devise new k-symplectic geometric methods to study their superposition rules, time independent constants and general properties. Our results are illustrated by examples of physical and mathematical interest. As a byproduct, we find a new interesting setting of application of k–symplectic geometry: systems of first-order ordinary differential equations.

References

[1] A. Awane, k–symplectic structures, *J. Math. Phys.* **33** (1992) 4046–4052.

[2] J.F. Cariñena, J. Grabowski and G. Marmo, Superposition rules, Lie theorem and partial differential equations, *Rep. Math. Phys.* **60** (2007) 237–258.

[3] J.F. Cariñena and J. de Lucas, *Lie systems: theory, generalisations, and applications.* Diss. Math. (Rozprawy Math.) **479**. Warsaw: Institute of Mathematics of the Polish Academy of Sciences, 2011.

Coadjoint orbits of the Hamiltonian group via symplectic reduction of the ideal fluid dual pair

Cornelia Vizman
West University of Timisoara, Romania
vizman@math.uvt.ro
Francois Gay-Balmaz

Dual pairs of momentum maps associated to commuting actions of two finite dimensional Lie groups have a nice property: symplectic reduction for one of the groups provides coadjoint orbits of the other group. We give a version of this result that works in infinite dimensions. We apply it to the ideal fluid dual pair due to Marsden and Weinstein, which consists of two momentum maps defined on the manifold of embeddings $Emb(S, M)$ of a manifold S endowed with a volume form into a symplectic manifold M. We get new coadjoint orbits of the Hamiltonian group of M, namely spaces of isotropic submanifolds of M endowed with volume forms.

Hamilton-Jacobi Theorems for Nonholonomic Reducible Hamiltonian Systems on a Cotangent Bundle

Hong Wang
Nankai University, China, Peoples Rep of China
hongwang@nankai.edu.cn
Manuel de Leon

In this talk, we first generalize the geometric Hamilton-Jacobi theorem for Hamiltonian system to the nonholonomic context, and obtain a Hamilton-Jacobi theorem for non-holonomic Hamiltonian system on the cotangent bundle of a configuration manifold, by using its distributional Hamiltonian system under a weaker condition. Then we generalize the above result to non-holonomic reducible Hamiltonian system with symmetry, as well as with momentum map, and obtain the Hamilton-Jacobi theorems for non-holonomic reduced Hamiltonian systems, by using the non-holonomic reduced distributional Hamiltonian systems. As an application of the theoretical results, we consider the motions of the constrained particle in space $\mathbb{R}^3$ and the vertical rolling disk, and give the Hamilton-Jacobi equations of the two systems as nonholonomic reduced distributional Hamiltonian systems.

Dirac Dynamical Systems with Symmetry and Applications to Nonholonomic Systems

Hiroaki Yoshimura
Waseda University, Japan
yoshimura@waseda.jp
Francois Gay-Balmaz

We will talk about reduction of Dirac structures by symmetry for nonholonomic systems on Lie groups with broken symmetry. We will review Lagrange-Dirac systems as well as Hamilton-Dirac systems over Lie groups and Lie-Dirac reduction of associated dynamics. Regarding dynamics of rigid body systems and fluids, we will show the reduction of Hamilton-Pontryagin principle and Lie-Dirac reduction with advected parameters by extending the so-called Euler-Poincare and Lie-Poisson reduction with advected parameters. In particular, we will show the Euler-Poincare-Dirac reduction with advective parameters and with nonholonomic constraints. Our theory will be demonstrated by some illustrative examples.

Multisymplectic manifolds, homotopy moment maps, and conserved quantities

Marco Zambon
ICMAT, Univesidad Autonoma de Madrid, Spain
marco.zambon@uam.es
Yael Fregier and Chris Rogers

Multisymplectic structures are higher generalizations of symplectic structures, where forms of higher degree are considered. We introduce a notion of moment map for such structures, based on Stasheff's notion of strong homotopy Lie algebra, and by relating it to equivariant cohomology we are able to produce many examples. Further, we discuss some geometric implications of our moment maps: the existence of conserved quantities, an induced hierarchy of moment maps, and reduction.

Markovian and non-Markovian stochastic deformations of Mechanics.

Jean-Claude Zambrini
Group of Mathematical Physics Univ. Lisbon, Portugal
zambrini@cii.fc.ul.pt
Remi Lassalle

When looking for stochastic deformations of various fundamental aspects of Mechanics ,the Markov property of the stochastic processes involved is almost always regarded as essential. However,there are many interesting cases where it is not sufficient. We shall describe some of these cases,in contrast with the Markovian scenarios.

Hamel Formalism and Variational Integrators

Dmitry Zenkov
North Carolina State University, USA

Hamel's formalism is a Lagrangian analogue of representation of Hamiltonian mechanics in non-canonical coordinates. This talk will elucidate the variational nature of Hamel's formalism and discuss applications to dynamics of mechanical systems with velocity constraints and structurally-stable variational integrators.

Special Session 107: Spatial and Temporal Heterogeneity in Reaction-Diffusion-Advection Models and Applications to Biology

Robert Stephen Cantrell, University of Miami, USA
Chris Cosner, University of Miami, USA
Yuan Lou, Ohio State University and Renmin University, USA
King-Yeung Lam, Ohio State University, USA

The special session aims to address some recent developments in mathematical modeling and analysis of reaction-diffusion-advection models in biology, with emphasis on spatial and temporal variability. Reaction-diffusion-advection models are ubiquitous tools to couple spatio-temporal interaction with biological dynamics. Quantifying the effect of such heterogeneities presents significant mathematical challenges. More generally, understanding biological processes in spatially and / or temporally varying environments can significantly enhance our understanding of how diversity is maintained in complex ecosystems and how organisms respond to different changes in a global scale.

Fisher's "Deterioration of the Environment" Universalized

Lee Altenberg
KLI for Evolution and Cognition Research, Austria
altenber@santafe.edu

R.A. Fisher argued that populations at a stationary state in the presence of natural selection must be subject to a "deterioration of the environment" in order to balance the increase in mean fitness due to the Fundamental Theorem of Natural Selection. Here it is shown that this "deterioration" is a universal feature of positive semigroups whose generators include operators of multiplication. Specifically, the non-multiplicative part of the generator must produce a net flow at equilibrium that is negative when weighted by the operator of multiplication. In biological applications, the operator of multiplication corresponds to fitness coefficients or heterogeneous growth rates, while the other parts of the generator correspond to diffusion, dispersal, mutation, recombination, sexual reproduction, or other transformations. As a consequence, the mean effect of these transformations at a stationary distribution is always to reduce fitness. This provides an intuitive explanation for the Reduction Principle for the evolution of reduced rates of transformation.

PDE to ODE: multiple timescales in reaction-advection-diffusion models

George Cosner
University of Miami, USA
gcc@math.miami.edu
Robert Stephen Cantrell, Mark Lewis, and Yuan Lou

Traditional reaction-advection-diffusion models in ecology assume that dispersal, population dynamics, and interactions between species all operate on the same time scale. However, in many cases, the time scale of dispersal may be faster than the time scale of population dynamics. One way to address that issue is to assume a pseudo-equilibrium hypothesis for dispersal, so that the spatial distribution of a populations can be described as a fixed spatial profile determined by its dispersal strategy, which is then normalized and multiplied by the total population to yield a population density. Interactions between individuals and species then can be described in terms of spatial averages weighted by the profiles arising from their dispersal strategies. This yields ordinary differential equations that still incorporate some of the effects of the dispersal strategies that organisms use. This talk will describe the formulation of such models and some results on the evolution of dispersal in that modeling context

Genetic consequences of range expansion: mathematical insights

Jimmy Garnier
Savoie University, France
jimmy.garnier@univ-savoie.fr

In this talk, I will present some reaction-dispersion equations with local and nonlocal dispersion terms and either monostable or bistable reaction terms. The goal of the talk is to present a new approach to study the inside dynamics of solutions which propagate in space. I will first focus on the reaction-diffusion traveling wave to present the notion of pulled and pushed traveling waves. Then, I will investigate the integro-diffrential traveling waves and the accelerating solutions of integro-differential equations. We will see that this approach gives some insights on the consequences of range expansion of population on their diversity.

Pulsating and generalized transition fronts in heterogeneous bistable media

Francois Hamel
Aix Marseille University, France
francois.hamel@univ-amu.fr
Weiwei Ding and Xiaoqiang Zhao

I will talk about the existence and qualitative properties of pulsating fronts and generalized transition fronts in heterogeneous spatially periodic bistable media. I will especially focus on global bounds for the mean speeds of transition fronts and on the influence of the underlying spatial scale on the existence of pulsating fronts.

Global dynamics of heterogeneous Lotka-Volterra competition diffusion systems

Xiaoqing He
National Tsing Hua University, Taiwan
sakula1213@gmail.com
Wei-Ming Ni

In this talk, we will discuss the joint effects of diffusion and spatial variation on the global dynamics of a classical Lotka-Volterra competition system. A complete understanding of the change in dynamics is obtained in terms of diffusion rates and competition coefficients. Some special cases will be discussed for a clearer understanding of the effects of spatial variation of the resource distribution.

Invading the ideal free distribution

King-Yeung Lam
Ohio State University, USA
lam.184@math.ohio-state.edu
Daniel Munther

Recently, the ideal free dispersal strategy has been proven to be evolutionarily stable in the spatially discrete as well as continuous setting. That is, at equilibrium a species adopting the strategy is immune against invasion by any species carrying a different dispersal strategy, other conditions being held equal. In this talk, we consider a two-species competition model where one of the species adopts an ideal free dispersal strategy, but is penalized by a weak Allee effect. We will show rigorously in this case that the ideal free disperser is invasible by a range of non-ideal free strategies, illustrating the trade-off between the advantage of being an ideal free disperser and the setback caused by the weak Allee effect. Moreover, a sharp integral criterion is given to determine the stability/instability of one of the semi-trivial steady state, which is always linearly neutrally stable due to the degeneracy caused by the weak Allee effect.

Non-local effects in an integro-PDE model from population genetics

Fang Li
East China Normal University, Peoples Rep of China
fangli0214@gmail.com

In this talk, we consider a genetic model of two alleles with complete dominance and partial panmixia. The model was proposed by Nagylaki in 2012 to model population with both short-distance and long-distance migration. Existence, stability, and multiplicity of steady state solutions are studied to understand the interaction of local diffusion, nonlocal diffusion and intensity of selection.

Dispersal in advective environments

Yuan Lou
Renmin University and Ohio State University, USA
lou.8@osu.edu
King-Yeung Lam, Frithjof Lutscher

We consider a two-species competition model in a one-dimensional advective environment, where individuals are exposed to unidirectional flow. The two species follow the same population dynamics but have different random dispersal rates and subject to a net loss of individuals from the habitat at the downstream end. We establish the existence of a critical advection speed for the persistence of a single species. For homogeneous advective environments with free-flow boundary conditions, we show that populations with higher dispersal rate will always displace populations with slower dispersal rate. In contrast, for hostile boundary conditions, it seems that there is a unique dispersal rate that is evolutionarily stable. Nevertheless, both scenarios show that unidirectional flow can put slow dispersers at a disadvantage and higher dispersal rate can evolve. We will also discuss further development in closed advective environments.

Modeling moments of a quantitative trait in a selective gradient

Judith Miller
Georgetown University, USA
jrm32@georgetown.edu

It is widely acknowledged that adaptation of quantitative (i.e. polygenic) traits plays a key role in the establishment and spread of invasive species, but few theoretical studies of such adaptation exist. Here we study reaction-diffusion systems (due to Kirkpatrick and Barton) and related, but more general, integrodifference systems governing the joint evolution of population density, trait mean and variance in a spatially heterogeneous environment. We describe conditions under which maladaptation contributes to extinction or range pinning or adaptation promotes the propagation of a traveling wave of invasion.

Asymptotic spreading in heterogeneous Fisher-KPP equations

Gregoire Nadin
Cnrs, Paris 6, France
nadin@ann.jussieu.fr
Henri Berestycki

In this article, we establish spreading properties for heterogeneous Fisher-KPP reaction-diffusion equations of the type:

$$\partial_t u - \Delta u = c(t,x)u(1-u)$$

for initial data with compact support, where the growth rate c is only assumed to be uniformly continuous and bounded in (t,x), without any specific assumption such as periodicity. Our aim is to localize the transition between the stable steady state 1 and the unstable one 0 in any direction $|e| = 1$. Namely, we construct two speeds such that

$$\lim_{t\to+\infty} u(t, wte) = 0 \text{ if } w > \overline{w}_e, \quad 1 \text{ if } w \in (0, \underline{w}_e).$$

The characterization of these speeds involve two new notions of generalized principal eigenvalues for linear operators in unbounded domains. It gives in particular an exact asymptotic speed of propagation for almost periodic, asymptotically almost periodic and radially periodic equations $\underline{w}_e = \overline{w}_e$ and explicit bounds on the location of the transition between 0 and 1 in spatially homogeneous equations. In dimension N, if the coefficients converge in radial segments, then $\underline{w}_e = \overline{w}_e$. and this set is characterized using some geometric optics minimization problem, which may give rise to non-convex expansion sets.

The shape of expansion induced by a line with fast diffusion in Fisher-KPP equations

Luca Rossi
Università di Padova, Italy
lucar@math.unipd.it
H. Berestycki and J.-M. Roquejoffre

We propose a system to describe the influence of a "road" with fast diffusion on biological invasions. Outside of the road a classical Fisher-KPP propagation with a different diffusion takes place. It is found that, if the ratio between the two diffusivities is above an explicit threshold, the road enhances the asymptotic speed of propagation in a cone of directions around the road. Outside this cone the speed of propagation coincides with the classical Fisher-KPP invasion speed. A description of the asymptotic shape of expansion is then derived.

Clines with directional selection and partial panmixia in an unbounded unidimensional habitat

Linlin Su
University of Vienna, Austria
linlin.su@gmail.com
Thomas Nagylaki

In geographically structured populations, global panmixia can be regarded as the limiting case of long-distance migration. The effect of incorporating partial panmixia into diallelic single-locus clines maintained by migration and arbitrary directional selection in an unbounded unidimensional habitat is investigated. The population density is uniform. Migration and selection are both weak; the former is homogeneous and symmetric. Under various conditions, we established for the equilibrium gene frequencies bounds; monotonicity; uniqueness; convexity-concavity; existence and multiplicity; noncrossing; and as the scaled panmictic rate increases, flattening or steepening.

Jointly approaching ideal free distribution of two species via different migration strategies

Youshan Tao
Donghua University, Peoples Rep of China
taoys@dhu.edu.cn
Yuan Lou and Michael Winkler

This talk addresses the global existence and asymptotic behavior of solutions to some reaction-diffusion-advection models for two competing species, where the species have the same population dynamics but different dispersal strategies. When one species possesses a combination of random dispersal and directed movement upward along its fitness gradient whereas the other species adopts random dispersal, the global existence of smooth solutions to the quasilinear parabolic system is established. When one species adopts the fitness-dependent dispersal but the other species does not disperse at all, we show the global existence of weak solutions to the degenerate parabolic-ODE system and further describe the asymptotic behavior of these weak solutions. In particular, we show that in the latter case the total population density approaches the so-called ideal free distribution in an appropriate sense.

Qualitative behavior of a reaction-advection-diffusion system modeling species competition

Qi Wang
Southwestern University of Finance and Economics, Peoples Rep of China
qwang@swufe.edu.cn
Chunyi Gai, Jingda Yan

We consider a diffusive Lotka−Volterra competition system with advection under homogeneous Neumann boundary conditions in smoothly bounded domains. We establish the global existence of bounded classical solutions for the system over one−dimensional domains. For multi−dimensional domains, globally bounded classical solutions are obtained for a parabolic−elliptic system under proper assumptions on the system parameters. Then we investigate the stationary problem over one−dimensional domains. Through bifurcation theories, the existence and stability of non−constant positive steady states are obtained. In the limit of large advection rate, we show that the reaction−advection−diffusion system converges to a shadow system of the competitor population density. The existence and stability of positive solutions to the shadow system have also been studied. Moreover, we construct positive interior−layer solutions to the shadow system when the crowding rate of the escaper and the diffusion rate of its inter-specific competitors are sufficiently small. These transition-layer solutions can be used to model the species segregation phenomenon. This talk is based on the recent work joint with my students Chunyi Gai and Jingda Yan at Southwestern University of Finance and Economics.

Special Session 108: Mathematics of Nonlinear Acoustics

Barbara Kaltenbacher, Alpen-Adria-Universitaet Klagenfurt, Austria

Research on nonlinear acoustics has recently been driven by the increasing number of industrial and medical applications of high intensity ultrasound ranging from ultrasound cleaning or welding via sonochemistry to lithotripsy and thermotherapy. Although the classical models of nonlinear acoustics such as the the Westervelt and the Kuznesov equation have already been devised in the 1960's and 70's, the physically correct and mathematically sound modelling of nonlinear wave propagation in this context is a currently a highly active field of research. An important prerequisite for reliable and well-founded numerical simulation and optimization of high intensity ultrasound devices is a mathematical analysis of the underlying partial differential equation (PDE) models in a general spatially three dimensional geometrical setting with appropriate boundary and initial conditions. While recently some results have been achieved concerning the classical models of nonlinear acoustics, an analyis of the qualitative and quantitative behavior also of recently developed models is crucial for assessing the required level of modelling for practically relevant applications. Another important issue is the coupling of nonlinear acoustics to other physical fields (excitation mechanisms, focusing devices, heat generation, interaction with kidney stones in lithotripsy). Also numerical simulation poses a major challenge due to nonlinearity, coupling to other physical fields, different spatial and temporal scales resulting from different wavelengths within the subdomains, and the fact that one often deals with open domain problems. Finally, the design of high intensity ultrasound devices leads to shape optimization and optimal control problems in the context of the above mentioned PDE models, with state and control constraints arising from physical and technical restrictions. The focus of this session will be on - modelling aspects - qualitative and quantitative analysis of PDE models - numerical simulation methods - control and optimization problems - applications in the context of nonlinear sound propagation.

Well-posedness and asymptotic behavior of solutions for the Blackstock–Crighton–Westervelt equation

Rainer Brunnhuber
Alpen-Adria-Universität Klagenfurt, Austria
rainer.brunnhuber@aau.at
Barbara Kaltenbacher

We consider the Blackstock–Crighton–Westervelt rotational model equation which is a fourth-order in space and third-order in time nonlinear partial differential equation and arises in the context of the modeling of nonlinear acoustic wave propagation in thermally relaxing viscous fluids. We use the theory of operator semigroups to investigate the linearization of the equation and show that the underlying semigroup is analytic which, together with a negative spectral bound of its generator, leads to exponential decay results for the linear homogeneous equation. Moreover, invoking the Banach fixed-point theorem, we prove local well-posedness of the Blackstock–Crighton model for sufficiently small initial data. Finally, we show how barrier's method is used to obtain global in time well-posedness and provide exponential decay results also for the nonlinear equation.

Analysis of a boundary control problem for the Moore-Gibson-Thompson equation

Francesca Bucci
Università di Firenze, Italy
francesca.bucci@unifi.it
Irena Lasiecka

The talk will focus on a boundary control problem for a (third order in time) Partial Differential Equation, referred to as Moore-Gibson-Thompson equation, which is a linearization of an established model for nonlinear propagation of waves in a thermally relaxing viscous fluid. The non-standard semigroup setup which arises from 'natural' choices of boundary controls—namely, the ones which are suggested by several applications—will be discussed. (The talk is based on ongoing work with Irena Lasiecka (Memphis, USA))

Avoiding degeneracy in the Westervelt equation by state constrained optimal control

Christian Clason
University Duisburg-Essen, Germany
christian.clason@uni-due.de
Barbara Kaltenbacher

The Westervelt equation, which describes nonlinear acoustic wave propagation in high intensity ultrasound applications, exhibits potential degeneracy for large acoustic pressure values. While well-posedness results on this PDE have so far been based on smallness of the solution in a higher order spatial norm, non-degeneracy can be enforced explicitly by a pointwise state constraint in a minimization problem, thus allowing for pressures with large gradients and higher-order derivatives, as is required in the mentioned applications. Using regularity results on the linearized state equation, well-posedness and necessary optimality conditions for the PDE constrained optimization problem can be shown via a relaxation approach by Alibert and Raymond.

Spectral Analysis and Stability Estimates for the Moore–Gibson–Thompson Equation

Richard Marchand
Slippery Rock University, USA
richard.marchand@sru.edu
Roberto Triggiani, Tim McDevitt

This presentation involves an abstract third-order equation motivated by the Moore–Gibson–Thompson Equation arising in high-intensity ultrasound. In its simplest form, the equation (with unbounded free dynamical operator) is not well-posed. However, a suitable change of variables permits one to show that it has a special structural decomposition, with a precise, hyperbolic-dominated part. Significant dynamical properties of the system, including spectral analysis and sharp stability estimates, will be presented and corroborated by numerical simulations.

Optimal regularity and long-time behavior of solutions for Kuznetsov's equation with inhomogeneous boundary data.

Stefan Meyer
University of Halle, Germany
stefan.meyer@mathematik.uni-halle.de
Mathias Wilke

We investigate a quasilinear initial-boundary value problem for Kuznetsov's equation

$$u_{tt} - c^2\Delta_x u - b\Delta_x u_t = k(u^2)_{tt} + \rho_0(v \cdot v)_{tt},$$

with a non-homogeneous Dirichlet boundary condition. This model describes the propagation of sound in a fluidic medium when the external pressure is prescribed. We prove that for small initial and boundary data there exists a unique global solution with optimal L_p-regularity. We show furthermore that the solution converges to zero at an exponential rate as time tends to infinity. Our techniques are based on maximal L_p-regularity for quasilinear parabolic equations. Part 1 - The linearized problem. The initial-boundary value problems for both Westervelt's and Kuznetsov's equations can be reduced to a common linearized problem with vanishing boundary data. We show that this problem is uniquely solvable with maximal regularity and the solutions decay exponentially. Similar assertions are proved for related linear problems which are needed to solve the nonlinear problem for Kuznetsov's equation.

Equations of nonlinear acoustics with nonlinear strong damping and Neumann as well as absorbing boundary conditions

Vanja Nikolic
Alpen-Adria-Universität Klagenfurt, Austria
vanja.nikolic@aau.at

Nonlinear acoustics has a wide range of important applications, one of which is the medical use of high intensity focused ultrasound in lithotripsy. Here we face the problem of a physically unbounded domain which will be truncated for numerical computations. Absorbing boundary conditions are then used to avoid reflections on the artificial boundary of the computational domain. Ultrasound excitation can be modeled by Neumann boundary conditions on the rest of the boundary.
In the simulation of a focusing lens immersed in an acoustic medium the problem of coupling regions with different material parameters arises, and so the existence of spatially less regular solutions becomes significant. On the other hand, in the Westervelt equation, which is a classical model in nonlinear acoustics, degeneracy of the coefficient $1 - 2ku$ needs to be avoided. Adding a nonlinear strong damping term to the Westervelt equation enables us to obtain an L_∞ estimate on u in order to avoid degeneracy of the coefficient $1 - 2ku$, while refraining from estimates on Δu (thus from too high regularity).
We will show local in time well-posedness of such a practically relevant problem for the Westervelt equation with nonlinear strong damping and Neumann as well as absorbing boundary conditions and give an outlook on acoustic-acoustic and elastic-acoustic coupling.

Shape optimization and free boundary problems

Gunther Peichl
University of Graz, Austria
gunther.peichl@uni-graz.at

Free boundary problems are challenging from a theoretical as well as numerical point of view. Efficient numerical solution strategies can be built on an equivalent formulation as a shape optimization problem. We briefly recall some aspects of shape optimization and discuss the shape gradient for shape optimization problems related to the Bernoulli free boundary problem and to a free boundary problem for the Stokes equation. We also apply the proposed techniques which allow to bypass the sometimes rather formal use of the shape derivative of the state variables to a shape optimization problem arising in lithotripsy where one is interested to find the shape of the excitation part of the boundary such that a prescribed pressure distribution is achieved.

Existence issues in nonlinear acoustics models

Petronela Radu
University of Nebraska-Lincoln, USA
pradu@math.unl.edu
Barbara Kaltenbacher, Rainer Brunnhuber

In the context of nonlinear wave equations, models of nonlinear acoustics involve different types of damping and source terms. I will discuss some challenges related to the presence of these nonlinearities and ways to overcome them and exemplify with some recent work in collaboration with B. Kaltenbacher and R. Brunnhuber.

Absorbing boundary conditions for the Westervelt equation

Igor Shevchenko
Imperial College London, England
i.shevchenko@imperial.ac.uk
Barbara Kaltenbacher

Many problems in science and engineering are naturally formulated in unbounded domains while numerical solutions to such problems require a finite region which needs special boundary conditions so that the boundary value

problem is well-posed and its solution is an accurate approximation to the restriction of the solution in the unbounded domain. These boundary conditions have to be transparent to or absorbing for solutions propagating outwards the boundary. Despite intensive research in the field of absorbing boundary conditions (ABCs), most results are obtained for linear equations with constant coefficients. Problems with variable coefficients have received much less attention, not to mention nonlinear models. Whereas some approaches to the construction of ABCs for nonlinear models exist, their use in concrete equations is out of the scope of most research works. The focus of this work is on ABCs for the Westervelt equation used as a basic acoustic model in various medical and industrial applications. The novelty of our work is in the derivation and analysis of high-order ABCs which have not been constructed so far. We provide well-posedness results for the Westervelt equation with zero and first order conditions as well as numerical experiments demonstrating the efficiency of the proposed ABCs.

Efficient time integration methods based on operator splitting and application to the Westervelt equation

Mechthild Thalhammer
University of Innsbruck, Austria
mechthild.thalhammer@uibk.ac.at
Barbara Kaltenbacher, Vanja Nikolic

In this talk, the approach of operator splitting for the efficient time integration of different classes of partial differential equations is discussed. In particular, a compact local error representation for the first-order Lie-Trotter splitting method is deduced and applied to the Westervelt equation, a nonlinear damped wave equation that arises in nonlinear acoustics as mathematical model for the propagation of sound waves in high intensity ultrasound applications. The resulting global error estimate confirms that the Lie-Trotter splitting method remains stable and that the nonstiff convergence order is retained in situations where the problem data are sufficiently regular. Numerical examples illustrate and complement the theoretical investigations.

Global uniqueness and Lipschitz stability in determining a damping coefficient of the MTG equation by one boundary measurement

Roberto Triggiani
University of Memphis, USA
rtrggani@memphis.edu
Shitao Liu

By means of just one boundary measurement exercised on a suitable portion of the boundary, we obtain the two canonical results in inverse theory for the MTG equation arising in high intensity ultrasound in terms of sharp data of the problem.

Optimal regularity and long-time behavior of solutions for Kuznetsov's equation with inhomogeneous boundary data, Part 2

Mathias Wilke
Martin-Luther University Halle-Wittenberg, Germany
mathias.wilke@mathematik.uni-halle.de
Stefan Meyer

We investigate a quasilinear initial-boundary value problem for Kuznetsov's equation

$$u_{tt} - c^2\Delta_x u - b\Delta_x u_t = k(u^2)_{tt} + \rho_0(v \cdot v)_{tt},$$

with a non-homogeneous Dirichlet boundary condition. This model describes the propagation of sound in a fluidic medium when the external pressure is prescribed. We prove that for small initial and boundary data there exists a unique global solution with optimal L_p-regularity. We show furthermore that the solution converges to zero at an exponential rate as time tends to infinity. Our techniques are based on maximal L_p-regularity for quasilinear parabolic equations. Part 2 - The nonlinear problem can be solved with the implicit function theorem, since its linearization has maximal regularity and therefore generates a topological isomorphism between the space of solutions and the space of data.

Special Session 109: Stochastic Partial Differential Equations

Michael Rockner, University of Bielefeld, Germany

Stochastic Partial Differential Equations (SPDE) and their applications is a relatively young field of Mathematics. In the past two decades it has, however, become one of the main research directions of Probability Theory, with rising activity across its entire spectrum. It combines the classical area of Partial Differential Equations with modern branches of Probability Theory, in particular, Stochastic Analysis, and thus constitutes one of the most prominent contact points between Analysis and Stochastics. Besides various other connections to pure mathematics (e.g. Differential Geometry, Dynamical Systems), one main focus of SPDE are its applications to the Sciences, in particular Physics, but also Biology and Chemistry. Another main area of applications is Economics, in particular Mathematical Finance. The aim of the session is to give an update on recent developments on SPDE and at the same time identify new frontiers with challenging open problems for the field, with emphasis on both theory and applications

Quasipotential and exit time for 2D Stochastic Navier-Stokes equations driven by space time white noise

Zdzislaw Brzezniak
University of York, England
zdzislaw.brzezniak@york.ac.uk
Sandra Cerrai and Mark Freidlin

We are dealing with the Navier-Stokes equation in a bounded regular domain D of $\mathbb{R}^2$, perturbed by an additive Gaussian noise $\partial w^{Q_\delta}/\partial t$, which is white in time and colored in space. We assume that the correlation radius of the noise gets smaller and smaller as $\delta \searrow 0$, so that the noise converges to the white noise in space and time. For every $\delta > 0$ we introduce the large deviation action functional $S^\delta_{0,T}$ and the corresponding quasi-potential U_δ and, by using arguments from relaxation and Γ-convergence we show that U_δ converges to $U = U_0$, in spite of the fact that the Navier-Stokes equation has no meaning in the space of square integrable functions, when perturbed by space-time white noise. Moreover, in the case of periodic boundary conditions the limiting functional U is explicitly computed. Finally, we apply these results to estimate of the asymptotics of the expected exit time of the solution of the stochastic Navier-Stokes equation from a basin of attraction of an asymptotically stable point for the unperturbed system.

On the Smoluchowski-Kramers approximation

Sandra Cerrai
University of Maryland, USA
cerrai@math.umd.edu
Michael Salins

We study the quasi-potential for a general class of damped semilinear stochastic wave equations. We show that, as the density of the mass converges to zero, the infimum of the quasi-potential with respect to all possible velocities converges to the quasi-potential of the corresponding stochastic heat equation, that one obtains from the zero mass limit. This shows in particular that the Smoluchowski-Kramers approximation is not only valid for small time, but, in the zero noise limit regime, can be used to approximate long-time behaviors such as exit time and exit place from a basin of attraction.

Gradient estimates for a Dirichlet problem in Hilbert spaces

Giuseppe da Prato
Scuola Normale Superiore, Pisa, Italy
daprato@sns.it

We are concerned with the following Dirichlet problem in a separable Hilbert space H

$$\begin{cases} u_t(t,x) = \frac{1}{2}\,\mathrm{Tr}\,[u_{xx}(t,x)] + \langle Ax, u_x(t,x)\rangle, \quad t>0,\ x\in\mathcal{O}, \\ u(t,x) = 0, \quad t>0,\ x\in\partial\mathcal{O}, \\ u(0,x)=\varphi(x), \quad x\in\overline{\mathcal{O}}, \end{cases}$$

where $\mathcal{O}$ is an open subset of H of the form $\mathcal{O} = G^{-1}((-\infty,0))$ of boundary $\partial\mathcal{O} = G^{-1}(0)$, where G is a given Borel function. We shall present some representation formulas for the gradient of $u(t,x)$ which only involves φ.

Fully-nonlinear SPDEs with rough path dependence

Peter Friz
TU and WIAS Berlin, Germany
P.K.Friz@gmail.com
Paul Gassiat, Pierre-Louis Lions and Panagiotis E. Souganidis

We shall briefly review the pathwise approach to fully non-linear stochastic partial differential equations due to Lions–Souganidis. In some cases (joint work Caruana, Diehl, Oberhauser) this was extended to rough path noise in the sense of Lyons. The focus of this talk will be on a fully non-linear evolution equations, with noise of the form $H(x, Du) \circ dB$ where the Hamiltonian is quadratic in Du and B can be an arbitrary, scalar continuous path.

Finite time extinction for stochastic sign fast diffusion and self-organized criticality

Benjamin Gess
University of Chicago, USA
gess@math.tu-berlin.de

We will first shortly review the informal derivation of a continuum limit for the Bak-Tang-Wiesenfeld model of self-organized criticality. This will lead to the stochastic sign fast diffusion equation. A key property of models exhibiting self-organized criticality is the relaxation of supercritical states into critical ones in finite time. However, it has remained an open question for several years whether the continuum limit - the stochastic sign fast diffusion - satisfies this relaxation in finite time. We will present a proof of this.

A Splitting Method for a Stochastic Schrödinger Equation

Wilfried Grecksch
Martin-Luther-University Halle-Wittenberg, Germany
wilfried.grecksch@mathematik.uni-halle.de

We construct a splitting method for a nonlinear one dimensional stochastic Schrödinger equation with bounded domain and homogeneous Neumann boundaries conditions. We approximate the solution of our problem by the sequence of solutions of two types of equations: one without stochastic integral term, but containing the Laplace operator and the other one containing only the stochastic integral term. The two types of equations are connected to each other by their initial values. We prove that the solutions of these equations both converge strongly to the variational solution of the Schrödinger equation. The idea of proof is based on [1].

References

[1] Grecksch, W., Lisei, H.: Approximation of Stochastic Nonlinear Equations of Schrödinger Type by the splitting method. Stoch.Anal.Appl. 31: 314-335, 2013

Approximation of an infinite dimensional sticky Ornstein–Uhlenbeck process via Dirichlet forms

Martin Grothaus
University of Kaiserslautern, Germany
grothaus@mathematik.uni-kl.de
Torben Fattler

Starting point is the dynamical wetting model, also known as Ginzburg–Landau dynamics with pinning and reflection competing on the boundary, on a bounded set. From the abstract point of view this is a distorted Brownian motion with sticky, reflecting boundary condition. Scaling limits of the corresponding invariant measures have been studied by Deuschel, Giacomin and Zambotti. In this talk we consider the scaling limit of the dynamics in the critical regime.

SPDE via Fourier analysis based rough path calculus

Peter Imkeller
Humboldt-Universitaet zu Berlin, Germany
imkeller@math.hu-berlin.de

In 1961, Ciesielski established a remarkable isomorphism of spaces of Hölder continuous functions and Banach spaces of real valued sequences. The isomorphism can be established along Fourier type expansions of (rough) Hölder continuous functions by means of the Haar-Schauder wavelet. We will use Schauder representations for a pathwise approach of the integral of one rough function with respect to another one, using Ciesielski's isomorphism. In a more general and analytical setting, this pathwise approach of rough path analysis can be understood in terms of Paley-Littlewood decompositions of distributions, and Bony paraproducts in Besov spaces. It allows a smooth approach of formal products of singular distributions, and consequently of BSDE with rough and multiplicative noise. This talk is based on work in progress with M. Gubinelli (U Paris-Dauphine) and N. Perkowski (HU Berlin).

Approximations and regularities for nonlinear stochastic ordinary and partial differential equations

Arnulf Jentzen
ETH Zurich, Switzerland
arnulf.jentzen@sam.math.ethz.ch
Sonja Cox, Martin Hairer, Martin Hutzenthaler, Xiajoie Wang

In this talk we analyze how smooth the solution of a nonlinear stochastic differential equation (SDE; by which we mean a stochastic ordinary or partial differential equation) depends in the strong L^p-sense on the initial value as well as the related question of convergence rates for numerical approximations of the considered nonlinear SDE. In the first part of this talk we give an example of an SDE with a globally bounded and smooth drift coefficient and a constant diffusion coefficient whose solution does in the strong L^p-sense not depend smoothly on the initial value. In addition, we demonstrate that the Euler-Maruyama approximation scheme converges without any arbitrarily small polynomial rate of convergence to the solution process of this SDE. In the second part of this talk we present a sufficient condition that ensures that the solutions of nonlinear SDEs depend smoothly on the initial values. Based on this regularity analysis, we then establish strong convergence rates for suitable numerical approximations of the considered nonlinear SDEs. We illustrate these results by a few example SDEs from finance, physics and biology. The first part of this talk is based on a joint work with Martin Hairer and Martin Hutzenthaler. The second part of this talk is based on joint works with Sonja Cox, Martin Hutzenthaler and Xiaojie Wang.

Numerical approximation of random and stochastic (partial) differential equations

Peter Kloeden
Goethe University, Germany
kloeden@math.uni-frankfurt.de

Higher order numerical schemes for stochastic differential equations (SODEs) can be derived systematically using stochastic Taylor expansions based on iterated applications of the Itô formula. For stochastic partial differential equations (SPDEs) there is no general Itô formula that can be used in this way. Nevertheless higher order temporal expansions for mild solutions of SPDEs are possible using Taylor-like expansions with an idea that was first used for pathwise random ordinary differential equations (RODEs). This will be illustrated first for RODEs and then extended to SPDEs. The same relationship between RODEs and SODES as well as RPDEs and SPDEs will be indicated as well as other issues that arise in their discretization.

Fluctuations over mesoscopic scaling limits

Yuri G Kondratiev
Bielefeld University, Germany
kondrat@mathematik.uni-bielefeld.de

We consider Markov dynamics of interacting particle systems in the continuum. A mesoscopic scaling for these dynamics leads to a kinetic description of the system in terms of a nonlinear evolution equation for the density of particles. We show that the fluctuations over this kinetic limit are described by means of certain SPDE derived from the form of the Markov generator for the considered dynamic.

Hörmander's theorem for stochastic partial differential equations

Nicolai Krylov
University of Minnesota, USA
krylov@math.umn.edu

We prove Hörmander's type hypoellipticity theorem for stochastic partial differential equations when the coefficients are only measurable with respect to the time variable. The need for such kind of results comes from filtering theory of partially observable diffusion processes, when even if the initial system is autonomous, the observation process enters the coefficients of the filtering equation and makes them time-dependent with no good control on the smoothness of the coefficients with respect to the time variable.

Well-posedness and Long Time Asymptotics of SPDE

Wei Liu
Jiangsu Normal University, Peoples Rep of China
weiliu@jsnu.edu.cn
Michael Roeckner, Benjamin Gess

In this talk we will first present some recent results on the well-posedness of SPDE with locally monotone coefficients, which generalize the classical results established by Pardoux, Krylov and Rozovskii etc. This extension provides a unified framework to analyze a large class of SPDEs such as stochastic Burgers type equations, stochastic 2D hydrodynamical systems, stochastic tamed 3D Navier-Stokes equations and stochastic equations of non-Newtonian fluids, which can not be included in the classical variational framework. The second part of this talk is to show the long time asymptotics of SPDE with locally monotone coefficients by proving the existence of random attractors. The approach is based on a construction of strictly stationary nonlinear Ornstein-Uhlenbeck processes, which also allows spatially much rougher noise than in existing works.

Stochastic PDEs with Gaussian Volterra Noise

Bohdan Maslowski
Charles University in Prague, Czech Rep
maslow@karlin.mff.cuni.cz

We will present some results on stochastic linear and bilinear equations in infinite dimensions where the driving process may be a general Gaussian process of Volterra type. Examples of such processes are fractional or Liouville fractional Brownian motion, multifractal Brownian motion or corresponding OU processes. Existence, uniqueness and large time behaviour of solutions will be established. In particular, some examples in which qualitative behaviour of solutions is significantly different from the standard white noise case will be discussed.

On the stochastic Cahn-Hilliard/Allen-Cahn equation with a sublinear diffusion coefficient

Annie Millet
University Paris 1, France
annie.millet@univ-paris1.fr
Dimitra Antonopoulou, Georgia Karali

The Cahn-Hilliard/Allen-Cahn equation with a noise perturbation is a simplified mean field model of stochastic microscopic dynamics associated with adsorption and desorption-spin flip mechanisms in the context of surface processes. For such an equation we consider a multiplicative space-time white noise with diffusion coefficient of sub-linear growth in dimension 1 up to 3. Using technics from semigroup theory and parabolic operators in the sense of Petrovskĭı, we prove the existence and uniqueness of the solution, as well as its path regularity. Our results are also valid for the stochastic Cahn-Hilliard equation with unbounded noise diffusion, for which previous results were established only in the framework of a bounded diffusion coefficient. The path regularity of the stochastic solution depends on the dimension and on that of the initial condition.

Stochastic heat equation with multiplicative colored noise

David Nualart
The University of Kansas, USA
nualart@ku.edu
Yaozhong Hu, Jingyu Huang and Samy Tindel

In this talk we discuss the stochastic heat equation on $\mathbb{R}^d$

$$\frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + u\dot{W},$$

where $\dot{W}$ is a mean zero Gaussian noise with covariance $E\left[\dot{W}_{t,x}\dot{W}_{s,y}\right] = \gamma(s-t)\,\Lambda(x-y)$, and γ and Λ are general nonnegative and nonnegative definite (generalized) functions satisfying some integrability conditions. The product $u\dot{W}$ can be interpreted in both the Skorohod and Stratonovich sense. We will present recent results on the existence and uniqueness of a solution and its Hölder continuity. Moreover we will establish Feynman-Kac formulas for the solution and for its moments, which allow us to derive moment estimates and intermittency properties.

Nonparametric Wick-Malliavin Approximations for Stochastic PDEs

Boris Rozovsky
Brown University, USA
Boris_Rozovsky@Brown.edu
R. Mikulevicius

We will discuss statistical and numerical properties of a new class of Wick- Malliavin approximations to nonlinear SPDEs. These approximations are non- parametric. That is, they are independent of the type of randomnes present in the underlying SPDE. Previously, Wick-Malliavin approximations to SPDEs were considered for Gaussian and Poisson types of forcing.

Generalized asymptotic couplings and convergence of transition probabilities

Michael Scheutzow
TU Berlin, Germany
ms@math.tu-berlin.de

A classical problem in the theory of Markov processes is to determine the existence and uniqueness of an invariant probability measure. If existence and uniqueness hold then the question whether all transition probabilities converge to the invariant measure is of interest. It has long been known that the strong Feller property together with irreducibility is sufficient for all these properties and in this case the transition probabilities do not only converge weakly but even in total variation. Unfortunately, the strong Feller property does not hold for many Markov processes with an infinite dimensional state space (like solutions of many SPDEs). We will show how generalized asymptotic couplings can be used to prove uniqueness of an invariant measure and weak convergence of transition probabilities.

A new approach to stochastic evolution equations with adapted drift

Mark Veraar
TU Delft, Netherlands
m.c.veraar@tudelft.nl
Matthijs Pronk

In this talk I will explain a new approach to stochastic evolution equations with an unbounded drift A which is dependent on time and the underlying probability space in an adapted way. It is well-known that the semigroup approach to equations with random drift leads to adaptedness problems for the stochastic convolution term. I will explain a new representation formula for the stochastic convolution which avoids integration of nonadapted processes. Connections with other solution concepts such as weak solutions will be given and the usual regularity properties will be shown. The approach can be applied in the study of semilinear problems with random drift.

Stochastic nonlinear Schrödinger equation

Deng Zhang
Bielefeld University, Germany
zhangdeng@amss.ac.cn
Viorel Barbu, Michael Röckner)

We present well-posedness resutlts in the energy space $H^1(\mathbb{R}^d)$ for the stochastic nonlinear Schrödinger equation with linear multiplicative noise, including also the non-conservative case. The exponents of the nonlinear term obtained here are optimal for the global well-posedness, hence this work sharpens earlier well-posedness results in the conservative case. Moreover, we also study the noise effects on blowup in the non-conservative case. In contrast to the conservative case, we prove that in the non-conservative focusing mass-(super)critical case, adding a large noise one can, with high probability, prevent blowup on the bounded time interval $[0, T]$ with $T < \infty$. In particular, for space-independent noise the explosion even can be prevented on the whole positive time interval with high probability.

Existence and uniqueness of solutions to stochastic functional differential equations in infinite dimensions

Rongchan Zhu
Beijing Institute of Technology, Peoples Rep of China
zhurongchan@126.com
Michael Röckner, Xiangchan Zhu

In this paper, we present a general framework for solving stochastic functional differential equations in infinite dimensions in the sense of martingale solutions, which can be applied to a large class of SPDEs with finite delays, e.g. d-dimensional stochastic fractional Navier-Stokes equation with delays, d-dimensional stochastic reaction equation with delays, d-dimensional stochastic porous media equation with delays. Moreover, under local monotone conditions for the nonlinear term we obtain the existence and uniqueness of strong solutions to SPDE with delays.

Three dimensional Navier-Stokes equation driven by space-time white noise

Xiangchan Zhu
Beijing Jiaotong University, Peoples Rep of China
zhuxiangchan@126.com

In this paper we use two different approaches to prove the local existence and uniqueness of solutions to the three dimensional Navier-Stokes equation driven by space-time white noise. One is based on M. Hairer's regularity structure. The other is the method of paracontrolled distribution, introduced recently by M. Gubinelli, P. Imkeller and N. Perkowski.

Special Session 110: Nonlinear Schrodinger Equations and Its Applications

Panayotis Kevrekidis, University of Massachusetts, Amherst, USA
Ricardo Carretero, San Diego State University, USA

The nonlinear Schrodinger (NLS) equation is used in a very large variety of physical systems since it describes at the lowest order the nonlinear propagation of modulated waves. A few of the most important applications of NLS equation emanate from the realm on nonlinear optics and Bose-Einstein condensates. The recent experimental realization of BECs and the ever growing control and experimental advances in nonlinear optical systems has ignited new and exciting developments. From the mathematical point of view, one of the most exciting aspects of these contexts is the broad range of possible configurations including: one to three spatial dimensions, one or many coupled fields, tunable external potentials, and temporally or even spatially variable nonlinearities, among many others. The aim of this special session is to bring together experts, as well as young researchers, working on the theory, the numerical simulation and the experimental study of nonlinear Schrodinger equations and their applications. The focus is to establish a fruitful discussion of the current state-of-the-art and an examination of future challenges and directions of interest. This should be a session appealing to theoretical physicists, experimental physicists and applied mathematicians alike and will be a vehicle for the exchange of ideas that could cross-fertilize different disciplines and promote the initiation of new collaborations that could address some of the pertinent open problems.

Microcavity polaritons: an experimental simulator for non-linear Schrodinger equations

Alberto Amo
CNRS-Laboratoire de Photonique et Nanostructures, France
alberto.amo@lpn.cnrs.fr

Solid-state systems appear today as excellent platforms to study a wide range of non-linear processes and to simulate non-linear Hamiltonians. In this sense, a remarkable system is that of polaritons in semiconductor microcavities. Polaritons are mixed light-matter quasiparticles arising from the strong coupling between excitons and photons confined in a two-dimensional semiconductor microcavity. They can be easily manipulated with lasers, detected using standard optical techniques, and present strong interactions. Additionally, they can condense in a coherent macroscopic state governed by a non-linear Schrodinger equation with strong losses due to the scape of polaritons in the form of photons out of the cavity. This condense state is, therefore, strongly out of equilibrium and it needs to be continuously pumped. Polaritons are thus an excellent physical system to study experimentally the solutions of a rich class of non-conservative non-linear Schrodinger-like equations. Here we will present experiments showing polariton dynamics in different external potentials, such as (i) two coupled wells in which we observe non-linear oscillations and macroscopic self-trapping, (ii) a hexagonal ring structure where spin-orbit coupling can be engineered, (iii) a honeycomb lattice where we have designed both Dirac and flat bands for polaritons.

Gross-Pitaevskii hierarchies and mean field limits for interacting Bose gases

Thomas Chen
University of Texas at Austin, USA
tc@math.utexas.edu
Natasa Pavlovic, Christian Hainzl, Robert Seiringer

This talk surveys some recent results related to the derivation of the cubic nonlinear Schroedinger equation in R^3, and the related Cauchy problem for Gross-Pitaevskii (GP) hierarchies. A new approach to the analysis of GP hierarchies using the quantum de Finetti theorem is discussed (joint work with C. Hainzl, N. Pavlovic and R. Seiringer).

Justification of Leading Order Quasicontinuum Approximations of Strongly Nonlinear Lattices

Christopher Chong
University of Massachusetts, Amherst, USA
chong@math.umass.edu
P.G. Kevrekidis, Guido Schneider

I will consider the leading order quasicontinuum limits of a one-dimensional granular chain governed by the Hertz contact law under precompression. The approximate model, a so-called p-system, which is derived in this limit is justified by establishing asymptotic bounds for the error with the help of energy estimates. The continuum model predicts the development of shock waves, which we investigate with the aid of numerical simulations.

Hysteresis in a quantized superfluid 'atomtronic' circuit

Mark Edwards
Georgia Southern University, USA
edwards@georgiasouthern.edu
Noel Murray

A ring Bose–Einstein condensate (BEC) with zero circulation ($m = 0$) stirred by a barrier will eventually jump to an $m = 1$ state when stirred faster than a certain critical speed, Ω_c^+. A ring BEC with $m = 1$ will drop to

$m = 0$ when stirred at a critical speed, Ω_c^-, which is lower than Ω_c^+. The loop areas, $\Omega_c^+ - \Omega_c^-$, of this hysteretic response of the BEC to stirring predicted by zero–temperature Gross–Pitaevskii equation (GPE) disagreed significantly with the results of a recent experiment. In the work reported here, we simulated this experiment with the phenomenologically damped GPE, [S. Choi, S. A. Morgan, and K. Burnett, Phys. Rev. A **57**, 4057 (1999)], and with the Zaremba–Nikuni–Griffin (ZNG) theory. The ZNG theory can account for finite–T, non–equilibrium dynamics. We compare the results of these simulations with the experimental data. The simulations show that a vortex/antivortex pair forms in the barrier region during the stirring and that this drives the hysteresis. We also show how the presence of an interacting, thermal cloud affects the dynamics of these pairs. We also simulate a ring condensate stirred by two barriers and find that the GPE matches the data much more closely.

Vortex nucleation and tunneling dynamics in Bosonic Josephson Junctions

Muntsa Guilleumas
Universitat de Barcelona, Spain
muntsa@ecm.ub.edu
R. Mayol, M. Abad, F. Piazza, D. M. Jezek, and A. Smerzi

We numerically analyze the relation between coherent tunneling and creation and annihilation of quantized vortex rings in a dilute Bose-Einstein condensate confined in a double-well trap. We show that while plasma oscillations are related to the dynamics of external vortex rings, in the self-trapping regime the vortices exist at the barrier and act as precursors of the flow inversion. We show that vortices are a crucial ingredient to completely understand the tunneling dynamics in a bosonic Josephon junction.

New criteria preventing Hamiltonian-Hopf bifurcations

Richard Kollar
Comenius University, Slovak Rep
richard.kollar@fmph.uniba.sk
Peter Miller

Linear stability of equilibria in the nonlinear Schrödinger equation and in general in Hamiltonian systems is characterized by a restriction of the spectrum of the linearized problem to the imaginary axis. Typically the underlying Hamiltonian is indefinite and thus stable equilibria under a perturbation may be subject to instability. Such a bifurcation associated with a collision of two purely imaginary eigenvalues of opposite Krein signature is named Hamiltonian-Hopf (HH) bifurcation. It is well-known that HH bifurcations are generic if opposite Krein signature eigenvalues are close to each other. On the other hand, there cases when under a particular perturbation eigenvalues pass each other on the imaginary axis and do not bifurcate were identified: it happens when the perturbation is definite (Krein & Ljubarski) or if the system has an additional symmetry and certain conditions are met (Melbourne, Delnitz & Marsden). Using the graphical Krein signature we present a new type of mechanism that prevents HH bifurcations. Although such a mechanism is non generic it may commonly appear in applications. Surprisingly the conditions appear to use information that may not be visible if one considers only the typical form of the eigenvalue problem. This is a joint work with Peter Miller (U Michigan).

Gap solitons in the spin-orbit coupled Bose-Einstein condensates

Vladimir Konotop
University of Lisbon, Portugal
vvkonotop@fc.ul.pt
Y. V. Kartashov, F. Kh. Abdullaev

I report a diversity of stable gap solitons in a spin-orbit coupled Bose-Einstein condensate in a Zeeman lattice. It is shown that the solitons, can be classified by the main physical symmetries they obey, i.e. symmetries with respect to parity (P), time (T), and internal degree of freedom, i.e. spin, (C) inversions. The conventional gap and gap-stripe solitons are obtained in lattices with different parameters. It is shown that solitons of the same type but obeying different symmetries can exist in the same lattice at different spatial locations. PT and CPT symmetric solitons have anti-ferromagnetic structure and are characterized respectively by nonzero and zero total magnetizations.

Dynamics and chaotic behavior of three non-co-rotating vortices in a confined Bose-Einstein condensate

Vassilis Koukouloyannis
University of Thessaloniki, Greece
vkouk@physics.auth.gr
P.G. Kevrekidis, N. Kyriakopoulos, G. Voyatzis, Ch. Skokos and H. Varvoglis

A system of three vortices with alternating charges in a confined Bose-Einstein condensate is studied. The vortices are considered as quasi-particles and their motion is described by a three-degrees of freedom Hamiltonian. This system possesses two integrals of motion, the energy (which is expressed through the Hamiltonian H) and the so-called angular momentum L of the system. We use the second of the integrals in order to reduce the system to a two-degrees of freedom one with L as a parameter. This way, we can produce Poincaré sections and calculate the corresponding scan maps by using the chaoticity index SALI as the primary tool of investigation. This way we study the general dynamical behavior of the system from completely regular to progressively chaotic one and acquire the percentages of the chaotic orbits for a wide range of values of h and L.

Controlling and exploring cold quantum gases on a chip

Peter Kruger
University of Nottingham, England
Peter.Kruger@nottingham.ac.uk
Fedja Orucevic, Thomas Barrett, Panos Kevrekidis

Recent developments in cooling and trapping neutral atoms with electromagnetic fields has led to rapid growth of the field of ultracold quantum gases with applications ranging from fundamental physics to quantum technology device developments. In particular, the critical temperature of Bose-Einstein condensation (BEC) can routinely be reached and surpassed in the laboratory. Such experiments provide an ideal testbed for studying the behaviour and dynamics of the gas, governed by a non-linear Schrödinger equation (Gross-Pitaevski equation). In this talk, we will introduce so called *atom chips*, featuring microfabricated conductor patterns used for controlling and manipulating atoms merely microns away from the chip surface. This concept provides large flexibility in shaping environments that are well suited for studies involving high-spatial resolution tailoring of quantum gases in combination with fast and precise temporal control. We will give a number of examples of the type of effect that can be studied in this context, including change in dimensionality of the system, soliton creation and dynamics and the impact of steps in the spatial profile of the non-linearity.

Propagation of polariton wavepackets

Fabrice Laussy
Universidad Autonoma de Madrid, Spain
fabrice.laussy@gmail.com
J. Restrepo Cuartas, D. Colas, G. Guirales, D. Vishnevsky, E. del Valle, L. Dominici, M. de Giorgi, S. Donati, D. Ballarini and D. Sanvitto.

Polaritons in microcavities can reach a regime where they are well described by a set of two coupled Schrödinger equations. This gives rise to quantum particles with a peculiar dispersion relation. We first give an overview of how such objects permit to revisit the physics of propagating Schrödinger wavepackets, with new insights such as the character of soliton-like propagation, properties such as self-healing, the nature of a Galilean boost for a wavefunction, generalization of fundamental concepts such as spacetime Rabi oscillations and other remarkable properties such as the emergence of sub-packets with properties defying the character of the whole. Their Physics can then be extended in countless directions and variations, with inclusion of ingredients such as dissipation, that can lead to negative effective mass, particle-particle interactions that turn one component into a Gross-Pitaevskii equation, external driving—coherent and/or incoherent—leading to parametric processes, condensation, dissipative solitons, etc. We focus on a few particular cases that illustrate the aforementioned scenarios and link the theoretical results to actual experimental observations, some of them as yet not fully understood.

Quasi-periodic solutions of quasi-periodically forced nonlinear Schrödinger equations with a given potential

Xuezhu Lu
Southeast University,, Peoples Rep of China
luxuezhu_apple@163.com
Junxiang Xu

In this talk, we consider one-dimensional quasi-periodically forced nonlinear Schrodinger equation $iu_t = u_{xx} - V(x)u + \varepsilon g(\omega t, x)|u|^2 u$ on $[0, \pi] \times \mathbb{R}$ under Dirichlet boundary condition, where ε is a small positive real number and $g(\omega t, x)$ is analytic with respect to all variables and quasi-periodic in time. We prove that for a prescribed analytic and periodic even potential $V(x)$, the above equation admits small-amplitude quasi-periodic solutions.

Bright solitons from defocusing nonlinearities - an overview

Boris Malomed
Tel Aviv University, Israel
malomed@post.tau.ac.il

The talk aims to give a review of recently obtained results which demonstrate that defocusing cubic media with spatially inhomogeneous nonlinearity, whose strength increases rapidly enough toward the periphery (faster than r^D in the D-dimensional space, $D = 1, 2, 3$, where r is the radial coordinate), can support a variety of stable solitons in all three dimensions, including one-dimensional (1D) fundamental and multihump states, 2D vortex solitons with arbitrary topological charges, and vortex tori (soliton gyroscopes) in 3D. Solitons maintain their coherence in the state of motion, oscillating in the effective nonlinear potential as robust quasiparticles.The 3D vortex tori exhibit stable precession, induced by the application of external torque. In addition to numerically found soliton families, particular solutions can be obtained in an exact analytical form, and accurate approximations are developed for the entire families by means of the variational and Thomas-Fermi approximations. Essentially the same mechanism for the self-trapping of bright solitons under the action of the spatially growing repulsive nonlinearity works in nonlocal media, and in discrete systems too. Furthermore, related numerical and analytical results demonstrate the existence of stable dissipative solitons in media with the uniform linear gain and nonlinear loss, whose local strength grows toward the periphery faster than r^D. Such 1D and 2D settings can be implemented in nonlinear optics and BEC. The 3D setting may be created in BEC.

Thomas-Fermi approximation in PT-symmetric potentials

Dmitry Pelinovsky
McMaster University, Canada
dmpeli@math.mcmaster.ca

For the stationary Gross–Pitaevskii equation with harmonic real and linear imaginary potentials in the space of one dimension, we construct the ground state solutions in the limit of large densities (large chemical potentials), where the solution degenerates into a compact Thomas–Fermi approximation. We find that the construction of the Thomas–Fermi approximation can be achieved with an invertible coordinate transformation and an unstable manifold theorem for planar dynamical systems. The persistence and justification of the Thomas-Fermi approximation are achieved with another coordinate transformation and reduction of the underlying problem to the Painlevé-II equation with a unique global Hastings-McLeod solution. Extensions of our results are discussed in the context of the stationary Gross–Pitaevskii equation with harmonic real and localized odd imaginary potentials.

a GdNLS model as an extensive resonant normal form for a Klein-Gordon chain and applications.

Tiziano Penati
Department of Mathematics, Italy
tiziano.penati@unimi.it
Simone Paleari

In the limit of small coupling in the nearest neighbour interaction, and small ℓ^2 norm, an extensive resonant normal form for the Klein-Gordon finite chain is constructed. It turns out to be a generalized discrete nonlinear Schrödinger (GdNLS) model, characterized by all neighbours couplings, both in the linear and nonlinear terms, with exponential decay of the coefficients with the distance between sites. Long time stability of true or approximated small amplitude Breathers can be obtained using variational arguments.

Gray Ring Solitons and Symmetry-Broken States in Two-Dimensional Polariton Condensates

Augusto Rodrigues
Universidade do Porto, Portugal
asrodrigues66@gmail.com
P.G. Kevrekidis, R. Carretero-González, J. Cuevas-Maraver, D. Frantzeskakis and F. Palmero

We consider the existence, stability and dynamics of nonlinear excitations, for a quasi-two-dimensional polariton condensate in the presence of pumping and nonlinear damping. In particular we report that, contrary to what is know, e.g., for the atomic BEC case, *stable* stationary gray ring solitons can be found for polariton condensates in suitable parametric regimes. We claim that these can be thought of as radial forms of Nozaki-Bekki holes. In other regimes, however, these may suffer symmetry breaking instabilities. The dynamical, pattern-forming implications of the above instabilities are explored through direct numerical simulations and, in turn, give rise to waveforms with triangular or quadrupolar symmetry, which we also explore.

Cubic-Quintic Long-Range Interactions With Double Well Potentials

Vassilis Rothos
Aristotle University of Thessaloniki, Greece
rothos@auth.gr
P.G.Kevrekidis, P. Tsilifis

In this talk, we examine the combined effects of cubic and quintic terms of the long range type in the dynamics of a double well potential. We obtain a reduced canonical description for the conjugate variables of relative population imbalance and relative phase between the two wells and proceed to a dynamical systems analysis of the resulting pair of ordinary differential equations. The relevant bifurcations, the stability of the branches and their dynamical implications are examined both in the reduced (ODE) and in the full (PDE) setting.

Asymptotic stability of solitary waves in generalized Gross-Neveu and Thirring models

Atanas Stefanov
University of Kansas, USA
stefanov@ku.edu
A. Comech, T. V. Phan

We establish asymptotic stability of solitary waves arising of the generalized Gross-Neveu and Thirring models.

Multisoliton Interactions for the Manakov System under Composite External Potentials

Michail Todorov
Technical University of Sofia, Bulgaria
mtod@tu-sofia.bg
V. S. Gerdjikov, A. V. Kyuldjiev

We analyze the dynamical behavior of the multisoliton train in adiabatic approximation of the perturbed Manakov system (MS)

$$i\mathbf{u}_t + \frac{1}{2}\mathbf{u}_{xx} + (\overline{\mathbf{u}}, \mathbf{u})\mathbf{u} = V(x)\mathbf{u}(x,t)$$

with composite external potentials of kind $V(x) = \sum_{s=1}^{N} c_s \mathrm{sech}^2(x - x_s)$ where x_s locate the positions of the small-amplitude ($|c_s| \ll 1$) wells/humps. We analyze the dynamics of multisoliton trains of Cauchy problem composed by MS and the initial condition

$$\mathbf{u}(x, t = 0) = \sum_{k=1}^{N} u_{k;1\mathrm{s}}(x, t = 0)\mathbf{n}_k,$$

where $u_{k;1\mathrm{s}}(x,t)$ is the 1-soliton solution of the scalar nonlinear Schrodinger equation with given velocity, amplitude, phase, and position, and $\mathbf{n}_k$ is normalized polarization vector (for details see [1]). We show that the dynamics of the multisoliton train is modeled by a perturbed complex Toda chain for the train parameters which generalize the results of [1]. Combining the analytic and numerical approach we focus also on the perturbation effects on the asymptotically free behavior, the bound state regime as well as on the mixed asymptotic regimes of the soliton trains and the transitions between them under the external potential. The results obtained extend the ones in Refs. [2,3,4]. The investigation is partially supported by the National Scientific Foundation of the Bulgarian Ministry of Education, Youth, and Science under grant DDVU02/71.

References

[1] V.S. Gerdjikov, E.V. Doktorov, and N.P. Matsuka. N-soliton Train and Generalized Complex Toda Chain for Manakov System, Theor. Math. Phys. 151(3), 762–773 (2007).

[2] M.D. Todorov and C.I. Christov. Impact of the Large Cross-modulation Parameter on the Collision Dynamics of Quasi-particles Governed by Vector NLSE, Math. Comp. Simul. 80(1), 46–55 (2009).

[3] V.S. Gerdjikov and M.D. Todorov. "On the Effects of Sech-like Potentials on Manakov Solitons," in 5th AMiTaNS'13, AIP CP1561, pp. 75–83, DOI: 10.1063/1.4827216, Melville, NY, 2013.

[4] V.S. Gerdjikov, and M.D. Todorov. "N-soliton Interactions for the Manakov System: Effects of External Potentials," in R. Carretero-Gonzalez et al. (eds.), Localized Excitations in Nonlinear Complex Systems, Nonlinear Systems and Complexity 7, pp.147–169, DOI: 10.1007/978-3-319-02057-0_7, Springer International Publishing Switzerland, 2014.

Nonlinear modes in a generalized PT-symmetric discrete nonlinear Schrödinger equation

Dmitry Zezyulin
University of Lisbon, Portugal
d.zezyulin@gmail.com
Dmitry Pelinovsky, Vladimir Konotop

We consider a generalized network of the discrete nonlinear Schrödinger type. The network consists of $2N$ sites where N is finite. The model is PT symmetric, i.e. it contains terms with gain and dissipation which compensate each other. The network accounts for the Kerr-type nonlinearity as well as for inter-site nonlinear coupling. Our results can be outlined as follows. (a) We obtain sufficient conditions of the unbroken and broken PT symmetry of the underlying linear problem. (b) We consider the nonlinear dynamics of the dimer model ($N = 1$) and show that even in the case of the unbroken PT symmetry there exist solutions unbounded in time. We also discover a new integrable configuration of the PT-symmetric dimer. All solutions of the integrable model are bounded in time provided that PT symmetry is unbroken. (c) We focus on the existence and stability of stationary nonlinear modes. A result of particular importance and novelty is the classification of all possible stationary modes in the limit of large amplitudes. We show that under certain conditions the system admits exactly $2^{N+1} - 2$ stationary modes (unique up to the gauge transformation) in the large amplitude limit.

Special Session 111: Computational Dynamics in Hamiltonian and Dissipative Systems

Fernando Blesa, Universidad de Zaragoza, Spain
Sergio Serrano, Universidad de Zaragoza, Spain
Arturo Vieiro, Universitat de Barcelona, Spain

The systematic analysis of dynamical systems responds to the scientific interest of a better understanding of the dynamics of concrete systems. An essential role in many areas of knowledge is played by systems with a Hamiltonian structure and/or by dissipative systems. A typical feature of Hamiltonian systems is the co-existence of regular and chaotic regimes in the phase space. Perturbations of Keplerian systems and gravitational n-body problems are well-known examples of Hamiltonian systems which, in particular, are systematically applied to many astrodynamical problems and chemical problems (e.g. the behaviour of atomic and molecular systems can be modelled as an n-body problem). Computational techniques for the detection/computation of chaotic regimes, periodic orbits, invariant tori, invariant manifolds associated to the hyperbolic objects, ... and their dependence with respect to parameters become essential in any (reasonable) explanation of the dynamics of the system. Dissipative systems show up different dynamical properties. Among these systems are, for example, the paradigmatic Lorenz problem obtained from a simplified climate model, the Rossler problem used in certain chemical reactions, different economic models, some models of laser with a chaotic response or the Hindmarsh-Rose model, used to study the neuronal activity. The (long-time) dynamics of such systems is determined by the presence of attractors (either periodic sinks, periodic orbits, invariant tori, strange attractors,...) that may co-exist in the phase space and evolve with respect the basic parameters. The combined use of different computational techniques is a basic tool to understand the dynamics of (both type of) these systems. These techniques include the systematic computation of chaos indicators, Lyapunov exponents, different invariants, the detection and analysis of essential bifurcations,... which can be combined (if needed) with the implementation of computer assisted proofs (interval analysis). This special session aims to present the main advances in computational techniques for analyzing dynamical systems and to show new results, about different dynamical systems, obtained by combining several computational techniques.

Production and Transfer of Energy and Information in Hamiltonian Systems

Chris Antonopoulos
University of Aberdeen/ICSMB, England
chris.antonopoulos@abdn.ac.uk
Ezequiel Bianco Martinez and Murilo Da Silva Baptista

In this talk I will discuss about some novel, recently published results [Antonopoulos et.al, 2014] that relate energy and information transfer with sensitivity to initial conditions in chaotic multi-dimensional Hamiltonian systems. I will show the relation among Kolmogorov-Sinai entropy, Lyapunov exponents, and upper bounds for the Mutual Information Rate calculated in the Hamiltonian phase space and on bi-dimensional subspaces. The main result is that the net amount of transfer from kinetic to potential energy per unit of time is a power-law of the upper bound for the Mutual Information Rate between kinetic and potential energies, and also a power-law of the Kolmogorov-Sinai entropy. Therefore, transfer of energy is related with both transfer and production of information. Finally, I will demonstrate the setup of an "experimental" implementation of a 1-dimensional communication channel based on a Hamiltonian system, and talk about the actual rate with which information is exchanged between the first and last particle of the channel.

Towards a symbolic quest into homoclinic chaos

Roberto Barrio
Universidad de Zaragoza, Spain
rbarrio@unizar.es
Tingli Xing and Andrey Shilnikov

In this talk we explore multi-fractal, self-similar structures of heteroclinic and homoclinic bifurcations of saddle singularities in the parameter space of a generic Z2-symmetric system. A computational technique based on the symbolic description utilizing kneading invariants is proposed for explorations of parametric chaos in systems with the Lorenz attractor. The technique allows for uncovering the stunning complexity and universality of the patterns discovered in the bi-parametric scans of the given models and detects their organizing centers – codimension-two T-points and separating saddles in the exemplary system: the Shimizu-Morioka model

A simple model for the location of Saturn F ring

Luis Benet
ICF, UNAM, Mexico
benet@fis.unam.mx
A. Jorba

Saturn's F ring is a fascinating example of a narrow eccentric ring that displays a rich dynamical structure: besides its non-zero eccentricity and sharp-edges, it has multiple components entangled in a complicated way which shows a variety of short-time features. The two shepherd moons, Prometheus and Pandora, influence importantly many of the short-time dynamical features observed. Yet, the location of the ring predicted by the shepherd theory does not correspond to the ring locationl. The confinement of Saturn's F-ring thus remains unexplained. In this talk, using a scattering approach, I shall present numerical results based on accurate long-time integrations on a realistic 5-body model for the occurrence and location of Saturn's F ring. Test particles that remain trapped and display some stability properties form a narrow elliptic ring displaying sharp edges. Comparison of our results with the observations is provided.

Chaos detection tools in Hamiltonian dynamics

Pablo Cincotta
Universidad Nacional de La Plata/CONICET, Argentina
pmc@fcaglp.unlp.edu.ar

In this talk I will discuss extensively the use of dynamical indicators that have proven to be efficient to investigate both regular and chaotic components of phase space of Hamiltonian systems. As it will be shown, they provide a clear picture of the resonance structure, the location of stable and unstable periodic orbits as well as a measure of hyperbolicity in chaotic domains which coincides with that given by the maximum Lyapunov characteristic exponent. Moreover, most dynamical indicators, based on the evolution of the so-called deviation vectors, are suitable to reveal the extremely thin chaotic layers around resonances and therefore, to investigate numerically the *diffusion* along a single resonance (Arnold diffusion?). Applications to discrete and continuous systems will be addressed, as well as an overview of the stability studies present in the literature encompassing quite different dynamical systems will be provided.

On the Dynamics of the Augmented Hill 3 Body Problem

Ariadna Farrés
Universitat de Barcelona, Spain
ariadna.farres@maia.ub.es
Àngel Jorba, Josep Maria Mondelo

In this talk we will describe the dynamics of the Augmented Hill 3 Body Problem (AH3BP), used to model the behaviour of a spacecraft propelled by a solar sail in the vicinity of an asteroids. The model includes the gravitational effect of both the Sun and the asteroid, as well as the solar radiation pressure due to the solar sail. Which depends on 3 parameters: the sail performance given by the sail lightness number β, and the sail orientation given by two angles α, δ. The AH3BP is Hamiltonian system. We will use different numerical tools to describe the bounded motion of the system. We will find equilibrium points, periodic orbits and invariant tori. We will try to give a complete description of the phase space properties and how this ones varies as we change the parameters in the system.

Automated detection of coherent Lagrangian vortices in extended two-dimensional flow domains

Florian Huhn
ETH Zurich, Switzerland, Switzerland
florian.huhn@imes.mavt.ethz.ch
D. Karrasch, G. Haller

The velocity field of a two-dimensional fluid flow defines a two-dimensional non-autonomous dynamical system for fluid particle motion. Coherent Lagrangian (i.e., material) vortices are generalized elliptic regions in this dynamical system. Haller and Beron-Vera have recently derived an objective variational principle for such coherent vortices, obtaining their boundaries as closed material lines with infinitesimally uniform stretching. The fluid in the interior of such a curve is transported by the flow without substantial deformation. Here, we discuss an automated detection method for such coherent vortex boundaries in spatially extended data sets. Our algorithm utilizes topological constraints on the singularities of the Cauchy-Green strain tensor field associated with the flow map. We demonstrate our algorithm by identifying coherent Lagrangian eddies in the Southern Ocean State Estimate (SOSE) data set.

Numerical continuation of invariant tori using averaging-extrapolation methods

Alejandro Luque
Universitat de Barcelona, Spain
luque@maia.ub.es
Jordi Villanueva

Frequency analysis was introduced by J. Laskar to study secular motions of the planets in the Solar system. A significant refinement of Laskar's methods, based in the simultaneous improvement of the frequencies and the amplitudes of the signal, was given given later by G. Gomez, J.M. Mondelo and C.Simo. On another front, a methodology to compute rotation numbers of invariant curves (and more general objects) has been introduced recently in several papers by A. Luque, J. Villanueva and T.M. Seara. The idea is to extrapolate the rotation number (and related quantities) from suitable averages of the iterates of an orbit. The goal of this talk is to present a methodology to compute the frequencies of a given quasi-periodic orbit. The construction is a generalization of the mentioned averaging-extrapolation approach to study rotation numbers and it allows us to compute with high precision the components of the frequency vector. The main advantage over other high precision methods is that we do not require to compute nor to refine the amplitudes of the signal. Moreover, using this methodology we can compute also derivatives of the frequencies with respect to parameters, so we can use a Newton method to compute and continue numerically quasi-periodic invariant tori. As an illustration, we will consider quasi-periodic motions close to the point L5 in an elliptic spatial restricted three-body problem.

An exploration of global properties of Area Preserving Maps

Narcis Miguel I Banos
Universitat de Barcelona, Spain
narcis@maia.ub.es
C. Simó, A. Vieiro

In this talk, we will present a methodology to investigate the processes that lead to chaos in area preserving maps (APM). Concretely, we will study some dynamical properties of orientation-preserving and orientation-reversing quadratic Hénon maps concerning the stability region, the size of the chaotic zones, its evolution with respect to parameters and the splitting of the separatrices of fixed and periodic points plus its role in the preceding aspects. Several techniques involving the computation of rotation numbers, estimates of the fraction of regular and chaotic points by means of Lyapunov exponent indicators and the measure of the splitting of the separatrices (using extended precision arithmetics when needed) will be used. These techniques can be also applied to study 2 degrees of freedom Hamiltonian systems in suitable Poincaré sections, in some level of energy. The role of Hénon maps in the diffusion which takes place in the Chirikov standard map for large values of the parameter will be stressed.

Flow reconstruction and invariant tori in the spatial three-body problem

Flora Sayas
Universidad Publica de Navarra, Spain
flora.sayas@unavarra.es
Jesús F. Palacián and Patricia Yanguas

We study the spatial three-body problem (a system of nine degrees of freedom). Our approach is based on a combination of averaging techniques with reduction theory with the aim of building a reduced Hamiltonian and a reduced phase space. The averaged Hamiltonian defines a system of one degree of freedom in the reduced space whose dimension is two which is a singular space embedded in $\mathbb{R}^3$. We consider every elliptic point in the reduced phase space and reconstruct it step by step to compute the corresponding KAM invariant tori that persist in the original space $\mathbb{R}^{12}$. In the cases where the equilibria are placed on the singular points of the two-dimensional reduced phase space we need to make a local analysis around the singularities, passing to a surface with no singular points. In the process of reconstruction we define intermediate spaces and sets of variables depending on the type of motions under consideration. Due to the scale in the perturbation, the Hamiltonian is too degenerate to apply classical results of KAM theory. Thus, we use a specific result by Han, Li and Yi for this kind of systems. We find Cantor families of five-dimensional KAM tori for the spatial three-body problem.

Organization of dissipative flows: Rössler model revisited

Sergio Serrano
Zaragoza University, Spain
sserrano@unizar.es
Roberto Barrio and Fernando Blesa

The Rössler model is a well-known problem among low-dimensional flows with chaotic behavior. Apart from regular and chaotic orbits, the Rössler system also has unbounded orbits with a transient (chaotic or regular) behavior. This fact makes a theoretical and numerical analysis of this problem more difficult. In this talk, we take Rössler model as paradigmatic example to show how the combined use of different numerical and analytical tools provides a deep insight of the organization of the bifurcation structure of the model, the origin of those structures and the mechanisms that lead to different behaviors of the flow. The main techniques that we combine and show in this talk include but not limited to biparametric Lyapunov exponent sweeps, parameter continuation to locate individual bifurcations of periodic and homoclinic orbits, the numerical computation of bounded regions and of different invariant sets.

Continuation of bifurcations of periodic orbits of dissipative PDEs.

Juan Sánchez Umbría
Universitat Politècnica de Catalunya, Spain
juan.j.sanchez@upc.edu
Marta Net

We consider the continuation of loci of bifurcations of periodic orbits in large scale dissipative systems depending on two parameters obtained after the discretization of PDEs. These kind of computations involve the integration of the variational equations up to order two when Newton-Krylov methods are employed. The actions of the Jacobian of the system, required by the linear solvers, are computed directly, instead of trying to approximate them by finite differences, for accuracy and convergence reasons. As a byproduct the methods allow also the continuation of bifurcations of fixed points. The application to the thermal convection of a binary fluid mixture will be used as an example of application. In our formulation the initial system consists of three partial differential equations for a streamfunction, and the perturbations of the temperature and the concentration of the denser component. This means that a system of twelve PDE's has to be integrated to compute the action of the Jacobian of the system for the bifurcation points.

On the computation of the abundance of attracting periodic orbits for families of 2D dissipative maps

Joan Carles Tatjer
Universitat de Barcelona, Spain
jcarles@maia.ub.es

TBA

Continuation of stable elliptic periodic orbits

Daniel Wilczak
Jagiellonian University, Poland
wilczak@ii.uj.edu.pl

We present an algorithm for rigorous validation of the existence and stability of branches of elliptic periodic orbits for area preserving maps. The method utilizes validated numerics for higher order derivatives of maps with analysis of normal form of the map at periodic point. As an application we give a computer assisted proof of the existence of wide branches of stable elliptic periodic orbits for the Michelson system and for the Henon-Heiles system.

Special Session 112: Nonlinear Dynamics in Neuroscience

Roberto Barrio, Universidad de Zaragoza, Spain
Toni Guillamon, Universitat Politecnica de Catalunya, Spain

Neuroscience is nowadays one of the most collaborative and active scientific research fields as it has been increasingly involving the participation of experts from other disciplines. In particular, computational and mathematical aspects of neuroscience are currently playing an important role both in modeling and replicating experimental findings and in explaining the underlying mechanisms of neurophysiological or cognitive processes. Differential equations are ubiquitous in the modeling of such phenomena and, consequently, nonlinear dynamics and dynamical systems techniques become fundamental sources of new mathematical and computational tools to study neuroscience models. The aim of this session is to present an overview of successful achievements in this rapidly developing collaborative field by putting together different types of applications of nonlinear dynamics (geometrical tools in dynamical systems, numerical methods, computational schemes, dynamical measures,...) to different problems in neuroscience (mononeuronal dynamics, network activity, cognitive problems,...). Additional emphasis will be put on experimental findings seeking for theoretical explanations. The final goal is spreading together mathematical methodology and neuroscience challenges and stimulating future cross-collaborations among participants.

Chaotic Breathing

Claude Baesens
University of Warwick, United Kingdom, Belgium
claude.baesens@warwick.ac.uk
Robert S MacKay

On exposure to opiates, preparations from rat brain stems have been observed to continue to produce regular expiratory signals but to fail to produce some inspiratory signals. The numbers of expirations between two successive inspirations form an apparently random sequence. Here we propose an explanation based on the qualitative theory of dynamical systems. A relatively simple scenario for the dynamics of interaction between the generators of expiratory and inspiratory signals produces pseudo-random behaviour of the type observed.

Describing chaotic structures in the Hindmarsh-Rose model of bursting neurons

Roberto Barrio
Universidad de Zaragoza, Spain
rbarrio@unizar.es
M. Angeles Martinez, Sergio Serrano, Marc Lefranc and Andrey Shilnikov

In this talk we study a plethora of chaotic phenomena in the Hindmarsh-Rose neuron model with the use of several computational techniques including the bifurcation parameter continuation, spike-quantification and evaluation of Lyapunov exponents in bi-parameter diagrams. We demonstrate how the organizing centers – points corresponding to codimension-two homoclinic bifurcations – along with fold and period-doubling bifurcation curves structure the biparametric plane, thus forming macro-chaotic regions resembling "onion bulb scales" and revealing spike-adding cascades that generate micro-chaotic structures due to the hysteresis. Moreover, we describe the topological changes in the structure of the chaotic attractors and their influence in the system.

Phase-Amplitude Response Functions for Transient-State Stimuli

Oriol Castejón
Universitat Politécnica de Catalunya, Spain
oriol.castejon@upc.edu
Antoni Guillamon, Gemma Huguet

The phase response curve (PRC) is a powerful tool to study the effect of a perturbation on the phase of an oscillator, assuming that the dynamics can be explained by the phase variable. However, factors like the rate of convergence to the oscillator or high stimulation frequency may invalidate this assumption and raise the question of how is the phase variation away from an attractor. Then the concept of isochron turns out to be crucial; from it, we propose an extension of advancement functions to the transient states by defining the Phase Response Functions (PRF) and the Amplitude Response Function (ARF). Using these we study the case of a pulse-train periodic stimulus, and compare the predictions given by the PRC-approach (a 1D map) to those given by the PRF-ARF-approach (a 2D map); we observe differences up to two orders of magnitude in favor of the 2D predictions. Apart from the comparison between 1D and 2D predictions, we also pay attention to bifurcations in the 2D maps that do not occur when using 1D maps. Summing up, we aim at enlightening the contribution of transient effects in predicting the phase response and showing the limits of the phase reduction approach.

Periodic solutions of the FitzHugh-Nagumo equations - a computer assisted proof

Aleksander Czechowski
Jagiellonian University, Poland
czechows@ii.uj.edu.pl
Piotr Zgliczyński

In this talk I will consider the FitzHugh-Nagumo slow-fast system arising from the travelling wave equation for the FitzHugh-Nagumo neuron model with diffusion. The existence of periodic solutions of this system is well known for timescales ratio parameter $\varepsilon \in (0, \varepsilon_0]$, ε_0 sufficiently small. The method I will present allows proving the existence of such solutions for ε_0 explicit, by combining rigorous numerical integration with the use of topological methods.

Canards in 3D piecewise-linear slow-fast systems; application to neuron models

Mathieu Desroches
Inria Paris-Rocquencourt Research Centre, France
mathieu.desroches@inria.fr
Antoni Guillamon, Enrique Ponce, Rafel Prohens and Antonio E. Teruel

In this talk, I will present recent results on canard-type solutions in piecewise-linear slow-fast systems in dimension 2 and 3. In particular, I will insist on the similarities and the differences with the smooth case. Applications to neuron models will be emphasised.

Dynamics of heteroclinic networks with diffusive gap-junction coupling

Irene Elices Ocon
Universidad Autonoma de Madrid, Spain
irene.elices@uam.es
Pablo Varona

Heteroclinic networks or neural ensembles that implement winnerless competition dynamics have recently been proposed to underlie encoding, coordination and execution of information at sensory, central and motor nervous systems. These kinds of networks are appropriate to describe information processing with transient neural dynamics by successive metastable states. Heteroclinic networks largely rely on topologies with asymmetric inhibitory connections. Using an activity rate model description, in this work we show that large-scale heteroclinic networks with inhibitory synapses and electrical gap-junction diffusive coupling give rise to coherent spatio-temporal patterns built of sequential activations.

Canard orbits in planar slow-fast piecewise linear systems

Soledad Fernandez Garcia
Inria, France
soledad.fernandez-garcia@inria.fr
Mathieu Desroches, Martin Krupa, Antonio E. Teruel

As it is well-known, the basis of most neuron models is to assume that a neuron behaves in first approximation as an electrical circuit. Moreover, electrical circuits have been shown to be faithfully modeled by piecewise linear systems. On the other hand, neuron models are characterized by different time scales, and canard phenomena (explosions, complex oscillations,...), have been extensively investigated in the context of smooth mathematical models of neuron. For that reason, we consider an interesting task the study of the existence of canards in piecewise linear slow-fast systems. In this work, we take a first step in this direction, by analyzing the existence of canard orbits in a family of planar slow-fast piecewise linear systems with three zones of linearity. In the near future, the purpose is trying to use the results obtained to provide models of neurons equally efficient to the smooth models, offering better mathematical tractability.

Piecewise-smooth stroboscopic maps in periodically driven spiking models

Albert Granados
Inria, France
albert.granados@inria.fr
Martin Krupa, Frédérique Clément

In this work we consider a general non-autonomous spiking model based on the integrate-and-fire model, widely used in neuronal modeling. Our unique assumption is that the system is monotonic, possesses an attracting subthreshold equilibrium point and is forced by means of periodic pulsatile (square wave) function.
In contrast to classical methods, in our approach we use the stroboscopic map instead of the so-called firing-map, and becomes a discontinuous map. By applying theory for piecewise-smooth systems, we avoid relying on particular computations and we develop a novel approach that can be easily extended to systems with other topologies (expansive dynamics) and higher dimensions.
We rigorously study the bifurcation structure in the two-dimensional parameter space formed by the amplitude and the duty cycle of the pulse. We show that it is covered by regions of existence of periodic orbits given by period adding structures. They completely describe all the possible spiking asymptotic dynamics and the behavior of the firing rate, which is a devil's staircase. Our results allow us to show that the firing-rate also follows a devil's staircase with non-monotonic steps when the frequency of the input is varied, and that there is an optimal response in the whole frequency domain.

Estimation of synaptic conductances: wrong procedures from experimental data and mathematical challenges

Antoni Guillamon
Universitat Politecnica de Catalunya, Spain
antoni.guillamon@upc.edu
Catalina Vich

A major problem in neuroscience is to unveil brain's connectivity. A local simplified situation is finding out which signal is receiving a single neuron subjected to a bombardment of synaptic inputs and discern the temporal contributions of excitation from those of inhibition. This quantitative information is important for the integrative properties of cortical neurons. Due to the multitude of synaptic contacts, obtaining direct measurements of the synaptic input becomes unrealistic. Therefore, inverse methods appear as an alternative to estimate the input conductances from experimental measurements. An extended strategy is filtering the voltage and then assume an input-output relationship. We have shown, using computational models, that this linearity hypothesis is unreliable both during spiking and in non-spiking regimes with ionic activity. We will lecture on the latest situation using a conductance-based model endowed both with an afterhyperpolarizing and low-subthreshold currents to show that the subthreshold activity can lead to significant errors in synaptic conductance estimation. Our results add a warning message about extracting conductance traces from intracellular recordings and the conclusions concerning neuronal activity that can be drawn from them. They also stimulate challenging questions in developing theoretical efficient methods, specially as an inverse problem in dynamical systems.

Mathematical and computational tools for phase control in transient-states of spiking neurons

Gemma Huguet
Universitat Politecnica de Catalunya, Spain
gemma.huguet@upc.edu
Oriol Castejón, Antoni Guillamon, Rafael de la Llave

Phase resetting curves (PRCs) constitute a powerful resource in time-control problems in biological processes. They predict the effect of a perturbation on the phase of an oscillator, assuming that it occurs on the attractor. However, factors like the rate of convergence to the oscillator, strong forcing or high stimulation frequency may invalidate the above assumption and raise the question of how is the phase variation away from an attractor. The computation of the phase advancement when we stimulate an oscillator which has not reached yet the asymptotic state (a limit cycle) relies on the concept of isochrons (the sets of points with the same asymptotic phase). In this talk, we will present efficient algorithms to compute limit cycles and their isochrons for planar vector fields. We formulate a functional equation for the parameterization of the invariant cycle and its isochrons and we show that it can be solved by means of a Newton method. Using the right transformations, we can solve the equation of the Newton step efficiently. We prove analytically the convergence of the algorithms. We will show some examples of the computations we have carried out for some well-known biological models.

Oscillations in the model of bipolar disorder.

Ekaterina Kutafina
AGH Krakow, Poland
ekaterina.kutafina@gmail.com
P. Szmolyan and I. Kosiuk

We study a phenomenological ODE model of the bipolar disorder which shows canard phenomena and mixed mode oscillations. The geometry of this four dimensional system is complicated, so the majority of results is based on numerical simulations. However some theoretical understanding could be gained by two small parameters approach combined with blow-up theory.

Computational tools for analysis of bursting polyrhythms in 3-cell CPG

Marcos Rodriguez
Centro Universitario de la Defensa de Zaragoza, Spain
marcos@unizar.es
Roberto Barrio, Sergio Serrano and Andrey Shilnikov

We study the formation of some rhythmic states in various 3-cell network motifs of a multifunctional central pattern generator (CPG) via several computational tools. The study is complemented with a detailed analysis of a single leech heart neuron, including bifurcations and Lyapunov exponents, that gives a “roadmap” for the basic neuron. The use of advanded GPU computing technologies is introduced.

Slow-fast n-dimensional piecewise linear differential systems

Catalina Vich
University of Balearic Islands, Spain
catilin75@hotmail.com
R. Prohens, A.E. Teruel

Slow-fast systems are a kind of system such that are used to modulate neural behaviours like bursts. The piecewise linear differential systems help to reproduce the dynamical richness of differential systems in the neuroscience modelling like global dynamics, forced behaviour, bifurcations and so on. However, they may also help to explain some of the previous phenomena. In this talk we present the slow-fast systems in n-dimensional piecewise linear framework providing some theorems about the existence, location and provenience of maximal and faux canard points.

Special Session 113: Normal Forms and Molecules in Motion Through Phase Space Bottlenecks

Florentino Borondo, Universidad Autonoma de Madrid and Instituto de Ciencias Matematicas, Spain
Rosa Maria Benito, Grupo de Sistemas Complejos, Universidad Politecnica de Madrid, Spain
Fabio Revuelta, Grupo de Sistemas Complejos, Universidad Politecnica de Madrid, Spain

A chemical reaction can be regarded as the process in which reactants are transformed into products, after passing through a transition state. This basic idea extends beyond chemistry to several other fields, such as celestial mechanics, where objects may be captured by a planet after passing through a transition state; stellar physics, where the rate of escape of stars from a cluster is regulated by a transition state, and in atomic physics, where electrons are responsible of atomic ionization. All of these seemingly disparate problems can be reconciled using methods developed in the context of Dynamical Systems Theory. The goal of this Minisymposium is to provide a forum for recent advances in the theory and application of transition state theory to be discussed with a view to mapping out fruitful future research directions.

Transition states and reaction rates in driven systems

Thomas Bartsch
Loughborough University, England
T.Bartsch@lboro.ac.uk
Galen Craven, Rigoberto Hernandez

When a chemical reaction is driven by an external field the transition state that the system must pass through as it changes from reactant and product—for example, an energy barrier—becomes time-dependent. We show that for periodic forcing the rate of barrier crossing can be determined through stability analysis of the non-autonomous transition state. Specifically, strong agreement is observed between the difference in the Floquet exponents describing stability of the transition state trajectory and the rates calculated by simulation of ensembles of trajectories. This result opens the possibility to extract rates directly from the intrinsic stability of the transition state, even when it is time-dependent, without requiring a numerically-expensive simulation of the long-time dynamics of a large ensemble of trajectories.

An exact description of invariant manifolds for laser-driven reactions via classical scattering theory

Daniel Blazevski
ETH Zurich, Institute for Mechanical Systems, Switzerland
blazevski@imes.mavt.ethz.ch
Rafael de la Llave; Jennifer Franklin

For systems subject to a large amplitude, transient perturbation – e.g. a laser-driven reaction – an exact description of the phase space structure for the kicked system can be given in terms that of the background system. The description comes from a wave operator based analogy of time-dependent scattering theory for general classical systems. For conservative models of reacting molecules, invariant stable and unstable manifolds have often been used to define transition states and dividing surfaces. For non-conservative laser-driven systems, stable and unstable manifolds in phase space vary in time, making the geometric view transition state theory more delicate. We use the description from scattering theory to compute such time-dependent stable and unstable manifolds for a laser-driven Henon-Heiles systems.

Survival probability distributions of microcanonical reactant ensembles in X-CN isomerization reactions

Pablo Garcia-Mueller
CIEMAT, Spain
pablo.garcia.muller@gmail.com
Rosa Maria Benito Zafrilla and Florentino Borondo

The classical Hamiltonian dynamics of X-CN three atomic systems have been thoroughly investigated in the past showing an intricate mixture of chaotic and regular phase space structures. The motion in the irregular component is dominated by the stickiness of trajectories close to the boundary circles separating regular from irregular regions. This stickiness significantly affects statistical quantities like Poincaré recurrences and isomerization reaction rates which exhibits algebraic behaviors. In this work, finite-time numerical calculations of $X--CN \leftrightharpoons CN--X$ isomerization reaction rates are carried out in order to assess possible universal non integer microcanonical reaction orders from the reactant decay laws.

Topological data analysis of macromolecular complexes

Marian Gidea
Yeshiva University, USA
Marian.Gidea@yu.edu

We will provide a brief description of persistent homology as a tool to analyze large data sets. We will illustrate this method by analyzing the density maps derived from electron microscopy for certain macromolecular complexes. We will also discuss some applications of this method to detection of critical transitions in noisy dynamical systems.

Transition State Theory in the molecular system LiNC/LiCN perturbed by a laser pulse

Henar Hernández
Universidad Politécnica de Madrid, Spain
henar.hmendiola@upm.es
J. C. Losada, R. M. Benito, A. Jorba, F. Borondo

In this talk we show the dynamical results obtained for the isomerization reaction LiNC/LiCN isolated and in the presence of a laser pulse. This study is based in the geometrical Transition State Theory (TST). We have develop an efficient and fast method for computing the normal forms that allows us to define the geometrical objects that lead the dynamics close to the characteristic solution (a fixed point in the isolated system, a periodic orbit when the system has been perturbed by a laser pulse). Once we have obtained these objects we have been able to compute reactive fluxes for pulses with different parameters. It has been proved that amplitude is the most influential parameter on the reaction dynamics and that increasing amplitude leads to greater fluxes and even to reactive fluxes at energies lowers than the ones in the isolated system.

Jet transport and normal forms

Angel Jorba
University of Barcelona, Spain
angel@maia.ub.es

In this talk we will focus on the use of the so-called jet transport technique to compute normal forms around periodic orbits of ordinary differential equations (ODEs). Jet transport is a technique to compute high order differentials of a given numerical algorithm, with respect to initial data and/or parameters. Combining this method with a numerical integrator for ODEs allows to compute Taylor expansions of Poincaré maps at a given fixed point. From this power expansion it is not difficult to compute the normal form at this point. Hence, this allows to compute high order approximations to the invariant manifolds around a given periodic orbit. We note that this approach can be used in situations that are far from integrable.

Normal forms and TST for wave packet dynamics

Andrej Junginger
University of Stuttgart, Germany
andrej.junginger@itp1.uni-stuttgart.de
Jörg Main, Günter Wunner

Variational approaches to quantum systems are a powerful framework for the efficient calculation of quantum dynamics. Therein, the original wave function is replaced by an appropriate trial wave function depending on a set of variational parameters, and the Schrödinger equation is approximately solved by applying a time-dependent variational principle. The latter defines a noncanonical Hamiltonian system, for which Darboux's theorem guarantees the local existence of canonical coordinates. In this talk, I will discuss how a normal form expansion in the variational space can be used to systematically construct such canonical coordinates in a natural way. The procedure has the advantage that the coordinates constructed are action-angle-variables in which the system becomes especially simple. As an application, a transition state theory for wave packet dynamics is presented. The treatment of quantum reaction dynamics is demonstrated for a model potential within the linear Schrödinger theory, and for Bose-Einstein condensates as nonlinear Schrödinger systems.

Chance and necessity in reactions under chaos and stochastic fluctuation

Tamiki Komatsuzaki
Hokkaido University, Japan
tamiki@es.hokudai.ac.jp

Chemical reactions are regarded as the most elementary prototype of change of the states or matter. How a rearrangement of atoms constituting a system occurs is one of the most intriguing fundamental questions from the day of alchemy. However, the question of why the system can change from one stable state to another is still unresolved. For example, in the case of isolated systems, i.e., many-degrees of freedom (dof) Hamiltonian systems, the total energy of the system should be larger than the barrier height on the potential. However, this is just only the necessary condition because even at the total energy higher than the saddle point energy, the system may go back to the state from which it climbs after wondering the region of the saddle. In other words, what origin differentiates the initial conditions which bring the system to the different states? Moreover, most of the reactions or conformational changes of biomolecules undergo in a thermally fluctuating environment. In this presentation, I overview some of recent progresses in our group and discuss the resonance condition between reactive and nonreactive dofs for systems under thermal fluctuation in terms of normal form theory for multidimensional Langevin systems, with some future perspectives.

Rate calculation with correlated noise

Fabio Revuelta
Universidad Politecnica de Madrid, Spain
fabio.revuelta@upm.es
Thomas Bartsch, R. M. Benito, F. Borondo

The usual identification of reactive trajectories for the calculation of reaction rates requires very time-demanding simulations, particularly if the environment presents memory effects. In this work, we develop a new method that permits the identification of reactive trajectories in a system under the action of a stochastic colored driving. This method is based on the perturbative computation of the invariant structures that act as separatrices for reactivity. Furthermore, using this perturbative scheme, we have obtained a formally exact expression for the reaction rate in multidimensional systems coupled to colored noisy environments.

Differentiable manifolds and Feynman's path integrals: efficient ways to investigate the role of quantum tunnelling in chemical systems

Judith Rommel
University of Cambridge, UK, England
jbr36@cam.ac.uk

Many catalytic processes involve hydrogen transfers such as enzymatically facilitated reactions in our bodies or the steam-reforming of methane during the Haber-Bosch process enhanced by metals. These hydrogen transfers are impacted by quantum effects such as atom tunnelling through the energy barriers of the reaction instead of going over it. Hydrogen tunnelling increases the reaction rates compared to the classical transition state theory model especially at low temperature. The imaginary free energy instanton method allows to investigate the role of tunnelling in high dimensional systems with several thousand degrees of freedom. The method is based on Feynman's path integral formalism to describe quantities from statistical quantum mechanics. The instanton is the most-likely tunnelling path obtained by optimising a ring polymer instead of running full quantum dynamics. In this talk the instanton method will (a) be explained by linking it to differential geometry, oscillatory integrals, and dynamical systems theory and (b) be applied to a couple of large chemical systems. The aim is to give a combined chemical and mathematical perspective on recent advances in quantum rate theory and the challenges to obtain efficient and reliable simulation methods.

The loss of the bottleneck property in High Energy Chemical Reactions

J. Pablo Salas
Universidad de La Rioja, Spain
josepablo.salas@unirioja.es
M. Iñarrea, J. Palacián and A. I. Pascual

In a chemical reaction, reactant and product regions are kept apart by a saddle point (a barrier). Thence, every reactive trajectory starts in the reactant region, crosses the barrier, and it enters in the product region. In this contest, Transition State Theory (TST) plays a fundamental role. The main assumption of this theory relies on the existence in phase space of a dividing surface located at the neighborhood of the barrier, the so-called Transition State (TS). A proper TS separates reactants from products, it is only crossed by reactive trajectories, and it is a non-recrossing surface in the sense that reactive trajectories must cross it only once. As long as the energy of the reaction remains close enough to the saddle point energy, the saddle point region present the so-called bottleneck structure and the TS preserves its non-recrossing property. However, as the energy increases, the bottleneck structure of the saddle region is lost and the non-recrossing property of the TS is lost due to the appearance of additional dynamical barriers in the saddle region. In this contest, the main subject of this talk is the study of the mechanisms for the appearance of those additional dynamical barriers.

The Quantum Normal Form approach to Transition State Theory

Roman Schubert
University of Bristol, England
roman.schubert@bristol.ac.uk
A. Goussev, H. Waalkens and S. Wiggins

Transition state theory describes certain classes of chemical reactions of molecules. In the realm of classical mechanics normal forms have been used for the qualitative and quantitative understanding of reaction dynamics as a passage through phase space bottlenecks. Recently we introduced a quantum version of this approach in which Quantum Normal Forms are used to approximate the quantum dynamics of the reaction, and to compute a local S-matrix which incorporates reaction rates. In this talk we will give an overview of this approach and discuss its application to some model systems.

Bifurcations of transition states in bimolecular reactions

Dayal Strub
University of Warwick, England
strubdayal@gmail.com
Robert MacKay

One way of finding rates of transport in Hamiltonian systems is to choose a dividing surface and compute the flux through it. This approach originated in physical chemistry, where it is used in transition state theory, but has since been applied to a multitude of fields. In this talk, we shall return to the original test-problems from the early days of transition state theory, namely bimolecular reactions in gaseous phase, and review them in the light of recent studies of the geometry and bifurcations of transition states. The reactions shall be considered in full generality, that is in 3D and with arbitrary angular momentum, and we shall find interesting sequences of Morse bifurcations of transition states and dividing surfaces.

Reaction Coordinate Switching Mechanism, on the possibility of its experimental verification and its quantum manifestation

Hiroshi Teramoto
Research Institute for Electronic Science / Hokkaido University, Japan
teramoto@es.hokudai.ac.jp
Mikito Toda and Tamiki Komatsuzaki

Reaction coordinate switching is a sudden switch between a reaction coordinate and a non-reactive coordinate triggered by a breakdown of a normally hyperbolic invariant manifold located in a vicinity of a reaction barrier. This phenomenon occurs only in systems with more than two degrees of freedom and thus it adds several novel features that cannot be seen in bifurcations of periodic orbit dividing surfaces in two-degrees-of-freedom systems. Moreover, it is also expected to be applicable in controls of chemical reactions. In this talk, we propose several methods to analyze the phenomenon. In addition, we address a possibility of its experimental verification and also its quantum manifestations in a hydrogen atom in crossed electric and magnetic fields.

References
[1] H. Teramoto, M. Toda and T. Komatsuzaki, Phys. Rev. Lett. 106, 054101 (2011).
[2] H. Teramoto, G. Haller and T. Komatsuzaki, Chaos 23, 043107 (2013).
[3] H. Teramoto, M. Toda and T. Komatsuzaki, in preparation to submit Theor. Chem. Acc.
[4] H. Teramoto, M. Toda and T. Komatsuzaki, Nonlinearity, submitted.
[5] H. Teramoto, M. Toda and T. Komatsuzaki, in preparation

Recollision scenario without tunneling: Role of the ionic core potential

Turgay Uzer
Georgia Institute of Technology, USA
tuzer@gatech.edu
A. Kamor, C. Chandre, F. Mauger

We present a purely classical recollision scenario, i.e., without tunneling, which, in contrast to the standard three-step model, takes into account the ionic core potential fully at all stages of the recollision process and is valid at all intensities. We find that a key periodic orbit drives the recollisions by guiding electrons away and back to the core. At sufficiently high intensity, we connect our scenario to the three-step model, and explain why the three-step model leads to good agreement with the cut-off in high harmonic generation despite neglecting the core potential after tunneling.

Construction of phase space structures governing reaction dynamics in rotating molecules

Holger Waalkens
University of Groningen, Netherlands
h.waalkens@rug.nl
Unver Ciftci

Recently, the phase space structures governing reaction dynamics in Hamiltonian systems have been identified and algorithms for their explicit construction have been developed. These phase space structures consist of invariant manifolds whose existence is induced by saddle type equilibrium points which are characteristic for reaction type dynamics. The main algorithms to construct the phase space structures are based on a Poincaré-Birkhoff normal form. Using tools from the geometric theory of Hamiltonian systems and their reduction, we show in this talk how this construction of these phase space structures can be generalized to the case of the relative equilibria of a rotational symmetry reduced N-body system. As rotations almost always play an important role in the reaction dynamics of molecules, the approach presented in this talk is of great relevance for applications.

Special Session 114: Nonstandard Analysis, Quantizations and Singular Perturbations

Imme V.D. Berg, Evora University,
Masanao Ozawa, Nagoya University,
Kiyoyuki Tchizawa, Institute of Administration Engineering, Japan

Nonstandard Analysis is effective in the context of discretizations and changes of scale, for example blow-ups. Notably in the case of singular perturbations and quantum-mechanics phenomena are studied which involve simultanuously the small and the observable. The different orders of magnitude of the parameters may then be related by general principles of nonstandard analysis, like the Transfer Principle. The present special session is devoted to several methods and applications developed in these settings.

Chatter Dynamics on Impulsive Surfaces in Impulsive Differential Systems via the Theory of flow switchability

Xilin Fu
Shandong Normal University, Peoples Rep of China
xilinfu@hotmail.com

Abstract In this talk, the phenomenon of free vibrations in LC circuit is introduced as well as some restrictions in the application of triode. Then we optimize the problems and present a certain kind of Van der Pol Equations which can be considered as a class of second-order impulsive switched systems. To investigate the chatter dynamics on such system, we turn to look for conditions that keep the complex pulse phenomena absent. We introduce several conceptions of theory of flow switchability and analyze the flow's dynamical behaviors such as transversal property at a boundary in the normal direction of separation surface by constructing generic mappings. Some sufficient conditions for the absence of pulse phenomena are obtained.

Canards existence in 3D and 4D singularly perturbed systems

Jean-Marc Ginoux
Laboratoire LSIS, CNRS, UMR 7296, Universite de Toulon, France
ginoux@univ-tln.fr
Lliber, Rossetto, Lozi, Chua

The aim of this work is to propose a new method for proving the existence of "canard solutions" for three and four-dimensional singularly perturbed systems with only one fast variable which does not require a center manifold reduction nor a blow-up technique i.e. a desingularization procedure for folded singularities. Contrary to previous works, this method uses the normalized slow dynamics and not the projection of the desingularized vector field. This method enables to state a unique generic condition for the existence of "canard solutions" for such three and four-dimensional singularly perturbed systems which is based on the stability of folded singularities of the normalized slow dynamics and not of the projection of the desingularized vector field. Application of this method to the famous three and four-dimensional memristor canonical Chua's circuits for which the classical piecewise-linear characteristic curve has been replaced by a smooth cubic nonlinear function according to the least squares method enables to prove the existence of "canard solutions" in such Memristor Based Chaotic Circuits. Moreover, extension of this method to the case of four-dimensional singularly perturbed systems with two fast and two slow variables will be briefly presented. Then, applications of this method to the famous coupled FitzHugh-Nagumo equations and to the Hodgkin-Huxley model will enable to prove as many previous works the existence of "canard solutions" in such system.

S-continuous financial time series composed from the delta-function

Shuya Kanagawa
Tokyo City University, Japan
sgk02122@nifty.ne.jp
Kiyoyuki Tchizawa

When the time series are decomposed as the fluctuation and the usual trend, it is possible to recompose S-continuous time series using a delta-function through nonstandard analysis. It is done under a weak integrability assumption. The representation of the financial time series can be applied to analysis of option prices.

On retarded canards: Complex oscillations in delayed slow-fast systems

Maciej Krupa
INRIA, France
maciej.krupa@inria.fr
Jonathan Touboul

We analyze canard explosions in delayed differential equations with a one-dimensional slow manifold. This study is applied to explore the dynamics of the van der Pol slow-fast system with delay self-coupling. In the absence of delays, this system provides a canonical example of a canard explosion. The presence of delays significantly enriches the dynamics, and varying the delay induces canard explosion, mixed mode oscillations as well as transitions to complex bursting periodic orbits. We show that as the delay is increased a family of 'classical' canard explosions ends as Bogdanov-Takens bifurcation occur at the folds points of the S-shaped critical manifold. Canard explosion and mixed-mode oscillations are investigated by means of geometric perturbation analysis, and bursting by means of slow-fast periodic averaging.

The "no waste" assumption and transversality conditions for discrete-time infinite-horizon problem

Takashi Nitta
Mie University, Japan
nitta@edu.mie-u.ac.jp
Dapeng Cai

The overtaking criterion commonly used when examining infinite-horizon optimization problems, corresponds to free terminal state finite-horizon problems. We modify this criterion by imposing a "no waste" assumption, under which the entire end-of-horizon stocks and used up before the end of planning horizon. We establish the necessary and sufficient conditions for optimality for a general discrete-time problem, under the modified optimality criterion.

Quantum Set Theory and Modal Interpretation of Quantum Mechanics

Masanao Ozawa
Nagoya University, Japan
ozawa@is.nagoya-u.ac.jp

In 1981, Takeuti introduced quantum set theory as a quantum counterpart of Boolean valued models of set theory by constructing a model of set theory based on quantum logic represented by the lattice of closed subspaces in a Hilbert space and showed that appropriate quantum counterparts of ZFC axioms hold in the model. In 2007, we extended the Takeuti formulation to construct a model of set theory based on the logic represented by the lattice of projections in an arbitrary von Neumann algebra and established a transfer principle that modifies every theorem of ZFC to a true statement for the model. Here, we discuss the following problem: In what model every theorem of ZFC holds with probability one in a given state. We call such a model as a beable universe. We determine all beable universes maximal under the condition that it contains a given observable and it is implicitly defined by the given state and observables. We also discuss the relation between those notions and a modal interpretation of quantum mechanics developed by Bub, Clifton, and Halvorson.

Canards in the coupled FitzHugh-Nagumo equations

Kiyoyuki Tchizawa
Institute of Administration Engineering, Japan
tchizawa@kthree.co.jp

In this talk, it is shown that there exist 4-dimensional canards in the coupled FitzHugh-Nagumo equations. There exists a 4-dimensional singular perturbation problem in this system. It was composed in 2001 by S.A.Campbell and K.T.. This system is described as a co-dimension 2 problem, that is, having two fast vector fields and two slow vector fields. The relation between the canards and the invariant manifold is provided.

Stochastic Euler method

Imme Van den Berg
University of Evora, Portugal
ivdb@uevora.pt

We study differential equations of the form

$$dX/dt = f(X),$$

where f may be only measurable. We intend to obtain more regularity and numerical feasability by the stochastic discretization

$$\delta x_t = f(x_t + w_t)\delta t.$$

The time-step δt is infinitesimal and the stochastic variables w_t are Lebesgue-measurable, independent and identically distributed, but are not necessarily standard: their variance may be infinitesimal. We give conditions to ensure that the solutions are almost surely infinitely close to the more regular system

$$\delta y_t = Ef(y_t + w_t)\delta t$$

Often the expectations are calculated by convolution. The regularizations obtained are of numerical interest for the original differential equation when f presents discontinuites or if there is no unqueness of solutions. The results are based on joint work with C. Lobry (Nice) and T. Sari (Montpellier).

Proof transformations for nonstandard analysis

Keita Yokoyama
Japan Advanced Institute of Science and Technology, Japan
y-keita@jaist.ac.jp

From the view point of mathematical logic, nonstandard analysis is based on the conservation results of nonstandard axioms. Here, conservation means that a theorem which is provable using nonstandard method is provable without using nonstandard method. This kind of conservation result is usually shown by model theory, thus we cannot know the information of a standard proof from a nonstandard proof. On the other hand, the conservation result can be also shown by proof-theoretic way, which gives a concrete proof transformation of nonstandard proofs into standard proofs. In this talk, I would like to give several examples of transformations of nonstandard proofs, and try to explain how we can understand nonstandard proofs from the view point of standard proofs.

Special Session 115: Mathematical Models of Chemotaxis

Jose Ignacio Tello, Universidad Politecnica de Madrid, Spain
Michael Winkler, Paderborn University, Germany

Chemotaxis is the ability of living organisms to orientate their movement towards or away a chemical substance. The term was introduced to describe cell migration observed during the early days of the development of microscopy in the XIX century. As the technology advanced, chemotactic mechanisms have turned out to be of outstanding relevance in numerous biological processes such as immune system response, embryonic development, tumor growth, or bacterial cluster formation, for instance. In the past decades, after the pioneering works of Keller and Segel in the 1970s, various chemotaxis processes have been described mathematically by using systems of evolutionary PDEs with nonlinear cross-diffusive terms as their most characteristic ingredient. Particular mathematical challenges originate from the fact that this chemotactic cross-diffusion can enforce singularity formation of solutions, thus letting even the basic questions of existence theory become nontrivial in many situations. Although such explosions have been detected in certain rather special chemotaxis systems under appropriate assumptions, a comprehensive understanding of these self-organizing features is still lacking. Accordingly, the analysis of chemotaxis systems still forms a very active field of mathematical research, involving steadily increasing expertise from various fields in PDE analysis. The proposed special session intends to showcase recent trends in the mathematics of chemotaxis systems. A particular focus will be on topics and techniques related to rigorous mathematical analysis, but also novel aspects in modeling and simulation will be addressed. We thereby aim at bringing together experts from different branches in the mathematics of chemotaxis, ranging from essentially mathematically motivated to mainly application-oriented, but also ranging from experienced specialists to auspicious young researchers, in order to provide rich opportunities for fruitful and multilateral discussion.

Long time behavior of a fully parabolic chemotaxis system with strong logistic reactions

Xueli Bai
East China Normal University, Peoples Rep of China
baixueli2012@gmail.com
Michael Winkler

In this talk, we deal with a fully parabolic Keller-Segel systems with strong logistic sources. We especially focus on the long time behavior of this system with one of the competition coefficient less than 1. We prove the global existence first and then show that the global solutions converge to either the positive steady state or semitrivial steady state depending on the competition coefficients.

Blowup in multidimensional chemotaxis

Piotr Biler
University of Wroclaw, Poland
Piotr.Biler@math.uni.wroc.pl
Grzegorz Karch, Jacek Zienkiewicz

We present some new results on the blowup of solutions in chemotaxis systems in multidimensional space. These results are presented for the simplest Keller-Segel model as well as for the model with the degradation term. Sufficient conditions for blowup are expressed in terms of suitable Morrey norms.

Global existence vs. blowup in a fully parabolic quasilinear 1D Keller-Segel system

Jan Burczak
Institute of Mathematics, Polsh Academy of Sciences, Poland
jb@impan.pl
Tomasz Cieslak, Cristian Morales-Rodrigo

We consider the one-dimensional fully parabolic Keller-Segel system with nonlinear diffusion. It turns out that it possesses global-in-time solutions, provided the nonlinear diffusion is equal to $\frac{1}{(1+u)^\alpha}$ with $\alpha < 1$, independently on the volume of the initial data. On the other hand, for any nonlinear diffusion integrable at infinity there exist initial data for which a solution blows up in a finite time. Thus the most interesting is the critical case of the diffusion equal to $\frac{1}{1+u}$. There we are able to show the global regularity result for initial masses smaller than a prescribed constant and we conjecture that the bound on the initial mass is of a technical nature.

Global bounded solutions of the higher-dimensional Keller-Segel system under smallness conditions in optimal spaces

Xinru Cao
Dalian University of Technology, Peoples Rep of China
caoxinru@gmail.com

The fully parabolic Keller-Segel system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v), & (x,t) \in \Omega \times (0,T), \\ v_t = \Delta v - v + u, & (x,t) \in \Omega \times (0,T), \end{cases}$$

is considered under Neumann boundary conditions in a bounded domain $\Omega \subset \mathbb{R}^n$ with smooth boundary, where $n \geq 2$. We derive a smallness condition on the initial data in optimal Lebesgue spaces which ensure global boundedness and large time convergence.

A simplified Keller-Segel model: construction of exact solutions for the Cauchy and Neumann boundary-value problems

Roman Cherniha
Nottingham University, England
cherniha@gmail.com
Maksym Didovych

This research is a natural continuation of our recent paper "Exact solutions of the simplified Keller-Segel model" published in Commun Nonlinear Sci Numer Simulat 2013; 18: 2960-2971. Here we show that the Keller-Segel type system in the case of one space variable is linearisable. The linearization procedure consists of a chain of substitutions, including the well-known Cole- Hopf substitution. Moreover, using the classical results for the linear heat equation and the Burgers equation, we construct exact solutions for the initial problem and the boundary-value problem with the zero fluxes in the case of the Keller-Segel system in question. Uniqueness of the solutions obtained is investigated and the relevant restrictions are derived. Finally, we present some results, including Lie symmetry invariance and particular exact solutions, for the model in the case of two space variables.

Critical mass in 2d volume filling Keller-Segel

Tomasz Cieslak
IMPAN, Poland
cieslak@impan.pl

In my talk I will review our recent common results with Christian Stinner related to the fully parabolic volume filling Keller-Segel model with a probability jump function given by

$$q(u) = (1+u)^{-\gamma} \ \ \gamma \geq 0.$$

The most interesting one is a critical mass phenomenon in dimension 2 which states that for $\gamma \geq 1$ the value of intial mass distinguishes between global-in-time bounded solutions for all the data with $m < 4(1+\gamma)$ in the case of radially symmetric solutions the critical value is $8(1+\gamma)$ an d an existence of solutions which become infinite when time goes to ∞, however they exist globally in time, if the initial mass of radially symmetric data exceeds $8(1+\gamma)$. For $0 < \gamma < 1$ we have a similar result, the only difference is that it is open whether in the supercritical case solutions which are known to be infinite blow up in finite time or they exist globally. For the precise results and proofs see T.Cieslak, C. Stinner, New critical exponents in a fully parabolic Keller-Segel and applications to volume filling models. Preprint.

Travelling waves in hybrid chemotaxis models

Radek Erban
University of Oxford, England
erban@maths.ox.ac.uk
Benjamin Franz, Chuan Xue, Kevin J. Painter

Hybrid models of chemotaxis combine agent-based models of cells with partial differential equation (PDE) models of extracellular chemical signals. I will discuss travelling wave properties of hybrid models of bacterial chemotaxis. Bacteria are modelled using an agent-based (individual-based) approach with internal dynamics describing signal transduction. In addition to the chemotactic behaviour of the bacteria, the individual-based model also includes cell proliferation and death. Cells consume the extracellular nutrient field (chemoattractant) which is modelled using a PDE. Mesoscopic and macroscopic equations representing the behaviour of the hybrid model are derived and the existence of travelling wave solutions for these models is established.

Behavior of solutions to parabolic-elliptic Keller-Segel systems with signal-dependent sensitivity

Kentarou Fujie
Tokyo University of Science, Japan
kentarou.fujie@gmail.com
Tomomi Yokota

In this talk, we deal with the Keller-Segel systems with signal-dependent sensitivity function. We especially focus on a model of chemotaxis processes where movement towards higher signal concentrations is inhibited at points where these concentrations are high. Such saturation effects are usually accounted for by introducing a signal-dependent sensitivity function. We study the behavior of solutions under some conditions on the initial date and the sensitivity function.

On the Patlak-Keller-Segel model with a nonlocal flux

Rafael Granero-Belinchón
University of California, Davis, USA
rgranero@math.ucdavis.edu
Yago Ascasibar, Jose M. Moreno, Rafael Orive

We consider the system of equations given by

$$\partial_t \rho = -\beta(-\Delta)^{\alpha/2}\rho + \nabla\cdot(\nabla u \rho) \qquad (1)$$
$$\Delta u = \rho - \langle \rho \rangle. \qquad (2)$$

This system of pde was proposed by Keller and Segel as a model for the motion of cells. It is assumed that these cells (ρ) move towards increasing concentrations of some chemical substance (u). The system (1)-(2) also appears as a model of gravitational collapse. Indeed the system (1)-(2) is very similar in spirit to the Zel'dovich approximation used in Cosmology to study the formation of large-scale structure in the primordial universe. Motivated by these problems, we addressed the analysis of (1)-(2). In particular we study the continuation criteria and the global existence of classical solution for small initial data. These results can be found in (Ascasibar, Granero, Moreno, *Physica D*, 2013). Furthermore, we study the one-dimensional, quasilinear, critical system

$$\partial_t \rho = -\partial_x(\beta(\rho)H\rho) + \partial_x(\partial_x u \rho)$$
$$\partial_x^2 u = \rho - \langle \rho \rangle,$$

where H denotes the Hilbert transform. We obtained global existence of weak solution, among other results. These results can be found in (Granero, Orive, *Submitted*, 2013). During this talk, we will try to motivate and explain the main results in these two papers.

Global-in-time solutions of the parabolic-parabolic Keller-Segel system on the plane

Ignacio Guerra
Universidad de Santiago de Chile, Chile
ignacio.guerra@usach.cl
Piotr Biler and Grzegorz Karch

It is well known that the parabolic-elliptic Keller-Segel system of chemotaxis on the plane has global-in-time regular non- negative solutions with total mass below the critical value 8π. Solutions with mass above 8π blow up in a finite time. Here we study the case of the parabolic-parabolic Keller-Segel where we proved that each mass may lead to a global-in-time-solution, even if the initial data is a finite signed measure.

A high order finite volume scheme for a model of chemotaxis

Arturo Hidalgo
Universidad Politecnica de Madrid, Spain
arturo.hidalgo@upm.es

In this work a finite volume scheme is applied to obtain the numerical solution of a mathematical model representing the cells movement due to chemotactic processes. The system of equations is of reaction-diffusion type with nonlinear source term. The problem is solved in a two-dimensional domain. In order to carry out the spatial reconstruction from cell averages, a high order WENO procedure is used. Time integration is achieved using a third order Runge-Kutta TVD scheme.

Existence of solutions to Keller-Segel-Stokes systems

Sachiko Ishida
Tokyo University of Science, Japan
s-ishida@rs.tus.ac.jp

We'd like to consider the existence of solutions (n, c, u, P) to Keller-Segel-Stokes systems (KSS), where components of the solution denote the bacteria density, the chemical concentration, the velocity field of the fluid and the pressure in the order of inputting. The analysis of this system is very difficult, but there are few results upon the global-in-time existence: As to the 2D case, there are results on the global existence in (KSS) with the linear diffusion (Δn) on $\mathbb{R}^2$ and with quasilinear degenerate diffusion $(\Delta n^m,\ m > 1)$ on bounded convex domains. Moreover in the 3D case, we know the global existence in (KSS) with $\Delta n^{\frac{4}{3}}$ on $\mathbb{R}^3$ and with Δn^m $(m > \frac{8}{7})$ on bounded convex domains. As can be seen from these, there are many open interesting problems. In this talk we'd like to discuss these problems.

Asymptotic behavior of solutions for an aerobatic model coupled to fluid equations

Kyungkeun Kang
Yonsei University, Korea
kkang@yonsei.ac.kr
Myeongju Chae and Jihoon Lee

We consider coupled system of Keller-Segel type equations and the incompressible Navier-Stokes equations in spatial dimension two. We show temporal decay estimates of solutions with small initial data and obtain their asymptotic profiles as time tends to infinity.

Global existence in time of a model of chemotaxis

Akisato Kubo
Fijita Health University, Japan
akikubo@fujita-hu.ac.jp
Jose Ignacio Tello

We deal with the following chemotaxis system with non-diffusive chemical gradients arising from mathematical biology and medicine for $u := u(x,t)$ and $v := v(x,t)$ $\frac{\partial u}{\partial t} - \Delta u = -\text{div}\,(\chi u \nabla v) + \lambda u(1-u-v), (x,t) \in \Omega \times (0,T)$ $\frac{\partial v}{\partial t} + h(v) = u$, where Ω is a bounded domain in R^n with the smooth boundary $\partial\Omega$, χ and λ are positive constants. We consider a intiial boundary value problem of the above system for the outer unit normal vector n $\quad \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0 \quad \text{in} \quad \partial\Omega \times (0,T) \quad u(x,0) = u_0(x), v(x,0) = v_0(x)$. We will discuss existence of global solutions in time and asymptotic behavior of solutions.

Thresholds on population density? - Not with chemotaxis (and slow enough diffusion)!

Johannes Lankeit
Universität Paderborn, Germany
jlankeit@math.upb.de

We define and (for $q > n$) prove uniqueness and an extensibility property of $W^{1,q}$-solutions to

$$\begin{aligned} u_t &= -\nabla \cdot (u \nabla v) + \kappa u - \mu u^2 \\ 0 &= \Delta v - v + u \\ \partial_\nu v &= \partial_\nu u = 0, u(0,\cdot) = u_0, \end{aligned}$$

in balls in $\mathbb{R}^n$, which we then use to obtain a criterion guaranteeing some kind of structure formation in a corresponding chemotaxis system - thereby extending recent results of Winkler to the higher dimensional (radially symmetric) case.

Global Wellposedness and Traveling Wave Solutions of Chemotaxis Models

Tong Li
University of Iowa, USA
tong-li@uiowa.edu

We investigate local and global existence, blowup criterion and long time behavior of classical solutions for a hyperbolic-parabolic system derived from the Keller-Segel model describing chemotaxis. Moreover, we establish the existence and the nonlinear stability of large-amplitude traveling wave solutions to the system of nonlinear conservation laws derived from Keller-Segel model.

Blow-up versus global behaviour in a chemotaxis model

Gabriela Litcanu
Institute of Mathematics "O. Mayer", Iasi, Romania
glitcanu@yahoo.com

We study the long time asymptotic behaviour for a mathematical model used to describe a class of aggregation phenomena. We investigate when blow-up occurs or whether globally solutions exist and how asymptotic dynamics depend on the parameters of the system, the initial data and the space dimension.

Angiogenesis models with chemotaxis

Cristian Morales-Rodrigo
University of Seville, Spain
cristianm@us.es

In this talk I will consider various models related to tumor angiogenesis. The models consist in various parabolic equations with transport terms as well as nonlinear boundary conditions. We will show global existence of solutions and we will study in detail the associated elliptic systems.

Asymptotic stability of a chemotaxis system with non-diffusive chemoattractant

Mihaela Negreanu
Complutense University, Spain
negreanu@mat.ucm.es
J. Ignacio Tello

We study the behavior of two biological populations "u" and "v" attracted by the same chemical substance which behavior is described in terms of second order parabolic equations. The model considers a logistic growth of the species and the interactions between them are relegated to the chemoatractant production. The system is completed with a third equation modeling the evolution of chemical. We assume that the chemical "w" is a non-diffusive substance and satisfies an ODE, more precisely, the system is presented by

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u\chi_1(w)\nabla w) + \mu_1 u(1-u), \\ v_t = \Delta v - \nabla \cdot (v\chi_2(w)\nabla w) + \mu_2 v(1-v), \\ w_t = h(u,v,w) \end{cases}$$

where $x \in \Omega$ and $t > 0$ under appropriate boundary and initial conditions in a n-dimensional open and bounded domain Ω. We consider the cases of positive chemosensitivities, not necessarily constant elements. The chemical production function h increases as the concentration of the species "u" and "v" increases. We first study the global existence and uniform boundedness of the solutions by using an iterative approach. The asymptotic stability of the homogeneous steady state is a consequence of the growth of h, χ_i and the size of μ_i. Finally, some examples of the theoretical results are presented for particular functions h and χ_i.

Global existence of solutions to a parabolic-parabolic chemotaxis system with subquadratic growth

Koichi Osaki
Kwansei Gakuin University, Japan
osaki@kwansei.ac.jp
Etsushi Nakaguchi

We are concerned with the global existence of solutions to a parabolic-parabolic chemotaxis system with growth in a smooth bounded domain in R^n. Our aim is to show a sufficient condition, in particular, some relations between the orders of degradation and secretion, for global existence. In this talk we will review the result of the case where $n = 2$ and 3(DCDS-B, special issue on chemotaxis, 2013), including the existence of attractors. We will discuss also the higher dimensional cases.

Competitive exclusion in a two-species chemotaxis model

Christian Stinner
University of Kaiserslautern, Germany
stinner@mathematik.uni-kl.de
Jose Ignacio Tello, Michael Winkler

We consider a mathematical model for the spatio-temporal evolution of two biological species in a competitive situation. Besides diffusing, both species move toward higher concentrations of a chemical substance which is produced by themselves. The resulting system consists of two parabolic equations with Lotka-Volterra-type kinetic terms and chemotactic cross-diffusion, along with an elliptic equation describing the behavior of the chemical. We study the question in how far the phenomenon of competitive exclusion occurs in such a context and identify parameter regimes for which indeed one of the species dies out asymptotically, whereas the other reaches its carrying capacity in the large time limit.

Uniqueness theorem on weak solutions to the Keller-Segel system of degenerate and singular types

Yoshie Sugiyama
Kyushu university, Japan
sugiyama@math.kyushu-u.ac.jp
Yoshiyuki Kagei, Tatsuki Kawakami, Masanari Miura

The Keller-Segel system contains several parameters which cause numerous structures such as linear, degenerate and singular type of PDE. In particular, the degenerate type contains the unknown function as the coefficients breaking down uniform ellipticity, which makes the problem more difficult in comparison with the other types. The Keller-Segel system itself is characterized as the parabolic-parabolic and parabolic-elliptic both of provide us an important research theme. Indeed, we need to handle these types in accordance with the characteristic features of equations. In this talk, we shall bring a focus onto the parabolic-parabolic and parabolic-elliptic Keller-Segel systems of the singular and degenerate types and show uniqueness of weak solutions in the class of Hoelder continuous functions.

Global dynamics of a coupled chemotaxis-haptotaxis system

Youshan Tao
Donghua University, Peoples Rep of China
taoys@dhu.edu.cn
Michael Winkler

This talk addresses a coupled chemotaxis-haptotaxis system modeling cancer cell invasion of surrounding tissue, which describes the interplays between the cancer cell density, the concentration of a matrix-degrading enzyme (MDE) and the density of extracellular matrix (ECM). In addition to random movement, cancer cells are supposed to bias their movement both towards increasing concentrations of urokinase plasminogen activator by chemotaxis, and towards increasing densities of the non-diffusible ECM through detecting the macromolecules adhered therein by haptotaxis. It is assumed that the cancer cells undergo birth and death in a logistic manner, competing for space with the ECM. The MDE is assumed to be produced by cancer cells, and to diffuse and decay, whereas the ECM is assumed to be degraded upon contact with MDE. We will discuss the global existence, boundedness and asymptotic behavior of solutions to the system, which strongly depend on the model parameters and initial conditions.

From Individual to Collective Dynamics in Argentine Ants

Maria Vela-Perez
Universidad Europea de Madrid; CEA, Spain
mvp_es@yahoo.es
M. A. Fontelos

Social insects are an important example of complex collective behavior. In particular, ant colonies develop different tasks as foraging, building and allocation [1]. While they search for food they deposit a pheromone that it is considered as a crucial element in the mechanism for finding minimal paths. The experimental observations suggest that the model should include the presence of pheromone and the persistence (tendency to follow straight paths in the absence of other effects). In our study, based on the experimental data described in [2], we develop a model in the plane to describe the behavior of Argentine Ants when foraging in the plane. Following the ideas explained in [3] we consider ants as random walkers. We treat them as pure random walkers when they detect an amount of pheromone that is below a certain threshold. The idea is that ants, once out of the nest, start foraging for food and do it following a random walk with the probability distribution for the change in direction that is fitted, from experimental data, to a distribution with fat tails. Once the ant detects an amount of pheromone concentration above the threshold, the motion changes to a reinforced random walk where a component of the change in the ant's direction is proportional to the gradient of the amount of pheromone.

References

[1] B. Holldobler and K. Wilson, *The ants*, Berlin: Springer, 1990

[2] A. Perna, et al. (2012) Individual rules for trail pattern formation in Argentine ants (Linepithema humile). PLOS Comput Biol 8(7):e1002592.

[3] M. Vela-Perez, et al. (2013), Ant foraging and geodesic paths in labyrinths: Analytical and computational results, J. Theo. Biol. 320, 100-112.

Quasilinear degenerate/singular parabolic system modeling biofilm and taking into account nutrient taxis.

Dariusz Wrzosek
University of Warsaw, Poland
darekw@mimuw.edu.pl

We analyze a prototype model of the colony of cells forming a biofilm. The model describes changes in biofilm density caused by nonlinear diffusion, food (nutrients) taxis and cell proliferation. The model is a system of two quasilinear degenerate/singular parabolic equations into which two thresholds are built in. One occurs at zero cell density level, the second one is related to the maximal density which the cells cannot exceed. Accordingly, both diffusion and taxis terms have degenerate or singular parts. This model extends a previously introduced degenerate biofillm model by combining it with the food-taxis. We give conditions for existence and uniqueness of weak global solutions and illustrate the model behavior in numerical simulations. The results are contained in the joint paper by H. Eberl, M. Efendiev, A.Zhigun and D.W. *Analysis of a degenerate biofilm model with a nutrient taxis term*, Discrete and Continuous Dynamical Systems 34 (1) 2014. 99-119

Nonlinear m-accretive operator approach to parabolic-elliptic chemotaxis systems with superlinear growth

Tomomi Yokota
Tokyo University of Science, Japan
yokota@rs.kagu.tus.ac.jp
Noriaki Yoshino

This talk presents a nonlinear m-accretive operator approach to parabolic-elliptic chemotaxis systems with nonlinear diffusion and superlinear growth. In the case of Lipschitz growth, Marinoschi (2013) established the existence of local-in-time weak solutions with sufficiently small initial data. The purpose of this talk is to show that in the case of superlinear growth there exists a local-in-time weak solution for any large data.

Special Session 116: Interacting Population on Social, Economic and Ecological Networks

Rosa Maria Benito, Universidad Politecnica de Madrid, Spain

Humans are structured in social networks which affect society in many ways: the functioning of free markets, contagion of diseases, communication patterns or political choices. The data collected from online communications such as Twitter or Facebook and cell phone calls has opened a new framework to the understanding of human behavior through the social networks in which individuals are embedded. However network structure is not unique to humans but common to all species in Nature. This workshop aims to bring together mathematicians and scientists of ecology, economics and social sciences so that they better understand the challenges faced by each discipline and how to best collaborate.

Sentiment in New York City: A High Resolution Spatial and Temporal View

Yaneer Bar Yam
New England Complex Systems Institute, USA
alfredo_morales_g@yahoo.es

Measuring public sentiment is a key task for researchers and policymakers alike. The explosion of available social media data allows for a more time-sensitive and geographically specific analysis than ever before. In this paper we analyze data from the micro-blogging site Twitter and generate a sentiment map of New York City. We develop a classier specifically tuned for 140-character Twitter messages, or tweets, using key words, phrases and emoticons to determine the mood of each tweet. This method, combined with geotagging provided by users, enables us to gauge public sentiment on extremely fine-grained spatial and temporal scales. We find that public mood is generally highest in public parks and lowest at transportation hubs, and locate other areas of strong sentiment such as cemeteries, medical centers, a jail, and a sewage facility. Sentiment progressively improves with proximity to Times Square. Periodic patterns of sentiment fluctuate on both a daily and a weekly scale: more positive tweets are posted on weekends than on weekdays, with a daily peak in sentiment around midnight and a nadir between 9:00 a.m. and noon.

Meritocracy and Topocracy in the Age of Networks

Javier Borondo
Universidad Politecnica de Madrid, Spain
fj.borondo@upm.es
F. Borondo, C. Rodriguez-Sickert, C. A. Hidalgo

A system is said to be meritocratic if the compensation and power available to individuals is determined by their abilities and merits. A system is topocratic if the compensation and power available to an individual is determined primarily by her position in a network. Here we introduce a model that is perfectly meritocratic for fully connected networks but that becomes topocratic for sparse networks-like the ones in society. In the model, individuals produce and sell content, but also distribute the content produced by others when they belong to the shortest path connecting a buyer and a seller. The production and distribution of content defines two channels of compensation: a meritocratic channel, where individuals are compensated for the content they produce, and a topocratic channel, where individual compensation is based on the number of shortest paths that go through them in the network. We solve the model analytically and show that the distribution of payoffs is meritocratic only if the average degree of the nodes is larger than a root of the total number of nodes. We conclude that, in the light of this model, the sparsity and structure of networks represents a fundamental constraint to the meritocracy of societies.

Networks of contacting linguistic domains

Jose Capitan
Consejo Superior de Investigaciones Cientificas, Spain
jcapitan@gmail.com
Jacob B. Axelsen, Susanna Manrubia

Human linguistic groups exhibit distinctive quantitative patterns, and most of them have a clear counterpart in ecological systems. Among those patterns, the distribution of home ranges' areas and the number of speakers of each group have been shown to follow log-normal distributions. In this contribution we analyze the topological properties of regional networks of neighboring linguistic groups. We show that such networks are topologically different from classical families of networks, and relate those properties with the concept of intervality in food webs. We propose a model to generate such networks based solely on the distribution of areas, and we test the hypothesis that the area distribution explains the empirical degree of intervality of real networks. In particular, degrees inherit the pattern observed for areas and degree distributions can be well fitted by log-normal functions. We find a very good agreement between model networks and empirical data.

A Non-linear Lead Lag Dependence Analysis of News and Return in a Corporate News Network

German Creamer
Stevens Institute of Technology, USA
gcreamer@stevens.edu

This paper evaluates the lead lag relationship between news and asset return in a corporate news network. We build a sequence of daily corporate news network for the period 2005-2011 using companies of the STOXX 50 index as nodes; the weights of the edges are the sum of the number of news items with the same topic by every pair of companies identified by the topic model methodology. The STOXX 50 includes the top 50 European companies by level of capitalization. We use the Brownian distance non-linear correlation and the Granger linear causality test to conduct a lead-lag analysis of asset return and news

that define the above network. We observe that the Brownian distance correlation determines relationships similar to those identified by the linear Granger causality test, and it also uncovers additional non-linear relationships between news and assets return.

Mobile phone data for measuring ethnical groups interaction

Werner Creixell
Universidad Tecnica Federico Santa Maria, Chile
werner.creixell@usm.cl
Alfredo Morales, Javier Borondo, Juan Carlos Losada, Rosa Maria Benito

Undeveloped countries has to deal with many social issues with a lack of basic infrastructure and reliable data sources for defining public policies. In this work we explore the use of mobile phone communication patterns to extract meaninful information for decision makers. We use the data for measuring the communication among different ethnical groups, information that can be very useful in places with risk of ethnical violence.

Competitivity Graphs: a new method to compare rankings

Regino Criado
Rey Juan Carlos University, Spain
regino.criado@urjc.es
E.Garcia, F. Pedroche and M.Romance

A new technique to analyze families of rankings will be shown. Specifically, it will be shown how to use some structural properties of a new class of graphs called competitivity graphs. Some particular examples will be provided.

A neutral model for Interbank lending networks

Sara Cuenda
Universidad Autonoma de Madrid, Spain
sara.cuenda@uam.es
Maximiliano Fernández, Javier Galeano and José A. Capitán

In the past decade several approaches to the description of the topology and statistical properties of interbank networks have been developed. Among the empirical networks, the most prevalent case study is the Italian interbank market, which has shown to be disassortative and to have fat-tailed distributions in degree, number of transactions and transaction sizes. However, data regarding interbank networks are not publicly available (when available). Thus, having a simple model that reproduces the topological properties of interbank networks would be very relevant in this field. In this work we model interbank market transactions as a temporal network based on an uncorrelated daily generation of the exposures between banks. We use empirical, publicly available data to draw transactional links each day. The aforementioned topological descriptors of the resulting aggregated network agree, qualitatively and quantitatively, with previous results reported in the literature.

New ideas about logistic approach for population dynamics of mutualistic interactions

Javier Galeano
Universidad Politecnica de Madrid, Spain
javier.galeano@upm.es
J. García-Algarra,J. M. Pastor, J.M. Iriondo, J. J. Ramasco

Classic papers declare that the plant-animal interactions have played a very important role in the generation of Earth's biodiversity. The importance of mutualism, the beneficial interaction between two species, for biodiversity maintenance can be supported by the fat that more than 90% of tropical plant species depend on animals for the dispersal of their seeds. These interactions are best represented by weighted mutualistic bipartite networks. Those networks have been repeatedly reported to show particular properties as truncated power law distribution of the degree or a nested structure. While several metrics for measuring nestedness in weighted mutualistic networks have been proposed, most dynamic models in the literature aim to reproduce just the binary nested structure ignoring the development of the weighted pattern. In this study, we have proposed a new model modifying the classical logistic equation with additional terms to take into account interspecies interactions. We have developed a model of mutualism based on the aggregation of benefit for each species in its equivalent growth rate:

$$\frac{1}{N_i^a}\frac{dN_i^a}{dt} = r_i + \sum_{k=1}^{n_p} b_{ik}\, N_k^p - \left(\alpha_i + c_i \sum_{k=1}^{n_p} b_{ik}\, N_k^p\right) N_i^a$$

$$\frac{1}{N_j^p}\frac{dN_j^p}{dt} = r_j + \sum_{\ell=1}^{n_a} b_{j\ell}\, N_\ell^a - \left(\alpha_j + c_j \sum_{\ell=1}^{n_a} b_{j\ell}\, N_\ell^a\right) N_j^p$$

where the superscripts stand for each of the class of species: animals and plants. $N_i(N_j)$ is the population of the species $i(j)$; $r_i\,(r_j)$ is the intrinsic growth rate of population. The rate of mutualistic interactions between a species i and another j is given by b_{ij}, which can be seen as elements of a matrix encoding the mutualistic interaction network. Note that the matrix is not necessarily symmetric if the benefit of the interaction is different for the two species involved. Following Velhurst's idea for the logistic equation, this implies that the friction term, α_i, must also depend on the mutualistic interactions. In order to keep the model simple, we assume that the effect of the mutualism on α is proportional to the benefit. Finally, c_i is a proportionality constant. With these equations, we have built a binomial stochastic simulator for the study of system dynamics. It allows the introduction of external perturbations such as step increases in mortality by plagues, removal of links between species due to evolution, or overlapping of a predator foodweb.

The elliptic model for communication fluxes

Carlos Herrera Yague
Univesidad Politecnica Madrid, Spain
carlos@hyague.es
CM Schneider, Z Smoreda, T Couronne, PJ Zufiria and MC Gonzalez

In this talked, a model (called Elliptic Model) is presented to estimate the number of social ties between two locations using population data in a similar manner transportation research does with trips. To overcome the asym-

metry of transportation models, the new model considers that the number of relationships between two locations is reversely proportional to the population in the ellipse whose focuses are in these two locations. The Elliptic Model is evaluated by considering the anonymous communications patterns of 25 million users from 3 different countries, where a location has been assigned to each user based on her most used phone tower or billing zip code. With this information, spatial social networks are built at three levels of resolution: tower, city and region for each of the three countries. The Elliptic Model reaches similar performance when predicting communication fluxes as transportation models do when predicting trips. This shows that human relationships are at least as influenced by geography as human mobility is

Limited communication capacity unveils strategies for human interaction

Giovanna Miritello
Telefonica Research, Spain
giovanna.miritello@uc3m.es
R.Lara, M.Cebrian, E.Moro

Social connectivity is the key process that characterizes the structural properties of social networks and in turn processes such as navigation, influence or information diffusion. Since time, attention and cognition are inelastic resources, humans should have a predefined strategy to manage their social interactions over time. However, the limited observational length of existing human interaction datasets, together with the bursty nature of dyadic communications have hampered the observation of tie dynamics in social networks. Here we develop a method for the detection of tie activation/deactivation, and apply it to a large longitudinal, cross-sectional communication dataset ($\approx$ 19 months, $\approx$ 20 million people). Contrary to the perception of ever-growing connectivity, we observe that individuals exhibit a finite communication capacity, which limits the number of ties they can maintain active. In particular we find that men have an overall higher communication capacity than women and that this capacity decrease gradually for both sexes over the lifespan of individuals (16-70 years). We are then able to separate communication capacity from communication activity, revealing a diverse range of tie activation patterns, from stable to exploratory: humans manage their interactions so that the number of open connections is kept constant through time and thus some individuals are *social keepers* (they maintain a clustered static network around them), other seem to undergo a *social exploring* strategy (in which many ties are formed and destroyed in time. Finally, we use computer simulations to investigate the possibility that social explorers have a competitive advantage towards information awareness because of their fast and distant tie formation dynamics. Counterintuitively, we found that social keepers receive information before social explorers do. The answer to this paradox is that, despite having a large variability in the social structure around a social explorer, the amount of time allocated to their contacts is low enough to produce countervailing effects and hinder information diffusion.

Efficiency of Human Activity on Information Spreading on Twitter

Alfredo Morales
Grupo de Sistemas Complejos. Universidad Politecnica de Madrid., Spain
alfredo.moralesg@alumnos.upm.es
J. Borondo, J.C. Losada, R.M. Benito

Understanding the collective reaction to individual actions is key to effectively spread information in social media. In this work we define efficiency on Twitter, as the ratio between the emergent spreading process and the activity employed by the user. We characterize this property by means of a quantitative analysis of the structural and dynamical patterns emergent from human interactions, and show it to be universal across several Twitter conversations. We found that some influential users efficiently cause remarkable collective reactions by each message sent, while the majority of users must employ extremely larger efforts to reach similar effects. Next we propose a model that reproduces the retweet cascades occurring on Twitter to explain the emergent distribution of the user efficiency. The model shows that the dynamical patterns of the conversations are strongly conditioned by the topology of the underlying network. We conclude that the appearance of a small fraction of extremely efficient users results from the heterogeneity of the followers network and independently of the individual user behaviour.

Trophic coherence determines food-web stability

Miguel Munoz
University of Granada, Spain
mamunoz@onsager.ugr.es
Samuel Johnson and Virginia Dominguez-Garcia

Why are large, complex ecosystems stable? Both theory and simulations of current models predict the onset of instability with growing size and complexity, so for decades it has been conjectured that ecosystems must have some unidentified structural property exempting them from this outcome. We show that *trophic coherence* – a hitherto ignored feature of food webs which current structural models fail to reproduce – is significantly more correlated with stability than are size or complexity. Furthermore, we prove that a maximally coherent network will always be stable. We also propose a simple model which, by correctly capturing the trophic coherence of food webs, accurately reproduces their stability and other basic structural features. Most remarkably, our model shows that stability can increase with size and complexity. This suggests a key to May's Paradox, and a range of opportunities and concerns for biodiversity conservation.

Is the Voter Model a model for voters?

Jose Ramasco
IFISC (CSIC-UIB), Spain
jramasco@ifisc.uib-csic.es

The voter model has been studied extensively as a paradigmatic opinion dynamics' model. However, its ability for modeling real opinion dynamics has not been addressed. We introduce a noisy voter model (accounting for social influence) with agents' recurrent mobility (as a proxy for social context), where the spatial and population diversity are taken as inputs to the model. We show that the dynamics can be described as a noisy diffusive process that contains the proper anisotropic coupling topology given by population and mobility heterogeneity. The model captures statistical features of the US presidential elections as the stationary vote-share fluctuations across counties, and the long-range spatial correlations that decay logarithmically with the distance. Furthermore, it recovers the behavior of these properties when the geographical space is coarse-grained at different scales from the county level through congressional districts and up to states. Finally, we analyze the role of the mobility range and the randomness in decision making which are consistent with the empirical observations.

Mapping the world trade web

M. Ángeles Serrano
Universitat de Barcelona, Spain
marian.serrano@ub.edu

Hidden geometries underlying real networks provide a simple and natural explanation for their large-scale structure and growth dynamics. These models assume a newtonian-like connection probability between nodes that encodes a trade-off between popularity and similarity, such that nodes closer in space –that is, more similar– are more likely to interact and nodes with more connections –that is, more popular– can reach further neighbors. Beyond their ability to simulate the observed topologies (including scale-free degree distributions, high levels of clustering, and self-similarity), these models enable a true cartography of real networks which can be embedded into a metric space using an optimization method. Applications to real systems range from metabolic networks to the Internet at the autonomous systems level. In economic systems, the gravity theory of trade flows can be reformulated to explain the observed topology of the world trade web in terms of a hidden metric space network model. Distances between two countries in the hidden geometry would correspond to an integrated measure that incorporates different distance attributes. We plan to map the world trade web such that the different countries are placed in relative positions according to not only their geographic location but also the actual aggregated barriers to international trade in the world.

Multi-layer structures of functional networks

Massimiliano Zanin
INNAXIS Foundation & Research Institute, Spain
mzanin@innaxis.org

In the last decade, complex network theory has evolved towards more comprehensive ways of representing complex systems: from simple mappings of real-world structures up to functional networks, in which links represent the presence of some kind of relationship between the time series describing pairs of nodes. The last frontier is the multi-layer representation, in which links of different nature are considered. It has extensively been shown that disregarding the multi-layer structure can distort the understanding that we have of the system, as the dynamics of even simple processes, *e.g.* propagation of failures or information, strongly depends on the chosen network representation. Nevertheless, less attention has been devoted to the relationship between functional and multi-layer networks. In this talk, I tackle this problem by dint of a multi-layer functional representation of the topology created by delay propagation in the European air transport system. By comparing different ways of projecting the dynamics and the topology, I show how any attempt at simplifying the representation results in catastrophic distortions of the characteristics of the system. Such results are relevant for understanding the limitations of the actual way of studying real-world systems, like for instance the human brain.

Special Session 117: Rigorous and Numerical Methods for Invariant Manifolds

Alex Haro, Universitat de Barcelona, Spain
Jordi-Lluis Figueras, Uppsala Universitetet, Sweden
Alejandro Luque, Universitat de Barcelona, Spain

Geometrically speaking, solutions of Dynamical Systems are organized according to hierarchies of invariant objects. This structure constitutes the skeleton of the dynamics in phase space. There is a long tradition of rigorous analytical results, starting with the work of H. Poincaré, dealing with existence and properties of such invariant objects (equilibria, periodic and quasi-periodic solutions, normally hyperbolic invariant manifolds, etc.). The use of analytical techniques is usually not enough to obtain a complete picture of this scenario in the study of a particular problem. In order to understand and rigorously characterize such objects, the corresponding dynamics on them and the connections between them, researchers started to develop algorithms for their effective computation in finite and infinite dimensional systems. In the last few years these efficient methods have led to methodologies for the rigorous validation of these objects with the assistance of computers (Computer-Assisted Proofs). In this session we plan to bring together researchers of all these aspects of invariant manifolds.

Rigorous verification of the first bifurcation problem of the Henon map

Zin Arai
Hokkaido University, Japan
zin@math.sci.hokudai.ac.jp
Yutaka Ishii

We study the first bifurcatoin problem of the real Hénon map and show that the boundary of the full horseshoe locus is characterized by a tangency. This is a generalization of the result obtained by Bedford and Smillie for the parameter region with very small Jacobian. To extend their results, we need to develop several topological techniques and we also use rigorous interval arithmetic to verify the topological conditions on the configuration of stable and unstable manifolds of the map. We note that although the result itself is on the real Hénon map, we need to work on the complex Hénon map to obtain the proof, and therefore the invariant manifolds we need to control are complex curves.

Existence and stability of traveling pulse solutions for the FitzHugh-Nagumo equation. Part I: Existence

Gianni Arioli
Politecnico di Milano, Italy
gianni.arioli@polimi.it
Hans Koch

The FitzHugh-Nagumo model is a reaction-diffusion equation describing the propagation of electrical signals nerve axons and other biological tissues. One of the model parameters is the ratio of two time scales, ranging from 1/1000 to 1/10 in typical simulations of nerve axons. Based on the existence of a (singular) limit at ratio=0, it has been shown that the FitzHugh-Nagumo equation admits a stable traveling pulse solution for sufficiently small ratio > 0. In the work presented here we prove the existence and stability of such a solution for the ratio 1/100. We cover both circular axons and axons of infinite length. Our method is non-perturbative and should apply to a wide range of other parameter values.

KAM estimates for quasi-periodic solutions of a conformally symplectic system

Renato Calleja
IIMAS-UNAM, Mexico
calleja@mym.iimas.unam.mx
Alessandra Celletti, Rafael de la Llave

Conformally symplectic systems send a symplectic form into a multiple of itself. They appear in mecanical systems with friction proportional to the velocity and as Euler-Lagrange equations of the time discounted actions common in economics. The conformally symplectic structure provides identities that we use to prove "a-posteriori" theorems that show that if we have an approximate solution which satisfies some non-degeneracy conditions, we can obtain a true solution close to the approximate one. The identities used to prove the theorem, also lead to very efficient algorithms and computer assisted proofs.

Parameterization Method for Computing Normally Hyperbolic Invariant Tori

Marta Canadell
universitat de barcelona, Spain
marta@maia.ub.es
A. Haro

In this talk we explain a numerical algorithm for the computation of normally hyperbolic invariant tori (NHIT) in families of discrete dynamical systems. The application of the parameterization method leads to solving invariance equations for which we use a Newton-like method adapted to the dynamics and the geometry of the invariant torus and its invariant bundles. The method computes the NHIT and its internal dynamics, which is a priori unknown. We apply the method to several examples, and explore the mechanism of breakdown of invariant tori.

Cone Condition Enclosure of Invariant Manifolds for Saddle-Center Fixed Points, with Application to the Restricted Three Body Problem

Maciej Capinski
AGH University of Science and Technology, Poland
mcapinsk@agh.edu.pl
Anna Wasieczko

We present a method for establishing invariant manifolds for saddle-center fixed points. The method is based on cone conditions, suitably formulated to allow for application in computer assisted proofs, and does not require rigorous integration of the vector field in order to prove the existence of the invariant manifolds. We apply our method to the restricted three body problem and show that for a given choice of the mass parameter, there exists a homoclinic orbit to one of the libration points.

Automatic reducibility methods for whiskered tori and their manifolds

Rafael de la Llave
Georgia Institute of Technology, USA
rafael.delallave@math.gatech.edu
G. Huguet, A. Haro

We present some methods that allow the simultaneous computation of whiskered tori and their stable and unstable manifolds in Hamitonian systems. The methods do not require transformation theory. The theorem has an a-posteriori format, which allows to validate numercal results. It also leads to very efficient algorithms requiring low storage and low operation count. Amusingly, the method to compute the tori and the whiskers may be faster than algorithms to compute just the torus.

Critical Asymmetric Tori in the Multiharmonic Standard Map

Adam Fox
Georgia Institute of Technology, USA
afox33@math.gatech.edu
James D. Meiss

Invariant circles play an important role as barriers to transport in the dynamics of area-preserving maps. KAM theory guarantees the persistence of some circles for near-integrable maps, but far from the integrable case all circles can be destroyed. A standard method for determining the existence or nonexistence of a circle, Greene's residue criterion, requires the computation of long-period orbits, which can be difficult if the map has no reversing symmetry. We use a quasi-Newton, Fourier-based scheme to numerically compute the conjugacy of a Diophantine circle conjugate to rigid rotation, and the singularity of a norm of a derivative of the conjugacy to predict criticality. We study near-critical conjugacies for families of rotational invariant circles in generalizations of Chirikov's standard map. A first goal is to obtain evidence to support the long-standing conjecture that when circles breakup they form cantori, as is known for twist maps by Aubry-Mather theory. A second goal is to support the conjecture that locally most robust circles have noble rotation numbers, even when the map is not reversible. Finally, we observe that the rotation number of the globally most robust circle generically appears to be a piecewise-constant function in two-parameter families of maps.

Shadowing Lemmas for Normally Hyperbolic Invariant Manifolds and Applications

Marian Gidea
Yeshiva University, USA
Marian.Gidea@yu.edu

We will discuss several shadowing lemmas-type of results for normally hyperbolic invariant manifolds and some applications to Hamiltonian and conservative systems.

Computer-assisted proofs in incompressible fluids

Javier Gomez-Serrano
Princeton University, USA
javier.gomez@icmat.es
Angel Castro, Diego Cordoba, Charles Fefferman, Francisco Gancedo, Rafael Granero-Belinchon, Alberto Martin Zamora

In this talk we will address some computer-assisted proofs and techniques applied to several interface problems in incompressible fluid dynamics, such as the α-patches, the inhomogeneous Muskat, and the water waves problem.

Existence and stability of traveling pulse solutions for the FitzHugh-Nagumo equation, Part II: Stability.

Hans Koch
The University of Texas at Austin, USA
koch@math.utexas.edu
Gianni Arioli

The FitzHugh-Nagumo model is a reaction-diffusion equation describing the propagation of electrical signals nerve axons and other biological tissues. One of the model parameters is the ratio of two time scales, ranging from 1/1000 to 1/10 in typical simulations of nerve axons. Based on the existence of a (singular) limit at ratio = 0, it has been shown that the FitzHugh-Nagumo equation admits a stable traveling pulse solution for sufficiently small ratio > 0. In the work presented here we prove the existence and stability of such a solution for the ratio 1/100. We cover both circular axons and axons of infinite length. Our method is non-perturbative and should apply to a wide range of other parameter values.

Lagrangian KAM tori far from integrability

Alejandro Luque
Universitat de Barcelona, Spain
luque@maia.ub.es
Alex Haro, Jordi-Lluis Figueras

KAM theory deals with the existence and persistence of invariant tori carrying quasi-periodic motion. Nowadays, KAM theory is a vast area of research that involves a large collection of methods and applications. The parametrization method, introduced during the last decade by Rafael de la Llave and collaborators, consists in adding a small function to a parametrization of an approximately invariant torus. This function is obtained by approximately solving the linearized invariance equation around the approximated torus (Newton-like iterative scheme). The approach is suitable for studying existence of invariant tori without using neither action-angle variables nor a perturbative setting. In this talk we present a KAM theorem using a common frame that unifies several works in the literature. We will consider some academic examples (the standard map and the Froeschle map) to illustrate the application of the result for specific values of the perturbation parameter (i.e. not arbitrarily small). Then, we will discuss how this methodology allows us to obtain invariant tori in a non-perturbative setting using computer-aided-methods.

Breakup of Invariant Tori in Volume-Preserving Maps

James Meiss
University of Colorado, USA
jdm@colorado.edu
A.M. Fox

Invariant tori are prominent features of Hamiltonian and symplectic dynamical systems that are integrable or nearly so. The celebrated KAM theorem implies the structural stability of certain "very irrational" (Diophantine) tori, and robustness is responsible for the long time correlations and slow transport in chaotic Hamiltonian dynamics. Each preserved torus has an associated rotation vector. When the rotation vector is a scalar (two-tori for flows, one-tori for maps), the torus can be approximated by a sequence of periodic orbits obtained by the continued fraction expansion. This leads to Greene's residue method for determining the breakup threshold of invariant circles in twist maps. A generalization to higher-dimensional tori is difficult because there is no satisfactory generalization of the continued fraction, and instead of a single "residue", there are multiple stability multipliers for periodic orbits. We study three-dimensional, volume-preserving maps with invariant two-tori. We generalize the residue criterion when the map is reversible. More generally, computations of the conjugacy to rigid rotation are used to compute the threshold. The local singular values of these functions gives evidence for the existence of remnants after breakup, analogous to the cantori of symplectic maps.

Some recent developments in parameterization of stable/unstable manifolds

Jason Mireles James
Rutgers/Florida Atlantic University, USA
jmireles@math.rutgers.edu

I will discuss some new applications of the Parameterization Method for invariant manifolds. In particular we examine how the Floquet normal form for a hyperbolic periodic orbit can be used in order to efficiently compute high order Fourier-Taylor expansions for the stable/unstable manifolds of the orbit. I will discuss an analogous normal form for fast/slow invariant sub manifolds which aids in the computation of the invariant linear bundles of the sub manifold. I will try to motivate the material with examples and computations.

Computing invariant manifolds of vector fields at fixed points using the parametrization method.

Josep-Maria Mondelo
Universitat Autonoma de Barcelona, Spain
jmm@mat.uab.cat
Alex Haro

In this talk, we will describe an algorithm for the computation of high-order power series expansions of invariant manifolds of fixed points of vector fields that is inspired in the parametrization method. It is very general, in the sense that works in any dimension (examples will be provided for 2, 4, 6) and provides a unified approach to the computation of different kinds of manifolds (like stable/unstable, center, or slow manifold inside a stable/unstable one) with different kind of expansions (as a graph, as a general parametrization with the reduced vector field in normal form, or a combination of both strategies). The actual implementation is especially efficient thanks to the use of automatic differentiation ideas and a recursive representation of polynomials in several variables. Three applications will be presented, that consist on the computation of expansions for: the 2D Lorenz manifold, the 4D center manifold of a collinear fixed point of the spatial, circular Restricted Three–Body Problem, and a 6D reparametrization of the full neighborhood of the same point of the same problem that non-linearly separates the central dynamics (periodic orbits, 2D tori, and chaotic dynamics in between) from the normal hyperbolic part (stable/unstable manifolds of all the previous objects).

Rigorous numerics for ODEs using Chebyshev series with connections to the parametrization method

Christian Reinhardt
VU University Amsterdam, Netherlands
c.p.reinhardt@vu.nl
Jean-Philippe Lessard, Jason Mireles-James and Jan Bouwe van den Berg

In the first part of this talk we present a computational method based on Chebyshev series to rigorously compute solutions of initial and boundary value problems of analytic nonlinear vector fields. The idea is to recast solutions as fixed points of an operator on the Banach space of geometrically decaying Chebyshev coefficients and to use the so-called radii polynomials to show the existence of a unique fixed point nearby an approximate solution. We illustrate the method by presenting a sample application to the solution of initial value problems in the Lorenz system. In the second part of the talk we show how an analogue approach can be used to rigorously solve the invariance equation occurring in the application of the parametrization method to the computation of the local stable manifold of a hyperbolic fixed point of an analytic ODE. We derive an equivalent fixed point problem on the space of geometrically decaying power series coefficients that we again tackle using the method of radii polynomials. In particular the resulting fixed point equation shares many structural features with the ODE case. We finish the talk with applications showing the performance of our approach.

The transition to complex-saddle in a Froeschlé-type map

Arturo Vieiro
Universitat de Barcelona, Spain
vieiro@maia.ub.es
E. Fontich, C. Simó

We consider a Froeschlé type 4D map (depending on suitable parameters) which is close to the time-1 map of a 2-d.o.f. Hamiltonian vector field. This map models the dynamics in a double resonance. The map has an elliptic fixed point that eventually, for some parameters, becomes a complex-saddle. It is interesting to study the behavior of the splitting of the 2D invariant manifolds of the complex-saddle point. We will show the exponentially small character of such splitting for a limit Hamiltonian vector field. For the 4D symplectic map it turns out that such splitting shows up a completely different behavior, being like the splitting of a quasi-periodic forced Hamiltonian. The theoretical results will be illustrated with several numerical examples, which involve the computation of normal forms, invariant manifolds, splitting of the separatrices, ..., both for the related Hamiltonian vector field and for the 4D map.

Geometric method for normally hyperbolic invariant manifolds

Piotr Zgliczynski
Jagiellonian University, Poland
umzglicz@cyf-kr.edu.pl
Maciej Capinski

We present a proof of a normally hyperbolic invariant manifold theorem for maps. The proof is conducted in the phase space of the system. The result is not perturbative. We give explicit conditions under which the existence and smoothness of a normally hyperbolic invariant manifold, together with its associated stable and unstable manifolds, is ensured inside of a given explicit bound. The assumptions are based on estimates on the map and on estimates of its derivative. Assumptions are formulated in a way, which allows for rigorous, interval-based, computer assisted verification.

Special Session 118: Transport Barriers in Unsteady Fluid Flows

Sanjeeva Balasuriya, University of Adelaide, Australia
Kathrin Padberg-Gehle, Technische Universitat Dresden, Germany
Wenbo Tang, Arizona State University, USA

Interest in determining and predicting transport barriers has exploded in the past few years, driven by environmental disasters (the Gulf oil spill) and biotechnological applications (controlling mixing at the micro-scale). The unsteady and finite-time nature of realistic flows has required new theoretical and computational methods for describing such barriers, for which a universally accepted definition remains elusive. Establishing connections between the diverse methods currently in usage, and evaluating their accuracy, is an ongoing endeavour. This special session brings together researchers advocating for a varied range of tools, and who offer theoretical, computational, experimental and observational perspectives to the issue.

Nonautonomous control of stable and unstable manifolds

Sanjeeva Balasuriya
University of Adelaide, Australia
sanjeeva.balasuriya@adelaide.edu.au
Kathrin Padberg-Gehle

The inverse problem of locating flow barriers is to ensure that they occur at user-specified time-varying locations. This will have particular applicability in controlling fluid transport at the micro/nano scales. Here, we present a first approach to this control problem. We derive the unsteady control velocity required to move steady stable and unstable manifolds to prescribed time-varying locations in 2D flows, and obtain rigorous error estimates. We demonstrate the efficacy of the theoretical approach through comparison with numerically determined nonautonomous invariant manifolds.

Oceanic applications of geodesic LCS theory

Francisco Beron-Vera
RSMAS, University of Miami, USA
fberon@rsmas.miami.edu

Several oceanic applications of the recent geodesic theory of Lagrangian Coherent Structures (LCS) will be discussed, including: forecasting of Lagrangian instabilities with a focus on oil spills and pollutant dispersal; inference of dynamical aspects of the general circulation of the ocean; calibration of ocean general circulation model simulations; identification and tracking of coherent eddies; computation of transport by coherent eddies; and suppression of transport by jets.

Characterizing tracer patterns in three-dimensional temporally aperiodic flows

Daniel Blazevski
ETH Zurich, Institute for Mechanical Systems, Switzerland
blazevski@imes.mavt.ethz.ch
George Haller

Detecting barriers to, and facilitators of, transport is a fundamental problem in studying the behavior of Lagrangian trajectories in a fluid. A recent extension of two-dimensional results provides definitions, characterizations and computational methods to extract the most locally repelling, attracting and shearling material surfaces. Such surfaces characterize tracer patterns seen in nature and experiments. In particular, the theory provides an objective definition of a Lagrangian vortex boundary as an outermost member of a family of most shearing cylindrical material surfaces. The detection of such a 3D vortex boundary yields an accurate estimate on the volume the vortex transports. We compute such vortices in kinematic models and a use a global circulation model to detect coherent three-dimensional Agulhas rings in the South Atlantic.

Braid dynamics of non-periodic trajectories

Marko Budišić
University of Wisconsin - Madison, USA
marko@math.wisc.edu
Jean-Luc Thiffeault

Most geometric and operator methods for analysis of flows depend on ability to evaluate the flow on a fairly dense set of points. When information about the flow is very sparsely sampled, e.g., oceanographic data coming from profiling floats, alternative methods are needed. The braid dynamics approach takes a sparse set of Lagrangian trajectories and represents it by an algebraic braid, a "data structure" that keeps track of a relative ordering of Lagrangian particles and the sequence of their interchanges over time. From previous work on stirring problems, it is known that this approach successfully estimates the amount of mixing in a flow when a particular set of periodic Lagrangian trajectories is used, e.g., trajectories of rods used to stir the flow. In this presentation, we discuss the efforts to extend the understanding of braid dynamics for problems in which a distinguished periodic trajectory set does not exist, or when it is not known a priori. Our intended application problems are oceanic flows, where trajectory data is often of finite length, sparse, non-periodic, and no distinguished trajectories are known.

Lagrangian coherent structures as ecological hotspots in the open ocean

Francesco D'Ovidio
CNRS/LOCEAN-IPSL, France
francesco.dovidio@locean-ipsl.upmc.fr

The identification of Lagrangian coherent structures provides a powerful conceptual framework for extracting key transport features from the velocity field at the ocean surface. When applied to satellite derived currents, this technique allows to track transport barriers and fronts induced by the mesoscale activity (i.e., interaction among vortices of 100km size, so-called mesoscale eddies). By comparing these physical features to biological data, like phytoplanktonic community structure and distribution of top predators, mechanisms of biophysical interactions can be explored. Here I will focus on the role of transport barriers induced by horizontal stirring, which can create quasi-isolated water patches with lifetimes long enough for specific phytoplanktonic community to emerge. These fluid dynamical niches are organized as a patchwork of 100km contrasted environments, that are eventually mixed together, hinting regions where hotspots of biodiversity could be dynamically generated. Similarly, semi-enclosed eddy cores may constitute key region where high concentration of phytoplanktonic biomass is transferred to higher trophic levels, creating regions of special interests for foraging animals like elephant seals, frigatebirds, and whales. Tracking these regions of ecological importance is of key interest for focusing conservation efforts in the open ocean.

Set Oriented Computation of Coherent Structures

Michael Dellnitz
University of Paderborn, Germany
dellnitz@uni-paderborn.de
Christian Horenkamp

Over the last years so-called *set oriented* numerical methods have been developed for the numerical analysis of dynamical systems. We will show how to make use of such techniques for the approximation of *transport processes* in ocean dynamics. In this context we will present new theoretical developments concerning these methods, and we will illustrate their applicability by the quantitative analysis of the movement of *Agulhas rings* into the South Atlantic Ocean.

Stretching fields in the ocean from finite-size Lyapunov exponents: Biological impacts of fluid transport

Emilio Hernandez-Garcia
IFISC (CSIC-University of the Balearic Islands), Spain
emilio@ifisc.uib-csic.es
J.H. Bettencourt, I. Hernandez-Carrasco, C. Lopez, V. Rossi

Stretching-line methods have become standard in the Lagrangian characterization of geophysical flows. Among them, local Lyapunov-exponent methods (finite-time and finite-size) have shown useful to locate barriers to transport, attracting manifolds, and similar organizing structures in fluid flows. Here we describe some recent applications of finite-size Lyapunov exponents (FSLEs) to characterize ocean transport processes. The identification of Lagrangian coherent structures is addressed, but we also use FSLEs for the original goal they were introduced, namely quantifying the intensity of dispersion or mixing at a particular spatial scale. Focusing on biological impacts of fluid transport, we explore the three-dimensional structure of Benguela eddies, of the Oxygen Minimum Zone in the Eastern Tropical Pacific, and the relationship between mixing intensity and biological productivity in upwelling areas.

Attraction-based computation of hyperbolic geodesic LCS

Daniel Karrasch
ETH Zürich, Switzerland
karrasch@imes.mavt.ethz.ch
George Haller

We show that in two-dimensional incompressible flows also attracting Lagrangian Coherent Structures (LCSs) can show up as ridges of the forward finite-size and finite-time Lyapunov exponent (FTLE) fields. This contradicts the common identification of attracting/repelling LCSs with ridges of some separation measure field computed in backward/forward time. We present further computational issues related to repulsion-based LCS approaches. Finally, based on the recently developed geodesic approach to hyperbolic LCSs and the singular value decomposition of the spatial derivative of the flow map, we give an efficient numerical scheme which yields more accurate approximations of hyperbolic LCSs than previous approaches.

Bifurcations of set-valued dynamical systems

Jeroen Lamb
Imperial College London, England
jswlamb@gmail.com
Alex Athorne, Xavier Dupuis, Martin Rasmussen, Christian Rodrigues

We discuss the dependence of minimal invariant sets of set-valued dynamical systems on parameters. Under mild assumptions which are often satisfied for random dynamical systems with bounded noise, in the context of which minimal invariant sets support stationary measures, we establish the fact that topological bifurcations of minimal invariant sets are discontinuous with respect to the Hausdorff metric, taking the form of lower semi-continuous explosions and instantaneous appearances. We also characterise these transitions by properties of Morse-like decompositions.

Pinned reaction fronts

John Mahoney
UC, Merced, USA
jmahoney3@ucmerced.edu
Kevin Mitchell

We take a low-dimensional dynamical systems approach to understanding the propagation of fronts in flows. In particular, we are motivated to connect with experiments involving chemical reaction fronts in fluid flows. An interesting feature of these systems is that with an appropriate balance between fluid flow and propagation, there can exist stable front configurations - much like a bunsen burner flame. We examine a simple model of fronts in flows, and find a variety of new results: The stable fronts are precisely the "burning invariant manifolds" recently discussed as one-way barriers in these systems. Certain flow conditions may lead to stable fronts composed of multiple interleaved manifolds. The set of stable states has a nontrivial poset structure. More can be said about ele-

mentary canonical flows such as single and double vortex flows and their bifurcations. We also attempt to extend this thinking to more statistical contexts, beginning to bridge the gap between theory and "natural" flow systems.

Lagrangian descriptors in geophysical flows

Ana Mancho
ICMAT, CSIC, Spain
a.m.mancho@icmat.es
S. Wiggins, J. Curbelo, C. Mendoza

In this presentation we report new techniques, which we refer to as Lagrangian descriptors, for revealing geometrical structures in phase spaces that are valid for aperiodically time dependent dynamical systems. These are based on the integration, for a finite time, along trajectories of an intrinsic, bounded, positive geometrical and/or physical property of the trajectory itself. We discuss a general methodology for constructing Lagrangian descriptors, and we support their performance by explicit calculations on a benchmark problem having a hyperbolic fixed point with stable and unstable manifolds that are known analytically. We also perform computations for an explicitly three dimensional, aperiodically time-dependent vector field and several geophysical flows. Comparisons of the performance of Lagrangian descriptors with both finite time Lyapunov exponents (FTLEs), finite size Lyapunov exponents (FSLEs) and finite time averages of certain components of the vector field ("time averages") are carried out. In all cases Lagrangian descriptors are shown to be both more accurate and computationally more efficient than these methods.

Finite-Time Transport in Volume-Preserving Flows

James Meiss
University of Colorado, USA
jdm@colorado.edu
B.A. Mosovsky and M.F.M. Speetjens

Finite-time transport between distinct flow regions is of great relevance to many scientific applications, yet quantitative studies remain scarce. The primary obstacle is computing the evolution of material volumes, which is often infeasible due to extreme interfacial stretching. We present a framework for describing and computing finite-time transport in n-dimensional (chaotic) volume-preserving flows that relies on the reduced dynamics of an $(n-2)$-dimensional "minimal set" of fundamental trajectories. This approach has essential advantages over existing methods: the regions between which transport is investigated can be arbitrarily specified; no knowledge of the flow outside the finite transport interval is needed; and computational effort is substantially reduced. We demonstrate our framework in 2D for an industrial mixing device, the "Rotated Arc Mixer".

Lagrangian coherent structures in reacting flows

Kevin Mitchell
University of California, Merced, USA
kmitchell@ucmerced.edu
John Mahoney

Recent theoretical and experimental investigations have highlighted the role of invariant manifolds, termed burning invariant manifolds (BIMs), as one-way barriers to reaction fronts propagating within flowing media. In its original context, BIM theory was restricted to time-independent or time-periodic flows. The present work extends these ideas to flows with general time-dependence, thereby constructing coherent structures that organize and constrain the propagation of fronts through general flows. This permits a much broader and physically realistic class of problems to be addressed. Our approach follows the recent work of Haller, Beron-Vera, and Farazmand, in which Lagrangian coherent structures (LCSs), relevant to purely advective transport, are characterized as curves of minimal Lagrangian shear.

Inertial particle dynamics in the ocean

Maria Olascoaga
RSMAS - U. Miami, USA
jolascoaga@rsmas.miami.edu
F. J. Beron-Vera, J. Trinanes, G. Haller

We consider inertial (i.e., finite-size and buoyancy) effects on the motion of particles advected by ocean velocities derived using satellite altimetry measurements. We find that cyclonic/anticyclonic coherent material eddies attract (repel) light/heavy finite-size particles in the northern (southern) hemisphere. Observational evidence (from the analysis of surface and subsurface drifting buoy trajectories, and distributions of macroscopic algae) so far collected provide support for our theoretical results.

Coherence Via Lagrangian Averaging

Nicholas Ouellette
Yale University, USA
nicholas.ouellette@yale.edu
Yang Liao

Recent years have seen the development of a vast number of approaches for detecting coherence in fluid flows, from simple methods based on thresholding to more complex tools rooted in the Lagrangian dynamics of the flow. Lagrangian methods are appealing, particularly for locating transport barriers, since they are based directly on the motion of material. But most current Lagrangian tools for the detection of coherent structures use only the material transport, without considering the wealth of other independent, dynamical information carried by fluid elements. I will discuss recent work on incorporating this additional information into a coherent-structures approach to flow analysis using Lagrangian averaging as a coherence filter, and will present results measured in an experimental quasi-two-dimensional laboratory flow.

On the time-evolution of finite-time coherent sets

Kathrin Padberg-Gehle
TU Dresden, Germany
kathrin.padberg@tu-dresden.de
Gary Froyland

Transport and mixing properties of nonautonomous dynamical systems over a finite-time interval can be described in the framework of finite-time coherent sets. These are regions in phase space that remain coherent while being minimally dispersive over the considered time span. Coherent sets can be efficiently detected and approximated numerically by set-oriented, transfer operator based methods. Here we will demonstrate the effects of finite-time duration, diffusion and directionality on the resulting structures and discuss several different constructions that are appropriate for evolving coherent sets in time.

Chaotic Advection, Barrier Destruction and Mixing in a 3D Model of an Ocean Eddy.

Larry Pratt
Woods Hole Oceanographic inst., USA
lpratt@whoi.edu
I. Rypina, T. M. Ozgokmen, P. Wang

We investigate the existence and stability of transport barriers in a fully three-dimensional Navier-Stokes flow in a rotating cylinder. This idealization of an isolated ocean eddy is driven from above by a surface stress and has both horizontal swirl and overturning. The invariant tori that act a material barriers in the steady, axi-symmetric case break when a symmetry-breaking disturbance becomes larger. Chaos is induced either by resonance or by the breakup of the central axis streamline into stable and unstable manifolds in 3D. We identify several distinct regimes of Lagrangian behavior and map these out in terms of the Ekman and Rossby numbers. We also use Eulerian constraints such as the Taylor-Proudman theorem to motivate the differences. We also calculate the acceleration of the stirring rate due to the chaos and find some surprising trends. A formula for the resonance width is derived, and this and a version of the KAM theorem are used to interpret our findings. We explore the diffusive affects of background turbulence on the integrity of the barriers and the rate of stirring.

Experimental studies of barriers to front propagation in advection-reaction-diffusion systems

Thomas Solomon
Bucknell University, USA
tsolomon@bucknell.edu

We present experimental studies of the behavior of reaction fronts propagating in laminar fluid flows. This is an issue with applications to a wide range of systems including microfluidic chemical reactors, cellular-scale processes in biological systems, and blooms of phytoplankton in the oceans. To analyze and predict the behavior of the fronts, we generalize tools developed to describe passive mixing. In particular, the concept of an invariant manifold is expanded to account for reactive burning. These "burning invariant manifolds" (BIMs) are barriers that locally block the motion of reaction fronts. Unlike invariant manifolds for passive transport, however, the BIMs are barriers for front propagation in one direction only. These ideas are tested and illustrated experimentally in a chain of alternating vortices, a spatially-random flow, and vortex flows with imposed winds.

Lagrangian transport in three-dimensional unsteady flows

Michel Speetjens
Eindhoven University of Technology, Netherlands
m.f.m.speetjens@tue.nl

Transport of scalar quantities (additives, chemical species, reactants, heat) in 3D deterministic flows is key to a wide variety of industrial and natural fluid flows of size extending from microns to hundreds of kilometers. Examples range from a diversity of technical applications based on laminar flows (mixing of viscous thermofluids, compact processing equipment, micro-fluidic systems) to deterministic representations of large-scale oceanographic and atmospheric flows. Fluid advection plays a central role in scalar transport and is an essentially Lagrangian process. This advances a Lagrangian perspective based on the properties of fluid trajectories as a natural way for its description. Continuity (in conjunction with momentum conservation) typically "organizes" these trajectories into coherent structures that geometrically determine the transport properties of a given flow. The formation of such coherent structures and its impact upon 3D transport is demonstrated by way of 3D unsteady flows that are experimentally realisable.

Biological Reactions in Coherent Structures

Wenbo Tang
Arizona State University, USA
wenbo.tang@asu.edu

Coherent structures are ubiquitous in environmental and geophysical flows. Recent advances in the identification of finite-time transport barriers have enabled the extraction of organizing templates for passive scalars in the limit of infinitesimal diffusion. In this presentation, we show, by using example of an autocatalytic reaction, that reaction processes can also be tied to finite-time Lagrangian measures, which varies quite significantly in different types of flow topologies. In particular, we consider the Fisher-Kolmogorov-Petrovsky-Piskunov (FKPP) equation embedded in a flow with two oscillating gyres. We show that a model based on finite-time Lagrangian stretching rates represents the variabilities from the actual simulations well. Some recent efforts to extend the theory to more complicated reaction processes will also be discussed.

Optimal mixing enhancement or mitigation by local perturbation

Tom Watson
UNSW, Australia
t.watson@unsw.edu.au
Gary Froyland, Cecilia Gonzalez Tokman

For a given map or flow, possibly time-dependent, we consider the problem of how to apply local perturbations to either optimally enhance or mitigate mixing. We develop an extremely flexible modelling approach, based on the Perron-Frobenius operator or transfer operator, posing this problem in the language of convex optimisation, and efficiently solving it with standard convex optimisation techniques. The perturbations applied satisfy physical constraints, such as preservation of the invariant measure of the dynamics (e.g., for incompressible fluid flow, the perturbations preserve volume), and a variety of other natural constraints can also be easily enforced. We can ask the optimiser to perturb so as to achieve complete mixing as quickly as possible by targeting the equilibrium distribution, or alternatively, we can specify a target non-equilibrium distribution; the latter may be of interest if one is trying to contain e.g. pollutants in a safe region.

Special Session 119: Dynamical Systems and Optimal Control

Cedric M. Campos, Universidad de Valladolid, Spain
Hasnaa Zidani, ENSTA-ParisTech, France

The study of dynamical systems, point particles that evolve within a finite or infinite dimensional space, has a long and broad history. In particular, along the late years of the past century, the theory has known a huge growth thanks to the introduction of new techniques coming from different branches of mathematics: differential geometry, topology, functional analysis, probability theory and, with the advent of computation, numerical analysis. In the present modern era, there is a crucial need of finding actual applications of the theory and further developing it in order to surpass the problems that appear on the go. A common and natural answer to this need is the control theory and optimization, which helps to shape the systems up to some extent according to different kinds of requirements, such as physical, economical or environmental. We therefore easily find in the literature applications of the geometric integration of Lagrangian and Hamiltonian mechanics to spatial industry, of game theory to economics or biology, of the Hamilton-Jacobi-Bellman equation to image processing and medicine, etc. In this session we aim to provide a space to discuss recent theoretical and applied results of dynamical systems with a special attention on optimal control.

Pontryagin's Maximum Principle for some families of PDEs.

Maria Barbero
Instituto de Ciencias Matematicas-UC3M, Spain
mbarbero@icmat.es
Miguel C. Muñoz Lecanda

An optimal control problem associated with the dynamics of the orientation of a bipolar molecule in the plane can be understood by means of tools in differential geometry. For first time in the literature k-symplectic formalism is used to provide the optimal control problems associated to some families of partial differential equations with a geometric formulation. A parallel between the classic formalism of optimal control theory with ordinary differential equations and the one with particular families of partial differential equations is established. This description allows us to state and prove Pontryagin's Maximum Principle on k-symplectic formalism. We also consider the unified Skinner-Rusk formalism for optimal control problems governed by an implicit partial differential equation.

On Optimal Control Problems with Mixed and Pathwise State Constraints

Andrea Boccia
Imperial College London, England
a.boccia@imperial.ac.uk
Richard B. Vinter, Maria do Rosario de Pinho

First order necessary conditions are presented for constrained, nonsmooth optimal control problems, which improve on available conditions in a number of respects. Most notably, they extend for the first time the 'stratified necessary conditions' framework of F. H. Clarke, to allow for both pure state constraints and mixed state and control constraints. They imply, as a special case, a Pontryagin-type Maximum Principle and a generalization of the Euler-Lagrange equation of the Calculus of Variations.

L^1-minimization in space mechanics: Old and new

Jean-Baptiste Caillau
Univ. Bourgogne & CNRS, France
jean-baptiste.caillau@u-bourgogne.fr
Z. Chen, Y. Chitour

As is well known in finite dimensions, the peculiarity of L^1-minimization is the sparsity of solutions. In the infinite dimensional setting of optimal control, this typically results in subarcs with zero control. An important application is the case of systems with varying mass; when this variation, corresponding to propellant consumption, is proportional to the control norm, minimization of the consumption is equivalent to minimizing the L^1-norm of the control. Such problems were investigated in the 60's looking for minimum fuel orbit transfers in space mechanics. Among others, Robbins chiefly addressed the existence and properties of second order singular arcs in the two-body controlled problem. In particular, fuel minimization exhibits the Fuller phenomenon. This was extensively studied by Marchal, then more recently by Zelikin and Borisov. Building upon the work of Simo et al, some modern approaches use the dynamical properties of the three body problem to devise low cost trajectories. A purely optimal control approach combining single shooting and homotopy method is used here to compute minimum fuel trajectories of the 2BP and 3BP. On the basis of ideas of Schattler, an analysis of the local optimality of these trajectories is proposed.

Geometric and numerical methods in the saturation problem of an ensemble of spin particles

Olivier Cots
INRIA, France
olivier.cots@inria.fr
B. Bonnard, P. Martinon

In this presentation, we analyse the saturation problem for an ensemble of spin in relation with the problem of RF-inhomogeneity in the contrast problem in Medical Resonance Imaging. The problem is modeled as a Mayer problem in optimal control and is analyzed using the Maximum Principle with algebraic-geometric methods complemented by numerical simulation using direct and indirect methods implemented in Hampath-Bocop softwares.

Infinite Horizon problems on Stratifiable State Constraints sets

Cristopher Hermosilla
Inria Saclay & ENSTA ParisTech, France
cristopher.hermosilla@inria.fr
Hasnaa Zidani

When dealing with state constrained problems it is usual to assume some Pointing Qualification Hypothesis as the introduced by Soner or Frankowska and Vinter. This type of assumption does not allow to treat cases where the only option for the trajectories are to stay at the boundary of the constraint. Here we attempt to cover this kind of problems by assuming that the state-constraint admits a sufficiently regular stratification and in each of these strata we can define a Lipschitz dynamic. We present a characterization of the value function for the infinite horizon problem in term of this stratification.

Global optimal feedbacks for stochastic quantized nonlinear event systems

Oliver Junge
Technische Universitaet Muenchen, Germany
oj@tum.de
Stefan Jerg, Marcus Post

We consider nonlinear control systems for which only quantized and event-triggered state information is available and which are subject to random delays and losses in the transmission of the state to the controller. We present an optimization based approach for computing globally stabilizing controllers for such systems. Our method is based on recently developed set oriented techniques for transforming the problem into a shortest path problem on a weighted hypergraph. We show how to extend this approach to a system subject to a stochastic parameter and propose a corresponding model for dealing with transmission delays.

The Tschebychev scalarization method for solving multi-objective optimal control problems

Helmut Maurer
University of Muenster, Germany
maurer@math.uni-muenster.de
Yalcin C. Kaya

We propose numerical methods for solving non-convex multi-objective optimal control problems with control and state constraints. We employ a scalarization technique which reduces the problem to a single-objective optimal control problem. In contrast to a standard weighted-sum scalarization we use a weighted Tschebychev scalarization that is particularly suited for non-convex problems. The weighted Tschebychev scalarization is surjective from the space of weights to the Pareto set (front). Solutions (obtained via discretization) of a sequence of scalarized problems yield an approximation of the Pareto front. The numerical method is illustrated on two numerically challenging problems involving tumor anti-angiogenesis and a fedbatch bioreactor. The control problems exhibit bang-bang and singular controls as well as boundary controls for the state constraints. Finally, we briefly discuss methods to optimize other cost functionals over the Pareto front.

Galerkin variational integrators in optimal control theory

Sina Ober-Bloebaum
University of Paderborn, Germany
sinaob@math.upb.de

In this talk, we derive and analyze variational integrators of higher order for the structure-preserving simulation and optimal control of mechanical systems. The construction is based on a space of polynomials together with Gauss and Lobatto quadrature rules to approximate the relevant integrals in the variational principle. The use of higher order schemes increases the accuracy of the discrete solution and thereby decrease the computational cost, while the preservation properties of the scheme are still preserved. The order of accuracy of the resulting variational integrators as well as stability properties are determined and demonstrated by numerical examples. Furthermore, by using theses integrators for the discretization of optimal control problems, we investigate the approximation order of the discrete adjoint equations resulting from the necessary optimality conditions of the discretized optimal control problem.

Optimal feedback control, linear first-order PDE systems, and obstacle problems

Pablo Pedregal
Universidad de Castilla-La Mancha, Spain
pablo.pedregal@uclm.es

We focus on the analysis and numerical approximation of the optimal feedback control mapping. Our perspective consists in looking at a typical optimal control problem in such a way that feasible controls are mappings depending both in time and space. In this way, the feedback form of the problem is built-in from the very beginning. Optimality conditions are derived for one such optimal mapping, which by construction is the optimal feedback mapping of the problem. In formulating such optimality conditions, costates in feedback form are solutions of linear, first-order transport systems, while optimal descent directions are solutions of appropriate obstacle problems. We illustrate these ideas by approximating such map in some simple, academic examples.

Limit solutions for systems with unbounded controls

Franco Rampazzo
Padova University, Italy, Italy
rampazzo@math.unipd.it
M.-Soledad Aronna

For a control Cauchy problem

$$\dot{x} = f(t,x,u,v) + \sum_{\alpha=1}^{m} g_\alpha(x)\dot{u}_\alpha \quad x(a) = \bar{x},$$

(notice the presence of the input's derivative) on an interval $[a,b]$, we propose the notion of Limit Solution x that verifies the following properties: i) x is defined for $\mathcal{L}^1$ inputs u and for standard, bounded measurable, controls v; ii) in the commutative case (i.e. when $[g_\alpha, g_\beta] \equiv 0$, for all $\alpha, \beta = 1,\ldots,m$), x coincides with the solution constructed via multiple fields' rectification; iii) x subsumes

former concepts of solution valid for the generic, noncommutative case. In particular, when u has bounded variation, we investigate the relation between limit solutions and (single-valued) graph completion solutions. Furthermore, we prove consistency with the classical Caratheodory solution when u and x are absolutely continuous. Even though some specific problems are better addressed by means of special representations of the solutions, we believe that various theoretical and practical issues call for a unified notion of trajectory. For instance, this is the case of optimal control problems, possibly with state and end-point constraints, for which no extra assumptions (like e.g. coercivity, boundedness, commutativity) are made in advance.

Optimal control of dynamical systems driven by impulsive controls

Zhiping Rao
RICAM, Linz, Austria
zhiping.rao@ricam.oeaw.ac.at

This work deals with a class of optimal control problems with impulsive controls. The optimization problem consists of trajectories of bounded variation satisfying an impulsive differential equation involving a measurable control and a second control with bounded variation. The dynamics of the trajectories depend not only on these two controls but also on the time derivative of the second control, which leads to the impulses in the trajectories. The definition of solutions to the impulsive dynamical system was introduced by Bressan and Rampazzo using the concept of Graph Completion and it has been studied recently by Aronna and Rampazzo in a more general setting. By means of the graph completion and reparametrization techniques, we study an auxiliary control problem and the properties of the value function. The characterization of the value function is provided via the Hamilton-Jacobi-Bellman approach and a verification theorem is given to test the optimality of a given control. Furthermore, we will discuss the problem with state constraints and conclude with some applications on mechanical systems.

Degenerate second order mean field games systems

Daniela Tonon
Ceremade Universite Paris Dauphine, France
tonon@ceremade.dauphine.fr
Pierre Cardaliaguet and Alessio Porretta

We consider degenerate second order mean field games systems with a local coupling. The starting point is the idea that mean field games systems can be understood as an optimality condition for optimal control of PDEs. Developing this strategy for the degenerate second order case, we discuss the existence and uniqueness of a weak solution as well as its stability (vanishing viscosity limit).

Special Session 120: Linear and Nonlinear Fourth Order PDE's

Jan Cholewa, University of Silesia in Katowice, Poland
Filippo Gazzola, Maths Department - Politecnico of Milan, Italy
Anibal Rodriguez-Bernal, Universidad Complutense de Madrid, Spain

Fourth order PDEs problems have a wide range of applications, from elasticity to phase transitions, to mention just a few. However compared with similar second order problems, they usually do not have maximum principles, a crucial tool that pushed forward the theory of such problems. Therefore, the mathematical theory for fourth order problems has not reached the same degree of development although it has witnessed a growing interest in recent times. This session is devoted to present some recent advances on higher order partial differential equations and is focused on various aspects of both elliptic and evolutionary models.

A Paneitz-type problem in pierced domain

Salomon Alarcon
Universidad Técnica Federico Santa María, Chile
salomon.alarcon@usm.cl
Angela Pistoia

We study the critical problem

$$\begin{cases} \Delta^2 u = u^{\frac{N+4}{N-4}} & \text{in } \Omega \setminus \overline{B(\xi_0,\varepsilon)}, \\ u > 0 & \text{in } \Omega \setminus \overline{B(\xi_0,\varepsilon)}, \\ u = \Delta u = 0 & \text{on } \partial(\Omega \setminus \overline{B(\xi_0,\varepsilon)}), \end{cases} \qquad (\mathrm{P}_\varepsilon)$$

where Ω is an open bounded domain in $\mathbb{R}^N$, $N \geq 5$, $\xi_0 \in \Omega$ and $B(\xi_0, \varepsilon)$ is the ball centered at ξ_0 with radius $\varepsilon > 0$ small enough. We construct solutions of (P_ε) blowing-up at ξ_0 as $\varepsilon \to 0$.

Spectral stability results for higher-order operators under perturbations of the domain

Jose Arrieta
Complutense University - ICMAT, Spain
arrieta@mat.ucm.es
Pier D. Lamberti

We analyze the spectral behavior of higher-order elliptic operators when the domain is perturbed. We give special attention to the bi-harmonic operator with the so-called intermediate boundary conditions and analyze its behavior when the boundary of the domain has some oscillatory behavior. We will show that there is a critical oscillatory behavior and that the limit problem depends on whether we are above, below or just sitting on this critical value.

Explicit estimates on the torus for the supnorm of solutions of the Swift and Hohenberg Equation in one and two space dimensions

Michele Bartuccelli
Surrey University (UK), England
m.bartuccelli@surrey.ac.uk

In this talk we will address the problem of obtaining explicit and accurate estimates of the sup-norm for solutions of the Swift and Hohenberg Equation (SHE) in one and two space dimensions. By using the best (so far) available estimates of the embedding constants which appear in the classical functional interpolation inequalities used in the study of solutions of dissipative partial differential equations, we have evaluated in an explicit manner the values of the sup-norm of the solutions of the SHE. In addition we have calculated the so-called time-averaged dissipative length scale associated to the above solutions.

Quantum diffusion equations

Mario Bukal
University of Zagreb, Croatia
mario.bukal@fer.hr

The talk will present nonlinear evolution equations of fourth and sixth order in spatial derivatives, which arise in the quantum modeling of semiconductor and plasma physics, and describe the evolution of densities of charged particles in a quantum fluid. We report on some basic results known about equations: local in time existence of positive classical, global in time existence and the long time behaviour of weak nonnegative solutions, as well as on respective techniques used in the analysis. Interesting structural properties, like dissipation of the physical entropy and formal gradient flow structure with respect to the Wasserstein distance will be also discussed.

Entire solutions for a class of fourth order semilinear elliptic equations with weights

Paolo Caldiroli
Università di Torino, Italy
paolo.caldiroli@unito.it
Gabriele Cora

We investigate the problem of entire solutions for a class of fourth order, dilation invariant, semilinear elliptic equations with power-type weights and with subcritical or critical growth in the nonlinear term. These equations define non compact variational problems and are characterized by the presence of a term containing lower order derivatives, whose strength is ruled by a parameter λ. We can prove existence of entire solutions found as extremal functions for some Rellich-Sobolev type inequalities. Moreover, when the nonlinearity is suitably close to the critical one and the parameter λ is large, symmetry breaking phenomena occur, and the asymptotic behavior of radial and non radial ground states can be somehow described. As a by-product result we obtain information on the sign of ground state solutions.

On the Cahn-Hilliard equation in $H^1(\mathbb{R}^N)$

Jan Cholewa
Silesian University, Poland
jan.cholewa@us.edu.pl
Anibal Rodriguez-Bernal

The Cahn-Hilliard equation is discussed with subcritically or some critically growing nonlinearities in $H^1(\mathbb{R}^N)$ and a dissipative mechanism is described. This involves a weak form of dissipativeness, in which case each individual solution is suitably attracted by the set of equilibria, and strong dissipativeness, for which we indicate that it cannot be in general expected in $H^1(\mathbb{R}^N)$. Two types of perturbations of the Cahn-Hilliard equation are also considered where the dissipative mechanism becomes strong enough to ensure the existence of a compact global attractor.

References
[1] J.W. Cholewa, A. Rodriguez-Bernal, On the Cahn-Hilliard equation in $H^1(\mathbb{R}^N)$, J. Differential Equations 253, 2012, 3678-3726.
[2] J.W. Cholewa, A. Rodriguez-Bernal, A note on the Cahn-Hilliard equation in $H^1(\mathbb{R}^N)$ involving critical exponent, preprint.

Korteweg-de Vries-Burgers system in R^N

Tomasz Dlotko
Institute of Mathematics, Silesian University, Poland
tdlotko@math.us.edu.pl

In [*J. Math. Anal. Appl.* 411 (2014), 853-872] we study Cauchy problem in R^N for the Korteweg-de Vries-Burgers system

$$\begin{aligned} u_t + \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \nabla\Phi(u) + \sum_{j=1}^{N} \frac{\partial}{\partial x_j} \sum_{i=1}^{N} \frac{\partial^{2p}}{\partial x_i^{2p}} u \\ = \alpha\Delta u + g(x,u), \end{aligned} \quad (1)$$

$$u(0,x) = u_0(x),\ t > 0,\ x \in R^N,$$

where $\alpha > 0$, $2p > N \geq 1$ and Φ is a scalar function of the vector $u(t,x) = (u_1(t,x), ..., u_m(t,x))$; ∇ denotes gradient with respect to u. Parabolic regularization technique is used to prove global in time solvability of (1); we study first its *parabolic regularization*:

$$\begin{aligned} u_t^\epsilon + \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \nabla\Phi(u^\epsilon) + \sum_{j=1}^{N} \frac{\partial}{\partial x_j} \sum_{i=1}^{N} \frac{\partial^{2p}}{\partial x_i^{2p}} u^\epsilon \\ = \alpha\Delta u^\epsilon + \epsilon(-1)^p (\Delta)^{p+1} u^\epsilon + g(x,u^\epsilon), \end{aligned} \quad (2)$$

$$u^\epsilon(0,x) = u_0(x),\quad t > 0,\ x \in R^N,$$

where $\epsilon > 0$ is the *viscosity coefficient*, that will later tend to 0^+. Certain estimates for solutions u^ϵ to (2) are extended next to the limit problem (**??**). The regularization effect of the Laplacian term is observed for the *viscous solutions* of (**??**) constructed in the paper. Asymptotic behavior, as $t \to \infty$, is finally discussed in the language of the global attractors.

References
[1] T. Dlotko, Nonlinear Analysis 74 (2011), 721-732.
[2] T. Dlotko, Chunyou Sun, J. Evol. Equ. 10 (2010), 571-595.
[3] M.E. Schonbek, S.V. Rajopadhye, Annales de l'I.H.P. 12 (1995), 425-457.
[4] Linghai Zhang, Proc. Royal Soc. Edinburgh 124A (1994), 263-271.

Existence and non-existence of solutions for a biharmonic partial differential equation with a Hessian nonlinearity

Carlos Escudero
Universidad Autónoma de Madrid & ICMAT, Spain
carlos.escudero@uam.es

We consider the initial-boundary value problem for the partial differential equation

$$u_t + \Delta^2 u = \det(D^2 u) + \lambda f(x,t),$$

where the forcing term obeys suitable summability conditions. This problem might present a gradient flow structure depending on the boundary conditions as well as the geometry of the spatial domain where it is posed. We will first summarize our results on the existence and regularity of stationary solutions. Then, for the evolution problem, we will show how to prove local existence of solutions for arbitrary data and global existence of solutions for small data. Depending on the boundary conditions and the concomitant presence of a variational structure in the equation as well as on the size of the data we prove blow-up of the solution in finite time and convergence to a stationary solution in the long time limit.

Existence and stability properties of entire solutions to a polyharmonic equation with exponential nonlinearity

Alberto Ferrero
Università degli Studi del Piemonte Orientale, Italy
alberto.ferrero@mfn.unipmn.it
Alberto Farina

We study existence and stability properties of entire solutions of a polyharmonic equation with an exponential nonlinearity namely $(-\Delta u)^m u = e^u$. This equation may considered a polyharmonic version of the well-known Gelfand equation $-\Delta u = e^u$. Most of our results deal with radial entire solutions: we study their existence and we provide some asymptotic estimates on their behavior at infinity. As a first result on stability we prove that stable solutions (not necessarily radial) in dimensions lower than the conformal one never exist. On the other hand, we prove that radial entire solutions which are stable outside a compact always exist both in high and low dimensions. In order to prove stability of solutions outside a compact set we prove some new Hardy-Rellich type inequalities in low dimensions.

Quasilinear fourth order elliptic equations modeling suspension bridges

Filippo Gazzola
Politecnico of Milano, Italy
filippo.gazzola@polimi.it
Mohammed Al-Gwaiz, Vieri Benci

A rectangular plate modeling the roadway of a suspension bridge is considered. Both the contributions of the bending and stretching energies are analyzed. The latter plays an important role due to the presence of the free edges. A linear model is first considered; in this case, separation of variables is used to determine explicitly the deformation of the plate in terms of the vertical load. Moreover, the same method allows us to study the spectrum of the linear operator and the least eigenvalue. Then the stretching energy is introduced without linearization and the equation becomes quasilinear; the nonlinear term also affects the boundary conditions. We consider two quasilinear models; the surface increment model (SIM) in which the stretching energy is proportional to the increment of surface and a nonlocal model (NLM) introduced by Berger in the 50's. The (SIM) and the (NLM) are studied in detail. According to the strength of prestressing we prove the existence of multiple equilibrium positions.

Cahn-Hilliard equations with inertial term and dynamic boundary conditions

Maurizio Grasselli
Politecnico di Milano, Italy
maurizio.grasselli@polimi.it

In order to account for separation processes in certain binary solutions, some physicists have proposed to modify the classical Cahn-Hilliard (CH) equation. This new nonlinear fourth order evolution equation is characterized by the presence of a second-order time derivative multiplied by a (small) inertial coefficient. Here we report on some recent results on this modified CH equation subject to dynamic boundary conditions. We also present some open problems.

Proper entropy solutions for nonlinear elliptic fourth-order equations with a strengthened coercivity and L^1-data

Alexander Kovalevsky
Institute of Applied Mathematics and Mechanics, Ukraine
alexkvl71@mail.ru

We give a series of results on the existence, uniqueness and summability of so called proper entropy solutions of the homogeneous Dirichlet problem for the equation

$$\sum_{|\alpha|=1,2} (-1)^{|\alpha|} D^\alpha A_\alpha(x, \nabla_2 u) = F(x,u) \text{ in } \Omega$$

where Ω is a bounded open set of $\mathbb{R}^n$ $(n > 2)$, $F : \Omega\times\mathbb{R} \to \mathbb{R}$ and $\nabla_2 u = \{D^\alpha u : |\alpha| = 1, 2\}$. It is assumed that the functions A_α satisfy some growth and monotonicity conditions and the following strengthened coercivity condition: for a. e. $x \in \Omega$ and for every $\xi = \{\xi_\alpha \in \mathbb{R} : |\alpha| = 1, 2\}$,

$$\sum_{|\alpha|=1,2} A_\alpha(x,\xi)\xi_\alpha \geq c\Big(\sum_{|\alpha|=1} |\xi_\alpha|^q + \sum_{|\alpha|=2} |\xi_\alpha|^p\Big) - g(x)$$

where $p \in (1, n/2)$, $q \in (2p, n)$, $c > 0$ and $g \in L^1(\Omega)$, $g \geq 0$. For the existence of the solutions, in particular, we require $F(\cdot, s) \in L^1(\Omega)$ for every $s \in \mathbb{R}$. In the case where $F(x, s) = f(x)$ and $f[\ln(1 + |f|)]^\sigma \in L^1(\Omega)$ or $f[\ln\ln(e + |f|)]^\sigma \in L^1(\Omega)$ with an arbitrary $\sigma > 0$, we describe dependence of summubility of the solutions on σ.

Reissner Mindlin system and Biharmonic equation: a spectral shape sensitivity analysis

Pier Domenico Lamberti
University of Padova, Italy
lamberti@math.unipd.it
Davide Buoso

We consider the eigenvalue problems for the Reissner-Mindlin system and the biharmonic equation, arising in the study of the free vibration modes of an elastic clamped plate. We discuss the interplay between the two problems with particular reference to spectral stability results and domain perturbation problems. We prove stability estimates for the variation of the eigenvalues upon variation of the domain. Our estimates are expressed in terms of the Hausdorff distance between the domains and are independent of the thickness of the plate. We also prove analyticity results for the dependence of the eigenvalues via 'transplantation' and establish Hadamard-type formulas. Finally, we address the problem of minimization of the eigenvalues in the case of isovolumetric domain perturbations. In the spirit of the Rayleigh conjecture for the biharmonic operator, we prove that balls are critical points with volume constraint for all simple eigenvalues and the elementary symmetric functions of multiple eigenvalues both of the biharmonic equation and of the Reissner-Mindlin system. Joint work with Davide Buoso.

Fourth order linear and nonlinear parabolic equations in Banach spaces in $\mathbb{R}^N$ with low regularity initial data

Carlos Quesada
Universidad Complutense, Spain
carlosqu@ucm.es
Aníbal Rodríguez-Bernal

We consider fourth order parabolic problems in $\mathbb{R}^N$ where the main operator is the bi-Laplacian. The equations we will consider have perturbations of the form $D^a(h(x)D^b u)$ with $a + b$, and h in the locally uniform spaces $\dot{L}^p_U(\mathbb{R}^N)$ which will be described in detail. We want to find the range of γ such that for $u_0 \in H^{4\gamma,q}(\mathbb{R}^N)$, $q \in (1,\infty)$, the problem has a unique solution. We prove that $\gamma \in I$, where I is an interval depending only on a, b, p, q and N. Furthermore the solution is given by an analytic semigroup $S(t)u_0$ and satisfies smoothing estimates between the space of initial data an the space of the solutions. Finally, we will briefly explain how to extend these results to a larger spaces such as the uniform Bessel spaces, and also how similar techniques can be used to consider non-linear terms.

On the dissipativity of some fourth order problems in R^N

Anibal Rodriguez-Bernal
U. Complutense, Madrid, Spain
arober@mat.ucm.es
J. Cholewa

We analyze the asymptotic behavior of solutions of some fourth order nonlinear problems in R^N. Due to the lack of maximum principles we use natural energy estimates for the analysis. Also the lack of compact Sobolev embeddings makes the asymptotic compactness of solutions a subtle problem. We will also discuss the role of the critical growth of the nonlinear term in the behavior of solutions.

Local asymptotic nondegeneracy for multi-bubble solutions to the biharmonic Liouville-Gel'fand problem in dimension four

Futoshi Takahashi
Osaka City University, Japan
futoshi@sci.osaka-cu.ac.jp
Hiroshi Ohtsuka

We consider the biharmonic Liouville-Gel'fand problem under the Navier boundary condition in four space dimension:

$$\begin{cases} \Delta^2 u = \lambda e^u & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega. \end{cases}$$

Under the nondegeneracy assumption of blow up points of multiple blowing-up solutions, we prove several estimates for the linearized equations and obtain some convergence result. The result can be seen as a weaker version of the asymptotic nondegeneracy of multi-bubble solutions, which was recently established by Grossi-Ohtsuka-Suzuki in two-dimensional Laplacian case. This talk is based on a joint work with H. Ohtsuka (Kanazawa University).

Remarks on a class of fourth order hyperbolic partial differential equations

Yongda Wang
Politecnico di Milano, Italy
yongda.wang@polimi.it

We discuss a kind of fourth order hyperbolic evolution problem

$$u_{tt} + \Delta^2 u + \mu u_t + h(x, y, u) = f(x, y, t), \text{ in } \Omega \times (0, T),$$

which is suggested as mathematical models for suspension bridges, whose roadway is viewed as a long narrow rectangular thin plate. We try to seek correct boundary conditions for the plate and look for solutions to these problems which exhibit the oscillations, the phenomenon visible in real bridges.

A fourth order parabolic equation modeling epitaxial thin film growth

Michael Winkler
University of Paderborn, Germany
michael.winkler@math.upb.de

We consider the Neumann problem for

$$u_t = -\Delta^2 u - \mu\Delta u - \lambda\Delta|\nabla u|^2 + f(x)$$

with $\mu \geq 0$ and $\lambda > 0$ and a given source function f, which has been proposed as a model for the evolution of a thin surface when exposed to molecular beam epitaxy. Numerical simulations suggest that even in the spatially one-dimensional setting, this problem may admit solutions for which the negative part u_- blows up with respect to the norm in L^∞, whereas u_+ apparently enjoys comparatively strong boundedness properties. The goal of the presentation is to point out some mathematical challenges stemming from this, and to describe how nevertheless a basic theory on global existence of appropriately defined weak solutions can be established. Moreover, some additional integral estimates are derived for these solutions which indicate that indeed u_+ enjoys quite favorable regularity properties, whereas u_- possibly might not. Finally, despite the lack of a genuine energy functional a statement on the large time behavior of solutions is presented under a suitable smallness condition on μ and $\|f\|_{L^\infty(\Omega)}$.

Special Session 121: Numerical Techniques for the Description of Charged Particles Transport

Francesco Vecil, Universitat de Valencia, Spain

The transport of charged particles plays a central role in the simulation of several models of fundamental interest for physics and engineering, in particular of semiconductor devices and plasma physics for nuclear fusion research. From the environmental point of view, the first topic is important as for saving energy and silicon, the second one as for avoiding the risks directly connected to other forms of energy production (like nuclear fission). The transistor is the fundamental block of any electronic device. In the seventies, the size of FETs inside a CPU was roughly 10000 nm, while now the commercial size is around 30 nm. The research is focused on obtaining 10 nm SOI devices, which requires an effort for the modeling and from the computational point of view in order to provide solvers that take into account all the relevant phenomena emerging at this scale. These solvers are divided essentially into two categories: Monte Carlo and deterministic. The first ones are widely used in the engineering community thanks to their efficiency and their implementation. Among the deterministic solvers, there exist different accuracy levels: the mesoscopic ones aim at being very precise despite their computational cost, while the hydrodynamic or fluid ones have a lower-dimensional domain and are hence faster but coarser; several are the strategies applied for the time-space discretization and for the time integration in the scientific community: Strang splitting, Runge-Kutta, semi-Lagrangian schemes, Finite Differences, Discontinuous Galerkin, waterbags, etc. A huge effort is being made in order to obtain energy from nuclear fusion. Instead of using very heavy atoms and extracting energy by bombarding them (nuclear fission), energy is extracted by fusioning light atoms. The advantages are clear in terms of safety, because there is no chain reaction to control, and environment, because no CO2 is produced, the radioactive waste products are much shorter-lived and the fuels could be produced locally. Much research has been developed in the last decades; the main problem to overcome now is how to control the instabilities propagating inside the reactor chamber for a plasma dense enough so as to achieve a reasonable gain factor ($Q \approx 10$ is the goal). The best model to describe the evolution of the plasma inside the reactor chamber would be a full 3D Vlasov-Maxwell system, which is 6D in the phase space thus unbearable from the numerical point of view. Therefore, reduced models are taken into account and simulated. In this special session we would like to achieve two objectives: by the one hand, sketch what the goals and the physics background are in the aforementioned fields; on the other hand, give an overview of the state-of-the-art numerical strategies used by the scientific community, namely by mathematicians, engineers and physicists, for the simulation of these problems.

Deterministic WENO-solvers for MOSFETs devices

María Cáceres
Universidad de Granada, Spain
caceresg@ugr.es
N. Ben Abdallah, J. A. Carrillo, F. Gámiz, A. Godoy, J. M Mantas, F. Vecil

Statistical models are used to describe electron transport in semiconductors at a mesoscopic level. The basic model is given by the Boltzmann transport equation for semiconductors in the semi-classical approximation coupled with Poisson's equation, since the electric field is self-consistent due to the electrostatics produced by the electrons and the dopants in the semiconductor. However, for small devices the quantum effects must be considered. And, in this case, the devices are described by the Schrödiger-Poisson-Boltzmann system. In this talk we show WENO-solvers for DG-MOSFETs in both versions: semi-classical approximation for large devices [2] and hybrid quantum-classical approximation for small devices [1,3].

References

[1] N. Ben Abdallah, M.J. Cáceres, J. A. Carrillo, F. Vecil, *A deterministic solver for a hybrid quantum-classical transport model in nanoMOSFETs*, J. Comput. Phys. 228 (2009) 6553-6571.

[2] J.M. Mantas, M.J. Cáceres, *Efficient deterministic parallel simulation of 2D semiconductor devices based on WENO-Boltzmann schemes*, Comput. Meth. Appl. Mech. Eng. 198 (2009) 693-704.

[3] F. Vecil, J. M Mantas, M. J. Cáceres, C. Sampedro, A. Godoy, F. Gámiz,*A parallel deterministic solver for the Schrodinger-Poisson-Boltzmann system in ultra-short DG-MOSFETs: Comparison with Monte-Carlo*, Preprint.

Asymptotic preserving implicit-explicit Runge-Kutta methods for non-linear kinetic equations

Giacomo Dimarco
university of Ferrara, Italy
giacomo.dimarco@unife.it
Lorenzo Pareschi

We discuss Implicit-Explicit (IMEX) Runge Kutta methods which are particularly adapted to stiff kinetic equations of Boltzmann type. We consider both the case of easy invertible collision operators and the challenging case of Boltzmann collision operators. We give sufficient conditions in order that such methods are asymptotic preserving and asymptotically accurate. Their monotonicity properties are also studied. In the case of the Boltzmann operator the methods are based on the introduction of a penalization technique for the collision integral. This reformulation of the collision operator permits to construct penalized IMEX schemes which work uniformly for a wide range of relaxation times avoiding the expensive implicit resolution of the collision operator. Finally we show some numerical results which confirm the theoretical analysis.

Nonlinear times solvers ans stability issues for reduced MHD models

Emmanuel Franck
Max Planck Institut fur Plasmaphisik, Germany
efranck21@gmail.com
M. Hoelzl, E. Sonnendruecker

In the context of the ITER magnetic fusion project it is important to simulate the behavior of edge instabilities in the Tokamak. Indeed these instabilities can damage wall components due to their extremely high energy transfer rate. Consequently, it is essential to estimate the amplitude of these instabilities and understand how control these. To simulate this phenomena, we use a 3D reduced MHD code in cylindrical geometry named Jorek. The full MHD model is given by

$$\begin{cases} \partial_t\rho + \nabla.(\rho\mathbf{v}) = 0 \\ \rho\partial_t\mathbf{v} + \rho\mathbf{v}.\nabla\mathbf{v} + \nabla(\rho T) = \mathbf{J}\times\mathbf{B} + \nu\triangle\mathbf{v} \\ \partial_t p + \mathbf{v}\nabla p + \gamma p\nabla\mathbf{v} = 0 \\ \partial_t\mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B} - \eta\mathbf{J}) \\ \nabla.\mathbf{B} = 0 \end{cases}$$

The reduced resistivity MHD models are designed to reduce the CPU cost using the properties of the plasma in the Tokamak configuration. The idea is to write the magnetic and velocity fields using a potential formulation adapted in the tokamak configuration and write the equations on these potentials. At the end the magnetic and velocity fields are given by

$$\mathbf{B} = \frac{F_0}{R}e_\phi + \frac{1}{R}\nabla\psi\times e_\phi$$

and

$$\mathbf{v} = -R\nabla u\times e_\phi + v_{||}\mathbf{B}$$

Firstly we propose some results about the rigorous derivation of the different reduced models and the conservation of the total energy (important for numerical stability). Secondly we propose to replace the implicit linear scheme used in Jorek by nonlinear time solvers as a Newton method or a continuation method with adaptive time stepping. We will show that these methods are more robust and efficient to compute correctly the physical instabilities. To finish we will propose some theoretical and numerical results on a physic preconditioning for the reduced MHD models. This preconditioning is based on a splitting between the diffusion terms and the hyperbolic systems and a parabolization of the hyperbolic coupling operators. We obtain a wave operator for the reduced MHD models which allows to design preconditioning operators.

An asymptotic preserving scheme for kinetic equation with anomalous diffusion limit

Helene Hivert
IRMAR - Universite de Rennes 1, France
helene.hivert@inria.fr
Nicolas Crouseilles, Mohammed Lemou

We consider a collisional kinetic equation with a heavy tail function for equilibrium, its asymptotic behaviour is known : it leads to the so-called anomalous diffusion equation, which is written with a fractional laplacian operator. In numerical computations, a problem often occurs: there is a relation linking the stiffness parameter ε and the time discretization. For small ε, having a numerical solution of the stiff kinetic equation takes computational time (and a phenomenon of numerical diffusion occurs). To get rid of these difficulties, we produce multi-scale schemes: if f^ε is a solution of the ε-dependant problem P^ε (here the kinetic equation), assuming that when ε goes to zero it converges to the solution f^0 of a limit problem P^0 (here the anomalous diffusion equation), the aim of an Asymptotic Preserving (AP) scheme is to permit the numerical resolution of the problem for the whole range of authorized ε with no condition linking the discretization parameters and ε. In the case of the kinetic equation with fractional diffusion limit, the difficulties for the numerical approximation arise from the non-local character of the limit equation and from the complexity of the methods to obtain the anomalous diffusion limit, even formally. Indeed, it seems that the usual approaches cannot be adapted to this asymptotic analysis. We propose a numerical method based on a Duhamel formulation of the initial equation. This gives us a strategy to formally obtain the limit equation and we are able to write a scheme, designed to be AP as required. The first numerical tests tend to confirm its efficiency: we can solve the initial equation for all the parameters, even very small, with no need of increasing the complexity of the discretization.

Hybrid classical-quantum electron transport models for strongly confined nanostructures

Clément Jourdana
Université Joseph Fourier, France
clement.jourdana@imag.fr
Paola Pietra

We propose hybrid classical-quantum approaches to study the electron transport in strongly confined nanostructures. The device domain is made of an active zone (where quantum effects are strong) sandwiched between two electron reservoirs (where the transport is considered highly collisional). So, a one dimensional effective mass Schrödinger system (used to describe the active zone) is spatially coupled with macroscopic collisional models (either a drift-diffusion model or a quantum drift-diffusion model). All the considered models contain effective quantities which retain the effects of the strong confinement and the atomistic information of the two dimensional transversal crystal structure. Self-consistent computations are performed coupling the hybrid transport equations with the resolution of a Poisson equation in the whole three dimensional domain. To illustrate these hybrid strategies, we present simulations of a gate-all-around single-walled Carbon Nanotube Field-Effect Transistor.

Parallelization of a Boltzmann-Schrödinger-Poisson solver for 2D nano DG-MOSFET device

Jose Miguel Mantas Ruiz
University of Granada, Spain
jmmantas@ugr.es
Francesco Vecil, María J. Cáceres, Carlos Sampedro, Andrés Godoy, Francisco Gámiz

The main characteristics of a parallel and distributed implementation of a fully deterministic solver for the Boltzmann-Schrödinger-Poisson system for partially-confined 2D Double Gate Metal Oxide Semiconductor Field Effect Transistors (DG-MOSFETs) are described. The numerical scheme is quite precise and presents several advantages compared to Monte-Carlo solvers. However, its computational implementation involves high demands for computational power due to the high dimensionality and the complexity of the collision operator. To overcome this inconvenience and make it possible to simulate 2D devices in reasonable times, the costliest parts of the numerical algorithm are parallelized for distributed-memory machines following a domain-decomposition approach. The parallel implementation uses functions of the Message-Passing Interface (MPI) to exchange data among the processes on the parallel platform, and of the Library of Iterative Solvers (LIS) to solve large sparse linear systems. We have chosen a data distribution strategy which enables a considerable reduction of the communication overheads because we perform almost no data movement at all for the main arrays of the solver being distributed among the processes. Several numerical experiments have been performed by simulating a DG-MOSFET devices on a SMP cluster in order to show and analyze the parallel performance of the solver.

Reduced model in velocity and hyperbolic approximation in space for the Vlasov equation

Laurent Navoret
Universite de Strasbourg, France
laurent.navoret@math.unistra.fr
Philippe Helluy, Nhung Pham

We present a reduction of the Vlasov equation into a space-only hyperbolic system: the distribution function is expanded on a finite-element basis in the velocity variables. Parallelized finite-volume schemes enable to solve the obtained system and achieve high performance. Moreover, this numerical approach has interesting conservation and stability properties. However, the numerical dissipation affects the precision of the finest resolved scales. In order to better control the high frequency oscillations in the velocity variables, the same numerical method is applied to the Fourier-transformed Vlasov equation.

Multi-Subband Ensemble Monte Carlo simulation of nanodevices for the 14nm node and beyond

Carlos Sampedro
University of Granada, Spain
csampe@ugr.es
F. Gámiz, A. Godoy, Marí a Cáceres, J. M Mantas, F. Vecil

With the current scenario of different candidates to lead the road to 14 nm node and beyond in the semiconductor industry, advanced device simulation represents an essential tool for the development of upcoming technological nodes in two ways: predicting the performance of different architectures and technological choices; and reducing the development stage in terms of cost and time. The Multi-Subband Ensemble Monte Carlo (MS-EMC) method is one of the best approximations to obtain a detailed description of both electrostatic and quantum phenomena in such small devices with an affordable computational cost. The possibility of studying the impact of each technological booster separately to explain experimental results becomes of paramount interest specially, nowadays, when it is difficult to isolate them in order to determine which one is the most effective in improving device performance. In this way, MSB-EMC simulations can offer shorter development times and easier implementation of new electrical compact models needed for the design and simulation of future circuits.

Special Session 122: Dynamics of Networks in Biology and Chemistry

Bernold Fiedler, Freie Universitaet Berlin, Germany
Atsushi Mochizuki, Theoretical Biology Laboratory, RIKEN, Japan

Recent progress in biology and chemistry provides many specific examples of systems which are described as dynamics involving networks. To understand the dynamics of such complex systems we need the power of existing and newly developing mathematics, as well as input from applications. Our intention is to bring together leading experts to survey the state of the art and discuss future trends in this exciting field of vast potential at the interface of graph theory, dynamical systems, and the Applied Sciences.
Links for our recent work in this area: http://link.springer.com/journal/10884/25/3/page/1#page-1
http://www.sciencedirect.com/science/article/pii/S0022519313002816

Signed graphs and network motifs

Fatihcan Atay
Max Planck Institute for Mathematics in the Sciences, Germany
fatay@mis.mpg.de

Signed graphs generalize network models by admitting positive and negative signs on the links, thus allowing two different types of interactions between the entities modeled by the nodes. This talk will study the Laplace operator on weighted signed graphs and its spectral properties. The operation of motif replication and its effect on the spectrum will be presented. A motif refers to a small subgraph whose occurrence in a graph is much higher (or lower) than a comparable random graph. Because of the deviation from a random graph, the presence of such motifs are often believed to be related to the function of biological and chemical networks. Based on a generic dynamical system defined on the graph, I will present analytical results on the role of different motifs on the dynamical behavior.

A spectral theory of linear operators on a Gelfand triplet and its application to the dynamics of coupled oscillators

Hayato Chiba
Kyushu University, Japan
chiba@imi.kyushu-u.ac.jp

The Kuramoto model is a system of ordinary differential equations for describing synchronization phenomena defined as a coupled phase oscillators. In this talk, an infinite dimensional Kuramoto model is considered. Kuramoto's conjecture on a bifurcation diagram of the system will be proved with the aid of a new spectral theory of linear operators based on Gelfand triplets.

Poster presentations of the minisymposium

Bernold Fiedler
Freie Universität Berlin, Germany
bernold.fiedler@googlemail.com

The following participants of the POSTER SESSIONS will briefly introduce their posters in short statements of five minutes, each, including discussion: Kouvaris, V.L. Maistrenko, Percie du Sert, Tang, Ujival.

Observability and hyperbolicity in coupled cells networks

Romain Joly
Institut Fourier (Grenoble), France
romain.joly@ujf-grenoble.fr

A coupled cells network is an ODE, in which the nonlinearity has a special structure: the interactions between the cells must follow the arrows of a given oriented graph. Thus, the possible dynamics of the coupled cells networks form a subclass of the dynamics of the general ODEs, strongly depending of the structure of the associated oriented graph and its symmetries. In this talk, we consider two related problems: - does the observation of one cell brings informations on the whole dynamics? For example, if the state of one cell is constant in time, is the whole network stationnary? - is it possible to study the generic dynamics of the coupled cells networks in the same spirit as the generic dynamics of ODEs have been studied? For example, are equilibrium points of coupled cells networks generically hyperbolic?

Jet lag can be avoided? : an oscillator network model for the circadian clock

Hiroshi Kori
Ochanomizu University, Japan
kori.hiroshi@ocha.ac.jp

Jet-lag symptoms arise from temporal mismatch between the internal circadian clock and external solar time. We know by experience that it takes about one week to recover from jet lag (i.e., reentrainment) after a long-distance trip. A group including one of the author reported in [1] that, in mice lacking the receptors of a certain neurotransmitter (KO mice), circadian rhythms of behavior and clock gene expression rhythms immediately reentrained to phase-shifted light-dark (LD) cycles. Still, the internal clock of such mice works normally under standard conditions. Experiments indicate that oscillation of clock gene expression in wild type mice significantly weakened after a large phase shift, whereas that in KO mice is robust. To uncover the mechanism of jet lag symptoms, we constructed an oscillator network model for the circadian master clock (the suprachiasmatic nucleus, SCN).We show that weaker coupling results in a quick response to jet lag.

References
[1] Y. Yamaguchi, T. Suzuki, Y. Mizoro, H. Kori, K. Okada, Y. Chen, J.M. Fustin, F. Yamazaki, N. Mizuguchi, J. Zhang, X. Dong, G. Tsujimoto, Y. Okuno, M. Doi, H. Okamura: Mice Genetically Deficient in Vasopressin V1a and V1b Receptors Are Resistant to Jet Lag, Science 342, pp. 85-90 (2013)

Dynamics of Coupled Maps in Heterogeneous Random Networks

Jeroen Lamb
Imperial College London, England
jswlamb@gmail.com
Tiago Pereira, Sebastian van Strien

We study expanding circle maps interacting in a heterogeneous random network. Heterogeneity means that some nodes in the network are massively connected, while the remaining nodes are only poorly connected. We provide a probabilistic approach which enables us to describe the effective dynamics of the massively connected nodes when taking a weak interaction limit. More precisely, we show that for almost every random network and almost all initial conditions the high dimensional network governing the dynamics of the massively connected nodes can be reduced to a few macroscopic equations. Such reduction is intimately related to the ergodic properties of the expanding maps. This reduction allows one to explore coherent properties of the network, such as hub synchronization.

Genericity and Bifurcations in Regular Networks

Reiner Lauterbach
University of Hamburg/Department of Mathematics, Germany
lauterbach@math.uni-hamburg.de

In joint work with M. Golubitsky (Bifurcations from Synchrony in Homogeneous Networks: Linear Theory, SIADS 8,40-75 (2009)) we investigated the generic structure of the linearization of the vectorfield at fully synchronous solutions in regular networks. Independent of the symmetry properties of the network the results show a structural similarity to the linearization of equivariant systems at fully symmetric solutions. These results have implications which allow statements similar to the equivariant branching lemma.

Here, we want to follow this approach further and show that some bifurcation results which are known for (certain) equivariant systems remain true in the context of regular networks.

Spreading speed for some two-component reaction-diffusion system

Hiroshi Matano
University of Tokyo, Japan
matano@ms.u-tokyo.ac.jp
Arnaud Ducrot, Thomas Giletti

In this talk I will discuss the spreading properties of solutions of a prey-predator type reaction-diffusion system. This system belongs to the class of reaction-diffusion systems for which the comparison principle does not hold. For such class of systems, little has been known about the spreading properties of the solutions. Here, by a spreading property, we mean the way the solution propagates when starting from compactly supported initial data. Among other things we show that propagation of the prey and the predator occurs with a definite spreading speed. Furthermore, quite intriguingly, the spreading speed of the prey and that of the predator are different in some situations. Next I will discuss the behavior of solutions "behind" the spreading fronts. If the corresponding ODE has a special type of Lyapunov function, one can show that the solution stabilizes to a steady-state. However, in a more general situation, the question is still largely open.

Dynamics of complex biological systems determined/controlled by minimal subsets of molecules in regulatory networks

Atsushi Mochizuki
RIKEN, Japan
mochi@riken.jp
Bernold Fiedler

Modern biology provides many networks describing regulations between many species of molecules. It is widely believed that the dynamics of molecular activities based on such regulatory networks are the origin of biological functions. In this study we develop a new theory to provide an important aspect of dynamics from information of regulatory linkages alone. We show that the "feedback vertex set" (FVS) of a regulatory network is a set of "determining nodes" of the dynamics. It assures that i) any long-term dynamical behavior of the whole system, such as steady states, periodic oscillations or quasi-periodic oscillations, can be identified by measurements of a subset of molecules in the network, and that ii) the subset is determined from the regulatory linkage alone. For example, dynamical attractors possibly generated by a signal transduction network with 113 molecules can be identified by measurement of the activity of only 5 molecules, if the information on the network structure is correct. We also demonstrate that controlling the dynamics of the FVS is sufficient to switch the dynamics of the whole system from one attractor to others, distinct from the original.

Dynamical properties of the MAP kinase cascade

Alan Rendall
University of Mainz, Germany
rendall@uni-mainz.de

This talk is concerned with mathematical models for the MAP kinase cascade, a pattern of chemical reactions of central importance in molecular biology. It is based on work done in collaboration with Juliette Hell. Existing numerical and heuristic work indicates that the ODE models for this biological system exhibit a variety of dynamical behaviour including more than one stable stationary solution for given parameter values, periodic solutions and chaos. It will be shown how a rigorous mathematical treatment of some of these features of the model can be given using bifurcation theory and geometric singular perturbation theory. A central aspect of this is the phenomenon of sequestration which leads to the appearance of networks with feedback as a limit of networks without manifest feedback.

Amplitude death in dynamical networks with time-varying delay

Eckehard Schoell
TU Berlin, Germany
schoell@physik.tu-berlin.de
A. Gjurchinovski, A. Zakharova

We study dynamical networks of Stuart-Landau oscillators including time-varying delay in the coupling and self-feedback. By generalizing the master stability function formalism to include variable-delay connections with high-frequency delay modulations, which is equivalent to distributed delay with an appropriate delay distribution, we analyze the regimes of amplitude death, i.e., stable steady states, in a ring network of oscillators, and demonstrate the superiority of the proposed method with respect to the constant delay case.

First steps towards the mathematical modeling of cytoskeletal networks

Angela Stevens
University of Münster (WWU), Germany
stevens@mis.mpg.de

The cellular cytoskeleton is a dynamic network of filamentous proteins which interacts, among others, with chemical signaling networks within the cell. To develop mechanical models which describ cell motion and polarization upon changing chemical signals, it is important to understand underlying principles that regulate the dynamics of the cytoskeleton. In this talk a first, spatially one dimensional model, for the (polarized) growth of the actin filaments of the cytoskeleton ist presented.

Origin and computational roles of intrinsic heterogeneity and spontaneous fluctuation in cortical networks

Jun-nosuke Teramae
Osaka University, Japan
teramae@ist.osaka-u.ac.jp

Neurons in cortical networks continuously show spontaneous firing even in the absence of sensory stimuli. The spontaneous firing is highly irregular and the firing rate is small. While it seems just background noise of cortical activity, recent experiments reported that spontaneous activity bidirectionally interacts with stimulus-evoked activity and actually affects animals perception. There has been much recent interest in the spontaneous irregular activity. However, the origin and functional roles of the activity remain almost completely unknown. In this study, by focusing on the recent experimental finding that amplitudes of postsynaptic potentials (PSP) between cortical neurons distribute with a highly skewed long-tailed distribution, we reveal that cortical networks can robustly generate the spontaneous irregular activity with neither internal noise source nor external input. We also find that spike information is transmitted in the cortical network most reliably by the help of the spontaneous activity. Our results identify a simple mechanism for internal noise generation supporting both stability and optimal spike-based communication between cortical neurons.

The Turing bifurcation in network systems: Snaking branches of localized patterns

Matthias Wolfrum
WIAS Berlin, Germany
wolfrum@wias-berlin.de
T. Wagenknecht, N. McCullen

We study the emergence of patterns in a diffusively coupled network that undergoes a Turing instability. It turns out that on large irregular networks there are fundamental differences to the case of continuous media. In particular, one can observe a huge variety of localized patterns, which are organized as snaking branches carrying an increasing number of differentiated nodes. However, in contrast to the classical snaking scenario, the irregular network structure leads to a complex and irregular structure of the snaking branches.

Special Session 123: Fractals

Raffaela Capitanelli, Sapienza Universita di Roma, Italy
Uta Freiberg, Universitaet Stuttgart, Germany

Aim of the session is to bring together scientists to discuss and exchange ideas on advanced research on the area of fractals. The topics include fractal geometry, analysis on fractals, stochastic processes on fractals.

Differential equations on fractal quantum graphs

Patricia Alonso Ruiz
Ulm University, Germany
patricia.alonso@uni-ulm.de
Daniel Kelleher, Alexander Teplyaev

One of the basic operators when studying differential equations is the Laplacian. When the underlying space is highly irregular, as happens with fractals, this operator needs to be redefined. In this talk we introduce a class of fractal spaces called fractal quantum graphs, which may not be self-similar, and obtain a Laplacian by means of Dirichlet forms.We will strongly use the theory of resistance forms developed by Kigami et al. Spectral asympotics for the eigenvalue counting function of this Laplacian in a particular type of such spaces are presented. We will also discuss some consequences of this result.

Obstacle problems on fractal sets

Raffaela Capitanelli
Sapienza-Roma, Italy
raffaela.capitanelli@uniroma1.it
M.A. Vivaldi

We deal with some obstacle problems on fractal sets. In particular, we present some approximation results for the solutions of these problems.

Numerical approximation of advection-diffusion problems in (pre)fractal domains.

Massimo Cefalo
DIAG - University La Sapienza of Rome, Italy
cefalo.m@gmail.com
Maria Rosaria Lancia

We consider the numerical approximation for a 2D advection-diffusion problem in a domain whose boundary is a prefractal mixture curve of Koch type. The presence of a prefractal boundary deteriorates the regularity of the solution and hence the rate of convergence of the numerical approximations. The two main difficulties arising in the numerical simulations of this type of problems are the generation of a suitable mesh to possibly achieve an optimal rate of convergence and the limitation of the computational costs. We obtain a priori error estimates. By exploiting some regularity results for the solution we build a mesh compliant with the so-called "Grisvard" conditions thus allowing to achieve an optimal rate of convergence.

Invariant (Fractal) Vector Measures

Ion Chitescu
University of Bucharest, Romania
ionchitescu@yahoo.com
Radu Miculescu,Loredana Ioana,Lucian Nita

Using a scalar valued integral of Hilbert valued functions with respect to Hilbert valued measures,different Monge-Kantorovich type distances on the space of Hilbert valued measures are introduced.Fixed points of Markov type operators on the space of vector valued measures (i.e.fractal measures) are studied.

Transmission problems and time-changed diffusions on irregular domains

Mirko D'Ovidio
Sapienza University of Rome, Italy
mirko.dovidio@sbai.uniroma1.it
Raffaela Capitanelli

We consider time-changed diffusions on fractal domains driven by generators with discontinuous coefficients. The PDE's connections are investigated and in particular some results on elliptic problems with transmission conditions are presented.

Trace and extension results for a class of ramified domains with fractal self-similar boundary

Thibaut Deheuvels
Ecole normale supérieure de Rennes, France
thibaut.deheuvels@ens-rennes.fr

We study some questions of analysis in view of the modeling of tree-like structures, such as the human lungs. More particularly, we focus on a class of planar ramified domains whose boundary contains a fractal self-similar part, noted Γ. We first study the Sobolev regularity of the traces on the fractal part Γ of the boundary of functions in some Sobolev spaces of the ramified domains. Then, we study the existence of Sobolev extension operators for the ramified domains we consider. In particular, we show that there exists $p* \in (1, \infty)$ such that there are $W^{1,p}$-extension operator for the ramified domains for every 1

Differential operators and generalized trigonomic functions on fractal subsets of the real line

Uta Freiberg
University of Stuttgart, Germany
uta.freiberg@mathematik.uni-stuttgart.de
Peter Arzt, Roland Etienne, Sabrina Kombrink

Second order differential operators of the form $d/d\mu d/dx$ are introduced on the real line. We discuss analytic properties and give spectral asymptotics for the case that μ is a self similar measure with compact support. Then we extend the results to some more general cases such as random fractal measures and self conformal measures. Moreover, we discuss eigenvalue problems on the real line with Cantor type set boundary and their discrete approximations.

Energy from spectral triples on the Sierpinski gasket

Daniele Guido
Univ. Roma Tor Vergata, Italy
guido@mat.uniroma2.it
F.Cipriani, T.Isola, J-L.Sauvageot

We construct a family of spectral triples for the Sierpiński Gasket K. For suitable values of the parameters, we determine the dimensional spectrum and recover the Hausdorff measure of K in terms of the residue of the volume functional $a \to \mathrm{tr}(a\,|D|^{-s})$ at its abscissa of convergence d_D, which coincides with the Hausdorff dimension d_H of the fractal. We determine the associated Connes' distance showing that it is bi-Lipschitz equivalent to the distance on K induced by the Euclidean metric of the plane, and show that the pairing of the associated Fredholm module with (odd) K-theory is non-trivial. When the parameters belong to a suitable range, the abscissa of convergence δ_D of the energy functional $a \to \mathrm{tr}(|D|^{-s/2}|[D,a]|^2\,|D|^{-s/2})$ takes the value $d_E = \frac{\log(12/5)}{\log 2}$, which we call *energy dimension*, and the corresponding residue gives the standard Dirichlet form on K.

Analysis and vector analysis on fractals

Michael Hinz
Bielefeld University, Germany
mhinz@math.uni-bielefeld.de

This talk will be a gentle introduction to some recent developments in the analysis on fractals, in particular to aspects of vector analysis and differential geometry. Based on a given energy form (or, equivalently, a given diffusion process) one can develop derivations, vector fields, tangent spaces and so on. We will provide some applications to PDE and explain some of our most recent results. Parts of the presented material are joint with Daniel Kelleher, Michael Roeckner, Luke Rogers and Alexander Teplyaev.

Hidden Variable Fractal Functions and Its Monotonicity Aspects

Saurabh Katiyar
Indian Institute of Technology Madras, Chennai, India
sbhkatiyar@gmail.com
A. K. B. Chand

Fractal interpolation that possesses the ability to produce smooth and nonsmooth interpolants is a novice to the subject of interpolation. Apart from appropriate degree of smoothness, a good interpolant should reflect shape properties, for instance monotonicity, inherent in a prescribed data set. To address this requirement, several shape preserving interpolation schemes are developed in the literature. There are a few articles concerned with intersection of these two fields, i.e., shape preserving fractal interpolation initiated by our group. Despite the flexibility offered by these shape preserving fractal interpolants, they are well-suited only for the representation of self-referential functions. We introduce a new class of monotone $\mathcal{C}^1$-cubic interpolants by taking full advantage of flexibility offered by the hidden variable fractal interpolation functions (HVFIFs). We achieve this using a two-step procedure enunciated in the following. Firstly, we associate an entire family of $\mathbb{R}^2$-valued continuous functions $\mathbf{f}[\mathbf{A}]$ parameterized by a suitable block matrix $\mathbf{A}$ with a prescribed function $f \in \mathcal{C}(I, \mathbb{R}^2)$. Secondly, we impose appropriate constraints on $\mathbf{A}$ so that $\mathbf{f}[\mathbf{A}]$ preserves monotonicity inherent in $\mathbf{f}$. This theory invoked to the $\mathcal{C}^1$-cubic spline HVFIF, which can be viewed as a fractal perturbation of the traditional $\mathcal{C}^1$-cubic spline, culminate with the desired monotonicity preserving $\mathcal{C}^1$-cubic HVFIF. The monotonicity preserving interpolation scheme developed herein generalizes and enriches its traditional nonrecursive counterpart and its fractal extension.

Ventsell problems in fractal domains

Maria Rosaria Lancia
Sapienza Università di Roma, Italy
maria.lancia@sbai.uniroma1.it
Paola Vernole

We consider a Ventsell problem for the heat equation in a fractal domain . We prove existence uniqueness and regularity results for the strict solution. In view of numerical approximation we study the corresponding approximating prefractal problems. We prove that the approximating prefractal solutions converge in a suitable sense to the limit fractal one.

Analysis, Dirac Operators and Geodesic Metrics on Fractal Manifolds, via Noncommutative Geometry

Michel Lapidus
University of California, Riverside, USA
lapidus@math.ucr.edu
Jonathan Sarhad

In this talk, we will report on recent work connecting aspects of geometric analysis on fractals and noncommutative fractal geometry. We construct spectral triples and Dirac operators on a class of fractals built on curves, including the Sierpinski gasket, the harmonic gasket (which is ideally suited for analysis on fractals and is a good model for a 'fractal manifold'), as well as suitable quantum graphs, Cayley graphs and other infinite graphs. This work is joint with Jonathan Sarhad (Math.arXiv:1207.6681v2 [math-ph], 2014, IHES preprint, IHES/M/12/22, 2012) and will be published in the "Journal of Noncommutative Geometry". It builds on earlier work, joint with Eric Christensen and Cristina Ivan (published in "Advances in Math.", vol. 217, 2008, pp. 1497-1507) in which we constructed geometric Dirac operators allowing us to recover the natural geodesic metric and the natural Hausdorff measure of the Euclidean Sieroinski gasket (as well of other fractals built on curves). It also builds on earlier work of the author in which, in particular, a broad research program was proposed for developing "noncommutaitve fractal geometry". The new advance in the present paper is that we can now deal with a significantly broader class of fractals, including the harmonic Sierpinski gasket (which can be viewed as a kind of "measurable Riemannian manifold", according to the recent work of Jun Kigami), allowing us to get one step closer to developing aspects of geometric analysis truly connected with the study of fractal manifolds and their intrinsic families of geodesic curves.

On a question of A. Kameyama

Radu Miculescu
University of Bucharest, Romania
miculesc@yahoo.com
Alexandru Mihail

A. Kameyama (see Distances on topological self-similar sets and the kneading determinants, J. Math. Kyoto Univ., 40 (2000), 603-674) introduced the concepts of topological self-similar set and of self-similar topological system and asked the following fundamental question: given a topological self-similar system, does there exist a metric on the topological self-similar set comparable to the topology such that all the functions of the self-similar topological system are contractions? Such a metric is called a self-similar metric. On the one hand, he provided a topological self-similar set which do not admit a self-similar metric and, on the other hand, he proved that every totally disconnected self-similar set and every non-recurrent finitely ramified self-similar set have a self-similar metric. The problem of the existence of a self-similar metric on a self-similar set was also studied by K. Hveberg, M. Barnsley, K. Igudesman, R. Atkins, M. Barnsley, A. Vince and D. Wilson. We modify the Kameyama's question (which, as we have seen, has a negative answer for an arbitrary topological self-similar system) by looking for phi-contractions instead of contractions and give an affirmative answer to the following question: given a topological self-similar system, does there exist a metric on the topological self-similar set comparable to the topology and a comparison function phi such that all the functions of the self-similar topological system are phi-contractions?

Existence and Uniqueness of Self-Similar Energies on Finitely Ramified Fractals

Roberto Peirone
Universita di Roma Tor Vergata, dipartimento di Matematica, Italy
peironemat.uniroma2.it

Self-similar energies on finitely ramified fractals correspond to the eigenvectors of a special nonlinear operator (called renormalization operator). I discuss some recent results on the existence of self-similar energies. In particular, on every fractal of this type there exists a self-similar energy provided we place suitable weights on the cells and consider a suitable level. I will also discuss some recent criteria for the uniqueness (up to a multiplicative constant) of the self-similar energy. In particular, on many non-tree fractals we have infinitely many normalized self-similar energies.

Fractals and the Jacobian Conjecture

Ronen Peretz
Ben Gurion University, Israel
ronenp@math.bgu.ac.il

Our primary motivation in this research originates from the two dimensional Jacobian Conjecture. The proposed program is to assume that the conjecture is false. This implies the existence of a semigroup of étale mappings and we intend to study its structure. A similar program in the past studied the structure of this semigroup with the structure of algebraic degree filtration. Here we impose a totally different structure on the semigroup. It is a fractal-like structure. We will describe our ideas for two different étale semigroups. We will start with the semigroup of the entire local homeomorphisms in one complex variable. After studying this classical case we will proceed to the case of the two dimensional étale polynomial mappings. The two theories will turn out to be different but still will share a few basic ingredients. We indicate how one might use the fractal structure we impose on the semigroup of étale mappings in order to solve the famous Jacobian Conjecture.

Uniform Restricted Range Approximation With Self-referential Functions

Viswanathan Puthan Veedu
Indian Institute of Technology Madras, India
amritaviswa@gmail.com
A. K. B. Chand and M. A. Navascue's

By an appropriate selection of elements in the iterated function system, the corresponding fractal interpolation function can be tailor-made to represent differentiable or nondifferentiable functions. The notion of fractal interpolation can be effectively used to obtain an entire class of self-referential functions f^α parameterized by a certain vector quantity α with f as its germ. For suitable values of the parameter, the fractal functions f^α simultaneously approximate and interpolate a prescribed continuous function f. The current article aims to identify appropriate values of the parameters so that the perturbed function f^α shares some properties such as regularity and positivity with the original function f. Consequently, elementary theorems on "property preserving fractal perturbation process" established in this article pave the way to shape preserving fractal approximation theory. Another problem exposed in this article is the approximation of a nonnegative function from below by nonnegative fractal splines. Further, some quantitative estimates in the copositive approximation of smooth functions by the fractal polynomials are broached. Fractal element introduced in the shape preserving approximant opens a wide range of choices for the selection of a uniform approximant corresponding to a given continuous function. Overall, the paper can be viewed as an earnest attempt to bridge the gap between the two fields that are developing in parallel - the theory of shape preserving approximation and fractal functions - for the one to benefit from the other.

Dirichlet forms for a jump-diffusion process on Koch's snowflake

Christian Seifert
Technische Universität Hamburg-Harburg, Germany
christian.seifert@tuhh.de
Uta Freiberg

We study Dirichlet forms subsets Ω of $\mathbb{R}^n$ describing diffusion processes. By introducing speed measures supported on subsets of Ω we obtain so-called singular diffusions. For the process this corresponds to a time change allowing for jumps. This setup yields an analytic description of a jump-diffusion process on Koch's snowflake.

Hamilton-Jacobi equations on neworks

Nicoletta Tchou
IRMAR- Rennes 1 university, France
nicoletta.tchou@univ-rennes1.fr
Yves Achdou, Salome Oudet

We consider continuous-state and continuous-time control problems where the admissible trajectories of the system are constrained to remain on a network. A notion of viscosity solution of Hamilton-Jacobi equations on the network has been proposed in earlier articles. Here, we propose a simple proof of a comparison principle based on arguments from the theory of optimal control.

Brennan's conjecture and weighted estimates on snowflake domains

Maria Agostina Vivaldi
Dipartimento di Scienze di Base e applicate per l'Ingegneria Sapienza Roma, Italy
maria.vivaldi@sbai.uniroma1.it
Raffaela Capitanelli

We establish regularity results in weighted Sobolev spaces for the solutions of the Dirichlet problems on snowflake domains by using suitable L^q-estimates for the gradient of the Green potential.

Special Session 124: Renormalization and Universality in Low-Dimensional Dynamics: From Computer Experiment to Proof. Dedicated to the Memory of Oscar Lanford III

Denis Gaydashev, Uppsala University, Sweden
Michael Yampolsky, University of Toronto, Canada

Renormalization and universality has emerged as one of the principal themes in low-dimensional dynamics. Historically, the study of renormalization has been motivated by computational experiments. The goal of the session is to bring together the leading experts and younger researchers working on both rigorous and computational aspects of renormalization and universality in low-dimensional dynamics, to explore the open problems and conjectures in this rapidly developing field.

Near parabolic renormalization for unicritcal maps

Arnaud Cheritat
CNR/Institut Math. Bordeaux, France
arnaud.cheritat@gmail.com

I will present a method to extend the proof of the existence of a compact invariant class for near parabolic renormalization, to a class that applies to unicritical maps.

Quasisymmetric rigidity of real maps

Trevor Clark
Imperial College London, England
t.clark@imperial.ac.uk
Sebastian van Strien

Quasisymmetric rigidity plays a crucial role in the renormalization theory of maps of the interval. I will discuss joint work with Sebastian van Strien on the quasisymmetric rigidity of smooth maps.

Supercentral limit theorems

Rafael de la Llave
Georgia Institute of Technology, USA
rafael.delallave@math.gatech.edu

We consider the scaling limits of noise in Siegel domains. (weak noise added for a long time). We show that the in the Siegel domain the scaling limits are Gaussian (central limit theorems) and any Gaussian can appear as one such scaling limit. In the boundary, all the scaling limits are rotationally invariant Gaussians.

The Julia set of the Feigenbaum map is poly-time computable

Artem Dudko
Stony Brook University, IMS, USA
artem149598@gmail.com
Michael Yampolsky

Let F(z) be the Feigenbaum fixed point of the period-doubling renormalization operator. We show that the Julia set of F is computable in polynomial time. This is the first example of a poly-time computable Julia set with a recurrent critical point. The talk is based on a joint work with Michael Yampolsky.

Period-doubling renormalization fixed points for area-preserving maps

Denis Gaidashev
Uppsala University, Sweden
gaidash@math.uu.se
P. Hazard

Existence of a period-doubling renormalization fixed point for area-preserving maps of a subset of the plane has been demonstrated in the 80's by J.-P. Eckmann, H. Koch and P. Wittwer using computer-assisted techniques. Since then, the progress towards an analytic proof of this fact has been slow and difficult. We will present an "almost analytic" existence proof which relies on the geometric features of the problem. This is a joint work with P. Hazard.

Renormalisability of Maps of Henon Type: Unbounded Combinatorics and Braid Equivalence

Peter Hazard
University of Toronto, Canada
p.hazard@utoronto.ca
Andre de Carvalho, Toby Hall

We discuss recent results on the structure of the space of Henon-like maps. We determine a mechanism by which distinct unimodal combinatorial types determine the same Henon "combinatorial type". More precisely we show that there are distinct unimodal combinatorial types are braid equivalent with equivalence realised by Henon-like maps. We then discuss some consequences of this.

On renormalizarion and rigidity for cyclic nonlinear interval exchange transformations

Konstantin Khanin
University of Toronto, Canada
khanin@math.toronto.edu

In this talk we shall report on recent progress in rigidity theory for nonlinear interval exchange transformations corresponding to cyclic permutations. Such maps can be viewed as circle homeomorphisms with multiple break points. We shall discuss both recent results on renormalizations of such maps in case of one break point (joint with S. Kocic and A. Teplinsky), and extension to the multiple-break setting (based on work in progress with A. Teplinsky).

Renormalization of critical and near-critical area-preserving maps

Hans Koch
The University of Texas at Austin, USA
koch@math.utexas.edu
Gianni Arioli

We describe the renormalization of analytic area-preserving maps and Hamiltonian flows. This includes the construction of the critical fixed point, a proof for the existence of non-smooth invariant tori, and recent work (so far mostly numerical) related to hyperbolicity.

Linearization of finitely differentiable flows on the multidimensional torus

Joao Lopes Dias
University of Lisbon, Portugal
jldias@iseg.utl.pt

We will discuss the C^s conjugacy between C^ℓ vector fields close to constant and diophantine vectors. We use a renormalization method based on a multidimensional continued fraction expansion in order to get bounds on ℓ and s.

The fixed point of parabolic renormalization

Michael Yampolsky
University of Toronto, Canada
yampol@math.toronto.edu

I will talk about our joint work with O. Lanford on the Inou-Shishikura fixed point f_* of the parabolic renormalization operator. In particular, I will discuss a renormalization-invariant class P of parabolic analytic germs $f(z)$ which have a maximal analytic continuation to a Jordan domain, with a specified branched covering structure, which contains the map f_*. I will also present a numerical computational scheme for f_*.

Special Session 125: Abstract Differential Equations and Related Topics

Toka Diagana, Howard University, United States
Claudio Cuevas, Universidade Federal de Pernambuco, Brazil
Juan J. Nieto, University of Santiago de Compostela, Spain

This special session is devoted to differential equations in Banach spaces and their applications.

On a family of nonlinear Volterra equations coming from the viscoelasticity theory

Bruno Andrade
ICMC-USP, Brazil
bruno00luis@gmail.com

In this talk we consider a family of nonlinear Volterra equations coming from the three dimensional viscoelasticity. Using the theory of abstract evolutionary integral equations, we analyze the existence of regular local mild solutions to the problem and their possible continuation to a maximal interval of existence. We also consider the problem of continuous dependence with respect to initial data.

A functional analytic approach of the attractivity in semilinear evolution equations

Joel Blot
SAMM University Paris 1, France
blot@univ-paris1.fr
Constantin Buse, Philippe Cieutat

We present a functional analytic approach of the attractivity in semilinea evolution equations which is based on the use of the space of the continuous functions which converge to zero at infinity. To realize that we establish results on the Nemytskii operators on such space and we treat the attractivity in several subclasses of solutions.

Asymptotic behavior for Volterra difference equations

Claudio Cuevas
Federal University of Pernambuco, Brazil
cch@dmat.ufpe.br
Mario Coquehuanca, Herme Soto

This work deals with the asymptotic behavior of solutions of a big class of non-linear Volterra difference equation. Concrete applications are given.

Self-similarity, symmetries and asymptotic behavior in Morrey spaces for a fractional wave equation

Marcelo de Almeida
Federal University of Sergipe, Brazil
nucaltiado@gmail.com
Lucas C. F. Ferreira

This paper is concerned with a fractional PDE that interpolates semilinear heat and wave equations. We show results on global-in-time well-posedness for small initial data in the critical Morrey spaces and space dimension $n \geq 1$. We also remark how to derive the local-in-time version of the results. Qualitative properties of solutions like self-similarity, antisymmetry and positivity are also investigated. Moreover, we analyze the asymptotic stability of the solutions and obtain a class of asymptotically self-similar solutions.

Existence Results for Some Nonautonomous Integro-differential Equations

Toka Diagana
Howard University, USA
tokadiag@gmail.com

In this talk we discuss some existence results for some nonautonomous integro-differential equations. Some applications will also be presented.

On certain classes of (not necessarily bounded) almost periodic functions with applications

Piotr Kasprzak
Adam Mickiewicz University, Poland
kasp@amu.edu.pl

Ever since their introduction by H. Bohr in mid-twenties, almost periodic functions of various types have been extensively studied by mathematicians. During the talk we are going to pay close attention to a certain class of not necessarily bounded almost periodic functions, which contains the class $AP(\mathbb{X})$, namely to the class of almost periodic functions in the sense of Levitan. Apart from discussing their basic properties, we are going to provide several results concerning convolution of Levitan almost periodic functions as well as necessary and sufficient conditions for a nonautonomous superposition operator to map the class of such functions into itself. Let us mention that the latter problem is closely connected with the question about natural topology on the class LAP. Furthermore, we are going also to discuss a new result concerning the asymptotic behaviour of certain Levitan almost periodic functions, whose proof required a completely new approach via continued fractions. Finally, we are going to indicate applications of the presented results to differential and integral equations.

Energy function method: modeling, methods and analysis

Gangaram Ladde
University of South Florida, Tampa, USA
gladde@usf.edu

In this work, we briefly outline the ideas of modeling, methods and analysis. In particular, we present a very general conceptual algorithm for finding solution processes of first order nonlinear stochastic differential equa-

tions. The presented conceptual algorithm is applied to several types of differential equations. Differential equations are solved in explicit or implicit forms. The scope of this approach is exhibited by solving special classes of stochastic differential equations in a systematic and unified way. Moreover, it is shown that the method of solving both deterministic and stochastic differential equations opens up a research area for young researchers, in particular serious undergraduate both mathematical sciences and interdisciplinary students.

Relative asymptotic equivalence between evolution difference equations

Hugo Leiva
Universidad de Los Andes, Venezuela
hleiva@ula.ve
Edgar Medina, Nelson Merentes and Hildebrando Rodrigues

In this paper we study the relative asymptotic equivalence between the solutions of the following two difference equations in a Banach space Z

$$y(n+1) = A(n)y(n), \quad x(n+1) = A(n)x(n) + f(n, x(n)),$$

where $y(n), x(n) \in Z$, $A \in l^\infty(\mathbb{N}, L(Z))$ and the function $f : \mathbb{N}\times Z \to Z$ is small enough in some sense. The generalized discrete dichotomy definition and a discrete version of Rodrigues Inequality play the main toll reaching our results, which is the following: Given a solution $y(n)$ of the unperturbed system, we provide sufficient conditions to prove that there exist a family of solutions $x(n)$ for the perturbed system such that

$$\|y(n) - x(n)\| = \circ(\|y(n)\|), \quad \text{as} \quad n \to \infty.$$

Conversely, given a solution $x(n)$ of the perturbed system having Lyapunov number $\alpha \in R$, we prove that, under certain conditions, there exist a family of solutions $y(n)$ for the unperturbed system, such that

$$\|y(n) - x(n)\| = \circ(\|x(n)\|), \quad \text{as} \quad n \to \infty.$$

Vector-valued weight pseudo almost automorphic functions and applications to evolution equations

Jin Liang
Shanghai Jiao Tong University, Peoples Rep of China
jinliang@sjtu.edu.cn
Ti-Jun Xiao

Of concern are vector-valued weighted pseudo almost automorphic functions taking values in Banach spaces and applications to evolution equations. We reveal several new properties of the vector-valued weighted pseudo almost automorphic functions by a new approach. Moreover, we establish new and general existence and uniqueness theorems for weighted pseudo almost automorphic solutions to some evolution equations. As one can see, our results improve essentially the existing corresponding results.

Fractional differential equations with uncertainty

Juan Nieto
University of Santiago de Compostela, Spain
juanjose.nieto.roig@usc.es

Differential equations of fractional order have recently proved to be valuable tools in the modeling of many physical and biological phenomena. There has also been a significant theoretical development in fractional differential equations in recent years. To model uncertainty, due to the imprecision inherent to the information available or the behavior of the dynamical system itself, it is useful the introduction of fuzzy sets in the model allowing a better description of the uncertainty in mathematical terms. We consider a fractional order differential equation with uncertainty. One of the approaches will be the use of a recent generalization of the classical Schauder Fixed Point Theorem to semilinear spaces.

Bounded mild solutions to fractional integro-differential equations in Banach spaces

Rodrigo Ponce
Universidad de Talca, Chile
rodrigo.poncec@gmail.com

In this talk, we consider the following equation with infinite delay

$$D^\alpha u(t) = Au(t) + \int_{-\infty}^{t} a(t-s)Au(s)ds + f(t, u(t)),\ t \in \mathbb{R}, \tag{1}$$

where A is a closed linear operator defined on a Banach space X, $a \in L^1(\mathbb{R}_+)$ is a scalar-valued kernel, f belongs to a closed subspace of the space of continuous and bounded functions, and for $\alpha > 0$, the fractional derivative is understood in the Weyl's sense. We study the existence and uniqueness of mild solutions for (1) where the input data f, is for example, almost periodic (resp. almost automorphic) and satisfies some Lipschitz type conditions. The unique mild solution u of (1) which is almost periodic (resp. almost automorphic) and is given by

$$u(t) = \int_{-\infty}^{t} S_\alpha(t-s)f(s, u(s))ds,\ t \in \mathbb{R}, \tag{2}$$

where $\{S_\alpha(t)\}_{t\geq 0}$ is the *α-resolvent family* generated by A. It is remarkable that, in the scalar case, that is $A = -\rho I$, with $\rho > 0$, some concrete examples of integrable α-resolvent families are showed.

A discrete-time approach for the asymptotic behavior of an exponentially bounded cocycle

Ciprian Preda
West University of Timisoara, Romania
preda@math.cornell.edu
Madalin Bunoiu

Roughly speaking, we prove that if the solution of the corresponding inhomogeneous variational di fference equation belongs to a sequence space (that has the ideal property and on which the right shift is an isometry) for every inhomogenity from a sequence space (with the same

properties), then the continuous-time solutions of the variational homogeneous diff erential equation will exhibit an exponential decay. Converse implications are also pointed out. This approach has many advantages among which we emphasize on the fact that the above conditions are very general (since the class of sequence spaces that we use includes almost all the known sequence spaces, as the classical spaces of p-summable sequences, sequence Orlicz spaces, etc.). Since we use a discrete-time technique we are not forced to require any continuity or measurability hypotheses on the trajectories of the exponentially bounded cocycle. Also, it is worth to mention that from discrete-time conditions we get informations about the continuous-time behavior of the solutions.

On abstract integro-differential equations with state dependent delay

Giovana Siracusa
UFS, Brazil
gisiracusa@gmail.com
Bruno de Andrade

Using topological tools we ensure that the solution set of an abstract integro-differential equations with state-dependent delay is a nonempty, compact and connected set. As application we consider our abstract results in the framework of integro-differential equations coming from viscoelasticity theory.

Abstract Integrodifferential Equations in Interpolation Spaces

Arlucio Viana
Universidade Federal de Sergipe, Brazil
arlucioviana@gmail.com
Bruno de Andrade

We consider the Cauchy problem

$$u' = Au + \int_0^t g(t-s, u(s))ds + f(t, u(t)), \; t > 0,$$
$$u(0) = u_0 \in X_1 = D(A),$$

where $A : D(A) \subset X_0 \longrightarrow X_0$ is a linear operator such that $-A$ is a sectorial operator, X_0 is a Banach space, and g and f are functions mapping X_1 into X_α, satisfying certain conditions. We purpose to present a result on existence and uniqueness of mild solutions for the above problem, when $\alpha \in (0, 1]$.

Special Session 127: Functional Inequalities and Variational Problems

Michinori Ishiwata, Osaka University, Japan
Bernhard Ruf, University of Milan, Italy
Futoshi Takahashi, Osaka City University, Japan

Variational methods are the key ingredients to produce several types of solutions to elliptic and also parabolic equations which are relevant to various fields of mathematics and mathematical physics. On the other hand, new types of functional inequalities which describe the subtle relationship between function spaces have been found and the research of these inequalities has been a subject in the field of real analysis. For example, recent study about Adams type higher-order Trudinger-Moser inequality and the related elliptic equations is an example of the fruitful collaborations of the above two fields. The main purpose of this Special Session is to encourage vital discussions between the groups of researches from the real analysis and PDE, and then to pursue the possible applications of such new functional inequalities to variational methods. Critical phenomena occurring from the non-compactness of embeddings, limiting profiles, concentration phenomena, and new types of scale-invariant inequalities will be subjects in this Session.

On the Hardy constant of non-convex planar domains: the case of the quadrilateral

Gerassimos Barbatis
National and Kapodistrian University of Athens, Greece
gbarbatis@math.uoa.gr
Achilles Tertikas

The Hardy constant of a simply connected domain $\Omega \subset R^2$ is the best constant for the inequality

$$\int_\Omega |\nabla u|^2 dx \geq c \int_\Omega \frac{u^2}{\operatorname{dist}(x, \partial\Omega)^2}\, dx\ , \qquad u \in C_c^\infty(\Omega).$$

After the work of Ancona where the universal lower bound 1/16 was obtained, there has been a substantial interest on computing or estimating the Hardy constant of planar domains. In this work we determine the Hardy constant of an arbitrary quadrilateral in the plane. In particular we show that the Hardy constant is the same as that of a certain infinite sectorial region which has been studied by E.B. Davies. Some other planar domains are also considered.

On Trudinger-Moser Type Inequalities with Logarithmic Weights

Marta Calanchi
University of Milan, Italy
marta.calanchi@unimi.it
Bernhard Ruf

Trudinger-Moser type inequalities for radial Sobolev spaces with logarithmic weights are considered. The precise Trudinger-Moser growths in dependence on the logarithmic terms, and the corresponding sharp Moser type exponents are determined. In a particular case a critical Trudinger-Moser growth of double exponential type is found.

Moser type inequalities in $\mathbb{R}^2$ and the zero mass case

Daniele Cassani
Università degli Studi dell'Insubria, Italy
daniele.cassani@uninsubria.it
Federica Sani, Cristina Tarsi

We investigate concentration and vanishing phenomena concerning Moser type inequalities in the whole plane which involve complete and reduced Sobolev norms. In particular we show that the critical Ruf inequality, which involves the complete Sobolev norm, is equivalent to an improved version of the subcritical Adachi-Tanaka inequality which we prove to be attained. Then, we consider the limiting space $\mathcal{D}^{1,2}(\mathbb{R}^2)$, completion of smooth compactly supported functions with respect to the Dirichlet norm $\|\nabla \cdot\|_2$, and we prove an optimal Lorentz-Zygmund type inequality from which can be derived classical inequalities in $H^1(\mathbb{R}^2)$.

Stability for parabolic quasiminimizers

Yohei Fujishima
Osaka University, Japan
fujishima@sigmath.es.osaka-u.ac.jp
Jens Habermann, Juha Kinnunen, Mathias Masson

We study parabolic quasiminimizers, which satisfies parabolic variational inequalities. In this talk, we show that, under a suitable regularity condition on the boundary, parabolic Q-quasiminimizers related to the parabolic p-Laplace equations with given boundary data are stable with respect to parameters Q and p.

Sharp decay estimates in Lorentz spaces for nonnegative Schrödinger heat semigroups

Norisuke Ioku
Ehime University, Japan
ioku@ehime-u.ac.jp
Kazuhiro Ishige, Eiji Yanagida

Let $H := -\Delta + V$ be a Schrödinger operator on $L^2(\mathbf{R}^N)$, where $N \geq 2$ and $V \in L^r_{\mathrm{loc}}(\mathbf{R}^N)$ with $r > N/2$. In this talk we focus on a nonnegative Schrödinger operator $H := -\Delta + V$ with a radially symmetric potential $V = V(|x|)$ behaving like $V(r) = \omega r^{-2}(1+o(1))$ as $r \to \infty$ with $\omega > -(N-2)^2/4$. We show the exact and optimal decay rates of the operator norm of the Schrödinger heat semigroup e^{-tH} in Lorentz spaces as $t \to \infty$.

On the effect of equivalent constraints on a maximizing problem associated with the Sobolev embedding in $\mathbb{R}^N$

Michinori Ishiwata
Fukushima University, Japan
ishiwata@sss.fukushima-u.ac.jp

In this talk, we consider the attainability of a maximizing problem

$$D := \sup_{\|u\|_{H^{1,N}_\gamma}=1} \left(\|u\|_N^N + \alpha \|u\|_p^p \right),$$

where $N \geq 2$, N_0 and $\|u\|_{H^{1,N}_\gamma} = \left(\|u\|_N^\gamma + \|\nabla u\|_N^\gamma \right)^{\frac{1}{\gamma}}$. The existence of a maximizer for D is closely related to the exponent γ. In fact, we show that the value

$$\alpha = \alpha_* := \inf_{\|u\|_{H^{1,N}_\gamma}=1} \left(\frac{1 - \|u\|_N^N}{\|u\|_p^p} \right)$$

is a threshold in terms of the attainability of D.

Group invariant positive solutions of the generalized Hénon equation

Ryuji Kajikiya
Saga University, Japan
kajikiya@ms.saga-u.ac.jp

We study the existence of symmetric positive solutions of the generalized Hénon equation

$$-\Delta u = f(x)u^p, \quad u > 0 \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega.$$

Here Ω is a bounded domain in $\mathbb{R}^N$, $f \in L^\infty(\Omega)$ and $f(x)$ may be positive or may change its sign. Let G be a closed subgroup of the orthogonal group $O(N)$. We assume that $f(x)$ and Ω are invariant under the action of G, that is, $f(gx) = f(x)$ for all $g \in G$, $x \in \Omega$ and $g(\Omega) = \Omega$ for all $g \in G$. For a proper closed subgroup H of G, we prove the existence of a positive solution which is H invariant but G non-invariant under suitable assumptions of H and G.

Existence of positive solutions of a semilinear elliptic equation with a dynamical boundary condition

Tatsuki Kawakami
Osaka Prefecture University, Japan
kawakami@ms.osakafu-u.ac.jp
Marek Fila, Kazuhiro Ishige

We consider the initial value problem for a semilinear elliptic equation with a dynamical boundary condition in the half space, which the space dimension is greater or equal to 2. In this talk we prove that there is a critical exponent for the existence of positive solutions. Furthermore, in the supercritical case we show that small solutions behave asymptotically like suitable multiples of the Poisson kernel. Moreover, we determine the optimal slow decay at spatial infinity for initial data giving rise to global bounded positive solutions.

Hardy-Sobolev Inequality for the Biharmonic Operator with Remainder Terms

Tommaso Passalacqua
Università degli Studi di Milano, Italy
tommaso.passalacqua@unimi.it
Bernhard Ruf

We prove that for $N \geq 5$, $\Omega \subset \mathbb{R}^N$ a bounded domain with boundary smooth, $1 \leq q < \frac{N}{N-4}$ and $0 \leq \tau < 4$, the critical Hardy-Sobolev constant C_{HS} does not depend on all the traces of the space $H^2(\Omega)$, i.e. we prove that the critical Hardy-Sobolev constant with Navier conditions coincides with the constant with Dirichlet conditions. Moreover, in the same assumptions, we prove two improvements of the Hardy-Sobolev inequality, with Navier and with Dirichlet boundary conditions. In the first case we prove that there exists a positive constant $C = C(N, q, \tau)$ such that the following critical Hardy-Sobolev inequality holds for any $u \in H^2_\vartheta(\Omega)$ and for any $1 \leq q < \frac{N}{N-4}$

$$\int_\Omega |\Delta u|^2 dx \geq C_{HS} \left(\int_\Omega \frac{|u|^\sigma}{|x|^\tau} dx \right)^{\frac{2}{\sigma}} + C \left(\int_\Omega |u|^q dx \right)^{\frac{2}{q}},$$

with $C_{HS} = C_{HS}(\tau)$ the critical Hardy-Sobolev constant. In the second case we prove that there exists a positive constant $C = C(N, q, \tau)$ such that for any $u \in H^2_0(\Omega)$ and for any $1 \leq q \leq \frac{N}{N-4}$

$$\int_\Omega |\Delta u|^2 dx \geq C_{HS} \left(\int_\Omega \frac{|u|^\sigma}{|x|^\tau} dx \right)^{\frac{2}{\sigma}} + C ||u||^2_{L^q_w(\Omega)}.$$

Both results are sharp in q.

Higher Order Functional Inequalities and the 1-Biharmonic Operator

Bernhard Ruf
University of Milan, Italy
bernhard.ruf@unimi.it
Daniele Cassani, Enea Parini, Cristina Tarsi

We study optimal embeddings for the space of functions whose Laplacian belongs to $L^1(\Omega)$, where $\Omega \subset \mathbb{R}^N$ is a bounded domain. This function space turns out to be strictly larger than the Sobolev space $W^{2,1}(\Omega)$ in which all second order derivatives are taken into account. In particular, in the limiting Sobolev case, when N = 2, we establish a sharp embedding inequality into the Zygmund space $L_{\exp}(\Omega)$. This result enables us to improve the Brezis-Merle regularity estimate for the Dirichlet problem $\Delta u = f(x) \in L^1(\Omega), u = 0$ on $\partial\Omega$. We then study the operator associated to related minimization problems, the *1-biharmonic operator* under various boundary conditions.

We consider the problem of finding the optimal constant for the embedding of the space

$$W^{2,1}_{\Delta}(\Omega) := \left\{ u \in W^{1,1}_0(\Omega) \,|\, \Delta u \in L^1(\Omega) \right\}$$

into the space $L^1(\Omega)$, where $\Omega \subset \mathbb{R}^n$ is a bounded convex domain, or a bounded domain with boundary of class $C^{1,\alpha}$. This is equivalent to find the first eigenvalue of the 1-biharmonic operator under (generalized) Navier boundary conditions. In this paper we provide an interpretation for the eigenvalue problem, we show some properties of the first eigenfunction, we prove an inequality of Faber-Krahn type, and we compute the first eigenvalue and the associated eigenfunction explicitly for a ball, and in terms of the torsion function for general domains.

Trudinger-Moser and Adams-type inequalities on the whole space $\mathbb{R}^N$

Federica Sani
Milano University, Italy
federica.sani@unimi.it
Nader Masmoudi

Adams' inequality is the complete generalization of the Trudinger-Moser inequality to the case of Sobolev spaces involving higher order derivatives. In this talk we discuss the optimal growth rate of the exponential-type function in Trudinger-Moser and Adams' inequalities when the problem is considered in the whole space $\mathbb{R}^N$.

Littlewood-Paley theory and Gagliardo-Nirenberg inequalities

Yoshihiro Sawano
Tokyo Metropolitan University, Japan
ysawano@tmu.ac.jp

In this talk, I plan to discuss the Gagliardo-Nirenberg inequalities in the various function spaces. The Gagliardo-Nirenberg inequality is an interpolation inequality and this fact allows us to assume less assumption. As an example of function spaces, I will take up Morrey spaces and variable Lebesgue spaces and I will obtain some interpolation inequalities of Gagliardo Nirenberg.

On a positive solution for (p, q)-Laplace equation with indefinite weight

Mieko Tanaka
Tokyo University of Science, Japan
tanaka@ma.kagu.tus.ac.jp

I provide existence and non-existence results of a positive solution for (p, q)-Laplace equation under the Dirichlet boundary condition.

Supercritical Schrödinger systems in dimension two

Cristina Tarsi
Università di Milano, Italy
Cristina.Tarsi@unimi.it
D. Cassani

We prove existence of variational solutions for the Hamiltonian coupling of nonlinear Schrödinger equations in the whole plane, when the nonlinearities exhibit supercritical growth with respect to the Trudinger-Moser inequality. We discover linking type solutions which have finite energy in a suitable Lorentz-Sobolev space setting.

Kinetic approximations to fractional diffusion equations

Elide Terraneo
University of Milano, Italy
elide.terraneo@unimi.it
G. Furioli, A. Pulvirenti, G. Toscani

We introduce a non-local linear kinetic equation which approximates a fractional diffusion equation. We then show that the solution to this approximation approaches the fundamental solution of the fractional diffusion (a Lévy stable law) at large times.

Boundedness of small data solutions to a chemotaxis system with non-diffusive memory

Yohei Tsutsui
University of Tokyo, Japan
tsutsui@ms.u-tokyo.ac.jp

We consider a chemotaxis system with logarithmic sensitivity and non-diffusing chemical substance. When the chemotatic sensitivity constant is in some interval, the existence of spatially bounded global solutions to the system was proved by Ahn and Kang. This talk gives the same result for any chemotatic sensitivity constant, assuming the smallness condition on the initial data. Although Ahn and Kang made use of a entropy functional, we achieve the result by using two standard theorems: the small data global existence theorem, in which solutions are spatially bounded in a weak sense, and the local existence theorem, in which solutions are spatially bounded.

Optimal embeddings on Sobolev-Lorentz-Zygmund type spaces

Hidemitsu Wadade
Gifu University, Japan
hidemitsuwadade@gmail.com

In this talk, we consider the embedding on the critical Sobolev-Lorentz-Zygmund type space $H^{\frac{n}{p}}_{p,q,\lambda_1,\cdots,\lambda_m}(\mathbb{R}^n)$ into the generalized Morrey space $\mathcal{M}_{\Phi,r}(\mathbb{R}^n)$ with an optimal Young function Φ. Furthermore, as an application of this embedding, we obtain the almost Lipschitz continuity for functions in $H^{\frac{n}{p}+1}_{p,q,\lambda_1,\cdots,\lambda_m}(\mathbb{R}^n)$. O'Neil's inequality and its reverse play an essential role for the proof of main theorems.

Special Session 128: How Do Complex Networks Improve Our Knowledge of Biology?

Jacobo Aguirre, Centro de Astrobiologia CSIC-INTA, 0
Javier M. Buldu, Universidad Rey Juan Carlos & Center for Biomedical Technology, Spain
Jose A. Capitan, Centro de Astrobiologia, CSIC-INTA, Spain

The discovery of the laws governing the structure and dynamics of Complex Networks is one of the greatest challenges of modern science. Nowadays, we have at hand many outstanding results in this research field, and large progress has been made in basic theoretical aspects, especially in the search of how the generated knowledge can be applied to the characterization and exploitation of real systems. Biology is one of the most benefited fields from these advances. Biological networks, in part due to the massive acquisition of data at the molecular and cellular levels, have constituted a relevant niche to test concepts of complexity and Graph Theory. At the same time, the characterization of real biological systems has fed back into new models, theoretical tools and numerical techniques, enriching our current knowledge of complex networked systems. In this session we aim to focus on the real contribution that Complex Network Theory has made to improve our knowledge of Biological processes, paying special attention on genotype, protein, and metabolic networks, brain networks, mesoscales in biological systems, ecological networks and food webs.

But... do complex networks really improve our knowledge of Biology?

Jacobo Aguirre
Centro de Astrobiologia CSIC-INTA, Spain
aguirreaj@cab.inta-csic.es

In recent years a wide variety of biological processes have constituted a fruitful niche to test concepts of complexity and Graph Theory, and for this reason have attracted the attention of many theoretical scientists. The question that we face in this open discussion is whether the results obtained in the field of Biology making use of complex networks: (i) really improve our knowledge of Biology (that is, provide the right answers to the right questions), and (ii) reach experimentalists and cast light on their work (and if not, why!)

Topology and dynamics of the zebrafish segmentation clock genetic network

Saul Ares
CNB - CSIC, Spain
saul.ares@csic.es
Christian Schröter, Luis G. Morelli, Alina Isakova, Korneel Hens, Daniele Soroldoni, Martin Gajewski, Frank Jülicher, Sebastian J. Maerkl, Bart Deplancke, Andrew C. Oates

During vertebrate embryogenesis, the rhythmic and sequential segmentation of the body axis is regulated by an oscillating genetic network termed the segmentation clock. We describe a new dynamic model for the core pace-making circuit of the zebrafish segmentation clock based on a systematic biochemical investigation of the network's topology and precise measurements of somitogenesis dynamics in novel genetic mutants. We show that the core pace-making circuit consists of two distinct negative feedback loops operating in parallel. To explain observed single and double mutant phenotypes of mutant embryos in our dynamic model, we postulate that the availability and effective stability of the dimers with DNA binding activity is controlled in a 'dimer cloud' that contains all possible dimeric combinations between the three factors. This feature of our model predicts that Hes6 protein levels should oscillate despite constant hes6 mRNA production, which we confirm experimentally using novel Hes6 antibodies. The control of the circuit's dynamics by a population of dimers with and without DNA binding activity is a new principle for the segmentation clock and may be relevant to other biological clocks and transcriptional regulatory networks.

Networks competing for centrality (and its biological implications)

Javier Buldu
Universidad Rey Juan Carlos, Spain
jmbuldu@gmail.com

Competition between systems that belong to a complex network relies on several dynamical and topological aspects such as their natural frequencies (or time scales), number of neighbors or location inside the network. Nevertheless, the fact that real networks are usually embedded in networks of networks (NoN) also has important consequences in competition processes. In the current contribution, we present a new framework to analyze the competition between interconnected networks for available resources as a struggle for network importance, measured via the eigenvector centrality. Our analytical and numerical results yield that different strategies must be played depending on the initial network structures. In particular, competitor networks can strikingly improve their outcome by selecting the adequate connector nodes or by rearranging their internal topology. Due to the existence of optimal strategies, we are able to define a competition parameter Ω that quantifies the benefit of the interconnection topology in real networks. These results can be applied to a large number of biological processes in networks related with evolutionary, spreading or diffusion processes.

From "description of" to "prediction with" biological networks

Roger Guimera
ICREA and U Rovira i Virgili, Spain
roger.guimera@urv.cat

In biological systems, individual components interact with each other giving rise to complex networks, which are neither totally regular nor totally random. Because of the interplay between network topology and dynamics, it is crucial to characterize the structure of complex networks.

Although during the last decade significant progress has been made in the study of complex networks, we are still far from the ultimate goals of understanding the precise mechanisms responsible for the observed topology, and evaluating the impact of the structure of the network on the dynamics of the system. The two main impairments to achieve these goals are: (i) most network data are very unreliable, that is, for most systems there is uncertainty as to what is the real structure of the network; and (ii) we lack the tools to extract the relevant information contained in the structure of networks, and to evaluate the impact of network structure on a system's dynamics. In my talk, I will discuss how we can use very general properties of complex networks to address these two very prominent problems, and even to go one step further and uncover previously unknown interactions. This opens the door to more accurate modeling of biological systems and even to radically new applications of network theory, including, for example, the prediction of novel drug interactions.

Evolutionary dynamics of defense mechanisms in simple host - pathogen systems

Jaime Iranzo
National Institutes of Health, USA
jaime.iranzosanz@nih.gov
Alexander E. Lobkovsky, Yuri I. Wolf, Eugene V. Koonin

The arms race between pathogens and hosts is a major evolutionary force. In this race, hosts have developed three qualitatively different types of defense mechanisms: (i) innate immunity systems, which recognize conserved pathogen features and evolve in a Darwinian way; (ii) adaptive immunity systems, which evolve during the life of the host and provide enhanced efficiency; (iii) host suicide, which prevents further spreading of the infection. Remarkably, these three mechanisms can be found in very different organisms, both unicellular and pluricellular. Moreover, defense mechanisms are usually found in association, e.g. innate and adaptive immune systems in vertebrates or CRISPR-Cas and cell suicide systems in prokaryotes. This contribution presents a computational model aimed at understanding which types of defense mechanisms evolve under distinct ecological conditions. Variable efficiencies and costs for innate immunity, adaptive immunity and suicide were considered, as well as multiple spatial structures for the host population and pathogen diffusion rates. Simulations reveal a collection of evolutionary behaviors characterized by the acquisition of one, two or all three classes of defense mechanisms. Moreover, they show that population structure is critical for the evolution of cell suicide. All these hints help us understand the distribution of defense mechanisms observed in nature.

Self-organized evolution of anatomical networks in neuronal in vitro cultures

I. Leyva
Universidad Rey Juan Carlos - Centro de Tecnologia Biomedica (UPM), Spain
inmaculada.leyva@urjc.es
D. de Santos-Sierra, I. Sendina-Nadal, J. A. Almendral, S. Anava, A. Ayali, D. Papo, S. Boccaletti

In vitro primary cultures of dissociated invertebrate neurons from locust ganglia are used to experimentally investigate the morphological evolution of assemblies of living neurons, as they self-organize from collections of separated cells into elaborated, clustered, networks. At all the different stages of the culture development, identification of the locations of neurons and neurites by means of a dedicated software allows to ultimately extract an adjacency matrix from each image of the culture. In turn, a systematic statistical analysis of a group of topological observables grants us the possibility of quantifying and tracking the progression of the main network's characteristics during the self-organization process of the culture. Our results point to the existence of a particular state corresponding to a small-world network configuration, in which several relevant graph's micro and mesoscale properties emerge. Finally, we identify the main physical processes ruling the culture's morphological transformations, and embed them into a simplified growth model qualitatively reproducing the overall set of experimental observations.

Using Parenclitic Networks to Evaluate the Loss of Brain Consistency

Johann Martinez Huartos
Universidad del Rosario, Technical University of Madrid, Colombia
johemart@gmail.com
P. Ariza, J. M. Pastor, M. Zanin, D. Papo, F. Maestu, R. Bajo, S. Boccaletti, J. M. Buldu

Increased variability in performance has been associated with the emergence of several neurological and psychiatric pathologies. However, whether and how consistency of neuronal activity may also be indicative of an underlying pathology is still poorly understood. Here we propose a novel method for evaluating consistency from non-invasive brain recordings. We evaluate the consistency of the cortical activity recorded with magnetoencephalography in a group of subjects diagnosed with Mild Cognitive Impairment (MCI), a condition sometimes prodromal of dementia, during the execution of a memory task. We use metrics coming from nonlinear dynamics to evaluate the consistency of cortical regions. A representation known as Parenclitic Networks is constructed, where atypical features are endowed with a network structure, the topological properties of which can be studied at various scales. Pathological conditions correspond to strongly heterogeneous networks, whereas typical or normative conditions are characterized by sparsely connected networks with homogeneous nodes. The analysis of this kind of networks allows identifying the extent to which consistency is affecting the MCI group and the focal points where MCI is specially severe. To the best of our knowledge, these results represent the first attempt at evaluating the consistency of brain functional activity using complex networks theory.

Competition and coexistence of multiple pathogens in metapopulation models

Sandro Meloni
University of Zaragoza, Spain
sandro@unizar.es

Interactions among multiple infectious agents are increasingly recognized as a fundamental issue in the understanding of key questions in public health, regarding pathogen emergence, maintenance, and evolution. The full description of host-multipathogen systems is however challenged by the multiplicity of factors affecting the interaction dynamics and the resulting competition that may occur at different scales, from the within-host scale to the spatial structure and mobility of the host population. Here we study the dynamics of two competing pathogens in a structured host population and assess the impact of both epidemiological parameters and the mobility patterns of the hosts population. We model the spatial structure of the population in terms of a metapopulation network and consider the most general case where the two pathogens can have different infectious duration and offer different levels of cross-immunity one to another. Via both mechanistic numerical simulations and a theoretical analysis we find that the model presents a very rich behavior with different dynamical regimes. We were able to fully characterize the regions of the parameters space where the two pathogens coexist at both systemic and single population level. In this scenario we also characterize, via a very simple analytical reasoning, the effects of the degree of cross-immunity between pathogens finding that for high hosts' mobility only two states are possible: one in which only the fastest pathogen survives and one in which the both diseases reach a similar fraction of the system. Finally, the present work sheds some light on the complex interactions between multiple infections spreading on the same population allowing for a deeper understanding of real world scenarios like the seasonal coexistence of different influenza strains.

Studying functional brain activity with complex network theory: potential, pitfalls and the way ahead

David Papo
CTB, Spain
papodav@gmail.com

Many physical and biological systems can be studied using complex network theory, a new statistical physics understanding of graph theory. Its recent application to the study of functional brain networks generated great enthusiasm as it allows addressing an entirely new set of experimental questions. However, complex network theory's potential in functional neuroimaging is still underexploited. At the same time, despite a high degree of generality, the theory was originally designed to describe systems profoundly different from the brain. Occasionally its wholesale application to neuroimaging turns out to be problematic. Furthermore, in its current form it does not adequately describe key aspects of brain activity. Neuroimaging will advance insofar as it will be able to promote parallel advances in network theory.

Multi-layer networks in evolutionary game theory.

Rubén J. Requejo
Universitat de Barcelona, Spain
rubenfisico@yahoo.es
LASAGNE team.

In this talk I will present recent results obtained in the LASAGNE project, which extend the evolutionary dynamics of three-strategies public goods games to multilayer networks. A general discussion on the motivation for such modelling will be carried out, and the results of agent based simulations showing the complex behaviour of the system will be shown.

Coupled games in interdependent networks with biased imitation

Marta Santos
Department of Physics & I3N, University of Aveiro, Portugal, Portugal
scully402@gmail.com
S. N. Dorogovtsev, J. F. F. Mendes

In this talk I will explore the evolutionary dynamics of two coupled two-person games played in distinct layers of an interdependent network. Individuals in the network revise their strategies by imitating neighbors from the same layer with probability p, and neighbors from the other layer with complementary probability 1-p. We provide results of extensive computer simulations for the case in which layers are modeled as regular random networks, and support this study with analytical results for coupled well-mixed populations.

Variability in mutational fitness effects prevents full lethal transitions in large quasispecies populations

Josep Sardanyes
Universitat Pompeu Fabra, Spain
josep.sardanes@upf.edu
Carles Simó, Regina Martóınez, Ricard V. Solé and Santiago F. Elena

The distribution of mutational fitness effects (DMFE) is crucial to the evolutionary fate of quasispecies. In this article we analyze the effect of the DMFE on the dynamics of a large quasispecies by means of a phenotypic version of the classic Eigen's model that incorporates beneficial, neutral, deleterious, and lethal mutations. By parameterizing the model with available experimental data on the DMFE of *Vesicular stomatitis virus* (VSV) and *Tobacco etch virus* (TEV), we found that increasing mutation does not totally push the entire viral quasispecies towards deleterious or lethal regions of the phenotypic sequence space. The probability of finding regions in the parameter space of the general model that results in a quasispecies only composed by lethal phenotypes is extremely small at equilibrium and in transient times. The implications of our findings can be extended to other scenarios, such as lethal mutagenesis or genomically unstable cancer, where increased mutagenesis has been suggested as a potential therapy.

Evolutionary search processes in networks of RNA genotypes

Michael Stich
Aston University, England
m.stich@aston.ac.uk
Susanna C. Manrubia

RNA molecules, through their dual identity as sequence and structure, are an appropriate model to study the genotype-phenotype map and evolutionary processes taking place in simple replicator populations. As RNA sequences mutate, they move at the same time dynamically on the regular network formed by sequences and on the network formed by the structures. We review some mathematical properties of the sequence-structure map, in particular the existence of common and rare structures. Then, through extensive simulations and statistical methods, we relate these properties with the number of replication events that an initially random population of sequences needs in order to find a predefined target structure through mutation and selection. For common structures, this search process turns out to be much faster than for rare structures. Furthermore, search and fixation processes are more efficient in a wider range of mutation rates for common structures, thus indicating that evolvability of RNA populations is not simply determined by abundance. As a result, although the population moves on the simple network of sequences, the sequence-structure map and the complex phenotype space are fundamental to understand the evolution of the population.

Special Session 129: Qualitative and Quantitative Techniques for Differential Equations Arising in Economics, Finance and Natural Sciences

Mariano Torrisi, University of Catania, Italy
Rehana Naz, Centre for Mathematics and Statistical Sciences Lahore School of Economics Lahore, Pakistan
Igor Leite Freire, UFABC, Brazil
Imran Naeem,Lahore University of Management Sciences (LUMS), Lahore Pakistan, Pakistan

The differential equations play a vital role in many disciplines from natural to social sciences. Most of physical laws in natural sciences are expressed in terms of differential equations. The Economists study dynamical systems of ordinary differential equations for sustainable Economic growth. Stochastic differential equations are the standard models for financial quantities important in financial market. Differential equations are mathematically studied from several different perspectives; this session will focus on the Qualitative and Quantitative techniques (including numerical methods) for ordinary differential equations, partial differential equations, fractional differential equations, difference equations, stochastic differential equations, integro-differential equations. Potential topics, of this session, include but are not limited to: Optimal control, Economic growth theory, Fractional equations modeling natural and economic models, Financial models e.g. Hamilton Jacobi equation, Hamilton Jacobi Bellman equations, Option models, Black Schole models etc, Equivalence transformations, classical and non classical symmetries , Reduction techniques and solutions , and linearization , Conserved quantities, Wave propagation, Stability analysis, Population dynamics and biological phenomena, Numerical techniques for special problems in modeling, Reaction diffusion equations, Mathematical methods for extended thermodynamics, Hyperbolic reaction diffusion equations, Mathematical biology, Fluid dynamics

Slip effects on steady MHD flow of a third-grade fluid in an annulus of rotating concentric cylinders

Shirley Abelman
University of the Witwatersrand, Johannesburg, So Africa
Shirley.Abelman@wits.ac.za

This study examines the influence of slip condition on steady MHD flow of a third-grade fluid in an annulus of rotating concentric cylinders. The nonlinear differential system is developed by using laws of mass and momentum. A numerical solution of the governing problem is obtained. The effects of emerging flow parameters are examined and discussed.

A Differential Model of Television Viewership

Mufid Abudiab
Texas A&M University-Corpus Christi, USA
mufid.abudiab@tamucc.edu
Aimee Maceyko

In our paper we modify the model of box office dynamics created by Edwards and Buckmire by expanding the application to network television series. A system of differential equations that models network television viewership and corresponding ad revenue will be introduced. Variables and parameters of our model will be relevant to network television business. The eigenvalue method will be used to solve the problem and the stability of the solution will be checked. Finally, based on the analytical solution and its stability, recommendations related to issues and characteristics that are unique to network television will be presented to people in this industry. Key words: Box office dynamics, mathematical modeling, network television series, eigenvalue method, stability.

Single wall and multi wall carbon nanotubes analysis in stenosed arteries

Noreen Akbar
DBS&H, CEME, National University of Sciences and Technology, Islamabad, Pakistan, Pakistan
noreensher@yahoo.com

Blood flow model is recycled to study the influence of magnetic field and carbon nanotubes in stenosed arteries. The carbon nanotubes for the blood flow with water as base fluid is not explored so far. The representation for the blood flow is through an axially non-symmetrical but radially symmetric stenosis. Symmetry of the distribution of the wall shearing stress and resistive impedance and their growth with the developing stenosis is another important feature of our analysis . Exact solutions have been evaluated for velocity, resistance impedance, wall shear stress and shearing stress at the stenosis throat. The graphical results of different type of tapered arteries (i.e converging tapering, diverging tapering, non-tapered artery) have been examined for different parameters of interest for single wall carbon nanotubes (SWCNT) and multi wall carbon nanotubes (MWCNT).

Nanofluid effects on peristaltic transport of a fourth grade fluid in the occurrence of inclined magnetic field

Safia Akram
National University of Sciences and Technology,, Pakistan
drsafiaakram@gmail.com

The effects of nanofluid and inclined magnetic field on peristaltic transport of a fourth grade fluid model are discussed. Mathematical modelling of fourth grade fluid and nanofluid model are given by taking two dimensional channel. Assumptions of long wave length are employed to simplify the governing equations of momentum, temperature and nano particle volume fraction. Homotopy perturbation technique is applied to calculate the solution of coupled nonlinear equations of temperature and nano particle volume fraction. Analytical solution is carried out

to calculate the solution of stream function. Pressure rise and pressure gradient on the cannel walls have been computed numerically. The graphical results are discussed to see the effects of various emerging parameters on different wave forms.

Group classification and conservation laws of some generalized Dullin-Gottwald-Holm equations

Maria Bruzon
University of Cadiz, Spain
m.bruzon@uca.es
Maria L Gandarias

The shallow water equations play a critical role in modeling and simulation of fundamental phenomena of natural science, as in storm surges that inundate coastal regions due to hurricanes or typhoons. In 2001 Dullin, Gottwald and Holm derived a new equation describing unidirectional propagation of surface waves on a shallow layer of water. In this paper we consider the problem of group classification and conservation laws of some generalized Dullin-Gottwald-Holm equations.

Nonlinear self-adjointness and conservation laws of the Korteweg-de Vries-Zakharov-Kuznetsov equation

Priscila Leal da Silva
UFABC, Brazil
pri.leal.silva@gmail.com
Igor Leite Freire

In this work we consider a (1+3) version of the Zakharov-Kuznetsov equation, an equation that plays an import hole in mathematical physics and can be viewed as a higher dimensional version of the Korteweg-de Vries equation. We obtain necessary and suficient conditions for the equation to be nonlinearly self-adjoint. Then, using its Lie point symmetry generators and the obtained substitutions, we establish some local conservation laws using Ibragimov's recent conservation theorem.

Environmental dispersion modelled by PDE: numerical approximations and computer simulations

Geraldo Diniz
Federal University at Mato Grosso, Brazil
geraldo@ufmt.br

In this paper we will present the main problems environmental dispersion in the region of Mato Grosso, strategies for Mathematical modelling for the problems and methods of numerical approximation of the solution. As an illustration, we will be present some computer simulations of problems already studied and the prospects for future work. In general, the equation that models the phenomenon dispersion, known transport equation, generally, can be modeled by:

$$\begin{aligned}&\{\text{variation of pollutant concentration}\}\\&\quad=\{\text{diffusion}\}-\{\text{transport}\}\\&\qquad-\{\text{degradation}\}+\{\text{source}\}\end{aligned}$$

where each of the terms listed above has been treated in literature by several authors. For simulations of scenarios, one gets the variational formulation the proposed by PDE classic model, whose discretizations in the space and time, has been made by the finite elements method (for spatial discretization), by Petrov/Galerkin scheme and Crank-Nicolson (for temporal discretization), for numerical approximation of the solutions. The program MATLAB^R has proven more efficient for implementation of numerical codes providing the graphic animations that will be shown.

A pre-existing fracture driven by a power-law fluid in permeable rock: Lie symmetry analysis and similarity solutions

Adewunmi Fareo
University of the Witwatersrand, Johannesburg, So Africa
Adewunmi.Fareo@wits.ac.za
DP Mason

The group invariant solutions for the evolution of a two dimensional fracture with non-zero initial length in permeable rock and driven by a laminar incompressible non-Newtonian fluid of power-law rheology are derived. With the aid of lubrication theory and the PKN approximation a nonlinear diffusion equation for the fracture half-width is derived. Since the fluid-rock interface is permeable the nonlinear diffusion equation contains a leak-off velocity sink term. A condition, in the form of a first order partial differential equation for the leak-off velocity, is obtained for the nonlinear diffusion equation to possess Lie point symmetries. The general form of the leak-off velocity is derived and the effect of power law rheology on the hydraulic fracturing is investigated.

Symmetries and conservation laws of a new third order equation

Igor Freire
UFABC, Brazil
igor.freire@ufabc.edu.br
Julio Cesar Santos Sampaio

In this talk we discuss about the Lie symmetry analysis of a dispersive equation recently discussed in the literature. We show that such equation is nonlinearly self-adjoint and the substitution is depending on the dispersion coefficient. From the symmetries we look for invariant solutions and also for local conserved quantities obtained from Ibragimov's approach.

Some conservation laws for a class of equations arising in financial mathematics.

Maria Gandarias
University of Cadiz, Spain
marialuz.gandarias@uca.es
Maria Santos Bruzon

Financial models were generally formulated in terms of stochastic differential equations. However, it was soon found that under certain restrictions these models could be written as linear evolutionary partial differemtial equa-

tion with variable coefficients. The Black and Scholes equation is an evolution equation dealing with random phenomena in the specific context of finance. The class of Hamilton-Jacobi-Bellman equations arises in the sphere of stochastic control theory. Conservation laws are useful in both general theory and the analysis of concrete systems. They are usually relevant for their physical interpretation and they are also useful to determine potential symmetries. S. Anco and G. Bluman gave a general treatment of a direct conservation law method for partial differential equations expressed in a standard Cauchy-Kovaleskaya form. One of the present authors has introduced the concept of weak self-adjoint equations. This definition generalizes4 the concept of self-adjoint and quasi self-adjoint equations that were introduced by N.H. Ibragimov. We have found a class of weak self-adjoint Hamilton-Jacobi-Bellman equations which are neither self-adjoint nor quasi self-adjoint. By using a general theorem on conservation laws proved by N.H. Ibragimov, the Lie symmetries of the Hamilton-Jacobi-Bellman equations derived by V. Naicker, K. Andriopulos and P.G.L. Leach and the concept of weak self-adjointness we find conservation laws for some of these partial differential equations.

Analysis of the unsteady separation phenomenon with slip boundary conditions in Navier-Stokes equation

Francesco Gargano
University of Palermo, Italy
francesco.gargano@unipa.it
Marco Sammartino, Vincenzo Sciacca

A numerical investigation on the Navier-Stokes equations with both the slip and no-slip boundary condition is carried out for an incompressible 2d flow. When the classical no-slip boundary condition is applied at the boundaries, the flow exhibits the phenomenon of the unsteady separation and, according to the *Re* number, two kind of interaction acting over different length scale develop in the boundary layer flow. Moreover all the interactions are characterized by the presence of several complex singularities in the wall shear stress at the boundary. By using a slip boundary condition, such as the Navier boundary condition whereby the slip velocity is proportional to the tangential viscous stress, the flow evolution exhibits a different behavior from the no-slip case. We show how this boundary condition acts on the separation evolution and on the characterization of the complex singularities of the wall shear. We carry out numerical simulations by using an efficient mixed spectral-finite differences numerical scheme as used in [1].

References
[1] F. Gargano. M. Sammartino, V. Sciacca, K.W. Cassel , *Analysis of complex singularities in high-Reynolds-number Navier-Stokes solutions*, J. Fluid. Mech, Accepted, 2014

Exact solutions for shout and reset options

Joanna Goard
University of Wollongong, Australia
joanna@uow.edu.au

A strike reset option is a European option with the added feature that it gives the holder the right to 'shout' during the life of the option and thus reset the strike to the then prevailing stock price. The related shout option is similar, but upon 'shouting' the holder resets the strike and locks in a profit.

Optimal portfolios based on weakly dependent data

Shuya Kanagawa
Tokyo City University, Japan
sgk02122@nifty.ne.jp
Hiroshi Takahashi and Ken-ichi Yoshihara

Let X_k be strictly stationary sequence of d-dimensional centered random vectors satisfying the strong mixing condition. Consider a stochastic difference equation

$$DP(t_k) = P(t_k - 1)(bt/n + s(t/n)^1/2X_k),$$

where s is the volatility and b is the trend. It is known that the solution of this difference equation converges almost surely to a Black-Sholes type model Z(t). The purpose of this talk is to estimate an approximation of a stock price model Z(t) with a random volatility using P(t) and consider optimal portfolios for Z(t).

Optimal Control Techniques for Epidemic Models

Adnan Khan
Lahore University of Management Sciences, Pakistan
adnan.khan@lums.edu.pk
Mudassar Imran

We consider deterministic models of transmission dynamics of epidemics based on SEIR type models. Various strategies for controlling the epidemics have been used by public health authorities; these include vaccination, quarantine and isolation. We discuss how ideas from the theory of optimal control are used to determine the optimal strategy. We illustrate the results on two different models, one for the transmission dynamics of Hepatitis C and the other for influenza.

Magnetohydrodynamic 3D flow of Maxwell fluid flow due to a horizontal stretched surface with convective boundary conditions

Muhammad Yousaf Malik
Department of Mathematics Quaid-i-Azam University, Islamabad, Pakistan
drmymalik@hotmail.com
Sohail Nadeem and Abdul Rehman

The study deals with the problem of steady, three-dimensional MHD boundary layer flow of a non-Newtonian Maxwell fluid flow due to a horizontal surface stretched exponentially in two lateral directions. The surface temperature is assumed to be distributed exponentially and possesses convective boundary conditions. The governing set of nonlinear partial differential equations along with the associated boundary conditions is simplified using a similarity transformation and the resulting set of ordinary differential equations is solved numerically. The effects of important physical parameters associated with the fluid flow and heat transfer are presented.

Entropy Dissipative Approximations of Cross-Diffusion Models

Pina Milisic
University of Zagreb, Croatia
pina.milisic@fer.hr
Ansgar Juengel

Partial differential equations arising in science and technology typically contain some structural information reflecting important physical properties such as positivity, mass and energy conservation, or entropy dissipation. Entropy dissipation is intensively used in the mathematical analysis of PDEs for the derivation of apriori estimates which represent a cruical tool in proving the existence of solutions and studing their long-time behaviour. Our aim is, using the entropy structure of the cross-diffusion system from population dynamics, to derive new numerical schemes which are structure-preserving. The used techniques are based on translation of the continuous entropy estimates to the discrete level with hope to obtain accurate and stable approximations. We introduce new one-leg multistep time discretizations of considered cross-diffusion model for which the existence of semi-discrete weak solutions is proved. The main features of the scheme are the preservation of the nonnegativity and the entropy-dissipation structure. The key ideas are to combine Dahlquist's G-stability theory with entropy-dissipation methods and to introduce a nonlinear transformation of variables which provides a needed quadratic structure in the equations. It is shown that G-stability of the one-leg scheme is sufficient to derive discrete entropy dissipation estimates. Furthermore, under some assumptions on the evolution operator, the second-order convergence of solutions is proved. In order to underline the theoretical results, some numerical experiments will be presented.

Lie Symmetries, Optimal systems and Exact solutions of some fractional differential equations

Imran Naeem
Lahore university of management sciences(LUMS), Lahore Pakistan, Pakistan
imran.naeem@lums.edu.pk
M. D. Khan

We study time-space fractional advection-dispersion (FADE) equation and time-space fractional Whitham-Broer-Kaup (FWBK) equation that have a significant role in hydrology. We introduce suitable transformations to convert fractional order derivatives to integer order derivatives and as a result these equations transform into partial differential equations (PDEs). Then the Lie symmetries and corresponding optimal systems of the resulting PDEs are derived. The symmetry reductions and exact independent solutions based on optimal system are investigated which constitute the exact solutions of original fractional differential equations.

The closed form solutions of some Economic growth models via partial Hamiltonian approach

Rehana Naz
Centre for Mathematics and Statistical Sciences Lahore School of Economics Lahore, Pakistan, Pakistan
rehananaz_qau@yahoo.com
Azam Chaudhry, Fazal M Mahomed

The closed form solutions of some Economic growth models are constructed models via newly developed partial Hamiltonian approach. The partial Hamiltonian approach can be used to obtain reductions and closed-form solutions via first integrals of current value Hamiltonian systems of ordinary differential equations arising in economic growth theory and other economic models. In this article, we derive the closed form solutions of a class of Ramsey Economic growth models and an endogenous growth model with health factor. Moreover, the stability of the equilibria and the transitional dynamics are also analyzed. The Economics interpretations are presented in detail.

A mathematical analysis of fluid motion in irreversible phase transitions

Gabriela Planas
Universidade Estadual de Campinas, Brazil
gplanas@ime.unicamp.br
Jose L. Boldrini and Luis H. de Miranda

We consider a model for the irreversible solidification process of certain materials by taking in consideration the effects of natural convection in molten regions. Such a model consists of a highly nonlinear system of partial differential equations coupled to a doubly nonlinear differential inclusion. The existence of weak-strong solutions for the system is proved, and certain mathematical effects of advection on the regularity of the solutions are discussed.

Classification of two-dimensional systems of third order ordinary differential equations by complex methods

Asghar Qadir
School of Natural Sciences National University of Sciences & Technology H-12, Islamabad, Pakistan, Pakistan
asgharqadir46@gmail.com
Hina M. Dutt, M. Safdarb

Lie's method of converting a second order scalar ordinary differential equation (ODE) to linear form by transformations of the independent and dependent variables (point transformations) had been extended to the third order. Separately, it had been shown that whereas for the second order ODEs there is a unique class of ODEs (with eight infinitesimal symmetry generators) for the third order there are three classes with four five and seven generators. Lie's linearization had also been extended to systems

of second order ODEs and it had been found that for the two-dimensional system there are five classes (with five, six, seven, eight and fifteen generators). Another development used the analyticity of complex scalar ODEs to linearize two-dimensional systems. Based on the use of geometric methods, which not only convert the second order system to linear form but also directly provide the solutions for them, it produced three of the five classes (with six, seven and fifteen generators). Here we use the complex methods for the classification of two-dimensional systems of third order ODEs. It provides the classification of the subset of linearizable two-dimensional systems of third order ODEs that corresponds to a scalar complex third order ODE. Some examples are also given.

Hypergeometric functions and transition dynamics in a model of optimal growth with a renewable natural resource

Jose-Ramon Ruiz-Tamarit
Universitat de Valencia, Spain
ramon.ruiz@uv.es

Special functions are commonly used in mathematical physics and computational mathematics to solve differential systems.The area of Special functions is not a new research area. Among their multiple uses, there is the case of the Gauss hypergeometric function, which allowed solving nicely the so-called hypergeometric differential equation. We use this class of functions in our inquiry about different models taken from the most recent endogenous growth literature, notably those applied to environmental problems and sustainable development. We claim that researchers in economic dynamics should use these powerful tools if possible, given that they can be decisive if one aims at getting beyond the computational and/or local approaches typically and intensively adopted in economics.

Stochastic models describing the generation of dsbs of DNA by radiation

Yasumasa Saisho
Hiroshima University, Japan
saisho@hiroshima-u.ac.jp

The double-strand break (dsb) of DNA is one of the most critical lesions leading to a variety of radiobiological effects such as cancer, cell death. In this talk, we reconsider the previously constructed and generally accepted mathematical models for dsb generation, and give a concrete mathematical basis for the generation of dsbs and the calculation of the number of induced dsbs, under the assumption of randomness in the dsb location in DNA and in the number of dsbs. These models enables us to calculate the dose dependence of dsb generation ([SI]). Reference [SI] Y. Saisho and A. Ito, Mathematical models of the generation of radiation-induced DNA double-strand breaks, Journal of Mathematical Biology 67-3, (2013), 717-736.

Approximations of stochastic differential equations by difference equations based on weakly dependent random vectors

Hiroshi Takahashi
Nihon University, Japan
takahashi_hs@penta.ge.cst.nihon-u.ac.jp
Shuya Kanagawa and Ken-ichi Yoshihara

In this talk, we present approximations of stochastic differential equations based on some weakly dependent sequences of random vectors. For stationary mixing sequences, many studies of strong Wiener approximation have been studied, and this method corresponds to Ito formula. In this direction, our method is an extension of the previous studies. We also discuss some applications to the Feynman-Kac representation and time series models related to mathematical finance.

Discontinuity waves in a two fluid model for debris flows avalanches

Mariano Torrisi
University of Catnia, Italy
torrisi@dmi.unict.it
Carmen Mineo

The increase, in the past few decades, of the phenomena of intense transport of geophysical mass flows, pushed the technical and scientific community to try to protect people, man-made structures, and the economic activities of affected areas. Typical examples of these dangerous and destructive natural phenomena are landslides, snow avalanches, pyroclastic flows, debris flows that are driven down a slope under the action of gravity Here we consider the two fluid model for debris flows introduced from Pitman and Le [1].After a wide discussion about the hyperbolicity of this model [2] we study the evolution of weak discontinuities. The critical time for some real cases of geophysical mass flows is obtained.

References

[1] E. B. Pitman and L. Le, *Phil. Trans. R. Soc. A*, **363**, 1573-1601 (2005)

[2] C. Mineo and M. Torrisi, *On the hyperbolicity of a twouid model for debris flows.* In: IUTAM-ISIMM Symposium on Mathematical Modeling and Physical Instances of Granular Flows. Reggio Calabria (Italy), September 14-18, 2009, Melville, New York: AIP Conference Proceedings, **1227**, 72-78 (2010)

Contributed Session 1: ODEs and Applications

A normalizing isospectral flow on complex Hessenberg matrices

Alessandro Arsie
The University of Toledo, USA
alessandro.arsie@utoledo.edu
Krishna Pokharel

After having briefly reviewed Lax flows on spaces of matrices, in this talk we discuss an isospectral flow that deforms any given complex upper Hessenberg matrix with simple spectrum to a normal upper Hessenberg matrix. Furthermore, we prove that if the spectrum of the initial condition is contained on a line l in the complex plane, then its omega-limit set is actually a tridiagonal normal matrix, possessing a special symmetry among the off-diagonal elements that depends on the slope of l. As a farther application, we show that the flow can be used to construct even dimensional tridiagonal real skew-symmetric matrices with given simple imaginary spectrum and with strictly positive elements on the secondary diagonals. The main technical aspect of the work is to prove that the omega-limit set of suitable initial conditions consists of a single point in the phase space.

On the regularized trace of an operator which has purely discrete spectrum

Ozlem Baksi
Yidiz Technical University, Turkey
baksi@yildiz.edu.tr

In this work we find the regularized trace of the operator L which is expressed by the differential equation

$$l(y) = -y''(x) + Ay(x) + Q(x)y(x)$$

with the boundary condition

$$y'(0) = y(\pi) = 0$$

In this differential equation: 1) The operator $A : D(A) \to H$ satisfies the conditions $A = A* >= I, A^(- 1) \in \sigma_{infinity}(H)$ 2) Q(x) is a kernel operator in the space H. Here, H is a Hilbert space and $\sigma_{infinity}(H)$ is kernel operator space from H to H

Implicit-Explicit multivalue methods for semi-discretized PDEs

Angelamaria Cardone
University of Salerno, Italy
ancardone@unisa.it

Many practical problems in science and engineering are modeled by PDEs, whose space-discretization gives raise to large systems of ordinary differential equations (ODEs), characterized by a stiff part and a non-stiff one. Significant examples are given by the discretization of advection-diffusion equations and by advection-reaction problems with a stiff reaction. Here we propose, for the numerical solution of such systems, implicit-explicit multivalue methods which treat the non-stiff part by an explicit multivalue method and the stiff part by a diagonally implicit multivalue method. We analyze the convergence and stability of these methods when implicit and explicit parts interact with each other. We look for methods with large absolute stability region, assuming that the implicit part of the method is A- or L-stable. Moreover we are interested in IMEX methods which are able to reproduce the qualitative behavior of the solution. We furnish examples of some IMEX methods, with optimal stability properties. Finally, we compare our methods with other existing ones, on some significant test examples.

Analysis of Dynamics in a Complex Food Chain with Ratio-Dependent Functional Response

Yaw Chang
UNC-Wilmington, USA
changy@uncw.edu
Michael Freeze, Wei Feng

We study a new model obtained as an extension of a three-species food chain model with ratio-dependent functional response. We provide non-persistence and permanence results and investigate the stability of all possible equilibria in relation to the ecological parameters. Results are obtained for the trivial and prey-only equilibria where the singularity of the model prevents linearization, and the remaining semi-trivial equilibria are studied using linearization. We provide a detailed analysis of conditions for existence, uniqueness, and multiplicity of coexistence equilibria, as well as permanent effect for all species. The complexity of the dynamics in this model is theoretically discussed and graphically demonstrated through various examples and numerical simulations.

Nearly conservative multi-value numerical methods for hamiltonian problems

Raffaele D'Ambrosio
University of Salerno, Italy
rdambrosio@unisa.it
Giuseppe De Martino, Beatrice Paternoster

This talk is devoted to the analysis of multi-value methods for the numerical integration of Hamiltonian problems. Even if the numerical flow generated by such a method cannot be symplectic, a concept of near conservation can be considered, i.e. G-symplecticity, which implies conjugate-symplecticity of the underlying one step method associated to the original multi-value scheme. It is known that multi-value methods introduce parasitic components in the numerical solution, deteriorating the overall accuracy and the ability of preserving the invariants of Hamiltonian systems; however, a remedy against the effects of parasitism is discussed, together with its long-term counterpart: indeed, a backward error analysis is presented, which permits to get sharp estimates for the parasitic solution components and for the error in the Hamiltonian. Symmetry of the numerical schemes is also analyzed as a practical tool for the construction of high order methods and an effective property for reversible mechanical systems. Numerical experiments confirming the theoretical expecations are given.

A Space Extension For Exponentially Fitted Runge-Kutta-Nystrom Methods

Ekin Deliktas
Istanbul Technical University, Turkey
edeliktas@itu.edu.tr
Turgut Ozis

In the last decade, there has been a lot of research on the exponentially/trigonometrically fitted Runge-Kutta-Nystrom methods for the numerical solution of initial value problems of ordinary differential equations with oscillatory or periodic solution. Such problems arise in different fields such as celestial mechanics, electronics, molecular dynamics, astrophysics.
In this paper, we consider special second order ordinary differential equations (ODEs) $y'' = f(x, y)$ whose solution can be expressed as linear combination of the set of functions $\{exp(\lambda x), exp(-\lambda x)\}$ or equivalently $\{sin(\omega x), cos(\omega x)\}$ when $\lambda = i\omega$, $\omega \in R$. We construct space extension by choosing new basis functions as combination of trigonometric and hyperbolic functions instead of standart basis functions. Our results are much better than the classical ones. Additionaly we note that our results and those of scientific literature are comparable.

Estimates of stability ecosystems in models of global processes

Irada Dzhalladova
Kyiv National Economic University, Ukraine
irada-05@mail.ru

We consider a system equations with quadratic right-hand side and asymptotically stable matrix in linear approximation.The system written in special vector-matrix form. Supposed the algorithm estimates the region of stability with quadratic right-hand side. The algorithm is based on using second method of Lyapunov quadratic function of the form [1]. The resulting estimates can be viewed as the analysis of conditions for stable functioning of ecosystems and develop a strategy for the survival of humanity [2,3].

References

[1] A.M. Ĺyapunov, General problem about moving stability, *Moscow.* 50 (1950) 472 p.

[2] Robert Smith?, Modelling Disease Ecology with Mathematics, *American Institute of Sciences* 29 (2008) , 189 p.

[3] I. A. Dzhalladova, Analysis of dynamic structure and estimates structure oscillation in demographic process , *K.-KNEU.* 87 (2012) 275-284.

Signalling network structure and its role in controlling overshoot in Escherichia coli chemotaxis signalling

Matthew Edgington
University of Reading, England
m.p.edgington@pgr.reading.ac.uk
Marcus J. Tindall

We consider a recent fourth order nonlinear ordinary differential equation model of the *Escherichia coli* chemotaxis signalling pathway. This is used by cells to communicate changes in their chemical environment to the flagellar motors responsible for controlling swimming behaviour. Recent experimental work has examined overshoot - a transient period during the chemotactic response in which the motor regulating protein exceeds its prestimulus concentration before returning to it. Here we use model reduction and stability analysis to identify features of the signalling cascade responsible for overshoot. We predict that signalling protein concentration is vital in controlling the overshoot response.

Invertibility of problems with reflection: reducing and solving the problem

F. Adrian F. Tojo
Universidade de Santiago de Compostela, Spain
fernandoadrian.fernandez@usc.es
Alberto Cabada

When dealing with first order linear problems with involutions it is always interesting to obtain a Green's function for the problem being considered. These kinds of problems appear frequently in the literature (see [1-4]). In some cases we can prove equivalence between problems with different involutions using a correspondence of involutions result. Using this kind of equivalence we can restrict our attention to the case with reflection. Studying those cases where the Green's function can be obtained through exponentiation we can obtain results which provide information on its sign and therefore maximum and anti-maximum principles.

References

[1] Cabada, A.; Tojo, F. A. F. Existence results for a linear equation with reflection, non-constant coefficient and periodic boundary conditions. Journal of Mathematical Analysis and Applications, 412-1 (2014), 529 - 546.

[2] Cabada, A.; Infante, G.; Tojo, F. A. F. Nontrivial Solutions of Perturbed Hammerstein Integral Equations with Reflections. Boundary Value Problems. 2013, 2013:86.

[3] Kotin, L.; Irving, E. J. On Matrices which Commute with their Derivatives. Linear and Multilinear Algebra, 1982, Vol. 12, pp. 57-72.

[4] Wiener, Joseph. Generalized solutions of functional-differential equations. World Scientific Publishing Co., Inc., River Edge, NJ, 1993.

A class of systems admitting an inverse integrating factor.

Natalia Fuentes
Huelva University, Spain
natalia.fuentes@dmat.uhu.es
Antonio Algaba, Cristobal Garcia and Manuel Reyes.

We use the normal form theory to analyze the existence of an inverse integrating factor of a class of systems, in general non-integrable, whose lowest-degree quasi-homogeneous term is a Hamiltonian vector field and whose Hamiltonian function only has simple factors over $\mathbb{C}[x, y]$. We solve the problem for a wide class of systems. This class of systems contains, for example, some systems with lineal part non-null and nilpotent systems, among others. Finally we apply our technique to two families of polynomial systems.

A new method to build a large scale, strongly coupled model as benchmark for ODE solvers

Thibaut-Hugues Gallois
IFPEN/Ecole Centrale Paris, France
gallois.thibauthugues@gmail.com
Jean Brac, Thierry Soriano

Let us consider large scale and strongly coupled models. To test ODE solvers and parallelization strategies, it is useful to build new benchmarks. Indeed, most of industrial large scale models ($\geq$ 1000 equations) are more or less coupled and from them it is difficult to build larger and more coupled models. This paper proposes a new method to build benchmark models as large and coupled as required from a physical initial one. The considered example is the one dimension heat conduction. - It is possible to adjust the model size as large as required by discretizing the spatial derivatives. - Couplings are added by changing the basis of the state variables of the model. Moreover, the analytical solution is known and used to compare results given by ODE solvers. Such benchmarks aim at comparing the behaviours of a solver A and a solver B by means of weakly and strongly coupled ODE models. Two different solvers may have close computation times for a weakly coupled model but shall give noticeable differences when the system is more coupled. These differences are often due to the fact that the parallelization strategies are different.

Fourth Painlevé hierarchies related to the Boussinesq hierarchy

Pilar Gordoa
Universidad Rey Juan Carlos, Spain
pilar.gordoa@urjc.es
J. M. Conde, A. Pickering

We consider a system of equations defined using the Hamiltonian operator of the Boussinesq hierarchy, as well as two successive modifications thereof. We are able to reduce the order of the three systems and also to give Bäcklund transformations between the integrated systems. We can also construct auto-Bäcklund transformations for the two modified systems. For a particular case, we identify generalized fourth Painlevé hierarchies, i.e., sequences of integrable equations having as base equation the fourth Painlevé equation. As a consequence we derive auto-Bäcklund transformations for these fourth Painlevé hierarchies as well as Bäcklund transformations between our Painlevé hierarchies. Moreover, the results on reduction of order provide a reduction of order of the scaling similarity reduction of the Boussinesq hierarchy.

Critical oscillation constant for half-linear differential equations with coefficients having mean values

Petr Hasil
Department of Mathematics and Statistics, Masaryk University, Brno, Czech Rep
hasil@mail.muni.cz
Michal Veselý

Aim of this talk is to show that the existence of the mean values of coefficients is sufficient for second order half-linear Euler-type differential equations to be conditionally oscillatory. We can explicitly find an oscillation constant even for the considered equations whose coefficients can change sign. These results cover known results concerning equations with periodic and almost periodic positive coefficients and extend them to larger classes of equations.

Analytical Integration of the Osculating Lagrange Planetary Equations in the Elliptic Orbital Motion

Denis Hautesserres
Centre National d'Etudes Spatiales, France
denis.hautesserres@cnes.fr

In the field of orbital motion perturbation methods, a half century of works has produced a lot of analytical theories. These theories are either based on Hamiltonian developments and series expansions of the perturbing functions (Brouwer), or use iterative approximation algorithms (Kozai). Generally speaking, analytical theories have difficulties to deal with high eccentric satellite orbits, due to series expansions or due to the large number of terms in closed forms. The present work comes back to the original Lagrange Planetary Equations. The goal is to solve the restricted three-body problem with a semi-analytical method accurate to deal with any high eccentric satellite orbit around an oblateness central body and perturbed by a third body. It is a simple method intellectually efficient to analyse each effect of the Sun and of the Moon over one orbital period. Moreover, a long duration simulation allows to observe the secular effects on the orbital parameters.The results of the method for solar and lunar effects on a Geostationary Transfer Orbit, a High Elliptic Orbit and for an eccentric Mean Earth Orbit are provided. As a conclusion, the high eccentric satellite orbit has been solved in an analytical way, but also as educational.

Fractal analysis of unit-time map and cyclicity of nilpotent singularities of planar vector fields

Lana Horvat Dmitrovic
University of Zagreb, Croatia
lana.horvat@fer.hr
Vesna Zupanovic

This article shows how fractal analysis of the unit-time map can be used in studying the cyclicity problem of nilpotent singularities. We study fractal properties such as box dimension and ε-neighbourhood of discrete orbits generated by the unit-time map. In the case of bifurcations of non-hyperbolic singularities, e.g. saddle-node or Hopf-Takens bifurcation, there is already known connection between the multiplicity of singularity and the

box dimension of the unit-time map or Poincaré map. We will also use previously known connection between the multiplicity of fixed points and the growth of ε-neighbourhoods of orbits. In this article we study how the ε-neighbourhood and box dimension of discrete orbits generated by the unit-time map near nilpotent singularities are connected to the known bounds for local cyclicity of singularities. In this analysis, we use the restriction of the unit-time map on the separatrices or characteristic curves, depending on the type of singularity. Main nilpotent singularities which are studied here are nilpotent node, focus and cusp. Moreover, we study fractal properties of the unit-time map for nilpotent singularities at infinity.

Global stability of a fixed point in Kolmogorov systems

Zhanyuan Hou
London Metropolitan University, England
z.hou@londonmet.ac.uk

In this paper, the split Lyapunov method for global stability of autonomous Lotka-Volterra systems is extended to system $\dot{x} = D(x)F(x)$, where $x \in \mathbf{R}_+^N$, $F : \mathbf{R}_+^N \to \mathbf{R}^N$ is at least C^1 and $D(x) = \text{diag}[x_1, \ldots, x_N]$. By using the positive definite property of an $(N-1) \times (N-1)$ variable matrix, criteria are established for an interior or boundary fixed point of the system to be globally asymptotically stable in forward or backward time with respect to certain set.

Transitions in a Logistic Population Model incorporating an Allee Effect

Majda Idlango
Al Jabal Al Gharbi University, Libya
Majda.Idlango@gmail.com
J J Shepherd, J A Gear

When describing the natural growth of single species populations, we note that some populations grow faster at high population density than at lower population density. In fact, it can occur that at low enough population densities, the population may decline, even to extinction. This phenomenon is well-known as an Allee effect. In most models describing this effect, the model parameters are constants, and the evolving populations show either a convergence to a nonzero limiting state (the carrying capacity), or to zero (extinguishment). However, when regular environmental changes are present, the problem parameters may be considered as slowly varying functions of time. In such cases, multitiming techniques allow us to construct leading order approximations to the evolving population in survival and extinction cases . With such environmental fluctuations, the possibility arises that the population may, at some point, move from a survival state to an extinction state, which can be termed a transition point In this paper, we analyse the specific case of a survival to extinction transition, for a logistic population model exhibiting an Allee effect. We show how the multiscaling approach can be combined with local asymptotic analysis at points where transition occurs, leading to an overall expression for the evolution of the population through this transition. We show good comparisons of these results with the results of numerical computations.

Trace formula of indefinite problem with one simple turning point

Aliasghar Jodayree Akbarfam
University of Tabriz, Iran
akbarfam@yahoo.com
Mahnaz Shahabie Oskoee, Hero Hassanpour

We consider the Dirichlet eigenvalue problem associated with the real weighted Sturm-Liouville equation

$$y'' + q(x)y = \lambda r(x)y$$

on the finite interval $[0, \pi]$. This eigenvalue problem will be called one turning point case provided the wieght function $r(x)$ contains one zero on $[0, \pi]$.
In this paper, we derive the regularized trace formula of indefinite Sturm-Liouville problem with one turning point for the Dirichlet boundary conditions.

Solving of two-point boundary value problem for loaded differential equations with impulse effect

Zhazira Kadirbayeva
Institute of Mathematics and Mathematical Modeling, Kazakhstan
apelman86pm@mail.ru

A linear two-point boundary value problem for loaded differential equations with impulse effect is considered on the interval $[0, T]$

$$\frac{dx}{dt} = A_0(t)x + \sum_{j=1}^{m} A_j(t)x(\mu_j) + f(t), \quad t \in (0,T) \backslash \{\theta_1, \theta_2, ..., \theta_m\}, \tag{1}$$

$$B_0 x(0) + C_0 x(T) = d, \quad d \in \mathbb{R}^n, x \in \mathbb{R}^n, \tag{2}$$

$$B_i \lim_{t \to \theta_i - 0} x(t) - C_i \lim_{t \to \theta_i + 0} x(t) = \varphi_i, \quad \varphi_i \in \mathbb{R}^n, i = \overline{1, m}. \tag{3}$$

The parametrization method is applied to research and solving of problem (1) - (3). The interval is divided into parts and additional parameters as values of the solution in the initial points of subintervals are entered. System of the linear algebraic equations concerning parameters is made on matrices of the loaded member, boundary condition and condition of impulse effect. Coefficients and the right part of system are defined by solutions of Cauchy problems for linear ordinary differential equations. Values of unknown function in the initial points of subintervals are found by solving of the system. Cauchy's problems for the ordinary differential equations on subintervals are solved by Runge-Kutta 4th order method.

Spectral parameter power series and transmutations for solving spectral problems

Vladislav Kravchenko
CINVESTAV, Mexico
vkravchenko@math.cinvestav.edu.mx

The spectral parameter power series (SPPS) method [1] is discussed together with some of its applications to solution of spectral problems for Sturm-Liouville equations. Its recently established relation [2] to transmutation operators is presented together with a powerful numerical technique based on the relation and developed in [3].

References

[1] V. V. Kravchenko, R. M. Porter Spectral parameter power series for Sturm-Liouville problems. Mathematical Methods in the Applied Sciences, 2010, v. 33, 459-468.

[2] H. Campos, V. V. Kravchenko, S. M. Torba Transmutations, L-bases and complete families of solutions of the stationary Schrodinger equation in the plane. Journal of Mathematical Analysis and Applications, 2012, v. 389, 1222–1238.

[3] V. V. Kravchenko, S. M. Torba Analytic approximation of transmutation operators and applications to highly accurate solution of spectral problems. Submitted, available at arxiv.org.

Locally adaptive regularisation for a meteorological inverse problem

Erik Lange
German Weather Service (DWD), Germany
erik.lange@dwd.de
Roland Potthast

Satellite radiances are used to probe temperature and humidity of the atmosphere. Based on radiative transfer models temperature and humidity in some atmospheric column are mapped into a spectrum of infrared radiations by a forward operator $\vec{r} = \vec{\mathcal{H}}\,[\vec{\varphi}]$. This leads to an *inverse problem*, which is *non-linear* and *severely ill-posed.*

We employ a weighted *Tikhonov regularisation* to invert the linearised equation, minimising:

$$J(\vec{\varphi}) \equiv \alpha\delta\vec{\varphi}^T\vec{B}^{-1}\delta\vec{\varphi} + \left[\vec{H}\delta\vec{\varphi} - \delta\vec{r}\right]^T\vec{R}^{-1}\left[\vec{H}\delta\vec{\varphi} - \delta\vec{r}\right]$$

with $\delta\vec{\varphi} \equiv \left[\vec{\varphi} - \vec{\varphi}^{(b)}\right]$ and $\delta\vec{r} \equiv \left[\vec{r} - \vec{r}^{(b)}\right]$, weight matrices $\vec{B}$ and $\vec{R}$, and regularisation parameter $\alpha > 0$, where $\vec{\varphi}^{(b)}$ is some given background profile.

For the real-data problem we develop a *locally adaptive regularisation*, where a family of global reconstructions, with different regularisation parameters, is calculated and the final reconstruction is derived from this family by a locally adaptive choice of the solution according to a *local optimality criterion.*

In this application the adaptive regularisation is better than any of the global reconstructions. Further, we derive a method to select subsets of the data (called *channel selection*) such that the instability of the inversion is minimised while the quality of reconstructions is maintained.

Dirichlet problem for differential equations involving dry friction

Hana Machu
Palacky University Olomouc, Czech Rep
machu.hana@gmail.com
Jan Andres

The effective criteria, in terms of coefficients a, b, c, will be presented for the solvability ot the following problem:

$$(D)\ \begin{cases} x'' + a\,x' + b\,x + c\,\mathrm{sgn}(x') = f(t, x, x'), \\ x(0) = 0,\ x(1) = 0. \end{cases}$$

Besides the existence, the localization of solutions to (D) will be established as well. Since our approach is based on Filippov's regularization of the discontinuous function $\mathrm{sgn}\,y$, and a subsequent application of Ky Fan's fixed point theorem to Kakutani-type multivalued operators, the right-hand side f need not be single-valued. For instance, it can be either Marchaud (i. e. globally u. s. c with convex, compact values) or upper-Caratheodory.

Inverse Problem of Astrodynamics

Yuri Menshikov
Dnepropetrovsk University, Ukraine
Menshikov2003@list.ru

The Le Verrier and Adams inverse problem is considered in this work as measurement inverse problem. This problem was reduced to solution of the same integral Volttera equations of first kind with inexact kernels. To obtain stable solutions of this incorrect problem the Tikhonov regularization method is used. An estimation of exact solutions of measurement inverse problems was suggested instead of finding approximate solutions. Special hypothesis was used to implement such approach to exact solutions. In the current work conditions at which the estimations of exact solutions exist were obtained. For obtaining more accurate estimations a method of choice of special operator in the governing equation of the problem was suggested. It was proved that this operator exists for any initial data of observations.

Numerical simulation of a SIS epidemic model based on a nonlinear Volterra integral equation

Eleonora Messina
University of Napoli "Federico II", Italy
eleonora.messina@unina.it

We consider a SIS epidemic model based on a Volterra integral equation and we compare the dynamical behavior of the analytical solution and its numerical approximation obtained by direct quadrature methods. We prove that, under suitable assumptions, the numerical scheme preserves the qualitative properties of the continuous equation and we show that, as the stepsize tends to zero, the numerical bifurcation points tend to the continuous one.

Generalized synchronization in a system of several non-autonomous oscillators coupled by a medium

Goncalo Morais
ISEL, Portugal
gnupost@gmail.com
Rogério Martins

An abstract theory on generalized synchronization of a system of several oscillators coupled by a medium is given. By generalized synchronization we mean the existence of an invariant manifold that allows a reduction in dimension. The case of a concrete system modelling the dynamics of a chemical solution on two containers connected to a third container is studied from the basics to arbitrary perturbations. Conditions under which synchronization occurs are given. Our theoretical results are complemented with a numerical study.

Uniform asymptotic stability implies exponential stability for half-linear differential systems with time-varying coefficients

Masakazu Onitsuka
Okayama University of Science, Japan
onitsuka@xmath.ous.ac.jp

We consider a two-dimensional half-linear differential system. In the special case, the half-linear system becomes the linear system $\mathbf{x} = A(t)\mathbf{x}$ where $A(t)$ is a 2×2 continuous matrix and $\mathbf{x}$ is a 2 dimensional vector. It is well-known that the zero solution of the linear system is uniformly asymptotically stable if and only if it is exponentially stable. However, in general, uniform asymptotic stability is not equivalent with exponential stability in the case of nonlinear systems. Despite the fact that the half-linear system is nonlinear, will uniform asymptotic stability guarantee exponential stability? In this talk, we give the answer to this question.

More is less: Improving connections leads to network failure

Jan Philipp Pade
HU Berlin, Germany
pade@math.hu-berlin.de
Tiago Pereira

We study synchronisation in directed networks of differential equations with interaction akin to diffusion. In many important applications the stability of synchronised states is vital for the functioning of the network. We show that, in contrast to undirected networks, the improvement of the coupling structure, for instance the introduction of a new edge, can lead to a network failure. Furthermore, we relate this effect to topological properties of the underlying digraph.

On the derivation of auto-Backlund transformations

Andrew Pickering
Universidad Rey Juan Carlos, Spain
andrew.pickering@urjc.es
P. R. Gordoa

Auto-Backlund tranformations are of use in the generation of solutions of ordinary differential equations. We show how, for certain classes of equation, auto-Backlund transformations may be derived. Examples include both scalar and matrix ordinary differential equations. Our approach is applicable not only to ordinary differential equations but also to other classes of equation such as differential-delay equations.

Asymptotic and Oscillatory Behaviour of the Solutions of a Difference Equation with Several Arguments

Sandra Pinelas
Academia Militar, Portugal
sandra.pinelas@gmail.com

In this paper, we study the asymptotic and oscillatory behavior of the solutions of a difference equation of the form

$$\Delta x(n) + \sum_{i=1}^{m} p_i(n)\, x(\tau_i(n)) = 0$$

where $(p_i(n))_{n\geq 0}$, $1 \geq i \geq m$ are sequences of real numbers, $(\tau_i(n))_{n\geq 0}$, $1 \geq i \geq m$ are sequences of integers such that

$$\tau_i(n) \leq n-1 \text{ for } n \in \mathbb{N} \text{ and}$$
$$\lim_{n\to+\infty} \tau_i(n) = +\infty,\ 1 \geq i \geq m$$

and Δ denotes the forward difference operator $\Delta x(n) = x(n+1) - x(n)$.

Oscillation Criteria For Nonlinear Neutral Differential Equations with Mixed Arguments

Sandra Pinelas
Academia Militar, Portugal
sandra.pinelas@gmail.com

This paper deals with the oscillation criteria for nonlinear neutral mixed type differential equations of the form

$$((x(t) + ax(t-\tau_1) - bx(t+\tau_2))^{\alpha})^{(n)} = q(t)x^{\beta}(t-\sigma_1) + p(t)x^{\gamma}(t+\sigma_2),$$

$$((x(t) - ax(t-\tau_1) + bx(t+\tau_2))^{\alpha})^{(n)} = q(t)x^{\beta}(t-\sigma_1) + p(t)x^{\gamma}(t+\sigma_2)$$

and

$$((x(t) + ax(t-\tau_1) + bx(t+\tau_2))^{\alpha})^{(n)} = q(t)x^{\beta}(t-\sigma_1) + p(t)x^{\gamma}(t+\sigma_2)$$

where n is a positive integer, a and b are nonnegative constants, τ_1, τ_2, σ_1 and σ_2 are positive real constants, $q(t), p(t) \in C([t_0,\infty),(0,\infty))$ and α, β and γ are ratios of odd positive integers with $\beta, \gamma \geq 1$.

Bifurcation without parameters in circuits with memristors: A DAE approach

Ricardo Riaza
Universidad Politécnica de Madrid, Spain, Spain
ricardo.riaza@upm.es
Ignacio García de la Vega

The recent design of a nanoscale memristor has had a great impact in electronics [4]. Nonlinear circuits including this and other related devices systematically exhibit non-isolated equilibrium points [3]. Therefore, bifurcation analyses in this context are naturally framed in the theory of bifurcations without parameters of Fiedler, Liebscher and Alexander [1]. In this talk we will provide a circuit-theoretic characterization of certain bifurcation phenomena; to do so we will need to reformulate first some theoretical results in the framework of differential-algebraic equations (DAEs) [2].

References

[1] B. Fiedler, S. Liebscher, and J. C. Alexander, Generic Hopf bifurcation from lines of equilibria without parameters: I. Theory, J. Differential Equations 167 (2000) 16-35.

[2] R. Riaza, Differential-Algebraic Systems, World Scientific, 2008.

[3] R. Riaza, Manifolds of equilibria and bifurcations without parameters in memristive circuits, SIAM J. Appl. Math. 72 (2012) 877-896.

[4] D. B. Strukov, G. S. Snider, D. R. Stewart and R. S. Williams, The missing memristor found, Nature 453 (2008) 80-83.

On the solution of equations in metric spaces

Rosana Rodríguez-López
University of Santiago de Compostela, Spain
rosana.rodriguez.lopez@usc.es

Some classical results are reconsidered in the context of dynamical systems defined on metric spaces. In particular, for this kind of systems, we construct certain approximate solutions and give an estimation for the distance between two of them which leads to the right-uniqueness in the lipschitzian case. Moreover, we deduce the existence of solution through a certain initial condition for the case of a complete space. For locally lipschitzian functions, we also discuss the global existence and uniqueness of solution. This extension of classical results in the field of differential equations in Banach spaces is useful in contexts such as fuzzy differential equations or even equations in metric spaces which are neither finite dimensional nor locally compact.

Lumpability of Differential Equations on Banach Spaces

Lavinia Roncoroni
Max Planck Institut, Germany
laviniaroncoroni@hotmail.it
Fatihcan Atay

We study the lumpability of linear differential equations on Banach spaces, namely, the possibility of projecting the dynamics by a linear reduction operator onto another (smaller, or simpler) state space in which a self-contained dynamical description exists. For systems whose evolution is described by a bounded linear operator, the lumpability condition is expressed as the invariance of the kernel of the reduction operator under the evolution operator. On the other hand, for systems defined by unbounded operators, additional conditions turn out to be needed, which are derived using methods from the theory of strongly continuous semigroups. Furthermore, the relation between lumpability and some key concepts in control theory, such as observability, are discussed. Some applications to particular systems are also indicated, including to delay differential equations.

Nehari solutions for superlinear boundary value problems

Felikss Sadirbajevs
Institute of Mathematics, University of Latvia, Latvia
felix@latnet.lv
Armands Gritsans

The Nehari solutions are those solutions of the boundary value problem $\{x(a) = 0 = x(b),\ x(t)$ has exactly n zeros in $(a, b)\}$ for a superlinear the second order ordinary differential equation which minimize the functional $\int_a^b x'^2(t)\,dt$. They definitely exist for differential equations of the form $x'' = -p(t)|x|^{2\varepsilon}x$, where $\varepsilon > 0$ and $p(t) \in C([a, b], (0, +\infty))$. It may happen that there are multiple Nehari solutions for a given n and there are other (non-minimizing) solutions of the BVP. Besides the Nehari solutions possess some peculiar properties discussed in *A. Gritsans and F. Sadyrbaev, Characteristic numbers of nonautonomous Emden-Fowler type equations, Math. Modelling and Analysis, vol. 11 (2006), 243 - 252* and Ryuji Kajikiya, Non-even least energy solutions of the Emden-Fowler equation. Proc. Amer. Math. Soc., Vol. 140, N 4, 2012, 1353-1362, where positive Nehari solutions were considered. We provide the results and computational evidences for *oscillatory* Nehari solutions.

Lie–Hamilton systems on the plane: classification, constants of motion and superposition rules

Cristina Sardon
University of Salamanca, Spain
cristinasardon@usal.es

We study *Lie–Hamilton systems* on the plane, i.e. systems of first-order differential equations describing the integral curves of a t-dependent vector field taking values in a finite-dimensional real Lie algebra of planar Hamiltonian vector fields with respect to a Poisson structure. Lie–Hamilton systems enjoy a plethora of properties, e.g. they admit their general solution expressed as a function, the so-called *superposition rule*, of a finite set of particular solutions and some constants. In obtaining their superposition rules and other properties, it is relevant the use of Poisson geometric methods and Poisson coalgebras. Lie–Hamilton systems are important because of their appearance in the physics, mathematics and biology literature. For example, they can be used to study Milne–Pinney, second-order Kummer–Schwarz, complex Riccati and Buchdahl equations, which occur in cosmology, relativity and classical mechanics; or in the investigation of Lotka–Volterra, predator-prey or growth of a viral infection models, more biologically biased. In this talk, I present recent research on Lie–Hamilton systems which is a compedium of two published papers. First, by relying on the classification of finite-dimensional real Lie algebras of planar vector fields given in [A. González-López, N. Kamran and P.J. Olver, *Proc. London Math. Soc.* **64**, 339 (1992)], we proceed to inspect which of such Lie algebras are Hamiltonian with respect to a Poisson structure. According to the definition of a Lie–Hamilton system, we obtain a complete classification of Lie–Hamilton systems on the plane. Further, we devise methods for the derivation of superposition rules, t-independent constants of motion and Lie symmetries for such Lie–Hamilton systems. Finally, we illustrate our results with several physically and mathematically motivated examples.

On the properties of solution sets for measure driven inclusions

Bianca Satco
Stefan cel Mare University, Romania
bianca.satco@eed.usv.ro

The theory of measure driven inclusions allows one to study in an unitary manner differential, difference and impulsive problems. Under less restrictive assumptions comparing to similar works, such as: [Silva, G.N., Vinter, R.B, Measure driven inclusions, JMAA 202 (1996)] or [Lygeros, J., Quincampoix, M., Rzezuchowski, T., Impulse differential inclusions driven by discrete measures, LNCS 4416 (2007)] (in particular: no Lipschitz continuity conditions) we present some properties of solution sets in the space of regulated functions. We focus on compactness and closure properties.

On the asymptotic behavior of eigenvalues of the sturm liouville operator

Yonca Sezer
Yildiz Technical University, Institutes of Natural and Applied Sciences, Faculty of Art and Scince, Turkey
ysezer@yildiz.edu.tr

Let L be operator which is formed by the differential expressions

$$\ell(y) = -(p(x)y'(x))' - q(x)y(x)$$

in the space $L_2[0,\infty)$, with the boundary condition $y(0) = 0$. In this work, asymptotic expression of the sum of squares of negative eigenvalues of the L operator has been investigated.

Generalized elementary functions

Antonín Slavík
Charles University in Prague, Czech Rep
slavik@karlin.mff.cuni.cz
Giselle Antunes Monteiro

We use the theory of generalized linear differential equations to introduce new definitions of the exponential, hyperbolic and trigonometric functions. We present some basic properties of these generalized functions, and show that the time scale elementary functions with Lebesgue integrable arguments represent a special case of our definitions.

A differential equations approach to implicit systems in arbitrary dimension

Dan Tiba
Institute of Mathematics, Bucharest, Romania
dan.tiba@imar.ro

We consider general systems of continuously differentiable implicit functions, in arbitrary dimension, and we construct a general parametrization procedure for the manifold giving their solution. In the critical case, when the usual nondegeneracy hypothesis is not fulfilled, we define the notion of generalized solution, prove its existence and some properties. The case of dimension two or three is particularly important and is based on the use of certain Hamiltonian systems. The extension to higher dimension is indicated as well. Besides its intrinsic interest, our main motivation comes from shape optimization applications (fixed domain methods).

Transmutation operators and complete systems of solutions of variable mass Klein-Gordon equations

Sergii Torba
CINVESTAV del IPN, Mexico
storba@math.cinvestav.edu.mx

An operator T is called a transmutation operator for pair of operators A and B if it is continuous and continuously invertible on a suitable topological space and satisfy the operator equality $AT = TB$. In the case when $A = -\partial^2 + q(x)$ and $B = -\partial^2$, a transmutation operator T can be realized in the form of the Volterra integral operator

$$Tu(x) = u(x) + \int_{-x}^{x} K(x,t)u(t)dt$$

with the integral kernel K satisfying the particular Goursat problem, whose exact solution is possible known only for some particular potentials. We propose another possible way to with the transmutation operators specifying the result of the action of the operator T on the powers of independent variable,

$$T[x^k] = \varphi_k(x), \quad k \in \mathbb{N}_{\not{\prime}},$$

where the functions φ_k are obtained by a simple procedure involving recursive integration and a particular solutions of the equation $-v'' + q(x)v = 0$. We construct a complete system of solutions of the variable mass Klein-Gordon equation

$$\Box u(x,t) - q(x)u(x,t) = 0$$

by applying the transmutation operator to the wave polynomials. An efficient numerical method for solving Cauchy problems for this Klein-Gordon equation is presented. The talk is based on a joint work with K. Khmelnitskaya, V. Kravchenko and S. Tremblay.

Non-almost periodic solutions of limit periodic and almost periodic systems

Michal Vesely
Department of Mathematics and Statistics, Masaryk University, Brno, Czech Rep
michal.vesely@mail.muni.cz
Petr Hasil

Limit periodic and almost periodic homogeneous linear difference systems are studied. The coefficient matrices of the considered systems belong to a given matrix group. We find a condition on the group under which the systems, which have non-zero (asymptotically) almost periodic solutions, form a dense subset in the space of all considered systems. The treated problem is discussed for the elements of the coefficient matrices from an arbitrary infinite field with an absolute value. Nevertheless, the presented results are new even for the field of complex numbers.

Fractal dimensions of oscillatory integrals

Domagoj Vlah
University of Zagreb, Croatia
domagoj.vlah@fer.hr
Jean-Philippe Rolin, Vesna Zupanovic

It is known that the asymptotics of oscillatory integrals has been related to singularities of the phase function. Abelian integrals and Fresnel integrals are well known classes of oscillatory integrals. Motivated by geometrical interpretation of Fresnel integrals by a spiral called the clothoid, we continue investigation concerning geometrical approach to oscillatory integrals. Using the point of view of fractal geometry, we consider oscillatory integrals depending on one parameter t. Oscillatority is measured by the box dimension of the plane curve parameterized by $x(t)$ and $y(t)$ that are the real and imaginary part of the integral, respectively. Also, the oscillatory dimension is defined as the box dimension of the graph $X(\tau) = x(1/\tau)$, near $\tau = 0$, where $x(t)$ for $t > 0$ is the graph of the real part of the integral. Analogously for $y(t)$ and the imaginary part of the integral. We investigate the connection between these dimensions and asymptotics of the integral.

Contributed Session 2: Modeling, Math Biology and Math Finance

2D and 3D-dynamic Representations of DNA Sequences

Dorota Bielinska-Waz
Medical University of Gdansk, Poland
djwaz@gumed.edu.pl
Piotr Waz

Methods known in the literature as *Graphical Representations of DNA Sequences* combine ideas from different areas of science. They allow for both graphical and numerical comparisons of DNA sequences. Contrary to frequently used methods based on the alignment of the sequences, graphical representations allow to consider each aspect of similarity separately. Another advantage of these methods is their low computing time. In this work we present two graphical representation methods: 2D- [1, 2, 3, 4] and 3D- [5] *Dynamic Representations of DNA Sequences*. A specific feature of these approaches is using the ideas of the classical dynamics. The issues related to the choice and the properties of the *descriptors*, i.e. the numerical characteristics of the graphs, are also discussed.

References

[1] D. Bielińska-Wąż, T. Clark, P. Wąż, W. Nowak, A. Nandy, *2D-dynamic Representation of DNA Sequences*, Chem. Phys. Lett. **442** (2007) 140-144.

[2] D. Bielińska-Wąż, W. Nowak, P. Wąż, A. Nandy, T. Clark, *Distribution Moments of 2D-graphs as Descriptors of DNA Sequences*, Chem. Phys. Lett. **443** (2007) 408-413.

[3] D. Bielińska-Wąż, P. Wąż, T. Clark, *Similarity Studies of DNA Sequences Using Genetic Methods*, Chem. Phys. Lett. **445** (2007) 68-73.

[4] P. Wąż, D. Bielińska-Wąż, A. Nandy, *Descriptors of 2D-dynamic Graphs as a Classification Tool of DNA Sequences*, J. Math. Chem. **52** (2014) 132-140.

[5] P. Wąż, D. Bielińska-Wąż, *3D-dynamic Representation of DNA Sequences*, J. Mol. Model. **xx** (2014) xx-xx, DOI: 10.1007/s00894-014-2141-8.

Global stability and oscillations for logistic type equations with discrete delay and asymptotically periodic suppression rate

Marek Bodnar
University of Warsaw, Poland
mbodnar@mimuw.edu.pl
Urszula Foryś, Monika J. Piotrowska

We discuss a single species dynamics governed by the logistic type delayed equation with an external suppression of the population size. We allow this external influence to be time dependent. In result, we obtain a non-autonomous equation. We study the asymptotic behaviour of the solutions, and derive conditions guaranteeing global stability of the zero steady state. We prove these conditions are necessary and sufficient for a periodic or asymptotically periodic suppression rate. Moreover, if the external influence is periodic and the zero steady state is repulsive, the existence of a periodic solution is shown. We also compare the results obtained for the non-autonomous case with known stability results for autonomous case. From the applications point of view, the logistic type equations are often used in the description of underlying tumour dynamics. In this case the external influence reflects the tumour treatment and the global stability of the zero steady state can be interpreted as the cure.

Mathematical models for dynamics of low grade gliomas and its response to radio- and chemotherapy

Magdalena Bogdanska
Uniwersytet Warszawski, Wydzial Matematyki, Informatyki i Mechaniki, Poland
m.bogdanska@mimuw.edu.pl
Victor Perez-Garcia, Marek Bodnar, Alicia Martinez-Gonzalez, Juan Belmonte-Beitia, Philippe Schucht, Luis Perez- Romasanta

Low grade gliomas (LGGs) is a term used to describe WHO grade II tumors of glial origin. They are highly infiltrative and proliferate slowly, but are generally incurable. Despite therapies used patients with LGGs eventually die because of transformation of LGGs into more aggressive, anaplastic forms. Treatment of LGGs brings many controversies to clinicians because LGGs usually occur in young and otherwise healthy patients. Therefore an object of treatment is not only to prolong time of survival, but also to delay minimiseside effects of aggressive therapies in such a way that patients maintain good quality of life as long as possible. We propose macroscopic models, based on Fisher-Kolmogorov equation, of growth of glioma and its response to main therapies used nowadays: radio- and chemotherapy. This strategy already gave very promising results and good agreement with experimental data for radiotherapy. We present the models for both types of therapies, and we show how our models reproduce main effects observed by clinicians. Due to obtained qualitative conformity with available medical data, it is possible to derive ideas that could be relevant in medical practice.

Comparison of Classification Methods; Kernel Discriminant Analysis, Artificial Neural Networks and Logistic Regression for Predicting Grasshopper Population

Isaias Chairez Hernandez
IPN CIIDIR Durango, Mexico
ichairez@ipn.mx
Tonghui Wang, J. Natividad Gurrola Reyes and Cipriano Garcia Gutierrez

Performance of multiple nonparametrics discriminant analysis; kernel discriminant analysis (KDA), logistic regression (LR) and artificial neural networks (ANN) were compared in predicting quartiles (QRT) and threshold (TRS) categories of grasshopper per square meter (training 33%, testing 33% and forecasting set 34%). Data was generated with generalized linear models from surveyed data in Durango México where grasshopper population per square meter was the dependent variable and ecological variables the independents. Percent classification error, received operating characteristic value and Pearson correlation coefficients among interpolated and predicted values were used to assess the model performance. Results showed that the generalized linear model that best fitted survey data was Negative binomial with the natural logarithm as a link function and R^2=0.91. Results established that ANN model, had the best performance in the TRS category, followed by KDA and LR. Grasshopper is part of the food chain in grassland ecosystems but in special ecological conditions the population increases and may cause damage to grassland and crops. Both ANN and KDA are efficient methods for population threshold predictions. Variable importance analysis showed that longitude, soil and precipitation were the variables that most influenced grasshopper per square meter.

A percolation approach to desertification transitions

Anna Maria Cherubini
Università del Salento, Italy
amcherubini@gmail.com
Raffaele Corrado and Cecilia Pennetta

We present the results of a theoretical study on the desertification transition in semi-arid ecosystems, aimed at identifying early transition indicators related to the time fluctuations of the vegetation patterns. To this purpose we performed numerical simulations based on a stochastic cellular automaton model for semi-arid ecosystems recently introduced in literature and we analysed the results in terms of percolation theory. For increasing values of the mortality rate m, related to the strength of external stresses acting on the ecosystem, we could follow its increasing degradation through different stages, characterised by different connectivity properties, up to the critical transition corre-sponding to the extinction of the vegetation and almost complete degradation of the soil, oc-curring at a critical value m_c. In order to find suitable indicators and precursors, we calculated the spanning probabilities and the percolation thresholds as functions of m according to different spanning criteria and we studied the time fluctuations of the sizes of the biggest vegetation cluster and the biggest non-vegetated cluster over the range of mortality values.

The role of network structure on brain activation dynamics

Jonathan Crofts
Nottingham Trent University, England
jonathan.crofts@ntu.ac.uk
Yi-Ping Lo, Reuben O'Dea, Cheol Han and Marcus Kaiser

Brain diseases such as epilepsy can be characterised by recurrent unpredictable instances of excessive or synchronous neuronal activity (seizures). Recent studies suggest that network topology is the primary factor affecting seizure initiation and propagation; however, the majority of theoretical studies in epilepsy employ uniform lattices in 1D or 2D with nearest-neighbour network connectivity, or with additional long-range connections obeying arbitrary rules (a small number of recent studies used DTI data to inform long-range connections). However, the cortex of the human brain is a highly convoluted 3D structure. Recently, we have shown that the cortical geometry of a rat brain influences spreading speed. Here, we define a neuronal network which represents accurately the *human cortex*, in order to study its influence on seizure dynamics. In this talk we restrict attention to a simple spreading model and nearest-neighbour connections, that is we limit our study to spreading on the brains surface – white matter fibre tracts between brain regions are not considered. Our results indicate that short-range connectivity associated with cortical structure, which we quantify by global and local gyrification index, influences significantly network synchronisation. Additionally, we indicate how our conclusions apply to a large cohort study, employing the enhanced NKI-Rockland Dataset.

Reaction-Diffusion Patterns on Networks

Isaac Donnelly
University of New South Wales, Australia
i.donnelly@unsw.edu.au
Bruce I. Henry, Chris N. Angstmann, Trevor A.M. Langlands

Motivated by the recent explosion of network science, we derive a generalized master equation for reaction-diffusive systems on a network. We also allow for time- and vertex-dependent forcing and edge weights. We use the Continuous Time Random Walk as a model for the underlying stochastic process of a particle. This allows for a range of Laplacian diffusion operators, including the standard graph Laplacian. This freedom of choice of Laplacian is a feature unique to diffusion on a network. We specifically investigate diffusive Turing patterns on a network and show hot spots occur, akin to spots in the continuum. Furthermore, in the case of subdiffusion, the patterns are sensitive to the anomalous exponent of the fractional diffusion operator. The structure of the network is also a significant component in the observed patterns. We investigate applications including the development of prion-like diseases in the brain, and the progression of global epidemics on the network.

A model for erythropoiesis adapted to individuals

Doris Fuertinger
Renal Research Institute, USA
doris.fuertinger@rriny.com
Franz Kappel

Treatment of anemia is a complicated but essential part in the care of patients suffering from end-stage renal disease. Individualization of anemia therapy is desirable. As a first step we adapt a comprehensive mathematical model describing erythropoiesis to individual hemodialysis patients. The reproduction of red blood cells is described using age-structured population equations. Parameters with high interindividual variability are chosen to be estimated using data that was gathered in a standard clinical setting. Limitations of the data available - frequency, measurement errors, quantities that can be measured - pose some difficulties for parameter identification. The approach chosen for parameter estimation is a discretize-optimize strategy. Hence, it is important that the numerical approximations preserve some characteristics of the original system. The approximation scheme used involves evolution semigroups and is based on the theory of abstract Cauchy problems. The performance of various cost-functionals incorporating several weighting functions is investigated for different lengths of sampling intervals. An appropriate choice of a combination of cost-functionals can reduce the observation time needed for parameter identification significantly.

References
[1] Fuertinger, D.H., Kappel, F., Thijssen, S., Levin, N. and Kotanko, P.: A model of erythropoiesis in adults with sufficient iron availability. J. Math. Biol. 66 (6), 1209 – 1240 (2013).

Mathematical modeling of risk preferences with application to portfolio optimization

Cristinca Fulga
Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Romania
fulga@csie.ase.ro

In this paper we address two issues considered as weaknesses of the Mean-Variance, respectively Mean-Risk, portfolio selection models: first is related to the amount of significant information contained in the return distributions, but ignored during the decision process, and second is the implicit assumption of neutrality at risk of the individual investor. We work with the forecast-ed empirical return distribution function; thus the higher moments of the return distributions and implicitly the information contained in them are not neglected. We propose two types of portfolio selection models incorporating the individual preferences: in the objective related to the portfolio return, respectively in the risk related objective. We evaluate the differences and the similarities between the three efficient frontiers corresponding to the two proposed models and the classical Mean-Variance model from three perspectives: firstly, we establish the extent to which the same securities appear in the models and the proportions they are present in each frontier, secondly, we calculate and compare the higher moments of efficient portfolios and lastly, we use two performance measures to compare the efficient portfolios. Our study shows that, by incorporating preferences, the Mean-Risk models provide efficient portfolios with better performances.

Dynamical interaction between cancer tumor and immune system of the organism

Victor Gotlib
Holon Institute of Technology, Israel
gotlib@hit.ac.il
R. Goot, G. Zharinov and E. Yakubov

2D dynamical system as model of interaction between cancer tumor and immune system of the organism is elaborated and investigated using qualitative methods of phase plane. The model basically is a competition-interaction one and consists of two ordinary differential equations with highly nonlinear right-hand sides. The right side is composed of exponential growth of the tumor and continuous piecewise differentiable functions for the organism immune system reaction. Such consumptions are reasonable from biological and medical point view on cell kinetics. The phase portrait of the system commonly contains two stationary points, depending on the system parameters. One of the points is of the saddle type and, placing relatively far from the biological realm. The second one may be varied as a spiral point or structurally stable centre point, being encircled by a limit cycle. It is shown that the system behavior in biological realm can go on 4 qualitatively different modes that correspond to 4 different phase portraits, which ones reflect the specific disease scenario.

Ecological Traps: Modeling habitat selection for mobile animals in heterogeneous landscapes

Jessica Hearns
University of Central Florida, USA
hearnsj@knights.ucf.edu
Lina Maria Sanchez Clavijo and Pedro Francisco Quintana-Ascencio

We used a research framework including sources, sinks, and ecological and perceptual traps to evaluate the role that habitat selection plays in native species adaptation to transformed landscapes. We built a simulation model in MATLAB to evaluate the correspondence between habitat selection and quality with forest generalist birds in tropical agroforestry plantations. We distinguished between three hypothesized possible roles for habitat selection (neutral, adaptive or maladaptive) and created a spatially-explicit population model to explore the consequences of the interplay between habitat selection, quality and availability. We use a novel and multi-scale methodological approach to link landscape structure, individual behavior, and population outcomes of mobile animals in heterogeneous landscapes. We tested theoretical predictions of habitat selection models using a study system to prompt generally applicable lessons for management and conservation. We found clear differences between scenarios of neutral, adaptive and maladaptive habitat selection consistent with current theoretical predictions and evaluated vital rates changes associated with these scenarios. The largest effects were associated with changes occurring during the transient demographics before all territories are occupied. We were able to evaluate how previous methodological limitations are hindering our ability to explore the ecological mechanisms allowing species persistence in anthropogenic and disturbed regions.

The persistence of competing plant species in a fire-dominated habitat: a mathematical model

Jessica Hearns
University of Central Florida, USA
hearnsj@knights.ucf.edu
Andrew Nevai, Pedro Francisco Quintana-Ascencio, and Eric Menges

We constructed a nonlinear dynamical system of difference equations to emulate the effect of different spatial arrangements and fire frequencies with several dominant Florida scrub species and their priority effect on the persistence of three subordinate herbaceous species. We present some stability analysis on the system however, due its complexity, we resort to numerical simulations in MATLAB which offers the benefit of exploring stochastic spatial arrangements and several fire frequency intervals intertwined with many competitive biological characteristics of Florida scrub, including allopathy. We are interested in persistence of various herbaceous species under spatial structure and fire frequency influences. In order to assess the persistence of the herbs, we vary managerial parameters that are not fixed by the biology of the plants: spatial structure of other scrub species, fire type (uniform, random, and kernel-based) and fire frequency (5, 10, or 15 year intervals). For each combination, we run simulations where one herb seed is randomly intro-

duced to the habitat. By defining herb persistence as an herb's ability to maintain a seedbank, we present several results. Many of these results are indicative of the field data. This study provides a solid framework under which suggestions for land managers can be made.

A mathematical model of skeletal muscle disease and immune response in the mdx mouse

Abdul Jarrah
American University of Sharjah, United Arab Emirates
ajarrah@aus.edu
Filippo Castiglione, Nicholas P. Evans, Robert W. Grange, and Reinhard Laubenbacher

Duchenne Muscular Dystrophy (DMD) is a genetic disease that results in the death of affected boys by early adulthood. In this talk, we present a simple mathematical model to investigate the role of the immune response in muscle degeneration and subsequent regeneration in the mdx mouse model of Duchenne muscular dystrophy. Our model suggests that the immune response contributes substantially to the muscle degeneration and regeneration processes. Furthermore, the analysis of the model predicts that the immune system response oscillates throughout the life of the mice, and the damaged fibers are never completely cleared.

Modelling of two-molecular reactions on supported catalysts

Pranas Katauskis
Vilnius University, Lithuania
pranas.katauskis@mif.vu.lt
V. Skakauskas

We consider a phenomenological model (Skakauskas, Katauskis, 2014) of two-molecular reactions on supported catalysts with spillover effect in two-dimensional in space case including: the bulk diffusion of reactants from a bounded vessel towards the adsorbent and the product bulk one from the adsorbent into the same vessel, adsorption and desorption of reactants molecules, and surface diffusion of adsorbed particles. Based on the Langmuir-Hinshelwood and Eley-Rideal surface reaction mechanisms the model is described by a system of partial and ordinary differential equations that are solved numerically. Simulations were performed using the finite difference technique. Three cases of reactants adsorption are considered: (i) each reactant can adsorb on the active in reaction catalyst surface and inactive support, (ii) one of reactants adsorbs on the catalyst surface while the other one adsorbs on the support, (iii) both reactants adsorb only on the support. The surface diffusion and catalytic surface size influence on the catalytic reactivity of a supported catalyst will be discussed.

References
[1] V. Skakauskas, P. Katauskis, Spillover in monomer-monomer reactions on supported catalysts – dynamic mean-field study, J. Math. Chem., 2014, DOI 10.1007/s10910-014-0315-3.

Effect of Machining on Surface Roughness of Pinus nigra Arnold Wood

Murat Kilic
Central Anatolia Forestry Research Institute, Turkey
kilicm@hacettepe.edu.tr

In this study,three surface treatment techniques (sawing with circular saw, planing and sanding) which are most preferred in the production of wood products were used. Surface roughness values of Pinus nigra Arnold wood samples subjected to surface treatments were determined. Experimental materials were taken from çamkoru Dr. Fuat AdalıResearch Forest. Trees were treated in Sample Preparation and Technology Laboratory of Wood and Non-Wood Forest Products Division in Central Anatolia Forestry Research Institute. Surface roughness values of Pinus nigra Arnold wood samples were determined according to some varieties such as cutting direction, kind of machine.

Turing patterns on multiplex networks

Nikos Kouvaris
University of Barcelona, Spain
nikos.kouvaris@ub.edu
Albert-Diaz Guilera

Stationary Turing patterns are studied in activator-inhibitor systems organized on multiplex (multilayer) networks, where activators and inhibitors occupy nodes in different layers/networks. They diffuse within their own layer through the paths defined by the intra-connectivity architecture, while they react across the layers with their counterparts defined by the inter-connectivity architecture. This separation of links in different layers has an important influence on the underlying reaction-diffusion process. Particularly, if the ratio be- tween the mean degrees of the inhibitors and the activators layers exceeds a certain threshold, the multiplex system undergoes a Turing instability even if the rates of the diffusional mobility are the same for both species. This self-organization phenomenon has been studied numerically by simulations on the actual multiplex networks as well as analytically by employing a mean-field theory.

Strategic games in a competitive market: feedback from the users' network

Natalia Kudryashova
University of Cambridge, UK
nk375@cam.ac.uk

We propose a mathematical model for the time dynamics of a competitive market, in which strategic decision making of the market players may be affected by the users' switching behavior, so that user choices and opinions can spread over the network. The model can be adopted for a particular market and take learning into account. We demonstrate applications of the model to the internet search market in the context of law and economics, where a user centered description is essential, for instance, in the US anti-trust proceedings.

Neural approximations to self-shaping vector fields

Xinhe Liu
Loughborough University, England
x.liu@lboro.ac.uk
Natalia B. Janson

Recently a new class of dynamical systems was proposed, whose velocity vector fields are flexible and self-organising under the influence of the externally applied signals, called self-shaping dynamical systems (SSDS) [1]. Such systems could serve a mathematical prototype of human learning, including memorisation, pattern recognition, categorisation and forgetting, all occurring simultaneously and automatically. They provide an alternative to a neural network (NN) description of learning, are potentially more powerful than, and lacking the known limitations of, the standard artificial NNs. We suggest that the performance of artificial NNs could be improved if their learning style matches that of SSDS. To implement the principle of SSDS by means of a NN, the state and the vector field of a SSDS are represented by a NN, and a neural approximation using Hopfield NN with Hebbian learning is obtained.

References

[1] N. B. Janson and C. J. Marsden, Self-shaping dynamical systems and learning, arXiv:1111.4443.

Modelling and analysis of filopodia extension regulated by VEGF-Delta-Notch signalling in angiogenic tip-cell selection

Sunny Modhara
University of Nottingham, England
stxsm18@nottingham.ac.uk
Markus Owen, Helen Byrne, Etienne Farcot, Martin Gering

Angiogenesis is the process by which new blood vessels form from pre-existing ones during, for example, wound healing or organismal development. Endothelial cells (ECs) that form blood vessels respond to tissue-derived growth factors, such as Vascular Endothelial Growth Factor (VEGF), by forming capillary sprouts. The sprouts are headed by tip-cells which migrate, via chemotaxis, towards the source of the growth factors, followed by proliferative stalk cells which maintain contact with the parent vessel. The process of tip-cell selection is regulated by interactions between the VEGF signalling pathway and Delta-Notch juxtacrine signalling with neighbouring cells. Here we develop dynamic, ordinary differential equation (ODE) models to investigate how these interactions modulate VEGF-induced extension of EC filopodia and how these filopodia in turn, help a population of ECs to sprout. We begin by neglecting filopodia elongation and focus on the signalling processes alone. We identify regions of parameter space in which there exist stable, spatially homogeneous solutions (all cells are identical) and others in which alternate cells express high (low) levels of Notch activity and VEGF receptors. We use linear stability analysis to demonstrate how the strengths of feedbacks in the model influence the patterns that emerge. We then show that the inclusion of filopodia growth into the model, via a variable representing filopodia length for each cell, can generate spatial instabilities (corresponding to tip-cell selection) not present in the absence of filopodial elongation. Thus filopodia can be a crucial ingredient in tip-cell selection. On the other hand, our analysis shows that VEGF gradients (as opposed to the absolute VEGF concentration) may not be required for tip-cell pattering. The results of such modelling may have implications in anti-angiogenic therapies used in wound healing or cancer treatment, for example.

Assessing test-and-vaccinate policy against rubella using an epidemic model

Hiroshi Nishiura
The University of Tokyo, Japan
nishiurah@gmail.com

From 2012, Japan has experienced a surge of rubella cases. Due to real-time vaccination during the course of the epidemic, there has been a shortage of rubella vaccines and Japan enforced an antibody test among women at child-bearing age before deciding their vaccination. Using an epidemic model, this test-and-vaccinate policy is evaluated. Final size equation using a linearized system with variable initial conditions is derived, permitting us to calculate the incremental cost-effectiveness ratio induced by test-and-vaccinate policy-based immunization. Even during non-linear phase, the derived model is shown to yield an appropriate future epidemic size.

The role of environmental variables on Dengue transmission: Insight from a mathematical model.

Sittisede Polwiang
Silpakorn University, Thailand
joddiers@hotmail.com

Dengue fever is one of the most serious health problems among the tropical countries. Mathematical models can explain deep insight into pattern of the transmission of the viruses from vectors to human. The epidemic is varied by environmental factors such as temperature and rainfall. We used the mathematical model with include seasonal variable in parameters to evaluate the effects of environment to the spread of the epidemic. The stability of equilibrium point and reproductive number show the impact of seasonal variable to the dengue transmission. The numerical solution was used to support the theoretical results. Since the hatching, biting rate of mosquito and the transmission rate from mosquito to human and vice versa depend on the environmental variables, seasonal variation of the disease may obtain.

Gene regulatory networks with stochastic noise

Arkadi Ponossov
NMBU, Norway
arkadi@nmbu.no

We suggest an explicit algorithm which directly incorporates intrinsic stochastic noise effects into a well-established deterministic formalism for describing gene regulatory networks with step (Heaviside) regulatory functions, where the dynamics of the corresponding piecewise linear system can explicitly be described between the thresholds. The basic technical tool consists in replacing the step regulatory functions by smooth Hill functions. The stochastic perturbations can then be defined as standard independent white noises whose diffusion coefficients are assumed, just for the sake of simplicity, to solely depend on the steepness parameter of the Hill functions. Discontinuities that arise when the steepness parameter becomes infinity are handled by applying the uniform version of the stochastic Tikhonov theorem for singularly perturbed systems suggested in [1].

References

[1] Y. Kabanov, S. Pergamenshchikov (2003) Two-Scale Stochastic Systems. Asymptotic analysis and control, Springer.

A Proposed Mathematical Model of Avian Influenza A, H7N9 for China

Teerapol Saleewong
King Mongkut's University of Technology Thonburi, Thailand
juspal@hotmail.com

The novel avian influenza A H7N9 virus, identified in mid-2013, spread rapidly in Eastern China. The outbreak expanded with a total of 135 laboratory confirmed cases reported by 12 August 2013. In this study we discuss our preliminaries studies of the development of mathematical model. We used influenza A H7N9 epidemic data from World Health Organization (WHO), in conjunction with the SEIR model (susceptible, infectious, recovered) to estimate the dynamics of an epidemic. The transmission rate was obtained by fitting the model to the cumulative number of local daily infected cases using the nonlinear ordinary least squares method. The duration of infectiousness were obtained from the published literature. Our proposed model and findings provide a relevant contribution towards understanding the characteristics of this avian influenza A H7N9, to design treatment and develop control strategies such as a vaccination program or quarantine policy.

Modelling of receptor-toxin-antibody interaction

Vladas Skakauskas
Vilnius University, Lithuania
vladas.skakauskas@maf.vu.lt
P. Katauskis, A. Skvortsov, P. Gray

The plant toxin ricin made from the seeds of the castor oil plant is one of the deadliest toxins to mamalian cells. To neutralise ricin effects the application of antibodies has been proposed. The development and production of antibodies is still an expensive process. In order to reduce this cost a modelling framework has been recently proposed (Skakauskas et al 2011, Skvortsov et al 2013) which enables extensive studies to increase the protective potential of antibodies. After penetrating into the cell ricin reaches the Endoplasmic Reticulum where it is separated into two chains. One of them is released into the cytosol, subsequently reaches ribosome, and damages the protein productiom machinery of the cell resulting in the cell death. Therefore it becomes important to estimate the ricin toxicological impact on the cell and evaluate the protective potential of the antibody. We consider cases where antibody is delivered inside and outside. Mathematical models based on PDEs for each cases were proposed and studied numerically.

References

[1] Skakauskas V, Katauskis P, Skvortsov A: A reaction-diffusion model of the receptor-toxin-antibody interaction. *Theor Biol Med Model* 2011, **8**:32.

[2] Skvortsov A, Gray P: A simple model for assessment of anti-toxin antibodies. *Biomed Res Int*, **2013**:230906.

Chronotaxic systems: deterministic dynamics which may look stochastic.

Yevhen Suprunenko
Lancaster University, England
y.suprunenko@lancaster.ac.uk
P. T. Clemson and A. Stefanovska

Deterministic, complex and non-autonomous oscillatory dynamics can often be misinterpreted as being either stochastic or chaotic when observed. We show that such a misinterpretation can be avoided when these systems resist external perturbations and therefore can be described as chronotaxic. These systems were recently introduced and named chronotaxic because their dynamics are ordered in time (from chronos - time and taxis - order). Their main feature is the stability in the oscillatory amplitude and phase, which originates from separate time-dependent point attractors (driven steady states) for each coordinate. In this presentation we introduce a generalised theory of chronotaxic systems, where the amplitude and phase dynamics do not need to be separable. Thus there is a unique time-dependent point attractor (driven steady state) in the system. We apply the theory of chronotaxic systems to the study of a human cardiorespiratory system and to the membrane potential dynamics of a living cell. In both cases it is still possible to identify a non-autonomous and deterministic component in the dynamics, despite appearing complex and stochastic-like.

2D and 3D-dynamic Representations of DNA Sequences

Piotr Waz
Medical University of Gdansk, Poland
phwaz@gumed.edu.pl
Dorota Bielinska-Waz

Methods known in the literature as *Graphical Representations of DNA Sequences* combine ideas from different areas of science. They allow for both graphical and numerical comparisons of DNA sequences. Contrary to frequently used methods based on the alignment of the sequences, graphical representations allow to consider each aspect of similarity separately. Another advantage of these methods is their low computing time. In this work we present two graphical representation methods: 2D- [1, 2, 3, 4] and 3D- [5] *Dynamic Representations of DNA Sequences*. A specific feature of these approaches is using the ideas of the classical dynamics. The issues related to the choice and the properties of the *descriptors*, i.e. the numerical characteristics of the graphs, are also discussed.

References

[1] D. Bielińska-Wąż, T. Clark, P. Wąż, W. Nowak, A. Nandy, *2D-dynamic Representation of DNA Sequences*, Chem. Phys. Lett. **442** (2007) 140-144.

[2] D. Bielińska-Wąż, W. Nowak, P. Wąż, A. Nandy, T. Clark, *Distribution Moments of 2D-graphs as Descriptors of DNA Sequences*, Chem. Phys. Lett. **443** (2007) 408-413.

[3] D. Bielińska-Wąż, P. Wąż, T. Clark, *Similarity Studies of DNA Sequences Using Genetic Methods*, Chem. Phys. Lett. **445** (2007) 68-73.

[4] P. Wąż, D. Bielińska-Wąż, A. Nandy, *Descriptors of 2D-dynamic Graphs as a Classification Tool of DNA Sequences*, J. Math. Chem. **52** (2014) 132-140.

[5] P. Wąż, D. Bielińska-Wąż, *3D-dynamic Representation of DNA Sequences*, J. Mol. Model. **xx** (2014) xx-xx, DOI: 10.1007/s00894-014-2141-8.

Contributed Session 3: Control and Optimization

Stablization of systems of hyperbolic conservation laws

Mapundi Banda
Stellenbosch University, So Africa
mkbanda@sun.ac.za
Mapundi Kondwani Banda, Michael Herty

We consider a stabilisation problem with boundary controls for hyperbolic conservation laws. A tool widely used for the study of these problems are suitable Lyapunov functions. For stability of the solution to the partial differential equation, the exponential decay of a Lyapunov function can be established. Analytical tools yield exponential decay of the continuous Lyapunov function. Suitable numerical discretisation for the boundary control problems of nonlinear hyperbolic partial differential equation are presented. Using a discrete Lyapunov function, exponential decay of the discrete solution of hyperbolic equations is proved. We analyse the numerical discretisation of these results and establish the conditions under which, in a numerical scheme, an exponential decay of the discrete solution can be observed. Furthermore, explicit decay rates depending on the numerical scheme, which are in many theoretical results not given, will be investigated. Mathematical proofs for explicit decay rates, using the discrete counterpart of the Lyapunov function, will be presented. The theoretical results will be accompanied by computational results.

Invariant Solutions of Bi-Level Games with Impulsive Dynamics

Elena Goncharova
Institute for System Dynamics and Control Theory, SB RAS, Russia
goncha@icc.ru
Maxim V. Staritsyn

We address two-person bilevel games with dynamics, described by a nonlinear measure differential equation (MDE) with controlled jumps. The Leader in our game operates with nonanticipative strategies (in the Varaiya-Roxin-Elliott-Kalton sense), while the Follower plays with open-loop controls. We study the issue on viability of a trajectory to MDE in a time-varying domain defined by a given family of set-valued functions. Practically, the Follower's actions here can be thought of as external disturbances of high intensity (of a shock type). As main results, we give suffcient conditions for the Leader's victory under any addmissible action of the Follower. The work was partially supported by RFBR, projects nos 13-08-00441, 14-01-31254, 14-08-00606.

Optimal Control of Incompressible Flow Driven Systems

Teresa Grilo
FCUP, Portugal
tgrilo@fc.up.pt
Fernando Lobo Pereira and Silvio Gama

The objective of this work is to develop a mathematical framework for the modeling, control and optimization of dynamic control systems whose state variable is driven by interacting ODEs and PDEs. This framework should provide a sound basis for the design and control of new advanced engineering systems arising in many important classes of applications, some of which may involve gliders and mechanical fishes. Until now, the research effort has been focused in gaining insight by applying necessary conditions of optimality for shear flow driven dynamic control systems which can be easily reduced to problems with ODE dynamics. Currently, we are extending this methodology to the two-dimensional incompressible Navier-Stokes flow by using vortex methods. In particular, we are studying the position of a particle when it is subject to the motion of a flow created by two vortices.

On Reachability Analysis for Nonlinear Systems with State Constraint

Mikhail Gusev
Institute of Mathematics and Mechanics, Russia
gmi@imm.uran.ru

The talk concerns the problem of approximating reachable sets for a nonlinear control system $\dot{x} = f(x, u(x)), u(t) \in P$, with state constraint $x(t) \in S$, which is given as the solution set of a system of smooth inequalities $S = \{x \in R^n : g_i(x) \leq 0, \ i = 1, ..., m\}$. An analogue of the penalty function method is proposed which consists in replacing the original system with state constraints by an auxiliary system (differential inclusion) $\dot{x} \in F_k(x)$ without state constraints. The right-hand side $F_k(x)$ of the auxiliary system is obtained by means of the restriction of the set of velocities $F(x) = f(x, P)$ of the original system in the neighborhood of the boundary of S. It depends on a scalar penalty parameter k. Under some standard assumptions on f and an inward pointing condition on the boundary of S, we prove that: 1) the mapping $F_k(x)$ is Lipschitz continuous on S for sufficiently large k; 2) a reachable set of the original system is approximated in the Hausdorff metric by reachable sets of the auxiliary system as $k \to \infty$. The estimates of the rate of convergence are given.

Delayed Feedback Control of unstable periodic orbits with arbitrary delay time

Hyok Jang
University of Science Pyongyang, Korea
eckehard.schoell@tu-berlin.de
Chol-Ung Choe, Ryong-Son Kim, Philipp Hövel, and Eckehard Schöll

We suggest a delayed feedback control scheme with arbitrary delay for stabilizing a periodic orbit, while maintaining the noninvasiveness of the controller. Since the constraint on the delay to be adjusted to the period of the unstable periodic orbit is not imposed, a richer structure of the dynamics can be observed: Not only weakly unstable, but also strongly unstable periodic orbits are stabilized and even stabilization of orbits with infinite period is achieved. The control mechanism is elucidated for the generic model of a subcritical Hopf bifurcation. A complete bifurcation analysis for the fixed point as well as the periodic orbit is presented and the stability domains are identified. Furthermore, we study the effects of distributed delayed feedback on the stabilization of periodic orbits, and show that larger variance of the delay distribution considerably enlarges the stabilization region in the parameter space.

Construction on exact control for a one dimensional heat equation with delay

Denys Khusainov
Kyiv University, Ukraine
d.y.khusainov@gmail.com

We prove an exact controllability for a one dimensional heat equations with delay in both lower and highest order term and non homogenous Dirichlet boundary conditions. Moreover, we give implicit representation of the control function steering the system into a given final state.

Control of orbital motion in a neighborhood of collinear libration point of the Earth-Sun system

Vasily Shmyrov
Saint-Petersburg State University, Russia
vasilyshmyrov@yandex.ru
Alexander S. Shmyrov

In this research we consider the controllable orbital motion of a spacecraft in a neighborhood of the collinear libration point L_1 of the Earth-Sun system. Libration point L_1 is unstable and that makes it necessary to apply a control action. We use a special approximation of the equations of the circular restricted three-body problem, the so-called Hill's equations as a mathematical model. Usually, in such studies have investigated the motion of a spacecraft near the basic halo orbit on which the spacecraft arrived from initial orbit. In this research we consider the problem of local motion in a neighborhood of L_1, in which the spacecraft is not necessarily at a given halo orbit. We built control laws, under which action motion of spacecraft in a neighborhood of L_1 is Lyapunov stable, the control laws for asymptotic stability with respect to part of the variables, and the control laws, which is optimal within the linear-quadratic problem of optimization. The properties of controllable movement are illustrated by numerical modeling.

Use of the collinear libration points for maneuvering in the near-Earth space

Dzmitry Shymanchuk
Saint-Petersburg State University, Russia
shymanchuk@mail.ru
Alexander S. Shmyrov

The use of the neighborhood of the collinear libration point (L_1 or L_2) of the Sun – Earth system has long been of practical importance in connection with projects implemented by NASA and ESA. A spacecraft motion is considered in a rotating frame within the Hill's problem of the circular restricted three-body problem. It is known that collinear libration points are unstable but the instability of libration points can be used as a positive factor when maneuvering a spacecraft in the near-Earth space, for example, while solving comet and asteroid impact hazards to monitor space objects that are dangerously approaching the Earth. Here, the problem of returning to the neighborhood of L_1 or L_2 arises. Returning to the neighborhood of a libration point is achieved through a control action. We offer a methodology for constructing control algorithms for the orbital motion of a spacecraft. This methodology is based on the properties of a specially introduced function of phase variables — the "hazard function". An algorithm for constructing a return trajectory is described. Numerical simulation results are given.

Dynamical systems with polynomial impulses

Maxim Staritsyn
Institute for System Dynamics and Control Theory of SB RAS, Russia
starmaxmath@gmail.com
Elena Goncharova

We study a BV-relaxation of dynamical systems with polynomial dependence on an unbounded control subject to an L_p-constraint. The relaxation is described by a discontinuous time reparameterization. Under some natural assumptions, we obtain an explicit representation of the relaxed system by a measure differential equation. For the relaxed system, we address an optimal impulsive control problem. For the quadratic case, we give necessary optimality conditions in the form of the Maximum Principle.
The work was partially supported by RFBR, projects nos 13-08-00441, 14-01-31254, 14-08-00606.

Numerical analysis of an optimal control problem governed by an elliptic variational inequality

Domingo Tarzia
CONICET & Univ. Austral, Argentina
DTarzia@austral.edu.ar
M.C. Olguin

The existence and uniqueness of a continuous distributed optimal control problem governed by an elliptic variational inequality and a quadratic cost functional was studied in Boukrouche - Tarzia, Comput. Optim. Appl., 53 (2012), 375-393. The goal of this paper is to study the numerical analysis of this continuous optimal control problem by using the finite element method with a regular triangulation with Lagrange triangles of type 1, h being the parameter of the finite element approximation which goes to zero. Then, we define the discrete elliptic variational inequality for the state system and the discrete cost functional for each positive h. We obtain the existence and uniqueness of the discrete optimal state system and control variable, and we prove that these elements convergent to the continuous optimal state system and control variable when h goes to zero.

Controllability properties of nonlocal semilinear time-varying delay control systems

Nutan Tomar
Indian Institute of Technology Patna, India
nktomar@iitp.ac.in
Suman Kumar

TBA

Observer design for semilinear descriptor systems

Nutan Tomar
Indian Institute of Technology Patna, India
nktomar@iitp.ac.in
Mahendra Kumar Gupta, Shovan Bhaumik

In the last few decades, many researchers have given a lot of attention on the analysis and design of descriptor systems as it is general enough to provide a solid understanding of the inner dynamics of any physical system. Many physical problems can be modeled as the system of differential algebraic equations (DAEs) that depicts a collection of relationships between variables of interest and some of their derivatives and can be written in the form of descriptor systems. Some real life example are electrical circuits, chemical control precess, constrained mechanical systems to name a few.
In the present work, we consider the rectangular linear time invariant descriptor system: $E\dot{x} = Ax + Bu + Df(Hx, u, t)$, $y = Cx$, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$, and $y \in \mathbb{R}^p$ are the state vector, the input vector and the output vector, respectively. E, $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times k}$, $D \in \mathbb{R}^{m\times n_d}$, $H \in \mathbb{R}^{n_h\times n}$, and $C \in \mathbb{R}^{p\times n}$ are known constant matrices, and the rank of $E = r < \max\{m, n\}$. f is a nonlinear operator.
A new method is proposed to design a Luenberger type observer for a class of above descriptor systems satisfying very less restrictive conditions. The method is based on the properties of restricted system equivalent, derived here from a given descriptor system by means of matrix theory. Coefficient matrices of the proposed observer are synthesized by using the feasibility of linear matrix inequality problem.

Contributed Session 5: Fixed Points and Topological Method

Impulsive systems with nonlocal initial conditions

Octavia-Maria Bolojan
Babes-Bolyai University of Cluj-Napoca, Romania
octavia.nica@math.ubbcluj.ro

In this work, we study the existence of solutions for nonlinear first order impulsive systems with nonlocal initial conditions. Our approach relies in the fixed point principles of Schauder and Perov, combined with a vector approach that uses matrices that converge to zero. We prove existence and uniqueness results for these systems. Some examples are presented in order to illustrate the theory.

Existence of mild solution for nonlocal impulsive integro-differential equation with state dependent delay

Jaydev Dabas
Indian Institute of Technology Roorkee, India

We establish the mild solution for an class of impulsive fractional integro-differential equation with state dependent delay. We prove the results of existence and uniqueness of solution by applying well known classical fixed point theorems.

Existence of locally attractive solution to fractional order quadratic functional integral equation

Bhalchandra Karande
Maharashtra Udayagiri Mahavidyalaya, India
bdkarande@rediffmail.com

In this article, we discuss the existence and uniqueness of locally attractive solution to fractional order quadratic functional integral equation in in Banach spaces. The results are obtained by using hybrid fixed point theorem. Some illustrative examples are also discussed.

On existence of locally attractive solution for for a nonlinear functional integral equation of fractional order

Bhalchandra Karande
Maharashtra Udayagiri Mahavidyalaya, India
bdkarande@rediffmail.com
S.P.Shinde

In this paper, we discuss the existence of solutions for a nonlinear functional integral equation of fractional order in R+ via a fixed point theorem due to B.C. Dhage. This equation will be carried out in the Banach space of real functions defined, continuous and bounded on an unbounded interval R+. Moreover, we show that solutions of this equation are locally attractive on R+.

Nonuniqueness and blowing up character of the solution to a nonlinear Abel type integral equation on the half line

Wojciech Mydlarczyk
Institute of Mathematics and Computer Science, Technical University of Wroclaw, Poland
wojciech.mydlarczyk@pwr.edu.pl

We are going to discuss a nonlinear integral equation of the Abel's type

$$u(t) = \int_{-\infty}^{t} (t-s)^{\alpha-1} r(s) g(u(s)) ds \quad (-\infty < t, 0 - \infty),$$

i.e. $u(t) = 0$ for $t \le t_0$ and $u(t) > 0$ for $t > t_0$. We also provide conditions under which the solution $u(t)$ is blowing up.

Coefficient bounds for certain subclasses of analytic functions

Oznur Ozkan Kilic
Baskent University, Turkey
oznur@baskent.edu.tr
Osman Altintas

In this paper, we introduce two subclasses $H(\alpha, A, B)$ and $K(\alpha, \mu, m, A, B)$ of the functions which are analytic in the open unit disk U and obtain coefficient bounds for these functions.

On some fractional problems with unbounded nonlinearity

Rosana Rodríguez-López
University of Santiago de Compostela, Spain
rosana.rodriguez.lopez@usc.es

By using fixed point results, we study the existence and uniqueness of solution to fractional differential equations with a nonlinearity which is unbounded on the strip $I \times \mathbb{R}$, where I is compact.

Cohomological index of Fuller type and its applications

Robert Skiba
Nicolaus Copernicus University, Poland
robo@mat.umk.pl
Wojciech Kryszewski

In this talk we are going to present a cohomological index of Fuller type for dynamical systems defined in Absolute Neighborhood Retracts satisfying the properties of Existence, Excision, Additivity, Homotopy and Topological Invariance. This notion is an extension of the classical (cohomological) fixed point index for single-valued maps. In particular, we will show that the constructed index is responsible for detecting of periodic orbits and stationary points of dynamical systems generated by differential equations without of uniqueness of solutions. This is a joint work with Wojciech Kryszewski.

Contributed Session 6: PDEs and Applications

Line segment cracks identification via the reciprocity gap principle and Fourier transform

Boukricha Abderrahman
University of Tunis El Mar, Tunisia
aboukricha@fst.rnu.tn
B. Maher

The problem of determining a crack by overspecified boundary data is considered. When complete data are available on the external boundary. A link that is established between the reciprocity gap functional and the Fourier transform of the temperature is introduced. If the crack is known (or assumed) to be a line, an explicit inversion formula is obtained and determination of the host line equation and the length of the crack in the two-dimensional (2D) situation. Numerical tests of the identification methods are proposed and show very good accuracies and significant computational costs.

A mathematical model of bimolecular catalytic reactions

Algirdas Ambrazevicius
Vilnius University, Lithuania
algirdas.ambrazevicius@mif.vu.lt
V. Skakauskas

Coupled systems of parabolic and ordinary differential equations usually are considered in the same domain. Heterogeneous unimolecular or multimolecular catalytic reactions are modeled by a coupled system of parabolic and ordinary differential equations [1]. Some of these equations are considered on a part of the boundary. We consider a system of two parabolic and three ordinary differential equations, the latter being determined on the boundary of the domain. This system describes the model of surface reactions between carbon monoxide and nitrous oxide. We prove the existence and uniqueness theorem of a classical solution to this model.

References

[1] A. Ambrazevicius. Reaction–diffusion problem with dynamical boundary condition, Nonlinear Analysis 95 (2014) 568-579

Stabilization of a hyperbolic/elliptic system modelling the viscoelastic-gravitational deformation in a multi-layered Earth

Alicia Arjona
European Center for Geodynamics and Seismology, Luxembourg
alicia.arjona@ecgs.lu
Jesús Ildefonso Díaz

In the last 30 years several mathematical studies have been devoted to the viscoelastic-gravitational coupling in stationary and transient regimes see e.g. (Rundle, 1980; Lanzano, 1982; Aki and Richards 2002; Arjona and Díaz, 2008) for static case and (Rundle, 1982; Fernández, Yu and Rundle, 1996; Arjona, Fernández and Díaz, 2014) for hyperbolic case. However in the best of our knowledge it lacks a mathematical study of the stabilization when t goes to infinity of a viscoelastic-gravitational models crustal deformations of multilayered Earth. Here we prove that, under some additional conditions on the data, the difference of the viscoelastic and elastic solutions converges to zero, as t goes to infinity, in a suitable functional space. Our proof uses a reformulation of the hyperbolic/elliptic system in terms of a nonlocal hyperbolic system leading to which we apply the La Salle invariance principle for a Lyapounov function involving the nonlocal terms.

On construction and investigation of dynamical models for elastic multi-structures

Gia Avalishvili
I. Javakhishvili Tbilisi State University, Rep of Georgia
gavalish@yahoo.com
Mariam Avalishvili, David Gordeziani

In the present paper we consider problems of vibration of elastic multi-structures consisting of several parts with different geometrical shapes. Many engineering constructions are multi-structures consisting of plates, shells and rods. Applying hierarchical dimensional reduction method we construct dynamical model for elastic multi-structure, which consists of elastic body with general shape, prismatic shell clamped in it and several rods. The prismatic shell is supported on rods clamped in the body. Along interfaces between body, prismatic shell and rods rigid contact conditions are given. The three-dimensional models of prismatic shell and rods are approximated by two-dimensional and one-dimensional ones, respectively, and dynamical models of multi-structure defined on the union of three-dimensional, two-dimensional and several one-dimensional domains are constructed. We prove the existence and uniqueness of solutions of the initial-boundary value problems corresponding to the obtained hierarchical models in suitable spaces. Moreover, we prove the convergence of the sequence of vector-functions of three space variables restored from the solutions of the constructed problems to the solution of the original three-dimensional problem in corresponding spaces and estimate the rate of convergence.

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Analysis of a certain model of thermo-visco-plasticity

Leszek Bartczak
University of Warsaw, Poland
lbartczak@mimuw.edu.pl

We will present the existence and uniqueness result for a model describing the phenomena in a deformed body which is exposed on temperature changes.

Fast Solution Methods for the Space-Time Fractional Diffusion Equations

Treena Basu
Rhodes College, USA
treena.basu@gmail.com
Hong Wang

Fractional diffusion equations model phenomena exhibiting anomalous diffusion that can not be modeled accurately by the second order diffusion equations. Because of the non-local property of fractional differential operators, the numerical methods have full coefficient matrices which require storage of $O(N^2)$ and computational cost of $O(N^3)$, where N is the number of grid points. Together we develop a fast finite difference method for the one-dimensional space and time fractional diffusion equation, which only requires storage of $O(N)$ and computational cost of $O(N \log N)$, while retaining the same accuracy and approximation property as the regular finite difference method. Numerical experiments are presented to show the utility of the method. For example, with 1024 computational nodes, the new scheme developed for the one-dimensional problem has about 40 times of CPU reduction than the standard scheme.

Trapped waves in a two-layer fluid bounded below by an elastic bed

Swaroop Bora
Indian Institute of Technology Guwahati, India
swaroop@iitg.ernet.in
Sunanda Saha

In the present paper, a hydroelastic model is developed to examine the trapped modes supported by a horizontal submerged cylinder placed in either of the layers of a two-layer fluid flowing over an elastic bottom at a finite depth. The elastic bottom is modeled as a thin elastic plate and is based on the Euler-Bernoulli beam equation. Using multipole expansion method, an infinite system of homogenous linear equations is obtained. For a fixed geometrical configuration and a specific arrangement of a set of other parameters, the frequencies for which the value of the truncated determinant is zero are numerically computed and the trapped wavenumbers corresponding to those frequencies are obtained by using the dispersion relation. These trapped modes are compared with those for which the lower layer is of infinite depth. We also look into the effect of the variation of elastic plate parameters on the existence of trapped modes. Significant difference is observed with respect to the existence and also in the pattern of the trapped modes between the present case and the one when the cylinder is placed in an infinite depth lower layer in a two-layer fluid.

The properties of matter-wave solitons with the minimum number of particles in two-dimensional quasiperiodic potentials

Gennadiy Burlak
UAEM, Mexico
gburlak@uaem.mx
Boris A. Malomed

The 2D matter-wave soliton families supported by an external potential is systematically studied, in a vicinity of the junction between stable and unstable branches of the families. In this case the norm of the solution attains a minimum, facilitating the creation of such excitation [1]. The model is based on the Gross-Pitaevskii equation for the self-attractive condensate loaded into a quasiperiodic (QP) optical lattice (OL). The same model applies to spatial optical solitons in QP photonic crystals. Dynamical properties and stability of the solitons are analyzed with respect to variations of the depth and wavenumber of the OL. In particular, it is found that the single-peak solitons are stable or not in exact accordance with the Vakhitov-Kolokolov criterion. The double-peak solitons, which are found if the OL wavenumber is small enough, are always unstable again splitting.

References

[1] Gennadiy Burlak, Boris A. Malomed, Phys.Rev. E, 85, 57601-57606 (2012).

Micro/Macroscopic Models for Control in Arterial Networks

Radu Cascaval
University of Colorado Colorado Springs, USA
radu@uccs.edu

We describe a Boussinesq-type system for modeling the dynamics of pressure-flow in arterial networks, considered as a 1d spatial network. The system captures the pulsatile behavior as well as various controls (boundary and distributed) present in physiological systems. Numerical solutions of the system of PDEs are compared with simplified models based on particle-tracking arguments, and are used to study flow optimization tasks, depending on the geometry and size of the network. Physiologically realistic control mechanisms are also tested in the context of these simplified models.

Frequency domain numerical spectral solutions for certain class of Partial Differential Equations

Vinita Chellappan
Indian Institute of Science, Bangalore, India
vinita@aero.iisc.ernet.in
S. Gopalakrishnan, V. Mani

In this paper we consider numerical solutions for different cases for the three dimensional Heat equation and the two dimensional time dependent Schrodinger equation along with the potential function. The numerical solution is obtained by using Laplace or Fourier approximation in the temporal domain and spectral approximation using Legendre and Chebyshev polynomials in the spatial domain and also using the Laplace spectral element method. The results of the numerical approximation are presented. It is compared and analyzed with the analytical results available. In the frequency domain we compute only FFT points upto $\frac{Ns}{2}$ for the heat equation whereas we require twice the number of FFT points for the Schrodinger equation. We then show in frequency domain using phase portraits that the modes oscillate in time for the Schrodinger equation whereas the modes decay in time for the Heat equation. Also, the eigen values for different frequencies in the frequency domain are computed to analyze the solution trajectory.

On a Mathematical Model of a Cusped Double-Layered Prismatic Shell

Natalia Chinchaladze
I. Javakhishvili Tbilisi State University, Rep of Georgia
chinchaladze@gmail.com

A vibration problem for a double-layered cusped prismatic shell in the zeroth approximation of Vekua's hierarchical models for elastic laminated prismatic shells (see, e.g., [1], [2]) is considered. The problem mathematically leads to the question of setting and solving BVPs for even order equations and systems of elliptic type with order degeneration in the statical case and of IBVP for even order equations and systems of hyperbolic type with order degeneration in the dynamical case. The well-posedness of the BVPs under the reasonable boundary conditions at the cusped edge and given displacements at the noncusped edge is studied. The classical and weak setting of the BVPs is considered. Appropriate weighted functional spaces are introduced. Uniqueness and existence results for the variational problem are proved. The structure of the constructed weighted space is described, its connection with weighted Sobolev spaces is established.

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References

[1] I.N. Vekua, Shell Theory: General Methods of Construction, Monographs, Advanced Texts and Surveys in Pure and Applied Mathematics, 25. Pitman, Boston, MA. 1985

[2] G. Jaiani, Cusped shell-like structures, SpringerBriefs in Applied Science and Technology, Springer, Heidelberg, 2011.

Steady Gravity-Capillary Surface Waves over a Bump – Critical Surface Tension Case

Jeongwhan Choi
Korea University, Korea
jchoi@korea.ac.kr
Shu-Ming Sun, S.I. Whang

We study steady forced surface waves on an incompressible, inviscid fluid in a two-dimensional channel with a small bump on a horizontal rigid flat bottom and nonzero surface tension on the free surface. It has been known that the wave motion is determined by a non-dimensional wave speed F, called Froude number, and a non-dimensional surface tension τ, called Bond number, where $F = 1$ and $\tau = 1/3$ are the critical values of F and τ, respectively. If $F = 1 + \lambda_1\epsilon^2$ and $\tau = 1/3 + \tau_1\epsilon$ with $\epsilon > 0$ a small parameter, then a time-dependent forced Kawahara (F-Kawahara) equation (also called forced fifth-order Korteweg-de Vries equation) can be derived to model the wave motion on the free surface. In the paper, the steady F-Kawahara equation is studied both theoretically and numerically. It is shown that if τ_1 is fixed, then there exists a cut-off value $\lambda_0 < 0$ of λ_1 such that for $\lambda_1 < \lambda_0$ there are two single-hump steady solutions, one with small amplitude and the other one with large amplitude. By using the unsteady F-Kawahara equation, it is obtained that the small steady solution is stable while the large one is unstable. Moreover, the steady F-Kawahara equation has many multi-hump solutions for large $\lambda_1 < 0$ and two-hump solutions appear unstable

An asymptotic and numerical study of of the vibration spectrum two beams coupled by a dissipative joint - a comparison of the Euler-Bernoulli and Timoshenko models

Matthew Coleman
Fairfield University, USA
mcoleman@fairfield.edu
Chris Abriola, Aglika Darakchieva and Tyler Wales

We perform a numerical and asymptotic study of the vibration spectra of two identical beams coupled via each of the four standard types of linear dissipative joint. We consider both the Euler-Bernoulli and Timoshenko models; numerical results are computed using the Legendre-tau spectral method, while analytic/asymptotic results are derived using the Wave Propagation Method of Keller and Rubinow. We provide a comparison of results for all 40 combinations of boundary and joint conditions, both within each model with an eye toward the asymptotic spectral equivalence of joints-and between the two models. It is found that, in every case, the spectrum consists of two branches or streams of frequencies, and that in a number of cases the spectra are, indeed, asymptotically equivalent. In addition, in many cases we find the existence of at least one branch that is asymptotically undamped. Further, it seems that the Euler-Bernoulli and Timoshenko models' predictions are different in the case of some of these undamped braches. The Euler-Bernoulli computations were performed by three undergraduate students (C. Abriola, A. Darakchieva, and T. Wales) during a summer Research Experience for Undergraduates at Fairfield University directed by the presenter (M. Coleman).

Fluid modeling of aircraft networks

Alexandra de Cecco
ONERA Toulouse, France
Alexandra.De_Cecco@onera.fr
Dufour Guillaume

This work focuses on the derivation of a system of partial differential equations in order to model the transit time of messages on AFDX networks such as those used in avionics. Obtaining a set of global equations allows to reduce the complexity of the problem. It also allows to give performance insights in short time, independently of the number of messages transiting on the network. In order to derive this so-called Fluid Model, we first describe the particulate behavior of the messages and we introduce a distribution function. By using an asymptotic analysis, we identify the equation solved by the distribu-

tion describing the global behavior of the system. Then, we derive a set of PDEs corresponding to the evolution of this distribution's first moments. The hypothesis of FIFO stacks in the network is enough to prove existence and uniqueness for both the kinetic equation and the set of resulting fluid PDEs. Moreover, by using a closure assumption defining the shape of the distribution, we also prove the equivalence between the kinetic scale and the fluid one. Finally, some numerical results illustrating the ability of the model to accurately capture the network performance will be shown.

Homoclinic snaking in a generalised Swift-Hohenberg equation

Andrew Dean
University of Leeds, England
matade@leeds.ac.uk

Homoclinic snaking, in which a multiplicity of localised states exists within a narrow region of parameter space, is observed in myriad experimental and theoretical contexts, and been the subject of much research activity in recent years. The majority of analytical work to date has been in the context of systems which are variational in time, and reversible and conservative in space; these properties are often lacking in real-world scenarios. The prototypical example of such an idealised system is the Swift-Hohenberg equation, in which the snaking phenomenon is well understood. We present work on a generalised Swift-Hohenberg equation, in which nonlinear terms involving derivatives are added, yielding a system lacking the aforementioned properties. The leading-order pattern-forming behaviour is derived using a multiple-scales analysis near bifurcation; this is extended to include exponentially small effects in order to correctly describe the snaking phenomenon. We compare the snaking bifurcations in our generalised equation with the classic picture, and discuss some of the complexities of analysing non-variational, non-conservative and irreversible systems.

Fractional Diffusion: A Continuous Time Random Walk Approach

Isaac Donnelly
University of New South Wales, Australia
i.donnelly@unsw.edu.au
Bruce I. Henry, Chris N. Angstmann, Trevor. A.M. Langlands

Fractional diffusion is characterized by a mean squared displacement of an ensemble of particles that scales non-linearly with time. This is a widely seen phenomena in experimental systems. We will present recent results in this area including the fractional Fokker-Planck equation, reaction-subdiffusion equations as well as corresponding equations in the network setting. These results were obtained using a Continuous Time Random Walk as a model for the underlying stochastic process of a particle. This approach allows for the incorporation of space and time dependent components in the separable jump and waiting time densities governing the behavior of a particle. By taking the diffusive limit, we are able to obtain the fore mentioned governing equations. Due to the discrete, non-local nature of a network, the governing equations allow for a range of interesting results not seen in the continuum. We will also investigate open problems in including specific types of spatial diffusion and forces.

Regularization of nonautonomous, non-linear ill-posed problems

Matthew Fury
Penn State Abington, USA
maf44@psu.edu

Ill-posed problems such as the backward heat equation are often treated with approximation methods since direct computation of a solution may be difficult. For example, given a differential operator A, Lattes and Lions's quasi-reversibility method $A - \epsilon A^2$, $\epsilon > 0$, may be applied to define an approximate well-posed model whose solution for any given initial data may be used to estimate a solution of the original ill-posed problem. In this presentation, we will consider the regularization of a non-linear, non-autonomous ill-posed problem $du/dt = A(t)u + h(t, u(t)), 0 \leq t \leq T, u(0) = x$ where $\{A(t)\}$, $0 \leq t \leq T$ is a family of operators in a Hilbert space. We will study the reason that alternate approximations to ones that are conventionally used may be needed; such approximations include a modified quasi-reversibility method $-\frac{1}{pT}\ln(\epsilon + e^{-pTA})$, $\epsilon > 0$, $p \geq 1$ recently introduced by Boussetila and Rebanni. This study may be applied to a wide class of nonlinear partial differential equations in L^2 spaces including the nonlinear backward heat equation with a time-dependent diffusion coefficient.

Efficiency over cylinders: Steiner symmetrization and shape differentiation

David Gomez-Castro
Universidad Complutense de Madrid, Spain
dgomez91@gmail.com
Jesus Ildefonso Diaz

We consider a parabolic reaction-diffusion mathematical model

$$\begin{cases} w_t - \Delta w + \lambda\beta(w) = f, & \mathbb{R}_+ \times G \\ w = w_0, & \{0\} \times G \\ w = 1, & \mathbb{R}_+ \times \partial G \end{cases}$$

which appears in applications, for example, on Chemical Engineer. We analyze the different efficiency of the domain, which is defined

$$\eta(t, G) = \frac{1}{|G|} \int_G \beta(w(t, x))\, dx$$

We extended the Steiner symmetrization techniques for domains $G = G' \times G''$ previously used by Alvino, Trombetti, Diaz and Lions (1996) for the elliptic linear problem coupled with techniques from Schwarz symmetrization in the nonlinear problem (see J.I. Diaz (1985)). We show that, for this kind of domain, $G' = B_R$ provides the lowest effectivity for fixed G'' and $|G'|$. However, we point out that, as a result of a theorem by C. Bandle in 1985 effective cylindrical reactors may have a circular basis as long as they are low (that is, it is contained between two close parallel hyperplanes), and generalize the result to maximal monotone operators on the limit case. For this class of domains we provide a sufficient condition for optimal effectiveness.

Analysis of a new semi-discrete finite element approximation of a linear fluid-structure interaction problem

Maria Gonzalez Taboada
Universidade da Coruña, Spain
maria.gonzalez.taboada@udc.es
Virginia Selgas

We study the interaction of a time-dependent Stokes fluid with an elastic structure through a fixed interface. We consider a variational formulation of the problem with the fluid velocity, the fluid pressure, the structural velocity and the Cauchy stress tensor as unknowns, and prove that it is well-posed under the usual regularity assumptions on the data. Then, we analyze the discretization by the finite element method. We prove that any conforming finite element approximation of the problem converges to the exact solution and we derived some error estimates. Using the backward Euler method for the time discretization, we obtain a fully discrete scheme. At every time step, we first solve a saddle point problem for the global velocity and the pressure; then, we compute an approximation of the Cauchy stress tensor by using an explicit formula. In order to solve the saddle point problem, we can consider any stable pair for the Stokes problem in the fluid domain and a finite element space for the structural velocity in the solid domain compatible on the interface with the finite element space for the fluid velocity. Displacements of the structure can be recovered by a quadrature formula.

Stabilization and a posteriori error analysis for convection-diffusion equations

Maria Gonzalez Taboada
Universidade da Coruña, Spain
maria.gonzalez.taboada@udc.es
Johan Jansson and Sergey Korotov

We introduce a new augmented dual-mixed variational formulation for a linear convection-diffusion equation with homogeneous Dirichlet boundary conditions. The approach is based on adding suitable Galerkin least squares type terms to the classical dual-mixed formulation of the problem. We prove that, for appropriate values of the stabilization parameters, that depend on the diffusion coefficient and the magnitude of the convective velocity, the new variational formulation and the corresponding Galerkin scheme are well-posed and a Céa estimate can be derived. This means that any conforming approximation will lead to a convergent scheme. In particular, we derived the rate of convergence when the flux and the concentration are approximated, respectively, by Raviart-Thomas and continuous piecewise polynomials. In addition, we develop an a posteriori error analysis of residual type and derive a simple a posteriori error estimator, which is reliable and locally efficient.

Compact Difference Schemes for Problems of Mathematical Physics

Vladimir Gordin
National Research University 'Higher School of Economics', Hydrometeorological Center of Russia, Russia
vagordin@mail.ru
E.A. Tsymbalov

We develop a general approach to compact difference schemes' construction. We approximate the differential problems $Au = f$ or $Au = Bf$ by difference one $Pu_h = Qf_h$ and determine the difference operators P and Q. The following typical for mathematical physics operators A are considered:

- Laplace op. Δ;
- Helmholtz op. $\Delta - q(\vec{x})$;
- diffusion op. with constant diffusion coefficient $\partial_t - D\Delta - q(\vec{x})$;
- diffusion op. with variable one $\partial_t - \partial_x D \partial_x - q(\vec{x})$;
- Schrödinger op. $\partial_t - iD\Delta - q(\vec{x})$;
- rod vibrations op. $\partial_t^2 - D\partial_t^2\partial_x^2 + C\partial_x^4$.

We constructed 4-th order CDSs and confirmed this order at various boundary and initial-boundary problems. We compare these CDSs with classic ones, e. g. with the Crank–Nicolson scheme. The relative high-order approximations for corresponding boundary and initial conditions were constructed, too. Also, these CDSs are applicable to some non-linear problems.

References
[1] V.A.Gordin. Mathematics, Computer, Weather Forecasting, and Other Scenarios of Mathematical Physics (in Russian). M., Fizmatlit, 2010, 2012.
[2] V. A.Gordin, E.A. Tsymbalov. Compact Difference Schemes for the Diffusion and Schrödinger Equations. Approximation, Stability, Convergence, Effectiveness, Monotony. Journal of Computational Mathematics, 2014 (to appear).

Hierarchical models for the plane and shell

Bakur Gulua
Sokhumi State University, Rep of Georgia
bak.gulua@gmail.com

In the present paper, by means of Vekua's hierarchical method for thin and shallow shells, the infinite system of second-order partial differential equations is obtained, when on upper and lower face surfaces displacements are assumed to be known. Then we consider $N = 1$ order approximation for plates. We obtain the elliptic system of equations of the twelfth order. The general solution of this system is expressed by six arbitrary analytic function of $z = x + iy$, where x and y are Cartesian coordinates. Concrete problem has been solved.

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Asymptotic behavior of some degenerate parabolic equations and the regularity of the solutions

Pelin Guven Geredeli
Hacettepe University, Turkey
pguven@hacettepe.edu.tr

In this work, we consider some degenerate parabolic equations in unbounded domain. Here, we study the long time behavior (in the sense of global attractor) of the weak solutions and also obtained the regularity of the solutions from this attractor using the well known Moser's iteration method.

H^1-Galerkin expanded mixed finite element Simulation for Sobolev equation in polymer flow

Chen Huanzhen
Shandong Normal University, Peoples Rep of China
chhzh@sdnu.edu.cn
Zhou Zhaojie

We present an H^1-Galerkin expanded mixed finite element method for a nonlinear Sobolev equation, which models an incompressible fluid flow process in subsurface polymer media. The method possesses the advantages of H^1-Galerkin and expanded mixed finite element methods, such as approximating the unknown, its gradient and flux directly, the finite element space being free of LBB condition, as well as avoiding the difficulties arising from calculating the inverse of a small diffusive coefficient when simulating the diffusive problems within a low permeability zone. We conduct theoretical analysis on the existence and uniqueness of the numerical solution and prove an optimal-order error estimate which is challenging in traditional H^1-Galerkin mixed methods for nonlinear problems. A numerical experiment is conducted to support the theoretical proven results.

Boundary layer theory for convection-diffusion equations in a circle

Chang-yeol Jung
UNIST, Korea
changyeoljung@gmail.com
Roger Temam

This article is related to the boundary layer theory for singularly perturbed convection-diffusion equations in the unit circle. Two characteristic points appear, $(\pm 1, 0)$, in the context of the equations considered here and singular behaviors may occur at these points depending on the behavior of the given function f at these points, namely, the flatness or compatibility of f at these points as explained below. Two previous articles addressed two particular cases: one dealt with the case where the function f is sufficiently flat at the characteristic points, the so-called compatible case; the other one dealt with a generic noncompatible case (f polynomial). This paper continues with the general case (f non-flat and non-polynomial) for which we additionally introduce new specific boundary layer functions of parabolic type.

Viscous Cahn-Hilliard equation in $\mathbb{R}^N$

Maria Kania-Blaszczyk
University of Silesia, Poland
mkania@math.us.edu.pl

Solvability of Cauchy's problem in $\mathbb{R}^N$ for an extended viscous Cahn-Hilliard equation will be discussed. The problem is considered first in standard Sobolev space $H^1(\mathbb{R}^N)$, next a notion of the 'H-solution' is introduced which is well adapted to the structure of the viscous Cahn-Hilliard equation. The Cauchy problem for an extended viscous Cahn-Hilliard equation has the form:

$$\begin{cases} (1-\nu)u_t = -\Delta(\Delta u + f(x,u) - \nu u_t), \ t > 0, \\ \hfill x \in \mathbb{R}^N, \\ u(0,x) = u_0(x), \end{cases}$$

where $\nu \in [0, 1)$ and the nonlinear term f fulfills the required regularity and growth assumptions.

References

[1] L. Cherfils, A. Miranville, S. Zelik, The Cahn-Hilliard equation with logarithmic potentials, Milan J. Math. 79 (2011), 561-596.

[2] J.W. Cholewa, A. Rodriguez-Bernal, On the Cahn-Hilliard equation in $H^1(\mathbb{R}^N)$, J. Diff. Equations 253 (2012), 3678-3726.

[3] T. Dlotko, C. Sun, Dynamics of the modified viscous Cahn-Hilliard equation in $\mathbb{R}^N$, Topol. Methods Nonlinear Anal. 35 (2010), 277-294.

[4] T. Dlotko, M.B. Kania, Chunyou Sun, Analysis of the viscous Cahn-Hilliard equation in $\mathbb{R}^N$, J. Differential Equations 252, 2012, 2771-2791.

[5] S. Gatti, M. Grasselli, A. Miranville, V. Pata, Hyperbolic relaxation of the viscous Cahn-Hilliard equation in 3-D, Math. Models Methods Appl. Sci. 15 (2005) 165-198.

Dynamics of bright and dark solitons in multicomponent Yajima-Oikawa system

Sakkaravarthi Karuppaiya
Bishop Heber College, India
ksakkaravarthi@gmail.com
K. Sakkaravarthi and T. Kanna

We investigate the dynamics of soliton propagation and their collision behavior in the multicomponent Yajima-Oikawa (m-YO) system describing the nonlinear interaction between a long wave and multiple short waves. We construct the bright and dark soliton solutions of m-YO system by using Hirota's direct method and investigate their propagation dynamics. We explore the collision dynamics of solitons and demonstrate the energy sharing collision of bright solitons appearing in the short-wave components. Also, we discuss the formation of dark soliton bound states and periodic structures.

On nonlinear elliptic equations of p(x)-Laplace type satisfying Cerami condition

Yun-Ho Kim
Sangmyung University, Korea
kyh1213@smu.ac.kr

We are concerned with nonlinear elliptic equations of $p(x)$-Laplacian type subject to Dirichlet boundary condition

$$\begin{cases} -\mathrm{div}(\phi(x,|\nabla u|)\nabla u) = \lambda f(x,u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with Lipschitz boundary $\partial\Omega$, the function $\phi(x,t)$ is of type $|t|^{p(x)-2}$ with continuous function $p : \overline{\Omega} \to (1,\infty)$ and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ satisfies a Carathéodory condition. The purpose of this talk is to show the existence of at least one solution, and under suitable assumptions, infinitely many solutions for the problem above without the well-known Ambrosetti-Rabinowitz type growth conditions by using Mountain pass theorem and Fountain theorem with Cerami condition.

A DDFV scheme for the simulation of heterogeneous viscous flows

Stella Krell
Univ. Nice Sophia Antipolis, France
krell@unice.fr
Thierry Goudon

We introduce a “Discrete Duality Finite Volume” scheme for the simulation of heterogeneous viscous flows. By contrast to standard Finite Volume methods, DDFV schemes are free of any “orthogonality” constraints on the mesh; they work with a large class of 2D or 3D meshes including non-conformal and distorted meshes. Therefore, the method is very appealing in order to handle complex geometries, or to be used in combination to mesh refinements methods to follow accurately regions of strong gradients. We consider models consistuted by the mass and momentum balance equations with an additional constraint on the divergence of the velocity field. A relevant example is given by the so-called Kazhikov-Smagulov system. At the discrete level, the difficulty consists in ensuring the compatibility between the mass conservation equation, the constraint on the velocity field, and the treatment of the convection terms in the momentum equation. We first focus on the case of a divergence free constraint since in this case the model is the well-known variable density incompressible Navier-Stokes system. We prove the existence of discrete solutions and establish the energy-stability of the scheme. We then give examples of more intricated constraints that can be treated and performed numerical simulations to illustrate the performances of the method.

A new model for Shape from Shading under Perspective Projection with Numerical Solution

Sanjeev Kumar
Indian Institute of Technology Roorkee, India
malikfma@iitr.ac.in
Anu Bala, Rama Bhargava

Shape from shading (SfS) is a technique in image processing which is used for the visualization of a single 2D image/photograph as a 3D model by solving a boundary value problem. In this, the solution of PDE is obtained as a function of reflected lights and shades is used to infer the shape of the surface in view. Until a few years ago, a very common assumption in the field of SfS was that image projection is orthographic. However, perspective projection is more realistic model when we talk about the formation of an image. This paper deals with a new formulation of SfS model for recovering the surface heights from the given shaded image. The perspective projection model of photometric stereo is used to utilize PDE techniques to model the problem. The uniqueness and existence of the solution of our model under various assumptions have been discussed. Two different numerical algorithms have been used to evaluate the validity of the proposed model with some synthetic images. Finally, some results have been given to demonstrate the method on real-life images. From the detailed experimental study, we conclude that using methodologies common in the field of Shape-from-Shading it is possible to recover more depth information for the photometric stereo problem under the more realistic perspective projection assumption.

Some fixed point theorems with applications in differential equations

Poom Kumam
King Mongkut’s University of Technology Thonburi, Thailand
poom.kumam@mail.kmutt.ac.th
Parin Chaipunya

Fixed point theory is a promising tool for modern mathematicians in showing existence, and sometimes, uniqueness. Also for the area of differential equations, fixed point theory has made it easier to study the existence and uniqueness of the solutions, or even the question of well-posedness. In this present talk, we will present some fixed point theorems, which eventually helps guarantee the existence of some classes of differential equations. In this case, we consider the non-homogeneous parabolic equations and fractional integral inclusion, for examples.

A multiplicity result for $p(x)$-Laplace type operator in Ω

Seung dae Lee
Sangmyung University, Korea
1107lsd@gmail.com
Yun-Ho Kim

In this talk, we study the following elliptic equations with variable exponents

$$\begin{cases} -\mathrm{div}(\phi(x,|\nabla u|)\nabla u) = \lambda f(x,u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

Under suitable conditions on ϕ and f, we establish the existence of at least three solutions of the above equations by using the three critical point theorem due to Ricceri.

Higher order Schrödinger and Hartree-Fock equations with Coulomb potential

Vanessa Lleras
Montpellier University, France
vlleras@univ-montp2.fr
R. Carles and E. Moulay

In this presentation, we extend the domain of validity of the higher order Schrödinger equations to the use of a Coulomb potential. We can use it for instance for the alpha particles which are fast charged semi-relativistic particles coming from the alpha decay and composed of 2 protons and 2 neutrons. Furthermore we develop higher order Hartree-Fock equations with bounded and Coulomb potentials which are able to take into account some relativistic effects in many particle physics, as for example the electrons of heavy atoms in quantum chemistry... The numerical simulations are in progress.

General homogenization of a bending-torsion theory for inextensible rods from 3D elasticity

Maroje Marohnic
University of Zagreb, Croatia
maroje.marohnic@math.hr
Igor Velcic

We derive, via Γ-convergence techniques, the equations of a homogenized bending rod starting from $3D$ nonlinear elasticity equations. The main assumption is that the energy is of the order h^2 (after dividing by the order h^2 of vanishing volume) where h is the thickness of the body. We do not presuppose any kind of periodicity and work in the general framework. The result shows that, on a subsequence, we always obtain the equations of bending-torsion rod and identifies, in an abstract formulation, the limiting quadratic form connected with that model. This is a generalization from periodic to non-periodic homogenization of a bending-torsion rod theory already present in the literature.

On the free vibration of shells. Theoretical approach and numerical approximations.

Jose Miguel Martinez Valle
Universidad de Cordoba, Spain
jmvalle@uco.es

The dynamic study of shells is a subject of undoubted interest. There are a lot of shell structures with different shapes such as cylindrical shells, elliptic paraboloid or spherical shells; however, the dynamic analysis of this kind of structures is rather limited. Most studies refer to shallow and thin shells. In this work, the equations that govern this problem are analyzed. It will be seen that these equations constitute a nonlinear system of differential equations whose analytical solution is only possible for very simple geometries, boundary conditions and loads. It is also described different finite element methods in order to perform the study of the vibrational frequencies of transverse oscillations of moderately thick shells and discussed the different kinds of mass matrices, lumped and consistent mass matrices.

Static and dynamic analysis of plates. Evolution and numerical approach.

Jose Miguel Martinez Valle
Universidad de Cordoba, Spain
jmvalle@uco.es

Static and dynamic analysis of plates is a topic of undoubted interest. Since Love formulated the famous biharmonic equation, there have appeared several theories including shear deformation. Reissner, Mindlin, Vlasov, between others, elaborated their technical calculation theories (first order shear deformation theory), with the objective of widening the application field of plates. Generally, all of those theories are characterised by the high level of mathematical complexity required to obtain solutions. Furthermore, the problems which are solved analytically only constitute several specific examples. In this talk, these theories are reviewed and their evolution and numerical approach are examined.

Existence Solutions for a Boundary Value Problem for Differential Equations of Arbitrary Order

Houas Mohamed
University of Khemis Miliana, Algeria
houasmed@yahoo.fr
Dahmani Zoubir

In this paper, we study the existence and uniqueness of solutions for a nonlinear fractional boundary value problem. New results are established using a Banach contraction principle and Schaefer's fixed point theorem.

Small perturbations of a nonlinear plate equation with p-Laplacian

Rodrigo Monteiro
University of Sao Paulo, Brazil
monteiro@icmc.usp.br
T. F. Ma

We consider plate equations with a small non-autonomous perturbation of type

$$u_{tt} + \Delta^2 u - \Delta_p u - \Delta u_t + f(u) = \epsilon g(x,t),$$

defined in bounded 2D domain. Under subcritical growth for f we prove the existence of global pullback attractors and upper-semicontinuity with respect to ϵ.

A constructive existence proof for a moving boundary fluid-composite structure interaction problem

Boris Muha
University of Zagreb, Croatia
borism@math.hr
Suncica Canic

Motivated by modeling blood flow in human arteries, we study a fluid-structure interaction problem in which the structure is composed of multiple layers, each with possibly different mechanical characteristics and thickness. In the problem presented in this talk the structure is composed of two layers: a thin layer modeled by the 1D wave equation, and a thick layer modeled by the 2D equations of linear elasticity. The flow of an incompressible, viscous fluid is modeled by the Navier-Stokes equations. The thin structure is in contact with the fluid, thereby serving as a fluid-structure interface with mass. The coupling between the fluid and the structure is nonlinear. The resulting problem is a nonlinear, moving-boundary problem of parabolic-hyperbolic-hyperbolic type. We will show that the model problem has a well-defined energy, and that the energy is bounded by the work done by the inlet and outlet dynamic pressure data. The spaces of weak solutions reveal that the presence of a thin fluid-structure interface with mass regularizes solutions of the coupled problem. We will present the main steps of a constructive proof of an existence of a weak solution for the considered problem. Theoretical results will be illustrated with numerical examples.

Controlling the kinetic energy spectrum in atmospheric turbulence

Keith Myerscough
CWI, Amsterdam, Netherlands
keith@cwi.nl
Jason Frank, Ben Leimkuhler

Small scale dissipation plays a vital role in establishing an equilibrium energy spectrum in 2D (atmospheric) turbulence. Because Global Circulation Models cannot represent these dissipative scales they require other methods to ensure dissipation at the smallest resolved scales. Such methods commonly involve the artificial dissipation of energy from these scales, not only resulting in a perturbed kinetic energy spectrum, but also prohibiting the growth of small scale instabilities. We propose attributing higher priority to known equilibrium statistical properties of fluid flow, at the expense of strict adherence to the model equations. In particular we augment the simulation of a 2D turbulent flow in a periodic box with a control such that the average kinetic energy spectrum is maintained when the resolution of the numerical method is insufficient to resolve the dissipative scales. The chosen target kinetic energy spectrum may be taken from data, from a high fidelity simulation of from theoretical predictions (Kolmogorov, KLB).

Homogenization of nonlinear systems

Antonio Jesús Pallares Martín
Universidad de Sevilla, Spain
ajpallares@us.es
Marc Briane, Juan Casado-Díaz, Manuel Luna-Laynez

The homogenization theory provides a mathematical framework for the analysis of composite materials with complete generality. In this work, we study the homogenization of systems of nonlinear equations. We seek conditions for the Γ-limit of a sequence of equi-coercitive upper unbounded functionals to be of the same type, at least for the regular functions. It is well known that in the case of equations, it is enough to assume weak compactness in L^1 of the coefficients. However, since that result is based on the maximum principle, this is not true for systems in general. Consequently, we obtain other conditions which are applicable to systems.

Dynamic Parameters Target Recognition for Port Protection System

Mohamad Nakcha
HIAST Institute, Syria
dnakshi@yahoo.com
Einas Alhaji, Mohamad Alchaita

Dynamic Parameters Target Recognition for Port protection System Abstract The dynamic system helps designers to reach solution for some hard situation. With noisy environments, the rise in underwater intrusion technologies promotes terrorist groups to increase their attacks. A dynamic monitoring system approach is discussed in this paper. This approach considers the site investigation using combining equipment types such as sonar, hydrophones, communication tools, multithreading software programs, dynamic parameters adjustments and so on, to get the maximum available data. After that, results data will be analyzed and sent to an appropriate processing part to detect any intruders' threat. Design simulation was developed and tested. Hardware prototype was built with new software versions. The recent trial result is shown, and trial results illustrated.

Zonal Controllability of fuzzy Semilinear system

Purnima Pandit
The M. S. University of Baroda, India
pkpandit@yahoo.com

Controllability is one of the important aspects for a dynamical system. In parabolic system controllability can be achieved using boundary or zonal control. The case of fuzzy zonal controller arises when there is imprecision regarding the zone over which the controller is applied. We discuss the results regarding the existence of such controller. The algorithmic approach for implementation for such a controller is given and illustrated with example.

A permutation encoding the connecting orbit structure of slowly non-dissipative systems

Juliana Pimentel
University of Lisbon, Portugal
pimenteljfs@gmail.com
Carlos Rocha

We consider non-dissipative dynamical systems generated by scalar reaction-diffusion equations. In particular, we address the recently introduced class known as slowly non-dissipative systems, which comprises those systems exhibiting blow-up only in infinite time. By extending known results, we are able to obtain a complete decomposition of the non compact global attractor. The existence of unbounded trajectories on the attractor requires the introduction of some objects interpreted as equilibria at infinity, yielding a more complex orbit structure than that appearing on dissipative systems. Under this setting, we still manage to determine the heteroclinic connections based on the Sturm permutation method.

On recovery of harmonic functions from partial boundary data respecting internal pointwise values

Dmitry Ponomarev
INRIA Sophia Antipolis, France
dmitry.ponomarev@inria.fr
Laurent Baratchart, Juliette Leblond

We consider the partially overdetermined boundary-value problem for Laplace PDE in a simply connected domain $\Omega \subset \mathbb{R}^2$ subject to the Dirichlet and Neumann data $u_0 \in W_{\mathbb{R}}^{1/2,2}(\Gamma)$, $\omega_0 \in W_{\mathbb{R}}^{-1/2,2}(\Gamma)$ on $\Gamma \subset \partial\Omega$. Assuming the domain boundary $\partial\Omega$ to be a Jordan curve of Hölder class, by conformal invariance, we may consider Ω to be the unit disk. To regularize this ill-posed Cauchy problem, we cast the following norm-constrained best approximation problem: given $f \in L^2(\Gamma)$ and a regularizing parameter $M > 0$, we search for $g \in H^2(\Omega)$, an analytic function in the disk with $L^2(\partial\Omega)$ trace, such that $\|g - f\|_{L^2(\Gamma)}$ is minimal and $\|g\|_{L^2(\partial\Omega\setminus\Gamma)} \leq M$. We investigate a more general formulation of this bounded extremal problem, when there is additional *a priori* information: we require the analytic function to take on prescribed pointwise values inside Ω. We prove well-posedness of this problem and propose a new computationally efficient method for its solution. The complex-analytic framework allows natural interpretation of pointwise interpolation conditions inside Ω. This may find numerous applications such that those where physical locations of measurements (sensor positions) are not well aligned along Γ but instead partly fall inside Ω. The present approach makes it possible to take into account such outlying measurements rather than discard them when constructing the interpolation functions u_0, ω_0 or else modify the boundary of the domain into a less regular one.

Absorbing boundary conditions and geometric integration for two coupled wave equations

Ana Portillo
Universidad de Valladolid, Spain
anapor@mat.uva.es
Isaías Alonso-Mallo

Initial value problems with two coupled wave equations are considered. The problem is discretized in space using fourth order implicit finite differences. In order to reduce the computation to a bounded domain, absorbing boundary conditions are deduced. Well posed systems with fifth order of absorption for the diagonalizable case, and third order of absorption for the non diagonalizable one, are obtained. When there are two very distinct velocities, a part of the solution is absorbed while another substantial part of the solution is still left inside the computational interval. This leads to longer time computation which supports the simultaneous use of geometric methods and absorbing boundary conditions. Advantages and disadvantages of the use of a symplectic time integrator are displayed by means of some numerical experiments.

Semi-discrete weakly damped nonlinear Klein-Gordon Schrodinger system

Marilena Poulou
University of Aegean, Greece
mpoulou@aegean.gr
Goubet O.

We consider a semi-discrete in time relaxation scheme to discretize a damped forced nonlinear Klein-Gordon Schrödinger system. This provides us with a discrete infinite-dimensional dynamical system. We prove the existence of a finite dimensional global attractor for this dynamical system.

An automatic cleaning algorithm for nonlinear wave equations

Nuria Reguera
Universidad de Burgos, Spain
nreguera@ubu.es
I. Alonso-Mallo

In this work we propose an automatic algorithm, based on a cleaning procedure, for detecting and generating solitary waves of nonlinear wave equations. It is an improvement of the iterative cleaning technique frequently used in the literature. Our algorithm carries out dynamic simulations in which the initial condition evolves with time into a main pulse along with smaller dispersive tails. The numerical solution is analyzed at each time step, checking conditions about the conservation of the amplitude and about the symmetry. Moreover, a cleaning strategy is applied in an automatic way for a suitable period of time inside an appropriate region of space until a solitary wave is obtained with the required precision.

The Asymptotic behavior of the Euler equations

Jaiok Roh
Hallym University, Korea
joroh@dreamwiz.com

In this talk, we will consider the two dimensional Euler equations in R^2. And, for a given smooth initial condition $u_0 \in L^p(R^2)$, with compact supported vorticity, we will show that L^p-norm of the smooth solution $u(t)$ can not decay faster than $t^{-\frac{1}{3}(1-\frac{2}{p})}$ as $t \to \infty$, for $2 \le p \le \infty$. Also, we will show that such solutions can not grow faster than $t^{\frac{1}{3}(\frac{2}{p}-1)}$ as $t \to \infty$, for $1 < p < 2$. Moreover, we will also look at for an exterior problem of the two dimensional Euler equations.

Heat content asymptotic propagation in compact domains with discontinuous transmission boundary conditions

Anna Rozanova-Pierrat
Ecole Centrale Paris, France
anna.rozanova-pierrat@ecp.fr
Claude Bardos, Denis Grebenkov

We consider a heat problem posed in $\mathbb{R}^n$ with discontinuous coefficients over $\partial\Omega$ and discontinuous boundary transmission conditions with a resistance coefficient λ:

$$\begin{aligned}
&\delta_t u^\pm - \operatorname{div}(D_\pm(x)\nabla u^\pm) = 0 \quad x \in \mathbb{R}^n,\ t > 0,\\
&u^+|_{t=0} = 1_\Omega(x), \quad u^-|_{t=0} = 0,\\
&D_-(x)\frac{\partial u^-}{\partial n}|_{\partial\Omega} = -\lambda(x)(u^+ - u^-)|_{\partial\Omega},\\
&D_+(x)\frac{\partial u^+}{\partial n}|_{\partial\Omega} = D_-(x)\frac{\partial u^-}{\partial n}|_{\partial\Omega},
\end{aligned}$$

where Ω represents a “hot body” and $\mathbb{R}^n \setminus \overline{\Omega}$ is a “cold” one. We obviously suppose that $D_-|_{\partial\Omega} \neq D_+|_{\partial\Omega}$, what excludes the case of continuous metric $g_-|_{\partial\Omega} = g_+|_{\partial\Omega}$ considered by P. Gilkey and K. Kirsten [Heat content asymptotics with transmittal and transmission boundary conditions, J. London Math Soc. 68:431–443, 2003].

On the validity of the Boltzmann equation: short range potentials and beyond

Chiara Saffirio
University of Zurich, Switzerland
chiara.saffirio@math.uzh.ch
M. Pulvirenti, S. Simonella

We will introduce the problem of the validity of the Boltzmann equation. After a brief review, we focus on the case of a classical system of N particles interacting by means of a short range potential. We show that, in the low–density limit, the system behaves as predicted by the associated Boltzmann equation. This is an extension of the unpublished thesis by King (appeared after the well known result of Lanford for a system of hard spheres). Our analysis applies to any stable and smooth potential. The results presented are obtained in collaboration with M. Pulvirenti and S. Simonella.

The basis of the space of spherical polynomials with quadratic forms

Ketevan Shavgulidze
Tbilisi State University, Rep of Georgia
ketevanshavgulidze@yahoo.com

A homogeneous polynomial $P(x) = P(x_1, x_2, \cdots, x_r)$ of degree ν with complex coefficients, satisfying the equation

$$\sum_{1\le i,j\le r} a^*_{ij}\frac{\partial^2 P}{\partial x_i \partial x_j} = 0$$

is called a spherical polynomial of order ν with respect to quadratic form $Q(x)$, where

$$Q(x) = \sum_{1\le i\le j\le r} b_{ij}x_i x_j.$$

In this paper the spherical polynomials with quadratic form of five variables are constructed and the basis of the space of these spherical polynomials is established.

Initial propagation of support in thin film flow equation

Andrey Shishkov
Institute of Appl.Math.and Mech.,Donetsk, Ukraine
aeshkv@yahoo.com

We study the behavior for small time of interfaces connected with supports of strong solutions to the Cauchy problem for thin-film equation with compactly supported initial function. We obtain in some sense sharp upper estimates of mentioned interfaces for initial functions with arbitrary asymptotic behavior near to the boundary of their supports.

Plane wave and traveling wave solutions induced by time-delay feedback in an oscillatory reaction-diffusion system

Michael Stich
Aston University, England
m.stich@aston.ac.uk

The supercritical Hopf bifurcation is one of the simplest ways that a stationary state in a nonlinear system can undergo a transition to sustained oscillations. If we are considering a reaction-diffusion system, the dynamics of a system that undergoes such a bifurcation is described by the complex Ginzburg-Landau equation (CGLE). The CGLE possesses a wide range of solutions, including regimes of spatiotemporal chaos. Time-delay feedback is a very efficient way to control chaotic states and replace them by regular solutions. In this work, plane and traveling waves are studied as solutions of a one-dimensional CGLE subjected to local and global time-delay feedback terms. The onset is described as an instability of the uniform oscillations with respect to spatially periodic perturbations. The solutions of the different wave patterns are given analytically and compared to numeric computations.

Exponential dependence of the lifetime of spiral chaos on system size

Kaori K. Sugimura
Ochanomizu University, Japan
sugimura.kaori@is.ocha.ac.jp
Hiroshi Kori

An excitable medium is known to show spatiotemporal chaos where the cores of spiral waves spontaneously generate or annihilate. Depending on initial conditions and parameter values, such chaotic dynamics arise only as a transient process. In this case, the system eventually arrives at a steady quiescent state after a transient time, which we refer to as lifetime. A previous numerical study demonstrated that the lifetime increases exponentially with the system size, but its mechanism has not been fully understood. To elucidate its mechanism, using the Baer model (which is a modified FitzHugh-Nagumo model), we investigate the dependence of various dynamical properties of excitable media on the system size. For both periodic and free boundary conditions, cores of the spiral waves (namely, defects) continue to generate or annihilate, and the number of defects vanishes when the system falls into a steady state.We analytically showed that the dependence of lifetime on the system size holds to be exponential under a mild condition for their functional forms. This type of the spatiotemporal chaos is believed to occur in the heart in fibrillation. We will discuss the implication of our study to this clinical treatment.

Boundary value problems in the linear theory of thermoelasticity for solids with double porosity

Merab Svanadze
Ilia State University, Rep of Georgia
svanadze@gmail.com

The theories of elasticity and thermoelasticity of double porosity media, as originally developed for the mechanics of naturally fractured reservoirs, has found applications in biomechanics. In the recent years, a great attention has been paid to the theories of poroelasticity taking into account the thermal effects. We shall consider the dynamical case of the coupled linear theory of thermoelasticity for solids with double porosity. The system of PDEs of the considered theory is first set up starting from motion equations, conservation of fluid mass, constitutive equations, extended Darcy's law for materials with double porosity and Fourier's law for heat conduction. The boundary value problems (BVPs) of the steady vibrations are investigated. The fundamental solution of system of equations of steady vibrations is constructed. The Green's formulas and the representations of general solution of equations of steady vibrations are obtained. The basic properties of plane waves and the radiation conditions for regular vector are established. The uniqueness theorems of the internal and external BVPs of steady vibrations are proved. The basic properties of surface (single-layer and double-layer) and volume potentials are established. The existence of regular solution of the BVPs by means of the boundary integral equation method and the theory of singular integral equations are proved.

Boundary integral equation method in the theory of thermoviscoelasticity for solids with voids

Maia Svanadze
Tbilisi State University and University of Goettingen, Rep of Georgia
maia.svanadze@gmail.com

In the present paper the linear theory of thermoviscoelasticity for Kelvin-Voigt materials with voids is considered. The fundamental solutions of the systems of equations of steady vibrations and quasi-statics are constructed by means of elementary functions and their basic properties are established. The formulas of representations of the general solutions for the systems of equations are established. The completeness of these representations of solutions is proved. The Green's formulas and integral representations of Somigliana type of regular vector and classical solution are obtained. The uniqueness theorems of the internal and external basic boundary value problems of are proved. The basic properties of thermoelastopotentials and singular integral operators are established. Finally, the existence theorems for classical solutions of the above mentioned boundary value problems are proved by using the boundary integral equation method and the theory of singular integral equations.

Numerical Analysis of One Partial Delay Differential Equation of Cell Replication and Maturation Process

Solodushkin Svyatoslav
Ural Federal University, Russia
solodushkin_s@mail.ru

We consider a first order partial differential equation with a time delay and a retardation of a state variable. Coefficients are assumed to be dependent on state variable. Equations of this type arise in a cell replication modelling when cells are structured by age and maturity. As a part of qualitative analysis we formulate the Cauchy problem, give a definition of a solution and provide conditions that sufficient for the existence, uniqueness and stability of a solution. We construct a family of grid schemes for the numerical solution of this equations and prove the convergence of the second-order method coordinatewise and do that of the first-order with respect to time. We apply constructed scheme for the numerical investigation of time-age-maturation structured cell population.

Existence and uniqueness of monotone and bounded solutions for a finite-difference discretization à la Mickens of the generalized Burgers-Huxley equation

Anna Szafrańska
Gdańsk University of Technology, Poland
aszafranska@mif.pg.gda.pl
J. E. Macías-Díaz

Departing from a generalized Burgers-Huxley partial differential equation, we provide a Mickens-type, nonlinear, finite-difference discretization of this model. The continuous system is a nonlinear regime for which the existence of traveling-wave solutions has been established previously in the literature. We prove that the method

proposed also preserves many of the relevant characteristics of these solutions, like the positivity, the boundedness, and the spatial and the temporal monotonicity. The main results provide conditions that guarantee the existence and the uniqueness of monotone and bounded solutions of our scheme. The technique was implemented and tested computationally, and the results confirm both a good agreement with respect to the traveling-wave solutions reported in the literature, and the preservation of the mathematical features of interests.

On the Lame-Clapeyron-Stefan problem with a convective boundary condition on the fixed face

Domingo Tarzia
CONICET & Univ. Austral, Argentina
DTarzia@austral.edu.ar

We obtain the equivalence for the two-phase Lame-Clapeyron-Stefan problem for a semi-infinite material between the temperature and convective boundary conditions at the fixed face, in the case that an inequality for the convective heat transfer coefficient is satisfied. Then, an inequality for the coefficient which characterizes the solid-liquid interface of the classical Neumann solution is also obtained. Moreover, for the corresponding one-phase problem the Solomon-Wilson-Alexiades mushy region model is studied when a convective or a heat flux boundary condition on the fixed face is imposed.

Linearized problem for viscous free surface flow

Kyoko Tomoeda
Setsunan University, Japan
tomoeda@mpg.setsunan.ac.jp
Yoshiaki Teramoto

We consider the linearized problem describing motion of a viscous incompressible fluid flow down an inclined plane under the effect of gravity. We formulate the problem for downward periodic disturbances from the laminar steady flow as an evolution equation in the product of some Sobolev spaces. It is crucial to study qualitative behavior of solutions that the operator arising in the linearized problem has compact resolvent operators and generates an analytic semigroup in the above space. In this talk we will talk about the construction of the resolvent operator.

A new way for decreasing of amplitude of wave reflected from artificial boundary condition for 1D nonlinear Shrodinger equation

Evgeny Trykin
Lomonosov Moscow State University, Russia
emtrykin@gmail.com
Vyacheslav A. Trofimov

In this report, we focus on computation performance enhancing at computer simulation of a laser pulse interaction with optical periodic structure (photonic crystal). As rule, a photonic crystal length is about 0.1-1 μm. In this case, the length of domain corresponding to linear propagation of laser pulse is many order of magnitude larger than the photonic crystal length. Therefore, decreasing the domain before the photonic crystal one can essentially increases a computation performance. With this aim, firstly, we provide a computation in linear medium and preserve the complex amplitude at chosen section along coordinate laser light propagates. Then we use this time-dependent value of complex amplitude as left boundary condition for the 1D nonlinear Schrödinger equation in decreased domain containing a nonlinear photonic crystal. Because we reformulate the problem we have to take into account two waves propagating in both direction. However, part of a laser pulse reflects from faces of photonic crystal layers. Therefore, we use artificial boundary condition. To decrease amplitude of the reflected wave we introduce in consideration some additional number of waves related with the equation under consideration.

Peculiarities of iteration process convergence assessment for semiconductor plasma generation problem by femtosecond pulse

Evgeny Trykin
Lomonosov Moscow State University, Russia
emtrykin@gmail.com
V.A. Trofimov, V.A. Egorenkov, M.M. Loginova

In this report, we develop conservative finite-difference scheme (CFDS) for the problem of semiconductor plasma generation induced by femtosecond laser pulse propagating in semiconductor placed in an external electric field. This process described by set of non-stationary 2D differential equations formulated to both free-electron concentration and ionized donor concentration together with Poisson equation for potential of electric field induced by plasma. Proposed CFDS is based on an original two-step iteration process which is applied to the set of equations. Its using results in both its applicability for problems with complicated boundary conditions and an asymptotic stability of the finite-difference scheme. Last property provides computations on long time interval without losing the conservatism of the finite-difference scheme. For solving the Poisson equation, we use additional iteration process. Thus, to solve the main problem it is necessary to achieve convergence of two iterative processes. Computer simulation results show that the choice of criterion for iterative process stopping can significantly affect on the accuracy of computation. For stopping of the global iteration process, we use the criterion, which is based on comparison of absolute and relative solution deviation at each iteration. For Poisson equation, the criterion of iteration stopping is: modulus of discrepancy is less than chosen accuracy.

Plate models in elasticity by simultaneous homogenization and dimensional reduction

Igor Velcic
University of Zagreb, Croatia
igor.velcic@fer.hr
Peter Hornung, Stefan Neukamm

We will start from $3D$ nonlinear elasticity and by means of Γ-convergence derive the models for bending plate, starting from the oscillating material with the oscillations of order $\varepsilon(h)$, where h is the thickness of the plate. Depending

on the relation between the oscillations of the material and thickness we obtain diffent models. We recognize different situations: $\varepsilon(h) \ll h$, $\varepsilon(h) \sim h$ and $\varepsilon(h)^2 \ll h \ll \varepsilon(h)$. In the case $\varepsilon(h) \sim h$ the model depends on the additional parameter $\gamma = \frac{h}{\varepsilon(h)}$. We also explain why is the regime $h \sim \varepsilon(h)^2$ critical.

Ambrosetti–Prodi-type problems for quasilinear elliptic equations with nonlocal boundary conditions on non-smooth domains

Alejandro Velez-Santiago
University of California, Riverside, USA
alejovelez32@gmail.com

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded Lipschitz domain, for $N \geq 2$. We investigate the solvability and regularity of the so called "Ambrosetti–Prodi problem" involving the p-Laplace operator, and a sort of nonlocal Neumann boundary conditions that involve a nonlocal Besov operator. Using a priori estimates, regularity theory, a sub-supersolution method, and the Leray-Shauder degre theory, we obtain a necessary condition for the non-existence of solutions (in the weak sense), the existence of at least one minimal solution, and the existence of at least two distinct solutions. Moreover, when the boundary value problem is solvable, we prove that the weak solutions are Hölder continuous over $\overline{\Omega}$.

Asymptotic analysis of Poisson's equation in thin domains with oscillatory boundaries

Manuel Villanueva-Pesqueira
Universidad Complutense de Madrid, Spain
manuelvillanueva@mat.ucm.es
José M. Arrieta

We analyze the limit behavior of solutions to Poisson's equation with Neumann boundary conditions in a thin domain with a highly oscillatory behavior. Our thin domains are given by

$$R^\epsilon = \{(x, y) \in \mathbb{R}^2 \mid x \in (0, 1),\, 0 < y < \epsilon\, G(x, x/\epsilon)\},$$

where G is a smooth function and $G(x, \cdot)$ is $l(x)-$periodic, with the function $l(\cdot)$ not being necessarily constant. Notice that, instead of the classical periodicity hypothesis, we consider thin domains where the oscillations are locally periodic in the sense that both the amplitude and the period of the oscillations may not be constant and actually they vary in space. We are interested in understanding the influence of both, the varying amplitude and period of the function G, in the limit homogenized problem. We obtain the asymptotic homogenized limit as the thickness parameter tends to zero and provide some correctors by extending the Unfolding Method to the locally periodic media. We will also present how to adapt the main ideas of this extension to other locally periodic cases like reticulated or perforated domains.

Homogenization for double porosity models for incompressible flow in a fractured reservoir

Anja Vrbaski
University of Zagreb, Croatia
avrbaski@math.hr
Mladen Jurak, Leonid Pankratov

A fractured reservoir is a kind of porous medium frequently encountered in hydrology and petroleum applications, which is characterized by the presence of two porous structures with strongly contrasted transport features: a continuous system of highly conductive fractures is intertwined by a disconnected periodic set of blocks of usual porous media. We consider incompressible fluid flows through a fractured reservoir. By applying the homogenization theory we derive models describing the global behavior of incompressible one- and two-phase flows in fractured porous media of standard double porosity type, and in porous media with thin fractures, where the thickness of the fractures is considered as an additional small parameter.

Determination of Heat Processes from Measurements at a Single Point

Kim Tuan Vu
University of West Georgia, USA
vu@westga.edu

Consider the multidimensional heat conduction problem described by

$$\begin{cases} u_t(x,t) = \Delta u(x,t) + q(x)u(x,t), \\ \qquad\qquad\qquad x \in \Omega \subset \mathbb{R}^m,\ m \geq 2, \\ u(.,t)|_{\partial\Omega} = 0, \quad t > 0, \\ u(x,0) = f(x) \in L^2(\Omega), \end{cases}$$

where Ω is a unknown connected bounded domain, and q is a unknown heat coefficient.
Let b be a fixed, but an arbitrary point on $\overline{\Omega}$. We prove, for the Dirichlet problem (), that the measurements $u(b,t)$ for $t \in (0,T)$, at a single point b, and for countably many initial data f, are enough for recovering the unique heat coefficient $q(x)$, and the domain Ω. Similar results are also obtained for the wave equation. Moreover, the proof also provides an explicit algorithm to recover q and Ω. In the one-dimensional case we show that for any $q \in L_1(0,l)$, we can recover a unique q and the length l of the interval after a finite number of measurements at the endpoint $x = 0$. If an upper bound for $\|q\|_1$ is known, then we would know explicitly the maximum number of required measurements. If, moreover, a lower bound of q is known a priori, a suitable choice of two initial conditions is enough to recover the coefficient q.

Asymptotic expansion of solutions to the drift-diffusion equation with critical dissipation

Masakazu Yamamoto
Hirosaki University, Japan
yamamoto@cc.hirosaki-u.ac.jp
Keiichi Kato, Yuusuke Sugiyama

We study the initial-value problem for the drift-diffusion equation which stands for the model of semiconductors. The dissipation of this equation is given by the fractional Laplacian. The fractional Laplacian is a generalization of the diffusion which is induced by the Brownian motion. When the exponent of dissipation is large, the drift-diffusion equation is a parabolic equation. In this case, by applying L^p-theory for parabolic equation, large-time behavior of solutions was already derived. On the other hand, if the dissipation is given by the half Laplacian, then our equation is of elliptic-type. Thus L^p-theory for parabolic-equation is useless in this case. Our aim is to show that the asymptotic profile of solutions to the drift-diffusion equation of elliptic-type is provided by the fundamental solution of the linear equation.

Global solutions to the 2D viscous, non-resistive MHD system with largebackground magnetic field

Ting Zhang
Zhejiang University, Peoples Rep of China
zhangting79@zju.edu.cn

In this talk, we consider the global existence and uniqueness of the solutions to the 2D viscous, non-resistive MHD system. For any initial data, if the background magnetic field is sufficiently large, then we can obtain the global strong solutions.

$$\begin{cases} \partial_t b + v \cdot \nabla b = b \cdot \nabla v, \quad (t,x) \in \mathbb{R}^+ \times \mathbb{R}^2, \\ \partial_t v + v \cdot \nabla v - \Delta v + \nabla p = b \cdot \nabla b, \\ \mathrm{div} v = \mathrm{div} b = 0, \\ b|_{t=0} = b_0, \ v|_{t=0} = v_0, \end{cases}$$

$$v(t,x) \to 0, b(t,x) \to (\frac{1}{\varepsilon}, 0)^\top, \text{ when } |x| \to \infty.$$

Cylindrical Korteweg-de Vries (cKdV) type equation for stratified fluid over a shear flow

Xizheng Zhang
Loughborough University, England
x.zhang7@lboro.ac.uk
Karima Khusnutdinova

We study the propagation of a ring wave in a stratified fluid over a prescribed shear flow. A weakly-nonlinear 2+1-dimensional long wave model is derived for internal and surface ring waves from the full set of Euler equations with background stratification and shear flow, written in cylindrical coordinates, subject to the boundary conditions typical for the oceanographic applications. Some particular cases of this equations are introduced. In the absence of the shear flow and stratification, the derived equation reduces to the cylindrical Korteweg -de Vries type equations obtained by V.D. Lipovskii and R.S. Johnson. The features of the ring waves are then studied numerically using a finite-difference code for the derived model.

Contributed Session 7: Bifurcation and Chaotic Dynamics

Wiggins chaos in linear nonautonomous discrete systems

Neda Abbasi
Shahid Beheshti University, Iran
n_abbasi@sbu.ac.ir
Mehdi Pourbarat

Suppose that X is a topological vector space, X_n's are its linear subspaces and the family $\{T_n\}_{n=0}^{\infty}$ with $T_n : X_n \to X_{n+1}$ is translated as a linear nonautonomous discrete system. We introduce a criterion that implies wiggins chaos in linear nonautonomous discrete system under some conditions.

Transition from Regular Structure to Chaos in 4D Spinor Type Instantons

Fatma Aydogmus
Istanbul University, Turkey
fatma.aydogmus@gmail.com
Eren Tosyali

Instantons have finite action with zero energy. They have been considered as configurations of quantum fields providing a tunnelling effect between the vacuums which have different topologies in space-time. In this study, we present the regular characterization of the four dimensional spinor type instantons in phase space and investigate the transition to chaos with external feedback.

Maximum number of limit cycles for generalized Lienard differential equations

Sabrina Badi
Guelma University, Algeria
badisabrina@yahoo.fr
Amar Makhlouf

Applying the averaging theory of first and second order to a class of generalized polynomial Lienard differential equations, we improve the known lower bounds for the maximum number of limit cycles that this class can exhibit.

Abrupt Bifurcations in Chaotic Scattering: view from the anti-integrable limit

Claude Baesens
University of Warwick, United Kingdom, Belgium
claude.baesens@warwick.ac.uk
Yi-Chiuan Chen and Robert S MacKay

Bleher, Ott and Grebogi found numerically an interesting chaotic phenomenon in 1989 for the scattering of a particle in a plane from a potential field with several peaks of equal height. They claimed that when the energy E of the particle is slightly less than the peak height E_c there is a hyperbolic suspension of a topological Markov chain from which chaotic scattering occurs, whereas for $E > E_c$ there are no bounded orbits. They called the bifurcation at $E = E_c$ an abrupt bifurcation to chaotic scattering. The aim of this paper is to establish a rigorous mathematical explanation for how chaotic orbits occur via the bifurcation, from the viewpoint of the anti-integrable limit, and to do so for a general range of chaotic scattering problems.

Conjugacy of symbolic spaces under (j,m)-decimation

Chih-Hung Chang
Feng Chia University, Taiwan
chihhung@mail.fcu.edu.tw
Jung-Chao Ban

This talk considers classification of symbolic spaces under (j, m)-decimation, where m is an integer and $1 \leq j \leq m$. The problem is raised by Abram and Lagarias [W. Abram and J. C. Lagarias, p-adic path set fractals and arithmetic, arXiv:1210.2478, 2013.] on the study of the *p-adic path set fractals and arithmetic* in number theory. Four symbolic spaces, namely, subshift of finite type, sofic shift, path set and p-path set fractal are considered herein. To compute their dimensions under (j, m)-decimation, we first establish their associated (j, m)-adjacency matrices. Such matrix is a rearrangement of the original one according to the pair (j, m). Then we form a new labeled graph by assigning suitable symbols on the edges of the graph induced from (j, m)-adjacency matrices. Finally, we proved that the classification problem can be derived by the *labeled (j, m)-adjacency matrices.* On one hand, the result extends the classical results on the symbolic spaces; On the other hand, it be can applied to the p-adic path set fractals on number theory.

The Lorenz system near the loss of the foliation condition

Jennifer Creaser
Auckland University, New Zealand
j.creaser@auckland.ac.nz
Bernd Krauskopf, Hinke M Osinga

The well-known Lorenz system is classically studied via its reduction to the one-dimensional Lorenz map, which captures the full behaviour of the dynamics of the system. The reduction requires the existence of a stable foliation. We study a parameter regime where this so-called foliation condition fails and the Lorenz map no longer accurately represents the dynamics. Hence, we study how the three-dimensional phase space is organised by the global invariant manifolds of saddle equilibria and saddle periodic orbits. We consider a previously unexplained phenomenon, found by Sparrow in the 1980's where the one-dimensional stable manifolds $W^s(p^{\pm})$ of the secondary equilibria $p^{\pm}$ flip from one side to the other. We characterise this geometrically as a bifurcation in the α-limit of $W^s(p^{\pm})$, which we call an α-flip. We find many such α-flips and, by following them in two parameters, show that each ends at a different co-dimension two bifurcation point, known as a T-point, many of which have not been found before. Moreover, we argue that the α-flip is a precursor to the loss of the foliation condition.

On the number of bifurcations of superstable periodic orbits for $f_c(x) = 1 - cx^2$

Bau-sen Du
Academia Sinica, Taiwan
dubs@math.sinica.edu.tw

Let $f_c(x) = 1 - cx^2$ be a one-parameter family of real continuous maps with parameter $c \geq 0$. For every positive integer m, let N_m denote the number of parameters c such that the point $x = 0$ is a (superstable) periodic point of $f_c(x)$ whose least period divides m (in particular, $f_c^m(0) = 0$). In this talk, we show that $\lim_{m\to\infty}(\log N_m)/m = \log 2$.

Chaos in nonautonomous discrete dynamical systems

Jana Dvořáková
Silesian University in Opava, Czech Rep
Jana.Dvorakova@math.slu.cz

We consider nonautonomous discrete dynamical systems $(I, f_{1,\infty})$ given by sequences $\{f_n\}_{n\geq 1}$ of surjective continuous maps $f_n : I \to I$ converging uniformly to a map $f : I \to I$ and study some aspects of chaotic behavior of such systems. Recently it was proved, among others, that generally there is no connection between chaotic behavior of $(I, f_{1,\infty})$ and chaotic behavior of the limit function f. We show that even the full Lebesgue measure of a distributionally scrambled set of the nonautonomous system does not guarantee the existence of distributional chaos of the limit map and conversely, that there is a nonautonomous system with arbitrarily small distributionally scrambled set that converges to a map distributionally chaotic a.e.

Towards theory of Liénard and Lorenz polynomial systems

Valery Gaiko
NAS of Belarus, Belarus
valery.gaiko@gmail.com

Using a new bifurcational geometric method, we solve the problem of the maximum number of limit cycles surrounding a singular point for an arbitrary planar polynomial dynamical system and Hilbert's Sixteenth Problem for the general Liénard polynomial system with an arbitrary (but finite) number of singular points. Applying this method, we study also three-dimensional polynomial systems and, in particular, complete the strange attractor bifurcation scenario in the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles.

Stability Analysis of Periodic Traveling Wave Solutions for Excitable Media

Mohammad Osman Gani
Meiji University, Japan
osmanganiju@gmail.com
Toshiyuki Ogawa

In this study, a two-variable reaction-diffusion system for excitable media such as cardiac cells, is proposed. We modify the standard FitzHugh-Nagumo equations without changing the slow manifold, but we change the velocity on each branch of the slow manifold instead. It is observed numerically that the periodic traveling wave loses its stability via a supercritical Hopf bifurcation. We investigate the existence of periodic traveling waves for the proposed model via the continuation of a numerically calculated periodic orbit. We use the method of continuation for studying these solutions in one dimension via the software package WAVETRAIN. A change of stability of Eckhaus type in periodic traveling waves in a two-dimensional parameter plane is observed. We calculate wave stability and the stability boundary in the two-dimensional parameter plane. Moreover, a comparison of wave stability between the proposed model and the standard FitzHugh-Nagumo model is reported. In two dimensions, we deduced that the onset of irregular spiral pattern appears when it crosses the Eckhaus stability boundary.

Skew products of interval maps over subshifts

Masoumeh Gharaei
University of Amsterdam, Netherlands
M.gharaei@uva.nl

We consider step skew products over transitive sub-shifts of finite type with interval fibers. We assume that the end points of the interval are fixed under the fiber maps and the skew product has positive Lyapunov exponents at the end points. Following Kleptsyn and Volk in Physical Measures for Nonlinear Random Walks on Interval , for an open and dense subset of such skew products we prove existence of a finite collection of disjoint attracting invariant graphs. In the proof we associate a random walk to the skew product and consider the relation between stationary measures and invariant measures of the skew product.

Microscale thermal convection

Roger Khayat
University of Western Ontario, Canada
rkhayat@uwo.ca
Daniel Stranges, John deBruyn

Rayleigh-Benard convection for a thin fluid layer is examined by assuming the fluid to be of the non-Fourier type. For a microscopically small gap, the thermal relaxation time is of the same order of magnitude as the process times. In this case, non-Fourier effects become important. Linear stability and nonlinear analyses are carried out. Comparison with experiment is also attempted.

Interactions of forward- and backward-time isochrons

Peter Langfield
University of Auckland, New Zealand
p.langfield@auckland.ac.nz
Bernd Krauskopf and Hinke Osinga

In the 1970s Winfree introduced the concept of an isochron as the set of all points in the basin of an attracting periodic orbit that converge to the periodic orbit in forward time with the same asymptotic phase. It has been observed that in slow-fast systems, such as the FitzHugh-Nagumo model, the isochrons of such systems can have complicated geometric features; in particular, regions with high curvature that are related to sensitivity in the system. In order to understand where these features come from, we introduce backward-time isochrons that exist in the basin of a repelling periodic orbit, and we consider their interactions with the forward-isochrons. We show that a cubic tangency between the two sets of isochrons is responsible for creating the high curvature features. We also present a normal-form-type model for this phenomenon.

Influence of a p-root singularity on piecewise-smooth maps

Roya Makrooni
Shahid Beheshti University, Iran
r_makrooni@sbu.ac.ir
Farhad Kkhellat

We consider a one-dimensional continuous piecewise-smooth map with p-root singularity. This map takes the form: $x_{n+1} = f(x_n)$ which is $ax_n + \mu$ for $x_n \leq 0$ and $bx_n^p + cx_n + \mu$ for $x_n \geq 0$, where a, b, c, d, μ are constant real numbers. All through the paper, we will consider p as the bifurcation parameter, i.e. the parameter being varied. For maps with a squared or $\frac{3}{2}$ term the stability and existence conditions of fixed points and period-2 orbits in the vicinity of the border-collision have been found analytically by Halse, et al (2003). When $p = \frac{1}{2}$, for different values of the parameters a, b, c, μ,the above map represents 1D approximation of the discrete-time model of impact oscillators with different physical parameters that has been studied by Nordmark, et al (2010). For $0 < p < 1$, these maps have a singular point at $x = 0$ and we will study how the singularity influences on dynamical behaviour which implies bifurcation when p varies to greater than 1.

A symmetry-preserving POD reduced order model to calculate bifurcations in a Rayleigh-Bénard convection problem

Francisco Pla Martos
Universidad de Castilla-La Mancha, Spain
Francisco.Pla@uclm.es
Henar Herrero and José Manuel Vega

In this work, a flexible Galerkin method based on proper orthogonal decomposition (POD) is applied to a Rayleigh-Bénard convection problem in the limit of infinite Prandtl number using the Rayleigh number as a bifurcation parameter. The fluid is confined between a lower solid plate and an upper non-deformable free surface, and the patterns are assumed to be periodic in the horizontal direction. Restricting to a half of a period, imposing symmetry conditions at the lateral boundaries (which breaks invariance under translations in the laterally infinite layer), and using the Boussinesq approximation, the nondimensional equations and boundary conditions can be written as

$$\begin{aligned}
&\nabla \cdot \mathbf{v} = 0 \quad \text{in } \Omega = [0, \Gamma] \times [0, 1], \\
&\theta \mathbf{e_z} - \nabla P + \frac{1}{\sqrt{R}} \Delta \mathbf{v} = 0 \quad \text{in } \Omega, \\
&\partial_t \theta + \mathbf{v} \cdot \nabla \theta - w = \frac{1}{\sqrt{R}} \Delta \theta \quad \text{in } \Omega, \\
&\mathbf{v} = \mathbf{0},\, \theta = 0 \quad \text{at } z = 0, \\
&\theta = \partial_z u = w = 0 \quad \text{at } z = 1, \\
&\partial_x \theta = \partial_x w = u = 0 \quad \text{at } x = 0 \text{ and at } x = \Gamma.
\end{aligned}$$

Here, $\mathbf{u} = (u, w)$ is the velocity and P and θ are the excess of pressure and temperature over their profiles at the conductive steady state, $\mathbf{e}_z$ is the unitary vector in the vertical direction, R is the Rayleigh number, and Γ is the aspect ratio. This problem exhibits a horizontal reflection symmetry which is exact and also an approximate vertical reflection symmetry, which is due to the large values of the Rayleigh number in the physically relevant regime. A reduced order model will be presented with two main ingredients, namely (i) the symmetries are accounted for in the calculation of the POD modes and (ii) advantage is taken of the property (already tested in related bifurcation problems) that POD manifold resulting from snapshots calculated in either Newton iterations or time-dependent runs for a particular value of the Rayleigh number also contain good approximations of the steady states other values of R. Using these and a basic continuation method on the reduced order model, the bifurcation diagram is calculated at a fairly low computational cost.

Heteroclinic tangencies near a Bykov cycle

Alexandre Rodrigues
Porto University, Portugal
alexandre.rodrigues@fc.up.pt
Isabel Labouriau

In this talk, we present a mechanism for the coexistence of hyperbolic and non-hyperbolic dynamics arising in a neighbourhood of a Bykov cycle where trajectories turn in opposite directions near the two nodes - we say that the nodes have different chirality. We show that in a C^2-open class of vector fields defined on a three-dimensional compact manifold, tangencies of the invariant manifolds of two hyperbolic saddle-foci occur at a full Lebesgue measure set of the parameters that determine the linear part of the vector field at the equilibria. This has important consequences: the global dynamics is persistently dominated by heteroclinic tangencies and by Newhouse phenomena, coexisting with hyperbolic dynamics arising from transversality. The coexistence gives rise to linked suspensions of Cantor sets, with hyperbolic and non-hyperbolic dynamics, in contrast with the case where the nodes have the same chirality.

Stability of synchronization in dissipatively driven Frenkel-Kontorova models

Sinisa Slijepcevic
University of Zagreb, Croatia
slijepce@math.hr

We consider dissipative driven dynamics of infinite one-dimensional elastic chains in a periodic potential (Frenkel-Kontorova models), given by equations

$$du_j/dt = W'(u_{j+1} - u_j) - W(u_j - u_{j-1}) + V'(u_j) + f(t),$$

where W is convex, V periodic, and f constant force (DC case) or periodic force (AC case). It is well-known by Aubry-Mather theory that when $f = 0$, the asymptotic dynamics is stationary and can be described as orbits of a 2D map. We show that for arbitrary force, there is a 2D attracting set. We then prove a numerically observed fact that synchronized dynamics is attracting: globally above the dynamical phase transition, and locally below it. Finally, we show that ideas from Hamiltonian dynamics (the Converse KAM theory) can be used to numerically find critical values of forces corresponding to dynamical phase transitions.

Simulation of complex dynamics using POD on the fly and residual estimates

Filippo Terragni
Universidad Carlos III de Madrid, Spain
fterragn@ing.uc3m.es
Jose Manuel Vega

Proper orthogonal decomposition (POD) is a very effective means to identify dynamical information contained in sets of snapshots resulting from temporal runs of dissipative systems of partial differential equations (SPDEs). With this information organized in POD modes, the SPDE can be Galerkin-projected onto the associated linear manifold. Quite frequently, this process gives a low dimensional system of ordinary differential equations that represents a very computationally efficient approximation of the SPDE. The snapshots may be calculated beforehand along the time dependent solution of interest. Albeit its much faster operation, the low dimensional system just re-calculates the already known dynamics. Instead, POD can be applied on the fly, combining a standard numerical solver with the low dimensional system, in an adaptive way, in interspersed intervals as either time or a bifurcation parameter is varied. Residual estimates help to make this adaptation both computationally efficient and robust. These ideas will be illustrated in several bifurcation scenarios, including quasi-periodic and chaotic dynamics, for the complex Ginzburg-Landau equation. In this application, the CPU time required by the standard solver alone is divided by a significant factor, or the order of ten.

Slow passage through a resonant bifurcation from a robust heteroclinic cycle

Tsung-Lung Tsai
National Changhua University of Education, Taiwan
ttlong23@cc.ncue.edu.tw
Hong-Fu Liu

Slow passage problem is a bifurcation problem in which the control parameter is time dependent. In many cases, there are interesting phenomena that bifurcation may not occur until the parameter beyond the value predicted from a static bifurcation analysis (delay effect) and that the delay in onset is dependent of the initial state of the system (memory effect). In this talk, we consider the standard 3-dimensional Lotka-Volterra system with an added higher order term to ensure that a resonant bifurcation could happen. Delay and memory effects in onset of bifurcation can be observed as expected when ramping a parameter linearly. However, we found that the the delay effect could be quite different from that reported in the literature when we transfer the system into the log-scale: the closer to the static resonant bifurcation point for the initial controlled parameter, the farther away from it for the onset of bifurcation. This suggests us to redefine the onset condition of bifurcation in a more rigorous way. Our result could also be applied to the well-known slow passage through a Hopf bifurcation because of the similarity of the bifurcation structures in these two cases.

On the Minimal Non-invertible Skew-products of 2-manifolds

Jakub Šotola
Silesian University in Opava, Czech Rep
Jakub.Sotola@math.slu.cz
Sergei Trofimchuk

In this talk we will present a new way to construct minimal non-invertible skew products of torus and we will positively answer the question whether there exists a non-invertible minimal map of the Klein bottle. The construction is based on a construction of minimal homeomorphism of Klein bottle by Parry and blowing-up techniques introduced by Hric and Jäger.

References

[1] Hric, R., Jäger, T.: A construction of almost automorphic minimal sets, Israel J. Math., to appear (2014).

[2] Kolyada, S., Snoha, Trofimchuk, S.: Noninvertible minimal maps, Fund. Math. **168**, 141-163 (2001)

[3] Parry, W.: A note on cocycles in ergodic theory, Contemp. Math. **28**, 343-350 (1974)

Poster Session

Perturbations of the boundary for a semilinear parabolic problem with nonlinear Neumann boundary conditions

Pricila Barbosa
University of Sao Paulo, Brazil
priimeusp@gmail.com
Pereira, A.L. ; Pereira, M.C.

The continuity of global attractors for dissipative systems under variation of the domain has been the subject of investigation by many authors. In particular, using techniques developed by D. Henry, the co-authors proved that continuity of attractors (upper and lower) holds for a class of semilinear parabolic problems with nonlinear Neumann boundary conditions under C^2-perturbations of the domain. Our aim here is to extend this result for less regular C^1 -perturbations. In order to achieve this goal, it is necessary to find a way to compare the solutions of the problems defined in different regions, which is done by bringing them back to a fixed region. To deal with the nonlinear boundary conditions, it has also been necessary to work in fractional power spaces with negative exponent.

Blow up and Global Solutions of a 6th order Cahn-Hilliard type equation

Benedict Boyle
University of Bath, England
bb255@bath.ac.uk

Higher order parabolic evolution equations have become a focus of interest in the last few years. In this talk we will discuss asymptotic and numerical approaches to investigate solutions to the Cauchy problem for the 6th order equation

$$u_t = \Delta^3 u - \Delta(|u|^{p-1}u) \tag{3}$$

for various values of $p > 0$. We will be primarily concerned with solutions that blow up in finite time, although some aspects of solutions that exist globally will also be addressed.

Liouville theorems for elliptic systems

Miguel Ángel Burgos Pérez
Universidad de La Laguna, Spain
miguelburgosperez@gmail.com
Jorge Garcia Melian

We consider the elliptic system

$$\begin{cases} -\Delta u + |\nabla u|^q = \lambda v^p \\ -\Delta v + |\nabla v|^q = \mu u^s \end{cases} \qquad \text{in } \mathbb{R}^N \setminus B_{R_0},$$

where $N \geq 3$, $q > 1$, $p, s > 0$, $\lambda, \mu > 0$. We are interested in analyzing the question of nonexistence of positive supersolutions of this system. For several ranges of the exponents involved we show that no positive supersolutions can exist. These ranges of nonexistence turn out to be optimal in some cases.

The stability problem and special solutions for the 5-components Maxwell-Bloch equations

Ioan Casu
West University of Timisoara, Romania
casu@math.uvt.ro
Petre Birtea

For the 5-components Maxwell-Bloch system the stability problem for the isolated equilibria is completely solved. Using the geometry of the symplectic leaves, a detailed construction of the homoclinic orbits is given. Studying the problem of invariant sets for the system we discover a rich family of periodic solutions in explicit form.

A model of the process of habitat fragmentation (loss of connectivity)

Fernando Cordova-Lepe
Universidad Catolica del Maule, Chile
fcordova@ucm.cl
Rodrigo Del-Valle

A process of habitat fragmentation of a population is modeled by a (non-autonomous) differential system that quantifies the abundance at each instant in patches forming. This postulated model is derived from assumptions of disturbance on demographic rates. In addition, some of its dynamic consequences are analyzed. Given a final state ($t = 1$) when the habitat of a population is completely divided into fragments A and B, from an initial ($t = 0$) fully pipelined, we can deduce:

$$\dot{x}_A(t) = r x_A(t) \left\{ 1 - \frac{x_A(t) + \eta(t) x_B(t)}{K_A + \eta(t) K_B} \right\} x_A(t)$$
$$\dot{x}_B(t) = r x_B(t) \left\{ 1 - \frac{\eta(t) x_A(t) + x_B(t)}{\eta(t) K_A + K_B} \right\} x_B(t),$$

where $x_A(t)$ and $x_B(t)$ represent the abundances of subpopulations in training, and $eta[0,1]\ to[0,1]$ is a decreasing function indicating a "degree" of environmental resistance from the patch B to A subpopulation, and vice versa. Notice how this system connects as process, not fragmentation ($\eta(0) = 1$) with for a completely decoupled system ($\eta(1) = 0$).

Globally solvable systems of complex vector fields

Cleber de Medeira
Federal University of Parana, Brazil
cleber3m@gmail.com
Bergamasco A.P. and Zani S. L.

In this work we consider a class of involutive systems of n smooth vector fields on the torus of dimension n + 1. We prove that the global solvability of this class is related to an algebraic condition involving Liouville forms and the connectedness of all sublevel and superlevel sets of the global primitive of a certain 1-form associated with the system.

Conditions for correct solvability of a first order linear differential equation with degeneracy at infinity

Lea Dorel
Bar Ilan University, Israel
lela@post.tau.ac.il
Leonid Shuster

We consider the first order linear differential equation

$$-y'(x) + q(x)y(x) = f(x), \; x \in R$$

where $f \in L_p(R)$, $p \in [1, \infty)$, $0 \leq q \in L_1^{loc}(R)$. Introduce the notation

$$q_0(a) = \inf_{x \in R} \int_{x-a}^{x+a} q(t)\, dt, \quad a \in [0, \infty).$$

For a continuous function θ such that $\theta(x) > 0$ for each $x \in R$, let

$$L_{p,\theta}(R) = \{f \in L_p^{loc}(R) : \|f\|_{p,\theta}^p = \int_{-\infty}^{\infty} |\theta(x)f(x)|^p\, dx$$

for some $a \in (0, \infty)$ is the minimal one to guarantee the fulfillment of conditions (i) and (ii). Here we consider the case when $\|q\|_{L_1(R)} = \infty$, $q_0(a) = 0$ for all $a \in [0, \infty)$ and find necessary and close to them sufficient conditions for θ and q under which (i) and (ii) are satisfied.

References

[1] Chernyavskaya N., *Conditions for correct solvability of a simplest singular boundary value problem*, Math. Nachr. **1** (2002), 5–18.

[2] Lukachev M., Shuster L., *On uniqueness of the solution of a linear differential equation without boundary conditions*, Functional Differential Equations **2** (2007), 337–346.

Analytical solutions of a cancer model

Jorge Duarte
Instituto Superior de Engenharia de Lisboa, Portugal
jduarte@adm.isel.pt
Cristina Januario, Nuno Martins, Carla Rodrigues, Josep Sardanyes

In this article we analytically solve the Itik-Banks tumor growth model by means of a recent technique for strongly nonlinear problems - the step homotopy analysis method (SHAM). This analytical algorithm, based on a modification of the standard homotopy analysis method (HAM) initially proposed by Liao, allows us to obtain a one-parameter family of explicit series solutions for the studied cancer model. These solutions describe the temporal dynamics of tumor cells interacting with healthy host cells and effector immune cells. The artificial parameter involved in the analytical method is particularly important, providing us with an elegant way to ensure convergent series solutions. Our analytical results are found to be in excellent agreement with the numerical simulations. We use the obtained analytical solutions to investigate the role of immune system activation due to antigen recognition. We found that an increase in the stimulation of immune cells by tumor cells makes the system to enter into chaotic dynamics. Our results are discussed in the context of tumor cells dynamics and persistence, as well as in the possible therapeutic consequences of the identified dynamics.

Controllability of parabolic equations arising in climatology

Giuseppe Floridia
Istituto Nazionale di Alta Matematica (INdAM), Italy
floridia.giuseppe@icloud.com

In this poster we address approximate controllability problems via multiplicative controls, motivated by our interest in some differential models for the study of climatology. One of the first attempts to model the interaction between large ice masses and solar radiation on climate is the Budyko-Sellers model. This model studies how extensive is the climate response to an event such as a sharp increase in greenhouse gases. The Budyko-Sellers model is interested in the role played by continental and oceanic areas of ice on climate change. The 1-d Budyko-Sellers model reduces to nonlinear degenerate parabolic equation with Neumann boundary condition. For this problem we will discuss recent results guaranteeing global approximate controllability in large time by bilinear control.

On the dynamical systems of the rheonomic Lagrangian mechanical systems

Camelia Frigioiu
"Dunarea de Jos" University of Galati, Romania
cfrigioiu@ugal.ro

In this paper one studies the dynamical system of a rheonomic nonconservative mechanical system, whose evolution curves are given, on the phase space $TM \times R$, by Lagrange equations of the form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial y^i}\right) - \frac{\partial L}{\partial x^i} = F_i(x, \dot{x}, t); \quad y^i = \frac{dx^i}{dt} = \dot{x}^i,$$

where $L(x, \dot{x}, t)$ is a regular rheonomic Lagrangian, $F_i(x, \dot{x}, t)$ are the components of an external force defined as a $d-$covector field on $TM \times R$. One can associate to the considered mechanical system a vector field S on $TM \times R$, which is the canonical semispray. Then all the geometric objects of the rheonomic Lagrangian mechanical system will be derived from S.

Limit cycles of generalized Liénard polynomial differential systems via averaging theory

Belén García
Universidad de Oviedo, Spain
belen.garcia@uniovi.es
Jaume Llibre, Jesús S. Pérez del Río

Using the averaging theory of first and second order we study the maximum number of limit cycles of the polynomial differential systems $\dot{x} = y$, $\dot{y} = -x - \epsilon(h_1(x) + p_1(x)y + q_1(x)y^2) - \epsilon^2(h_2(x) + p_2(x)y + q_2(x)y^2)$, which bifurcate from the periodic orbits of the linear center $\dot{x} = y$, $\dot{y} = -x$, where ϵ is a small parameter. If the degrees of the polynomials h_1, h_2, p_1, p_2, q_1 and q_2 are equal to n, then we prove that this maximum number is $[n/2]$ using the averaging theory of first order, where $[\cdot]$ denotes the integer part function; and this maximum number is at most n using the averaging theory of second order.

Limit cycles in a Gause type prey-predator model with a rational non-monotonic functional response

Eduardo Gonzalez-Olivares
Instituto de Matematicas, Pontificia Universidad Catolica de Valparaiso,, Chile
ejgonzal@ucv.cl

In this work, a Gause type predator-prey model is analyzed, considering a non-monotonic functional response . One of the main tarjet is to establish the number of limit cycles surrounding a positive fixed point of sytem, showing the existence of two concentric limit cycles. It is also shown the system has two equilibrium points at inside of the first quadrant for a wide subset of parameter values, where one is always a saddle point. When this points collapses a cusp point due to a Bogdanov-Takens bifurcation is obtained. Other behaviours of system are given and in particular the model predicts the populations can coexist for all parameters value since $(0, 0)$ is saddle point, but a great probability of extinction of predadors exists, because the singularity $(K, 0)$ is a local atractor. Then, the phenomenon of bistability appears since two singularities can be local attractor at the first quadrant, or else, a stable limit cycle coexists with a locally asymptotically stable point.

Elliptic systems involving multi-singular inverse square potentials and critical exponents

Tsing-San Hsu
Chang Gung University, Taiwan
tshsu@mail.cgu.edu.tw

In this poster, we investigate a singular elliptic system, which involves multi-singular inverse square potentials and multiple critical exponents. By employing variational methods, the existence of positive solutions to the system is established.

A receptor-based model with coexistence of Diffusion-Driven Instability and hysteresis

Steffen Härting
Heidelberg University, Germany
steffen.haerting@bioquant.uni-heidelberg.de

The aim of this poster is to contribute to the understanding of stability of steady states of systems of ordinary differential equations coupled to reaction-diffusion equations. Such systems arise naturally from the modeling of biological phenomena such as cell-receptor-ligand binding, so called receptor-based models. We present conditions for stability of irregular, discontinuous steady states of a system of one ordinary differential equation coupled to one reaction-diffusion equation. Moreover, we present a model exhibiting Diffusion-Driven Instability and stable discontinuous irregular steady states, i.e. de-novo formation of discontinuous, non-periodic stable steady states. The presentation is based on a joint work with A. Marciniak-Czochra (University of Heidelberg) and I. Takagi (Tohoku University).

A simple population dynamics model of tumor and immune cells under the effect of a combination therapy

Rym Jaroudi
Tunis el manar University, Tunisia
rym.jaroudi@gmail.com
Amira Kebir, Slimane ben Miled, Jean Clairambault

Cancer appears as a result of genetic and epigenetic events and is characterized by a proliferation of uncontrolled tissue cells to form a tumor mass. Tumor cells carry specific antigens recognized by the immune system that triggers an anti-tumor immune response and then cancer competes with the immune system. Although chemotherapy and immunotherapy have different mechanisms of action and toxicities, These two therapies can be combined. The combination therapy is a treatment that uses more than one anticancer drug to prevent resistance mechanisms of cancer cells and to increase treatment effectiveness. In this ODE 's model , we will consider two cell populations: a population of tumor cells and a population of cytotoxic effector cells , namely NK cells or CD8 + T cells. Our goal will be to highlight the effect of a combination therapy..

Canard-Mediated Mixed-Mode Oscillations in a 4D Model of GnRH Pulse and Surge Generator

Elif Koksal
INRIA, France
elif.koksal@inria.fr
Mathieu Desroches, Frédérique Clément

We present canard-mediated mixed-mode oscillations (MMOs) arising in a model for the alternating pulse and surge pattern of gonadotropin releasing hormone (GnRH) secretion. The model is formed by two FitzHugh-Nagumo (FHN) oscillators that evolve on different timescales, with a feedforward coupling from the slow one (regulating system) to the fast one (secreting system). The resulting 4D model involves three different timescales. So far, global and local features of the model have been studied in the context of slow-fast dynamics and mixed-mode oscillations where folded singularities and associated canard trajectories have a particular importance. For instance, so-called secondary canards due to a folded node are responsible for the presence of a plateau with small oscillations in the model output, after the surge and before the return to pulsatility. In this poster, we focus on another canard-mediated slow-fast transition in the system, occurring in between the end of the pulsatility phase and the beginning of the surge. We study the effect of the type of the folded singularity on this delayed transition.

Multiple solutions for the biharmonic elliptic equation with Sobolev critical exponent

Hueili Lin
Chang Gung University, Taiwan
hlin@mail.cgu.edu.tw

The biharmonic equation is a fourth-order partial differential equation. It is well-known that the weak solutions of the biharmonic elliptic equation with Sobolev critical exponent are the critical points of the corresponding energy functional. Under some conditions, by using the variational method, we study the mountain pass lemma and Palais-Smale theory to show the existence of nontrivial solutions for this equation in a bounded domain. Moreover, according to the properties of coefficient function f, we discuss the multiplicity of nontrivial solutions for this equation.

One-dimensional thermal mathematical model for evaluation of the effect of the single glazing in the air temperature of an enclosure under weather conditions.

Juan Miguel Lirola Perez
Universidad Politecnica de Madrid, Spain
juanmi.lirolaperez@upm.es
L. Tello, B. Lauret, M.Khayet.

In this work, we propose a mathematical model for the evolution of temperatures in the interior of a cube with a single glass face. Considerations of conduction, convection and radiation are taken into account. The validation of the model is obtained comparing the results with the experimental gathered data. The approximate solution is obtained by using the method of finite differences.

Dalí canards and singular Hopf bifurcation in the spike-adding mechanism of the Hindmarsh-Rose burster

Megret Lucile
INRIA & Pierre et Marie Curie University, France
lucile.megret@gmail.com
Mathieu Desroches, Jean-Pierre Françoise, Maciej Krupa

As we observe the electrical activities of the secretory cells (GnRH neurons, β-cells of the pancreas,...) we observe an alternation between an *active* phase and a *quiescent* one. A lot of mathematical models have been developed to represent this behavior. They are *dynamical systems* which take the form :

$$\dot{x} = f(x, \alpha),$$

where $x \in \mathbb{R}^n$ ($n \geq 3$) are the variables and $\alpha \in \mathbb{R}^m$ are the parameters. For such a system to have bursting solutions, at least two fast variables and one slow variable are needed; the Hindmarsh-Rose (HR) model is one of the most famous bursters. Even if it is overall well known, many things remain to be understood in the spike adding mechanism. In this poster we present a work about two mathematical phenomena which occur as a spike is added to the burst in the HR model. The first one is the *Dalí canard* which is based on the form of the center manifold near the upper fold of the fast nullcline. The second one is a *singular Hopf bifurcation* that we treat with a new method based on the study of the integrable normal form of this bifurcation.

3-D chimera states in oscillatory networks

Volodymyr Maistrenko
National Academy of Sciences of Ukraine, Kyiv, Ukraine
maistren@nas.gov.ua
Oleksandr Sudakov, Yuri Maistrenko

We report the first observation of three-dimensional chimera states of the following types: *incoherent steaks* (i.e. "tubes" of spiral vorticity) *incoherent ball*, and *incoherent tubes*. The dynamical system of concern is governed by nonlocaly coupled Kuramoto network of N^3 oscillators uniformly distributed on a 3Dim torus. The coupling is supposed to be long-ranged and isotropic: each oscillator is coupled with equal strength to all its nearest neighbors within range R, and with phase shift α. We find that 3-D chimera states exist in a wide region of the parameter space (R, α), where they co-exist with regular space-time regimes. Cascades of 3-D chimera states with increasing number of streaks and other chimera types are obtained in analogy with 1-D case. The massive calculations were performed at the computer cluster "CHIMERA" (http://nll.biomed.kiev.ua/cluster) and GRID infrastructure. The contribution comprises video presentation illustrating the origin and evolution of the 3-D chimera states.

Intrinsic Shape in Dynamical Systems

Zoran Misajleski
"Cyril and Methodius" University, Rep of Macedonia
misajleski@gf.ukim.edu.mk
Martin Shoptrajanov, Nikita Shekutkovski

The interaction between topology and dynamical systems will be considered through shape theory tools. Namely, using the intrinsic approach to shape, the notion of proximate net will be introduced as a sequence of near continuous functions which converge in homotopical sense and a natural way of producing one in a given flow will be discussed. The central part of the presentation will be to generalize the shape-attractor theorem for Morse decomposition and to give the strong shape version, using the intrinsic approach.

On the failure of Kronecker's density theorem for powers of an algebraic number

Maurizio Monge
Universidade Federal do Rio de Janeiro, Brazil
maurizio.monge@im.ufrj.br
N. Dubbini

We will present a quantitative estimate on the failure of Kronecker's density theorem for the subgroup of the torus generated by the vector formed by m powers of an algebraic number, when m is big. We prove that the resulting subgroup is epsilon-dense, where epsilon is related to the Mahler measure of the algebraic number. The problem is motivated by a problem in control theory, where we assume that only the integral part of the behaviour

is known. The estimate on the density is proved to be best-possible up to a constant, for m big enough; this optimality is proved by means of a result on linear recurrences of finite length, and estimates on the determinant of Toeplitz matrices. We formulate a conjecture on the constant provinding the best possible estimate, relating our problem to algebraic dynamical systems on the torus.

Positive solutions of a nonlinear elliptic problem involving the Dirichlet energy

Daisuke Naimen
Oska City University, Japan
d12sax0J51@ex.media.osaka-cu.ac.jp

In this poster, we investigate a nonlinear elliptic problem involving the Dirichlet energy (called a Kirchhoff type problem, in common). We shall see several new effects of a nonlocal coefficient on the well-known existence, nonexistence and uniqueness results for semilinear elliptic problems. Our results include the critical case.

Can collagen fibers in the fish scales associate and align autonomously?

Etsushi Nakaguchi
Tokyo Medical and Dental University, Japan
nakaguti.las@tmd.ac.jp

The scale of bony fish, such as goldfish or carp, has layer structure of two layers; the upper one is the calcified layer, and the lower the fibrous layer. The fibrous layer is formed by a large number of stratified thin sheets of collagen fibers with no cell inside. The collagen fibers are produced as molecules by the osteoblasts (bone formative cells) located at the floor of the fibrous layer, associate after secretion, and then align to form thin sheets; but the mechanisms are unknown. We present a model of autonomous association and alignment system of fiber molecules. The distribution of molecules is approximated by continuously distributed dipoles, and is assumed to form some potential fields for self-attraction and orientation. We then describe the model by a system of parabolic and elliptic PDEs like chemotaxis systems. Some mathematical analysis and numerical simulation will be also demonstrated in the presentation.

Numerical analysis of the diffraction problem on prefractal impedance grating based on boundary integral equations

Kateryna Nesvit
Karazin Kharkiv National University, Ukraine
nesvit.k@gmail.com

The models which are given in this paper are an approximation of real fractal antennas in 2D. Transverse electric and magnetic wave diffraction problems on prefractal impedance strips are considered in this paper. This type of problem leads to boundary integral equations with logarithmic singularities. The main idea of the present paper is to numerically analysis both cases based on their mathematical model. A discrete mathematical model of the boundary integral equations has been developed with the help of special quadrature formulas with the nodes in the nulls of Chebyshev polynomials. This research is completely aimed at performing numerical experiments with the help of an efficient discrete singularities method.

A mixed problem with an integral two-space-variables condition for parabolic equation with the bessel operator

Taki Eddine Oussaeif
Oum El Bouaghi, University, Algeria
taki_maths@live.fr
Bouziani Abdelfatah

In this paper, we study a mixed problem with an integral two-space-variables condition for parabolic equation with the bessel operator. The existence and uniqueness of the solution in functional weighted Sobolev space are proved. The proof is based on a priori estimate “energy inequality” and the density of the range of the operator generated by the problem considered.

Convex Billards on Surfaces of Constant Curvature

Sonia Pinto-de-Carvalho
Universidade Federal de Minas Gerais, Brazil
sonia@mat.ufmg.br
Luciano Coutinho dos Santos

Let S be the Euclidean plane, the hyperbolic plane or a hemisphere of the unit sphere. The convex billiard problem consist on the free motion of a particle inside a geodesically convex bounded region on S, making elastic collisions at the impacts with the boundary. This problem defines a class of area preserving twist diffeomorphisms, giving rise to 2-dimensional conservative dynamical systems. In this work we will present the generic dynamics of such billiards, and show they exhibit the richness of the dynamics of conservative systems.

Intrinsic Shape in Dynamical Systems

Martin Shoptrajanov
“Cyril and Methodius” University, Rep of Macedonia
martin@pmf.ukim.mk
Nikita Shekutkovski, Zoran Misajleski

The interaction between topology and dynamical systems will be considered through shape theory tools. Namely, using the intrinsic approach to shape, the notion of proximate net will be introduced as a sequence of near continuous functions which converge in homotopical sense and a natural way of producing one in a given flow will be discussed. The central part of the presentation will be to generalize the shape-attractor theorem for Morse decomposition and to give the strong shape version, using the intrinsic approach.

Some Remarks on Globally Hypoelliptic Pseudodifferential Operators on $\mathbb{T} \times M^n$

Fernando Silva
UFPR, Brazil / Univ. Cagliari, Brazil
favilasi@gmail.com
Todor Gramchev, Alexandre Kirilov

In this work we study the global hypoellipticity of a class of operators of type

$$L = D_t + a(t)Q(x, D) + ib(t)P(x, D), \quad D_t = i^{-1}\partial_t,$$

where $(t, x) \in \mathbb{T} \times M^n$, a, b are real smooth functions on $\mathbb{T}$, and $P(x, D), Q(x, D)$ are self-adjoint first order pseudodifferential operators, defined on a compact smooth Riemannian manifold M^n. Furthermore, we request that discrete spectrum of P, $\sigma(P) = \{\lambda_j\} \subset \mathbb{R}$, satisfies $|\lambda_j| \to +\infty$ when $j \to +\infty$. In this talk, under the commutation hypothesis $[P, Q] = 0$, we will present necessary and sufficient conditions on the symbols of the operators P and Q to ensure global hypoellipticity of L.

Mass concentration in a nonlocal model of clonal selection

Jan-erik Stecher
Heidelberg University, Germany
jan-erik.stecher@bioquant.uni-heidelberg.de
Piotr Gwiazda, Anna Marciniak-Czochra

Self-renewal is a constitutive property of stem cells. Testing the cancer stem cell hypothesis requires investigation of the impact of self-renewal on cancer expansion. To understand better this impact, we propose a mathematical model describing dynamics of a continuum of cell clones structured by the self-renewal potential. The model is an extension of the finite multi-compartment models of interactions between normal and cancer cells in acute leukemias. It takes a form of a system of integro-differential equations with a nonlinear and nonlocal coupling, which describes regulatory feedback loops in cell proliferation and differentiation process. We show that such coupling leads to mass concentration in points corresponding to maximum of the self-renewal potential and the model solutions tend asymptotically to a linear combination of Dirac measures. Furthermore, using a Lyapunov function constructed for a finite dimensional counterpart of the model, we prove that the total mass of the solution converges to a globally stable equilibrium. Additionally, we show stability of the model in space of positive Radon measures equipped with flat metric. The analytical results are illustrated by numerical simulations.

Extinction of solutions for higher order parabolic equations with double nonlinearity

Kateryna Stiepanova
IAMM of NASU, Ukraine
kitti_dob@rambler.ru

The poster is devoted to the behavior of energy (generalized) solutions for a wide class of nonlinear higher order parabolic equations. We investigate the Cauchy-Dirichlet problem for parabolic equations with nonlinear homogeneous principal part and degenerate nonlinear absorption potential. The main focus of our study is the long-time extinction effect of solutions. Modifying the semi-classical technic of [1], we find sufficient conditions for the extinction of solutions to the mentioned problem above.

References

[1] Y. Belaud, A. Shishkov, Extinction of solutions of some semilinear higher order parabolic equations with degenerate absorption potential, *Journal of Differential Equations*, **10**, (2010), N 4, p. 857–882.

Synchronization in coupled dynamical systems on quasirandom graphs

Xuezhi Tang
Drexel University, USA
xt32@drexel.edu
Georgi S. Medvedev

Quasirandom graphs share many structural properties with truly random Erdős-Rényi graphs. In this work, we study synchronization in coupled dynamical systems on certain quasirandom graphs, including complete, Paley, and Erdős-Rényi random graphs. For these models, we estimate the synchronization rate and robustness of synchrony to noise.

Chimeras on a ring of phase oscillators coupled through bipartite topology

Sangeeta Ujjwal
Jawaharlal Nehru University, India
sangeeta.ujjwal@gmail.com
Awadhesh Prasad and Ramakrishna Ramaswamy

We study a system of phase oscillators on a bipartite network, with N_1 oscillators in one partition, and $N_2(\geq N_1)$ in the other. The coupling between oscillators in different partitions is nonlocal. Different types of chimeric states are observed as a function of the ratio $\gamma \equiv N_1/N_2$ in the range $1 \leq \gamma \leq 0$. When $N_1 = 1$, namely when the network has the star topology, a fully desynchronized state results.

On a green roof energy balance model.

Maria Vilar
Universidad Politecnica de Madrid, Spain
marialuisa.vilar.guerrero@alumnos.upm.es
L. Tello, C. Bedoya

We study a problem of green roof energy balance. One of the most known green roof's benefit is the energy reduction. The problem we have studied focuses on the thermal behavior of extensive green roofs. The space domain of the model has been divided in two layers: the soil and the canopy. We present a model including roof's shape and first results about its numerical approximation.

Fractional Stochastic Differential Equations and Space Weather

James Wanliss
Presbyterian College, USA
wanliss@gmail.com
V.V. Anh, Z.-G. Yu, Y. Wang, D. Mao

Space weather involves the complex solar-terrestrial plasma and electrodynamic interactions at altitudes above 80-km around the earth. In this paper we explore the characteristics and develop a stochastic model for the horizontal component of the geomagnetic field at 22 stations of the global magnetic observatory framework called INTERMAGNET. Our model is in the form of a fractional stochastic differential equation. We estimate model parameters on the basis of observations and cluster the data in terms of the degree of fractional differentiation and the alpha-stability coefficient. We find different clusters corresponding to geomagnetic latitudes which map to regions in the magnetosphere with vastly different physics in operation.

Graph Theoretical Analysis of Dynamic Brain Networks in the Resting State

Chia-Yen Yang
Ming Chuan University, Taiwan
cyyang@mail.mcu.edu.tw
Ching-Po Lin

Connections between human brain regions have been extensively studied at a functional level. Much of those evidences suggested that functional interactions are mediated by synchronization of oscillations. For that reason, the use of synchronization likelihood (SL) has been one of the most suitable algorithms in highly nonlinear and nonstationary brain networks. In many EEG and MEG studies, the SL patterns was measured statistically. Therefore, this study used synchronization likelihood to establish small-world functional networks and a simple method for constructing quasi-dynamic graphs by dividing a long-term static graph into a sequence of subgraphs that each had a timescale of 1 s. Our results indicated that SL maps were not exactly the same between eyes-closed and eyes-open rest in each frequency band, especially for the widespread alpha oscillations which had high functional connectivity during eyes-closed condition. Parameters C and L indicate that graph properties could differ with brain frequency rhythms, with higher frequencies have a lower small-worldness. For further investigation, dynamic brain network might give more information about specific processing capabilities from several spatially separated units, where low frequencies represent the essential foundation and high frequencies represent cognitive processing.

Matings with the Basilica

Jonguk Yang
University of Toronto, Canada
jonguk.yang@mail.utoronto.ca

I will present an overview of the family of quadratic rational maps with a 2- periodic superattracting orbit. Many of these maps can be described as the mating of two quadratic polynomials, one of which is the basilica. For this reason, the family has attracted much recent attention, and the parameter space picture is now nearly completely understood. I will survey the known results, with particular emphasis on my own contribution to the topic: the mateability of Siegel quadratic polynomials of bounded type with the basilica polynomial.

An extended model describing the hypothalamus-pituitary-thyroid axis - analysis of stability with respect to delay

Beata Zduniak
Warsaw University of Life Sciences, Poland
beata.zduniak@wp.pl

In the paper, we consider the extended mathematical model describing the hypothalamus-pituitary-thyroid homeostatic mechanism in endocrine system. We introduce to this system two types of couplings and delay. In our model feedback controls the secretation of thyroid hormons and delay reflects time lags required for transportation of the hormons. The influence of delayed feedback on the stability behaviour of the system is discussed. This system of equations describes normal activity of the thyroid and also a couple of types of malfunctions (e.g. Hashimoto's throiditis). We compare the results for the thyroid-pituitary axis model. Numerical simulations are presented to support analysis results.

Student Paper Competition Session

On maximum and comparison principles for parabolic problems with p-Laplacian

Vladimir Bobkov
University of Rostock, Germany
bobkovve@gmail.com
Peter Takac

We consider initial-boundary value problems for nonlinear parabolic equations with p-Laplacian

$$\partial_t u - \Delta_p u - \lambda |u|^{p-2} u = f(t, x),$$

where $\lambda \in \mathbb{R}$ is a spectral parameter. Problems of this type attract a lot of attention in the last decades. This is due to the fact that solutions to such problems possesses many unusual qualitative effects, which are not observed in the linear case ($p = 2$), e.g., extinction in finite time, simultaneously backward, elliptic and forward Harnack's estimates, violation of the strong maximum principle and Hopf's lemma, etc. In this talk we will speak about current state of affairs and recent progress in maximum and comparison principles for such kind of parabolic problems. Some outstanding issues will be discussed.

Strict self-adjointness and shallow water models

Priscila Leal da Silva
Centro de Matemática, Computação e Cognição, CMCC, UFABC, Brazil
priscila.silva@ufabc.edu.br
Igor Freire

In this work we study the modified Novikov equation using group methods. The com- plete group classification is carried out. Then, from the point symmetry generators we find the one- parameter group of local diffeomorfisms which preserves the equation. By using the Lie symmetry generators we also obtain exact solutions to the considered equation. It is also proved that only one non-trivial conservation law can be established using Ibragimov's recent developments.

A finite information KAM theorem

Piotr Kamienski
Jagiellonian University, Poland
piotr.kamienski@ii.uj.edu.pl

We present a KAM type theorem for two-dimensional tori which does not assume the classical diophantine condition, instead it is based on finite approximation of the rotation vector. Specifically, we prove that for all $p > 0$ a KAM theorem holds with a perturbation threshold ε_*, which depends only on $N(p)$ first "digits" of the continued fraction expansion of the rotation frequency ω provided that ω belongs to a set of probabilistic measure $1 - p$. The quantities are explicitly computed, which makes the method amenable for computer assisted proofs. Our reasoning relies on a careful analysis of the small divisors - the ones which are "truly small" are related to the continued fraction expansion of ω, which can be controlled statistically in spirit of the theorem on Khintchine's constant.

An equation decomposition based tailored finite point method for linearized incompressible flow in 2D space

Ye Li
Tsinghua University, China
yeli11@mails.tsinghua.edu.cn
Houde Han, Zhongyi Huang

In this work, we use an equation decomposition technique to transfer the Oseen equations to a system of two elliptic equations in the cases with/without body force. These two equations are decoupled in the interior domain. We show the stability of the new problem and found the kernel spaces of the new system. Furthermore, we show the equivalence of it to the original problem. Then we design the tailored finite point method to the new problem which could give good approximations on coarse mesh whatever the Reynolds number is small or large. Especially, we use several benchmark examples to show the advantages of our method comparing with the traditional MAC scheme. This paper is preparing for studying the Navier-Stokes equations with high Reynolds number.

The critical problem of Kirchhoff type elliptic equations in dimension four

Daisuke Naimen
Osaka City University, Japan
d12sax0J51@ex.media.osaka-cu.ac.jp

In this paper we consider the critical problem of Kirchhoff type elliptic equations in dimension four.

$$(P) \begin{cases} -\left(1 + b\int_\Omega |\nabla u|^2 dx\right)\Delta u = \lambda u^q + u^3, \\ \qquad\qquad\qquad\qquad u > 0 \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^4$ is a bounded domain with smooth boundary $\partial\Omega$, $b \geq 0$, $\lambda > 0$ and $1 \leq q < 3$. We prove the existence of solutions of (). Our main tool is the variational method. In the proof, a typical difficulty occurs in showing the PS condition because of the lack of the compactness of the Sobolev embedding $H_0^1(\Omega) \hookrightarrow L^4(\Omega)$. Furthermore the interaction between the Kirchhoff type nonlocal principal part, $(b\int_\Omega |\nabla u|^2 dx)\Delta u$, and the critical nonlinearity, u^3, is crucial. It is because, in view of the corresponding energy, both will have the same exponent 4 ! These make our argument so delicate and challenging. To accomplish the proof, we utilize the concentration compactness analysis of PS sequences together with the method of the construction of bounded PS sequences. Consequently several new existence phenomena induced by the nonlocal coefficient are obtained.

Strichartz estimates and smooth attractors for wave equations with fractional damping in bounded domains

Anton Savostianov
University of Surrey, England
a.savostianov@surrey.ac.uk
Sergey Zelik

We consider Dirichlet problem for a semi-linear wave equation with damping term $(-\Delta)^\alpha \partial_t u$, where $\alpha \in [0, \frac{1}{2}]$, in a bounded smooth domain $\Omega \subset \mathbb{R}^3$, assuming initial data from usual energy space $H_0^1(\Omega) \times L^2(\Omega)$. First, we establish control of $L^5([0,T]; L^{10}(\Omega))$ norm of solutions for corresponding linear non-autonomous problem in terms of energy norm, which does not follow from energy estimate as well as Strichartz estimates for pure wave equation. Then treating semi-linear equation as perturbation of the linear problem we establish its well-posedness in the class of energy solutions with finite $L^5_{loc}(\mathbb{R}_+, L^{10}(\Omega))$ norm. Moreover, we show that solutions from the mentioned class possess smoothing property analogous to solutions of parabolic equation. Finally we show that dynamical system generated by these solutions possesses a smooth global attractor.

Reaction-diffusion model aided understanding of pattern formation of inflorescence

Yoshitaro Tanaka
Meiji University, Japan
ds31003@meiji.ac.jp
Masayasu Mimura, Hirokazu Ninomiya

One of the most fascinating patterns in plants should be the pattern of inflorescence. Inflorescence is the distribution of florets or seeds in plants. Douady and Couder introduced a physical model using the distances of each primordium and explained the mechanism of the formation of inflorescence. In this talk, we propose a reaction-diffusion model for inflorescence and re- veal an intrinsic relationship between our reaction-diffusion model and the Douady-Couder model by using singular limit analysis. Furthermore, we draw a global bifurcation diagram numerically to explain the pattern formation of inflorescence as the growth rate of the plant is varied.

Well-posedness and exponential equilibration of a volume-surface reaction-diffusion system with nonlinear boundary coupling

Bao Tang
Institute for Mathematics and Scientific Computing, University of Graz, Austria
baotangquoc@gmail.com
Klemens Fellner, Evangelos Latos

We consider a model system consisting of two reaction-diffusion equations, where one specie diffuses in a volume while the other specie diffuses on the surface which surrounds the volume. The two equations are coupled via nonlinear reversible Robin-type boundary conditions for the volume specie and a matching reversible source term for the boundary specie. As a consequence the total mass of the species is conserved. The considered system is motivated by models for asymmetric stem cell division. We first prove the existence of a unique weak solution via an iterative method of converging upper and lower solutions to overcome the difficulties of the nonlinear boundary terms. Secondly, we show explicit exponential convergence to equilibrium via an entropy method after deriving a suitable entropy entropy-dissipation estimate.

Long Time Behavior of the Forced Critical Surface Quasi-geostrophic Equation

Andrei Tarfulea
Princeton University, USA
tarfulea@math.princeton.edu
Peter Constantin, Vlad Vicol

We prove the existence of a compact global attractor in $H^1(\mathbb{T}^2)$ for the dynamics of the forced critical surface quasi-geostrophic equation (SQG). After a transient time, the solution is bounded in C^α and H^1 by higher regularity norms of the forcing term f and independently of the initial data. The attractor also has finite fractal (box-counting) dimension.

Slow-fast n-dimensional piecewise linear differential systems

Catalina Llompart Vich
University of Balearic Islands, Spain
catilin75@hotmail.com
R. Prohens, A.E. Teruel and C. Vich

Slow-fast systems are a kind of system such that are used to modulate neural behaviours like bursts. The piecewise linear differential systems help to reproduce the dynamical richness of differential systems in the neuroscience modelling like global dynamics, forced behaviour, bifurcations and so on. However, they may also help to explain some of the previous phenomena. In this talk we present the slow-fast systems in n-dimensional piecewise linear framework providing some theorems about the existence, location and provenience of maximal and faux canard points.

List of Contributors

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