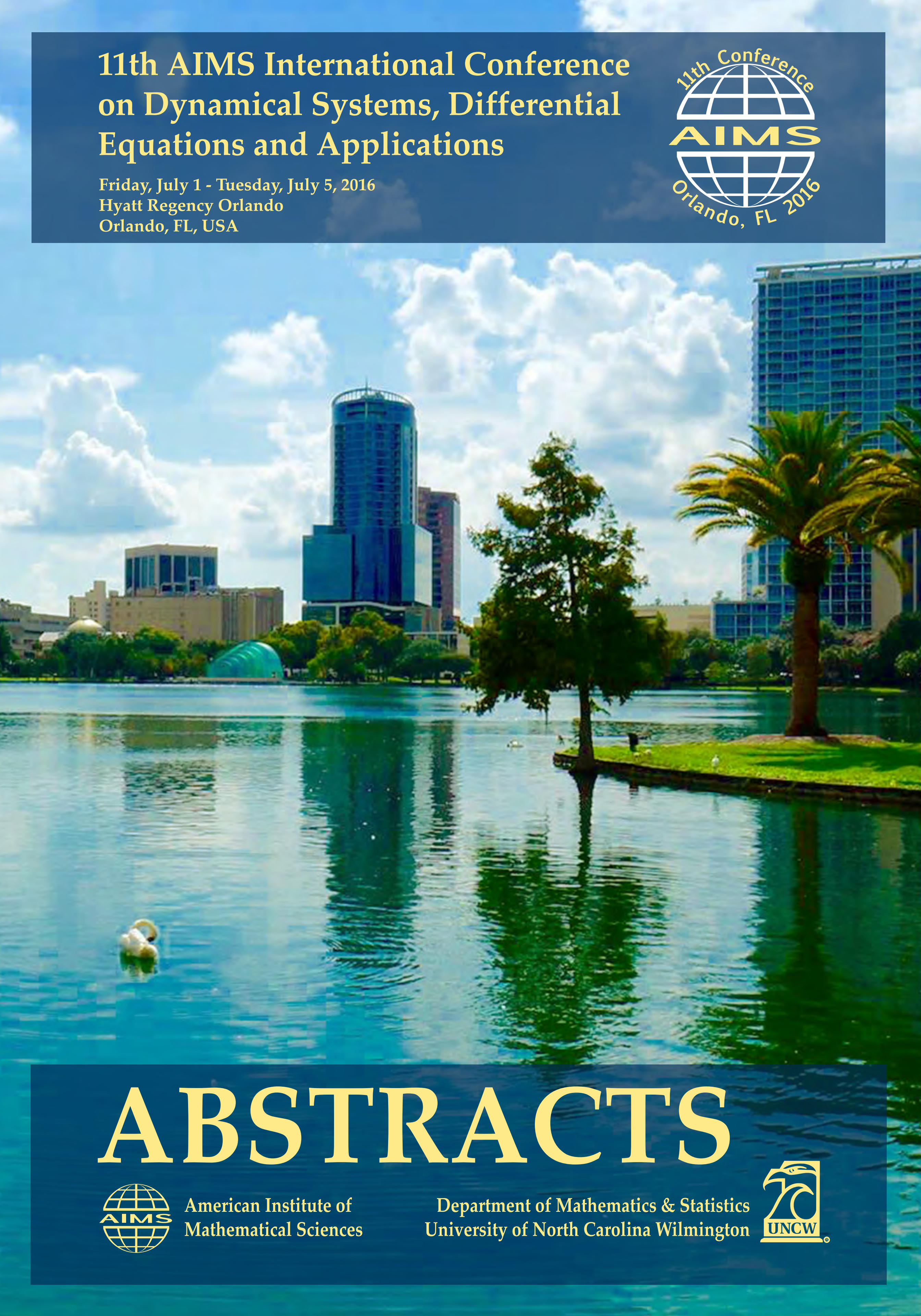
11th AIMS International Conference on Dynamical Systems, Differential Equations and Applications

Friday, July 1 - Tuesday, July 5, 2016
Hyatt Regency Orlando
Orlando, FL, USA



ABSTRACTS

American Institute of Mathematical Sciences

Department of Mathematics & Statistics
University of North Carolina Wilmington

The 11th AIMS Conference on Dynamical Systems, Differential Equations and Applications

July 1 – July 5, 2016
Orlando, Florida, USA

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The University of North Carolina Wilmington

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Gail Wolkowicz

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Contributed Sessions

Invited Plenary Lectures

Suncica Canic
University of Houston, USA
http://www.math.uh.edu/∼canic/

Suncica Canic earned her Ph.D. in 1992 in the area of nonlinear hyperbolic conservation laws from the Department of Applied Mathematics and Statistics at SUNY Stony Brook. Upon her move to the University of Houston in 1999, she began collaborating with several medical specialists at the Texas Medical Center in Houston on problems related to cardiovascular treatment and diagnosis. She was honored for her research by the National Science Foundation as Distinguished MPS Lecturer in 2007, and received the US Congressional Recognition for Top Women in Technology in 2006. Her research received local and national media attention, and was featured in several publications by NSF, NIH, and AMS. Canic was also invited to present a Congressional Briefing on Applied Mathematics, on Capitol Hill on December 6th, 2011. She serves on the Board of Governors of the Institute for Mathematics and its Applications in Minneapolis, and was the Program Director of the SIAM Activity Group on Partial Differential Equations. In 2014 she was elected Fellow of the Society for Industrial and Applied Mathematics. She is the only woman who holds a prestigious Cullen Distinguished Professorship position at the University of Houston.

Fluid-Composite Structure Interaction and Blood Flow

Abstract
Fluid-structure interaction problems with composite structures arise in many applications. One example is the interaction between blood flow and arterial walls. Arterial walls are composed of several layers, each with different mechanical characteristics and thickness. No mathematical results exist so far that analyze existence of solutions to nonlinear, fluid-structure interaction problems in which the structure is composed of several layers. In this talk we summarize the main difficulties in studying this class of problems, and present a computational scheme based on which a proof of the existence of a weak solution was obtained. Our results reveal a new physical regularizing mechanism in FSI problems: inertia of thin fluid-structure interface with mass regularizes evolution of FSI solutions. Implications of our theoretical results on modeling the human cardiovascular system will be discussed.

This is a joint work with Boris Muha (University of Zagreb, Croatia), and with Martina Bukac (U of Notre Dame, US). Numerical results with vascular stents were obtained with S. Deparis and D. Forti (EPFL, Switzerland).

Alessio Figalli
The University of Texas at Austin, USA
http://www.ma.utexas.edu/users/figalli/

Figalli received his master degree in mathematics from the Scuola Normale Superiore di Pisa in 2006, and earned his doctorate in 2007 under the supervision of Luigi Ambrosio at the Scuola Normale Superiore di Pisa and Cédric Villani at the École Normale Supérieure de Lyon. In 2007 he was appointed Chargé de recherche at the French National Centre for Scientific Research, in 2008 he went to the École polytechnique as Professeur Hadamard. He has been a professor at University of Texas at Austin since 2009. Starting from 2013 he holds the R. L. Moore Chair. Amongst his several recognitions, Figalli has won an EMS Prize in 2012, he has been awarded the Peccot-Vimont Prize 2011 and Cours Peccot 2012 of the Collège de France, and has been appointed Nachdiplom Lecturer in 2014 at ETH Zürich. His main research interests include Partial Differential Equations and Calculus of Variations.

From Isoperimetry to Random Matrices

Abstract
The optimal transport problem consists in finding the cheapest way to transport a distribution of mass from one place to another. Apart from its natural applications in economics, optimal transport maps provide “efficient” changes of variables that have been used to investigate the stability of minimizers to geometric/functional inequalities. However, in some cases, optimal maps may not alway been the “right” choice and other changes of variables may be more suitable. For instance, this happens to be the case in the study of universality in random matrix theory. In this talk I’ll give an overview of these results.

Irene Fonseca
Carnegie Mellon University, USA
http://irenefonseca.weebly.com/

Irene Fonseca is an educator and researcher in applied mathematics and is currently the Director of Carnegie Mellon University's Center for Nonlinear Analysis (CNA). In recognition for her contributions to the advancement of research in her area of expertise, Irene Fonseca was bestowed a knighthood in the Military Order of St. James (Grande Oficial da Ordem Militar de Santiago da Espada) by the then-President of Portugal, Jorge Sampaio, in 1997. Irene Fonseca received the Mellon College of Science chair in Mathematics in 2003 and was appointed to the rank of University Professor in 2014. In 2012, she was elected President of the Society for Industrial and Applied Mathematics (SIAM), one of the largest organizations dedicated to mathematics and computational science in the world. Irene Fonseca was also named a Fellow of SIAM in 2009 and a Fellow of the American Mathematical Society in 2012. Irene Fonseca's recent work is focused on variational techniques as they apply to contemporary problems in materials sciences and computer vision, including the mathematical study of ferroelectric and magnetic materials, composites, thin structures, phase transitions, epitaxy and dislocations, and image segmentation and denoising in imaging science.

A Chromaticity-Brightness Model for Color Images Denoising in a Meyer's "u + v" Framework

Abstract
A variational model for imaging denoising aimed at restoring color images is proposed. The model combines Meyer's "u+v" decomposition with a chromaticity-brightness framework, and is expressed in terms of a minimization of energy integral functionals depending on a small parameter $\varepsilon > 0$. The asymptotic behavior as $\varepsilon \to 0^+$ is characterized, and convergence of infima, almost minimizers, and energies are established. In particular, an integral representation of the lower semicontinuous envelope, with respect to the L^1-norm, of functionals with linear growth and defined for maps taking values on a compact manifold is provided.

Michael Ghil
Ecole Normale Supérieure, Paris, France
University of California Los Angeles, USA
http://research.atmos.ucla.edu/tcd/GHIL/

Michael Ghil obtained his Ph.D. from New York University's Courant Institute of Mathematical Science with Peter D. Lax in 1975. He is a Distinguished Professor of Geosciences (emeritus) at the Ecole Normale Supérieure, Paris, past Head of its Geosciences Department (2003–2009) and founder of its Environmental Research and Teaching Institute. He is also a Distinguished Research Professor at the University of California, Los Angeles, where he was Chair of the Department of Atmospheric Sciences (1988–1992) and Director of the Institute of Geophysics and Planetary Physics (1992–2003). Ghil is a founder of theoretical climate dynamics, as presented in his Springer-Verlag (1987) book with Steve Childress, as well as of advanced data assimilation methodology, as presented in the Springer-Verlag (1981) book co-edited with Lennart Bengtsson and Erland Källén. He has applied systematically ideas and methods from dynamical systems theory to planetary-scale flows, atmospheric and oceanic. Ghil has used these methods to proceed from simple flows with high temporal regularity and spatial symmetry to the observed flows, with their complex behavior in space and time. His studies of climate variability on many time scales have used a full hierarchy of models, from the simplest "toy" models all the way to atmospheric, oceanic and coupled general circulation models. Ghil has worked on Climate Dynamics, Dynamical and Complex Systems, Extreme Events, Numerical and Statistical Methods, and (most recently) Mathematical Economics. He is the author or editor of a dozen books and author or co-author of over 300 research and review articles. Many of the latter can be found on the web site of his research group at UCLA, http://www.atmos.ucla.edu/tcd/. His honors and awards include the L.F. Richardson Medal of the European Geosciences Union (EGU, 2004), the E.N. Lorenz Lecture of the American Geophysical Union (2005), a Plenary Lecture at the 7th International Congress on Industrial and Applied Mathematics (ICIAM 2011), the Alfred Wegener Medal of the EGU (2012), and Membership in the Academia Europaea (1998).

A Mathematical Theory of Climate Sensitivity or, A Tale of Deterministic and Stochastic Dynamical Systems

Abstract
The climate system is nonlinear, complex and variable on many scales of time and space. It is typically studied across a hierarchy of models from low-dimensional systems of ordinary differential equations (ODEs) to infinite-dimensional systems of partial and functional differential equations (PDEs and FDEs). The theory of differentiable dynamical systems (DDS) has provided a road map for climbing this hierarchy and for comparing theoretical results with observations. The climate system is also subject to time-dependent forcing, both natural and anthropogenic, e.g. volcanic eruptions and changing greenhouse gas concentrations. Hence increased attention has been paid recently to applications of the theory of non-autonomous and random dynamical systems (NDS and RDS). This talk will review the road from the classical DDS applications to low-dimensional ODE climate models to current efforts at applying NDS and RDS theory to non-autonomous FDE and stochastic PDE models. The debt of the lecturer and of his co-authors over the years to Peter D. Lax is immense, and a modest tribute will be paid to Peter's contributions to pure and applied mathematics.

Martin Hairer
The University of Warwick, UK
http://www.hairer.org

Martin Hairer received his PhD in theoretical physics in 2001 from the University of Geneva. He then moved to the University of Warwick where he became Associate Professor in 2004. In 2009, he was appointed at the Courant Institute in New York and then moved back to the University of Warwick as a full professor in 2010. His main areas of research are stochastic dynamics, stochastic analysis, and stochastic partial differential equations. In recent years, his emphasis was mainly on developing the theory of regularity structures which provides a robust framework in which to interpret large classes of stochastic PDEs whose mathematical meaning had so far been unclear. His work has been distinguished with a 2008 Whitehead prize, a 2013 Fermat prize and a 2014 Fields medal.

Evolution on Random Loops

Abstract
A “rubber band” constrained to remain on a manifold evolves by trying to shorten its length, eventually settling on some minimal closed geodesic, or collapsing entirely. It is natural to try to consider a noisy version of such a model where each segment of the band gets pulled in random directions. Trying to build such a model turns out to be surprisingly difficult and generates a number of nice geometric insights, as well as some beautiful algebraic and analytical objects. We will survey some of the main results obtained on the way to this construction.

Anatole Katok
Penn State University, USA
http://www.personal.psu.edu/axk29/

Entropy in Dynamical Systems: Complexity, Flexibility and Rigidity

Several interrelated concepts of entropy as well as closely related notions of Lyapunov characteristic exponents play a central role in the modern theory of dynamical systems.Those notions give quantitative expression of the measure of exponential complexity present in a deterministic system. After a general review of those concepts and principal relations between them I will discuss results and open problems related to two complimentary phenomena. of flexibility and rigidity. The general paradigm of flexibility can be rather vaguely formulated as follows:

Under properly understood general restrictions within a fixed class of smooth dynamical systems quantitative dynamical invariants take arbitrary values.

Precise calculations are possible only in very few cases, primarily of algebraic nature such as homogeneous or affine systems. Most known constructions are perturbative and hence at best would allow to cover a small neighborhood of the values allowed by the model, or more often, not even that, since those models are often "extremal". So establishing flexibility calls for *non-perturbative or large perturbation constructions* in large families to cover possible values of invariants.
On the other hand, there is the rigidity paradigm that is better developed. It has several aspects and in the case of classical systems with discrete and continuos time one of them is related to these quantitative characteristics os exponential complexity:

Particular values of entropies or Lyapunov exponents or relations between those determine algebraic or similar models within a broad class of systems.

Rigidity becomes more common and even prevalent when one passes from classical systems to systems with multi-dimensional time.

Wei-Ming Ni
University of Minnesota, USA
East China Normal University, China

Wei-Ming Ni received his B.S. from National Taiwan University and Ph.D. from Courant Institute, New York University. He is currently on the faculty of University of Minnesota, and the Director of the Center for PDE at East China Normal University in Shanghai. His research interests are mainly in elliptic and/or parabolic equations/systems, including: Symmetry properties of solutions, solutions on entire space and their stability properties, peak solutions (spiky Turing patterns), and interactions between spatial and/or temporal heterogeneities and diffusion.

From Logistic Equation to Lotka-Volterra Competition-Diffusion System

Abstract
In this lecture I shall report some of the recent progress on the logistic equation and 2×2 Lotka-Volterra competition-diffusion system when spatial heterogeneity and/or temporal periodicity are present.

Stan Osher
University of California Los Angeles, USA
http://www.math.ucla.edu/∼sjo/

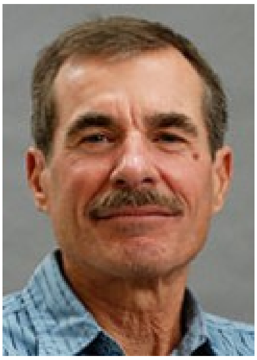
Stanley Osher is a Professor of Mathematics, Computer Science, Chemical Engineering and Electrical Engineering at UCLA. He is also an Associate Director of the NSF-funded Institute for Pure and Applied Mathematics at UCLA. He received his MS and PhD degrees in Mathematics from the Courant Institute of NYU. Before joining the faculty at UCLA in 1977, he taught at SUNY Stony Brook, becoming professor in 1975. Professor Osher is one of the most highly cited researchers in both mathematics and computer sciences with an h index of 100(according to google scholar). In recent years he has averaged one citation per hour. He has received numerous academic honors and co-founded three successful companies, each based largely on his own (joint) research.
He has co-invented and/or co-developed the following widely used algorithms:

1. Essentially nonoscillatory (ENO), weighted essentially nonoscillatory (WENO) and other shock capturing schemes for hyperbolic systems of conservation laws and their analogues for Hamilton-Jacobi equations;
2. The level set method for capturing dynamic surface evolution;
3. Total variation and other partial differential based methods for image processing;
4. Bregman iterative methods for L1 and related regularized problems which arise in compressive sensing, matrix completion, imaging and elsewhere;
5. Diffusion generated motion by mean curvature and other threshold dynamics methods.

Professor Osher has been elected to the US National Academy of Science and the American Academy of Arts and Sciences. He was awarded the SIAM Pioneer Prize at the 2003 ICIAM conference and the Ralph E. Kleinman Prize in 2005. He was awarded honorary doctoral degrees by ENS Cachan, France, in 2006 and by Hong Kong Baptist University in 2009. He is a SIAM and AMS Fellow. He gave a one hour plenary address at the 2010 International Conference of Mathematicians. He also gave the John von Neumann Lecture at the SIAM 2013 annual meeting. He is a Thomson-Reuters highly cited researcher-among the top 1% from 2002-2012 in both Mathematics and Computer Science. In 2014 he received the Carl Friedrich Gauss Prize from the International Mathematics Union-this is regarded as the highest prize in applied mathematics. His current interests involve information science which includes optimization, image processing, compressed sensing and machine learning and applications of these techniques to the equations of physics, engineering and elsewhere.

Overcoming the Curse of Dimensionality for Certain Hamilton-Jacobi (HJ) Equations Arising in Control Theory and Elsewhere

Abstract
It is well known that certain HJ PDE's play an important role in analyzing continuous dynamic games and control theory problems. The cost of standard algorithms, and, in fact all PDE grid based approsimations is exponential in the space dimension and time, with huge memory requirements. Here we propose and test methods for solving a large class of HJ PDE relevant to optimal control without the use of grids or numerical approximations. Rather we use rhe classical Hopf formulas for solving initial value problems for HJ PDE. We have noticed that if the Hamiltonian is convex and positively homogeneous of degree one that very fast methods (related to those used in compressed sensing) exist to solve the resulting optimization problem. We seem to obtain methods which are polynomial in dimension. We can evaluate the solution in very high dimensions in between 10^{-4} and 10^{-8} seconds per evaluation on a laptop. The method requires very limited memory and is almost perfectly parallelizable. In addition, as a step often needed in this procedure, we have developed a new and equally fast and efficient method to find, in very high dimensions, the projection of a point exterior to a compact set A onto A. We can also compute the distance to such sets much faster than fast marching or fast sweeping algorithms. The term "curse of dimensionality" was coined by Richard Bellman in 1957 when he did his pioneering work on dynamic optimization.



Hal Smith
Arizona State University, USA
http://math.la.asu.edu/∼halsmith/

Hal L. Smith obtained his PhD in mathematics at the University of Iowa and is currently Professor of Mathematics at Arizona State University. He has held visiting professor positions at the University of Utah, Brown University, Georgia Institute of Technology, and the University of Minnesota. His research interests include nonlinear analysis, differential equations, dynamical systems, and applications to the biological sciences.

Monotone Dynamical Systems: Reflections on New Advances & Applications

Abstract
I offer some reflections on recent developments in a very select portion of the now vast subject of monotone dynamical systems. Continuous time dynamics generated by cooperative systems of ordinary differential equations, delay differential equations, parabolic partial differential equations, and control systems are the main focus. Results are included which the author feels have had a major impact in the applications. These include especially the theory of competition between two species or two teams and the theory of monotone control systems.

Gang Tian
Peking University, Peoples Rep of China and Princeton University, USA
http://bicmr.pku.edu.cn/∼tian/
http://www.math.princeton.edu/directory/gang-tian

Dr. Gang Tian has made fundamental contributions to geometric analysis, complex geometry and symplectic geometry. He did his undergraduate study at Nanjing University in China, received his MS at Peking University and PhD at Harvard University. He is now a distinguished professor at Peking University and an E. Higgins professor at Princeton University. Dr. Gang Tian established completely the existence of Kähler-Einstein metrics on compact complex surfaces. He proved that the deformation of Calabi-Yau manifolds is unobstructed. Together with Ruan, he established a mathematical theory of the quantum cohomology and Gromov-Witten invariants. He was one of pioneers in constructing virtual cycles. With Liu, he solved the Arnold Conjecture in the non-degenerate case . He introduced the K-stability which has been further developed and has become a central notion in the theory of geometric stability. He initiated the Analytical Minimal Model program through Kahler-Ricci flow, referred as the Song-Tian program. Together with J. Morgan, amongst others, Dr. Gang Tian helped verifying the proof of the Poincare Conjecture and the Geometrization Conjecture. In 2012, he gave a solution for the Yau-Tian-Donaldson Conjecture in the case of Fano manifolds which has been a driving force in Kahler geometry for last few decades Dr. Gang Tian won Alan T. Waterman Award in 1994 and Veblen Prize in 1996. He spoke twice at the International Congress of Mathematics in 1990 and 2002. He was elected to the National Academy of China in 2001 and the American Academy of Arts and Science in 2004.

Einstein Equation and Kähler Geometry

Abstract
In this general talk, I will present some recent advances in Kähler geometry. The study of Kähler-Einstein metrics was initiated by E. Calabi in the 50's. In the 70's, Yau's solution for the Calabi conjecture settles the case when the scalar curvature is zero. Aubin and Yau solved the case when the scalar curvature is negative. Since then, it has been a very challenging problem to study the existence problem for Kähler-Einstein metrics with positive scalar curvature. Recently, a deep connection has been established between this existence and a variant of the geometric invariant theory I started in the 90s and advocated since then.

Special Session 1: Nonlinearity in Climate and the Geosciences, A Special Session Honoring Peter D. Lax

Michael Ghil, ENS, France and UCLA, USA
Mickael D. Chekroun, UCLA, USA
Shouhong Wang, Indiana University, USA

Dynamical systems have played a major role in the early development of modern, post-WWII climate dynamics, with the work of E.N. Lorenz, H.M. Stommel, G. Veronis and others. Today, this role extends to other branches of the geosciences, and the emphasis is shifting from the use of autonomous dynamical systems to non-autonomous and random ones. This session will cover the applications of dynamical systems, in the broadest sense, to climate and the geosciences. The applications will span a hierarchy of models, from the simplest, autonomous systems of ordinary differential equations to functional, partial and stochastic ones. The session will be inspired by the elegance of Peter Lax's contributions to many pertinent areas of nonlinear mathematics, including but not restricted to numerical methods for evolution problems and the highly nonlinear Korteweg-DeVries equation.

Rogue Waves

Jerry Bona
University of Illinois at Chicago, USA
Gustavo Ponce, Jean-Claude Saut, Christof Sparber

Rogue waves, or giant waves is the name given to a certain class of oceanic phenomena that involve surface waves which are large compared to man made objects such as ships or oil platforms. They seem to be localized in both space and time. The lecture will focus on one possible route to their generation.

Non-Markovian Reduced SDEs for Markovian SPDEs

Mickael Chekroun
University of California, Los Angeles, USA
Honghu Liu, Shouhong Wang

In this talk, a novel approach to deal with the parameterization problem of the "small" spatial scales by the "large" ones for stochastic partial differential equations (SPDEs), will be discussed. This approach relies on stochastic parameterizing manifolds (PMs) which are random manifolds aiming to provide — in a mean square sense — the optimal approximate small-scale parameterizations. Backward-forward systems will be introduced to determine such PMs in practice. These auxiliary systems will be used for the analytic derivation of non-Markovian reduced stochastic differential equations (SDEs) with random coefficients. It will be shown that these random coefficients allow in certain circumstances for a striking restoring of the missing information due to the coarse-graining, namely to parameterize what is unresolved. Noise-induced transitions and stochastic bifurcations will serve as illustrations, including the case of additive noise. This talk is based on a joint work with Honghu Liu and Shouhong Wang.

Discrete Stochastic Parametrization for Dynamical Systems

Alexandre Chorin
UC Berkeley, USA
Fei Lu, Kevin Lin

Many problems in science are described by equations that cannot be solved in full because the equations are either uncertain or overly complex, but where one is interested in predicting only a small subset of observed variables. Under these conditions, it is natural to formulate simple models for the variables of interest and correct them by terms inferred from the observations. I will describe a fully discrete stochastic approach to this problem, which simplifies both the inference from the observations and the solution of the resulting equations. However, in the discrete approach, like in any other, it is necessary to identify models that are parsimonious in computing effort. I will present several ways of ensuring parsimony, based on ideas from differential equations and from machine learning. The examples include the Lorenz 96 model and the Kuramoto-Sivashinski equation.

Detection and Tracking of Coherent Structures in Stochastic and Deterministic Models of the Atmosphere and Ocean

Gary Froyland
University of New South Wales, Australia

In complex, unsteady, geophysical flows, features that persist on moderate timescales provide crucial information on dynamics and transport, which in turn is important for climate models. The non-autonomous and/or stochastic nature of many geophysical models means that classical dynamical systems approaches are inappropriate to diagnose coherent features in these flows. I will describe a general method, based on global mixing minimisation, for identifying coherent structures in non-autonomous, compressible, and possibly stochastic flows, with illustrations from atmosphere and ocean models in two and three dimensions. I will discuss a second method, compatible with the first, but based on curve or surface minimisation, for deterministic flows.

The Wind-Driven Ocean Circulation: Applying Dynamical Systems Theory to a Climate Problem

Michael Ghil
ENS, Paris, and UCLA, France

The large-scale, near-surface flow of the mid-latitude oceans is dominated by the presence of a larger, anticyclonic and a smaller, cyclonic gyre. The two gyres share the eastward extension of western boundary currents, such as the Gulf Stream or Kuroshio, and are induced by the shear in the winds that cross the respective ocean basins.
We study the low-frequency variability of this wind-driven, double-gyre circulation in mid-latitude ocean basins, subject to time-constant, purely periodic and more general forms of time-dependent wind stress. Both analytical and numerical methods of nonlinear dynamics are applied in this study. Recent work has focused on the application of non-autonomous and random forcing to double-gyre models. We discuss the associated pullback and random attractors and the non-uniqueness of the invariant measures that are obtained.
This talk reflects collaborative work with a large and still increasing number of people. Please visit http://www.atmos.ucla.edu/tcd/ for their names, affiliations, and respective contributions.

How to Uncover Coherent Water Masses in an Unsteady Ocean?

George Haller
ETH Zurich, Switzerland

Transport by mesoscale oceanic eddies is broadly viewed as important for climate modelling. The accurate assessment of this transport relies on the identification of coherent water masses in otherwise turbulent velocity fields. Recent progress in nonlinear dynamical systems enables such an identification by revealing analogues of generalized elliptic (or KAM) regions in non-autonomous, finite-time dynamical systems. We show how these mathematical results provide objective (i.e., frame-invariant) identification of unsteady material vortices in the ocean. We illustrate the results by uncovering highly coherent Lagrangian eddies in two-dimensional satellite altimeter data and in three-dimensional numerical ocean models.

Books by and about Peter Lax

Reuben Hersh
University of New Mexico, USA

I will try to convince you to read books by and about Peter Lax.

Variational Principles for Stochastic Parameterisations in Geophysical Fluid Dynamics

Darryl Holm
Imperial College London, England
Colin Cotter, Dan Crisan, Franco Flandoli

In recent work, Holm derived novel stochastic fluid dynamics equations aimed at geophysical fluid dynamics applications by developing a new class of stochastic variational principles. I will present some on-going research on the theoretical and numerical analysis of the solutions of these nonlinear stochastic partial differential equations.

El Niño Dynamics: Its Beauty and Challenges

Fei-Fei Jin
University of Hawaii at Manoa, USA

El Niño refers to major climate events that recur every few years, with ocean surface temperatures often reaching a few degrees Celsius warmer than normal over a vast area of the Tropical Pacific. These events reorganize the Earth's weather and climate patterns, causing global environmental and socio-economic impacts. It was first recognized 50 years ago that El Niño results from an instability involving basin-scale dynamic interaction between two geophysical fluids, namely the atmosphere and oceans. In the last 30 years, the dynamic system approach to the coupled atmosphere-ocean climate system has led to a much deeper understanding of this most influential, as well as best observed, simulated, and now predicted climatic phenomenon. This approach has allowed the climate dynamicist to unravel the basic mechanisms for many key aspects of El Niño, including the nature of the instability that causes it, its phase locking with the solar annual cycle, its striking asymmetry and its frequency cascade; it has also allowed us to uncover El Niño's hidden mathematical beauty. The latter echoes Peter Lax's statement that mathematics has a mysterious unity which really connects seemingly distinct parts, which is one of the glories of mathematics. In this talk honoring Peter Lax's work and its impact on the climate sciences, I will present a brief review of the progress of El Niño dynamics and its remaining challenges.

On the Predictability Limit of Tropical Cyclone Intensity

Chanh Kieu
Indiana University, USA

Weather has long been projected to possess limited predictability due to the inherent chaotic nature of the atmosphere, which may result in an entirely different state of the atmosphere after 2 weeks regardless of how small initial errors are. Because of the critical dependence of such a range of predictability on the underlying dynamics of atmospheric flows, a natural question is how far in advance can we predict tropical cyclone (TC) intensity, given that the TC rotational dynamics is highly axisymmetric at the meso scale (30-350 km)? In this study, we will examine the growth of TC intensity errors at the maximum potential intensity (MPI) limit in a phase space of TC basic scales, using an axisymmetric TC model. It will be shown that there exists an MPI attractor at the maximum intensity limit that all TC orbits will converge into, regardless of TC initial conditions. Direct estimation of the leading Lyapunov exponent shows that the MPI attractor is not only an attracting set but also chaotic in nature. This finding of the chaotic MPI attractor suggests an upper bound on the predictability limit of the TC intensity, which prevents the 4-5 day absolute intensity errors in TC model forecasts from being reduced below a threshold of 8-10 ms^-1.

A Discrete-Time Approach to Stochastic Model Reduction for Chaotic Dynamical Systems

Kevin Lin
University of Arizona, USA
Fei Lu, Alexandre Chorin

In computational modeling of complex dynamical phenomena, it is often useful to be able to construct simpler, reduced models that nevertheless capture key dynamical features of interest. One well-studied strategy is to fit parametric families of stochastic models to data. In recent work, Chorin and Lu proposed a novel, discrete-time approach that has certain appealing features. I will report on applications of this discrete-time approach to the Kuramoto-Sivashinsky PDE and other prototypical chaotic dynamical systems, and discuss some of the issues that arise and how they can be overcome.

Galerkin Approximations of Nonlinear Delay Differential Equations Revisited

Honghu Liu
Virginia Tech, USA
Mickael Chekroun, Michael Ghil, Shouhong Wang

Delay differential equations (DDEs) are widely used in many fields such as the biosciences, climate dynamics, control theory, and engineering. In particular, certain DDEs or more general differential equations with retarded arguments can be derived from hyperbolic partial differential equations that support wave propagation. In this talk, we revisit the approximation problem of DDEs by ODE systems. A key ingredient in our construction is a special polynomial basis whose elements are orthogonal with respect to a measure adjoining a point mass. The associated Galerkin scheme enjoys some nice properties that help reduce the derivation of the corresponding convergence results to essentially very basic functional analysis exercises. Analytic formulas are also available within this approach, which simplify the numerical treatment. The efficiency of the method will be illustrated on several examples, one of which has solutions that recall Brownian motion.

Climate As a Problem of Nonequilibrium Statistical Mechanics

Valerio Lucarini
University of Hamburg, Germany

The climate is a complex, chaotic, non-equilibrium system featuring a limited horizon of predictability, variability on a vast range of temporal and spatial scales, instabilities resulting into energy transformations, and mixing and dissipative processes resulting into entropy production. Despite great progresses, we still do not have a complete theory of climate dynamics able to encompass instabilities, equilibration processes, response to changing parameters of the system, and coupling across scales of motion. We will outline some possible applications of the Ruelle response theory, showing how it allows for setting on firm ground and on a coherent framework concepts like climate sensitivity, climate response, climate tipping points, and for constructing parametrizations.. We will show results for comprehensive global climate models.
V. Lucarini, R. Blender, C. Herbert, F. Ragone, S. Pascale, J. Wouters, Mathematical and Physical Ideas for Climate Science, Reviews of Geophysics 52, 809-859 (2014)

Modeling of the Humid Atmosphere

Roger Temam
Indiana University, USA
Makram Hamouda, Joseph Tribbia, Xiaoyan Wang

The humid atmosphere is a multi-phase system, made of air, water vapor, cloud-condensate, and rain water (and possibly ice / snow, aerosols and other components). The possible changes of phase due to evaporation and condensation make the equations nonlinear, non-continuous (and non-monotone in the framework of nonlinear partial differential equations). We will discuss some modeling aspects, and some issues of existence, uniqueness and regularity for the solutions of the considered problems, making use of convex analysis and variational inequalities.

The Impact of Threshold Nonlinearities on Atmospheric Predictability

Joseph Tribbia
NCAR, USA

We consider the problem of atmospheric predictability in both the idealized circumstance of dry atmospheric dynamics and the more realistic situation in which threshold nonlinearity exists because of the possibility of a phase change due to the presence

of moisture. Recent research has reached conflicting conclusions as to whether such step function nonlinearities substantially modify the classical implications of quasi-geostrophic turbulence of exponential predictability. Alternatively, it has been posited that moist dynamics can result in a much more rapid cascade of error and thus result in a true "butterfly effect" as originally suggested by Lorenz in analysis of the implications of his 1963 result on deterministic non-periodic flow. A critical comparison of the nonlinear cascade of error between the dry and moist cases will be presented.

The Low-Frequency Variability of the Ocean-Atmosphere Coupled System - a Dynamical Systems Perspective

Stephane Vannitsem
Royal Meteorological Institute of Belgium, Belgium

The low-frequency variability (LFV) of the atmosphere at mid-latitudes develops on a wide range of time scales. One particularly interesting indicator of this variability is the North Atlantic Oscillation index measuring the fluctuations of predominant weather patterns in the course of the years over the Atlantic and Western Europe. The source of variability is, however, controversial and several possibilities have been envisaged, including oceanic and coupled ocean-atmosphere variability and stratospheric warming, possibly related to ENSO in the tropical Pacific.
Recently we have demonstrated that genuinely coupled LFV can emerge in a very simple low-order, nonlinear, coupled ocean-atmosphere model. This LFV concentrates on and near a long-periodic, attracting orbit. This orbit combines atmospheric and oceanic modes, and it arises for large values of the meridional gradient of radiative input and of the frictional coupling. Chaotic behavior develops around this orbit as it loses its stability. This behavior is still dominated by the LFV on decadal and multi-decadal time scales that is typical of oceanic processes. Furthermore, this natural coupled mode is still present as the number of variables is increased in the model. In this talk, we will present the dynamics of this type of reduced-order models, together with their predictability.

Interplay Between Mathematics and Physics

Shouhong Wang
Indiana University, USA

In this talk I shall present two examples demonstrating the symbiotic interplay between physics and mathematics. We start with a general principle that dynamic transitions of all dissipative systems can be classified into three categories: continuous, catastrophic and random. We shall illustrate this principle with an application of the theory to thermohaline circulation. Then we present a basic principle: the principle of interaction dynamics (PID), which takes the variation of the action functional under energy-momentum conservation constraint. With PID, we derive new gravitational field equations, providing a unified theory for dark matter and dark energy. In addition, the PID offers a simple first principle approach for introducing Higgs fields.

A Tree-Graph Approach to Selected Problems of Nonlinear Dynamics

Ilya Zaliapin
University of Nevada Reno, USA
Yevgeniy Kovchegov, Maxim Arnold

Graph-theoretic (network) representation has proven to be effective in many areas of science, prominently including geophysics. This talk focuses on nonlinear dynamics problems addressed using the simplest networks - tree graphs. Trees provide a close approximation to many natural structures and processes, including river networks, spread of disease or information, transfer of gene characteristics, dynamics of particles with localized interactions, etc. This would sound like a trivial observation if not for the following fact. Despite apparent diversity, a majority of rigorously studied branching structures belong to a two-parametric Tokunaga self-similarity class and exhibit Horton scaling. The Horton scaling is a counterpart of the power-law size distribution of system's elements. The stronger Tokunaga constraint ensures that different levels of a hierarchy have the same probabilistic structure. I will introduce a dynamic stochastic framework for studying self-similar trees and discuss so-called Horton-Smoluchowski differential equations that generalize Smoluchowski description of coalescent phenomena in a tree-oriented framework. This provides a new characterization for the classical Kingman's coalescent process and suggests a novel view on general coalescence process. I also show how a tree representation facilitates studies of self-similar solutions of the inviscid Burgers' equation, and cluster dynamics in the systems of interacting particles.

Special Session 2: Emergence and Dynamics of Patterns in Nonlinear Partial Differential Equations

Danielle Hilhorst, Université de Paris Sud, France
Yoshihisa Morita, Ryukoku University, Japan
Junping Shi, College of William and Mary, USA

The solution structures of many nonlinear partial differential equations reveal the emergence and the evolution of very exciting patterns. Such nonlinear models come from various fields of mathematical science, including material science as well as life sciences. In this session, we will bring together recent studies on solutions of nonlinear partial differential equations related to pattern formation, dynamics, and bifurcations, presenting new aspects of solutions capturing nonlinear phenomena together with underlying solution structures.

Nonlinear Elliptic Systems with Mixed Interactions Between Components

Jaeyoung Byeon
KAIST, Korea

For a nonlinear elliptic system where repulsive and attractive interactions coexist between components, the existence of a least energy vector solution and its pattern formation for large interactions will be introduced.

Convection and Linear Determinacy

Elaine Crooks
Swansea University, Wales
Ameera Al-Kiffai

In reaction-diffusion-convection models, convective effects due to, for instance, movement of insects in wind, or the motion of chemotactic cells, etc, can clearly have major impact on the behaviour of solutions, and in particular, introduce asymmetry in spreading speeds and front propagation due to the presence of first-order spatial derivatives. We will discuss sufficient conditions for linear determinacy, meaning that a spreading speed into an unstable state equals a value predicted by the linearization of the travelling-wave problem about that state, for classes of monostable reaction-diffusion-convection equations and co-operative systems in one space dimension. Separate conditions for spreading to the left and to the right are clearly needed. Various examples will be presented, illustrating the effect on linear determinacy of the interplay between the reaction and convection terms and diffusion parameters, and the symmetry-breaking caused by convection.

Constant Solutions, Ground State Solutions and Radial Terrace Solutions

Yihong Du
University of New England, Australia, Australia

In this talk, I will discuss the long-time behavior of nonnegative solutions of the Cauchy problem

$$u_t - \Delta u = f(u)(x \in R^N, t > 0),\ u(x,0) = u_0(x)(x \in R^N),$$

where u_0 is nonnegative with compact support, and f is a smooth function satisfying $f(0) = 0$. Suppose that $u(x,t)$ is a globally bounded solution, we will describe its behavior as $t \to \infty$ in the space $V_1 := L^\infty_{loc}(R^N)$ and in the space $V_2 := L^\infty(R^N)$. Under rather general conditions on f, we show that in V_1 the limit is a constant solution or a ground state solution of the corresponding elliptic problem over R^N, and in V_2, the long-time behavior of $u(\cdot,t)$ is described by a radial terrace solution.
This talk is based on joint works with Peter Polacik and with Hiroshi Matano.

Trend to Equilibrium for a Reaction-Diffusion System Modelling Enzyme Reaction

Jan Elias
University Paris-Sud, France

A spatio-temporal evolution of the species occurring in an enzyme reaction possessing some mass conservation properties is considered. In particular, we study the large time behaviour of solutions of a four-component non-linear reaction-diffusion system to the unique steady state. The system is equipped with the Laplacians for the diffusive motion of the species and the reaction terms which are obtained from the law of mass action kinetics applied to the enzyme reaction. A main tool used in the proposed large time analysis is an entropy method known from kinetic theory which is capable to provide explicitly computable rates of the convergence.

Forced Traveling Waves of the Fisher-KPP Equation in Moving Envrionment

Jian Fang
Harbin Institute of Technology, Peoples Rep of China

In this talk, I will investigate the type, existence and multiplicity of forced traveling waves for the following Fisher-KPP equation with a moving reaction term

$$u_t = u_{xx} + u(a(x - ct) - u), \quad x \in \mathbb{R}$$

under certain conditions on continuous function a. This talk is based on joint works with Henri Berestycki and with Yijun Lou and Jianhong Wu.

On Certain Models of Liquid Crystals

Eduard Feireisl
Czech Academy of Sciences, Prague, Czech Rep
E.Rocca, G.Schimperna, A.Zarnescu

We discuss mathematical properties of a model of liquid crystals, where the equation for the Q tensor is of hyperbolic type containing second material derivative. We show local solvability in the class of strong solutions and discuss global existence of weak solutions with a defect measure. Finally, we show that these weak solutions must coincide with a strong solution emanating from the same initial data as long as the latter exists.

Convergence of a Finite Volume Scheme for a First Order Conservation Law Involving a Q-Wiener Process

Yueyuan Gao
University Paris-Sud, France
Tadahisa Funaki, Danielle Hilhorst, Hendrik Weber

We study a time explicit finite volume method with a monotone scheme for a first order conservation law with a multiplicative source term involving a Q-Wiener process. We present some a priori estimates including a weak BV estimate. After performing a time interpolation, we prove two entropy inequalities satisfied by the discrete solution and show that the discrete solution converges up to a subsequence to a stochastic measure-valued entropy solution of the conservation law in the sense of Young measures as the maximum diameter of the volume elements and the time step tend to zero.

Localized States in Ohta-Kawasaki Models

Nir Gavish
Technion - IIT, Israel
Arik Yochelis, Idan Versano

The Ohta-Kawasaki model is a nonlocal Cahn-Hilliard model for pattern formation driven by competing long- and short-range interactions. The model is commonly used to describe pattern formation in diblock copolymer systems. Recently, we have developed an Otha-Kawasaki type model for ionic liquids. This equation involves more general, possibly asymmetric, long- and short-range interactions. In this talk, I will present a systematic study of pattern formation in the classical and the extended Otha-Kawasaki model, which reveals the effect of asymmetric interactions on pattern formation. Specifically, we focus on new spatially localized states in 1D and 2D in both infinite and finite domain sizes. We show that such states exist in both the classic and the extended Otha-Kawasaki model, and describe their dependence upon domain size.

Numerical Simulations of the Primitive Equations with Humidity and Saturations Above Mountain

Youngjoon Hong
University of Illinois, Chicago, USA

New avenues are explored to study the two dimensional inviscid primitive equations of the atmosphere with humidity and saturation, in presence of topography and subject to physically plausible boundary conditions for the system of equations. The filtering of the gravity waves produces a compatibility condition similar to the condition of incompressibility for the Navier-Stokes equations, which we treat in a similar manner. In that respect, a version of the projection method is introduced to enforce the compatibility condition on the horizontal velocity field, which comes from the boundary conditions. The resulting scheme allows for a significant reduction of the errors near the topography when compared to more standard finite volume schemes.

Traveling Waves in a Reaction-Diffusion System Describing Smoldering Combustion

Hirofumi Izuhara
University of Miyazaki, Japan
Ekeoma Rowland Ijioma, Masayasu Mimura

Combustion is a fast oxidation process and exhibits diverse behavior according to experimental conditions. When there is no natural convection of gas such as experiments in the space shuttle and in a vertically confined system, it is observed that unexpected finger-like smoldering combustion develops. In this talk, a reaction-diffusion-advection system that describes smoldering combustion is studied from the viewpoint of computer-aided analysis. In particular, we focus on the traveling wave solutions of the system, which play a role of a characteristic propagation of combustion.

The Role of the Microenvironment in Regulation of CSPG-Driven Tumor Growth: Invasive and Non-Invasive Gliomas

Yangjin Kim
Konkuk University, Korea
Hyun-Geun Lee

Glioblastoma (GBM) is one of the most lethal type of brain cancer with poor survival time. GBM is characterized by infiltration of the cancer cells through the brain tissue while lower grade gliomas and other non-neural metastatic types form self-contained non-invasive lesions. Glycosylated chondroitin sulfate proteoglycans (CSPGs), acting as critical regulators of the tumor microenvironment, dramatically govern the spatiotemporal status of resident reactive astrocytes and activation of tumor-associated microglia. In this paper we develop a mathematical model to investigate the effect of the CSPG distribution on regulation of a fundamental switch between two distinct patterns: invasive and non-invasive tumors. We show that the model's predictions agree with experimental results for a spherical glioma. The model specifically predicts that noninvasive tumor lesions are highly associated with a thick extracellular matrix (ECM) containing rich CSPGs, while the absence of glycosylated CSPGs results in diffusely infiltrative tumors. It is also shown that heavy CSPGs can drive the exodus of resident reactive astrocytes from the main tumor mass, and direct inhibition of tumor invasion by the astrogliotic capsule, leading to encapsulation of non-invasive lesions. However, stable residence of reactive astrocytes from GBM in the absence or low level of CSPGs presents a microenvironment favorable to diffuse infiltration due to loss of the primary (CSPG-induced cell-ECM bonding) and secondary (astrogliotic capsule) inhibitors. The mathematical model presents the clear role of the key tumor microenvironment in brain tumor invasion.

Construction of Dialogical Control

Ryo Kobayashi
Hiroshima University, Japan
Akio Ishiguro, Hitoshi Aonuma and Koichi Osuka

We introduce a challenge to construct a novel control policy for mobile robots. Conventional control theory made great successes by separating the plant and the environment and regarding the interaction between them as a disturbance. Such a treatment works well when the environment is well-known and well-defined, such a situation is typically achieved in factories. But mobile robot is not the case, because he/she must rush into the unknown environment. Under such situations, interaction between robots and environment can no more be regarded as a disturbance. We consider that some new control policy is needed to make robots move around in the complex world like animals do. We are proposing a new control policy named dialogical control, which means that the environment is not necessarily an enemy but a friend and we can have dialogue with an environment. Our approach includes three basic concept - hierarchical control, *Tegotae* control and Ying-Yang control. In this presentation, we mainly introduce a concept of *Tegotae* control and its potentials.

Existence and Non-Existence of Non-Constant Stationary Patterns for Certain Prey-Predator Type Reaction Diffusion Systems with a Cross-Diffusion Effect

Kazuhiro Kurata
Tokyo Metropolitan University, Japan
Sohei Omata

In this talk, we consider a stationary problem for certain prey-predator type reaction-diffusion systems with a cross-diffusion effect on a bounded domain in $\mathbf{R}^n$ under homogeneous Neumann boundary condition. This system does not have Turing's diffusion-driven instability, but has a cross-diffusion-driven instability. So, we are interested in the cross-diffusion effect and present results on the existence and non-existence of non-constant stationary patterns. We also study the limiting problem when the strength of the cross-diffusion parameter β tends to infinity. Numerically, we can see spiky patterns in the full system and have some rigorous evidence at least in the limiting system.

Traveling Wave Solutions of Lotka-Volterra Competition Systems with Nonlocal Dispersal in Periodic Habitats

Wan-Tong Li
Labzhou University, Peoples Rep of China
Xiongxiong Bao, Wenxian Shen

This talk is concerned with the existence, uniqueness and asymptotic stability of space periodic traveling wave solutions of the Lotka-Volterra competition system with nonlocal dispersal and space periodic dependence. Here we assume that the system admits two semitrivial space periodic equilibria $(u_1^*(x), 0)$ and $(0, u_2^*(x))$, where $(u_1^*(x), 0)$ is linearly and globally stable and $(0, u_2^*(x))$ is linearly unstable with respect to space periodic perturbations. This is the joint work with Xiongxiong Bao and Wenxian Shen.

Emergence and Transition of Clonal Selection Patterns in Acute Leukemias

Anna Marciniak-Czochra
University of Heidelberg, Germany

Motivated by clonal selection observed in acute myeloid leukemia (AML), we propose mathematical models describing evolution of a multiclonal and hierarchical cell population. The models in a form of partial and integro-differential equations are applied to study the role of self-renewal properties and growth kinetics during disease development and relapse. It is shown how resulting nonlinear and nonlocal terms may lead to a selection process and ultimately to therapy resistance. The solutions of the model tend asymptotically to Dirac measures multiplied by positive constants. Additionally, we show stability of the model in the space of positive Radon measures equipped with the flat metric (bounded Lipschitz distance). The results are compared to AML patient data. Model based interpretation of clinical data allows to assess parameters that cannot be measured directly. This might have clinical implications for future treatment and follow-up strategies.

The Phase-Field Crystal Model with Logarithmic Nonlinear Term

Alain Miranville
University of Poitiers, France

Our aim in this talk is to discuss the well-posedness of the phase-field crystal model with a logarithmic nonlinear term. In particular, we prove the existence and uniqueness of variational solutions, based on a variational inequality.

Mathematics of Cell-Cell Adhesion: Experiments, Modeling and Analysis

Hideki Murakawa
Kyushu University, Japan

Cell adhesion is the binding of a cell to another cell or to an extracellular matrix component. This process is essential in organ formation during embryonic development and in maintaining multicellular structure. Armstrong, Painter and Sherratt [J. Theor. Biol. 243 (2006), pp. 98-113] proposed a nonlocal advection-diffusion system as a possible continuous mathematical model for cell-cell adhesion. Although the system is attractive and challenging, it gives biologically unrealistic numerical solutions. We identify the problems and change underlying idea of cell movement from “cells move randomly to “cells move from high to low pressure regions. Then we provide a modified continuous model for cell-cell adhesion. Numerical experiments illustrate that the modified model is able to replicate not only the Steinberg’s cell sorting experiments but also some phenomena which can not be captured at all by Armstrong-Painter-Sherratt model. Furthermore, we give theoretical results about the modified model.

Stability of Traveling Waves for Some Bistable Lattice Dynamical System

Ken-Ichi Nakamura
Kanazawa University, Japan
Toshiko Ogiwara

In this talk, we will study traveling waves for some bistable system on a one-dimensional lattice arising in a model of competing species. We will discuss the stability of monotone traveling waves.

Spreading Fronts in the Anisotropic Allen-Cahn Equations on $\mathbb{R}^n$

Mitsunori Nara
Iwate University, Japan
Hiroshi Matano, Yoichiro Mori

We consider the Cauchy problem for the anisotropic Allen-Cahn equation on $\mathbb{R}^n$ with $n \geq 2$ and study the large time behavior of the solutions with spreading fronts. Our result states that, under some mild assumptions on the initial value, the solution develops a well-formed front whose position roughly coincides with the spreading Wulff shape.

Large Time Behavior, Lyapunov Functionals and Rearrangement Theory for a Nonlocal Differential Equation

Thanh Nam Nguyen
National Institute for Mathematical Sciences, Korea
Danielle Hilhorst, Philippe Laurencot

We consider an initial-boundary value problem for a nonlocal parabolic equation of bistable type and study possible sharp transition layers that the solution may develop at a very early stage. It turns out that such transition layers can be investigated via the structure of the omega limit set of the corresponding nonlocal ordinary differential equation. We prove that for a large class of initial functions, the omega limit set of the nonlocal ordinary differential equation only contains one element. Furthermore, that element is a step function taking at most two values. The proof bases on the rearrangement theory and the existence of infinitely many Lyapunov functionals.

Coupling Surface Diffusion with Grain Boundary Migration: Wetting and Dewetting

Amy Novick-Cohen
Technion-IIT, Israel
Vadim Derkach, Eugen Rabkin, Arcady Vilenkin

Solid state wetting and dewetting can be reasonably modeled by surface diffusion, and depending on the initial conditions and the parameters, can result in the formation of a variety of patterns. In the context of thin nanocrystalline specimens, the situation becomes more complicated since the wetting/dewetting surface is then coupled with a network of grain boundaries which can also migrate. Features of interest include: void formation, oscillation, acceleration, and fingering. In this lecture we will report primarily on numerical studies, which allow us to test various conjectures based on experiment. Our numerical studies open the door to a variety of analytical questions.

Nonlinear Fronts in the Swift-Hohenberg Equation As a Model for Phyllotaxis

Matt Pennybacker
University of New Mexico, USA
Alan C. Newell, Patrick Shipman

Some of the most spectacular patterns in the natural world can be found on members of the plant kingdom. Furthermore, the regular configurations of organs on plants, collectively called phyllotaxis, exhibit a remarkable predisposition for Fibonacci and Fibonacci-like progressions. Starting from a biochemical and mechanical growth model similar to the classic Swift-Hohenberg equation, we discuss the ways in which nearly every property of phyllotaxis can be explained as the propagation of a pushed pattern-forming front.

The Dynamics of Front-Like Solutions of Parabolic Equations on the Real Line

Peter Polacik
University of Minnesota, USA

We examine the behavior of bounded front-like solutions of reaction-diffusion equations on the real line. First, we give a general result on the approach of such solution to a propagating terrace, or, a stacked system of traveling fronts. Then we draw some consequences of this result, including in particular a quasiconvergence property of the solutions.

Dynamics and Bifurcation of Multicomponent Amphiphilic Membranes

Keith Promislow
Michigan State University, USA
Qiliang Wu

Abstract: Polymer chains are typically hydrophobic, the addition of functional groups to the backbone adds regions of hydrophilicity. The amphiphilic material (both hydrophobic and hydrophilic) has a strong affinity for solvent, imbibing it to self assemble charge-lined networks which serve as charge-selective ion conductors in a host of energy conversion applications. We present a continuum model for the free energy of an amphiphilic mixture. The associated gradient flows admit dynamic competition between network morphologies of distinct co-dimension. We present a model for multicomponent amphiphilic mixtures that permits competitive geometric evolution for co-dimension 1 bilayers and co-dimension two pore morphologies, present an analysis of the associated spectral problems, and describe rigorous existence results for pearled morphologies.

Multiscale Modelling and Analysis of the Mechanical Properties of Biological Tissues

Mariya Ptashnyk
University of Dundee, Scotland
Andrey Piatnitski, Brian Seguin

In this talk the derivation and analysis of microscopic models for the mechanics of biological tissues that take into account the interactions between the microscopic structure, mechanical properties and the chemical processes will be presented. The strongly coupled systems of nonlinear reaction-diffusion-convection equations for chemical processes and the equations of linear elasticity or poroelasticity for deformations will constitute the microscopic models. Using homogenization techniques we will derive the macroscopic models for the mechanics of biological tissues. The analysis and numerical simulations of the macroscopic equations will demonstrate the patterns in the interactions between mechanical stresses and chemical processes.

The Existence of Fast-Decay Ground States to a Weakly-Coupled Elliptic System

Yuanwei Qi
University of Central Florida, USA
Zhi Zheng, Xinfu Chen, Shoulin Zhou

This article studies the existence of positive solutions to a weakly-coupled elliptic system in R^N which originates from self-similar solutions of a parabolic system $u_t = \triangle u + v^p, \;\; v_t = \triangle v + u^q$, where $p, q > 1$. It is shown that there exist ground state solutions with exponential decay as $|x| \to \infty$ as well as positive solutions on a finite ball with zero Dirichlet boundary condition when (p, q) is subcritical. On the other hand, it is proved that when (p, q) is supercritical, both types of solutions do not exist under some technical conditions.

Homogenization of Cahn-Hilliard-Type Equations Via Gradient Structures

Sina Reichelt
Weierstrass Institute Berlin, Germany
Matthias Liero

In the paper [1] we discuss two approaches to evolutionary Γ-convergence of gradient systems in Hilbert spaces. The formulation of the gradient system is based on two functionals, namely the energy functional and the dissipation potential, which allows us to employ Γ-convergence methods. In the first approach we consider families of uniformly convex energy functionals such that the limit passage of the time-dependent problems can be based on the theory of evolutionary variational inequalities as developed by Daneri and Savaré 2010. The second approach uses the equivalent formulation of the gradient system via the energy-dissipation principle and follows the ideas of Sandier and Serfaty 2004. We apply both approaches to rigorously derive homogenization limits for Cahn–Hilliard-type equations. Using the method of weak and strong two-scale convergence via periodic unfolding, we show that the energy and dissipation functionals Γ-converge. In conclusion, we will give specific examples for the applicability of each of the two approaches.

References

[1] M. Liero and S. Reichelt, Homogenization of Cahn–Hilliard-type equations via evolutionary Γ-convergence, WIAS Preprint No. 2114 (2015).

Exclusive Regions Connected by Diffusion

Thomas Seidman
UMBC, USA

With 2 components $u, v \geq 0$ on $t > 0, x \in D$ the DE $u_t = -uv = v_t$ leads to exclusive regions: extinction of the (initially) smaller component. After the initial transient in $u_t - \epsilon\Delta u = -uv$, etc. one considers the evolution of a spatial pattern of regions. [This is complicated even by the possibility of s simple perturbation through other components (as, for the 1-dim case with an additional perturbing component w with $uv \mapsto w$ and $uw \mapsto 0$); in that case it can be shown rigorously that this problem is well posed.] More interesting geometry arises computationally in higher dimensions. This represents collaborations with A.Muntean and others.

Front Propagation and Symmetrization for the Fractional Fisher-KPP Equation

Andrei Tarfulea
University of Chicago, USA
Jean-Michel Roquejoffre

We prove strong gradient decay estimates for solutions to the multi-dimensional Fisher-KPP equation with fractional diffusion. It is known that this equation exhibits exponentially advancing level sets with strong qualitative upper and lower bounds on the solution. However, little has been shown concerning the gradient of the solution. We prove that, under mild conditions on the initial data, the first and second derivatives of the solution obey a comparative exponential decay in time. We then use this estimate to prove a symmetrization result, which shows that the reaction front flattens and quantifiably circularizes, losing its initial structure.

Analysis and Optimization for Edge-Emitting Semiconductor Heterostructures

Marita Thomas
Weierstrass Institute Berlin, Germany

This contribution discusses results on the existence of local-in-time classical solutions for edge-emitting semiconductor heterostructures both in 2D and 3D. Electronics of the semiconductor is governed by the van Roosbroeck system, consisting of the Poisson equation for the electrostatic potential and a system of drift-diffusion equations for the carrier transport. To describe lasing effects of a semiconductor device, the van Roosbroeck system is coupled with the equations of optics, given by a Helmholtz-type eigenvalue problem and an ODE for the photon balance. Based on this coupled system optimization strategies for the light emission of a device are obtained. This contribution is based on joint work with K. Disser, D. Peschka, J. Rehberg, and N. Rotundo (all WIAS Berlin).

Numerical Analysis for Reaction Diffusion Models of Spreading Phenomena with Free Boundary

Takeo Ushijima
Tokyo University of Science, Japan

In recent years, several reaction diffusion models with free boundary are proposed to describe the spreading of species or the transmission of diseases. In theses models the free boundary with Stefan like condition is concerned and it describes the motion of the front of the spreading phenomena. In this talk, we will discuss numerical methods for these models.

Nonplanar Traveling Fronts in Reaction-Diffusion Equations with Combustion and Non-KPP Monostable Nonlinearities

Zhi-Cheng Wang
Lanzhou University, Peoples Rep of China

In this talk we concern with nonplanar traveling fronts in reaction-diffusion equations with combustion nonlinearity and non-KPP monostable nonlinearity. Our study contains two parts: in the first part we establish the existence and stability of V-shaped traveling fronts in $\mathbb{R}^2$, and in the second part we establish the existence and stability of traveling fronts of pyramidal shape in $\mathbb{R}^3$.

Analysis of Coupled Degenerate Reaction-Diffusion SIR Model Describing Population Dynamics of Fox Rabies

Fengqi Yi
Harbin Engineering University, Peoples Rep of China

A coupled degenerate reaction-diffusion SIR model for the overall dynamics of the interaction between fox populations and rabies is considered. The model helps to explain the epidemiological patterns observed in Europe, including 3 to 5 year cycle in fox populations infected with rabies. We provide some global analyses of the model, including the asymptotic behaviors of the system. In addition, we show the existence of oscillatory behaviors due to Hopf bifurcation in the homogeneous model described by a set of ordinary differential equations. The existence of traveling wave solutions of the reaction-diffusion rabies epidemic model is also discussed.

Special Session 3: Nonlinear Evolution PDEs, Interfaces and Applications

Alain Miranville, University of Poitiers, France
Gunduz Caginalp, University of Pittsburgh, USA
Maurizio Grasselli, Politecnico di Milano, Italy

Many issues in applied science can be formulated as interface problems which can be regarded as limiting cases of evolution equations exhibiting transition layers. The study of phase field or diffusive interface problems, Allen-Cahn and Cahn-Hilliard equations have been an active area for the past few decades. This has also been an additional motivation for studying general nonlinear evolution equations. This session will focus on the mathematical properties (well-posedness, regularity, stability, asymptotic behavior of solutions, ...) and applications of these equations.

Surface Tension, Higher Order Phase Field Equations and Homogeneous Equations

Gunduz Caginalp
Univ of Pittsburgh, USA

Higher order phase field equations describe detailed structure such as complex anisotropy. The surface tension derived from these equations has some basic properties that may provide insight into interface formation. Using dimensional analysis one can show that surface tension has some homogeneity properties. In this talk some open problems related to these issues will be presented.

Analytical and Numerical Solutions of the Phase Field Crystal Model for the Interface Dynamics

Peter Galenko
University of Jena, Germany
I.G. Nizovtseva, K.R. Elder

Analytical and numerical methods are used to predict the dynamic evolution of a crystal-liquid interface [1]. In this work, the dynamics of the phase field crystal model is analyzed using complex amplitude wave representation. It is shown that if the phases of the amplitudes are constant, the model reduces to a non-linear Allen-Cahn equation that can be solved analytically for traveling waves. Numerical solutions of the phase field crystal model are compared with analytically obtained traveling wave solutions. These solutions can be used for the problem of crystal lattice selection at the front of crystalline phase invading homogeneous liquid state [2].

References

[1] P.K. Galenko, F.I. Sanches, and K.R. Elder, Traveling wave profiles for a crystalline front invading liquid states: analytical and numerical solutions. Physica D: Nonlinear Phenomena 308 (2015) 1-10.

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Generalized Regularized Long Wave Equation with White Noise Dispersion

Olivier Goubet
Univeriste de Picardie Jules Verne, France
Min Chen, Youcef Mammeri

In this talk, we address the generalized BBM equation with white noise dispersion which reads

$$du - du_{xx} + u_x \circ dW + u^p u_x dt = 0,$$

in the Stratonovich formulation, where $W(t)$ is a standard real valued Brownian motion. We first investigate the initial value problem for this equation. We then prove theoretically and numerically that for a deterministic initial data, the expectation of the L^∞_x norm of the solutions decay to zero at $O(t^{-\frac{1}{6}})$ as t approaches $+\infty$, by assuming that $p > 8$ and that the initial data is small in $L^1_x \cap H^4_x$. This decay rate matches the one for solution of the linear equation with white noise dispersion.

Coupled Navier-Stokes and Phase Field Models for Interface Dynamics in Coating Process.

Ludovic Goudenège
CNRS, France
Isabelle Cantat, Sebastian Minjeaud, Jacopo Seiwert

A coating process, for instance the film deposition by withdrawing a substrate from a bath with solution, is a complex phenomenon. In order to study and simulate it, we need at least two equations. One for the dynamic of the fluid and one for the interface between the different species.
For this reason we present a phase-field model of Allen-Cahn or Cahn-Hilliard type coupled with the Navier-Stokes equations. But we are concerned by the dynamics of surfactant at the interface, so we add a new equation to the coupled system in order to take into account the effects of surfactant.
We will present several models for surfactant dynamics and study their impact on the evolution of the interface. In particular we will present the adequation between numerical simulations and experimentations with respect to some classical well-known physical laws.

On Micro-Macro Models for Two-Phase Flow with Dilute Polymeric Solutions – Modeling and Analysis

Günther Grün
University of Erlangen-Nürnberg, Germany
Stefan Metzger

By methods from nonequilibrium thermodynamics, we derive a diffuse-interface model for two-phase flow of incompressible fluids with dissolved noninteracting polymers. The polymers are modeled by dumbbells subjected to finitely extensible, nonlinear elastic (FENE) spring-force potentials. Their density and orientation are described by a Fokker-Planck-type equation which is coupled to a Cahn-Hilliard and a momentum equation for phase-field and gross velocity/pressure. Henry-type energy functionals are used to describe different solubility properties of the polymers in the different phases or at the liquid-liquid interface. Taking advantage of the underlying energetic/entropic structure of the system, we prove existence of a weak solution globally in time. As a by-product in the case of Hookean spring potentials, we derive a macroscopic diffuse-interface model for two-phase flow of Oldroyd-B-type liquids. We discuss extensions of the model to take the interaction between polymer and fluid interface orientation into account (amphiphilic surfactant) and provide some numerical experiments.

Mathematical Analysis of a Parabolic System for Chemotactic E.coli Colonies

Danielle Hilhorst
CNRS and Univ. Paris-Sud Paris-Saclay, France
R. Celinski, G. Karch, M. Mimura

We consider an initial-boundary value problem describing the formation of colony patterns of bacteria Escherichia coli. This model consists of reaction-diffusion equations coupled with the Keller-Segel system from the chemotaxis theory in a bounded domain, supplemented with zero-flux boundary conditions and with nonnegative initial data. We answer questions about the global in time existence of solutions as well as on their large time behavior. Moreover, we show that the solutions of a related model may blow up in finite time.

Convergent Finite Difference Scheme for Compressible Navier-Stokes Equations in Three Dimensions

Radim Hošek
Czech Academy of Sciences, Czech Rep

Motivated by works of Karper and co-authors we suggest a numerical scheme for solving compressible Navier-Stokes equations in three spatial dimensions that is based on finite difference method. The scheme is applied to a domain with simple cubic geometry. We mimick the existence proof developed by Lions and Feireisl to show that a sequence of numerical solutions on grids with decreasing discretization parameter converges weakly (up to a subsequence) to a weak solution of the compressible Navier-Stokes system. The main difficulties of the proof compared to the standard Karper's FEM/DG scheme will be pointed out.

Global Dynamics of Boundary Droplets for the 2-D Mass-Conserving Allen-Cahn Equation

Jiayin Jin
Georgia Institute of Technology, USA
Peter W. Bates

In this talk I will present how to establish the existence of an invariant manifold of bubble states for the mass-conserving Allen-Cahn equation in two space dimensions, and give the dynamics of the center of the bubbles.

A Diffuse Interface Tumour Model with Chemotaxis and Active Transport

Kei Fong Lam
University of Regensburg, Germany
Harald Garcke, Emanuel Sitka, Vanessa Styles

We consider a thermodynamically consistent diffuse interface model for tumour growth, which couples a Cahn-Hilliard system and a reaction-diffusion equation. The system of PDEs models the growth of a tumour in the presence of a nutrient and surrounded by host tissue. A new feature of the model is the inclusion of transport mechanisms such as chemotaxis and active transport through specific choices of the fluxes. We will first give a heuristic derivation of a simplified model and discuss how chemotaxis and active transport are modelled, along with some results regarding the well-posedness of the system. Then, we will present the more general model, which is a Cahn-Hilliard-Darcy system coupled to a convection-reaction-diffusion equation for the nutrient. The effects of including the transport mechanisms and fluid flow will be demonstrated with numerical computations, and if time permitting, we will discuss some results regarding the existence of weak solutions to the general model. This is joint work with Harald Garcke, Emanuel Sitka (Regensburg) and Vanessa Styles (Sussex).

On the Viscous and Non-Viscous Cahn-Hilliard and Allen-Cahn Equations

Ahmad Makki
LMA- Universite de Poitiers, France
Alain Miranville

We discuss the well-posedness and the asymptotic behaviour of the viscous Allen-Cahn (resp. Cahn-Hilliard) and the non-viscous anisotropic Cahn-Hilliard (resp. Allen-Cahn) equations. In particular, these models contain a regularization term, called Willmore regularization. Also, we give some numerical simulations which show the effects of the viscosity term on the anisotropic and isotropic Cahn-Hilliard equations.

A Phase-Field Model for Thermal Binary Alloys: Asymptotic Behaviour

Pedro Marin-Rubio
Universidad de Sevilla, Spain
Gabriela Planas, Jose Real

In this talk I will present some recent results on existence, but with uniqueness issue unknown, to a phase-field model for a thermal binary alloy. After that, using some techniques from multivalued analysis, the long-time behaviour of dynamical systems associated the problem will be analysed. In particular, existence and regularity of attractors and structure of omega-limit sets will be discussed.

Existence of Pulses for a Monotone Reaction-Diffusion System

Martine Marion
Ecole Centrale de Lyon, France
V. Volpert

We discuss the existence of pulses for a monotone reaction-diffusion system of two equations. The result is applied to prove the existence of pulses for the system of competition of species in population dynamics.

On Stable Splitting Schemes for Phase-Field Models with Ion Transport

Stefan Metzger
FAU Erlangen-Nürnberg, Germany
Günther Grün

We present an energy-stable, decoupled discrete scheme for a recent model (see [1]) supposed to describe electrokinetic phenomena in two-phase flow with general mass densities. This model permits to describe the effect of contact line hysteresis. It considers several kinds of species with different charges and solubility properties, which may react with each other. Exposing the fluids to an electric field induces ion motion and therefore influences the fluid flow. Topological changes in the droplet configuration are practically important features to be captured by this approach.

As the model couples Nernst–Planck-equations with momentum and Cahn–Hilliard phase-field equations in a nonlinear way, the formulation of a stable, efficient and fully practical numerical scheme is not obvious.

Using a subtle approximation of the velocities arising in the convective terms, we manage to derive a stable splitting scheme allowing to decouple the equations for the phase-field, the species, and the electro-static potential from the momentum equations (cf. [2]).

In the case of a fluid independent dielectric permittivity, we end up with three blocks which can be treated sequentially. By establishing a discrete counterpart of the continuous energy estimate, we will show that this splitting approach does not affect the stability of the scheme. Finally, we shall present numerical simulations showing species induced fluid motion and droplet break-up to underline the full practicality of this approach.

References

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[2] S. Metzger, On numerical schemes for phase-field models for electrowetting with electrolyte solutions, *PAMM*, **15**, 715–718 (2015).

Dynamic Boundary Conditions for Allen–Cahn Type Equations with Singularities

Ryota Nakayashiki
Chiba University, Japan
Pierluigi Colli, Gianni Gilardi, Ken Shirakawa

This study is suported by CNR-JSPS Joint Project "Innovative variational methods for evolution equations". In this talk, we consider coupled systems of nonlinear variational inequalities. Each system consists of an Allen-Cahn type equation in a bounded spatial domainΩ , and another Allen-Cahn type equation on the boundary $\Gamma := \partial\Omega$, and also, these equations are transmitted via a dynamic boundary condition. In particular, the equations in Ω are derived from nonsmooth free-energies, and hence, the diffusions in Ω are provided in quasi-linear forms with singularities. The objective of this study is to build a mathematical method which enable to deal with the quasi-linear and singular situations. On this basis, we will set the focus of the discussion on the well-posedness of our systems in L^2-based framework, and the continuous associations among different systems brought by Γ-convergences of free-energies.

Coupled Surface Diffusion and Grain Boundary Migration in Three Grain Systems

Amy Novick-Cohen
Technion-IIT, Israel
Vadim Derkach, John McCuan

The stability of thin nanocrystalline films is critical in numerous applications. We consider the relative motion of three grains with prescribed volume, which are constrained to lie within a triangular geometry which can be used to produce a thin film "tiling" of the plane via mirror refection. In our model, the exterior surface of the grains evolve by surface diffusion motion and the boundaries between the grains evolve by motion by mean curvature. Drawing conclusions based on existence of steady states, relative energies, and the results of numerical simulations, we demonstrate that many phenomena are truly nonlinear.

On the Viscous Cahn-Hilliard-Navier-Stokes Equations with Dynamic Boundary Conditions

Madalina Petcu
University of Poitiers, France
Laurence Cherfils

The presentation concentrates on the study of the viscous Cahn-Hilliard-Navier-Stokes model, endowed with dynamic boundary conditions, from the theoretical and numerical point of view. We start by deducing results on the existence, uniqueness and regularity of the solutions for the continuous problem. Then we propose a space semi-discrete finite element approximation of the model and we study the convergence of the approximate scheme. We also prove the stability and convergence of a fully discretized scheme, obtained using the semi-implicit Euler scheme applied to the space semi-discretization proposed previously. Numerical simulations are also presented to illustrate the theoretical results.

Convergence to Equilibrium for Finite Element Discretizations of the Modified Phase Field Crystal Equation

Morgan Pierre
University of Poitiers, France
Maurizio Grasselli

The phase field crystal equation is a conservative Swift-Hohenberg equation which has been employed to model and simulate the dynamics of crystalline materials. The modified version considered here can account for elastic interactions. We propose a fully discrete approximation of this model based on a Galerkin approximation in H^1 for the phase function and a second-order energy stable time discretization. We analyze the properties of this approximation (well-posedness, stability, convergence as the discretization parameters tend to 0). In particular, we will prove that the discrete solution converges to a steady state as time goes to infinity.

Optimal Control of the Dendrite Structure Using Magnetic Field

Amer Rasheed
Lahore University of Management Sciences (LUMS), Pakistan
Aziz Belmiloudi

We present the optimal control of a phase field model, recently developed by A. Rasheed and A. Belmiloudi [1], which represents the effect of magnetic field on the evolution of dendrite during the solidification process of a binary alloy in an isothermal environment. The aim of this study is to control the desired dynamics of the dendrite by using magnetic field as a control variable. In the control problem, the cost functional measures the distance between the calculated and desired dynamics. We have established the existence results and optimality conditions along with the adjoint problem.

References

[1] A. Rasheed and A. Belmiloudi, Modeling and simulations of dendrite growth using phase-field method with magnetic field effect, Communications in Computational Physics, 14(2), pp. 477 - 508, 2013.

A Convergent, Energy Stable, and Efficient Hexagonal Finite Difference Scheme for the Phase Field Crystal Amplitude Expansion Model

Steven Wise
The University of Tennessee, USA
Zhen Guan, Vili Heinonen, John Lowengrub, Cheng Wang

The phase field crystal model is a sixth order, nonlinear parabolic PDE describing crystal dynamics at the atomic length scale, but on diffusion time scales. It can describe point defects, grain boundaries, and elastically mediated phase transformations. I will describe a reformulation of the model using the technique of amplitude expansions, which reduces the model to a system of fourth-order equations, but introduces several complexities. I will briefly describe some properties of the PDE solutions and will then investigate an energy stable and convergent numerical method for the amplitude model based on a nonstandard finite difference discretization of space. I will show a number of numerical examples demonstrating the advantages of the scheme. This is joint work with Zhen Guan, Vili Heinonen, John Lowengrub, and Cheng Wang.

Long-Time Behaviour of the Plate Equation with Nonlocal Nonlinearity

Sema Yayla
Hacettepe University, Turkey
Zehra Arat, Azer Khanmamedov

We consider Cauchy problems for the semilinear plate equations with nonlocal nonlinearities $f(\|\nabla u\|_{L^2(\mathbb{R}^n)})\Delta u$ and $f(\|u(t)\|_{L^p(\mathbb{R}^n)})\,|u|^{p-2}\,u$. In the case of nonlocal nonlinearity $f(\|\nabla u\|_{L^2(\mathbb{R}^n)})\Delta u$, we prove the existence of the global attractor and establish the regularity and finite dimensionality of this attractor under the strict positivity condition on the damping coefficient. For the plate equation with nonlinearity $f(\|u(t)\|_{L^p(\mathbb{R}^n)})\,|u|^{p-2}\,u$, we prove the existence and regularity of the global attractor under the weaker condition on the damping coefficient. Namely, in this case we require the strict positivity condition on the damping coefficient only in the exterior of some ball.

Special Session 4: Classical and Geophysical Fluid Dynamics

Youngjoon Hong, University of Illinois at Chicago, USA
Madalina Petcu, Université de Poitiers, France
Roger Temam, Indiana University, USA

Over the past decades, mathematics and geophysics have found successful interactions through the study of the Euler, Navier-Stokes and convection-diffusion equations, and their geophysical counterparts the Boussinesq and Primitive Equations. The main purpose of this workshop is to bring together specialists in the field and to exchange ideas on these problems. This special session focuses on recent progresses in the development and applications of classical and geophysical fluid dynamics. In particular, the session aims to address some important issues such as the properties of solutions, existence, regularity, stability and asymptotic behaviors arising from nonlinear differential equations.

Some Dynamical Problems Arising in Blood Flow

Jerry Bona
University of Illinois at Chicago, USA
Chun-Hsiung Hsia, Daniela Jasso-Valdez

Pulmonary arterial hypertension is an insidious disease that currently has poor prospects for treatment. In an effort to better understand this disease, a model is developed for blood flow in the relevant part of the body. The model is shown to have at least some predictive capability. A plan is then outlined for using the model in conjunction with laboratory data to begin fathoming the remodeling that is a hallmark of the condition, and which is oftentimes the cause of premature death.

The Role of the Pressure in the Partial Regularity Theory for Weak Solutions of the Navier-Stokes Equations

Diego Chamorro
Universite d'Evry, France
Pierre-Gilles Lemari'e-Rieusset, Kawther Mayoufi

We study the role of the pressure in the partial regularity theory for the weak solutions of the Navier-Stokes equations. By introducing the notion of dissipative solutions, due to Duchon and Robert, we will provide a generalization of the Caffarelli, Kohn and Nirenberg theory. Our approach gives a new enlightenment of the role of the pressure in this theory in connection to Serrin's local regularity criterion.

Low-Dimensional Approximate Solutions of Hamilton-Jacobi-Bellman Equations, and Applications to the Optimal control of nonlinear evolution equations

Mickael Chekroun
University of California, Los Angeles, USA
Axel Kröner, Honghu Liu

In this talk a new approach to approximate the solutions of Hamilton-Jacobi-Bellman (HJB) equations, will be presented. The focus of our study is concerned with the optimal control of nonlinear evolution equations in Hilbert spaces. The approach fits into the long tradition of seeking for slaving relationships between the small scales and the large ones but differ by the introduction of a new type of manifolds to do so, namely the finite-horizon parameterizing manifolds introduced by Chekroun and Liu (ACTA Appl. Math., 2015, **135**, pp. 81-144). A control in feedback form will be derived from the reduced HJB equations associated with the corresponding optimal control problem for the surrogate system. Rigorous error estimates between the solutions of the corresponding HJB equations and the approximate ones will be also derived and application to fluid problems discussed. This talk is based on a joint work with Axel Kroner and Honghu Liu.

The Barotropic Quasi-Geostrophic Equation Under a Free Surface

Qingshan Chen
Clemson University, USA

When the length scale of the flow is on the same order of the Rossby deformation radius, the classical rigid-lid assumption is no longer valid, the impact of the free surface deformation on the the vorticity field is no longer negligible, and therefore it has to be accounted for in the model. In this talk, we present some new results concerning the well-posedness of the barotropic quasi-geostrophic equation under a free surface. The connection of this model to other more complex and more realistic models will also be discussed.

Two-Point Boundary Value Problem for a Higher Order Nonlinear Dispersive Wave Equation

Hongqiu Chen
University of Memphis, USA
Shu-Ming Sun, Juan-Ming Yuan

The focus of the talk is the higher order nonlinear dispersive equation which models unidirectional propagation of small amplitude long waves in dispersive media. The specific interest is in the initial, two point-boundary value problem. With proper requirement on initial and boundary condition, we show local and global well posedness.

Determining Wavenumber for Fluid Equations

Alexey Cheskidov
UIC, USA

We introduce a determining wavenumber for the forced 3D Navier-Stokes equations (NSE) defined for each individual solution. Even though this wavenumber blows up if the solution blows up, its time average is uniformly bounded by Kolmogorov's dissipation wavenumber for all solutions on the global attractor. A similar approach in the two-dimensional case is consistent with the prediction of Kraichnan's theory of turbulence.

Determining Modes for the Surface Quasi-Geostrophic Equations

Mimi Dai
University of Illinois Chicago, USA
Alexey Cheskidov

We introduce a determining wavenumber for the surface quasi-geostrophic (SQG) equation
defined for each individual trajectory and then study its dependence on the force. While in the subcritical and critical cases this wavenumber has a uniform upper bound, it may blow up
when the equation is supercritical. A bound on the determining wavenumber provides determining modes, which in some sense measure the number of degrees of freedom of the flow, or resolution needed to describe a solution to the SQG equation.

Optimal Ship Forms Based on Michell's Wave Resistance: Numerical Aspects

Julien Dambrine
University of Poitiers, France
Morgan Pierre, Germain Rousseaux

The slender body approximation, first introduced by Rankine for the study of a potential flows around a thin obstacle in an infinite domain states that a thin obstacle has the same effect on the flow as a distribution of source/well doublets on a line. This model has been extended later for linear free surface flows and lead to the explicit formulas of Michell for the computation of the wave making resistance of a slender ship moving with an uniform velocity in deep waters. This formula provides the wave making resistance as a quadratic function of the hull offset function. Although very well known and widely used for the prediction of the wave-making resistance of simple ships, very few attempts of hull optimisation have been made through this model. The goal of this talk is to illustrate the theoretical issues of ship hull optimisation with some numerical calculations.

Boundary Layers of the Navier-Stokes Equations

Gung-Min Gie
University of Louisville, USA

In this talk, we discuss some recent progresses on boundary layer analysis of the Navier-Stokes equations.

Energy and Potential Enstrophy Flux Constraints in Quasi-Geostrophic Models

Eleftherios Gkioulekas
University of Texas Rio Grande Valley, USA

We investigate an inequality constraining the energy and potential enstrophy flux spectra in two-layer and multi-layer quasi-geostrophic models. Its physical significance is that it can diagnose whether any given multi-layer model that allows co-existing downscale cascades of energy and potential enstrophy can allow the downscale energy flux to become large enough to yield a mixed energy spectrum where the dominant k^{-3} scaling is overtaken by a subdominant $k^{-5/3}$ contribution beyond a transition wavenumber k_t situated in the inertial range. The validity of the flux inequality implies that this scaling transition cannot occur within the inertial range, whereas a violation of the flux inequality beyond some wavenumber k_t implies the existence of a scaling transition near that wavenumber. This flux inequality holds unconditionally in two-dimensional Navier-Stokes turbulence, however, it is far from obvious that it continues to hold in multi-layer quasi-geostrophic models, because the dissipation rate spectra for energy and potential enstrophy no longer relate in a trivial way, as in two-dimensional Navier-Stokes. We derive the general form of the energy and potential enstrophy dissipation rate spectra for a generalized symmetrically coupled multi-layer model. From this result, we prove that in a symmetrically coupled multi-layer quasi-geostrophic model, where the dissipation terms for each layer consist of the same Fourier-diagonal linear operator applied on the streamfunction field of only the same layer, the flux inequality continues to hold. It follows that a necessary condition to violate the flux inequality is the use of asymmetric dissipation where different operators are used on different layers. We explore dissipation asymmetry further in the context of a two-layer quasi-geostrophic model and derive upper bounds on the asymmetry that will allow the flux inequality to continue to hold. Asymmetry is introduced both via an extrapolated Ekman term, based on a 1980 model by Salmon, and via differential small-scale dissipation. The results given are mathematically rigorous and require no phenomenological assumptions about the inertial range. Sufficient conditions for violating the flux inequality, on the other hand, require phenomenological hypotheses, and will be explored in future work.

Rotating Fluids and Boundary Layers

Makram Hamouda
Indiana University, USA
Soumaya Ben Chaabane, Mahdi Tekitek

We investigate in this talk the boundary layers appearing for a fluid under moderate rotation when the viscosity is small. The fluid is modeled by time-dependent rotating Stokes equations also known as the Stokes-Coriolis equations. The fluid is considered in an infinite channel with periodicity on the lateral boundary and Dirichlet boundary conditions on the top and bottom of the channel. First, we analytically derive the correctors which describe the sharp variations at large Reynolds number i.e. small viscosity). Second, thanks to a new finite volume method (NFVM) we give the numerical solutions of the Stokes-Coriolis system at small viscosity $(10^{-2}-10^{-10})$. The NFVM can be applied to a large class of singular perturbation problems.

Optimal Ship Forms Based on Michell's Wave Resistance: Theoretical Aspects

Morgan Pierre
University of Poitiers, France
Julien Dambrine, Germain Rousseaux

In 1898, J.H. Michell provided a formula for the wave resistance of a thin ship moving at constant speed in calm water. By adding a simple viscous resistance term (proportional to the area of the immerged hull and to the square of the speed), we obtain a model for the total resistance of water to the motion of the ship. This allows us to seek for optimal ship hulls expressed in a parametric form, with parameters in the region of the vertical plane of symmetry. We investigate some mathematical properties of the optimal hull, first when the domain of parameters is fixed, and then when this domain varies.

Two-Dimensional Bifurcation in the Whitham Equation with Surface Tension

Filippo Remonato
NTNU - Norwegian University of Science and Technology, Norway
Henrik Kalisch, Mats Ehrnström, Matthew Johnson

The capillary Whitham equation

$$u_t + u^2 + K * u = 0,$$

with the convolution kernel defined as

$$K(x) = \mathcal{F}^{-1}\left(m_T(\xi)\right)(x),$$
$$m_T(\xi) = \sqrt{\frac{(1+T\xi^2)\tanh(\xi)}{\xi}},$$

models small-amplitude shallow water waves with surface tension. Here $T \in \mathbb{R}$ represents the capillarity.
In 2011, Ehrnström, Escher and Wahlèn proved the existence of small amplitude sheets of steady solutions bifurcating from a two-dimensional kernel for the water-wave problem with vorticity.
A similar result has been given for irrotational waves, but with surface tension, by Toland and Jones.
In this talk we show that an analogous result holds for the capillary Whitham equation, establishing global one-dimensional and local two-dimensional bifurcation analytically. In addition, we present several numerical results showing the profile of the waves along different branches.

On Nontraditional Quasi-Geostrophic Equations

Antoine Rousseau
Inria Chile, France
Carine Lucas, James C. McWilliams

We present nontraditional models where the so-called traditional approximation on the Coriolis force is removed. In the derivation of the quasi-geostrophic equations, we obtain new terms in δ/ε, where δ (aspect ratio) and ε (Rossby number) are both small numbers. We provide here some rigorous crossed-asymptotics with regards to these parameters, prove some mathematical results and provide physical properties on the nontraditional models, and situate them among traditional ones.

Dynamic Transitions of Quasi-Geostrophic Channel Flow

Taylan Sengul
Marmara University, Turkey
Henk Dijkstra, Jie Shen, Shouhong Wang

The main aim of this talk is to describe the dynamic transitions in flows described by the two-dimensional, barotropic vorticity equation in a periodic zonal channel, one of the cornerstone dynamical model of the ocean and atmospheric circulation.
The equation admits a steady state solution which represents a zonal jet. In this talk, the recent advances in this problem which addresses the stability problem of the bifurcated periodic solutions will be considered. In particular, it will be shown that the modelled flow exhibits either a continuous or catastrophic transition as the basic zonal jet loses its stability.

Precipitating Quasi-Geostrophic Equations

Leslie Smith
University of Wisconsin, Madison, USA

A persistent challenge in atmospheric science is to understand the dynamical role of water substance, tightly linked with the complicated dynamics of clouds and convection. This work presents a framework for theory and modeling of precipitating convection on mid-latitude large scales, analogous to the dry quasi-geostrophic (QG) equations. A precipitating QG approximation is derived from a distinguished limit of parent equations combining asymptotically fast cloud microphysics and a Boussinesq core. The presence of phase changes from water vapor to liquid water and vice versa leads to important differences from the dry QG case: (i) the appropriately defined potential vorticity (PV) changes across phase boundaries, (ii) the evolution for the PV must be coupled to an evolution for a thermodynamic quantity combining temperature and water, and (iii) the inversion to find the pressure involves a nonlinear elliptic operator. Some examples will be discussed.

Numerical Algorithms for Stationary Statistical Properties of Dissipative Dynamical Systems

Xiaoming Wang
Florida State University, USA

It is well-known that physical laws for large chaotic dynamical systems are revealed statistically.
The main concern of this manuscript is numerical methods for dissipative chaotic infinite-dimensional dynamical systems that are able to capture the stationary statistical properties of the underlying dynamical systems.
We show that the main ingredients in ensuring the convergence of the long time statistical properties of the numerical schemes are: (1) uniform dissipativity of the scheme in the sense that the union of the global attractors of the numerical approximations is pre-compact in the phase space; (2) convergence of the solutions of the numerical scheme to the solution of the continuous system on the unit time interval [0,1] modulo an initial layer, uniformly with respect to initial data from the union of the global attractors. The two conditions are reminiscent of the Lax equivalence theorem where stability and consistency are needed for the convergence of a numerical scheme.
Applications to the complex Ginzburg-Landau equation and the two-dimensional Navier-Stokes equations in a periodic box will be discussed.

Special Session 5: Spatial and Evolutionary Aspects in Ecology and Epidemiology

Yuan Lou, Ohio State University, USA and Renmin University, Peoples Rep of China
King-Yeung Lam, Ohio State University, USA
Steve Cantrell, Chris Cosner, University of Miami, USA

Spatial heterogeneity is indispensable in explaining many important phenomena in ecology and epidemiology, e.g. the persistence of complicated foodwebs and the definition of the basic reproduction number in a spatial habitat. A natural and challenging question is to understand the effect of space in the evolution of various processes including animal behavior and the spread of diseases. Prompted by these considerations, a variety of mathematical models has been proposed and studied, including PDE models, patch models, integral-differential models, etc. In this special session we will bring together people across different modeling frameworks both to review the current state-of-the-art of the area, and to discuss the emerging theoretical questions and their mathematical challenges.

On Competition of Species with Different Diffusion Strategies

Elena Braverman
University of Calgary, Canada
Md. Kamrujjaman, L. Korobenko

We study the interaction between two species choosing different types of dispersal in a heterogeneous environment, where in addition intrinsic growth rates and carrying capacities can be different. As a particular case we assume that one of them moves in the direction of most per capita available resources while the other choose a different type of diffusion. Then, only the spatial profile of the intrinsic growth rate matters: if they coincide, together with spatially heterogeneous carrying capacities, then competitive exclusion of the latter population is inevitable. However, the situation may change if intrinsic growth rates for the two populations have different spatial forms, or carrying capacities are different.
We also study a Lotka system describing two populations which can compete or cooperate, and each of them chooses its diffusion strategy as the tendency to have a distribution proportional to a certain positive prescribed function. We prove that higher diffusion coefficients are detrimental while higher growth rates, as well as lower resources sharing, are beneficial for population survival, either for similar diffusion strategies or whenever one of the species chooses the carrying capacity driven dispersal. We consider the case when the choice of dispersal strategies guarantees coexistence, and compare different diffusion strategies. The case of several species is discussed (work in progress).

Population Genetic PDE Models of Dispersal and Spatially Varying Selection

Reinhard Buerger
University of Vienna, Austria
Linlin Su

In population genetics we study how the genetic composition of biological populations is shaped by ecological and genetic factors such as selection, mutation, recombination, or migration. This talk will focus on PDE models of a spatially distributed population dispersing in a heterogeneous environment. We extend classical and recent results, which assume a single gene locus under selection, by considering two recombining loci under selection. This does not only add biological realism, but also substantial mathematical difficulties caused by the more complicated structure of the underlying PDEs. In particular, the equilibrium structure depends strongly on the recombination rate. We use perturbation techniques to explore the existence, stability and other properties of stationary solutions, called clines, for various limiting cases. Several open problems will be mentioned. This talk will be based on joint work with Dr. Linlin Su.

Spatial Sorting: Travelling Waves and Accelerating Fronts

Vincent Calvez
CNRS & ENS de Lyon, France

I will present several aspects of dispersal evolution during expansion of an invasive species: 1) existence of travelling waves for a minimal reaction-diffusion-selection model in the case of bounded dispersal rates; 2) front acceleration in the case of unbounded dispersal rates. Interestingly, analysis is performed for a population dynamics equation which is structured with respect to both space and dispersal rate.

Resident-Invader Dynamics in Infinite Dimensional Systems

Robert Stephen Cantrell
University of Miami, USA
Chris Cosner, King-Yeung Lam

We discuss an extension of the Tube Theorem from adaptive dynamics to infinite dimensional contexts, including that of reaction-diffusion equations.

The Reduction Principle, the Ideal Free Distribution, and The Evolution of Dispersal Strategies

Chris Cosner
University of Miami, USA

The problem of understanding the evolution of dispersal has attracted much attention from mathematicians and biologists in recent years. For reaction-diffusion models and their nonlocal and discrete analogues, in environments that vary in space but not in time, the strategy of not moving at all is often convergence stable within many classes of strategies. This is related to a "reduction principle which states that in general dispersal reduces population growth rates. However, when the class of feasible strategies includes strategies that generate an ideal free population distribution at equilibrium (all individuals have equal fitness, with no net movement), such strategies are known to be evolutionarily stable in various cases. Much of the work in this area involves using ideas from dynamical systems theory and partial differential equations to analyze pairwise invasibility problems, which are motivated by ideas from adaptive dynamics and ultimately game theory. The talk will describe some past results and current work on these topics.

Propagation Phenomena in Nonlocal Monostable Equation: Travelling Wave Vs Acceleration

Jerome Coville
INRA, France
M. Alfaro

I will present some recent results on existence/non-existence of travelling wave solution for the homogeneous nonlocal equation

$$\partial_t u(t,x) = J \star u(t,x) - u(t,x) + f(u(t,x)), \quad \mathbb{R}^+ \times \mathbb{R}$$

when f is a monostable degenerate function and J a probability density, i.e. f such that $f(0) = 0 = f'(0) = f(1)$ and $f > 0$ in (0,1). In this situation, the existence of a travelling wave is conditioned by the tailed of J and the behaviour of f near 0. When $f(s) = s^p(1-s)$, I will present the curve that describes the region where Travelling waves exists or not. I will also present an optimal criteria for the existence of TW when f is of ignition type.

A Patched Malaria Model: Implications for Control

Patrick de Leenheer
Oregon State University, USA

We consider the dynamics of a mosquito-transmitted pathogen in a multi-patch Ross-Macdonald malaria model with mobile human hosts, mobile vectors, and a heterogeneous environment. We show the existence of a globally stable steady state, and a threshold that determines whether a pathogen is either absent from all patches, or endemic and present at some level in all patches. Each patch is characterized by a local basic reproduction number, whose value predicts whether the disease is cleared or not when the patch is isolated: patches are known as "demographic sinks" if they have a local basic reproduction number less than one, and hence would clear the disease if isolated; patches with a basic reproduction number above one would sustain endemic infection in isolation, and become "demographic sources" of parasites when connected to other patches. Sources are also considered focal areas of transmission for the larger landscape, as they export excess parasites to other areas and can sustain parasite populations. We show how to determine the various basic reproduction numbers from steady state estimates in the patched network and knowledge of additional model parameters, hereby identifying parasite sources in the process. This is useful in the context of control of the infection on natural landscapes, because a commonly suggested strategy is to target focal areas, in order to make their corresponding basic reproduction numbers less than one, effectively turning them into sinks. We show that this is indeed a successful control strategy -albeit a conservative and possibly expensive one- in case either the human host, or the vector does not move. However, we also show that when both humans and vectors move, this strategy may fail, depending on the specific movement patterns exhibited by hosts and vectors.

Effects of Diffusion on Total Biomass in Heterogeneous Continuous and Discrete-Patch Systems

Donald Deangelis
U. S. Geological Survey, USA
Wei-Ming Ni, Bo Zhang

Theoretical models of populations on a system of two connected patches previously have shown that, when the two patches differ in maximum growth rate and carrying capacity, and in the limit of high diffusion, conditions exist for which the total population size at equilibrium exceeds that of the Ideal Free Distribution, which predicts that the total population would equal the total carrying capacity of the two patches. However, this result has only been shown for the Pearl-Verhulst growth function on two patches and for a single-parameter growth function in continuous space. Here we provide a general criterion for total population size to exceed total carrying capacity for three commonly used population growth rates for both heterogeneous continuous and multi-patch heterogeneous landscapes with high population diffusion. We show that a necessary condition for this situation is that there is a convex positive relationship between the maximum growth rate and the carrying capacity, as both vary across a spatial region. This relationship occurs in some biological populations, though not in others, so the result has ecological implications.

Hawks, Doves, and Population Dynamics

Theodore Galanthay
Ithaca College, USA
Vlastimil Krivan

The hawk-dove model describes a game of conflict where two players (perhaps members of a large population) compete for some benefit. We have created an ordinary differential equations model that incorporates population dynamics, and we approach the study of this game through the lens of dynamical systems. Hawks and doves are consumers who compete for a common resource. We subdivide hawks into three classes: searchers, handlers, and fighters. Doves are subdivided into two classes: searchers and handlers. In this talk, I will present preliminary results of the analysis of this model system and discuss its ecological importance.

Disease Persistence for a Ross-Macdonald Model in an Environment with Identical Patches

Daozhou Gao
Shanghai Normal University, Peoples Rep of China
Chris Cosner, P. van den Driessche

Based on a multipatch Ross-Macdonald model, we study the role of human and/or mosquito movement on malaria spread between an arbitrary number of identical patches. By using a theorem on line-sum symmetric matrix, we establish an eigenvalue inequality on the product of a class of nonnegative matrices and then apply it to prove that the basic reproduction number of the multipatch model, R_0, is always greater than or equal to that of the model of each patch in isolation. Biologically, this means that host and/or vector movement promotes the persistence of malaria in an environment with identical patches.

Spatial Dynamics for Lattice Differential Equations with a Shifting Habitat

Changbing Hu
University of Louisville, USA
Bingtuan Li

We study a lattice differential model that describes the growth and spread of a species in a shifting habitat. We show that the long term behavior of solutions depends on the speed of the shifting habitat edge c and a number $c^*(\infty)$ that is determined by the maximum linearized growth rate and the diffusion coefficient. We demonstrate that if $c > c^*(\infty)$ then the species will become extinct in the habitat.

Spatiotemporal Virulence Dynamics During an Epidemic in the Presence of Multiple Evolutionary Trade-Offs

Paul Hurtado
University of Nevada, Reno, USA

In recent years, empirical studies have fueled development of mathematical theory that describes the evolutionary dynamics of pathogen characteristics as they spread among their hosts. Virulence, or the degree to which a pathogen causes host morbidity or mortality, is often the primary parasite trait of interest. One of the primary mechanisms driving virulence dynamics is the well known Transmission-Virulence trade-off in which parasite strains exhibit a strong correlation between how sick they make their hosts (virulence) and how easily they are transmitted. Here I will present results describing how a second, newly described Movement-Virulence trade-off (between host movement and pathogen virulence) can alter those spatiotemporal virulence dynamics. These results have important implications for studying spatiotemporal epidemic data and may help explain virulence dynamics during the epidemic phase of many emerging infectious diseases.

Population Dynamics of Honeybees with Varroa Destructor As Parasite and Virus Vector: the Potential Effects of foraging behavior

Yun Kang
Arizona State University, USA
Komi Messan, Krystal Blanco, Talia Davis, Ying Wang, Gloria DeGrandi-Hoffman

The worldwide decline in honeybee colonies during the past 50 years has often been linked to the spread of the parasitic mite Varroa destructor and its interaction with certain honeybee viruses carried by Varroa mites. In this talk, we study a honeybee-mite-virus model and a honeybee-mite model with dispersal to have a better understanding of the synergistic effects of parasitism and virus infections as well as the foraging behavior of honeybees on honeybee population dynamics and its persistence. Interesting findings from our work include: (a) Due to Allee effects experienced by the honeybee population, initial conditions are essential for the survival of the colony. (b) Low adult honeybees to brood ratios have destabilizing effects on the system which generate fluctuating dynamics that lead to a catastrophic event where both honeybees and mites suddenly become extinct. This catastrophic event could be potentially linked to Colony Collapse Disorder (CCD) of honeybee colonies. (c) Virus infections may have stabilizing effects on the system, and parasitic mites could make disease more persistent. (d) How may foraging behavior of honeybees stabilize/destabilize the honeybee population dynamics in the presence of parasitic mites. Our model illustrates how the synergy between the parasitic mites and virus infections consequently generates rich dynamics including multiple attractors where all species can coexist or go extinct depending on initial conditions. Our findings may provide important insights on honeybee viruses and parasites and how to best control them.

Dirac Concentrations in an Integro-PDE Arising from Evolution of Dispersal.

King-yeung Lam
The Ohio State University, USA
Wenrui Hao, Yuan Lou

We consider an integro-PDE model for a population structured by the spatial variables and a trait variable affecting the dispersal coefficients. Competition for resource is local in spatial variables, but nonlocal in the trait variable. We focus on the asymptotic profile of positive steady state solutions. Our result shows that in the limit of small mutation rate, the solution remains regular in the spatial variables and yet concentrates in the trait variable and forms Dirac concetrations (i) at one boundary point; (ii) the interior; or (iii) at both boundary points. The main techniques are the perturbed test function approach for viscosity solution, a Liouville result on a cylinder, and elliptic DeGiorgi-Nash-Moser estimates for the obligue derivative problem. Finally, connections to notions and concepts in evolutionary game theory will be discussed. This is joint work with Wenrui Hao (MBI) and Yuan Lou(Ohio State).

Cross Diffusion Models in Population Biology

Yuan Lou
Renmin University/Ohio State, Peoples Rep of China
Wei-Ming Ni, Michael Winkler, Shoji Yotsutani

Cross-diffusion system is an important class of reaction-diffusion problems. At the individual level, the basic underlying assumption for cross-diffusion is that the transition probability only depends upon departure conditions, e.g., population density and environmental condition at the departure location. We will discuss the Shigesada-Kawasaki-Teromoto model for two competing species. This talk is based on joint works with Wei-Ming Ni, Michael Winkler, Shoji Yotsutani.

Special Solutions of the Kirkpatrick-Barton System

Judith Miller
Georgetown University, USA

In an influential study, Kirkpatrick and Barton developed and numerically solved a partial differential equation model for the joint evolution of population density and the mean of a quantitative trait in time and space. Stationary localized solutions, or pinned states, of this system model the setting of species range limits through “genetic swamping“; traveling waves model the advance of an invasive species subject to spatially heterogeneous selection. We prove existence of both types of solutions and present numerical evidence for a spreading speed.

Stray Cat Population Dynamics

Andrew Nevai
University of Central Florida, USA
Jeff Sharpe

Stray cat populations cause ecological destruction and spread many diseases in places that people live. Here, we describe a mathematical model for their population dynamics. The gender-based model includes kittens, adult females and adult males. A net reproduction number R_0 distinguishes betweeen population extinction ($R_0 < 1$) and population persistence ($R_0 > 1$). In a separate talk, the model will be extended to include the spatial movement of adult males between patches and the spread of feline leukemia.

Algae-Herbivore Interactions with Allee Effect and Chemical Defense

Samares Pal
University of Kalyani, India
Joydeb Bhattacharyya

Macroalgae exhibit a variety of characteristics that provide a degree of protection from herbivores. One characteristic is the production of chemicals that are toxic to herbivores. The toxic effect of macro-algae on herbivorous reef fish is studied by means of a spatio-temporal model of population dynamics with a non-monotonic toxin-determined functional response of herbivores. The growth rate of macro-algae is mediated by Allee effect. We see that under certain conditions the system is uniformly persistent. Conditions for local stability of the system are obtained with weak and strong Allee effects. We observe that in presence of Allee effect on macro-algae, the system exhibits complex dynamics including Hopf-bifurcation and saddle-node bifurcation.

Effects of Asymmetric Movement on Population Dynamics

Zhisheng Shuai
University of Central Florida, USA

Spatial heterogeneity and spatial movement play an important role in population dynamics. Spatial movement in heterogeneous environment or networks could be symmetric or asymmetric (biased). The effects of symmetric and asymmetric movement on population dynamics will be investigated using several ecological and epidemiological models from the literature.

A Nonlocal Spatial Population Model on Continuous Time and Space

Fan Zhang
Florida Atlantic University, USA
Chris Cosner

We formulate an integro-differential model based on Levins' metapopulation model, on a continuous heterogeneous environment. Nonlocal dispersal can be used to describe the spatial movement of seeds, juveniles and offspring produced by adult individuals such as sessile animals and plants. The dispersal kernel allows long distance movement on continuum habitat. In this model, we are interested in the coexistence status when assuming both species use the same resource which is the space that an individual could occupy. It is a noticeable effect that the spatially heterogeneity could promote coexistence while assuming one competitor is stronger than the other one. We are also interested in the evolution of dispersal strategies between two similar species and show that the ideal free dispersal strategy is evolutionarily stable in this model.

Special Session 6: Numerical Approximation of Fractional and Integral Differential Equations

Jie Shen, Purdue University, USA
Chuanju Xu, Xiamen University, Peoples Rep of China

Progress in the last two decades have demonstrated that many phenomena in various fields of science, mathematics, engineering, bioengineering, and economics are more accurately described by using fractional derivatives and/or integral differential equations, and they are emerging as a new powerful tool for modeling many difficult type of complex systems. However, due to their non-local and singular nature, it is a tremendous challenge to design accurate and efficient numerical methods for solving fractional and integral differential equations. The main objective of this special session is to bring together experts for fractional and integral differential equations to present their latest advances and discuss future directions.

Analysis of Two-Grid Methods for Miscible Displacement Problem by Mixed Finite Element Methods

Yanping Chen
South China Normal University, Peoples Rep of China

The miscible displacement of one incompressible fluid by another in a porous medium is governed by a system of two equations. One is elliptic form equation for the pressure and the other is parabolic form equation for the concentration of one of the fluids. Since only the velocity and not the pressure appears explicitly in the concentration equation, we use a mixed finite element method for the approximation of the pressure equation. In order to find a stable finite element discretization method method, we use different discretization method for the concentration equation, such as finite element method with characteristic; mixed finite element method with characteristic; expanded mixed finite element method with characteristic etc. To linearize the discretized equations, we use one (two) Newton iterations on the fine grid in our methods. Firstly, we solve an original non-linear coupling problem. Then, solve a linear system on the fine grid and while in second method we make a correction on the coarse grid between one (two) Newton iterations on the fine grid. We obtain the error estimates of two-grid method, it is shown that coarse space can be extremely coarse and we achieve asymptotically optimal approximation. Finally, numerical experiment indicates that two-grid algorithm is very effective.

Deriving the PDE Governing the Functional Distribution of the Anomalous Paths and Its Multiresolution Galerkin method

Weihua Deng
Lanzhou University, Peoples Rep of China
Xiaochao Wu, Eli Barkai

. Anomalous dynamics widely appear in cell migration, the motion of mRNA molecules and lipid granules in live cells. Since the pioneering work of Montroll and Weiss in 1965, CTRW models have become a pillar of statistical physics, especially in characterizing the anomalous dynamics. When the jump length and/or waiting times of the CTRW obey(s) the distribution of power law (with divergent first or second moment), it describes the anomalous diffusion, i.e., the second moment of the particle's trajectories is a nonlinear function of time t. For performing more deep research, more often the jump lengths and/or waiting times need to be tempered. This is because that the lifetime is finite and physical space is bounded; sometimes it is the reason that we expect to model the observed experimental phenomena of the transition between anomalous dynamics and normal ones. In this talk, we will derive the PDE governing the functional distribution of the tempered anomalous paths, and discuss the multiresolution Galerkin method of the derived PDE.

Spectral Collocation Methods for Fractional Boundary Value Problems

Zeng Fanhai
Brown University, USA
George Em Karniadakis, Zhiping Mao

We develop spectral collocation methods for variable-order fractional differential equations with two endpoint singularities. In order to develop the spectral collocation methods, we derive three-term recurrence relations for both integrals and derivatives of the weighted Jacobi polynomials of the form $(1+x)^{\mu_1}(1-x)^{\mu_2}P_j^{a,b}(x)\,(a,b,\mu_1,\mu_2>-1)$, which leads to the desired differentiation matrices. We apply these new differentiation matrices to construct collocation methods to solve fractional boundary value problems and fractional partial differential equations with two endpoint singularities. Some theoretical explanations are investigated to illustrate that the new methods can achieve better accuracy than the existing numerical methods. We demonstrate that the singular basis enhances greatly the accuracy of the numerical solutions by properly tuning the parameters μ_1 and μ_2, even for cases that we do not know explicitly the form of singularities in the solution at the boundaries.

Memory Independent Predictor-Corrector Method for Solving Differential Equations of Fractional Order

Bongsoo Jang
UNIST, Korea
Thien Binh Nguyen

An accurate and efficient new class of predictor-corrector schemes are proposed for solving nonlinear differential equations of fractional order. By introducing a new prediction method which is explicit and of the same accuracy order as that of the correction stage, the new schemes achieve a uniform accuracy order regardless of the values of fractional order α. In cases of $0 < \alpha < 1$, the new schemes significantly improve the numerical accuracy comparing with other predictor-corrector methods whose accuracy depends on α. Furthermore, by computing the memory term just once for both the prediction and correction stages, the new schemes reduce the computational cost of the so-called memory effect, which make numerical schemes for fractional differential equations expensive in general. Both 2nd-order scheme with linear interpolation and the high-order 3rd-order one with quadratic interpolation are developed and show their advantages over other comparing schemes via various numerical tests.

Well-Conditioned Fractional Collocation Methods Using Fractional Birkhoff Interpolation Basis

Yujian Jiao
Shanghai Normal University, Peoples Rep of China
Li-Lian Wang, Can Huang

The purpose of this talk is twofold. Firstly, we provide explicit and compact formulas for computing both Caputo and Riemann-Liouville fractional pseudospectral differentiation matrices(F-PSDM) of any order at general Jacobi-Gauss-Lobatto points. We show that in the Caputo case, it suffices to compute F-PSDM of order $\mu \in (0, 1)$ to compute that of any order $k + \mu$ with integer $k \geq 0$, while in the modified RL case, it is only necessary to evaluate a fractional integral matrix of order $\mu \in (0, 1)$. Secondly, we introduce suitable fractional JGL Birkhoff interpolation problems leading to new interpolation polynomial basis functions with remarkable properties: (i) the matrix generated from the new basis yields the exact inverse of F-PSDM at interior JGL points; (ii) the matrix of the highest fractional derivative in a collocation scheme under the new basis is diagonal; and (iii) the resulted linear system is well-conditioned in the Caputo case, while in the modified RL case, the eigenvalues of the coefficient matrix are highly concentrated. In both cases, the linear systems of the collocation schemes using the new basis can solved by an iterative solver within a few iterations.

Time Stepping Schemes for Fractional Diffusion

Bangti Jin
University College London, England
Raytcho Lazarov, Zhi Zhou

In this talk we discuss the time stepping schemes for fractional diffusion, which involves a fractional derivative in time. The nonlocality of the fractional derivative poses significant challenge in the development and analysis of robust numerical schemes for efficiently simulating fractional diffusion. I shall describe recent progresses, e.g., the L1 scheme and convolution quadrature, together with finite element in space. Throughout numerical results will be presented to illustrate the convergence theory.

Fourier Spectral Method for Fractional Cahn-Hilliard Equation

Zhiping Mao
Brown University, USA
Mark Ainsworth

In this presentation, I am going to present a fractional Cahn-Hilliard equation(FCHE) which possess mass conservation law and energy law. The stability and L_∞ boundness are studied. Fourier Galerkin method is applied to solve FCHE and the convergence is analyzed. Numerical tests are showed to study the convergence of the scheme and the behavior of the FCHE.

Extended Spectral Method for Solving Singular Problems

Jie Shen
Purdue University, USA
Sheng Chen

The usual spectral methods will provide high-order accuracy for problems with smooth solutions. However, they may not work well for problems with singular solutions due to various facts such as corner singularities, non-matching boundary conditions, non-smooth coefficients. For many singular problems, it is possible to determine the forms of a few leading singular terms. It is expected that we can improve the convergence rate by adding these singular terms into the spectral basis. However, the new system with added singular terms is usually ill conditioned and hard to solve. We present a new extended spectral-Galerkin method which allows us to split it into two separate problems: one is to find an approximation for the smooth part by a usual spectral method, the other is to determine an approximation to the singular part with k terms by solving a $k \times k$ system. So the new method is very easy to implement, very efficient and is capable of providing very accurate approximations for a class of singular problems.

A Direct Spectral Method for Optimal Control Problems Governed by the Time Fractional Diffusion Equation

Shengzhu Shi
Harbin Institute of Technology (HIT), Peoples Rep of China
Boying Wu, Jiebao Sun, Wenjuan Yao

This paper is devoted to designing and analyzing an efficient numerical method for unconstrained convex distributed optimal control problems governed by time fractional diffusion equation. Unlike the general method focusing on the first order optimality condition, we approximate the optimal control problem directly with polynomials and turn the problem into a nonlinear problem (NLP). The error estimate for the spectral approximation is derived and some numerical experiments are presented.

A Fractional Phase-Field Model for Two-Phase Flows with Tunable Sharpness: Algorithms and Simulations

Fangying Song
Brown University, USA
Chuanju Xu, George Em Karniadakis

We develop a fractional extension of a mass-conserving Allen-Cahn phase-field model that describes the mixture of two incompressible fluids. The fractional order controls the sharpness of the interface, which is typically diffusive in integer-order phase-field models. The model is derived based on an energy variational formulation. An additional constraint is employed to make the Allen-Cahn formulation mass-conserving and comparable to the Cahn-Hilliard formulation but at reduced cost. The spatial discretization is based on a Petrov-Galerkin spectral method whereas the temporal discretization is based on a stabilized ADI scheme both for the phase-field equation and for the Navier-Stokes equation. We demonstrate the spectral accuracy of the method with fabricated smooth solutions and also the ability to control the interface thickness between two fluids with different viscosity and density in simulations of two-phase flow in a pipe and of a rising bubble. We also demonstrate that using a formulation with variable fractional order we can deal simultaneously with both erroneous boundary effects and sharpening of the interface at no extra resolution.

Fast and Accurate Numerical Methods for Fractional Partial Differential Equations

Hong Wang
University of South Carolina, USA

Fractional PDEs provide an adequate and accurate description of transport processes that exhibit anomalous diffusion and long-range space-time interactions. Computationally, because of the nonlocal property of these models, the numerical methods often generate dense stiffness matrices. Traditionally, direct methods were used to solve these problems, which require O(N3) computations (per time step) and O(N2) mememy, where N is the number of unknowns. We go over the development of accurate and efficient numerical methods for FPDEs, which has an optimal order storage and almost linear computational complexity. These methods were developed by utilizing the structure of the stiffness matrices. No lossy compression or approximation was used. Hence, these methods retaining the same accuracy and approximation/conservation property of the underlying numerical methods.

Special Session 8: New Trends in Calculus of Variations and Partial Differential Equations

Irene Fonseca, Carnegie Mellon University, USA
Giovanni Leoni, Carnegie Mellon University, USA

Variational formulations of physical and biological questions are central in applied mathematics. The goal of this minisymposium is to provide a forum to discuss recent progress and promising directions in calculus of variations, geometric measure theory, partial differential equations, and applications to science and engineering. Thematic areas of focus will include: variational problems involving material defects, multi-scale problems, thin structures, homogenization and mixing, and interfacial phenomena.

Cohesive Behaviour Arising in Homogenization of Mumford-Shah Type Functionals

Marco Barchiesi
Universita di Napoli Federico II, Italy
Giuliano Lazzaroni, Caterina Ida Zeppieri

I will discuss the effective properties of a composite material whose micro-structure is constituted by a brittle material with periodically distributed soft inclusions. In particular, I will show that the presence of the soft inclusions gives rise, in the limit, to a cohesive behaviour and a toughening phenomenon.

The Rigidity Problem for Symmetrization Inequalities

Filippo Cagnetti
University of Sussex, England

We will discuss several symmetrizations (Steiner, Ehrhard, and spherical symmetrization) that are known not to increase the perimeter. We will show how it is possible to characterize those sets whose perimeter remains unchanged under symmetrization. We will also characterize rigidity of equality cases. By rigidity, we mean the situation when those sets whose perimeter remains unchanged under symmetrization, are trivially obtained through a rigid motion of the (Steiner, Ehrhard or spherical) symmetral.
We will achieve this through the introduction of a suitable measure-theoretic notion of connectedness, and through a fine analysis of the barycenter function for a special class of sets. These results are obtained together with several collaborators (Maria Colombo, Guido De Philippis, Francesco Maggi, Matteo Perugini, Dominik Stoger).

Lower Semicontinuity of a Class of Integral Functionals on the Space of Functions of Bounded Deformation

Gianni dal Maso
SISSA, Italy
Gianluca Orlano, Rodica Toader

We study the lower semicontinuity of some free discontinuity functionals with linear growth defined on the space of functions with bounded deformation. The volume term is convex and depends only on the Euclidean norm of the symmetrized gradient. We introduce a suitable class of surface terms, which make the functional lower semicontinuous with respect to L^1 convergence.

Homogenization in the Framework of $\mathcal{A}$-Quasiconvexity with Variable Coefficients

Elisa Davoli
University of Vienna, Austria
Irene Fonseca

A homogenization result for a family of integral energies will be presented, where the field under consideration are subjected to periodic first order oscillating differential constraints in divergence form. The talk is based on the theory of $\mathcal{A}$-quasiconvexity with variable coefficients and on two-scale convergence tecniques.

Recent Applications of Quantitative Stability to Convergence to Equilibrium

Alessio Figalli
UT Austin, USA

Geometric and functional inequalities play a crucial role in several PDE problems. In view of applications to PDEs, a natural question is the following: suppose that a function almost solve the Euler-Lagrange equation associated to some functional inequality. Is this function close to one of the minimizers? In this setting, one has to face the presence of bubbling phe-

nomena. In this talk I'll give a overview of these general questions using some concrete examples, and then present some recent applications to the asymptotic behavior for some fast diffusion equation related to the Yamabe flow.

Domain Formation in Membranes Near the Onset of Instability

Gurgen Hayrapetyan
Ohio University, USA
I. Fonseca, G. Leoni, B. Zwicknagl

The formation of microdomains, also called rafts, in biomembranes is believed to be attributed to the surface tension of the membrane. In order to model this phenomenon, a family of energy functionals involving a coupling between the local composition and the local curvature was proposed by Seul and Andelman. We discuss the Γ-convergence of this family of energy functionals, involving nonlocal as well as negative terms in the regime of strong surface tension.

Formulas for Relaxed Disarrangement Densities

Marco Morandotti
SISSA, Trieste, Italy
A. C. Barroso, J. Matias, D. R. Owen

Structured deformations provide a multiscale geometry that captures the contributions at the macrolevel of both smooth geometrical changes and non-smooth geometrical changes (disarrangements) at submacroscopic levels. Recently, Owen and Paroni evaluated explicitly some relaxed energy densities arising in Choksi and Fonseca's energetics of structured deformations. In this talk, we will show that a different approach to the energetics of structured deformations, that due to Baìa, Matias, and Santos, confirms the roles of the relaxed densities established by Owen and Paroni. In doing so, we give an alternative, shorter proof of Owen and Paroni's results, and we establish additional explicit formulas for other measures of disarrangements.

Large Deviations, Gradient Flows, and Taking Limits

Mark Peletier
TU Eindhoven, Netherlands
Giovanni Bonaschi, Giuseppe Savarè

It is now well understood that there is a strong connection between gradient flows on one hand and large-deviation principles on the other hand. In a sense, this connection takes the form of a single functional that characterizes both the large deviations and the gradient-flow behaviour.
In this talk, which is work with Giovanni Bonaschi and Giuseppe Savare, I will show how this insight produces a unification of both structure and method. I will focus on a very simple stochastic system, and show how the large-deviation rate functional is related to a generalized gradient flow - with a parameter. By taking various limits in this parameter we recover both linear gradient-flow behaviour and rate-independent behaviour.
The unification lies in the fact that we can base our entire discussion on this one functional. It characterizes the structure and is also the main actor in the limit-taking, leading both to compactness and to characterization of the limit. This work shows how the connection between large deviations and gradient flows is not only philosophically interesting but also provides tools for analysis.

A Variational Approach to Existence and Uniqueness of Crystalline Curvature Flows

Marcello Ponsiglione
Sapienza University of Rome, Italy
Antonin Chambolle, Massimiliano Morini

In this seminar I will describe a new approach to existence and uniqueness of crystalline curvature flows. The results are valid in any dimension and for arbitrary, possibly unbounded, initial closed sets. The comparison principle is obtained by means of a suitable weak (distributional) formulation of the flow, while the existence of a global-in-time solution follows via a minimizing movements approach.

Slow Motion for the One-Dimensional Swift-Hohenberg Equation

Matteo Rinaldi
Carnegie Mellon University, USA
Gurgen Hayrapetyan

The behavior of solutions of the Swift–Hohenberg equation in a bounded interval $I \subset \mathbb{R}$ with periodic boundary conditions is studied. Combining results from Γ–convergence and ODE theory it is shown that solutions that start L^1–close to a jump function v, remain close to v. This can be achieved by regarding the equation as the L^2–gradient flow of a given energy functional and studying the asymptotic behavior of solutions of its Euler–Lagrange equation. The linearization of such equation provides almost sharp estimates on the tail of the associated energy.

On the Structure of PDE-Constrained Measures and Applications

Filip Rindler
University of Warwick, England
Guido De Philippis

Vector-valued measures satisfying a PDE constraint appear in various areas of nonlinear PDE theory and the calculus of variations. Often, the shape of singularities that may be contained in these measures, such as jumps or fractal parts, is of particular interest. In this talk, I will first motivate how variational problems in crystal plasticity naturally lead to such PDE-constrained measures and how their shape is physically relevant. Then, I will present a recent general structure theorem for the singular part of any vector-valued measure satisfying a linear PDE constraint. As applications, we obtain a simple new proof of Alberti's Rank-One Theorem on the shape of derivatives of functions with bounded variation (BV), an extension of this theorem to functions of bounded deformation (BD), and a structure theorem for families of normal currents. Further, our structure result for currents implies the solution to the conjecture that if every Lipschitz function is differentiable almost everywhere with respect to some positive measure (i.e. the Rademacher theorem holds with respect to that measure), then this measure has to be absolutely continuous relative to Lebesgue measure. This is joint work with Guido De Philippis (SISSA Trieste).

On the Shape of the D-Dimensional Cahn-Hilliard Energy Landscape

Maria Westdickenberg
RWTH Aachen, Germany
Michael Gelantalis and Alfred Wagner

For mean values close to -1, it is easy to see that the constant state is a local energy minimizer of the Cahn-Hilliard energy. As already described in the seminal work of Cahn and Hilliard, stochastic fluctuations lead to nucleation of small, droplet-shaped regions of $+1$, which may then grow and coalesce. Moreover, whether the regions of $+1$ grow or shrink should depend on whether their mass is large enough to form a so-called critical nucleus. We describe recent (deterministic) work on the Cahn-Hilliard energy landscape in the regime of mean value close to -1 and large system size. Our work leads to quantitative bounds on the volume and approximate droplet-shape of a candidate for the critical nucleus. Our methods involve Gamma-limits, quantitative isoperimetric inequalities, and Steiner symmetrization.

Special Session 9: Stochastic Modeling in Fluid Dynamics: Theory and Approximation

Chuntian Wang, University of California, Los Angeles, USA
Honghu Liu, Virginia Tech, USA
Roger Temam, Institute for Scientific Computing and Applied Mathematics, Indiana University Bloomington, USA

Stochastic models of fluid dynamics play a significant role in science, with numerous applications in e.g. climate science, geophysics and engineering. Moreover, sitting at the interface between probability theory, mathematical analysis and theory of parabolic and hyperbolic partial differential equations, these problems provide interesting and challenging mathematical complications. Motivated by the need from both the outside and inside of the mathematical community, our session will focus on the recent advances in stochastic fluid dynamics modeling, especially those derived from mathematical physics like plasma physics, fluid dynamics and thermomechanics. We aim to bring together researchers to discuss such models from both the theoretical and applied points of view, with topics including the regularity behavior of solutions, the stochastic dynamics and the stochastic numerical implementations.

Abridged Determining Parameters and Data Assimilation for the MHD Equations with Applications to Statistical Solutions

Animikh Biswas
UMBC, USA

We establish the existence of determining modes, nodes and data assimilation technique for the two dimensional MHD equations when only one component of the velocity and magnetic fields are observed. We discuss applications to the determination of statistical solutions inspired by the recent work of Foias, Mondaini and Titi.

Multi-Locality and Fusion Rules on the Generalized Structure Functions in Two-Dimensional and Three-Dimensional Navier-Stokes Turbulence

Eleftherios Gkioulekas
University of Texas Rio Grande Valley, USA

Using the fusion rules hypothesis for three-dimensional and two-dimensional Navier-Stokes turbulence, we generalize a previous non-perturbative locality proof to multiple applications of the nonlinear interactions operator on generalized structure functions of velocity differences. We shall call this generalization of non-perturbative locality to multiple applications of the nonlinear interactions operator "multilocality". The resulting cross-terms pose a new challenge requiring a new argument and the introduction of a new fusion rule that takes advantage of rotational symmetry. Our main result is that the fusion rules hypothesis implies both locality and multilocality in both the IR and UV limits for the downscale energy cascade of three-dimensional Navier-Stokes turbulence and the downscale enstrophy cascade and inverse energy cascade of two-dimensional Navier-Stokes turbulence. We stress that these claims relate to non-perturbative locality of generalized structure functions on all orders, and not the term by term perturbative locality of diagrammatic theories or closure models that involve only two-point correlation and response functions.

Stochastic Swift-Hohenberg Equation with Degenerate Linear Multiplicative Noise

Marco Hernandez
Indiana University, USA
Kiah Wah Ong

We study the dynamic transition of the Swift-Hohenberg equation (SHE) when linear multiplicative noise acting on a finite set of modes of the dominant linear flow is introduced. Existence of a stochastic flow and a local stochastic invariant manifold for this stochastic form of SHE are both adressed in this work. We show that the approximate reduced system correspoding to the invariant manifold undergoes a stochastic pitchfork bifurcation, and obtain numerical evidence suggesting that this picture is a good approximation for the full system as well.

Compressible Fluid Flows Driven by Stochastic Forcing

Martina Hofmanova
Technical University Berlin, Germany

We study the Navier-Stokes equations governing the motion of an isentropic compressible fluid in three dimensions driven by a multiplicative stochastic forcing. In particular, we consider a stochastic perturbation of the system as a function of momentum and density. We establish existence of a finite energy weak martingale solution as well as a weak-strong uniqueness principle and several singular limit results. The talk is based on joint works with Dominic Breit and Eduard Feireisl.

Non-Markovian Reduced Equations for Stochastic PDEs

Honghu Liu
Virginia Tech, USA
Mickael Chekroun, Shouhong Wang

In this talk, a new approach to deal with the parameterization problem of the small spatial scales by the large ones for stochastic partial differential equations will be discussed. This approach relies on stochastic parameterizing manifolds (PMs) which are random manifolds aiming to provide—in a mean square sense—approximate parameterizations of the small scales by the large ones. Backward-forward systems will be introduced to give access to such PMs as pullback limits depending on the time history of certain approximations of the low modes dynamics. It will be shown that the corresponding pullback limits can be efficiently determined in practice, leading in turn to an operational procedure for the derivation of non-Markovian reduced equations able to achieve good modeling performances. Examples will then be presented to illustrate that the memory effects conveyed by such reduced systems can play a key role to capture noise-induced transitions or large excursions caused by the noise.

Stochastic Systems of Diffusion Equations with Polynomial Reaction Terms.

Phuong Nguyen
Indiana University, USA
Du Pham

In this work, we consider the stochastic version of the diffusion equations with polynomial reaction terms of arbitrary degrees forced by a multiplicative noise . We first establish the existence local in time of the martingale solution and then derive the pathwise uniqueness to imply the existence of a pathwise solution for a limited period of time. The proofs rely on the Skorohod representation, the Gyongy-Krylov theorem and a stopping time argument.

Martingale and Pathwise Solutions to the Stochastic Bingham Fluid Equations with Multiplicative Noise.

Du Pham
UTSA, USA

In this talk, the martingale and pathwise solutions of the stochastic Bingham fluid are studied. We also further show that in two dimension, when the yield limit of the stochastic Bingham fluid goes to 0 then the pathwise solutions of the Bingham equations will convergence to the pathwise solutions of the Navier Stokes equations in probability.

Invariant Measure Concentrated on $C^\infty(\mathbb{T})$ for the Benjamin-Ono Equation

Mouhamadou Sy
Université de Cergy Pontoise, France

The existence of an invariant measure for a PDE allows to describe its large time dynamics via the Poincarè recurrence theorem. In the context of the Benjamin-Ono equation, such a measure was constructed in any Sobolev space (Deng-Tvzetkov-Visciglia). This allows to establish a reccurence in time property of the periodic in space BO solutions belonging to Sobolev spaces. However, these measures neglect the space C^∞. In this talk we construct a measure on H^3 invariant under the flow of the BO equation and concentrated on the space C^∞. We give then a description of the long time behavior of the infinitely smooth in space BO solutions. We developp for this equation the stochastic-perturbation/dissipation approach already used in other contexts. We will discuss some qualitative properties of this measure. At the end, we will present briefly a measure for the Klein-Gordon equation.

CLT and Large Deviation in a Toy Quadratic Model: Whether to Use a Gaussian Approximation?

Molei Tao
Georgia Tech, USA

We consider a simple system evolved under both dissipation and quadratic inputs from a fast Ornstein-Uhlenbeck process. A Gaussian approximation can be obtained via multiscale analysis, and the result is analogous to central limit theorem and has its own domain of applicability. However, this approximation fails to correctly characterize rare events in the slow-fast system. An appropriate characterization will be demonstrated by the establishment of a large deviation principle. Joint work with Eric Vanden-Eijnden.

The Stampacchia Maximum Principle for Stochastic Partial Differential Equations and Applications

Roger Temam
Indiana University, USA
Mickael Chekroun, Eunhee Park

Using the truncation technique of Stampacchia, we establish the positivity of the solutions of certain stochastic partial differential equations, and other properties as appropriate. Among the applications, we show how to prove the existence of positive solutions of equations in nonlinear population dynamics, where the nonlinearity is an even power of the solution u.

Time Discrete Approximation of Weak Solutions for Stochastic Equations of Geophysical Fluid Dynamics and Applications

Chuntian Wang
University of California, Los Angeles, USA
Nathan Glatt-Holtz, Roger Temam

As a first step towards the numerical analysis of the stochastic primitive equations of the atmosphere and the oceans, we study here their time discretization by an implicit Euler scheme. From the deterministic point of view the 3D Primitive Equations are studied in their full form on a general domain and with physically realistic boundary conditions. From the probabilistic viewpoint we consider a wide class of nonlinear, state dependent, white noise forcings which may be interpreted in either the Ito or the Stratonovich sense. The proof of convergence of the Euler scheme, which is carried out within an abstract framework covers the equations for the oceans, the atmosphere, the coupled oceanic-atmospheric system as well as other related geophysical equations. We obtain the existence of solutions which are weak in both the PDE and probabilistic sense, a result which is new by itself to the best of our knowledge.

Special Session 10: Complex Systems and Nonlinear Dynamics

Jung-Chao Ban, National Dong Hwa University, Taiwan
Kuo-Chang Chen, National Tsing Hua University, Taiwan

The Hamiltonian systems, ODE and PDE provide powerful frameworks for the modelling of the complex systems. Meanwhile, the nonlinear phenomena of complex systems also inspire many interesting problems in mathematical analysis. This special session is concerned with the nonlinear analysis on various fields of complex dynamical systems. Related topics including the analysis on fractal geometry, N-body problem, and traveling waves solutions in partial differential equations. The aim of the proposed session is to exchange our ideas and discuss recent advances in such fields.

Entropy of Tree Shifts of Finite Type

Jung-Chao Ban
National Dong Hwa University, Taiwan

This talk studies the entropy of tree shifts of finite type with and without boundary conditions. We demonstrate that computing the entropy of a tree shift of finite type is equivalent to solving a system of nonlinear recurrence equations. Furthermore, the entropy of binary Markov tree shifts defined on two symbols is either 0 or ln2. Meanwhile, the realization of entropy of one-dimensional shifts of finite type is elaborated, which indicates that tree shifts are capable of rich phenomena. Considering the influence of three different types of boundary conditions, say, the periodic, Dirichlet, and Neumann boundary conditions, the necessary and sufficient condition for the coincidence of entropy with and without boundary condition are addressed.

On the Topology of Tree-Shifts

Chih-Hung Chang
National University of Kaohsiung, Taiwan

Topological behavior, such as chaos, irreducibility, and mixing of a one-sided shift of finite type, is well elucidated. Meanwhile, the investigation of multidimensional shifts, for instance, textile systems is difficult and only a few results have been obtained so far. This paper studies shifts defined on infinite trees, which are called tree-shifts. Infinite trees have a natural structure of one-sided symbolic dynamical systems equipped with multiple shift maps and constitute an intermediate class in between one-sided shifts and multidimensional shifts. We have shown not only an irreducible tree-shift of finite type, but also a mixing tree-shift that are chaotic in the sense of Devaney. Furthermore, the graph and labeled graph representations of tree-shifts are revealed so that the verification of irreducibility and mixing of a tree-shift is equivalent to determining the irreducibility and mixing of matrices, respectively. This extends the classical results of one-sided symbolic dynamics. A necessary and sufficient condition for the irreducibility and mixing of tree-shifts of finite type is demonstrated. Most important of all, the examination can be done in finite steps with an upper bound.

Syzygies of the N-Center Problem

Kuo-chang Chen
National Tsing Hua University, Taiwan
Guowei Yu

In this talk we consider the N-center problem with collinear centers and identify syzygy sequences which can be realized by minimizers of the Lagrangian action functional. In particular, we show that the number of such realizable syzygy sequences of length L for the 3-center problem is at least $F_{L+2} - 2$, where F_n is the Fibonacci sequence. Moreover, with fixed length L, the density of such realizable syzygy sequences of length L for the N-center problem approaches 1 as N goes to infinity. This is a joint work with Guowei Yu.

Lie Symmetry to Nonlinear Oscillator Systems

Zhaosheng Feng
University of Texas-Rio Grande Valley, USA

In this talk, we apply the method of Lie point symmetry to study a generalized second-order nonlinear ordinary differential equation, which includes several physically important nonlinear oscillators such as force-free Helmholtz oscillator, force-free Duffing and Duffing-van der Pol oscillators, modified Emden-type equation and its hierarchy, generalized Duffing-Van der Pol oscillator equation hierarchy, and so on, and investigate the integrability properties of this rather general equation. We identify and classify several new integrable cases for arbitrary values of exponents, which determines the tangent vector as well as the infinitesimal generator. Using the Lie point symmetry reduction method, we find the infinitesimal generator and canonical coordinates. Combining them with the inverse transformation, we obtain the first integrals of the second-order nonlinear ODEs under certain parametric conditions. Our results show that many classical integrable nonlinear oscillators can be derived as subcases of our results and significantly enlarge the list of integrable equations that exists in the contemporary literature.

A Note on Zeros of the First Kind Bessel Functions

Cheng-hsiung Hsu
National Central University, Taiwan

In this talk, we consider the zeros' distribution of the first kind Bessel functions $J_\nu(z)$ of order $\nu \geq 1$. The problem arises from the conjecture given in our previous work which considered the existence of smooth solutions for one-dimensional compressible Euler equation with gravity. Expanding the smooth solution as a power series of parameters, we were confronted with the investigation of the zeros' distribution of $J_\nu(z)$. This is a joint work with Chi-Ru Yang.

Global Structure and Regularity of Solutions to the Eikonal Equation

Tianhong Li
Chinese Academy of Sciences, Peoples Rep of China

The Eikonal equation is regarded as a very important equation in geometric optics. It is derivable from Maxwell's equations of electromagnetics by WKB methods, and provides a link between physical (wave) optics and geometric (ray) optics, and also has many applications in optimal control, path planing and etc. It is a Hamilton-Jacobi equation with Hamiltonian $H(P) = |P|$, which is not strictly convex and smooth. The regularizing effect of Hamiltonian for the Eikonal equation is much weaker than strictly convex Hamiltonians. There are many new phenomena, for instance, the appearance of contact discontinuity. In this talk, the set of nondifferentiable points is studied. We also discuss the regularity of u in the compliment of the set of nondifferentiable points.

Periodic Solutions of a Prescribed-Energy Problem for a Singular Hamiltonian System

Mitsuru Shibayama
Kyoto University, Japan

We study the existence of periodic solutions for a prescribed-energy problem of a Hamiltonian system whose potential function has a singularity at the origin like $1/|q|^\alpha (q \in \mathbb{R}^N)$.
It is known that there exists a generalized periodic solution which may have collisions, and the number of possible collisions has been estimated. In this talk we obtain new estimation of the number of collisions. Especially we show that the obtained solution has no collision if $N \geq 2$ and $\alpha > 1$.

Birkhoff Normal Form for Null Form Wave Equations

Chi-Ru Yang
McMaster University, Canada
Walter Craig, Amanda French

In this talk, we will construct third order Birkhoff normal forms transformations for the class of wave equations on R^n for $n \geq 3$ which are both Hamiltonian PDEs and null forms. We will identify the null condition as the vanishing of the three-wave interaction coefficients on the cubic order resonant variety. The main point of the construction is that the normal forms transformation is a continuous mapping of an appropriate Sobolev space which removes the quadratic nonlinear terms of the equation, and this in turn gives a new proof via canonical transformations of the global in time existence theorems of S. Klainerman and J. Shatah for null form wave equations for small data. This is a progress report with Walter Craig (McMaster University and the Fields Institute) and Amanda French (Haverford).

Special Session 11: PDEs with Applications in Biology, Fluid Mechanics and Material Sciences

Yi Li, California State University at Northridge, USA
Yuanwei Qi, University of Central Florida, USA
Jack Xin, UC Irvine, USA

This session will bring experts of theoretical and numerical PDEs from several areas of applications to communicate up to date progress on evolutionary equations modeling Turing Pattern Formation, Population Dynamics and Infection Disease, Chemical Waves and Combustion, Turbulent Flame and G-equation and Surface Reaction in Material Sciences. In particular, both theoretical and numerical results will be presented to stimulate collaboration to study challenge problems as teams in future.

Bifurcation for a Free Boundary Problem Modeling the Growth Tumors with the Nonlinear Proliferation/death Rate

Fengjie Li
China University of Petroleum, Peoples Rep of China

In this paper, we study bifurcations for a free boundary problem modeling the growth of tumors under the action of inhibitors. We introduce the effect of the drug into the net proliferation / death rate which is nonlinear for the unknown function in the tumor region. Symmetry-breaking solutions bifurcating from the radially symmetric stationary solutions are obtained.

Analysis of Some Nonlinear Elliptic Systems

Congming Li
Shanghai Jiao Tong University, Peoples Rep of China
Wenxiong Chen, Ze Cheng, Genggeng Huang, Yutian Lei, Chao Ma, John Villabvert

This talk is on the well-known Hardy-Littlewood-Sobolev and the related Schrodinger and Riesz type systems. We present a brief survey on some important known results, a short introduction of some of our recent results, and also a brief description of some basic problems that we are interested. Beyond the basic qualitative properties such as the existence, non-existence, and classification of positive solutions, we are also interested in the integrability, asymptotic at infinite, and symmetries of positive solutions.

The Stability of Travelling Waves for General Autocatalytic Reaction Systems

Yi Li
California State University, Northridge, USA
Yaping Wu

We investigate the following autocatalytic reaction system

$$\begin{cases} u_t = d\triangle u - u^q v^p \\ v_t = \triangle v + u^q v^p. \end{cases} \tag{1}$$

System (1) can describe the following autocatalytic step in isothermal, autocatalytic chemical reaction schemes

$qA+pB \to (p+q)B$, isothermal reaction rate $ku^q v^p$

where the autocatalysis B is stable and does not decay to further product of reaction,

u: concentration of reactant A,
v: concentration of autocatalyst B.

Cubic autocatalytic step:

$$2A + B \to 3B, \text{ rate } k_1 a^2 b,$$

System (1) also arises in thermal-diffusive combustion problems, and mathematical biology (e.g. epidemic models).

Turbulent Flame Speeds in G-Equation Models and ABC Flow

Yu-Yu Liu
National Cheng Kung University, Taiwan

In turbulent combustion theory, the G-equation models are level set Hamilton-Jacobi equations where the motion of the flame front is described by a prescribed flow velocity and a laminar velocity. Besides the constant laminar flame speed, the curvature and the strain terms may be added into the laminar velocity for the flame stretching effect. I will report the current numerical study on G-equation models in three dimensional space with flow velocity chosen as the Arnold-Beltrami-Childress (ABC) flows. The numerical methods are finite difference WENO schemes on monotone Hamiltonian.

Soret and Dufour Effects on Natural Convetion from a Cone in Viscoelastic Fluid by the Linearization Method

Gilbert Makanda
Central University of Technology, So Africa
Preciuos Sibanda

The paper applies non-perturbation successive linearization method to solve the nonlinear coupled boundary value problem on Soret and Dufour effects from a cone in viscoelastic fluid. The partial differential equations are transformed into a system of ordinary differential equations which are then solved using the successive linearization method (SLM). The boundary layer velocity, temperature and concentration profiles are numerically computed for different viscoelastic, concentration, buoyancy, Soret and Dufour numbers. The numerical method is based on the Chebyshev spectral collocation technique. Many researches have been carried out on Dufour and Soret effects but did not consider natural convection in viscoelastic fluids under linear surface temperature. The results are computed and compared with other results in the literature showing a strong agreement and efficiency of the SLM. These comparisons show the robustness of the successive linearization method and that it can be used as an alternative in solving nonlinear boundary value problems.

Bifurcation Curves for 1D Prescribed Mean Curvature Equations

Hongjing Pan
South China Normal University, Peoples Rep of China
Ruixiang Xing

In this talk we will discuss some recent progress on bifurcation curves for one-dimensional prescribed mean curvature problems. Notice that the well-known Bernstein-Nagumo condition is violated in this case. The special structure of the mean curvature operator produces some remarkable effects. The length parameter plays a key role. By various examples, we show the rich diversity of bifurcation curves for the quasilinear problem, different from semilinear cases.

Stochastic Partial Functional Differential Equations with Delay

Peter Pang
National University of Singapore, Singapore

We will present our recent results on the existence and uniqueness of strong solutions to stochastic partial functional differential equations (SPFDEs) with locally monotone coefficients, locally Lipschitz nonlinearity, and time delay. We note that, while SPFDEs have important applications, they are far less studied than SPDEs and SFDEs. Our results extend and widen the applicability of those of Liu-Röcker (2010), Caraballo et al (2000), and Taniguchi et al (2002). We illustrate the applicability of our results by applying them to a stochastic 2D Navier-Stokes equation with time delay, and a stochastic Nicholson's blowflies equation with time delay.

Traveling Wave to Gray-Scott Systems

Yuanwei Qi
University of Central Florida, USA
Xinfu Chen, Yajing Zhang, Zhi Zheng

In this talk I shall present some recent works on reaction-diffusion systems which arise from Gray-Scott type of models of pattern formation. We show the existence, multiplicity and special features such as oscillation of the traveling wave solutions. Applications to models in population dynamics and infection diseases will also be discussed.

A Multi-Scale Stochastic Finite Element Method for Random Heterogeneous Materials and Its Validation Using Immersed Interface Finite Element Method

Lihua Shen
University of Calgary, Canada
Xi Xu, Changfeng Wang, Wenting Jin

In this talk, a multi-scale stochastic finite element method for random heterogeneous materials is presented. To show its validation, Monte Carlo method combined with the immersed interface finite element method is used to simulate a type of simple random heterogeneous materials. The numerical results from the multi-scale stochastic finite element method, standard finite element method and the immersed interface finite element method are compared.

Symmetry-Breaking Bifurcation for a Free Boundary Problem Modeling Solid Tumour Growth with ECM and MDE Interactions

Ruixiang Xing
Sun Yat-sen University, Peoples Rep of China
Hongjing Pan

In this talk, we deal with a free boundary problem modeling solid tumour growth with ECM and MDE interactions. This problem has a unique radially symmetric stationary solution. We show that symmetry-breaking solutions bifurcate from the radially symmetric stationary solutions.

Motion by Mean Curvature for a Second Order Gradient Theory

Nung Kwan (Aaron) Yip
Purdue University, USA
Drew Swartz

We prove in a radially symmetric geometry, the convergence in the sharp interfacial limit, to motion by mean curvature of a second order gradient model for phase transition. This is in spirit similar to the classical Allen-Cahn theory of phase boundary motion. However the corresponding dynamical equation is fourth order thus creating some challenging difficulties for its analysis. A characterization and stability analysis of the optimal profile are performed which are in turn used in the proof of convergence of an asymptotic expansion.

Flame Propagation Enhanced by the ABC Flow

Yifeng Yu
UC Irvine, USA
Jack Xin, Andrej Zlatos

G-equation is a well known model in turbulent combustion. its simplest form is a convex but not coercive Hamilton-Jacobi equation. The associated effective Hamiltonian can be viewed as the turbulent flame speed. A significant problem is to understand the dependence of the turbulent flame speed on the velocity of the ambient fluid. In this talk, I will show that if the flow velocity is given by the 1-1-1 Arnold-Beltrami-Childress (ABC) flow, the turbulent flame speed grows linearly with respect to the flow velocity. This is based on a joint work with Jack Xin and Andrej Zlatos.

Classification of Blowup Solutions for a Degenerate Parabolic Equation with Nonlinear Gradient Terms

Zhengce Zhang
Xian Jiaotong University, Peoples Rep of China
Yan Li

In this talk, we will present our recent results on L^∞ blowup and gradient blowup. We study the degenerate parabolic equations with nonlinear gradient terms. In the different ranges of reaction exponents, we give the complete classification of blowup results including L^∞ blowup, gradient blowup and the global-in-time existence.

The Study the Global Stabilities of Diffusive Predator-Prey Systems of Holling-Tanner Type and of Lesile Type

Yi Zhu
University of Central Florida, USA
Yuanwei Qi

In this paper we study the global stabilities of diffusive predator-prey systems of Holling-Tanner Type and of Lesile type in a bounded domain$\Omega \subset R^N$ with no-flux boundary condition. By using a novel approach, we establish much improved global asymptotic stability of the unique positive equilibrium solutions. We also show the result can be extended to more general type of systems with heterogeneous environment and/or other kind of kinetic terms.

Special Session 12: Propagation Phenomena in Reaction-Diffusion Systems

Hirokazu Ninomiya, Meiji University, Japan
Masaharu Taniguchi, Okayama University, Japan

This special session is concerned with mathematical analysis on propagation phenomena and pattern formation in reaction-diffusion systems. Related topics are equilibrium states, traveling waves, spiral waves and asymptotic behavior of solutions in reaction-diffusion systems as well as the related free boundary problems. The aim of this special session is to exchange our ideas and promote our knowledge and understanding on this subject.

Inside Dynamics of Positive Solutions in Some Non-Local Equations

Jerome Coville
INRA, France
Olivier Bonnefon, Jimmy Garnier, Lionel Roques

In this talk, I will present some joint work with Olivier Bonnefon, Jimmy Garnier and Lionel Roques concerning the inside dynamics of positive solutions of some non-local equations. The notion of inside dynamics has been recently used to characterise the genetic structure of a colonizing population modeled by a traveling wave solution of a homogeneous reaction diffusion equation. The notions of pushed and pulled fronts were at this purpose introduced. I will present some extensions of the notions of pushed/pulled fronts to the case of positive solutions of some homogeneous non-local reaction diffusion equations and the classification that we were able to achieve in a monostable situation.

On Existence of Wavefront Solutions in Mixed Monotone Reaction-Diffusion Systems with Time Delays

Wei Feng
University of North Carolina Wilmington, USA
Weihua Ruan, Xin Lu

We study the existence of traveling wave solutions in a general class of mixed quasi-monotone reaction-diffusion systems with time delays. First, by applying the Schauder Fixed Point Theorem, we prove the existence of a traveling wave solution between classically defined upper and lower solutions. For better applications of the upper-lower solution method on various real-life models, the existence result is further extended under weak form or piecewise smooth upper-lower solutions. In several reaction-diffusion systems with time delays, we apply our main result to establish the existence of traveling wave solutions flowing towards the positive or coexistent states under reasonable conditions on ecological parameters.

Stability of Standing Planar Spot Solutions in Three-Component FitzHugh-Nagumo Systems

Hideo Ikeda
University of Toyama, Japan

Various localized planar patterns are observed in many reaction-diffusion systems. Especially, two-component systems are well studied so far and several mathematical results are obtained. But, even if such stationary localized planar solutions (stationary spot solutions) exist stably, one do not get stable traveling spot solutions which are bifurcated from stationary spot solutions. This implies that such traveling spot solutions seem to be unstable in two-component systems if they exist. In this talk, we will show the existence of stationary spot solutions and the stability properties of them in three-component FitzHugh-Nagumo systems and consider the possibility of the supercritical drift bifurcation. Note that these results are already obtained by Heijster and Sandstede via the formal analysis (Physica D 275(2014),19-34).

Reduction Approach to a Reaction-Diffusion System for Collective Motions of Camphor Boats

Kota Ikeda
Meiji University, Japan
Ei Shin-Ichiro

The collective motion of camphor boats in the water channel exhibits both a homogeneous and an inhomogeneous state, depending on the number of boats. The motion of each camphor boat is described by a traveling pulse in a reaction-diffusion model proposed in Nagayama et al. (2004), in which camphor boats are assumed to be interact each other by the change of surface tension by diffusive molecules on the water surface. In order to verify the inhomogeneous motion of camphor boats, we have to study the linearized eigenvalue problem and see the destabilization of the homogeneous flow. However, the eigenvalue problem is too difficult to analyze even if the number of camphor boats is 2. Then we would like to derive a reduced system from the original model and analyze it by applying the center manifold theorem. Several reaction-diffusion systems can generate a solution with a pulse shape. Pulse-pulse interaction is treated mathematically in Ei et al. (2002), in which a reduced system of an ODE form is derived

from a reaction-diffusion model by applying a center manifold theorem. Since the delta functions naturally arise in our model, the theory established in L2-framework cannot be applied directly. In this talk, we modify the previous results in Ei et al. (2002) and propose a new approach of reduction to systems with the delta function.

Exponential Stability of a Traveling Wave for an Area Preserving Curvature Motion with Two Endpoints Moving Freely on a line

Shimojo Masahiko
Okayama University of Science, Japan
Kagaya Takashi

The asymptotic behavior of solutions to an area-preserving curvature flow of planar curves in the upper half plane is concerned. Two endpoints of the curve slide along the horizontal axis with prescribed fixed contact angles. By establishing an isoperimetric inequality, we prove the global existence of the solution. We then study the asymptotic behavior of solutions with concave initial data near a traveling wave.

Spreading Speed and Asymptotic Profiles of Solutions for Nonlinear Free Boundary Problems in High Space Dimension.

Hiroshi Matsuzawa
National Institute of Technology, Numazu College, Japan
Yihong Du, Maolin Zhou

In this talk we consider nonlinear diffusion equations $u_t = \Delta u + f(u)$ with Stefan free boundary conditions. When the nonlinear term $f(u)$ is of monostable, bistable or combustion type, these problems are used to describe the spreading of a biological or chemical species with the free boundary representing the expanding front. In this talk, we consider radially symmetric solutions. Then the Stefan free boundary conditions are reduced to $h‘(t) = -\mu u_r(t, h(t))$ where $\{x \in \mathbf{R}^N : |x| = h(t)\}$ is the free boundary. We show that when spreading occurs, a logarithmic shifting appears in sharp estimate of $h(t)$ when $N \geq 2$, while such a logarithmic shifting does not appear when $N = 1$.

Existence of Traveling Wave Solutions to Curvature Flows with External Driving Force

Harunori Monobe
Meiji University, Japan
Hirokazu Ninomiya

Mean curvature flow is a geometric flow of hypersurfaces. In 2, 3-dimensional Euclidean spaces, mean curvature flow with external driving force is naturally appeared as a mathematical model describing various physical and biological phenomena, e.g., soap films, grain boundaries, ray optics and singular limit of population dynamics. In this talk, we consider the existence of traveling wave solutions, for curvature flow with external driving force, which is a Jordan curve in 2-dimensional space. When there exist traveling wave solutions, we intend to make reference to the shape of traveling waves too.

Large Time Behavior of the Solutions with Spreading Fronts in the Allen-Cahn Equation

Mitsunori Nara
Iwate University, Japan
Hiroshi Matano

In this talk, we consider the Cauchy problem for the Allen-Cahn equation $u_t = \Delta u + f(u)$ on $\mathbb{R}^n$ with $n \geq 2$, where the nonlinearity f is of bistable and unbalanced type. We show that, under some assumptions on the inital value, the solution develops a well-fromed front whose coutours are nearly shperical and are spreading as time goes to infinity.

Mean Curvature Flow of Entire Graphs Evolving Away from the Heat Flow

Xuan Hien Nguyen
Iowa State Univ., USA
Gregory Drugan

We present two initial graphs over the entire $\mathbb{R}^n$, $n \geq 2$ for which the mean curvature flow behaves differently from the heat flow. In the first example, the two flows stabilize at different heights. With our second example, the mean curvature flow oscillates indefinitely while the heat flow stabilizes. These results highlight the difference between dimensions $n \geq 2$ and dimension $n = 1$, where Nara and Taniguchi proved that entire graphs in $C^{2,\alpha}(\mathbb{R})$ converge to solutions to the heat equation with the same initial data.

Layered Interface Systems and Its Dynamics

Hirokazu Ninomiya
Meiji University, Japan
H. Mitake and K. Todoroki

The dynamics of three dimensional interface motion is often complicated. To reduce the dimension will be helpful to understand it. For this purpose, we will introduce a new system of two dimensional interface equations. We call it layered interface system. In the talk I will explain how to derive it and its dynamics especially for mean curvature flow.

Micro Phase Separation in Higher Dimensions

Yoshihito Oshita
Okayama University, Japan

We study the equation for the distribution of the particle radii obtained by taking the homogenization limit of the evolution equation describing the micro phase separation phenomenon in higher dimensions.

Traveling Wave Solutions of Quasilinear Reaction-Diffusion Systems with Mixed Quasi-Monotonicity

Weihua Ruan
Purdue University Calumet, USA
Wei Feng, Xin Lu

We study the existence of traveling wave solutions in a general class of mixed quasi-monotone reaction-diffusion systems with nonlinear diffusions in the form

$$\partial u_i/\partial t = \nabla \cdot (D_i(u_i)\nabla u_i) + f_i(\mathbf{u}), \; -\infty < x < \infty, \; t > 0$$

for $i = 1, \ldots, n$. Each function f_i is quasi-monotone increasing for some components of $\mathbf{u} = (u_1, \ldots, u_n)$ and decreasing for other components of $\mathbf{u}$. Such systems model reaction-diffusion processes with a density driven diffusion mechanism. Under certain general conditions we prove the existence of a traveling wave solution that is between a pair of coupled upper and lower traveling wave solutions. An example of quasilinear Lotka-Volterra population model is given as an illustration of application.

Asymptotic Behavior of Solutions of Virus Dynamics Models with Diffusion

Toru Sasaki
Okayama University, Japan
Takashi Suzuki

In this talk, we consider the asymptotic behavior of solutions of virus dynamics systems with diffusion, for example,

$$\begin{aligned}
\frac{\partial u_1}{\partial t} &= d_1\Delta u_1 + \lambda - mv_1 - \beta u_1 u_3, \\
\frac{\partial u_2}{\partial t} &= d_2\Delta u_2 + \beta u_1 u_3 - au_2, \\
\frac{\partial u_3}{\partial t} &= d_3\Delta u_3 + aru_2 - bu_3,
\end{aligned}$$

with $u_i = u_i(x,t)$, $t \geq 0$, $x \in \Omega$, where Ω is a bounded set in $\mathbf{R}^3$, with smooth boundary. Here u_1, u_2, and u_3 denote the populations of uninfected cells, of infected cells, and of infectious agents, respectively. On the boundary of the domain, we impose the homogenuous Neumann condition. We can use the Lyapunov function proposed in Korobeinikov (Bull. Math. Biol., 2004), which deals with the ODE system for spatially homogeneous solutions.

An (N-1)-Dimensional Convex Compact Set Gives an N-Dimensional Traveling Front

Masaharu Taniguchi
Okayama University, Japan

We study traveling fronts to the unbalanced Allen–Cahn equation or cooperation-diffusion systems in the N-dimensional Euclidean space for N that is larger or equals to 3. We consider (N-2)-dimensional smooth surfaces as boundaries of strictly convex compact sets in in the (N-1)-dimensional Euclidean space, and define an equivalence relation between them. We prove that there exists a traveling front associated with a given surface and show its stability. The associated traveling fronts coincide up to phase transition if and only if the given surfaces satisfy the equivalence relation.

Traveling Waves and Stability of Reaction Diffusion Equations

Rong Yuan
Beijing Normal University, Peoples Rep of China

In this talk, we would like introduce some results about the asymptotically stability and the convergence rate of the pyramidal traveling fronts.

Special Session 13: Chemotactic Cross-Diffusion in Complex Frameworks

Michael Winkler, University of Paderborn, Germany
Dariusz Wrzosek, University of Warsaw, Poland

Chemotaxis is a ubiquitously observed mechanism of interaction between individuals (cells or organisms) in biological systems. Typically the chemotactic interaction is mediated be a chemical agent whose density gradient determines the direction of movement of the individuals. Chemotaxis mechanisms are known to play outstanding roles in numerous processes of self organization at all levels of complexity of biological systems. According to various modeling approaches, an adequate mathematical description thereof requires the study of parabolic PDE systems involving certain cross-diffusive terms as their most characteristic ingredient. Important results in the literature identify situations in which in the framework of basic models, such chemotactic cross-diffusion indeed enforces spontaneous generation of structures or space-time patterns e.g. in the sense of stabilization toward non-constant equilibria, or even in the sense of singularity formation. The goal of the proposed special session is to present up-to-date results in the analysis of chemotaxis systems, with emphasis on qualitative properties of solutions such as boundedness and asymptotic behavior or the occurrence of blow-up or pattern formation. According to recent developments in both modeling and analysis, a particular focus will be on models for chemotactic cross-diffusion in complex frameworks, including couplings to further mechanisms such as cell proliferation, haptotaxis, or also interaction with a surrounding fluid. By planning to bring together experienced and young researchers working in these areas, we intend to stimulate a fruitful sharing of ideas, and thereby to initiate new directions of future research.

Concentration Waves of Bacteria at the Mesoscopic Scale

Vincent Calvez
CNRS & ENS de Lyon, France

Concentration waves of swimming bacteria Escherichia coli were described in his seminal paper by Adler (Science 1966). These experiments gave rise to intensive PDE modelling and analysis, after the original model by Keller and Segel (J. Theor. Biol. 1971), and the work of Alt and co-authors in the 80s. Together with Bournaveas, Perthame, Raoul and Schmeiser, we have revisited this old problem from the point of view of kinetic transport equations. This framework is very much adapted to the so-called run-and-tumble motion, in which any bacteria modulate the frequency of reorientation (tumble) – and thus the duration of free runs – depending on chemical variations in its environment.
In this talk, I will present existence results for solitary waves both at the macroscopic scale, and at the mesoscopic scale. The macroscopic problem consists of a drift-diffusion equation derived from the kinetic equation after a suitable diffusive rescaling, coupled to two reaction-diffusion equations. Mathematical difficulties arise at the mesoscopic scale, where the proof of existence of travelling waves require a refined description of spatial and velocity profiles.
I will also present numerical simulations done in collaboration with Gosse and Twarogowska, in order to illustrate some unexpected behavior of the mesoscopic problem.

A New Approach to the Large Time Behavior in Chemotaxis Logistic Model

Xinru Cao
Remin University of China, Peoples Rep of China

In this talk, we present a new approach towards the large time behavior of chemotaxis model with logistic source.

Reaction Enhancement by Chemotaxis

Elio Espejo
Universidad Nacional de Colombia, Colombia
Ramon Plaza; Carlos Malaga, Takashi Suzuki

An interesting problem arising in many contexts of math-biology is the study of the relevance of chemotaxis in reaction-diffusion processes. I will approach this problem through a mathematical model representing the fertilization process of corals. As a result we obtain a system of partial differential equations describing the cell dynamics being affected by three basic phenomena: diffusion, chemotaxis and a surrounded flux. First we prove that our model has in general global classical solutions. Next, we compare the assymptotic behavior with and without chemotaxis. We show the relevance of chemotaxis after making a systematic adaptation of the well-known moments technique on bounded domains (usually used for proving blow-up in Keller-Segel systems), to analyze the behavior of the cell dynamics when the chemotactic signal increases. The proposed method seems to be readily adaptable to many other models specially when working in two dimensions. In the context of the proposed model, our analysis shows that in the two dimensional case the remaining fraction of unfertilized eggs at any given time $t > 0$ becomes arbitrarily small if the chemotaxis signal is sufficiently large.

Global Solvability in 2D Keller-Segel-Stokes Systems with Decaying Sensitivity

Kentarou Fujie
Tokyo University of Science, Japan

In this talk, we consider two-dimensional Keller-Segel-Stokes systems with decaying sensitivity (e.g. logarithmic sensitivity). It is shown that for all reasonable regular initial data, the system has a global and bounded solution. The proof is based on the localization method, which was established in Fujie-Senba (2016).

Global Existence of Smooth Solutions to the SKT System in High Dimensional Spaces

Luan Hoang
Texas Tech University, USA
Truyen V. Nguyen, Tuoc V. Phan

We investigate the global time existence of smooth solutions for the Shigesada-Kawasaki-Teramoto (SKT) system of cross-diffusion equations of two competing species in population dynamics. If there are self-diffusion in one species and no cross-diffusion in the other, we show that the system has a unique smooth solution for all time in bounded domains of any dimension. We obtain this result by deriving global $W^{1,p}$-estimates of Calderòn-Zygmund type for a class of nonlinear reaction-diffusion equations with self-diffusion. These estimates are achieved by employing Caffarelli-Peral perturbation technique together with a new two-parameter scaling argument.

Boundedness and Large Time Behavior of an Attraction-Repulsion Chemotaxis Model with Logistic Source

Haiyang Jin
South China University of Technology, Peoples Rep of China
Shishi Jie

In this talk, we will study an attraction-repulsion Keller-Segel chemotaxis model with logistic source. When repulsion cancels attraction, the relation of the damping parameters θ and the space dimension n is established to ensure the existence of classical solution with uniform-in-time bound. Furthermore, for the classical logistic source $f(u) = \mu u(1-u)$, we also study the large time behavior of solution under some conditions on the parameters.

Long-Term Behaviour in a Chemotaxis-Fluid System with Logistic Source

Johannes Lankeit
Paderborn University, Germany

We consider the coupled chemotaxis Navier-Stokes model with logistic source terms

$$
\begin{aligned}
n_t + u \cdot \nabla n &= \Delta n - \chi \nabla \cdot (n \nabla c) + \kappa n - \mu n^2 \\
c_t + u \cdot \nabla c &= \Delta c - nc \\
u_t + (u \cdot \nabla) u &= \Delta u + \nabla P + n \nabla \Phi + f, \qquad \nabla \cdot u = 0
\end{aligned}
$$

in a bounded, smooth domain$\Omega \subset \mathbb{R}^3$ under homogeneous Neumann boundary conditions for n and c and homogeneous Dirichlet boundary conditions for u and with given functions $f \in L^\infty(\Omega \times (0,\infty))$ satisfying certain decay conditions and$\Phi \in C^{1+\beta}(\bar{\Omega})$ for some $\beta \in (0,1)$.

We construct weak solutions and prove that after some waiting time they become smooth and finally converge to the semi-trivial steady state $(\frac{\kappa}{\mu}, 0, 0)$.

Decay Rates of Solutions for 2-D Chemotaxis-Navier-Stokes System

Yuxiang Li
Southeast University, Peoples Rep of China
Zhang Qingshan

We consider an initial-boundary value problem for the incompressible chemotaxis-Navier-Stokes equation on a bounded domain of two dimension, which describes the spatio-temporal evolution of populations of swimming aerobic bacteria. It is shown by Winkler [ARMA 2014] that the solution converges to a constant stationary state. In the present talk, we show that the convergence is exponential.

Blow-Up in Finite Time for Solutions in Chemotaxis Systems with a Source Term

Monica Marras
University of Cagliari, Italy
S.Vernier-Piro, G.Viglialoro

We investigate a class of parabolic-parabolic Keller-Segel type system in a bounded domain in R^N, with $N = 2,\ 3$, under different boundary conditions, with time dependent coefficients, a nonlinear cross diffusion and a positive source term. The solutions may blow up in finite time T and under appropriate assumptions on data, we derive an explicit lower bound for blow-up time.

On a Chemotaxis System with Singular Chemotactic Sensitivity and A Non-Diffusible Chemical

Cristian Morales-Rodrigo
Univ. Sevilla, Spain
M. Winkler

In this talk we consider, in higher dimensions, a system of partial differential equations with singular chemotactic sensitivity. We will prove the existence of global weak solutions and their asymptotic behavior.

Quantized Blowup Mechanism for 2D Smoluchowski-Poisson Equation: Stationary, Infinite Time, and Finite time

Takashi Suzuki
Osaka University, Japan

We study the Smoluchowski-Poisson equation in $2D$ bounded domain with smooth boundary. The solution is positive, provided with total mass conservatin and free energy decreasing. Its stationary state is the Boltzmann-Poisson equation, where the quantized blowup mechanism is observed with the singular limits described by the Hamiltonian of point vortices. First, blowup in infinite time arises with this singular limit. Concerning the blowup in finite time, next, collapse mass quantization arises with possible collision of sub-collases. Furthermore, total free energy is bounded if and only if any blowup point is simple, which is equivalent to the localized type II blowup rate.

On a Chemotaxis System with Nonlocal Terms

J. Ignacio Tello
Universidad Politecnica de Madrid, Spain

We will present result of a system of partial differential equations describing the evolution of a population under chemotactic effects with nonlocal reaction terms. By introducing global competitive/cooperative factors in terms of the mass of the population, we obtain, for a range of parameters, results of the existence of solutions and the asymptotic behavior.

Explicit Lower Bound of Blow-Up Time to a Parabolic Chemotaxis System with Nonlinear Cross-Diffusion

Stella Vernier-Piro
UNICA, Italy
Youshan Tao

Let us consider the chemotaxis model

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(h(u)\nabla v), & x\in B_1(0),\, t>0,\\ v_t = \Delta v - v + u, & x\in B_1(0),\, t>0,\end{cases}$$

under homogeneous Neumann boundary conditions in a unit ball $B_1(0)\subset\mathbb{R}^3$ centered at the origin, with $\chi>0$.
Under the assumption that $h(u) = (u(u+1)^{m-1}$, $m\in\mathbb{R}$, $(u(x,0),v(x,0)) = (u_0(|x|),v_0(|x|))\in C^0(\bar{B}_1(0))\times W^{1,\infty}(B_1(0))$, whenever $m\in[\frac{2}{3},2]$, the blow-up time of a classical solution to the corresponding initial-boundary problem has an explicit lower bound measured in terms of χ, $\int_{B_1(0)} u_0^p$ and $\int_{B_1(0)}|\nabla v_0|^{2q}$ for appropriate $p>1$ and $q>1$. The global classical solution exists bounded if m

Some Results on a Parabolic-Parabolic Keller-Segel System with Logistic Term

Giuseppe Viglialoro
University of Cagliari, Italy

This talk is concerned with a fully parabolic Keller-Segel system, modeling chemotaxis phenomena, which is defined in a n-dimensional (bounded and smooth) domain and whose source term is controlled by a sort of logistic function. Precisely, in line with the contribution "Chemotaxis with logistic source: Very weak global solutions and their boundedness properties (Winkler, J. Math. Anal. Appl., 348, (2008), 708-729)", dealing with the parabolic-elliptic version of the aforementioned problem, our main goal is to give a result of existence of solutions for the presented system and discuss some boundedness properties of such solutions. In addition, we also present numerical simulations which inspire further questions related to this model.

Boundary Layers Arising from Chemotaxis Models

Zhian Wang
Hong Kong Polytechnic University, Hong Kong
Hongyun Peng, Changjiang Zhu

In this work, we consider an initial-boundary value problem of a hyperbolic system which was derived from a chemotaxis system with singular sensitivity modeling the initiation of angiogenesis. By imposing (time-dependent) Dirichlet boundary conditions, we show that as the chemical diffusion vanishes, the solution becomes very sharp around the boundary and hence exhibits the boundary layer phenomenon. We also numerically illustrate that the boundary layer is only a transient behavior: exists in short-time but vanishes in large-time.

Interspecies Competition and Chemorepulsion

Dariusz Wrzosek
Warsaw University, Poland

Classical Lotka-Volterra model of competition is extended to account for the random dispersal of individuals and for their capability to avoid encounters with competitors by means of a chemo-sensory reaction to the smell of rivals (chemorepulsion). We consider the case of diffusing and non-diffusing repellent and study the existence of global in time solutions, existence of non-constant steady states and long time-behaviour. The models are compared with other models of interspecies competition.

How Strong a Logistic Damping Can Prevent Blow-Up for the Minimal Keller-Segel Model?

Tian Xiang
Renmin University of China, Peoples Rep of China

We study a general class of fully parabolic Keller-Segel chemotaxis systems with growth sources, under homogeneous Neumann boundary conditions in a multi-dimensional bounded domain. It is recently known that blowup is possible even in the presence of superlinear growth restrictions. Here, we first derive several characterizations on the growth versus boundedness.. Then we apply these criteria to the minimal KS model with a logistic source $\kappa u - \mu u^2$ and obtain a quantitative description of the competition between chemotactic aggregation and logistic damping, and, in particular, obtain an explicit formula independent of the size of the domain, initial data, κ and Sobolev embedding constants for the logistic damping rate μ_0 such that blow-up is completely ruled out if $\mu > \mu_0$. This offers a quantized effect of the logistic source on the prevention of blow-ups and hence improves existing results in this regard. Moreover, μ_0 tends to infinity when either diffusion coefficient of cells or chemicals shrinks to zero. Therefore, small diffusion, especially, degenerate or nonlinear diffusion, enhances the possibility of the occurrence of unbounded solutions. This gives a clue on how to produce blow-up solutions for Keller-Segel chemotaxis models with logistic sources.

Boundedness in a Quasilinear Parabolic-Parabolic Keller-Segel System Via Maximal Regularity

Tomomi Yokota
Tokyo University of Science, Japan
Sachiko Ishida

We consider a quasilinear degenerate parabolic-parabolic Keller-Segel system. Global existence of weak solutions to the system on the whole space was studied by Sugiyama-Kunii (JDE, 2006) and the result was improved by Ishida-Yokota (JDE, 2012), while global existence and boundedness of solutions to the nondegenerate system on bounded convex domains was established by Tao-Winkler (JDE, 2012) and the convexity assumption was removed by Ishida-Seki-Yokota (JDE, 2014) not only in the case of nondegenerate diffusion but also in the case of degenerate diffusion. The purpose of this talk is to present a simple way to prove global existence and boundedness of solutions via maximal regularity.

Global Solutions of a Chemotaxis System Without Gradient-Sensing.

Changwook Yoon
Yonsei University, Korea
Yong-Jung Kim

We consider a Keller-Segel model describing cell aggregation phenomena. The model is formulated under the assumption that microscopic scale bacteria does not sense the macroscopic scale concentration gradient of chemical. We focus on the global existence, uniform boundedness of solutions and the steady states having aggregation structures.

Special Session 14: Nonlinear Evolution Equations and Related Topics

Mitsuharu Otani, Waseda University, Japan
Tohru Ozawa, Waseda University, Japan

This session will focus on the recent developments in the theory of Nonlinear Evolution Equations and Related Topics including the theory of abstract evolution equations in Banach spaces as well as the studies (the existence , regularity and asymptotic behaviour of solutions) of various types of Nonlinear Partial Differential Equations.

Properties of Solutions of Nonlinear Evolution Equations

Zhivko Athanassov
Bulgarian Academy of Sciences, Bulgaria

We establish results on the existence, uniqueness, and regularity of solutions of (E) $du(t)/dt = Au(t) + f(t,u(t))$ in a Banach space. We are concerned with the case when A does not generate a semi-group. Our assumptions are motivated by the applications to parabolic boundary value problems of higher order in t. The higher order equation is reduced to a first order system of the form (E) by introduction the t - derivatives as new unknowns. The approach adopted in this talk has been employed in the linear case by S. Agmon and L. Nirenberg.

Existence of Three Nontrivial Solutions for a Nonlinear Fractional Laplacian Problem

Fatma Gamze Duzgun
Hacettepe University, Turkey

We consider the problem

$$\begin{cases} (-\Delta)^s u = f(x,u) & \text{in} \quad \Omega \\ u = 0 & \text{in } \Omega^c, \end{cases} \tag{1.1}$$

where$\Omega \subset \mathbb{R}^N$ $(N > 1)$ is a bounded domain with a C^2 boundary, $s \in (0,1)$, and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Caratheodory function. The fractional Laplacian operator is defined for any sufficiently smooth function $u : \mathbb{R}^N \to \mathbb{R}$ and all $x \in \mathbb{R}$ by

$$(-\Delta)^s u(x) = C_{N,s} \lim_{\varepsilon \to 0^+} \int_{\mathbb{R}^N \setminus B_\varepsilon(x)} \frac{u(x) - u(y)}{|x-y|^{N+2s}}\, dy, \tag{1.2}$$

where $B_\varepsilon(x)$ is the open ball of radius $\varepsilon > 0$ centered at x and $C_{N,s} > 0$ is a suitable normalization constant. For the existence of three nontrivial solutions of problem (1.1), we make use of the second deformation theorem and some spectral properties of $(-\Delta)^s$ if $f(x,\cdot)$ is sublinear at infinity and make use of the mountain pass theorem and Morse theory if $f(x,\cdot)$ is superlinear at infinity.

Lifespan of Strong Solutions to the Periodic Nonlinear Schrödinger Equations Without Gauge Invariance

Kazumasa Fujiwara
Waseda University, Department of Pure and Applied Physics, Japan
Tohru Ozawa

A lifespan estimate and sharp condition of the initial data for finite time blowup for the periodic nonlinear Schrödinger equations without gauge invariance are presented. The condition for finite time blowup is sharp from a viewpoint of the average of initial data and a simple and direct explanation how the condition comes into play in blowup in a general settings are given. Moreover, an explicit upper bound of the life span of solutions is obtained.

Well-Posedness for a Generalized Derivative Nonlinear Schrödinger Equation

Masayuki Hayashi
Waseda University, Japan
Tohru Ozawa

We study the Cauchy problem for a generalized derivative nonlinear Schrödinger equation with the Dirichlet boundary condition. We establish the local well-posedness results in the Sobolev spaces H^1 and H^2. Solutions are constructed as a limit of approximate solutions by a method independent of a compactness argument. We also discuss the global existence of solutions in the energy space H^1.

Analytic Smoothing Effect for a Quadratic System of Schrödinger Equations in the Framework of Mass Sub Critical

Gaku Hoshino
Waseda University, Japan

We consider the Cauchy problem for a system of nonlinear Schrödinger equations in the framework of mass sub critical setting.
In particular, we study the analytic smoothing effect for the solutions to the Cauchy problem for a system of nonlinear Schrödinger equations under the mass resonance condition in the framework of mass sub critical setting with large data.

On the Cauchy Problem for the Nonlinear Schrödinger Equation with Initial Data with Infinite L^2-Norm

Ryosuke Hyakuna
Waseda University, Japan

In this talk, we consider the initial value problem for the nonlinear Schrödinger equation:

$$\mathrm{i}u_t + \Delta u + |u|^{\alpha-1}u = 0, u(0,x) = u_0(x) \in L^p(\mathbb{R}^n).$$

We are interested in the problem of the solvability of this Cauchy problem for classes of initial date with infinite L^2-norm. In this talk, we focus our attention on L^p-space, and $\mathcal{F}L^p$-spaces and investigate well-posedness and existence of global solutions.

On Some Nonlinear Evolution Equations and Application to Mathematical Models

Akisato Kubo
Fujita Health University, Japan

In this talk we consider initial-Neumann boundary value problem of nonlinear evolution equations with strong dissipation and proliferation arising from mathematical biology and physics formulated as

$$(NE)\begin{cases} u_{tt} = D\nabla^2 u_t \\ \quad +\nabla\cdot(\chi_1(u_t, e^{-u})\chi_2(u,\nabla u)\nabla u) \\ \quad +\mu_1 u_t(1-u_t) \\ \quad \text{for } (x,t)\in\mathbf{\Omega}\times(0,\infty) \quad (1) \\ \dfrac{\partial}{\partial\nu}u|_{\partial\mathbf{\Omega}} = 0 \quad \text{on} \quad \partial\mathbf{\Omega}\times(0,\infty) \quad (2) \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x) \quad \text{in } \mathbf{\Omega} \quad (3) \end{cases}$$

where constants D, μ_1 are positive, $\mathbf{\Omega}$ is a bounded domain in R^n with a smooth boundary $\partial\mathbf{\Omega}$ and ν is the outer unit normal vector. Under some regularity and boundedness conditions of the coefficient $\chi_i(u_t, e^{-u}), i = 1, 2$ of (1), we derive the energy estimate of (NE), which enables us to show the global existence in time and asymptotic behavior of the solution. We apply our result to related mathematical models and discuss the property of solutions of them. Ebihara [1] considered (1) in a more general form and proved a local existence in time of the solution. In [2] we deal with (NE) in case $\chi_2(u_t, e^{-u}) \equiv 1$and show global existence in time of solutions.

References

[1] Y. Ebihara, On some nonlinear evolution equations with the strong dissipation, II, J. Differential Equations, 34, 339-352, 1979.

[2] A. Kubo, H. Hoshino and K. Kimura, Global existence and asymptotic behaviour of solutions for nonlinear evolution equations related to tumour invasion, AIMS Proceedings 2015, 733-744, 2015.

Solvability of the Complex Ginzburg-Landau Equation

Takanori Kuroda
Waseda University, Japan
Mitsuharu Ôtani

We are concerned with the existence of a solution to the initial value problem for the complex Ginzburg-Landau equation with the Dirichlet boundary value condition of the form:

$$\begin{cases} u_t - (\lambda + i\alpha)\Delta u + (\kappa + i\beta)|u|^{q-2}u - \gamma u = f \\ \qquad\qquad\qquad\qquad (t,x)\in[0,T]\times\Omega, \\ u(t,x) = 0 \quad (t,x)\in[0,T]\times\partial\Omega, \\ u(0,x) = u_0(x) \quad x\in\Omega. \end{cases}$$

Here$\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, $T > 0$, u is an unknown complex-valued function and u_0 is a given complex-valued function. Given parameters are $\lambda > 0$, $\kappa, \alpha, \beta, \gamma \in \mathbb{R}$, $2 < q$ and f is an external force. In this talk, we mainly discuss the case where $\kappa < 0$.

Asymptotic Behavior of Some Gradient-Like Systems

Yoji Kurosaki
Waseda University, Japan
Mitsuharu Ôtani

We consider the following second order gradient-like system:

$$\ddot{u}(t) + g(\dot{u}(t)) + \nabla F(u(t)) = f(t), \tag{1}$$

where $F : \mathbb{R}^N \to \mathbb{R}$ is analytic, $g : \mathbb{R}^N \to \mathbb{R}^N$ is locally Lipschitz and coercive with $g(0) = 0$ and $f : \mathbb{R}^+ \to \mathbb{R}$ quickly decays to zero as t tends to infinity. We give a sufficient condition to assure that any bounded global solution u of (1) converge to an equilibrium point as t tends to infinity. In this proof we use the Łojasiewicz inequality.

A Priori Decay Estimates for Traveling Wave Solutions to a Nonlocal Dispersive Model for Shallow Water

Long Pei
Norwegian University of Science and Technology, Norway
Gabriele Bruell and Mats Ehrnstrom

For the Whitham equation, as a model for shallow water, we prove that if its traveling wave solutions with supercritical wave speed are continuous on R and tend to 0 at infinity, then these solutions will decay exponentially fast at infinity. Based on the a priori decay estimates and the method of moving planes, we can further prove that those traveling wave solutions are symmetric solutions with only one crest over the real line.

Global Attractors and Weak Exponential Attractors for Strongly Damped Wave Equations with Nonlinear Hyperbolic Dynamic Boundary Conditions

Joseph Shomberg
Providence College, USA
P. Jameson Graber

We discuss the well-posedness and asymptotic behavior of a strongly damped semilinear wave equation equipped with nonlinear hyperbolic dynamic boundary conditions. Well-posedness is due to some recent semigroup results, which here were carried out in the presence of a parameter distinguishing whether the underlying operator is analytic, $\alpha > 0$, or only of Gevrey class, $\alpha = 0$. For $\alpha \in [0, 1]$, we establish the existence of a global attractor; the family of global attractors is upper-semicontinuous as $\alpha \to 0^+$. We also prove the existence of a weak exponential attractor (a finite dimensional compact attracting set in the weak topology of the phase space). This result ensures the corresponding global attractor also possesses finite fractal dimension in the weak topology; moreover, the dimension is independent of the perturbation parameter α. In both settings, attractors are found under minimal assumptions on the nonlinear terms.

Stepanov-Like Weighted Asymptotic Behavior of Solutions to Some Stochastic Differential Equations in Hilbert spaces

Chao Tang
Sichuan University, Peoples Rep of China
Y. K. Chang

we first introduce the notation and properties of S^2-weighted pseudo almost automorphy for stochastic processes. And then, we apply the results obtained to consider the existence and uniqueness of S^2-weighted pseudo almost automorphic solutions to some stochastic differential equations in a real separable Hilbert space under global Lipschitz conditions. Moreover, we also investigate asymptotic behavior of solutions to a stochastic differential equation under S^2-weighted pseudo almost automorphic coefficients without global Lipschitz conditions. Our main results extend some known ones in the sense of square-mean weighted pseudo almost automorphy or S^2- pseudo almost automorphy for stochastic processes.

Time Periodic Problem of a System Describing Double-Diffusive Convection Phenomena in the Whole Space

Shun Uchida
Waseda University, Japan
Mitsuharu Ôtani

We consider the time periodic problem of the following equations which describe double-diffusive convection phenomena in some porous medium.

$$\begin{cases} \partial_t \vec{u} = \nu\Delta\vec{u} - a\vec{u} - \nabla p + \vec{g}T + \vec{h}C + \vec{f_1} \\ \partial_t T + \vec{u}\cdot\nabla T = \Delta T + f_2 \\ \partial_t C + \vec{u}\cdot\nabla C = \Delta C + \rho\Delta T + f_3 \\ \nabla\cdot\vec{u} = 0 \end{cases} \quad \text{in } \mathbb{R}^N \times [0, S],$$

where $N = 3, 4$ and $S > 0$ denotes the time period. Unknown functions are $\vec{u}$, T, C and p, which represent the fluid velocity, the temperature, the concentration of solute and the pressure respectively.
As for given data, a, ν, ρ are positive constants and $\vec{g}$, $\vec{h}$ are constant vectors. Moreover, $\vec{f_1}$, f_2, f_3 designate given external forces.
The main purpose of this talk is to construct a periodic solution of the system without the smallness condition of given data $\vec{f_1}$, f_2, f_3 via the convergence of solutions for some approximate equations in bounded domains.

Uniqueness of Limit Flow for a Class of Quasi-Linear Parabolic Equations

Tatsuya Watanabe
Kyoto Sangyo University, Japan
Marco Squassina

In this talk, the issue of uniqueness of the limit flow for a class of quasi-linear parabolic equations

$$iu_t + \Delta u + u\Delta u^2 - u + |u|^{p-1}u = 0 \quad \text{in } \mathbb{R}^N \times (0, \infty)$$

is discussed. We shall investigate conditions which guarantee that the global solutions decay at infinity uniformly in time and their entire trajectory approaches a single steady state as time goes to infinity. We also consider a characterization of solutions which blow-up, vanish or converge to a stationary state for initial data of the form $\lambda\varphi_0$.

Special Session 15: Special Session on Monotone Dynamical Systems and Applications

Hal L. Smith, Arizona State University, USA
Janusz Mierczynski, Wroclaw University of Technology, Poland

The theory of monotone dynamical systems grew out of the much earlier and well-developed monotone methods and comparison theory largely through the work of M.W. Hirsch and H. Matano in the 1980s. Essentially, the theory focuses on the implications of order-preservation of the (semi-) flow map on the asymptotic behavior of solutions. Typically, the asymptotic behavior of order-preserving dynamical systems is much simpler than for generic systems. Applications of the theory to systems of ordinary differential equations, delay differential equations, parabolic partial differential equations, and to the discrete-time dynamics generated by monotone maps and systems of difference equations have grown rapidly. Mathematical models in biology and chemical reaction dynamics are a rich source of applications because state variables are often intrinsically nonnegative and therefore the requirement that the dynamics be order-preserving is less restrictive. Recently, the theory has been extended to a control theory setting that includes system inputs and outputs and to random dynamical systems as well to stochastic games. New applications have arisen by identifying novel order relations preserved by special classes of dynamical systems. This special session will focus on these recent developments in both theory and applications.

Carrying Simplicies of Continuous and Discrete-Time Systems

Stephen Baigent
UCLL, England

I will describe the invariant carrying simplex of competitive ecological systems and how it may be used to study the stability of fixed points. Both continuous- and discrete-time models will be discussed. I will show how, in some cases, the carrying simplex can be shown to be the graph of a convex or concave function. Lastly, I will show that an analogue of the carrying simplex exists for ecological models that are not necessarily monotone.

Persistence of Aquatic Insect Populations Subject to Flooding

Patrick de Leenheer
Oregon State University, USA

We consider a logistically growing aquatic insect population that is subject to flooding events which follow a Poisson process. Each flooding event has two opposite effects. On the one hand, the insect population is decreased instantaneously due to washout; this is modeled by multiplying the current population level by a random factor taking values in $[0,1]$. On the other hand, the flood also increases the carrying capacity -also modeled by a random factor, but this one taking values in $[1,\infty)$. Following this increase, the carrying capacity relaxes back to a baseline. This model is motivated by the fact that right after each flood, the habitat available for species re-growth has increased. But with time, the flood plains dry out until a new flooding event occurs. The central question addressed here is how various ecological parameters, in conjunction with the random disturbance characteristics, affect the persistence of the aquatic species.

Bistable Traveling Waves for Monotone Semiflows

Jian Fang
Harbin Institute of Technology, Peoples Rep of China
Xiao-Qiang Zhao

In this talk, I will first recall some classical results on the existence of traveling waves connecting two stable steady states for typical reaction-diffusion equations and their analogues. Such an existence result is then established for a class of bistable evolution systems in homogeneous/periodic environment from a monotone dynamical system point of view. Finally, the obtained results are illustrated with concrete models that may arise from population ecology.

Open Differential Positive Systems: Attractors and Interconnection

Fulvio Forni
University of Cambridge, England

The novel notion of differential positivity extends linear positivity to the nonlinear setting: a nonlinear system is differentially positive if its linearization along trajectories leaves a cone (field) invariant. A monotone system is differentially positive with respect to a constant cone. Differential positivity extends monotonicity to manifolds, replacing order relations with local orders. The property strongly restricts the asymptotic nonlinear behavior, through suitable extensions of Perron-Frobenius theory and of results on contraction of the Hilbert metric. We illustrate these technical points by revisiting classical properties of monotone systems and by providing novel results for systems on manifolds. We also discuss the notion of differential positivity for open systems. We study the dependence of the system attractors on (locally) ordered inputs. We illustrate the notion of positivity preserving interconnections, a key step towards a robust interconnection theory for networks of differentially positive systems.

Group Actions on Monotone Skew-Product Semiflows

Mats Gyllenberg
University of Helsinki, Finland

We discuss a general framework of monotone skew-product semiflows under a connected group action. In a prior work, a compact connected group G-action has been considered on a strongly monotone skew-product semiflow. Here we relax the strong monotonicity and compactness requirements, and establish a theory concerning symmetry or monotonicity properties of uniformly stable 1-cover minimal sets. We then apply this theory to show rotational symmetry of certain stable entire solutions for a class of nonautonomous reaction-diffusion equations on R^n, as well as monotonicity of stable traveling waves of some nonlinear diffusion equations in timerecurrent structures including almost periodicity and almost automorphy.
The talk is based on joint work with Feng Cao and Yi Wang.

Monotone Semidynamical Systems with Dense Periodic Points

Morris Hirsch
University of Wisconsin, Madison, USA

Let $\mathbf{R}^n$ be ordered by a solid, pointed closed convex cone K. Let $\mathsf{F} := \{\mathsf{F}_t\}$ be an order-preserving semi-dynamical system in a connected open set $X \subset \mathbf{R}^n$, with either continuous time ($t \in \mathbf{R}_+$) or discrete time ($t \in \mathbf{N}$). Assume henceforth: Periodic points are dense in X. This implies F is nonchaotic in a strong sense:

Proposition Every point has a neighborhood in which F is uniformly equicontinuous.
But I think there is a much stronger result:

Conjecture F is globally periodic.
This has been verified in several settings:

Theorem The conjecture holds in the following cases: $n \leq 3$ and time is discrete, $n \leq 4$ and time is continuous, K is polyhedral.

In particular, the conjecture holds when K is an orthant. Since a globally periodic flow in a polyhedral cone is trivial, we have:

Corollary For a nontrivial cooperative or competitive vector field in the positive orthant, there is a nonempty open set containing no periodic point.

Competition for Two Essential Resources with Internal Storage and Periodic Input

Sze-Bi Hsu
National Tsing-Hua University, Taiwan
Feng-Bin Wang, Xiao-Qiang Zhao

We study a mathematical model of two species competing in a chemostat for two internally stored essential nutrients , where the nutrients are added to the culture vessel by way of periodic forcing functions. For the case of single species growth, we apply the theory of monotone dynamical systems to show that the population is washed out if a sub-threshold criterion holds, while there is a global stable positive periodic solution if a super-threshold criterion holds. For the case of competition of two species, we prove that when there is a mutual invasion of both semi-trivial periodic solution , both uniform persistence and existence of periodic coexistence state are established.

The Decomposition Formula for Stochastic Lotka-Volterra Systems with Identical Intrinsic Growth Rate and Its Applications to Stationary Motions

Jifa Jiang
Shanghai Normal University, Peoples Rep of China
Chen Lifeng, Dong Zhao, Zhai Jianliang

We exploit the long-run behavior for stochastic Lotka-Volterra systems (SLVS) with identical intrinsic growth rate both in pull-back trajectory and in stationary motion, which is motivated by convection in a rotating layer. It is first proved so-called the *decomposition formula* : every solution process for SLVS is expressed in terms of a solution for the deterministic Lotka-Volterra system (DLVS) without noise perturbation multiplied by an appropriate solution process of the scalar logistic equation with the same type noise perturbation. By virtue of this formula, it is verified that every pull-back omega-limit set is an omega-limit set for DLVS multiplied by the random equilibrium of the scalar stochastic logistic equation. We still investigate the weak convergence for the transition probability function of solution process as the time tends to infinity. It is shown that a bounded orbit for the DLVS deduces the existence of stationary measure for SLVS supported in a cone decided by the closure of this orbit. This makes us to construct many examples to possess a continuum of ergodic stationary measures or multiple isolated ergodic stationary measures. Suppose that the DLVS is dissipative. Then we prove that the set of stationary measures with small noise intensity is tight, and that their limiting measures in weak topology are invariant with respect to the DLVS as the noise intensity tends to zero, whose supports are contained in the Birkhoff center of the DLVS. Finally we provide the complete classification for three dimensional competitive SLVS with identical intrinsic growth rate in both stationary motions and trajectories. Every omega limit set for DLVS produces an ergodic stationary measure for SLVS, which makes us to get all ergodic stationary measures and furthermore all stationary measures. Precisely, there are exactly 37 dynamic scenrios in terms of competitive coefficients. Among them, each pull-back trajectory in 34 classes is asymptotically stationary, but possibly different stationary solution for different trajectory in same class. In each of these 34 classes, every equilibrium for DLVS produces an ergodic stationary measure, all stationary measures are obtained by all convex combination of all such ergodic stationary measures. Two of the remain classes possess stochastic closed orbits, and a continuum of ergodic stationary measures supported in cones decided by periodic orbits for DLVS respectively, which weakly converge to the Haar measures of periodic orbits as the noise intensity tends to zero. In the final class, almost every pull-back trajectory cyclically oscillates around the boundary of the stochastic carrying simplex which is characterized by three unstable stationary solutions, the realized trajectories produce a family of stationary measures which weakly converge to an invariant measure supported in the three unstable axial equilibria. This rigorously proves that a stochastic version for so called statistical limit cycle exists and that the turbulence in a fluid layer heated from below and rotating about a vertical axis is robust under stochastic disturbances.

A Remark on Global Dynamics of Competition Systems in Ordered Banach Spaces

King-Yeung Lam
The Ohio State University, USA
Daniel Munther

A well-known result in [Hsu-Smith-Waltman, Trans. AMS (1996)] states that in a competitive semiflow defined on $X_+ = X_{1,+} \times X_2,+$, the product of two cones in respective Banach spaces, if $(u_*, 0)$ and $(0, v_*)$ are the global attractors in $X_{1,+} \times \{0\}$ and $\{0\} \times X_{2,+}$ respectively, then one of the following three outcomes is possible for the two competitors: either there is at least one coexistence steady state, or one of $(u_*, 0)$, $(0, v_*)$ attracts all trajectories initiating in the order interval $I = [0, u_*] \times [0, v_*]$. However, it was demonstrated by an example that in some cases neither $(u_*, 0)$ nor $(0, v_*)$ is globally asymptotically stable if we broaden our scope to all of X_+. In this paper, we give two sufficient conditions that guarantee, in the absence of coexistence steady states, the global asymptotic stability of one of $(u_*, 0)$ or $(0, v_*)$ among all trajectories in X_+. Namely, one of $(u_*, 0)$ or $(0, v_*)$ is (i) linearly unstable, or (ii) is linearly neutrally stable but zero is a simple eigenvalue. Our results complement the counter-example mentioned in the above paper as well as applications that arise in practice.

Mean Field Dynamics in Social Networks

Chjan Lim
Rensselaer Polytechnic Institute, USA
Weituo Zhang

This talk will present several interesting dynamics and bifurcations that arise in social networks. Mean field models are often in the form of simple dynamical systems with well defined center manifolds. A large subclass have monotonic properties which have implications for social consensus.

Dispersal in Advective Environments

Yuan Lou
Renmin University/Ohio State, Peoples Rep of China
King-Yeung Lam, Frithjof Lutscher, Peng Zhou

We consider some mathematical models in advective environments, where individuals are exposed to unidirectional flow, with the possibility of being lost through the boundary. We study the persistence and range for a single species. We also consider the evolution of dispersal in such advective environments. Our analysis suggests that, in contrast to the case of no advection, slow dispersal is generally selected against in advective environments, and fast or intermediate dispersal rate will be favored.

Criteria for the Existence of Principal Eigenvalue of Time Periodic Cooperative Linear Systems with Nonlocal Dispersal

Wenxian Shen
Auburn University, USA
Xiongxiong Bao

This talk is concerned with the criteria for the existence of principal eigenvalues of time periodic cooperative linear nonlocal dispersal systems with Dirichlet type, Neumann type or periodic type boundary conditions. It first introduces the definition of principal eigenvalues of such cooperative systems. Next, it shows that a time periodic cooperative linear nonlocal dispersal system has a principal eigenvalue in the following cases: the nonlocal dispersal distance is sufficiently small; the spatial inhomogeneity satisfies the so called vanishing condition; or the spatial inhomogeneity is nearly globally homogeneous. It should be pointed out that a cooperative linear nonlocal dispersal system may not have a principal eigenvalue. Finally, it discusses some applications of the established principal eigenvalue theory.

On Heteroclinic Cycles of Competitive Maps Via Carrying Simplices

Yi Wang
University of Sci&Tech of China, Peoples Rep of China
Jifa Jiang, Lei Niu

In this talk, we concentrate on the effects of heteroclinic cycles and the interplay of heteroclinic attractors or repellers on the boundary of the carrying simplices for low-dimensional discrete-time competitive systems. We first present an alternative theorem on the existence of the carrying simplex for the competitive mapping. Several concrete discrete-time competition models are further analyzed, which do admit heteroclinic cycles. The criteria on the stability of the heteroclinic cycle for each model are also given.

Monotone Semiflows with Respect to High-Rank Cones on a Banach Space

Jianhong Wu
York, Canada
Lirui Feng, Yi Wang

We consider semiflows in general Banach spaces motivated by monotone cyclic feedback systems or differential equations with integer-valued Lyapunov functionals. These semiflows enjoy strong monotonicity properties with respect to cones of high ranks, which imply order-related structures on the ω-limit sets of precompact semi-orbits. We show that for a pseudo-ordered precompact semi-orbit the ω-limit set Ω is either ordered, or is contained in the set of equilibria, or possesses a certain ordered homoclinic property. In particular, we show that if Ω contains no equilibrium, then Ω itself is ordered and hence the dynamics of the semiflow on Ω is topologically conjugate to a compact flow on $\mathbb{R}^k$ with k being the rank. We also establish a Poincarè-Bendixson type Theorem in the case where $k = 2$. All our results are established without the smoothness condition on the semiflow, allowing applications to such cellular or physiological feedback systems with piecewise linear vector fields and to such infinite dimensional systems where the C^1-Closing Lemma or smooth manifold theory has not been developed.

Asymptotic Behaviour, Spreading Speeds and Travelling Waves of Some Dynamical Systems

Xingfu Zou
University of Western Ontario, Canada
Taishan Yi

I will report some recent results on asymptotic behavior, spreading speeds and existence/non-existence of travelling waves of some dynamical systems in form of discrete-time dynamical system. As a byproduct, we obtain some results on the global attractivityof nontrivial constant fixed point and on the existence of non-constant fixed point. We then apply the main results to three model systems: (i) a spatially nonlocal integro-difference equation; (ii) a reaction-diffusion equation with spatial nonlocality and time delay in the reaction term; (iii) an equation with nonlocal diffusion and delayed non-monotone nonlinearity in the reaction term. The obtained results for these three equations improve some existing ones by removing the symmetry of the kernel functions and relaxing the conditions on the nonlinear terms.

Special Session 16: Dissipative Systems and Applications

Georg Hetzer, Auburn University, USA
Wenxian Shen, Auburn University, USA
Lourdes Tello Del Castillo, Universidad Politecnica de Madrid, Spain

Dissipative systems arise in many applications. The special session will feature talks from infinite dynamical systems theory and random dynamical systems to evolutionary partial differential equations and numerical simulation. The scope of applications covers reaction-diffusion systems with local and nonlocal dispersal, ecology, and climate modeling.

Analyticity in Time and Space for a System of Semilinear Evolution Equations with Variable Coefficients

Falko Baustian
University of Rostock, Germany

We consider the classical Cauchy problem for a strongly parabolic system of M semilinear partial differential equations of order $2m$ with analytic coefficients

$$\begin{cases} \frac{\partial \mathbf{u}}{\partial t} + \mathbf{P}\left(x,t,\frac{1}{i}\frac{\partial}{\partial x}\right) & = \mathbf{f}^{(0)}\left(x,t;\left(\frac{\partial^{|\beta|}\mathbf{u}}{tial x^{\beta}}\right)_{|\beta|\le m}\right) + \\ & + \begin{cases} 0 & \text{if } m=1; \\ \sum_{j=1}^{N} \frac{\partial}{\partial x_j}\mathbf{f}^{(j)}\left(x,t;\left(\frac{\partial^{|\beta|}\mathbf{u}}{\partial x^{\beta}}\right)_{|\beta|\le m}\right) & \text{if } m\ge 2; \end{cases} \\ & \text{for } (x,t)\in\mathbb{R}^N\times(0,T); \\ \mathbf{u}(x,0) & = \mathbf{u}_0(x) \text{ for } x\in\mathbb{R}^N, \end{cases}$$

but with the initial data $\mathbf{u}_0$ only in the real interpolation space $\mathbf{B}^{s;p,p}(\mathbb{R}^N)$.
We show analyticity in the space and time variables of the unique strict solutions $\mathbf{u}$. For the analyticity in time we investigate an abstract nonlinear Cauchy problem and operators with the maximal L^p-regularity property. For the analyticity in space we approximate the initial value with analytic functions and use suitable estimates of the corresponding solutions on a complex domain.
The results are generalisation of the linear case in P. Takac: Space-Time analyticity of weak solutions to linear parabolic systems with variable coefficients, Journal of Functional Analysis 236, 50–88, 2012.

Positive Solutions in Logistic Problems with Nonlinear Mixed Boundary Conditions a Spatial Heterogeneities

Santiago Cano-Casanova
Comillas Pontifical University, Spain

In this talk will be analyzed the existence, uniqueness and stability of the positive solutions of a very general class of logistic problems containing spatial heterogeneities in the PDE and on the boundary conditions and with a nonlinear flux on the boundary, with positive, negative or changing sign along it. The results will be given in terms of the nodal behaviour and sign of the potencials appearing in the PDE and on the boundary conditions. The boundary conditions considered are Dirichlet on a component of the boundary and nonlinear Robin or Neumann boundary conditions on the other. The main techniques used to achieve the results are bifurcation, continuation, monotonicity and blow up methods.

Random Versus Stochastic Lattices Dynamical Systems

Tomas Caraballo
Universidad de Sevilla, Spain

The objective of this talk is to report on recent advances in the topic of random dynamical systems generated by stochastic lattice differential systems. We will focus on problems containing additive and multiplicative noise and will emphasize the differences when considering a finite number of noisy terms at each node (essentially the same noise in each node) or a different noisy perturbation at each one. We will show how these systems generate a random dynamical system possessing a random attractor.

Controlled Explosions: Dynamics After Blow-Up Time for Semilinear Problems with a Dynamic Boundary Condition

Alfonso Casal
Technical University of Madrid, Spain
Jesus Ildefonso Diaz, Gregorio Diaz, Jose Manuel Vegas

It is well know that in many nonlinear dynamical problems the maximal existence time T_{maximal} of solutions is defined trough the blow-up time of some of the norm of the solution as, e.g., $\|u(T_\infty)\|_{L^\infty} = +\infty$. Nevertheless, in the case of some ordinary differential equations it is possible to control the blow-up in such a way that the solution of the controlled equation let well defined after the blow-up time T_∞ (see, A.C. Casal, J.I. Dìaz, J.M. Vegas, Complete recuperation after the blow up time for semilinear problems.Discrete and Continuous Dynamical Systems. Dynamical Systems, Differential Equations and Applications, AIMS Proceedings, doi:10.3934/proc.2015.0223 2015 pp. 223-229).

The main goal of this work is to extend such control process to some semilinear boundary value problem of the type

$$\begin{cases} -\Delta y(r,t) + |y(r,t)|^{m-1}\, y(r,t) = 0 & \text{in } B_R(0) \times (0, T_{\text{maximal}}\ ), \\ \frac{\partial y}{\partial t}(R,t) + \frac{\partial y}{\partial n}(R,t) = |y(r,t)|^{p-1}\, y(r,t) + u(t) & \text{on } \partial B_R(0) \times (0, T_{\text{maximal}}\ ), \end{cases}$$

where $m, p > 0$, $B_R(0)$ is the ball of $\mathbb{R}^N$ of radius R and n is the normal unit exterior vector. In a first part we assume that no control is applied $(u(t) \equiv 0)$ and show that if $p > (m+1)/2 > 1$ then T_∞

Spatial Population Models with Fitness Based Dispersal

Chris Cosner
University of Miami, USA

Traditional continuous time models in spatial ecology typically describe movement in terms of linear diffusion and advection, which combine with nonlinear population dynamics to produce semilinear equations and systems. However, if organisms are assumed to move up gradients of their reproductive fitness, and fitness is density dependent (for example logistic), the resulting models are quasilinear and may have other novel features. This talk will describe some models involving fitness dependent dispersal and some results and challenges in the analysis of such models.

Global and Nonglobal Existence of Solutions of Source Types of Degenerate Parabolic Equations with a Singular Absorption: complete quenching phenomenon.

Anh Dao Nguyen
Ton Duc Thang University, Vietnam
Jesus Ildefonso Diaz

We consider nonnegative solutions of source type of degenerate parabolic equations with a singular absorption term:

$$\partial_t u - (|u_x|^{p-2} u_x)_x + u^{-\beta} \chi_{\{u>0\}} = f(u,x,t), \text{ in } I \times (0,T),$$

with the homogeneous zero boundary condition, in that $p > 2$, $\beta \in (0,1)$, and $I = (x_1, x_2)$ is an open interval in $\mathbb{R}$. To show the existence result, we prove a pointwise estimate for $|u_x|$, the so called gradient estimate in N-dimension. We also consider the global and non-global existence of solutions of the above equation. Moreover, we prove that any solution must vanish identically after a finite time if either the initial data, or the source term is small enough.

Inverse Problems for Parabolic Equations Arising in Thermochronology

Dmitry Glotov
Auburn University, USA
Willis E. Hames, A. J. Meir, Sedar Ngoma

The reconstruction of the temperature history of minerals allows geologists to better understand various processes within Earth's crust. One dating method is based on the decay of radiogenic potassium to stable argon. The concentration of argon, which is subject to thermally activated diffusion, is governed by a parabolic equation with a time-dependent diffusion coefficient. We study the inverse coefficient and related inverse source problems for this equation. To ensure uniqueness we introduce an integral constraint motivated by the type of data reported in geological literature. For the constrained problem, we establish existence and uniqueness. We propose a scheme for solving the equations numerically and report on the observed rates of convergence.

Stability Analysis for Positive Solutions for Classes of Semilinear Elliptic Boundary Value Problems with Nonlinear Boundary conditions

Jerome Goddard II
Auburn University Montgomery, USA
R. Shivaji

In this talk, we will investigate the stability properties of nontrivial positive steady state solutions of semilinear initial-boundary value problems with nonlinear boundary conditions. In particular, we will employ a Principle of Linearized Stability for this class of problems to prove sufficient conditions for stability and instability of positive steady state solutions. These results shed some light on the combined effects of the reaction term and the boundary nonlinearity on stability properties. If time permits, we will also discuss existence results and provide complete bifurcation curves in the case of dimension one.

On the Well Posedness of a Dissipative Nonlinear System for Li-Ion Batteries

David Gomez-Castro
Universidad Complutense de Madrid, Spain
J.I. Díaz, A.M. Ramos

We study a dissipative model developed in the framework of Li-ion batteries, which is a leading candidate for the generation of automomotive and aerospace applications. After presenting some details on the model, we prove the existence and uniqueness of its solutions. We consider a pseudo-two dimensional model coupled to a lumped thermal model, that follows the macrohomogeneous approach presented by Newman in 1973. So far, only some numerical studies

of special cases of the nonlinear system or some analysis of the linearized system have been made. Here we prove, for the first time in the literature, the existence and uniqueness of solutions of the complete nonlinear model. The model can be considered as strongly nonlinear since it consists of several coupled nonlinear elliptic and degenerate parabolic equations in different spatial dimensions and also involves some nonlinear boundary conditions of Robin type.

Numerical Approximation of Regions of Attraction of Equilibrium Solutions in a Climate EBM

Arturo Hidalgo
Universidad Politecnica de Madrid, Spain

We study an energy balance model arising in Climatology. The model is sensible to the fluctuations of the Solar constant. The number of equilibrium states depends on the Solar constant and the shape of the planetary coalbedo (one of the nonlinearity of the model). In this work, we study the regions of attraction of the stationary states by numerical approximation based upon the finite volume method with Weighted Essentially Non-Oscillatory (WENO) reconstruction. This results are in collaboration with L. Tello (UPM).

On Limiting Behavior of Stationary Measures for Stochastic Evolution Systems with Small Noise Intensity

Jifa Jiang
Shanghai Normal University, Peoples Rep of China
Chen Lifeng, Dong Zhao, Zhai Jianliang

This talk presents the limit behavior of a family of stationary measures for various stochastic evolution systems, which include stochastic ordinary differential equations, stochastic functional differential equations and stochastic partial differential equations driven by either Brown motion or Lévy process with small noise intensity as well as stochastic approximation with constant step. Under mild regular conditions, it is proved that as noise intensity tends to zero, any limit measure of a tight family for stationary measures in sense of weak convergence is an invariant measure of the corresponding systems without noise, whose support is contained in the Birkhoff center. This result plays a role both in understanding concentrating location for stationary measures with small noise and in stability for deterministic systems undergoing noise perturbation. The proof is uniform, and is divided into three steps: probability convergence, invariance for limiting measure and the support for limiting measure. Several examples are constructed to illustrate peculiar property: the supports for limiting measures are saddles, however, almost every trajectory for corresponding deterministic systems without noise is asymptotic to cycles connecting saddles.

Attractors for the General Shigesada-Kawasaki-Teramoto Models on Planar Domains

Dung Le
University of Texas at San Antonio, USA

The existence of global and exponential atrractors is established for generalized Shigesada-Kawasaki-Teramoto models on planar domains. The cross diffusion and reaction can have polynomial growth of any order. If time permits we will discuss the case when the self diffusion is large and show that the dynamics of the cross diffusion system will be described by that of the corresponding ODE system.

A Small-Gain Theorem for Nonlinear Stochastic Systems with Inputs and Outputs

Xiang Lv
Shanghai Normal University, Peoples Rep of China
Jifa Jiang

This paper studies a small gain theorem for nonlinear stochastic equations driven by additive white noise in both trajectories and stationary distribution. Motivated by the most recent work of Freitas and Sontag, we firstly define the "input-to-state characteristic operator" $K(u)$ of the system in a suitably chosen input space via backward Itô integral, and then for a given output function h, define the "gain operator" as the composition of output function h and the input-to-state characteristic operator $K(u)$ on the input space. Suppose that the output function is either order-preserving or anti-order-preserving in the usual vector order and the global Lipschitz constant of the output function is less than the absolute of the negative principal eigenvalue of linear matrix. Then we prove the so-called "small gain theorem": the gain operator has a unique fixed point, the image for input-to-state characteristic operator at the fixed point is a globally attracting stochastic equilibrium for the random dynamical system generated by the stochastic system. Under the same assumption for the relation between the Lipschitz constant of the output function and maximal real part of stable linear matrix, we prove that the stochastic system has a unique stationary distribution, which is regarded as a stationary distribution version of small gain theorem. These results can be applied to stochastic cooperative, competitive and predator-prey systems, or even others.

A Robust and Efficient Adaptive Multigrid Solver for the Optimal Control of Phase Field Formulations of Geometric Evolution laws

Anotida Madzvamuse
University of Sussex, England
F. Yang, C. Venkataraman, V. Styles

In this talk, I will present a novel solution strategy to efficiently and accurately compute approximate solutions to semilinear optimal control problems, focusing on the optimal control of phase field formulations of geometric evolution laws. The optimal control of geometric evolution laws arises in a number of applications in fields including material science, image processing, tumour growth and cell motility. Despite this, many open problems remain in the analysis and approximation of such problems.
In the current work we focus on a phase field formulation of the optimal control problem, hence exploiting the well developed mathematical theory for the optimal control of semilinear parabolic partial differential equations. Approximation of the resulting optimal control problem is computationally challenging, requiring massive amounts of computational time and memory storage. The main focus of this work is to propose, derive, implement and test an efficient solution method for such problems.
The solver for the discretised partial differential equations is based upon a geometric multigrid method incorporating advanced techniques to deal with the nonlinearities in the problem and utilising adaptive mesh refinement. An in-house two-grid solution strategy for the forward and adjoint problems, that significantly reduces memory requirements and CPU time, is proposed and investigated computationally. Furthermore, parallelisation as well as an adaptive-step gradient update for the control are employed to further improve efficiency. Along with a detailed description of our proposed solution method together with its implementation we present a number of computational results that demonstrate and evaluate our algorithms with respect to accuracy and efficiency. A highlight of the present work is simulation results on the optimal control of phase field formulations of geometric evolution laws in 3-D which would be computationally in-feasible without the solution strategies proposed in the present work.

On Some Nonlinear Problems in Poroelasticity

Amnon Meir
Southern Methodist University, USA

Poromechanics is the science of energy, motion, and forces, and their effect on porous material and in particular the swelling and shrinking of fluid-saturated porous media. Modeling and predicting the mechanical behavior of fluid-infiltrated porous media is significant since many natural substances, for example, rocks, soils, clays, shales, biological tissues, and bones, as well as man-made materials, such as, foams, gels, concrete, water-solute drug carriers, and ceramics are all elastic porous materials (hence poroelastic). In this talk I will describe some nonlinear problems in poroelasticity and their mathematical analysis. I will also describe finite element based numerical methods for approximating solutions of (nonlinear) model problems in poroelasticity, and the available a-priori error estimates.

On Doubly Nonlinear Evolution Equations with Nonpotential Or Dynamic Relation Between the State Variables

Jochen Merker
HTWK Leipzig University of Applied Sciences, Germany
Ales Matas

This talk is about doubly nonlinear evolution equations of the form $\frac{d}{dt}Bu + Au = f$, where A, B are nonlinear operators and B does not admit a potential. A particular case are systems of doubly nonlinear reaction-diffusion equations

$$\frac{\partial v}{\partial t} - \operatorname{div}\left(a(\nabla u)\right) = f\,,$$

where u is vector-valued and the operator $Au = -\operatorname{div}\left(a(\nabla u)\right)$ may be degenerate or singular. After a review of non-potential static relations $v = b(u)$ the case of an additional dynamic equations for u determining the relation between u and v is discussed, like e.g. $\frac{\partial u}{\partial t} - \Delta u = \frac{1}{\epsilon}(v - b(u))$ with a relaxation time $\epsilon > 0$.

On the Uniqueness of Weak Solutions for a Traffic Flow Model in Two Lanes in One Direction

Juan Francisco Padial
Universidad Politecnica de Madrid, Spain

We study a the traffic flow on long highways with tow lines in same direction. Assuming that the drivers continuously adjust their speed towards what they consider to be ideal value under the local traffic conditions; we propose a new mathematical model in terms of a class of nonlinear parabolic systems with zero–order coupling. Let $\rho_1\,(t,x)$ be the density in the slower lane, $\rho_2\,(t,x)$ be the density in the faster lane at any time t and position x. If we assume that it is possible for the cars to change lanes, we have the following doubly nonlinear degenerate parabolic system

$$\begin{cases} \frac{\partial}{\partial t}\rho_1 - \frac{\partial}{\partial x}\left(\kappa_1 \frac{\partial}{\partial x}\rho_1 - V_{1*}(\rho_1)\right) = F(t,x,\rho_1,\rho_2), \\ \frac{\partial}{\partial t}\rho_2 - \frac{\partial}{\partial x}\left(\kappa_2 \frac{\partial}{\partial x}\rho_2 - V_{2*}(\rho_2)\right) = -F(t,x,\rho_1,\rho_2), \end{cases}$$

where κ_i is a positive quantity which tells how much the change in the number of cars is considered by the driver and $V_{i*}(\rho_i) = \rho_i W_{i*}(\rho_i)$ with W_{i*} be the corresponding flow speeds for $i = 1, 2$. The net effect of time changes in density and space changes in flow at a point x and a time t are equal to the exchange between the two lanes and it is described by function F (see, e. g., GAZIS, D.C., *Traffic Science.* John Wiley & Son, New York 1974; HABERMAN R., *Mathematical Models: Mechanical Vibrations, Population Dynamics and Traffic Flow.* Prentice Hall, 1977). Several types of functions F can be chose: a model in which lane changes were proportional to the square of the density in the lane that was the source of departing vehicles and the first power of the emptiness (unused space) of the lane–admitting vehicles, can be expressed by $F(\rho_1, \rho_2) := -\alpha_1\rho_1^2[\gamma_2 - \rho_2] + \alpha_2\rho_2^2[\gamma_1 - \rho_1]$ where γ_i are the maximum allowable densities in the respective lanes.
We develop a method based in doubling of the time variable to prove the uniqueness of weak solution. This technique is inspired in a method introduced by S.N. Kruzhkov in order to prove a L^1–contraction property for entropy solutions of hyperbolic problems.
(*) Partially Supported by the Project MTM2014–57113 of DGISPI (Spain)

Quantitative Concentration of Stationary Measures

Zhongwei Shen
University of Alberta, Canada
Min Ji, Yingfei Yi

This talk concerns stationary measures of Fokker-Planck equations associated with a system of ordinary differential equations perturbed by "small" multiplicative white noises. I will present some quantitative results on the concentration of stationary measures as noises vanish. In particular, the stability or instability of local attractors and local repellers will be presented. This directly relates to the stability or instability of the ODE system under small white noise perturbations.

Global Extinction of Population with Allee Effect in Advective Environment

Junping Shi
College of William and Mary, USA

We show that in a spatial population model with strong Allee effect, the population becomes extinct no matter how large the initial population is, if the advection is strong enough. Hence the extinction equilibrium is globally asymptotically stable, which is quite different from the case of small or no advection that the dynamics is bistable. We will show results in both ODE patch model and reaction-diffusion-advection model.

On Compact Support Solutions to Parabolic Problems with the P-Laplacian

Peter Takac
University of Rostock, Germany
Jiri Benedikt, Petr Girg, Lukas Kotrla

The validity of the weak and strong comparison principles for degenerate parabolic equations with the p-Laplacian operator will be discussed for $p > 2$ and $1 < p < 2$.
This problem is reduced to the comparison of the trivial solution ($\equiv 0$) with a nonnegative solution $u(x, t)$. The problem is closely related also to the question of uniqueness of a nonnegative solution via the weak comparison principle. In this presentation, for $p > 2$ realistic counterexamples to the uniqueness of a nonnegative solution, the weak comparison principle, and the strong maximum principle are constructed with a nonsmooth reaction function that satisfies neither a Lipschitz nor an Osgood standard uniqueness condition. Nonnegative multi-bump solutions with spatially disconnected compact supports and zero initial data are constructed between sub- and super-solutions that have supports of the same type.

On a Green Roof Mathematical Model

Lourdes Tello
Universidad Politecnica de Madrid, Spain

We study a problem of parabolic type based on an energy balance for buildings with vegetation cover (green roof). The model represents the evolution of the temperature in two layers of a green roof: the vegetation layer and the soil layer. The model is based on an energy balance and it includes the interactions between these two layers. The model includes different shapes of green roof. The spatial domain is a surface and the unknowns are the temperature in a vegetal layer and the temperature in the substrate. We present a model including roof's shape and the mathematical treatment of it. Part of this work is in collaboration with M.L. Vilar (UPM).

On a Parabolic-ODE System with Chemotactic Terms

J. Ignacio Tello
Universidad Politecnica de Madrid, Spain

We consider a competitive system of differential equations describing the behavior of two biological species. The system is weakly coupled and one of the species has the capacity of diffuse and moves to-

ward the higher concentration of the second species following its gradient, the density function satisfies a second order parabolic equation with chemotactic terms. The second species does not have motility capacity and satisfies an ordinary differential equation. We prove that the solutions are uniformly bounded and exist globally in time. The asymptotic behavior of solutions is also studied for a range of parameters and initial data.

Dynamics of Almost-Periodic Forced Parabolic Equations on the Circle

Yi Wang
University of Sci&Tech of China, Peoples Rep of China
Wenxian Shen, Dun Zhou

We consider the skew-product semiflow which is generated by an almost-periodic forced scalar reaction-diffusion equation on the circle.The structure of the minimal set M is thoroughly investigated under the assumption that the center space associated with M is no more than 2-dimensional. Such situation naturally occurs while, for instance, M is hyperbolic or uniquely ergodic. It is shown in this paper that M is a 1-cover of the hull $H(f)$ provided that M is hyperbolic. If $\dim V^c(M) = 1$ (resp. $\dim V^c(M) = 2$ with $\dim V^u(M)$ being odd), then either M is an almost 1-cover of $H(f)$ and topologically conjugate to a minimal flow in $\mathbb{R} \times H(f)$; or M can be (resp. residually) embedded into an almost periodically (resp. almost automorphically) forced circle-flow $S^1 \times H(f)$.

Persistence Criteria for the Nonlocal Model and Applications

Xiaoxia Xie
Illinois Institute of Technology, USA
Wenxian Shen

Long range dispersal is a common phenomenon in biology and ecology. To have a better understanding of the evolution of biodiversity in some ecosystem, there is a need to understand the influence of nonlocal dispersal on the survival/persistence of a population.
In this talk, I will report on a recent study concerning persistence criteria in some nonlocal models on temporal and spatial heterogeneous environment.
I will first present some spectral theory of the associated eigenvalue problem, such as the existence of the principal eigenvalue. Secondly, I will show some results of the eigenvalue problem with indefinite weight functions, which have practical importance in the context of reserve design or pest control.

Transition Fronts of Fisher-KPP Equations in Locally Spatially Inhomogeneous Patchy Environments

Aijun Zhang
Drexel University, USA
Erik Van Vleck

The current paper is devoted to the study of spatial propagation dynamics of species in locally spatially inhomogeneous patchy environments or media. For a lattice differential equation with monostable nonlinearity in a discrete periodic media, it is well-known that there exists a minimal wave speed such that a traveling wave exists if and only if the wave speed is above this minimal wave speed. We shall show that strongly localized spatial inhomogeneous patchy environments may prevent the existence of transition fronts. Transition fronts may exist in weaklier localized spatial inhomogeneous patchy environments but only in a finite range of speeds, which implies that it is plausible to obtain a maximal wave speed of existence of transition fronts.

Special Session 17: Quantitative Geometric and Functional Inequalities and New Trends in Nonlinear PDEs

Alessio Figalli, UT Austin, USA
Emanuel Indrei, CMU, USA
Enrico Valdinoci, WIAS, Germany

The aim of this special session is two-fold: first, to discuss quantitative versions of various geometric and functional inequalities and their applications; second, to identify new trends in nonlinear partial differential equations, including the study of free boundary problems and geometric PDEs.

Systems of Partial Differential Equations Arising from Population Dynamics: Free Boundary Problems As a Result of segregation.

Veronica Rita Antunes de Soares Quitalo
Purdue University, USA
Luis Caffarelli, Stefania Patrizi, Monica Torres

In this talk we will present our latest results for two phase free boundary problems arising from population dynamics. We will focus on fully nonlinear systems with local interaction and linear systems with a (non local) long range interaction.

Mean Value Theorems for and Obstacle Problems on Riemannian Manifolds

Brian Benson
Kansas State University, USA
Ivan Blank, Jeremy LeCrone

In Euclidean space, mean value theorems for the Laplacian are used to prove many fundamental results in PDE such as maximum principles, Harnack inequalities, interior estimates, and semicontinuity of weakly superharmonic functions. In 1998, Caffarelli discussed a proof of the mean value theorem for the Laplacian which differs from the usual approach. Specifically, this different approach could be generalized to a divergence-form elliptic operator when a Green's function for the operator and a solution to a related obstacle problem are known. Generalizing results of many authors, a mean value theorem for the Laplacian on a strongly non-parabolic Riemannian manifold is given by Ni. Motivated by the work of Ni and the usefulness of mean value theorems, we will discuss adapting Caffarelli's approach to the setting of Riemanian manifolds to prove mean value theorems for elliptic operators.

Classes of Improved Moser-Trudinger Inequalities and Their Geometric Applications

Alessandro Carlotto
ETH, Switzerland

Moser-Trudinger inequalities concern, roughly speaking, the exponential integrability of functions belonging to suitable Sobolev spaces and naturally appear in a variety of geometric contexts, ranging from the conformal to the Kaehler setting. After a general introduction, I will describe some recent variations on these themes motivated by the curvature prescription problem in the category of Riemann surfaces with conical singularities. More specifically, I will briefly present an important class of improved, scaling-invariant inequalities which lie at the heart of the variational/Morse-theoretic approach of the author and A. Malchiodi to the singular uniformization problem.

Further Time Regularity for Parabolic Equations

Hector Chang-Lara
Columbia University, USA
Dennis Kriventsov

In contrast with second order parabolic equations, a particular phenomena of nonlocal models is the loss of time regularity. Even for the fractional heat equation a sudden change in time of the boundary data (over the complement of the domain) gets immediately noticed by the nonlocal term and usually creates a discontinuity for the time derivative of the solution. Together with D. Kriventsov, we recently established a priori Schauder estimates for nonlocal, fully nonlinear parabolic equations, addressing the regularity of the time derivative of the solution under mild assumption on the boundary data. Our result extend to new estimates for second order equations.

On a Fractional Monge-Ampere Operator

Fernando Charro
Universitat Politecnica de Catalunya, Spain
Luis Caffarelli

In this talk we consider a fractional analogue of the Monge-Ampere operator. Our operator is a concave envelope of fractional linear operators that are affine transformations of determinant one of a given multiple of the fractional Laplacian. We set up a relatively simple framework of global solutions prescribing data at infinity and global barriers. In our key estimate, we show that the operator remains strictly elliptic, which allows to apply known regularity results for uniformly elliptic operators and deduce that solutions are classical.

A-Free Rigidity and Applications to the Compressible Euler System

Elisabetta Chiodaroli
EPFL, Lausanne, Switzerland
E. Feireisl, O. Kreml, E. Wiedemann

Szèkelyhidi and Wiedemann showed that any measure-valued solution to the incompressible Euler equations in several space dimensions can be generated by a sequence of exact solutions. This means that measure-valued solutions and weak solutions are substantially the same for incompressible Euler, thus leading to a very large set of weak solutions. In this talk we address the corresponding problem for the compressible Euler system: can every measure-valued solution to the compressible Euler equations be approximated by a sequence of weak solutions? We show that the answer is negative: generalizing a well-known rigidity result of Ball and James, we give an explicit example of a measure-valued solution for the compressible Euler equations which can not be generated by a sequence of distributional solutions. We also give an abstract necessary condition in the form of a Jensen-type inequality for measure-valued solutions to be generated by weak solutions. The dichotomy between weak and measure-valued solutions in the compressible case is in contrast with the incompressible situation.

Deficit Estimates for the Gaussian Logarithmic Sobolev Inequality

Max Fathi
UC Berkeley, USA
Michel Ledoux, Emanuel Indrei

I will present a few results on estimating the deficit in the Gaussian logarithmic Sobolev inequality, as well as in Talagrand's transport-entropy inequality. These estimates have some applications in the study of long-time behavior of diffusion processes.

Global Well-Posedness of the High Dimensional Maxwell-Dirac Equation for Small Critical Data

Cristian Gavrus
UC Berkeley, USA
Sung-Jin Oh

We discuss the global well-posedness of the massless Maxwell-Dirac equation in Coulomb gauge on R^{1+d} ($d \geq 4$) for data with small scale-critical Sobolev norm, as well as modified scattering of the solutions. Main components of our proof are A) uncovering null structure of Maxwell-Dirac in the Coulomb gauge, and B) construction of a microlocal-parametrix for the underlying paradifferential magnetic Dirac equation. A key step for achieving both is to exploit and justify a deep analogy between Maxwell-Dirac and Maxwell-Klein-Gordon (for which an analogous result was proved earlier by Krieger-Sterbenz-Tataru). This is joint work with Sung-Jin Oh.

Approximation Schemes for Optimal Constants and Extremal Functions in Sobolev Inequalities

Ryan Hynd
University of Pennsylvania, USA

We employ discrete and continuous time flows to approximate optimal constants and functions for which equality holds in various inequalities such as Poincarè's and Morrey's inequality. The discrete time scheme is based on the inverse iteration method for square matrices and the continuous time flow is a particular type of doubly nonlinear evolution.

Convergence of Thresholding Schemes for Mean-Curvature Flow

Tim Laux
MPI MIS Leipzig, Germany
Felix Otto, Drew Swartz

The thresholding scheme, a time discretization for mean-curvature flow was introduced by Meriman, Bence and Osher in 1992. In the talk we present new convergence results for several variants of this scheme, in particular in the multi-phase case with arbitrary surface tensions. The proofs are based on the interpretation of the thresholding scheme as a minimizing movements scheme by Esedoglu and Otto in 2014. This interpretation means that the thresholding scheme preserves the structure of (multi-phase) mean-curvature flow as a gradient flow w. r. t. the total interfacial energy. More precisely, the thresholding scheme is a minimizing movements scheme for an energy that Γ-converges to the total interfacial energy. In this sense, our proof is similar to the convergence results of Almgren, Taylor and Wang in 1993 and Luckhaus and Sturzenhecker in 1995, which establish convergence of a more academic minimizing movements scheme. Like the one of Luckhaus and Sturzenhecker, ours is a conditional convergence result, which means that we have to assume that the time-integrated energy of the approximation converges to the time-integrated energy of the limit. This is a natural assumption, which is however not ensured by the compactness coming from the basic estimates.

The Shape of Capillarity Droplets in a Container

Mihalia Cornelia
University of Texas at Austin, USA
Francesco Maggi

The capillarity droplet problem concerns the minimization of the Gauss free energy of a set, where the Gauss free energy is defined as the sum of the surface and potential energies of that set. In this talk, we consider minimizers of the Gauss free energy for a liquid droplet bounded in a container with $C^{1,1}$ boundary. We discuss a quantitative description of the shape of global minimizers and their regularity properties in the small volume regime. This work was performed in collaboration with Francesco Maggi.

The Sharp Quantitative Euclidean Concentration Inequality

Connor Mooney
UT Austin, USA
Alessio Figalli, Francesco Maggi

A special case of the Brunn-Minkowski inequality states that, among sets with fixed volume in $\mathbb{R}^n$, balls have r-neighborhoods of smallest volume for each $r > 0$. We will discuss a sharp quantitative version of this result.

A Strong Form of the Quantitative Wulff Inequality

Robin Neumayer
University of Texas at Austin, USA

For a set E that almost minimizes perimeter among sets of the same volume, quantitative isoperimetric inequalities measure how close E is to the unique perimeter minimizer. A recent paper of Fusco and Julin gives a quantitative isoperimetric inequality where the deficit in the inequality controls the oscillation of the boundary of E. In this talk, we will generalize this result to the anisotropic case, where perimeter is weighted with respect to some fixed convex set K.

Differential Calculus in Wasserstein-Orlicz Spaces

Levon Nurbekyan
King Abdullah University of Science and Technology (KAUST), Saudi Arabia
Alessio Figalli

In this work, we study metric properties of the Wasserstein-Orlicz spaces of probability measures. We generalize results by L. Ambrosio, N. Gigli and G. Savarè on gradient flows in p-Wasserstein spaces to the case of Wasserstein-Orlicz spaces and apply these methods to study certain classes of parabolic PDEs.

A Min-Max Formula for Lipschitz Operators That Satisfy the Global Comparison Principle.

Russell Schwab
Michigan State University, USA
Nestor Guillen

We investigate Lipschitz maps, I, mapping $C^2(D) \to C(D)$, where D is an appropriate domain. The global comparison principle (GCP) simply states that whenever two functions are ordered in D and touch at a point, i.e. $u(x) \leq v(x)$ for all x and $u(z) = v(z)$ for some $z \in D$, then also the mapping I has the same order, i.e. $I(u,z) \leq I(v,z)$. It has been known since the 1960s, by Courrège, that if I is a linear mapping with the GCP, then I must be represented as a linear drift-jump-diffusion operator that may have both local and integro-differential parts. It has also long been known and utilized that when I has the GCP and is both local and Lipschitz it will be a min-max over linear and local drift-diffusion operators, with zero nonlocal part. In this talk we discuss some recent work that bridges the gap between these situations to cover the nonlinear and nonlocal setting for the map, I. These results open up the possibility to study Dirichlet-to-Neumann mappings for fully nonlinear equations as integro-differential operators on the boundary as well as have implications for existence and uniqueness theory for weak solutions of integro-differential equations. This is joint work with Nestor Guillen

Special Session 18: Traffic Flow Models and Their Application in Traffic Engineering

Benedetto Piccoli, Rutgers University, USA
Benjamin Seibold, Temple University, USA

Traffic engineering is undergoing a fundamental transformation: modern sensor technologies (e.g. smartphones) have caused a surge in new types of traffic data; and autonomous vehicles and wireless vehicle-vehicle communication give rise to completely new means of controlling traffic flow. With this rise of high-quality traffic data, and high-fidelity means of traffic control, sophisticated mathematical traffic models become increasingly important in actual engineering practice. This special session brings together traffic modeling researchers who are also interested in practical perspectives, with engineers who are using traffic models as fundamental tools in their research and engineering solutions. Senior and junior faculty, postdoctoral researchers, and students, will present and discuss recent research on traffic flow theory, traffic computation, state estimation and prediction, and the control of traffic flow.

Multi-Jam Solutions and Lagrangian Data Assimilation in Traffic-Flow Models

Paul Carter
Brown University, USA
Courtney Cochrane, Peter Leth Christiansen, Joseph DeGuire, Gaoyang Fan, Yuri B. Gaididei, Carlos Gorria, Emma Holmes, Melissa McGuirl, Patrick Murphy, Jenna Palmer, Bjorn Sandstede, Laura Slivinski, Mads Peter Sorensen, Jens Starke, Chao Xia

The aims of this talk are twofold. Firstly, a microscopic optimal velocity model of traffic flow is presented which allows for the spontaneous formation of traffic jam solutions. Connections with the fundamental diagram of traffic flow are also discussed. Secondly, a study of data assimilation for traffic flow models is presented; the method allows for the assimilation of both Eulerian (sensor) and Lagrangian (GPS) data and works well in different traffic scenarios using either ensemble Kalman or particle filters. The algorithm is also demonstrated to be capable of estimating parameters and assimilating real traffic data as well as data obtained from the aforementioned microscopic model.

A Fast Algorithm for Computing Solutions to the LWR Traffic Flow Model with Internal Conditions

Christian Claudel
UT Austin, USA
Michele Simoni

In this article, we focus on the problem of computing the solutions to the LWR traffic flow model with internal conditions. The main issue arising in this type of computation is the need for one to know the parameters (actual velocity, passing rate) of the moving boundary condition at all times. These parameters both impact and are impacted by the solution around the internal condition, which makes this problem difficult to solve. To date, the incorporation of internal conditions in the LWR model is done on a ad-hoc basis, in situations where the parameters of these internal conditions are easy to determine, but no algorithm exist to incorporate an arbitrary number of internal conditions into the LWR model. Using an equivalent formulation of the problem based on an Hamilton Jacobi equation, we show that the moving boundary conditions can be modeled as a hybrid system with three states, and that the computation of the parameters of the internal conditions can be done efficiently by using a semi-analytical decomposition of the solutions, which we outline. We then present a few possible applications of this framework, from the optimization of the trajectories of slow vehicles to optimal signal timing.

High-Order Methods Devoid of Velocity Overshoots for Second-Order Traffic Models

Shumo Cui
Temple University, USA
Benjamin Seibold

Traditional finite volume methods, applied to the Aw-Rascle-Zhang (ARZ) model or generic second order models (GSOM), may lead to spurious overshoots in the velocity. The fundamental reason is that neither velocity nor momentum is conservative variables of the model. If left unattended, numerical defects would first emerge as a velocity overshoot at contact waves and later contaminate the solution in a gradually enlarging vicinity. In this talk, we present an efficient and flexible velocity correction approach, which can be added to most numerical schemes as a black box substep. Moreover, the proposed correction method is only triggered near the contact discontinuities and preserves the formal order of the underlying scheme. The performance and robustness of the proposed method are demonstrated in a series of numerical examples.

Traffic Flow Modeling and Simulations with Moving Bottlenecks

Maria Laura delle Monache
Rutgers University, USA

We consider a strongly coupled PDE-ODE system that describes the influence of a slow and large vehicle on road traffic. The model consists of a scalar conservation law accounting for the main traffic evolution, while the trajectory of the slower vehicle is given by an ODE depending on the downstream traffic density. The moving constraint is expressed by an inequality on the flux, which models the bottleneck created in the road by the presence of the slower vehicle. We prove the existence of solutions to the Cauchy problem for initial data of bounded variation. Moreover, we introduce two numerical methods for the tracking of the slower vehicle trajectory on a road. The first algorithm is a finite volume scheme that uses a locally non uniform moving mesh that tracks the slower vehicle. The second method is a conservative scheme that uses a reconstruction technique. We perform numerical tests and compute numerically the order of convergence.

A Nonstandard Second-Order Formulation of the LWR Model

Wenlong Jin
UC Irvine, USA

It is well known that the LWR model can be considered a limit of many second-order continuum models with a zero relaxation time. In this study, we present a nonstandard second-order formulation of the LWR model, which can be considered as a limit of a numerical scheme. The difference between the new formulation and the traditional formulation is discussed. Then by converting the second-order model into the Lagrangian coordinates, we demonstrate that its discrete version is the same as that of the LWR model, which was derived based on the Hopf-Lax formula. In addition, we introduce a new criterion, absolutely collision-free condition, for the well-definedness of the discrete version. With numerical examples, we demonstrate that the new model admits smooth solutions but has shock waves as the limit case. Potential applications of the new formulation are discussed.

Traffic Regulation Via Controlled Speed Limit

Benedetto Piccoli
Rutgers University, USA
Maria Laura Delle Monache, Francesco Rossi

We present an optimal control problem for traffic regulation via variable speed limit. The traffic flow dynamics is described with the Lighthill-Whitham-Richards (LWR) model with Newell-Daganzo flux function. We aim at minimizing the L^2 quadratic error to a desired outflow, given an inflow on a single road. We first provide existence of a minimizer and compute analytically the cost functional variations due to needle-like variation in the control policy. Then, we compare three strategies: instantaneous policy; random exploration of control space; steepest descent using numerical expression of gradient. We show that the gradient technique is able to achieve a cost within 10% of random exploration minimum with better computational performances.

Mathematical Models for Capacity Drop Phenomena

Benjamin Seibold
Temple University, USA

Most contemporary traffic models are based on a flow vs. density relationship ("fundamental diagram') that is continuous. In contrast, many traffic flow control heuristics (e.g., ramp metering) are based on a capacity drop argument: for a certain density (or density range), traffic flow may exist in a high flow ("free flow') or in a low flow ("congested') state; thus, when transitioning from free flow to congestion, the capacity of the flow drops. We demonstrate how such a drop in the fundamental diagram, or even hysteresis behavior, can be included in continuum traffic models. Specifically, we first show how not to model the phenomenon, and then present two suitable way how to actually do it: one describing a straight road, and another one a bottleneck.

Robust Control of Autonomous Vehicle Trajectories

Jonathan Sprinkle
University of Arizona, USA
Rahul Bhadani, Shumo Cui, Benjamin Seibold

In this paper we describe a robust treatment of tracking trajectories with an autonomous vehicle. In employing autonomous behaviors for traffic control there will inevitably be disturbances introduced through model error, non-planar surfaces, sensor noise, and delay in both sensing and actuation. We describe how we address these issues through robust control techniques. The trajectories we follow include position and orientation as part of their specification: but the most interesting aspect of these trajectories is the time-varying description of the state. This is opposed to a traditional approach of following a trajectory at any speed (with expected error in all dimensions of the state vector), as long as the speed does not exceed a maximum value. However, for traffic control to reduce traffic waves, most of the dampening approaches are time-varying trajectories. With this in mind, it becomes necessary to consider the delay of following the reference trajectory, and how this may affect drivers in the flow. We include simulation data demonstrating the results, as well as data from a full-sized robotic Ford Escape.

Control of Microscopic Traffic Flow Via a Single Autonomous Vehicle

Raphael Stern
University of Illinois at Urbana-Champaign, USA

This work focuses on traffic control to locally mitigate instabilities that adversely affect fuel consumption (e.g., stop-and-go waves) via precise velocity control of a single autonomous vehicle (AV) on the highway. The approach is to use the sensors onboard the AV to detect congestion events, and then close the loop by carefully following prescribed velocity controllers that are demonstrated to stabilize the traffic flow. Specifically, this work considers the use of a feedback controller to prescribe the necessary acceleration profile to dampen traffic waves. The main finding is that even a single autonomous vehicle may substantially reduce undesirable traffic waves in its vicinity when properly controlled.

On Some Constant-Speed and Stop-And-Go Solutions of a Car-Following Model

Eugen Stumpf
University of Hamburg, Germany

In this talk we consider a simple optimal velocity car-following model describing the dynamics of infinitely many cars moving, one after another, along a single lane road. Using a traveling wave ansatz and applying a time transformation, we deduce a parameter-dependent delay differential equation whose solutions form wave front solutions for the traffic model. In this way, we will not only be able to prove the existence and analyze the stability of constant-speed solutions, but we will also be able to address the appearance of stop-and-go traffic. The obtained results are in accordance with earlier studies of a similar car-following model on a circular road where the existence of stop-and-go traffic can be evidently observed in experiments.

Efficient Traffic State Estimation and Incident Detection

Dan Work
University of Illinois at Urbana-Champaign, USA
Ren Wang

This talk links the traffic state estimation problem with the traffic incident detection problem, resulting in a unified framework to solve both problems simultaneously. The joint problem is posed as a hybrid state estimation problem, where a continuous variable denotes the traffic state and a discrete model variable identifies the location and severity of an incident. The hybrid state is estimated using a multiple model particle filter to accommodate the nonlinearity and switching dynamics of the traffic incident model, and is validated in simulation and with field data. The joint framework can improve both incident detection capabilities and post incident traffic state estimation. Compared to our earlier work, a new variation of a multiple model particle filter is proposed which reduces the computation time by three orders of magnitude.

Special Session 19: Modern Applications of Mathematical and Computational Sciences

Katie Newhall, University of North Carolina at Chapel Hill, USA
Rafail Abramov, University of Illinois at Chicago, USA
Gregor Kovacic, Rensselaer Polytechnic Institute, USA

This special session encompasses a broad scope of modern applications of applied and computational mathematics, as well as mathematical science. Covered topics include applications of dynamical systems in neuroscience and population dynamics, applications in fluid dynamics, microbiology, electromagnetism, turbulence and other related areas.

Diffusive Boltzmann Equation, Its Fluid Dynamics, Couette Flow and Knudsen Layers

Rafail Abramov
University of Illinois at Chicago, USA

In the current work we propose a diffusive modification of the Boltzmann equation. This naturally leads to the corresponding diffusive fluid dynamics equations, which we numerically investigate in a simple Couette flow setting. This diffusive modification is based on the assumption of the "imperfect" model collision term, which is unable to track all collisions in the corresponding real gas particle system. The effect of missed collisions is then modeled by an appropriately scaled long-term homogenization process of the particle dynamics. The corresponding diffusive fluid dynamics equations are produced in a standard way by closing the hierarchy of the moment equations using either the Euler or the Grad closure. In the numerical experiments with the Couette flow, we discover that the diffusive Euler equations behave similarly to the conventional Navier-Stokes equations, while the diffusive Grad equations additionally exhibit Knudsen-like velocity boundary layers. We compare the simulations with the corresponding Direct Simulation Monte Carlo (DSMC) results. Argon and the air are studied as examples.

Instability of Steep Ocean Waves and Whitecapping.

Sergey Dyachenko
University of Illinois, USA
Alan C. Newell

Wavebreaking in deep oceans is a challenge that still defies complete scientific understanding. Sailors know that at wind speeds of approximately 5 m/sec, the random looking windblown surface begins to develop patches of white foam ('whitecaps') near sharply angled wave crests. We idealize such a sea locally by a family of close to maximum amplitude Stokes waves and show, using highly accurate simulation algorithms based on a conformal map representation, that perturbed Stokes waves develop the universal feature of an overturning plunging jet. We analyze both the cases when surface tension is absent and present. In the latter case, we show the plunging jet is regularized by capillary waves which rapidly become nonlinear Crapper waves in whose trough pockets whitecaps may be spawned.

On-Site and Off-Site Bound States of the Discrete Nonlinear Schroedinger Equation and The Peierls-Nabarro Barrier

Michael Jenkinson
Rensselaer Polytechnic Institute, USA
Michael I. Weinstein

We construct several families of symmetric localized standing waves (breathers) to the one-, two-, and three-dimensional discrete nonlinear Schroedinger equation (DNLS) with cubic nonlinearity using bifurcation methods about the continuum limit. Such waves and their energy differences play a role in the propagation of localized states of DNLS across the lattice. The energy differences, which we prove to exponentially small in a natural parameter, are related to the Peierls-Nabarro Barrier in discrete systems, first investigated by M. Peyrard and M.D. Kruskal (1984). These results may be generalized to different lattice geometries and inter-site coupling parameters. Finally, we discuss the local stability properties of such bound states. This is joint work with Michael I. Weinstein.

Synchronizing Cortical Dynamics Via Gap Junctions Between Excitatory Neurons

Jennifer Kile
Rensselaer Polytechnic Institute, USA
Gregor Kovacic, David Cai

Brain networks are known to give rise to global oscillations that are linked to synchronized neuronal activity, which has been shown to contribute to cognitive processes such as motor performance, learning and memory. Electric coupling through gap junctions may facilitate the emergence of synchronized oscillations in the cortex, and influence their properties. Electrical synapses, or gap junctions, connect the cytosolic contents of coupled neurons, allowing the direct transfer of electrical signals between the cells. While such synapses between interneurons in the cortex have been studied, electric coupling between ex-

citatory, pyramidal neurons have only recently been discovered. In order to further study these hypotheses, we have developed a detailed, comprehensive model with both synaptic and electric coupling for both excitatory and inhibitory neurons using a modified version of the Hodgkin-Huxley equations. The network incorporates local synaptic connections between pairs of neurons using known models from the literature to describe the kinetics of the synapses. This model also includes gap junctions between inhibitory neurons, as well as pairs of excitatory neurons. Through this model, we examine the resulting dynamical regimes from the inclusion of both electric and synaptic connections, with a specific interest in the emergence and properties of synchrony.

Hydrodynamic Limits of Grain Coarsening in Two Dimensions

Joe Klobusicky
RPI, USA
Govind Menon, Robert Pego

In two dimensions, areas of individual grains in foams and metals evolve at constant rates based on their topologies. When a grain vanishes, neighboring grains may gain or lose sides to satisfy the cell network's trivalency requirement. Various curvature based and mean field models handle the problem of side redistribution differently. In this talk, we compare these models, and also their corresponding nonlinear transport equations which describe evolution for a grain's area and topology. We also introduce a particle system, through piecewise deterministic Markov processes, which captures both deterministic drift of grain areas and random mean field side reassignment. We test this model against others with stochastic simulations on a large number of grains.

Modeling of Gas Flow Networks with Stochastic Load: Theory and Numerical Experiment

Alexander Korotkevich
University of New Mexico, USA
S.A. Dyachenko, M. Chertkov, V.V. Lebedev, S.N. Backhause

We propose a method for gas transportation network modelling based on operator splitting technique. This approach is stable for cases with loops in the graph of the network. For the fluctuating consumption in the network, which is a standard situation for heterogeneous power generation networks, where gas powered generators work as a power sources when solar or wind power station is not generating due to weather change, we demonstrate growth of pressure fluctuations with time. Theoretical computations are supported by numerical simulations using different physical models of the network.

Is Our Sensing Compressed?

Gregor Kovacic
Rensselaer Polytechnic Institute, USA
Victor Barranca, Douglas Zhou, David Cai

Considering many natural stimuli are sparse, can a sensory system evolve to take advantage of sparsity? We show significant downstream reductions in the numbers of neurons transmitting stimuli in early sensory pathways might be a consequence of sparsity. Our work points to a potential mechanism for transmitting stimuli related to compressed-sensing (CS) data acquisition. Through simulation of integrate-and-fire point-neuron models and reconstruction via firing-rate models, we examine the characteristics of networks that optimally encode sparsity and the role of receptive fields in stimulus sampling.

Metastable Transitions in Sleep-Wake Networks

Peter Kramer
Rensselaer Polytechnic Institute, USA
Anthony Trubiano, Fatih Olmez, Jung-Eun Kim, Janet Best

We have developed a computational neuronal network model for sleep-wake transitions which exhibits both a power law region and an exponential tail in the probability density function for the time between sleep/wake switches. Such power-law and exponential features are seen in experimental recordings of young mammals. We analyze the nature of the transitions contributing to both the power law and exponential region, and what distinguishes them.

Symmetrization of Rare Event Samplers for Stochastic Differential Equations

Andrew Leach
University of Arizona, USA
Kevin Lin, Matthias Morzfeld

Many interesting behaviors in stochastic differential equations (SDE) occur infrequently and are difficult to observe through direct simulation. Recently, optimal control based sampling methods have been proposed for efficient simulation of rare events in SDE. We analyze the performance of these techniques when the noise parameter is small, and show that the relative variance of such methods is order one in the noise parameter. Moreover, we show that this order can be improved by a symmetrization procedure akin to antithetic variates. We illustrate our small noise analysis with numerical examples, and compare the control based samplers to other methods.

Wave Patterns in an Excitable Neuronal Network

Christina Lee
Oxford College of Emory University, USA
Gregor Kovacic

This talk describes a study of spiral- and target-like waves traveling in a two-dimensional network of integrate-and-fire neurons with close-neighbor coupling. The individual neurons are driven by Poisson trains of incoming spikes. Each wave nucleates as a result of a fluctuation in the drive. It begins as a target or a spiral, and eventually evolves into a straight "zebra" -like grating. Some of the waves contain defects arising from collisions with other waves. The wavelength and wave speed of the patterns were investigated, as were the temporal power spectra of the oscillations experienced by the individual neurons as waves were passing through them.

The Effective Dispersion Relation of the Nonlinear Schrodinger Equation

Katelyn Leisman
Rensselaer Polytechnic Institute, USA
Gregor Kovacic, David Cai

The linear part of the Nonlinear Schrödinger Equation (NLS) ($iq_t = q_{xx}$) has dispersion relation $\omega = k^2$. We don't necessarily expect solutions to the NLS to behave nicely or have any kind of effective dispersion relation, since we expect nonlinear waves to be strongly coupled and not sinusoidal in time. However, I have seen that solutions to the NLS are actually weakly coupled and are often nearly sinusoidal in time with a dominant frequency, often behaving similarly to modulated plane waves. In fact, when I look at long-time average of either a solution with many soliton like pulses or with many unstable modes, the power spectral density does indicate a quadratic dispersion relation that has been shifted by a constant proportional to the amplitude of the initial condition: $\omega = k^2 - 2A$ where $A = \frac{\|\hat{q}(k,0)\|^2}{2\pi}$.

Nonlinear Combining of Multiple Laser Beams in Multimode Optical Fiber

Pavel Lushnikov
University of New Mexico, USA
Natalia Vladimirova

We simulate combining of 127 laser beams into diffraction-limited beam by beam self-focusing (collapse) in multimode optical fiber with fiber diameter about 1mm. Beams with total power above critical are combined in near field at the entrance. Beam quality of the combined beam is not affected by moderate fluctuations of the combining beams phases.

First-Passage Time to Clear the Way for Receptor-Ligand Binding in a Crowded Environment

Jay Newby
University of North Carolina, Chapel Hill, USA

I will present theoretical support for a hypothesis about cell-cell contact, which plays a critical role in immune function. A fundamental question for all cell-cell interfaces is how receptors and ligands come into contact, despite being separated by large molecules, the extracellular fluid, and other structures in the glycocalyx. The cell membrane is a crowded domain filled with large glycoproteins that impair interactions between smaller pairs of molecules, such as the T cell receptor and its ligand, which is a key step in immunological information processing and decision-making. A first passage time problem allows us to gauge whether a reaction zone can be cleared of large molecules through passive diffusion on biologically relevant timescales. I combine numerical and asymptotic approaches to obtain a complete picture of the first passage time, which shows that passive diffusion alone would take far too long to account for experimentally observed cell-cell contact formation times. The result suggests that cell-cell contact formation may involve previously unknown active mechanical processes.

The Causes of Metastability and Their Effects on Transition Times

Katie Newhall
UNC Chapel Hill, USA

Many experimental systems can spend extended periods of time relative to their natural time scale in localized regions of phase space, transiting infrequently between them. This display of metastability can arise in stochastically driven systems due to the presence of large energy barriers, or in deterministic systems due to the presence of narrow passages in phase space. To investigate metastability in these different cases, we take the Langevin equation and determine the effects of small damping, small noise, and dimensionality on the dynamics and mean transition time.

Modeling the Evolving Oscillatory Dynamics of the Rat Locus Coeruleus Through Early Infancy

Mainak Patel
College of William and Mary, USA
Badal joshi

The mammalian locus coeruleus (LC) is a brainstem structure that displays extensive interconnections with numerous brain regions, and in particular plays a prominent role in the regulation of sleep and arousal. Postnatal LC development is known to drastically alter sleep-wake switching behavior through early infancy, and, in rats, exerts its most significant influence from about postnatal day 8 to postnatal day 21 (P8-P21). Physiologically, several dramatic changes are seen in LC functionality through this time period. Prior to P8, LC neurons are extensively coupled via electrical gap junctions and chemical synapses, and the entire LC network exhibits synchronized ~0.3 Hz subthreshold oscillations and spiking. From P8 to P21, the network oscillation frequency rises up to ~3 Hz (at P21) while the amplitude of the network oscillation decreases. Beyond P21, synchronized network oscillations vanish and gap junction coupling is sparse or nonexistent. In this work, we develop a large-scale, biophysically realistic model of the rat LC and we use this model to examine the changing physiology of the LC through the pivotal P8-P21 developmental period. We find that progressive gap junction pruning is sufficient to account for all of the physiological changes observed from P8-P21. Furthermore, we discuss the relevance of this work to the physiology of sleep-wake cycling behavior and the applications of this work to sleep-wake neurophysiology that will be explored in our future modeling efforts.

Pattern and Defect Formation by Ion Bombardment of Binary Substrates

Matt Pennybacker
University of New Mexico, USA
Patrick Shipman, R. Mark Bradley

When a solid surface is bombarded with a broad ion beam, a wide variety of self-assembled nanoscale patterns can emerge, including nanodots arranged in hexagonal arrays of remarkable regularity. We discuss a theory that explains the formation of the strikingly regular hexagonal arrays of nanodots that can occur when binary materials are bombarded. In this theory, the coupling between a surface layer of altered chemical composition and the topography of the surface is the key to the observed pattern formation. We analyze how a soft mode related to the mean sputter yield can lead to defect formation and give rise to less ordered patterns.

Idealized Models of Insect Olfaction

Pamela Pyzza
Ohio Wesleyan University, USA
Gregor Kovacic, David Cai

The functionality of olfaction is known to be shared among a range of phyla from insects to mammals, and the locust is an important animal model for studying olfaction. Experiments suggest that odors, detected through receptors on its antennas, are relayed to sensory neurons in the antennal lobe. There, they trigger a series of synchronous oscillations, followed by slow dynamical modulation of their firing rates, which slowly subside after the stimulus has been removed. We model the effects of a white-smell-type odor using an integrate-and-fire network and firing-rate model, with both fast excitatory and inhibitory and slow inhibitory conductance responses. The fast inhibitory conductance response, together with the excitation, creates initial oscillations, and the slow component then damps the oscillations and aids in the creation of the slow firing rate patterns that follow yet later. We propose a possible mechanism for generating this dynamical sequence to be a slow passage through a saddle-node-on-a-circle bifurcation.

Waveaction Spectra for Fully Nonlinear MMT Model

Michael Schwarz
Rensselaer Polytechnic Institute, USA
David Cai, Gregor Kovacic, Peter Kramer

We investigate a version of the Majda-McLaughlin-Tabak model of dispersive wave turbulence where the linear term in the time derivative is removed. We consider driven-damped and undriven, undamped cases of the model. Our theoretical predictions for the waveaction spectrum, which are made using statistical mechanical methods as well as arguments reminiscent of Kolmogorov's theory of turbulence, are found to agree with time dynamics simulations.

Transverse Instability of Electron Plasma Waves Study Via Direct 2+2D Vlasov Simulations

Denis Silantyev
University of New Mexico, USA
Pavel Lushnikov, Harvey Rose

Transverse instability could be viewed as initial stage of electron plasma waves (EPWs) filamentation. We performed direct 2+2D Vlasov-Poisson simulations of collisionless plasma to systematically study the growth rates of oblique modes of finite-amplitude EPW depending on its amplitude, wavenumber, angle of the oblique mode wavevector relative to the EPW's wavevector and the configuration of the trapped electrons in the EPW. Simulation results are compared to the theoretical predictions of simplified models.

Special Session 20: Models for Treatment of Prostate Cancer

Fabio Milner, Arizona State University, USA
Yang Kuang, Arizona State University, USA

The session will be focused on the description, analysis, and calibration of various models of treatment of prostate cancer in different stages. Most prostate cancers depend on the hormone testosterone to grow. About one-third of prostate cancer patients require hormone therapy when their cancer spreads beyond the prostate or recurs after treatment. Some models will be concerned with intermittent ablation therapy, others with continuous ablation therapy when resistance develops and a new line of treatment is added to suppress intramural testosterone production. Models are calibrated using clinical data from different sources that typically track the Prostate-Specific Antigen (PSA) concentration in patient blood.

Predicting the Outcome of Prostate Cancer Patients Undergoing Intermittent Androgen Deprivation Therapy

Rebecca Everett
North Carolina State University, USA
A. M. Packer, Y. Kuang

Prostate cancer is often treated by intermittent androgen deprivation therapy since prostate cells depend on androgens for proliferation and survival. We present a mathematical model that uses the Droop equation to apply the idea of androgen as a limiting nutrient. Using clinical data, we test the model's predictive accuracy and predict whether or not a patient can undergo another off-treatment cycle, thus improving their quality of life. Ideas for future work will be discussed.

The Potential for Immunotherapy in Combination with Androgen Ablation for the Treatment of Prostate Cancer

Harsh Jain
Florida State University, USA

Androgen ablation is the standard treatment for metastatic prostate cancer; but the onset of castration resistant disease has necessitated the development of alternative treatments like immunotherapy, wherein the hosts immune system is trained to target the tumor. New evidence points to a potential for synergy with androgen ablation. We will investigate the potential for such a combination using a detailed mathematical model that includes temporal dynamics of key cell players and chemokines together with detailed intracellular pathways relevant to drug action.We will also discuss the limited success of anti-cancer vaccines like Provenge due to immune modulation by the prostate cells and therefore the need for a third kind of drug, namely those that target this cross talk.

Data Assimilation in Mathematical Models of Cancer Growth and Treatment

Eric Kostelich
Arizona State University, USA
Javier Baez, Yang Kuang

This talk will consider some applications of data assimilation on models of tumor growth and treatment. The ensemble Kalman filter and its variants provide a computationally efficient way to estimate initial conditions and parameters and their associated uncertainties in dynamical systems. I will also discuss some results regarding identifiability and bias in the model variables. These concepts will be illustrated by assimilating clinical trial data into a differential equation models of prostate cancer growth and treatment.

Parameter Distribution in Prostate Cancer Models

Fabio Milner
Arizona State University, USA
Maria Peters

Mathematical models of prostate cancer—and of many other biological systems—often involve a large number of unknown parameters, posing the problem of their identification. Since very often there is no way of knowing their exact values, different techniques are used to fit the models to individual cases in an effort to predict the evolution of the particular system. Reliable estimates for the parameters using these techniques require a significant amount of collected data. In contrast, a database of many individual cases containing each a relatively small number of data points, amounts to a dataset large enough for reliably identifying *population parameter distibutions* rather than single values for each parameter. Although estimated parameter distributions cannot be used to predict cancer progression in individual cases, they can be a useful tool in identifying general features or behavior of the system. To track progression of prostate cancer it is common to use blood concentration of prostate specific antigen (PSA). The Hirata model predicts PSA levels based on 10 free parameters and the initial values of three different prostate cancer cell populations. We use MCMC (Markov Chain Monte Carlo) methods to identify parameter distributions for those 10 parameters.

Evolutionary Stable Strategies in Nutrient-Dependent Prostate Cancer Growth

John Nagy
Arizona State University, USA
Kirsten Karr, David Ung, Karl Lundin

Cancer cells require nutrient concentrations to be within a fixed range to maintain viability. When concentrations become too low, cells face nutrient deficiency, or starvation; when too high, cells can suffer metabolic costs associated with toxicity, secretion or detoxification. However, the relationship between tumor growth and nutrient concentrations is complex and difficult to quantify. We have developed a mathematical model which explores this relationship, focusing on androgen as the nutrient in question. This model extends the Portz et al. prostate cancer model to include costs of androgen dependence and independence. We consider two iterations of this model: one that features a constant per-capita death rate, and one that assumes that cell morality is a unimodal function of intracellular nutrient concentration. Here we show, using the theory of adaptive dynamics, that the interplay between costs and benefits of castration resistance drive natural selection toward specific Evolutionary Stable Strategies (ESS). These results suggest targets for potential improvement of clinical outcomes by manipulating the selective environment within advanced prostate malignancies.

Global Dynamics of a Model of Joint Hormone Treatment with Dendritic Cell Vaccine for Prostate Cancer

Erica Rutter
Arizona State University, USA
Yang Kuang

Androgen deprivation therapy (ADT) is often used to treat advanced prostate cancer. This type of hormone therapy is effective initially, but eventually castration-resistant prostate cancer emerges, which cannot be treated with ADT. Intermittent androgen deprivation therapy is suggested as an alternative which may lessen side effects, lower therapy costs, and potentially increase time to treatment resistance. Immunotherapy is often used once patients develop resistance to hormone therapy, in the form of dendritic cell vaccines. We model hormone therapy (intermittent and continual ADT) in conjunction with dendritic cell vaccine therapy. We numerically investigate the effect scheduling of dendritic cell vaccines has on patient longevity. From basic local analysis, we determine a personalized dendritic cell vaccine dosage for disease-free equilibrium stability. We examine a quasi-steady state system and classify the global dynamics.

Special Session 21: Bifurcations and Asymptotic Analysis of Solutions of Nonlinear Models

Jann-Long Chern, Central University, Taiwan
Yoshio Yamada, Waseda University, Japan
Shoji Yotsutani, Ryukoku University, Japan

The aim of this special session is to exchange recent results, ideas and techniques on nonlinear elliptic and parabolic PDEs, including reaction-diffusion systems and free boundary problems, from mathematical physics, chemical reactions, mathematical biology, medical science and some other fields. In particular, we are interested in the global bifurcation structure for such models. Combinations of numerical simulations and theoretical approaches with asymptotic analysis will be very useful to understand the nonlinear phenomena together with underlying structure of solutions. We will give opportunities to both established and junior researchers working in the related area to present their recent results.

On the Uniqueness and Structure of Solutions to the System Arising from Maxwell-Chern-Simons O(3) Sgma Model

Zhi-You Chen
National Changhua University of Education, Taiwan
Jann-Long Chern

In this talk, we prove the uniqueness of topological multivortex solutions for the self-dual Maxwell-Chern-Simons O(3) sigma model with Chern-Simons coupling parameter sufficiently large and the charge of electron either sufficiently small or large. Besides, we also establish the sharp region of flux-pairs for the non-topological solutions and provide the classification of radial solutions of all types for single vortex-point case.

Stability of Equilibria for Epidemiological Models with Temporary Acquired Immunity

Yoichi Enatsu
Tokyo University of Science, Japan

To theoretically understand the disease transmission in the population, the asymptotic behavior of the solutions of epidemiological models have been widely studied. In particular, incorporating immunity to a disease of recovered individuals has now been considered to be an important concept of the modelling. In this talk, we introduce the recent works on the stability of equilibria of the model governed by a class of nonlinear delay differential equations. We also discuss the effect of the waning immunity on the stability of an endemic equilibrium of the model with two constant delays that represent latency time and infectious period, via the linearization of the model at the equilibrium.

Bifurcation from Equilibrium Points for a Modified Swift-Hohenberg Equation

Jongmin Han
Kyung Hee University, Korea
Yuncherl Choi, Taeyoung Ha, Doo Seok Lee

In this talk, we consider the dynamical bifurcation of a modified Swift-Hohenberg equation(MSHE). As the control parameter crosses a critical value, it is shown that the MSHE bifurcates from a trivial solution to an attractor which determines the long time dynamics of the system. Using the center manifold analysis, we describe the bifurcated attractor in detail.

On the Stability of Time Periodic Solutions of the Primitive Equations

Chun-Hsiung Hsia
National Taiwan University, Taiwan
Ming-Cheng Shiue

In this joint work with Ming-Cheng Shiue, we consider the stability of time periodic solutions for the primitive equations with periodic forcing terms. We provide a global stability analysis for the small force case.

The Domain Geometry and The Bubbling Phenomenon of Rank Two Gauge Theory

Hsin-Yuan Huang
National Sun Yat-sen University, Taiwan
Lei Zhang

Let G be the Green's function on a flat torus. One intriguing mystery of G is how the number of its critical points is related to blowup solutions of certain PDEs. In this talk, I will show that if fully bubbling solutions of Liouville type to Chern-Simons Model with two Higgs particles exist, the G has exactly three critical points. In addition, the necessary and sufficient conditions for the existence of fully bubbling solutions with multiple bubbles will be presented.

Shadow System Approach to a Plankton Model Generating Harmful Algal Bloom

Hideo Ikeda
University of Toyama, Japan
Masayasu Mimura, Tommaso Scotti

Spatially localized blooms of toxic plankton species have negative impacts on other organisms via the production of toxins, mechanical damage, or by other means. Such blooms are nowadays a worldwide spread environmental issue. To understand the mechanism behind this phenomenon, a two-prey (toxic and nontoxic phytoplankton)-one-predator (zooplankton) Lotka-Volterra system with diffusion has been considered in a previous paper. Numerical results suggest the occurrence of stable non-constant equilibrium solutions, that is, spatially localized blooms of the toxic prey. Such blooms appear for intermediate values of the rate of toxicity μ when the ratio D of the diffusion rates of the predator and the two prey is rather large. In this paper, we consider a one-dimensional limiting system (we call it a shadow system) in $(0, L)$ as $D \to \infty$ and discuss the existence and stability of non-constant equilibrium solutions with large amplitude when μ is globally varied. We also show that the structure of non-constant equilibrium solutions sensitively depends on L as well as μ.

Singular Solutions to the Equation of the Scalar-Field Type on the Unit Sphere

Yoshitsugu Kabeya
Osaka Prefecture Unviersity, Japan
Soohyun Bae, Jann-Long Chern, Shoji Yotsutani

We consider the nonlinear elliptic equation of the form$\Lambda\ u - \lambda u + u^p = 0$ on the unit sphere $\mathbb{S}^n \subset \mathbb{R}^{n+1}$ with $n \geq 2$ and with λ being a real parameter, where Λ stands for the Laplace-Beltrami operator on the unit sphere. We discuss the existence of positive singular solutions and the structure of solutions to the equation.

Generation of Singularity and Large Time Behaviors of Solutions for a Free Boundary Problem of a Reaction-Diffusion equation

Yuki Kaneko
Waseda University, Japan
Yoshio Yamada

We discuss a free boundary problem for a reaction-diffusion equation in multi-dimensions. This problem may be used to model the spreading of biological species, where unknown functions are population density and spreading front of the species. If an initial function is defined in an annulus, the inner boundary possibly shrinks until it touches the origin and disappears (called Singularity). Hence it is appropriate to introduce a weak solution to study the problem after the disappearance of the inner boundary, and we study large-time behaviors. Putting a monostable nonlinearity as a reaction term and assuming that the outer boundary is sufficiently far from the inner boundary, we will show that Singularity appears in a finite time. Moreover we will derive some information on spreading and vanishing for large time behaviors.

Secondary Bifurcation for a Nonlocal Allen-Cahn Equation

Kousuke Kuto
University of Electro-Communications, Japan
Tatsuki Mori, Tohru Tsujikawa, Shoji Yotsutani

This talk is concerned with the Neumann problem of the 1D stationary Allen-Cahn equation with a nonlocal term. Since this nonlocal term is given as the average of the unknown function, each odd-symmetric solution counteracts the nonlocal effect, and thereby, becomes a stationary solution of the Allen-Cahn equation without nonlocal term. Namely, the set of odd-symmetric solutions of the nonlocal problem forms a bifurcation branch of the Chafee-Intante problem which emanates from a pitchfork bifurcation point on the branch of the trivial solution. Our main result reveals that the nonlocal term induces a symmetry breaking bifurcation point on the branch of odd-symmetric solutions. This talk also mentions the uniqueness of such secondary bifurcation points and the global behavior of the bifurcation branch of asymmetric solutions.

Some Well-Posedness Problem for the Compressible Navier-Stokes Equations

Ying-Chieh Lin
National University of Kaohsiung, Taiwan

In this talk we concern with the isentropic compressible Navier-Stokes equations with density dependent viscosity on the n-dimensional torus $\mathbb{T}^n$ (n = 2 or 3) and with initial density vanishing somewhere inside $\mathbb{T}^n$. Let Ω (t) denote the domain where the fluid density is positive at time t. We assume that the boundary of Ω moves along with the fluid velocity. We will present our strategy to get the local well-posedness of the problem.

The Fisher-KPP Equation with a Free Boundary and A Moving Boundary

Hiroshi Matsuzawa
National Institute of Technology, Numazu College, Japan

In this talk, we concern with free boundary problem of Fisher-KPP equation

$$u_t = u_{xx} + u(1-u), t > 0, \ ct < x < h(t),$$

where $c > 0$ is a given constant, $h(t)$ is a free boundary which is determined by the Stefan-like condition. This model may be used to describe the spreading of a new or invasive species with population density $u(t,x)$ over a one dimensional habitat. The free boundary $x = h(t)$ represents the spreading front. In this model, we impose zero Dirichlet boundary condition at left boundary $x = ct$. This means that the left boundary of the habitat is a very hostile environment for the species and that the habitat is eroded away by the left moving boundary at a constant speed c.
In this talk we will give a trichotomy result, that is, for any initial data, one of the three situations (vanishing, spreading and transition) happens. This result is related to the result of Fisher-KPP equation with a shifting-environment, obtained by Du, Wei and Zhou.

Convergence of the Hydrodynamical Limit for Generalized Carleman Models

Hironari Miyoshi
Waseda University, Japan
Masayoshi Tsutsumi

We consider the initial-boundary value problem for a 2-speed system of first order semilinear hyperbolic equations with the homogeneous boundary conditions. We establish the existence of global weak solutions in L^1 by the theory of nonlinear contraction semigroups. Using the monotone method and the div-curl lemma, we investigate the hydrodynamic limit of the sum of the solutions of the hyperbolic system and show that the limit verifies the doubly nonlinear parabolic equations.

Global Structure of Stationary Solutions to a Cell Polarization Model

Tatsuki Mori
Ryukoku University, Japan
Kousuke Kuto, Tohru Tsujikawa, Shoji Yotsutani

Various cell polarization models are proposed by S.Ishihara, et al. (Phys. Rev. E 75 015203(R), 2007) and M.Otsuji, et al. (PLoS Compt. Biol. 3: e108, 2007). We investigate global structure of stationary solutions to a cell polarization model proposed by Y.Mori, A.Jilkine and L.Edelstein-Keshet (SIAM J.Appl Math, 2011), which is closely related with results by them. We study both infinite and finite diffusion coefficient case. We show several mathematical and numerical results.

Spectral Comparison in a Generalized Phase-Field Type System

Yoshihisa Morita
Ryukoku University, Japan

We are concerned with the system $\alpha u_t - \beta v_t = d\Delta u + f(u) + v, \ \ \gamma u_t + \delta v_t = \Delta v$, in a bounded domain with the Neumann boundary conditions, where $\alpha, \beta, \gamma, \delta$ and d are positive parameters and $f(u)$ is smooth. The system allows a Lyapunov function and the stationary problem is reduced to a scalar equation with a nonlocal term. We study the linearized eigenvalue problem of an equilibrium solution, and by a spectral comparison method we show that the unstable dimension coincides with that of the simpler eigenvalue problem of the scalar equation. This talk is based on a joint work with S.Jimbo and a joint work with E.Latos and T.Suzuki.

On the Equilibrium States of an Inextensible Elastic Ring Under the Uniform Pressure

Minoru Murai
Osaka city University Advanced Mathematical Institute, Japan
Waichiro Matsumoto, Shoji Yotsutani

We consider a variational problem proposed by Tadjbakhsh-Odeh to study the equilibrium states of an elastic inextensible ring under a uniform pressure. Takagi-Watanabe mathematically investigate the structure of the solutions of the Euler-Lagrange equation for this problem. They gave very interesting results for it.
However, there are some parts to be clarified. In this talk, we will give a complete answer to the global structure of the solutions.

Chaotic Traveling Pulses in Some Reaction-Diffusion System

Masaharu Nagayama
Hokkaido University, Japan
Kei-Ichi Ueda, Masaaki Yadome

We consider the extended Gray-Scott model as one of the reaction diffusion system which have a excitable property with a mon-stable equilibrium point. This model has an oscillatory traveling pulse which bifurcate form a traveling pulse. Using a global bifurcation calculation, we find that the oscillatory traveling pulse is destabilized by peirod-doubling bifurcation. As a result, we discover the oscillatory traveling pulse to make a chaotic motion, and we make clear that the solution is chaos by calculating the maximal Lyapunov exponent.

Asymptotic Behaviour of Equilibrium States of Reaction-Diffusion Systems with Mass Conservation

Tien-Tsan Shieh
National Taiwan University, Taiwan
Jann-Long Chern, Yoshihisa Morita

We study stationary solutions for a 1-D cross-diffusion system, a conceptual model for cell polarity,

$$\begin{aligned} u_t &= d\Delta u - g(u+v) + v, \\ v_t &= \Delta v + g(u+v) - v \end{aligned}$$

where $g(w) = w/(w+1)^2$. In particular, we are interested in asymptotic behaviours of the solution for the system as $d \to 0$. We show that the solution with a minimizing energy exhibits the phenomena of mass concentration. Using a blow-up technique, we also find the limiting profile for the solution.

Multiple-Angle Formulas of Generalized Trigonometric Functions with Two Parameters

Shingo Takeuchi
Shibaura Institute of Technology, Japan

Generalized trigonometric functions with two parameters were introduced by Dràbek and Manàsevich to study an inhomogeneous eigenvalue problem of the p-Laplacian. Concerning these functions, no multiple-angle formula has been known except for the classical cases and a special case discovered by Edmunds-Gurka-Lang, not to mention addition theorems. In this talk, we will present new multiple-angle formulas which are established between two kinds of the generalized trigonometric functions, and apply the formulas to generalize classical topics related to the trigonometric functions.

Classification of Standing Wave Solutions to a Coupled Schrodinger System

Yong-Li Tang
Feng Chia University, Taiwan
Zhi-You Chen

In this talk, a Schrodinger-type system is considered. We establish the existence and uniqueness of solutions for the Dirichlet boundary value problem of the system. Besides, the nonexistence of ground state solutions under certain conditions on nonlinearities, and a complete structure of various types of solutions are also provided.

On the Maximizing Problem Associated with Trudinger-Moser Type Inequalities

Hidemitsu Wadade
Kanazawa University / Institute of Science and Engineering, Japan
Michinori Ishiwata

In this talk, we consider the existence and non-existence of maximizers associated with Trudinger-Moser type inequalities. Recently, A Trudinger-Moser inequality was derived by B. Ruf, JFA, 219 (2005) for the two spacial dimension and extended to the higher spacial dimensions by Y. Li-B. Ruf, Indiana UMJ, 57 (2008), which are in-homogeneous type inequalities in the whole space. In the papers Li-B. Ruf, Indiana UMJ, 57 (2008) and M. Ishiwata, Math. Ann. 351 (2011), the authors considered the variational problems associated with these Trudinger-Moser type inequalities and proved that the existence and non-existence results depending on the exponents appearing in the exponential type integrals. We revisit the existence and non-existence problems for the above inequalities and clarify the effects of the norm-normalization to the structure of these variational problems.

Limiting Classification on Linearized Eigenvalue Problems for 1-Dimensional Scalar Field Equation

Tohru Wakasa
Kyushu Institute of Technology, Japan

We are interested in classical linearized eigenvalue problems for one dimensional reaction-diffusion equations with a small diffusion parameter. In the previous work by Wakasa and Yotsutani, the case of bistable nonlinearities has been investigated, and asymptotic formulas of eigenvalues and eigenfunctions as the parameter tends to zero are obtained. It can be observed from the asymptotic formulas that all of eigenpairs are classified into the finite number of group, which is characterized by the associated limit problem. In this talk we will focus on the scalar field equations, and will investigate the limiting classifications on the eigenvalue and eigenfunctions associated with the spike solutions.

Nonlinear M-Accretive Operator Theoretic Approach to Parabolic-Parabolic Keller-Segel Systems

Noriaki Yoshino
Tokyo University of Science, Japan

In this talk, we deal with parabolic-parabolic chemotaxis systems by using the theory for nonlinear m-accretive operators. In the case of parabolic-elliptic type, an approach by nonlinear m-accretive operators to the systems was developed and the solvability of the systems was established by Marinoschi (2013), and Yokota and Yoshino (2014, 2015). However the case of parabolic-parabolic type was left unsolved. We develop the theory to establish existence of solutions to the parabolic-parabolic chemotaxis systems.

Special Session 22: Dynamics and Games

Alberto Adrego Pinto, University of Porto, Portugal
Michel Benaim, Institut de Mathematiques, Universite de Neuchatel, Switzerland

The session aims to bring together world top researchers and practitioners from the fields of Dynamical Systems, Game Theory and applications to such areas as Biology, Economics, Engineering, Energy, Natural Resources and Social Sciences. This session is organized by the founders and editors-in-chief of the Journal of Dynamics and Games (JDG), published by the American Institute of Mathematical Sciences (AIMS).

Anosov Diffeomorphisms and Self-Renormalizable Sequences

Joao Almeida
LIAAD - INESC TEC and Polytechnic Institute of Braganca, Portugal
A.A. Pinto

We consider a hyperbolic toral automorphism $A : \mathbb{T} \to \mathbb{T}$ induced by the matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL(2, \mathbb{Z}),$$

where $\mathbb{T} = \mathbb{R}^2/\mathbb{Z}^2$.
We use Adler, Tresser and Worfolk decomposition of A to give an explicit construction of the stable and unstable C^{1+} self-renormalizable sequences.

Bayesian-Nash Equilibria in Theory of Planned Behavior

Joao Almeida
LIAAD - INESC TEC and Polytechnic Institute of Braganca, Portugal
L. Almeida, J. Cruz, H. Ferreira, J.P. Almeida, B. Oliveira, Alberto A. Pinto

We construct a model, using Game Theory, for the Theory of Planned Behavior and we propose the Bayesian-Nash Equilibria as one of many possible mechanisms to transform human intentions into behavior decisions. We show that saturation, boredom and frustration can lead to the adoption of a variety of different behavior decisions, as opposed to no saturation, which leads to the adoption of a single consistent behavior decision.

Coupled Drift-Diffusion Stochastic Processes As a Model for Decision Making

Reggie Caginalp
University of Pittsburgh, USA
Brent Doiron

We consider N drift-diffusion escape processes, and allow them to interact via a kick at the escape time. The influence of the interaction on the number of processes that escape through a certain gate is investigated. A two-body case is first discussed, and it is investigated with both Monte-Carlo simulations as well as solving the Fokker-Planck equation. Monte-Carlo simulations for a large number of these coupled processes are then considered.

Externality Effects in the Formation of Societies

Abdelrahim Mousa
Birzeit University, Israel
R. Soeiro, A. Pinto

We study a finite decision model where the utility function is an additive combination of a personal valuation component and an interaction component. Individuals are characterized according to these two components (their valuation type and externality type), and also according to their crowding type (how they influence others). We study how positive externalities lead to type symmetries in the set of Nash equilibria, while negative externalities allow the existence of equilibria that are not type-symmetric. In particular, we show that positive externalities lead to equilibria having a unique partition into a minimum number of societies (similar individuals using the same strategy, see [27]); and negative externalities lead to equilibria with multiple societal partitions, some with the maximum number of societies.

Cournot Competition with Uncertainty in the Production Costs

Bruno Oliveira
FCNA Universidade do Porto and INESC TEC, Portugal
Joana Becker Paulo, Allberto A. Pinto

In an economy with a single sector, under Cournot competition with complete information, firms choose the optimal quantities that maximize their profits. This maximum is a unique perfect Nash equilibrium and depends on the values of the parameters of the firms, in particular, their production costs. We study a static game where uncertainty is on the production costs, that can be either high or low. Before production starts, firms are uncertain of the production cost of the other firm and are certain of the value of their production cost. There are six distinct possible cases. For each case, we have characterized the Nash Equilibria and have obtained explicit formulas for the output quantities of each firm and their respective profits.

Prices in Random Exchange Markets and Cobb-Douglas Utility

Bruno Oliveira
FCNA Universidade do Porto and INESC TEC, Portugal
A. Yusuf, B. Finkenstadt, A. N. Yannacopoulos, A. A. Pinto

We study a random matching economy, where pairs of participants trade two goods and follow Cobb-Douglas utility functions. Under the appropriate symmetry conditions, depending on the initial distribution of endowments and the agents preferences, we show that the sequence of bilateral prices converges to the Walrasian price for this economy. Additionally, we study the effect of an asymmetry in the preferences on the difference between the bilateral price and the Walrasian price for this economy. Moreover, we associate a selfishness factor to each participant in this market. This brings up a game alike the prisoner's dilemma, where trade may occur at a price different from the bilateral, with advantage to the more selfish participant, or trade may not even be allowed. We discuss how the selfishness affects the sequence of prices and the increase in utility.

Impact of Nash and Social Equilibria in an International Trade Model

Alberto Pinto
University of Porto, Portugal
F. Martins, M. Choubdar, J. Zubelli

We study an international trade model consisting of a strategic game in the tariffs of the governments. We consider a two-stage game where, at the first stage, governments of each country choose their tariffs competitively or socially for certain utilities that are relevant economic quantities, such as total output produced by the home firm, total quantity in the home market, inverse demand, consumer's savings, profits of the firms, custom revenue of the country and welfare of the country. In the second stage, firms choose competitively (Nash) their home and export quantities. We compare the competitive (Nash) tariffs with the social tariffs and classify the game according to the coincidence or not of these equilibria for each utility. The lack of coincidence of these equilibria is a main difficulty in international trade that can be partially dealt with the use of trade agreements.

Local Market Structure in a Hotelling Town

Alberto Pinto
University of Porto, Portugal
Telmo Parreira, J. P. Almeida

We develop a theoretical framework to study the location-price competition in a Hotelling-type network game, extending the Hotelling model, with linear transportation costs, from a line (city) to a network (town). We show the existence of a pure Nash equilibrium price if, and only if, some explicit conditions on the production costs and on the network structure hold. Furthermore, we prove that the local optimal localization of the firms are at the cross-roads of the town.

A Mathematical Model of Radicalization

Manuele Santoprete
Wilfrid Laurier University, Canada
Connell McCluskey

Radicalization is the process by which people come to adopt increasingly extreme political or religious ideologies. In recent years radicalization has become a major concern for national security because it can lead to violent extremism. Governments and security services are making a substantial effort to better understand the radicalization process and to identify the psychological, social, economic, and political circumstances that lead to violent extremism. It is in this context that this talk attempts to describe radicalization mathematically by modelling the spread of extremist ideology as the spread of an infectious disease. This is done by using a compartmental epidemiological model. We try to use this model to evaluate the effectiveness of some strategies to counter violent extremism.

Special Session 23: Numerical Methods for Phase-Field Models

Xiaoming Wang, Florida State University, USA
Steven Wise, University of Tennessee Knoxville, USA

Phase field models are becoming ever more important in the study of many multi-phase physical, chemical or biological processes. There has been a recent surge in the development of fast and accurate numerical methods for various phase field models that are of importance in applications. The purpose of this mini-symposium is to provide a platform for experts to report the state of the art progress and discuss future directions in numerical methods relevant to phase field models.

New Epitaxial Thin Film Models and Numerical Approximation

Wenbin Chen
Fudan University, Peoples Rep of China
Zhenhua Chen, Jin Cheng, Yanqiu Wang

This paper concerns new continuum phenomenological model for epitaxial thin film growth with three different forms of the Ehrlich-Schwoebel current. Two of these forms were first proposed by Politi and Villain and then studied by Evans, Thiel and Bartelt. The other one is completely new. Following the techniques used in Li and Liu, we present rigorous analysis of the well-posedness, regularity and time stability for the new model. We also studied both the global and the local behavior of the surface roughness in the growth process. The new model differs from other known models in that it features a linear convex part and a nonlinear concave part, and thus by using a convex-concave time splitting scheme, one can naturally build unconditionally stable semi-implicit numerical discretizations with linear implicit parts, which is much easier to implement than conventional models requiring nonlinear implicit parts. Despite this fundamental difference in the model, numerical experiments show that the nonlinear morphological instability of the new model agrees well with results of other models , which indicates that the new model correctly captures the essential morphological states in the thin film growth process.

A Second Order in Time Finite Element Scheme for the Cahn-Hilliard-Navier-Stokes Equation

Amanda Diegel
Louisiana State University, USA
Steven Wise, Cheng Wang, Xiaoming Wang

In this talk, we present a second order in time mixed finite element method for the Cahn-Hilliard equation coupled with a Navier-Stokes flow that models phase separation and coupled fluid flow in immiscible binary fluids in two and three dimensions. We will discuss the main results of the numerical scheme including the following. We show that our scheme is unconditionally energy stable with respect to a spatially discrete analogue of the continuous free energy of the system. Additionally, we show that the discrete phase variable is bounded in $L^\infty(0,T;L^\infty)$ and the discrete chemical potential is bounded in $L^\infty(0,T;L^2)$, for any time and space step sizes, in two and three dimensions, and for any finite final time T. We subsequently prove that these variables converge with optimal rates in the appropriate energy norms in both two and three dimensions.

Thermodynamically Consistent Modeling and Computations for Two-Phase Flows with Variable Density

Zhenlin Guo
University of California Irvine, USA
Ping Lin, John Lowengrub, Steven Wise

In this talk, we will present a phase-field model for binary incompressible fluid with thermocapillary effects, which allows for the different properties (densities, viscosities and heat conductivities) of each component while maintaining thermodynamic consistency. The governing equations of the model including the Navier-Stokes equations with additional stress term, Cahn-Hilliard equations and energy balance equation are derived within a thermodynamic framework based on entropy generation, which guarantees thermodynamic consistency. A sharp-interface limit analysis is carried out to show that the interfacial conditions of the classical sharp-interface models can be recovered from our phase-field model. Some numerical examples for the multiphase flows with and without thermocapillary effects will be presented. The results are compared to the corresponding analytical solutions and existing numerical results as validations for our model.

Decoupled Unconditioanlly Stable Schemes for Cahn-Hilliard-Darcy Type Equations

Daozhi Han
Indiana University Bloomington, USA
Xiaoming Wang

In this talk, we will present first-order and second-order, decoupled, unconditionally energy stable numerical schemes for solving the Cahn-Hilliard-Darcy type equations. The Darcy equations treated here include the classical Darcy equation and a variation of it with the time derivative retained. Numerical examples will be presented as well.

Continuum Models of Network Formation in Ionomer Membranes

Keith Promislow
Michigan State University, USA

Ionomer membranes, in particular Nafion, are well-known to be hysteric materials which display long time-scales associated with transient behavior, time-scales that are far outside the reach of even the most coarse-grained particle based simulations. An understanding of the mechanisms behind the transients states of Nafion is most readily obtained by the development of relatively simple models, based upon a dissipation of a free energy, which resolves the competition between various morphological states. The Functionalized Cahn-Hilliard free energy incorporates solvation energy of pendant ionic groups against interfacial bending energy and various contributions to the solvent-phase pressure, to develop an diffuse-interface expression for the interfacial free energy within functionalized-polymer/solvent mixtures. We present this free energy, a multiscale analysis of the solvent phase morphology, discuss challenges in the numerical resolution of the model, as well as identifying several possible scaling regimes for these most complex materials.

Characterizing the Stabilization Size for Semi-Implicit Fourier-Spectral Method to Phase Field Equations

Zhonghua Qiao
The Hong Kong Polytechnic University, Hong Kong
Dong Li, Tao Tang

Recent results in the literature provide computational evidence that stabilized semi-implicit time-stepping method can efficiently simulate phase field problems involving fourth-order nonlinear diffusion, with typical examples like the Cahn-Hilliard equation and the thin film type equation. The up-to-date theoretical explanation of the numerical stability relies on the assumption that the derivative of the nonlinear potential function satisfies a Lipschitz type condition, which in a rigorous sense, implies the boundedness of the numerical solution. In this work we remove the Lipschitz assumption on the nonlinearity and prove unconditional energy stability for the stabilized semi-implicit time-stepping methods. It is shown that the size of stabilization term depends on the initial energy and the perturbation parameter but is independent of the time step. The corresponding error analysis is also established under minimal nonlinearity and regularity assumptions.

A Diffuse Interface Model for Two-Phase Ferrofluid Flows

Abner Salgado
University of Tennessee, USA
Ricardo H. Nochetto, Ignacio Tomas

A ferrofluid is a liquid which becomes strongly magnetized in the presence of applied magnetic fields. It is a colloid made of nanoscale monodomain ferromagnetic particles suspended in a carrier fluid. These particles are suspended by Brownian motion and will not precipitate nor clump under normal conditions. Ferrofluids are dielectric and paramagnetic.
There are two well established PDE models used as a mathematical description for the behavior of ferrofluids: the Rosensweig and Shliomis models. These deal with one-phase flows, which is the case of many technological applications. However, some applications arise naturally in the form of a two-phase flow: one of the phases has magnetic properties while the other one does not (magnetic manipulation of microchannel flows, microvalves, magnetically guided transport, etc.).
We develop a model describing the behavior of two-phase ferrofluid flows using phase field techniques and present an energy-stable numerical scheme for it. For a simplified version of this model and the corresponding numerical scheme we prove, in addition to stability, convergence and, as a consequence, existence of solutions. With a series of numerical experiments we illustrate the potential of these simple models and their ability to capture basic phenomenological features of ferrofluids such as the Rosensweig instability.

Decoupled, Linear and Energy Stable Schemes for Phase-Field Models

Jie Shen
Purdue University, USA

I shall present unconditionally energy stable, decoupled numerical schemes which only require solving a sequence of linear elliptic equations at each time step for solving this coupled nonlinear system, and show ample numerical results which not only demonstrate the effectiveness of the numerical schemes, but also validate the flexibility and robustness of the phase-field model.

Long-Time Stability of a Regularized Family of Models for Homogeneous Incompressible Two-Phase Flows

Florentina Tone
University of West Florida, USA
T. Tachim Medjo, C. Tone

In this talk we present results on the stability of the fully implicit Euler scheme for a regularized family of models for an incompressible two-phase flow model. More precisely, we consider the time discretisation scheme and with the aid of the discrete Gronwall lemma and of the discrete uniform Gronwall lemma we prove that the numerical scheme is stable.

A Second Order Accurate and Efficient Numerical Scheme for the Cahn-Hilliard-Darcy System

Xiaoming Wang
Florida State University, USA
Daozhi Han

We propose a novel second order in time, decoupled and unconditionally stable numerical scheme for solving the Cahn-Hilliard-Darcy (CHD) system which models two-phase flow in porous medium or in a Hele-Shaw cell. The scheme is based on the ideas of second order convex- splitting for the Cahn-Hilliard equation and pressure-correction for the Darcy equation. We show that the scheme is uniquely solvable, unconditionally energy stable and mass-conservative. Ample numerical results are presented to gauge the efficiency and robustness of our scheme.

An Arbitrary-Lagrangian-Eulerian-Phase-Field Method for Contact-Line Dynamics on Moving Particles

Pengtao Yue
Virginia Tech, USA

In this talk, we will present a hybrid Arbitrary-Lagrangian-Eulerian(ALE)-Phase-Field method for the direct numerical simulation of multiphase flows where fluid interfaces, moving rigid particles, and moving contact lines coexist. Practical applications include Pickering emulsions, froth flotation, and biolocomotion at fluid interface. An ALE algorithm based on the finite element method and an adaptive moving mesh is used to track the moving boundaries of rigid particles. A phase-field method based on the same moving mesh is used to capture the fluid interfaces; meanwhile, the Cahn-Hilliard diffusion automatically takes care of the stress singularity at the moving contact line when a fluid interface intersects a solid surface. To fully resolve the diffuse interface, mesh is locally refined at the fluid interface. All the governing equations, i.e., equations for fluids, interfaces, and particles, are solved implicitly in a unified variational framework. As a result, the hydrodynamic forces and moments on particles do not appear explicitly in the formulation and an energy law holds for the whole system. In the end, we will present some results on the water entry problem and capillary interaction between floating particles, with a focus on the effect of contact-line dynamics.

On Energy-Stable Schemes for Complex-Fluid Models

Jia Zhao
University of South Carolina, USA
Qi Wang, Xiaofeng Yang

Complex fluids are fluids whose micro-structure have impact on the fluid macroscopic properties, which include complex fluid mixtures of different types. In this talk, I will first present a systematic development of a general hydrodynamic model for complex fluid system using the generalized Onsager relation. Then, a semi-discrete scheme to solve this general model, which satisfies the discrete energy dissipation law, will be presented. Specific tricks on linearizing and decoupling the schemes will be presented for particular reduced models. In the end, several 3D simulations will be shown to illustrate the effectiveness of our schemes.

Special Session 24: SPDEs/SDEs and Stochastic Systems with Control/Optimization and Applications

Wanyang Dai, Nanjing University, Peoples Rep of China

We will discuss theory, methods, and numerical schemes of stochastic partial differential equations (SPDEs), stochastic ordinary differential equations (SDEs), and general stochastic dynamical systems. Furthermore, we will also talk about their interactions with and applications in stochastic optimal controls and stochastic differential games, queueing networks and discrete event systems, statistical mechanics and quantum physics, service and financial systems, computer and communication networks, etc.

Particle Representations for Stochastic Partial Differential Equations with Boundary Conditions

Dan Crisan
Imperial College London, England
C. Janjigian, T. G. Kurtz

I discuss a weighted particle representation for a class of stochastic partial differential equations with Dirichlet boundary conditions. The locations and weights of the particles satisfy an infinite system of stochastic differential equations (SDEs). The evolution of the particles is modelled by an infinite system of stochastic differential equations with reflecting boundary condition and driven by independent finite dimensional Brownian motions. The weights of the particles evolve according to an infinite system of stochastic differential equations driven by a common cylindrical noise W and interact through V, the associated weighted empirical measure. When the particles hit the boundary their corresponding weights are assigned a pre-specified value. We show the existence and uniqueness of a solution of the infinite dimensional system of stochastic differential equations modeling the location and the weights of the particles. We also prove that the associated weighted empirical measure V is the unique solution of a nonlinear stochastic partial differential equation driven by W with Dirichlet boundary condition. The work is motivated by and applied to the stochastic Allen-Cahn equation. This joint work with C. Janjigian and T. G. Kurtz.

A Unified System of SPDEs with Levy Jumps Vs. Stochastic Differential Games

Wanyang Dai
Nanjing University, Peoples Rep of China

We study a unified system of stochastic partial differential equations (SPDEs) with Levy jumps in a forward-backward coupling manner. The partial differential operators in its drift, diffusion, and jump coefficients are in time-variable and position-parameters over a domain (e.g., a hyperbox or a manifold). A solution to the system is defined by a 4-tuple random vector-field process evolving in time. Since our unified system is a general-dimensional vector one with general nonlinearity and general high-order, the popular computation (e.g., integration by parts) based proof method can not be applied. Thus, we develop an approach to prove the well-posedness of an adapted 4-tuple strong solution to the system in a topological space and under a sequence of generalized local linear growth and Lipschitz conditions. As the further investigation of our system, we formulate a non-zero-sum stochastic differential game (SDG) problem with general number of players. By a 4-tuple solution to the system, we get a Pareto optimal Nash equilibrium policy to the SDG. In addition, illustrative examples from quantum physics, statistical mechanics, and queueing networks are also presented.

Stochastic Systems with Memory and Jumps

Giulia di Nunno
University of Oslo, Norway
D.R. Banos, F. Cordoni, L. Di Persio, E. Rose

Stochastic systems with memory naturally appear in life science, economy, and finance. We take the modeling point of view of stochastic functional delay equations and we study these structures when the driving noises admit jumps. Our results concern existence and uniqueness of strong solutions, estimates for the moments and the fundamental tools of calculus, such as the Ito formula. We study the robustness of the solution to the change of noises. Specifically, we consider the noises with infinite activity jumps versus an adequately corrected Gaussian noise. Our techniques include tools of infinite dimensional calculus and the stochastic calculus via regularization.

Optimal Life-Insurance Selection and Purchase Within a Market of Several Life-Insurance Providers

Abdelrahim Mousa
Birzeit University, Israel
D. Pinheiro, A. A. Pinto

We consider the problem faced by a wage-earner with an uncertain lifetime having to reach decisions concerning consumption and life-insurance purchase, while investing his savings in a financial market comprised of one risk-free security and an arbitrary number of risky securities whose prices are determined by diffusive linear stochastic differential equations. We assume that life-insurance is continuously available for the wage-earner to buy from a market composed by a fixed number of life-insurance companies offering

pairwise distinct life-insurance contracts. We characterize the optimal consumption, investment and life-insurance selection and purchase strategies for the wage-earner with an uncertain lifetime and whose goal is to maximize the expected utility obtained from his family consumption, from the size of the estate in the event of premature death, and from the size of the estate at the time of retirement. We use dynamic programming techniques to obtain an explicit solution in the case of discounted constant relative risk aversion (CRRA) utility functions.

Differentiability of Stochastic Flows and Sensitivity Analysis of Reflected Diffusions in Convex Polyhedral Domains

Kavita Ramanan
Brown University, India
David Lipshutz

Differentiability of flows and sensitivity analysis are classical topics in dynamical systems. However, the analysis of these properties for constrained processes, which arise in a variety of applications, is challenging due to the discontinuities in the dynamics at the boundary of the domain, and is further complicated when the boundary is non-smooth. We show that the study of both flows and sensitivities of constrained processes in convex polyhedral domains can be largely reduced to the study of directional derivatives of an associated map, called the extended Skorokhod map, and we introduce an axiomatic framework to characterize these directional derivatives. In addition, we establish pathwise differentiability of a large class of reflected diffusions in convex polyhedral domains and show that they can be described in terms of certain constrained stochastic differential equations with time-varying domains and directions of reflection. This is joint work with David Lipshutz.

Backward Uniqueness for a Class of SPDE

Michael Röckner
Bielefeld University, Germany
Viorel Barbu

We present recent results on backward uniqueness of solutions to stochastic semilinear parabolic equations and also for the tamed 3D Navier-Stokes equations driven by linear multiplicative Gaussian noises. In the first case we use a rescaling transformation to reduce the SPDE to a random PDE. Applications to approximate controllability of nonlinear stochastic parabolic equations with initial controllers are given. The method of proof relies on the logarithmic convexity property known to hold for solutions to linear evolution equations in Hilbert spaces with self-adjoint principal part.

Stochastic Control of Path-Dependent Systems, Application to the Principal-Agent Problem

Nizar Touzi
Ecole Polytechnique, France
Dylan Possamai, Jaksa Cvitanic

We consider a general formulation of the Principal-Agent problem. Our approach is the following: we first find the contract that is optimal among those for which the Agent's value function allows for the dynamic programmin approach. We then show the optimization over this restricted family represents no moss of generality. Hence, this reduces the non-zero sum stochastic differential game to a standard stochastic control problem which can then be analyzed by standard tools of control theory.
Our proofs rely on the Backward Stochastic Differential Equations approach to Non-Markovian stochastic control, and more specifically, on the recent extensions to the second order case.

Backward Stochastic Dynamics with a Subdifferential Operator and Non-Local Parabolic Variational Inequalities

Phillip Yam
Chinese University of Hong Kong, Hong Kong
Alain Bensoussan, Yiqun Li

In this talk, we introduce the first systematic study on the unique existence of the solution of backward stochastic dynamical variational inequalities (BSDVI) on a general complete filtered probability space. On the one hand, penalized method is used with a novel application of the backward-Gronwall inequality to construct the a-priori estimates of the unknown martingale, including that of its indeterminate quadratic variation, adapted to the general filtration. On the other hand, the unique existence of the weak solution of the parabolic variational inequality is also established by making an association with a suitable BSDVI. It should be emphasized that the concept of viscosity solution could not be adopted since the potential function involved is non-local.

Robust Dynkin Game

Song Yao
University of Pittsburgh, USA
Erhan Bayraktar

We analyze a robust version of the Dynkin game over a set P of mutually singular probabilities. We first prove that conservative player's lower and upper value coincide (Let us denote the value by V). Such a result connects the robust Dynkin game with second-order doubly reflected backward stochastic differential equations. Also, we show that the value process

V is a submartingale under an appropriately defined nonlinear expectations up to the first time τ_* when V meets the lower payoff process. If the probability set P is weakly compact, one can even find an optimal triple (P_*, τ_*, γ_*) for the value V_0.

Exponential Convergence for 3D Stochastic Primitive Equations of the Large Scale Ocean

Dong Zhao
Academy of Mathematics and Systems Science CAS, Peoples Rep of China

In this paper, we consider the ergodicity for the three-dimensional stochastic primitive equations of the large scale oceanic motion. We proved that if the noise is sufficiently smooth and non-degenerate, the weak solutions converge exponentially fast to equilibrium. Moreover, the uniqueness of invariant measure is stated.

Method of Evolving Junctions (MEJ) for Optimal Control with Constraints

Haomin Zhou
Georgia Tech, USA
Shui-Nee Chow, Magnus Egerstedt, Wuchen Li, and Jun Lu

We design a new stochastic differential equation (SDE) based algorithm to efficiently compute the solutions of a class of infinite dimensional optimal control problems with constraints on both state and control variables. The main ideas include two parts. 1) Use junctions to separate paths into segments on which no constraint changes from active to in-active, or vice versa. In this way, we transfer the original infinite dimensional optimal control problems into finite dimensional optimizations. 2) Employ the intermittent diffusion (ID), a SDE based global optimization strategy, to compute the solutions efficiently. It can find the global optimal solution in our numerical experiments. We illustrate the performance of this algorithm by several shortest path problems, the frogger problem and generalized Nash equilibrium examples.

On Feller and Strong Feller Properties of Regime-Switching Jump Diffusions

Chao Zhu
University of Wisconsin-Milwaukee, USA
Fubao Xi

This work considers the martingale problem for a class of weakly coupled Lèvy type operators. It is shown that under some mild conditions, the martingale problem is well-posed and uniquely determines a strong Markov process (X, Λ). The process (X, Λ), called a regime-switching jump diffusion with Lèvy type jumps, is further shown to posses Feller and strong Feller properties via the coupling method.

Special Session 25: Applied Analysis and Dynamics in Engineering and Sciences

Thomas Hagen, University of Memphis, USA
Florian Rupp, German University of Technology, Oman

The goal of this session is to bring together mathematicians who work in different areas of applied mathematics and might thus not meet and exchange ideas and points of view. Consequently, the session program addresses a cross section of theoretical and computational developments and their applications to fluid dynamics, solid mechanics and life sciences. Areas of interest include the theory of differential equations, in particular evolution equations and stochastic differential equations, stability and asymptotics, control theoretic issues, numerical results and computational methods, and related aspects.

Rational Decays for Fluid-Structure PDE Models

George Avalos
University of Nebraska-Lincoln, USA
Roberto Triggiani

In this paper, we consider a fluid-structure PDE model of longstanding interest within the mathematical and biological sciences. Here, a Stokes system and vector-valued wave equation comprise the coupled PDE system under study; these respective PDE components come into contact via a boundary interface. For this fluid structure system, our main result is as follows: Under an appropriate geometric assumption which precludes imaginary point spectrum for the associated semigroup generator, then for smooth initial data - i.e., data in the domain of said generator - the corresponding solutions decay at a certain polynomial rate.

On the Stationary Solutions of Non-Isothermal Film Casting with Unknown Frost Point

Shaun Ceci
Le Moyne College, USA
Thomas Hagen

In this talk, we will examine a one-dimensional model for film casting, an industrial process used to manufacture thin sheets and films from a highly viscous polymer melt. The flow is assumed to be nonisothermal and dominated by viscous forces with temperature-dependent viscosity and an a priori unknown frost point. In particular, we will focus our discussion on the existence, uniqueness, and linear stability of stationary solutions to the governing equations.

Computation of Lyapunov Functions by Convex Optimization

Sigurdur Hafstein
Reykjavik University, Iceland

A Lyapunov function for a dynamical system delivers valuable information on the system's qualitative behavior. The algorithmic computation of Lyapunov functions for nonlinear systems has received considerable attention in the last two decades and there has been major progress in the methods developed. We give an overview of some of the methods developed and precent some recent developments in the computation of control- and ISS (input-to-state stability) Lyapunov functions for nonlinear systems.

Surface Tension Driven Flow in Networks

Thomas Hagen
University of Memphis, USA

In this presentation we discuss the dynamics and stability of surface tension driven fluid flow in networks of channels. The mechanism behind the dynamics of such flows is volume scavenging of capillary drops due to changes in pressure. We study the resulting droplet coarsening for Newtonian and non-Newtonian fluid models in the underlying gradient-like flow equations. An important aspect of the study is the occurrence of heteroclinic orbits connecting stationary solutions.

Basins of Attraction in Stochastically Excited Systems

Florian Rupp
German University of Technology in Oman, Oman

We will discuss approximation techniques for basins of attractions of stochastically asymptotically stable equilibrium points in dynamical systems generated by Stochastic and Random Ordinary Differential Equations. Hereby special attention will be given to an extension of the deterministic sums of square method to compute suitable level sets of stochastic Lyapunov functions were we give error bounds for the approximation and validate the quality of the approximation. Our examples will in particular cover first examples from earthquake and offshore engineering.

Approximate Fourier-Series Solutions of Stochastic Sine-Gordon Equations

Henri Schurz
SIU, USA

An analysis of approximate Fourier solutions of modified stochastic Sine-Gordon equations is presented. We prove existence and uniqueness of Fourier series solutions under Dirichlet-type boundary conditions. For this purpose, make use of Lyapunov-functionals (energy-type based methods), truncate and control the dynamics of the original infinite-dimensional stochastic system by finite-dimensional systems of stochastic differential equations. We arrive at uniform energy-type estimates for its series solutions. If time permits, we shall also discuss some stability results. The advantage is seen by a direct preparation for the use and qualitative control of adequate numerical methods.

Semigroup Well-Posedness for the Total Linearization of a Free-Boundary Hydro-Elastic Interaction

Daniel Toundykov
University of Nebraska-Lincoln, USA
Lorena Bociu, Jean-Paul Zolesio

We investigate wellposedness of a linearization around a steady regime of a free-boundary fluid-structure interaction model. The hydro-elastic equations and the free boundary were linearized together which results in a system rather different from the classical coupling of the Stokes flow and linear elastodynamics. New terms emerge on the common interface, some of them involving boundary curvatures. We proceed to establish that the associated evolution operator generates a strongly continuous semigroup.

Numerical Solutions of a Class of Singular Neutral Functional Differential Equations on Graded Meshes

Janos Turi
UTD, USA
Pedro Perez-Nagera

In this talk we present case studies to illustrate the dependence of the rate of convergence of numerical schemes for singular neutral equations (SNFDEs) on the particular discretization employed in the computation. Based on our numerical experiments we observe that the ideal mesh (i.e., resulting in the highest achievable rate of convergence) for the SNFDE under consideration is a discretization corresponding to equal integrals of its kernel function.

Existence of Periodic and Multiple Spike Standing Waves in Coupled Reaction-Diffusion Systems

Fu Zhang
Cheyney University of Pennsylvania, USA
Stuart P. Hastings

In this talk we present our proof of the existence of periodic patterns in a coupled homogeneous reaction-diffusion system employing topological shooting arguments and the existence of multiple spike standing waves in a coupled non-homogeneous system by a perturbation method.

Special Session 26: Hamiltonian Systems and the Planetary Problem

Gabriella Pinzari, University of Naples "Federico II", Italy

The study of Hamiltonian systems is of relevant interest for Physics. These may consist of classical systems, namely with a finite number of degrees of freedom, like the N-body problem, the planetary problem, the rigid body, billiards, the spin-orbit system, or, more generally, of extended systems, with a infinite number of degrees of freedom, like, for example, the Schrödinger equation, the wave equation, the Euler equations of hydrodynamics. Since the early 50s, many robust techniques have been applied firstly to classical and next also to extended systems, like the theorem of Kolmogorov, Moser and Arnold, the theorem of Nekhorossev, Arnold's instability for systems with more than two degrees of freedom, splitting of separatrices, variational techniques, Mather theory. In this special session, we aim to gather specialist in this field, to outline the status of the art and perspectives.

Diffusion Along Chains of Normally Hyperbolic Cylinders

Marian Gidea
Yeshiva University, USA
Jean-Pierre Marco

We consider chains of 3-dimensional normally hyperbolic invariant cylinders with boundary. We assume that the unstable manifolds of each cylinder intersects both the stable manifold of the same cylinder, and the stable manifold of the next cylinder in the chain. We make some further assumptions on these intersections, that amount to the existence of a certain family of locally defined scattering maps on each cylinder, and on the dynamics restricted to each cylinder. Under these assumptions we prove the the existence of diffusion orbits that drift along the cylinder chains. Our approach is geometric, extending a method introduced by Moeckel on constructing connecting orbits inside a zone of instability for a twist map on the annuls. The motivation of our work resides with the a priori stable case of the Arnold diffusion problem.

The Scattering Map in a Piezoelectric Energy Harvester

Albert Granados
Technical University of Denmark, Denmark
Tere Seara

In this talk we consider an energy harvesting system based on two piezoelectric oscillators modeled by duffing equations. When forced to oscillate, for instance when driven by a small periodic vibration, the oscillators create an electrical current which charges an accumulator (a capacitor or a battery). The electrical circuit also couples the oscillators adding an extra dimension to the system. We aim to identify trajectories that benefit the absorption of energy from the source and somehow optimize the energy harvester. To this end, we use techniques in which is based a common approach for the study of Arnold diffusion, typically associated with celestial dynamics. In the absence of dissipation (given by the damping in the duffing equations), coupling and forcing, the system possesses a 3-dimensional Normally Hyperbolic Manifold. We use a modified Melnikov method to study the existence of 4-dimensional homoclinic intersections in the presence of coupling and the small periodic forcing. We then study the scattering map associated with homoclinic excursions allowing us to identify trajectories injecting energy from the source to one of the oscillators.

Arnold Diffusion of Charged Particles in ABC Magnetic Fields

Alejandro Luque
Instituto de Ciencias Matem'aticas, Spain
Daniel Peralta-Salas

In this talk we prove the existence of diffusing solutions in the motion of a charged particle in the presence of an ABC magnetic field. The equations of motion are modeled by a 3DOF Hamiltonian system depending on two parameters. For small values of these parameters, we obtain a normally hyperbolic invariant manifold and we apply the so-called geometric methods for a priori unstable systems developed by A. Delshams, R. de la Llave, and T.M. Seara. We characterize explicitly sufficient conditions for the existence of a transition chain of invariant tori having heteroclinic connections, thus obtaining global instability (Arnold diffusion). We also check the obtained conditions in a computer assisted proof. This is a joint work with Daniel Peralta-Salas.

On the Lax-Oleinik Semigroup of Some Gravitational Problems

Ezequiel Maderna
Universidad de la Republica, Uruguay

The Lax-Oleinik semigroup associated to a Tonelli Lagrangian on a compact manifold gives a very fruitful link between the dynamics of the Euler-Lagrange flow and the viscosity solutions of the Hamilton-Jacobi equation. More precisely, the invariant sets coming from the Aubry-Mather theory can be characterized in terms of the fixed points of the Lax-Oleinik semigroup, or weak KAM solutions. I will show in this talk that this method also works for more general Lagrangian systems with singularities, like several gravitational problems.

Oscillatory Orbits in the Restricted Elliptic Planar Three Body Problem

Pau Martin
Universitat Politecnica de Catalunya, Spain
Marcel Guardia, Tere M Seara

The restricted planar elliptic three body problem models the motion of a massless body under the Newtonian gravitational force of two other bodies, the primaries, which evolve in Keplerian ellipses. A trajectory is called oscillatory if it leaves every bounded region but returns infinitely often to some fixed bounded region. We prove the existence of such type of trajectories for any values for the masses of the primaries provided the eccentricity of the Keplerian ellipses is small.

From Moser's Normal Form to Dissipative KAM Theory. an Application to the Spin-Orbit Problem.

Jessica Elisa Massetti
Université Paris-Dauphine and IMCCE, Observatoire de Paris, France

In 1967 J. Moser established a powerful normal form theorem for real analytic perturbations of vector fields possessing an invariant reducible quasi-periodic torus of Diophantine frequencies. From this normal form, in some particular cases issued from Hamiltonian Mechanics and its dissipative versions issued from Celestial Mechanics, we show the existence of particular remarkable normal forms: à la Herman and à la Rüssmann. Through these normal forms, it's possible to deduce KAM-type results if the system depends in an opportune way on a sufficient number of free parameters - internal or external to it. The persistence result is hence obtained through a technique of elimination of parameters, set up by Rüssmann, Herman and other authors in the 80s-90s. In this geometric frame the dissipative spin-orbit problem of Celestial Mechanics (recently presented by Celletti-Chierchia and Locatelli-Stefanelli), can more easily be handled: deducing the existence of quasi-periodic attractors becomes a particular case of small dimension. Moreover, the process of elimination of parameters highlights relations among dissipation, frequency and perturbation proper to this system and brings out a better understanding of their role, opening the way to a global study in the parameters' space on the persistence of different kinds of motions under perturbation.

Beatings for the NLS Equation

Michela Procesi
Università di Roma tre, Italy
Emanuele Haus

We prove the existence and stability of a class of simple quasi-periodic solutions for the NLS equation on tori which exhibit energy transfer phoenomena.

The Problem of Global Regularity for Water Waves

Fabio Pusateri
Princeton University, USA

We will discuss some recent works on the problem of global regularity for the water waves equations, focusing in particular on the role played by resonances and normal forms in the understanding of the long-time behavior.

Numerical Study of the 3:1 Resonance with Application to Diffusion in the RTBP

Pablo Roldan
ITAM, Mexico

Astronomical observations show that the Main Asteroid Belt has some gaps corresponding to those asteroids in mean-motion resonance with Jupiter. This physical phenomenon can be explained in terms of Arnold diffusion. We consider the classical (planar, circular) Restricted Three Body Problem, modeling the Sun-Jupiter-Asteroid system, and study the geometric structure of the $3:1$ resonance numerically. Namely, we compute the Normally Hyperbolic Invariant Manifold of resonant periodic orbits, its associated (un)stable manifolds, two different homoclinic manifolds, and their corresponding splitting functions.

An Approximation Theorem in Classical Mechanics

Cristina Stoica
Wilfrid Laurier University, Waterloo, Canada

A theorem by K. Meyer and D. Schmidt says that "The reduced three-body problem in two or three dimensions with one small mass is approximately the product of the restricted problem and a harmonic oscillator (Transactions AMS, 352, 2000). This theorem was used to prove dynamical continuation results from the classical restricted circular three-body problem to the three-body problem with one small mass. We examine the analogue statement in a broader class mechanical systems and state a definition of restricted problems. We state and prove a similar theorem applicable to a larger class of mechanical systems. We present applications to the spherical double pendulum with a small mass at the free end, the spatial (N+1)-body systems with one small mass, and gravitationally coupled systems formed by a rigid body and a small point mass.

Positive Lyapunov Exponents for Some Randomly Perturbed 2D Conservative Maps

Jinxin Xue
University of Chicago, USA
Alex Blumenthal, Lai-Sang Young

Positive Lyapunov exponent is an important characterization of the exponential instability and chaos in dynamical systems. However, it is a well-known hard problem to prove positive Lyapunov exponents in conservative concrete systems. On the other hand, conservative maps appears naturally by taking the Poincare return map in Hamiltonian systems, and the Chirikov standard map can be considered as a model of the Poincare map near separatrix in Hamiltonian systems with two degrees of freedom. In this talk, we show the positive Lyapunov exponent of a class of two dimensional maps with the help of a tiny random perturbation.

Normally Hyperbolic Laminations in a Priori Unstable Systems

Ke Zhang
Department of Mathematics, University of Toronto, Canada
Vadim Kaloshin, Jianlu Zhang

We construct normally hyperbolic laminations for an a priori unstable Hamiltonian system, using the separatrix maps of Treschev. Normally hyperbolic laminations provide a model of stochasticity in Arnold diffusion.

Special Session 27: Advances in the Mathematical Modeling of Failure Phenomena and Interfaces in Materials

Marco Morandotti, SISSA - International School for Advanced Studies, Italy
Marco Barchiesi, Università di Napoli "Federico II", Italy
José Matias, Instituto Superior Técnico, Universidade de Lisboa, Portugal

In recent years, there has been an ever-increasing interest in the development of mathematical techniques capable to describe the interplay between microscopic and macroscopic theories in material science. Bridging different (length and time) scales is crucial to grasp the fine structural behavior of the equilibrium configurations. The challenge is two-fold: capturing the emergence of microstructure from meso- and macroscopic models, and deriving effective macroscopic models from microscopic ones. Tackling these issues is interesting both for the mathematical and for the engineering communities: for the former, this involves dealing with the minimization of non-convex and non-local energies; for the latter, it provides formalization and validation of experimental models. This special session will focus on current research topics including dislocation theory, pattern formation, fracture mechanics, plasticity, and their numerical implementation.

Ground States for a Ternary System with Coulomb Interaction

Marco Bonacini
University of Bonn, Germany
Hans Knuepfer

We study a variational model where two phases - interacting via attractive and repulsive Coulomb forces - are embedded in a third homogeneous phase, describing for instance systems of copolymer-homopolymer blends or of surfactants in water solutions. The energy of the system is the sum of a local interfacial contribution and a nonlocal interaction of Coulomb type. We establish existence and regularity properties of global minimizers, together with a full characterization of minimizers in the small mass regime. Furthermore, we prove uniform bounds on the potential of minimizing configurations, which in turn imply some qualitative estimates about the geometry of minimizers in the large mass regime.

Cohesive Fracture Evolutions: Existence Results and Applications

Filippo Cagnetti
University of Sussex, England

I will start by recalling an abstract existence theorem for the time evolution of cohesive fractures. The above result has some interesting features, but it is not easily implementable by a computer. I will then discuss a recent work, in collaboration with Marco Artina, Massimo Fornasier, and Francesco Solombrino (from Technical University of Munich), in which the model is modified, in such a way that numerical simulations can be done.

Periodic Critical Points of the Ohta-Kawasaki Functional

Riccardo Cristoferi
Carnegie Mellon University, USA

In this talk we present some new observations about periodic critical points and local minimizers of a non-local isoperimetric problem arising in the modeling of diblock copolymers. In particular, by using a purely variational procedure, we show that it is possible to construct (locally minimizing) periodic critical points whose shape resemble that of any given strictly stable constant mean curvature (periodic) hypersurface.

Existence and Uniqueness of Dynamic Evolutions for a Peeling Test in Dimension One

Gianni dal Maso
SISSA, Italy
Giuliano Lazzaroni, Lorenzo Nardini

In this paper we present a one-dimensional model of a dynamic peeling test for a thin film, where the wave equation is coupled with a Griffith criterion for the propagation of the debonding front. Our main results provide existence and uniqueness for the solution to this coupled problem under different assumptions on the data.

Wulff Shape Emergence in Graphene

Elisa Davoli
University of Vienna, Austria
Paolo Piovano, Ulisse Stefanelli

Graphene samples are identified as minimizers of configurational energies featuring both two- and three-body atomic-interaction terms. This variational viewpoint allows for a detailed description of ground-state geometries as connected subsets of a regular hexagonal lattice. We investigate here how these geometries evolve as the number n of carbon atoms in the graphene sample increases. By means of an equivalent characterization of minimality via a discrete isoperimetric inequality, we prove that ground states converge to the ideal hexagonal Wulff shape as $n \to +\infty$. Precisely, we show that ground states deviate from such hexagonal Wulff shape by at most $Kn^{3/4} + o(n^3/4)$ atoms.

Ground States of a Two Phase Model with Cross and Self Attractive Interactions.

Lucia de Luca
Technical University of Munich, Germany
Marco Cicalese, Matteo Novaga, Marcello Ponsiglione

We consider a variational model for two interacting species (or phases), subject to cross and self attractive interactions. We show existence and several qualitative properties of minimizers. Depending on the strengths of the attractive forces, minimizers can exhibit different behaviors: phase mixing or phase separation with nested or disjoint phases. For the special case of Coulomb interaction forces, we fully characterize the ground state configurations, by giving its explicit shape.

Second Order Gamma-Convergence for the Modica Mortola Functional

Irene Fonseca
Carnegie Mellon University, USA
Gianni Dal Maso, Gurgen Hayrapetyan, Giovanni Leoni, Matteo Rinaldi, Barbara Zwicknagl

The asymptotic behavior of an anisotropic Cahn-Hilliard functional with prescribed mass and Dirichlet boundary condition is studied when the parameter that determines the width of the transition layers tends to zero. The first order term in the asymptotic development by Gamma-convergence is well-known, and is related to a suitable anisotropic perimeter of the interface. Here it is shown that, depending on symmetry and growth hypotheses on the double well potential, the second order term in the Gamma-convergence expansion is zero. Slow motion is addressed, and related estimates of the rate of convergence of solutions of the associated Allen-Cahn equation to the minimum value are discussed.

Linear and Nonlinear Stability of Bilayers Under the Functionalized Cahn-Hilliard Gradient Flow

Gurgen Hayrapetyan
Ohio University, USA
K. Promislow

Functionalized energies, such as the Functionalized Cahn-Hilliard, model phase separation in amphiphilic systems, in which interface production is energetically favorable, but is limited by competition for surfactant phase, which wets the interface. This is in contrast to classical phase-separating energies, such as the Cahn-Hilliard, in which interfacial area is energetically penalized. We discuss the linear and nonlinear stability of bilayer interfaces under an associated mass-preserving gradient flow. In particular, for sufficiently small perturbations of radial bilayers, we show that there is a unique decomposition into a non-radial bilayer and a decaying remainder, and that as the remainder decays the perturbed interface relaxes back radial symmetry through a sharp-interface motion governed to leading order by a linearized Willmore flow.

Integral Representation for Functionals Defined on SBD^p in Dimension Two

Flaviana Iurlano
IAM, University of Bonn, Germany
Sergio Conti, Matteo Focardi

We present an integral representation result for functionals with growth conditions which give coercivity on the space $SBD^p(\Omega)$, for$\Omega \subset \mathbb{R}^2$. The space SBD^p of functions whose distributional strain is the sum of an L^p part and a bounded measure supported on a set of finite $\mathcal{H}^1$-dimensional measure appears naturally in the study of fracture and damage models. Our result is based on the construction of a local approximation by $W^{1,p}$ functions. We also obtain a generalization of Korn's inequality in the SBD^p setting.

Rigidity of Discrete Energies with Surface Scaling: Interactions Beyond Nearest Neighbours Versus Non-Interpenetration

Giuliano Lazzaroni
SISSA, Trieste, Italy
Roberto Alicandro, Mariapia Palombaro

We present some discrete models for crystals with surface scaling of the interaction energy. We assume that at least nearest and next-to-nearest neighbour interactions are taken into account. Our purpose is to show that interactions beyond nearest neighbours have the role of penalising changes of orientation and, to some extent, they may replace the positive-determinant constraint that is usually required when only nearest neighbours are accounted for.

A Model for Dislocations in Epitaxially Strained Elastic Films

Giovanni Leoni
Carnegie Mellon University, USA
Nicola Fusco, Irene Fonseca, Massimiliano Morini

We present a variational model for nucleation of dislocations in epitaxially strained films. This is joint work with Nicola Fusco, Irene Fonseca, and Massimiliano Morini.

Optimal Location of Dislocations in a Crystal with Prescribed External Strain

Ilaria Lucardesi
SISSA, Trieste, Italy
M. Morandotti, R. Scala, D. Zucco

Dislocations are point defects appearing in crystals and explain the observation of plastic deformations. In this talk I propose a location problem for screw dislocations, in presence of a Dirichlet boundary condition, which prevents dislocations to migrate to the boundary and leave the domain. The study is carried out with techniques of Gamma-convergence and PDEs, in the framework of the so-called "core-radius approach".

Variational Models for Dislocations at Semi-Coherent Interfaces

Marcello Ponsiglione
Sapienza University of Rome, Italy
Silvio Fanzon, Mariapia Palombaro

We discuss some simple variational models for dislocations at semi-coherent interfaces. The energy functional describes the competition between two terms: a surface energy induced by dislocations and a far field elastic energy, spent to decrease the amount of needed dislocations. We prove that the former scales like the surface area of the interface, the latter like its diameter. The proposed continuum model is deduced from the semi-discrete theory of dislocations. Even if we deal with finite elasticity, linearized elasticity naturally emerges in our analysis since the far field strain contribution vanishes as the interface size increases.

A Bridging Mechanism in the Homogenization of Brittle Composites with Soft Inclusions

Caterina Ida Zeppieri
University of Muenster, Germany
Marco Barchiesi, Giuliano Lazzaroni

We study the limit behavior of the energy associated with a purely brittle composite whose microstructure is characterized by soft inclusions periodically embedded in a stiffer matrix. We exhibit an elementary micro-geometry for the composite which gives rise, in the limit, to a cohesive-zone energy.

Special Session 28: Recent Developments Related to Conservation Laws and Hamilton-Jacobi Equations

Laura Caravenna, Università di Padova, Italy
Annalisa Cesaroni, University of Padova, Italy
Hung Vinh Tran, University of Wisconsin Madison, USA

The session focuses on some recent developments of first order nonlinear Partial Differential Equations, and in particular conservation laws, Hamilton-Jacobi equations and related topics such as dynamical properties and homogenization. Recently the joint analysis of conservation laws and Hamilton-Jacobi equations on heterogeneous structures has received an increasing attention. This includes problems on networks and their applications to modeling of traffic flows, homogenization in periodic and random media, dynamical properties of solutions, etc. One of the main motivation for problems on networks is the application to dynamic models of traffic flow, e.g. the flowing of cars on a highway or of gas along pipelines or of packages of data on telecommunication networks. The established mathematical framework of these models consists of single conservation laws, systems of conservation or balance laws running on a network modeled as a topological graph. These models are widely used also in engineering: it is an area where fundamental studies are interesting but which is also directly related to applications. More recently, a complementary analysis of the network dynamics based on Hamilton-Jacobi equations has also been developed. The current trend consists of proposing new models/problems, studying their well-posednesses, dynamical properties (optimal control formulas, large time behaviors), and related homogenization problems. Numerical approaches are as well of great interest. Homogenization problems are about finding averaged (effective) properties of solutions to inhomogeneous equations depending on small parameters and set in self averaging media. The area is moving fast in various perspectives: qualitative and quantitative properties of the effective equations, rates of convergences, stochastic homogenization of front propagations, and non-convex Hamilton-Jacobi equations, etc. Moreover, there have been a lot of developments connecting homogenization and problems on networks such as homogenization on junction framework, Hamilton-Jacobi equations on a network as a limit of a singularly perturbed problem in optimal control defined on thin strips around the network. The aim of this session is to bring together mathematicians working on conservation laws and Hamilton-Jacobi from different backgrounds and perspectives, including homogenization. It will be a great occasion to present the new advances and directions of different research groups, in order to interact and to improve the understanding of these exciting topics that have been considered intensively in the last few years.

Global Existence of Prandtl-Meyer Reflection and Optimal Regularity Results

Myoungjean Bae
POSTECH, Korea
Gui-Qiang G. Chen, Mikhail Feldman

Prandtl (1936) first employed the shock polar analysis to show that, when a steady supersonic flow impinges a solid wedge whose angle is less than a critical angle (i.e., the detachment angle), there are two possible configurations: the weak shock solution and the strong shock solution, and conjectured that the weak shock solution is physically admissible. The fundamental issue of whether one or both of the strong and the weak shocks are physically admissible has been vigorously debated over several decades and has not yet been settled in a definite manner. In this talk, I address this longstanding open issue and present recent analysis to establish the stability theorem for steady weak shock solutions as the long-time asymptotics of unsteady flows for all the physical parameters up to the detachment angle for potential flow.

An HJB Equation and Regularity Results for a Time-Optimal Control Problem in the Space of Probability Measures

Giulia Cavagnari
University of Trento, Italy
Antonio Marigonda

We present some results related to the study of a time-optimal control problem in the space of probability measures endowed with the topology induced by the Wasserstein metric. Such a formulation seems to be quite natural to model situations in which the knowledge of the initial state is only probabilistic as it happens when measurements are affected by noises, or to describe the behaviour of multi-agent systems. Potential applications are problems arising in pedestrian dynamics where a possible objective consists in steering a mass of people outside a room in the minimum amount of time. The dynamics is given by a *controlled* continuity equation, which can be seen as a superposition of admissible curves of an underlying optimal control problem, while the target set is defined by duality. Through suitable definitions of sub/super-differentials we formulate an Hamilton-Jacobi-Bellman equation solved, in this suitable viscosity sense, by the generalized minimum time function $\tilde{T}(\cdot)$, provided its continuity. We discuss also some attainability results and we give further con-

ditions yielding Lipschitz-continuity regularity of $\tilde{T}$. The main open problem remains the formulation of an HJB equation requiring less regularity on $\tilde{T}$, and to prove a comparison principle, granting uniqueness of the solution.

Semi-Geostrophic System with Variable Coriolis Parameter

Jingrui Cheng
University of Wisconsin-Madison, USA
Michael Cullen, Mikhail Feldman

The semi-geostrophic system (abbreviated as SG) is a model of large-scale atmospheric/ocean flows. Previous works about the SG system have been restricted to the case of constant Coriolis force, where we write the equation in dual coordinates and solve. This method does not apply for variable Coriolis parameter case. We develop a time-stepping procedure to overcome this difficulty and prove local existence and uniqueness of smooth solutions to SG system. This is joint work with Michael Cullen and Mikhail Feldman.

Loss of Regularity for Linear Transport Equations

Gianluca Crippa
University of Basel, Switzerland
Giovanni Alberti, Anna L. Mazzucato

For a linear transport equation

$$\partial_t u + b \cdot \nabla u = 0$$

with a Lipschitz velocity field b, the classical Cauchy-Lipschitz theory ensures propagation in time of the (Lipschitz) regularity of the initial datum. Although for less regular (Sobolev or BV, for instance) velocity fields a well-posedness theory for this equation is by now available (based on seminal results by DiPerna-Lions and Ambrosio), it turns out that the issue of the propagation in time of the regularity is much more delicate. In this talk I will report on a joint work with Alberti and Mazzucato, in which Sobolev velocity fields and smooth initial data are constructed, in such a way that any fractional regularity of the solution is instantaneously destroyed. Connections to mixing phenomena in fluids will also be mentioned.

Stochastic Homogenisation for Degenerate Hamilton-Jacobi Equations.

Federica Dragoni
Cardiff University, Wales
Claudio Marchi, Paola Mannucci

In the talk I Investigate the limit behaviour for a family of Cauchy problems for Hamilton-Jacobi equations describing a stochastic microscopic model. The Hamiltonian considered is not coercive in the total gradient. The Hamiltonian depends on a lower dimensional gradient variable which is asso- ciated to a Carnot group structure. The rescaling is adapted to the Carnot group structure, therefore it is anisotropic. Under suitable stationary-ergodic assumptions on the Hamiltonian, the solutions of the stochastic microscopic models will converge to a function independent of the random variable: the limit function can be characterised as the unique viscosity solution of a deter- ministic PDE. The key step will be to introduce suitable lower-dimensional constrained variational problems.

Shock Reflection Problem: Existence and Properties of Solutions

Mikhail Feldman
University of Wisconsin-Madison, USA
Myoungjean Bae, Gui-Qiang Chen, Wei Xiang

We discuss shock reflection problem for compressible gas dynamics, and von Neumann conjectures on transition between regular and Mach reflections. Then we will talk about existence and regularity of regular reflection solutions for potential flow equation up to the detachment angle, and geometric properties of the free boundary (shock curve), including its convexity. Our approach is to reduce the shock reflection problem to a free boundary problem for a nonlinear equation of mixed elliptic-hyperbolic type. We will also discuss known results and open questions regarding uniqueness and stability. The talk is based on the joint works with Gui-Qiang Chen, Myoungjean Bae and Wei Xiang.

On Cell Problems for Hamilton-Jacobi Equations with Non-Coercive Hamiltonians and Its Application to Homogenization Problems

Nao Hamamuki
Hokkaido University, Japan
Atsushi Nakayasu, Tokinaga Namba

We study a cell problem arising in homogenization for a Hamilton-Jacobi equation whose Hamiltonian is not coercive. We introduce a generalized notion of effective Hamiltonians by approximating the equation and characterize the solvability of the cell problem in terms of the generalized effective Hamiltonian. Under some sufficient conditions, the result is applied to the associated homogenization problem. We also show that homogenization for non-coercive equations fails in general.

Homogenization of Hamilton-Jacobi Equations in Dynamic Random Environments

Wenjia Jing
The University of Chicago, USA
Panagiotis E. Souganidis, Hung V. Tran

We consider stochastic homogenization of Hamilton-Jacobi equations in dynamic random environments, where the coefficients of the equations, namely the Hamiltonian and, for second order equations, the diffusion matrix, are highly oscillatory in space and time. I will discuss how to generalize the metric approach of stochastic homogenization developed for static random environment to the dynamic random setting, when uniform continuity (uniform with respect to the scale of oscillation and the random realization) of the minimal cost function is available. This talk is based on joint work with P.E. Souganidis and H.V. Tran.

Scalar Conservation Laws with Markov Initial Data

David Kaspar
Brown University, USA
Fraydoun Rezakhanlou

The inviscid Burgers‘ equation has the remarkable property that its dynamics preserve the class of spectrally negative Lèvy initial data, as observed by Carraro and Duchon (statistical solutions) and Bertoin (entropy solutions). Further, the evolution of the Lèvy measure admits a mean-field description, given by the Smoluchowski coagulation equation with additive kernel. In this talk we discuss ongoing efforts to generalize this result to scalar conservation laws, a special case where this is done, and a connection with integrable systems. Includes work with F. Rezakhanlou.

Numerical Approximation of Ensemble Based Solutions to Incompressible Flow Equations

Filippo Leonardi
ETH Zurich, Switzerland
S. Mishra, Ch. Schwab

We are interested in the behaviour of ensemble of solutions of incompressible Navier-Stokes and Euler equations and in algorithms for the approximation of those solutions. In the context of viscous flows, following the introduction of a proper notion of statistical solution for the vorticity formulation, we present an efficient and convergent vorticity-based finite difference algorithm for the approximation of such solutions, exploiting efficient Multi-level Monte Carlo techniques. For this algorithm, we are able to prove convergence rates under suitable assumptions. For inviscid flows, we present a similar algorithm, based on single-level Monte Carlo approximations, for the computation of approximations of admissible measure valued solutions: a notion of solution arising naturally when considering a vanishing viscosity approach to incompressible Euler equations.

Stochastic Homogenization of Reaction-Diffusion Equations in Isotropic Media

Jessica Lin
University of Wisconsin-Madison, USA
Andrej Zlatos

We consider reaction-diffusion equations with combustion nonlinearity in stationary ergodic and isotropic environments in dimensions $d \leq 3$. We prove existence of asymptotic, deterministic speeds of propagation for solutions with both spark-like and front-like initial data. This leads to a general stochastic homogenization result which shows that on average, the large-scale large-time behavior is governed by a deterministic Hamilton-Jacobi equation modeling front propagation. Applications include predicting the evolution of forest fires in random heterogeneous media. This talk is based on joint work with Andrej Zlatos.

Is a Nonlocal Diffusion Strategy Convenient for Biological Populations in Competition?

Annalisa Massaccesi
Universität Zürich, Switzerland
Enrico Valdinoci

We study the convenience of a nonlocal dispersal strategy in a reaction-diffusion system with a fractional Laplacian operator. We show that there are circumstances - namely, a precise condition on the distribution of the resource - under which a nonlocal dispersal behavior is favored. In particular, we consider the linearization of a biological system that models the interaction of two biological species, one with local and one with nonlocal dispersal, that are competing for the same resource. We give a simple, concrete example of resources for which the equilibrium with only the local population becomes linearly unstable. In a sense, this example shows that nonlocal strategies can become successful even in an environment in which purely local strategies are dominant at the beginning, provided that the resource is sufficiently sparse. Indeed, the example considered presents a high variance of the distribution of the dispersal, thus suggesting that the shortage of resources and their unbalanced supply may be some of the basic ingredients that favor nonlocal strategies.

On Rate of the Convergence for the Vanishing Discount Problem

Hiroyoshi Mitake
Hiroshima University, Japan
Kohei Soga

We will discuss on the vanishing discount problem for Hamilton-Jacobi equations. Recently, it has been proved that the whole family of solutions v^λ of the discount problem with the factor λ converges to a solution of the ergodic problem as $\lambda \to 0$. In this talk, we will consider some of specific case in $1D$ and particularly discuss on a rate of the convergence.

Conservation Laws on Networks

Benedetto Piccoli
Rutgers University, USA
Maria Laura Delle Monache

We present recent and less recent results on the theory of conservation laws on topological graphs, including existence and continuous dependence of solutions. Moreover, we will show applications to different areas such as vehicular traffic, supply chains and water channels.

A Level Set Approach to the Crystalline Mean Curvature Flow of Surfaces

Norbert Pozar
Kanazawa University, Japan

The crystalline mean curvature flow is a motion of a surface with normal velocity law $V = f(\nu,\kappa_\sigma)$, where f is a given function of the normal vector ν and the so-called crystalline mean curvature κ_σ. This problem appears in models of crystal growth formulated as a gradient flow of the surface energy when the surface energy density is not differentiable. A characteristic feature of the evolution is the appearance of flat parts of the surface, the facets of a crystal, on which the crystalline curvature is a nonlocal quantity. These facets are usually preserved by the flow, but they might also break or bend. Because of this phenomenon, even a local-in-time well-posedness had been open in dimensions higher than two except in special cases like convex initial data. We introduce a new notion of viscosity solutions for the level set formulation of the crystalline mean curvature flow. In three dimensions, we prove a comparison principle, stability and well-posedness of the initial value problem for arbitrary bounded crystals. This talk is based on joint work with Yoshikazu Giga from University of Tokyo.

A Counterexample Concerning Regularity Properties for Systems of Conservation Laws

Laura Spinolo
IMATI-CNR, Pavia, Italy
Laura Caravenna

In 1973 Schaeffer established a result that applies to scalar conservation laws with convex fluxes and can be loosely speaking formulated as follows: for a generic smooth initial datum, the admissible solution is smooth outside a locally finite number of curves in the (t, x) plane. Here the term "generic" should be interpreted in a suitable technical sense, related to the Baire Category Theorem. My talk will aim at discussing a recent explicit counterexample that shows that Schaeffer's Theorem does not extend to systems of conservation laws.

Inverse Problems, Non-Roundness and Flat Pieces Ofthe Effective Burning Velocity from an Inviscid Quadratic Hamilton-Jacobi Model

Yifeng Yu
UC Irvine, USA
Wenjia Jing, Hung V. Tran

I will talk about some finer properties of the effective burning velocity from a combustion model introduced by Majda and Souganidis in 90s. We proved that when the dimension is two and the flow of the ambient fluid is either weak or very strong, the level set of the effective burning velocity has flat pieces. Implications on the effective flame front and other related inverse type problems will also be discussed. This is a joint work with Wenjia Jing and Hung Tran.

On the Camassa-Holm Type Equations

Qingtian Zhang
Penn State University, USA
Alberto Bressan, Geng Chen, Mingjie Li

I will talk about the uniqueness of energy conservative weak solutions to Camassa-Holm and two-component Camassa-Holm equations, as well as the generic regularity of those solutions. Time permitting, I'll also mention the well-posedness of cubic Camassa-Holm equations.

Special Session 29: Advances in Theory & Application of Reaction Diffusion Models

Jerome Goddard II, Auburn University Montgomery, USA
Ratnasingham Shivaji, University of North Carolina Greensboro, USA

Application of reaction diffusion models is seemingly endless with their use naturally arising in disciplines such as biology, ecology, chemistry, geology, physics, and engineering. Reaction diffusion models have recently become even more useful in modeling physical and biological phenomena due to many important developments in the study of their dynamics. A key tool in understanding the dynamics of such models requires detailed investigation of the structure solutions to the corresponding parabolic and elliptic partial differential equations. This investigation yields interesting nonlinear initial-boundary and boundary value problems of varied types. Even though the study of reaction diffusion models has had a rich mathematical history dating back to the 1960s, much is still not known about the structure of solutions to such problems. Several techniques have been developed and successfully used to solve these problems including, iterative monotone methods, sub-super solutions, topological degree theory, and variational methods, among others.

A Nonlinear Model of Cancer Tumor Treatment with Cancer Stem Cells

Kristen Abernathy
Winthrop University, USA
Hannah Horner, Alexander Middleton

We present a system of six nonlinear ordinary differential equations which model the interactions between normal, cancer, endothelial, and cancer stem cell populations, as well as chemotherapy agent and anti-angiogenic agent concentrations. With analysis, it is shown that chemotherapy, with the co-administration of anti-angiogenic treatment, can produce three states: persistence of cancer, cancer recurrence, and a cure state. Results are supported by numerical simulations and bifurcation diagrams. We conclude with a discussion of the role of traveling waves in cancer dynamics and possible extensions for future work.

A Mathematical Model of Cancer Stem Cell Driven Tumor Growth with Radiation and Chemotherapy Treatment

Zach Abernathy
Winthrop University, USA
Savannah Bates, Rebecca Santorella

In this talk, we build a tumor model that incorporates the cancer stem cell hypothesis with chemotherapy and periodic radiation treatment using aspects of current models. We calculate conditions for the existence and local stability of equilibria in the case of no treatment as well as constant radiation with and without chemotherapy. Additionally, for periodic radiation treatment, sufficient conditions for the existence of cancer persistence and cure state periodic solutions are established. Conditions for global stability of the periodic cure state are also derived using a Lyapunov function. Numerical simulations demonstrate that treatments targeting cancer stem cells are more effective in eradicating cancer.

Analyticity in Time for an Abstract Nonlinear Evolutionary Problem

Falko Baustian
University of Rostock, Germany

We consider an abstract nonlinear Cauchy problem

$$\begin{cases} \frac{\mathrm{d}u}{\mathrm{d}t} - A(t,u(t))u(t) & = f(t,u(t)) + g(t), \\ u(0) & = u_0, \end{cases}$$

with the initial value in some real interpolation space. We investigate existence of a strict solution and analyticity in time for operators with the maximal L^p-regularity property. The results are applied to a general strongly parabolic system of semilinear partial differential equations. For this system we can show analyticity in time and also in the space variables. To prove the analyticity in space we approximate the initial value with analytic functions and use suitable estimates of the corresponding solutions on a complex domain. The results are a generalisation of the linear case in P. Takac: Space-Time analyticity of weak solutions to linear parabolic systems with variable coefficients, Journal of Functional Analysis 236, 50–88, 2012.

On the Solvability of Asymptotically Linear Systems at Resonance

Maya Chhetri
UNC Greensboro, USA
Petr Girg

We employ the Lyapunov-Schmidt method to discuss the solvability of asymptotically linear system at resonance at the simple eigenvalue of the corresponding linear eigenvalue problem. We consider nonlinear perturbations that are vanishing at infinity, unbounded but sublinear at infinity and those satisfying Landesman-Lazer type conditions for systems. Our approach allows us to treat systems that do not have variational structure and are coupled in linear as well as nonlinear part.

On Existence of Multiple Positive Solutions for Elliptic Equations and Systems

David Costa
University of Nevada Las Vegas, USA
Alfonso Castro, Ratnasingham Shivaji

After reviewing some of the literature on existence of positive solutions for elliptic equations, we present a new result on existence of multiple positive solutions for a class of elliptic systems in variational form.

Modeling the Effects of U-Shaped Density Dependent Dispersal Via Reaction Diffusion Equations

Jerome Goddard II
Auburn University Montgomery, USA
R. Shivaji

Dispersal is broadly defined as movement from one habitat patch to another and typically is considered to encompass three stages: emigration, inter-patch movement, and immigration. Dispersal can have both beneficial and detrimental effects on the persistence of spatially structured systems. Recent empirical results indicate that certain organisms‘ emigration from a patch is dependent on their own density– known as density dependent emigration. In fact, a U-shaped relationship between density and emigration has been observed in several organisms in field studies. To date, little is known about the patch-level consequences of such a dispersal strategy. In this talk, we will discuss a population model built upon the reaction diffusion framework that is designed to model the patch-level effects of U-shaped density dependent emigration. In particular, we will discuss the existence and stability properties of positive steady state solutions to this model for a one-dimensional patch. A brief discussion regarding ecological conclusions of the model's predictions will also be presented.

Trajectory Attractor for a Reaction-Diffusion Problem from Climate Modeling

Georg Hetzer
Auburn University, USA

Energy balance climate models lead to reaction-diffusion problems with slow diffusion and a set-valued reaction term on the 2-sphere. A hysteresis term accounts for a frequent repetition of sudden and fast warming followed by much slower cooling as observed from paleoclimate proxy data. Existence of a trajectory attractor and the lack of attractor stability will be discussed.

Bounded Solutions of Nonlinear Parabolic Equations in Unbounded Domains

Nsoki Mavinga
Swarthmore College, USA

We are concerned with the existence of full-bounded solutions for nonlinear parabolic equations where the space domain is unbounded with compact boundary (such as $\mathbb{R}^N$ which has empty boundary, or an exterior space-domain) and the time domain is actually the entire real line. We give an example which shows that the classical comparison principle does not hold in general on such domains. We establish L^∞ a-priori estimates for solutions to linear boundary value problems and derive a comparison/weak-maximum principle. We then define the notion of sub and super-solutions in this case, and by using comparison techniques, a-priori estimates and nonlinear approximation methods, we derive the existence results.

Positive, Radial Solutions for a Superlinear Semipositone p-Laplacian Problem on the Exterior of a Ball

Quinn Morris
University of North Carolina at Greensboro, USA
Inbo Sim, Ratnasingham Shivaji

We prove the existence of positive radial solutions to a class of semipositone p-Laplacian problems on the exterior of a ball subject to Dirichlet and nonlinear boundary conditions. Using variational methods we prove the existence of a solution, and then use a priori estimates to prove the positivity of the solution.

Multiple Strong Solutions and Bifurcation Structure for Diffusion Equations with Nonlinear Boundary Flux

M. Nkashama
University of Alabama at Birmingham, USA
N. Mavinga

We shall present multiplicity results for (strong) solutions of second order elliptic partial differential equations with nonlinear boundary conditions which include among others the so-called Steklov type problems (i.e., harmonic-function solutions). We impose asymptotic conditions on the boundary-nonlinearity and let the boundary-parameter vary. Our asymptotic conditions include the so-called ‘very-strong-resonance‘ conditions as well as oscillatory behavior. We set up the problem as a nonlinear ‘normal derivative trace‘ equation on the boundary, and proceed to establish *a priori* estimates and prove multiplicity results (for large-norm solutions) when the parameter belongs to a (nontrivial) continuum of real numbers. The proofs are based on degree theory, continuation methods and bifurcation from infinity techniques.

A Priori Bounds and Branches of Positive Solutions for Subcritical Semilinear Elliptic Systems

Rosa Pardo
Universidad Complutense de Madrid, Spain
Nsoki Mavinga

We provide a-priori L^∞-bounds for classical positive solutions of subcritical semilinear elliptic systems in bounded convex domains$\Omega \subset \mathbb{R}^N$. The critical case for a system $-\Delta u = f(v), -\Delta v = g(u)$, with Dirichlet boundary conditions is the so called critical hyperbola, that is $f(t) = t^p$, $g(s) = s^q$ with $\frac{1}{p+1} + \frac{1}{q+1} = \frac{N-2}{N}$. We prove that all classical positive solutions are a priori L^∞-bounded where the nonlinearities are slightly below the critical hyperbola, specifically $f(v) = \dfrac{v^p}{[\ln(e+v)]^\alpha}$, $g(u) = \dfrac{u^q}{[\ln(e+u)]^\beta}$ with $\dfrac{1}{p+1} + \dfrac{1}{q+1} = \dfrac{N-2}{N}$ and $\alpha, \beta > \dfrac{2}{N-2}$. Our analysis provides a new class of nonlinearities for which classical positive solutions of Hamiltonian elliptic systems are a priori bounded.

In [1] the authors prove the existence of a-priori bounds for positive solutions of elliptic equations $-\Delta u = f(u)$ with Dirichlet homogeneous boundary conditions, when $f(u) = u^{\frac{N+2}{N-2}}/\ln(e+u)^\alpha$, with $\alpha > 2/(N-2)$, see [1, Corollary 2.2].

References

[1] A. Castro and R. Pardo, A priori bounds for Positive Solutions of Subcritical Elliptic Equations, Rev. Mat. Complut. **28** (2015), 715-731.

[2] N. Mavinga and R. Pardo, A priori bounds and existence of positive solutions for subcritical semilinear elliptic systems, Preprint.

Steklov-Robin Eigencurves

Stephen Robinson
Wake Forest University, USA
Dr. Mauricio Rivas

We investigate nontrivial solutions of the boundary value problem

$$-\Delta u = \mu\, m_2 u \text{ in}\Omega$$
$$\tfrac{\partial u}{\partial \nu} + bu = \lambda\, m_1 u \text{ on } \partial\Omega$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $(\lambda, \mu) \in \mathbb{R}^2$, and the coefficient function b and the weights m_1, m_2 lie in appropriate L^p-spaces. In particular we characterize the sequence of *eigencurves* $(\lambda, \mu_n(\lambda))$ associated with the problem and then prove several results concerning the properties of those curves.

Spreading Speeds and Semi-Wave Solutions of Diffusive KPP Equations with a Free Boundary in Time Almost Periodic Environments

Wenxian Shen
Auburn University, USA
Fang Li, Xing Liang

The current talk is concerned with spreading speeds and semi-wave solutions of diffusive KPP equations with a free boundary in time almost periodic environments. It first discusses the criteria for the spreading to occur in such equations. It then provides a characterization of the spreading speeds and shows the existence of almost periodic semi-wave solutions in the case that the spreading occurs. It should be pointed out that the spreading may not occur in diffusive KPP equations with a free boundary.

On Radial Solutions for Singular Combined Superlinear Elliptic Systems on Annular Domains

Ratnasingham Shivaji
University of North Carolina at Greensboro, USA
D. Hai

We prove the existence of a large positive solution to the system

$$\begin{cases} -(r^{N-1}\phi_1(u'))' = \lambda r^{N-1} f_1(v),\ a < r < b, \\ -(r^{N-1}\phi_2(v'))' = \lambda r^{N-1} f_2(u),\ a < r < b, \\ \quad u(a) = 0 = u(b), v(a) = 0 = v(b), \end{cases}$$

where $a > 0$, λ is a small positive parameter, $f_i : (0, \infty) \to \mathbb{R}$ are continuous and $\lim_{z\to\infty} \dfrac{\phi_1^{-1}(f_1(c(\phi_2^{-1}(f_2(z)))))}{z} = \infty$ for all $c > 0$.

Three Positive Solutions for One-Dimensional P-Laplacian Problem with Sign-Changing Weight

Inbo Sim
University of Ulsan, Korea
Satoshi Tanaka

We show that one-dimensional p-Laplacian with a sign-changing weight which is subject to Dirichlet boundary condition has three positive solutions suggesting suitable conditions on the weight function and nonlinearity. Proofs are mainly based on the directions of a bifurcation.

A Uniqueness Result for a Semipositone p-Laplacian Problem on the Exterior of a Ball

Byungjae Son
University of North Carolina at Greensboro, USA
Ratnasingham Shivaji, Inbo Sim

We consider steady state reaction diffusion equations on the exterior of a ball :

$$\begin{cases} -\Delta_p u = \lambda K(|x|) f(u) & \text{in } \Omega_E, \\ u = 0 & \text{on } |x| = r_0, \\ u \to 0 & \text{when } |x| \to \infty, \end{cases}$$

where$\Delta_p z := \mathrm{div}(|\nabla z|^{p-2}\nabla z)$, $p \in (1, n)$, $\lambda > 0$, $r_0 > 0$ and $\Omega_E := \{x \in \mathbb{R}^n \mid |x| > r_0\}$. Here the weight function $K \in C^1([r_0, \infty), (0, \infty))$ satisfies $\lim_{r\to\infty} K(r) = 0$, and the reaction term $f \in C^1[0, \infty)$ is strictly increasing and satisfies $f(0)$ is negative, $\lim_{s\to\infty} f(s) = \infty$, $\lim_{s\to\infty} \frac{f(s)}{s^{p-1}} = 0$ and $\frac{f(s)}{s^q}$ is nonincreasing on $[a, \infty)$ for some $a > 0$ and $q \in (0, p-1)$. We establish the uniqueness of nonnegative radial solution for $\lambda \gg 1$.

On the Infinite Propagation Speed in Parabolic Problems with the p-Laplacian in a Domain for $p < 2$

Peter Takac
University of Rostock, Germany
Jiri Benedikt, Petr Girg, Lukas Kotrla

The validity of the weak and strong comparison principles for degenerate parabolic partial differential equations with the p-Laplace operator$\Delta_p(u) = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ will be discussed for $1 < p < 2$ (the "singular" case). This case is entirely different from the "degenerate" case $p > 2$ that allows for nonnegative "multi-bump" solutions with a spatially compact support and zero initial data. For $1 < p < 2$ we consider the special case of comparing the trivial solution ($\equiv 0$, by hypothesis) with a nontrivial nonnegative solution $u(x,t)$ that starts from nontrivial nonnegative initial data $u_0(x)$. We will show that, even for a doubly nonlinear parabolic problem, there is a (sufficiently short) time-interval $(0, T_0)$ such that the solution $u(x,t)$ becomes positive immediately, i.e., $u(x,t) > 0$ for all $(x,t) \in \Omega \times T_0$. The spatial domain $\Omega \subset \mathbb{R}^N$ is arbitrary, possibly unbounded. The previous work was focused on the case $\Omega = \mathbb{R}^N$. This case is much easier, as we are able to construct a subsolution – a spherical wave – travelling arbitrarily fast. The case$\Omega \neq \mathbb{R}^N$ requires the use of a rather sophisticated local result to obtain $u(x,t) > 0$ for all $(x,t) \in \Omega \times T_0$.

Special Session 30: High Order Numerical Methods for Partial Differential Equations

Wei Wang, Florida International University, USA
Zhongming Wang, Florida International University, USA

High order numerical methods have attracted considerable attention in scientific computation and engineering community due to their accuracy. This minisymposium aims to present the most recent developments in the design, analysis, implementation and applications of high order methods. Topics may include discontinuous Galerkin (DG) method and weighted essentially non-oscillatory (WENO) method as well as their applications in fluid dynamics, kinetic theory, quantum mechanics, biophysics and electromagnetics.

Recovering Exponential Accuracy in Spectral Methods Involving Piecewise Smooth Functions

Zheng Chen
Oak Ridge National Laboratory, USA
Chi-Wang Shu

Spectral methods achieve exponential accuracy both on the approximation level and for solving partial differential equations, if the solution is analytic and periodic. Lack of periodicity results in poor pointwise accuracy and $\mathcal{O}(1)$ errors near the discontinuity, even though the function is analytic. Such behavior is the so-called Gibbs Phenomenon. With Gegenbauer reconstruction, the function with discontinuities can be recovered with exponential accuracy inside each subinterval of analyticity. These techniques have been widely used as post-processing methods in many fields. The methods highly rely on the analyticity of the function, and thus fail in the case that the function has a singularity. The techniques have been extended via transformations to recover the exponential accuracy point-wisely for functions which have unbounded derivative singularities at end points, from the knowledge of point values on standard collocation points or the first $2N + 1$ Fourier coefficients. With this new reconstructions as post-processing methods, we are able to obtain exponential accuracy of spectral methods applied to linear transport equations involving such functions. Numerical results and applications will be shown in this talk.

Local Discontinuous Galerkin Method for Khokhlov-Zabolotskaya-Kuznetsov (KZK) Equation

Ching-Shan Chou
Ohio State University, USA
Weizhou Sun, He Yang, Yulong Xing

Acoustic pulses with high intensity are widely used in medical ultrasonics and sonar systems. Finite amplitude effects by high-intensity sound cannot be predicted by linear acoustical theory, and consideration of the combined effects of diffraction, absorption and nonlinearity is require to study finite amplitude wave propagation in thermoviscous fluid from an acoustic source of finite size. Numerical simulations are challenging due to the non-smoothness of the solutions. Here we present the LDG scheme for this type of equations, and we will show the stability analysis and numerical experiments.

A New Multiscale Discontinuous Galerkin Method for the One-Dimensional Stationary Schrodinger Equation

Bo Dong
University of Massachusetts Dartmouth, USA
Chi-Wang Shu, Wei Wang

We develop and analyze a new multiscale discontinuous Galerkin (DG) method for one-dimensional stationary Schrodinger equations with open boundary conditions which have highly oscillating solutions. Our method uses a smaller finite element space than the WKB local DG method proposed in Wang and Shu (J Comput Phys 218:295–323, 2006) while achieving the same order of accuracy with no resonance errors. We prove that the DG approximation converges optimally with respect to the mesh size h in L^2-norm without the typical constraint that h has to be smaller than the wave length. Numerical experiments were carried out to verify the second order optimal convergence rate of the method and to demonstrate its ability to capture oscillating solutions on coarse meshes in the applications to Schrodinger equations.

A Hybrid Eulerian-Lagrangian Method for Free Boundary Problems

David Kelly
Florida International University, USA

In this paper the author considers the numerical solution of time-dependent free boundary problems. Free boundary problems are boundary-value problems where part of the boundary is unknown and thus forms part of the solution. An efficient, high-order, hybrid Eulerian-Lagrangian method for the time evolution of the free boundary will be outlined. An application of the method to predict the evolution of fully non-linear water waves over arbitrary bathymetry will be presented.

A Conservative Sweeping Method for Enforcing Maximum Principle

Yuan Liu
Mississippi State University, USA

In this talk, we will talk about a conservative bound-preserving sweeping procedure. The main advantage is the simplicity of implementation while maintaining the high order of accuracy. Numerical examples are provided to show the performance and efficiency of the procedure.

High-Order Methods for Traveltime and Amplitude with Application in Geometrical Optics

Songting Luo
Iowa State University, USA

We present efficient methods to compute high order accurate traveltime and amplitude with applications in geometrical optics. Efficient factorization approaches are presented to resolve the upwind source singularities such that high-order methods can be designed and applied effectively to obtain high-order accurate traveltimes and amplitudes. With high accurate traveltimes and amplitudes, we present an efficient method, namely the fast Huygens sweeping method, to solve the Helmholtz equation in the high frequency regime. Numerical examples verify the effectiveness of the methods.

Multi-Level Monte Carlo Method for Stochastic Optimal Control Problems

Ju Ming
Beijing Computational Science Research Centre, Peoples Rep of China
Qiang Du, Qi Sun

In this lecture, we consider the implementation of multi-level Monte Carlo (MLMC) method to an elliptic optimal control problem with uncertain coefficients. Sample size formulas at each level of MLMC from the perspective of optimization, i.e., minimizing the computational error/cost with given computational cost/error, were derived. A gradient-based optimization algorithm using MLMC-based finite element method was proposed and compared to the results obtained by classical Monte Carlo method that employs many more degrees of freedom. These comparisons show the effectiveness and feasibility of the use of MLMC for obtaining accurate optimal solutions, which are required to construct statistical moments associated with the quantity of interest (QoI), of the stochastic control problem at low cost.

High-Order DG-FEM for Micro-Macro Partitioned Kinetic Models

James Rossmanith
Iowa State University, USA

The dynamics of gases can be simulated using kinetic or fluid models. Kinetic models are valid over most of the spatial and temporal scales that are of physical relevance in many application problems; however, they are computationally expensive due to the high dimensionality of phase space. Fluid models have a more limited range of validity, but are generally computationally more tractable than kinetic models. One critical aspect of fluid models is the question of what assumptions to make in order to close the fluid model. In this work we develop a high-order discontinuous Galerkin finite element method (DG-FEM) for a so-called micro-macro decomposition approximation of the kinetic equations. The micro-macro decomposition approach allows us to obtain accurate solutions of the fluid model, but instead of forcing a particular moment-closure approximation, which would typically only have a limited range of validity, this approach directly solves a version of the kinetic equations and uses this solution to provide a closure for the fluid equations. The proposed approach in this work makes use of an efficient semi-Lagrangian DG method for solving the kinetic portion of the update. The resulting numerical method is validated on several standard test cases.

An Entropy Satisfying Discontinuous Galerkin Method for Nonlinear Fokker-Planck Equations

Zhongming Wang
Florida International University, USA
Hailiang Liu

We propose a high order discontinuous Galerkin (DG) method for solving nonlinear Fokker-Planck equations with a gradient flow structure. For some of these models it is known that the transient solutions converge to steady-states when time tends to infinity. The scheme is shown to satisfy a discrete version of the entropy dissipation law and preserve steady-states, therefore providing numerical solutions with satisfying long-time behavior. The positivity of numerical solutions is enforced through a reconstruction algorithm, based on positive cell averages. For the model with trivial potential, a parameter range sufficient for positivity preservation is rigorously established. For other cases, cell averages can be made positive at each time step by tuning the numerical flux parameters. A selected set of numerical examples is presented to confirm both the high-order accuracy and the efficiency to capture the large-time asymptotic.

Well-Balanced Discontinuous Galerkin Methods for the Euler Equations Under Gravitational Fields

Yulong Xing
University of California Riverside, USA

Hydrodynamical evolution in a gravitational field arises in many astrophysical and atmospheric problems. Improper treatment of the gravitational force can lead to a solution which oscillates around the equilibrium. In this presentation, we propose a recently developed well-balanced discontinuous Galerkin method for the Euler equations under gravitational fields, which can maintain the isothermal and polytropic hydrostatic equilibrium states exactly. Some numerical tests are performed to verify the well-balanced property, high-order accuracy, and good resolution for smooth and discontinuous solutions.

Third Order Maximum-Principle-Satisfying Direct Discontinuous Galerkin Methods for Time Dependent Convection Diffusion equations on unstructured triangular meshes

Jue Yan
Iowa State University, USA

We develop 3rd order maximum-principle-satisfying direct discontinuous Galerkin methods for convection diffusion equations on unstructured triangular mesh. We carefully calculate the normal derivative numerical flux across the cell boundary and prove that, with proper choice of parameter pair in the numerical flux formula, the quadratic polynomial solution satisfies the strict maximum principle. The numerical solution is bounded within the given range and third order accuracy is maintained. There is no geometric restriction on the mesh and obtuse triangles are allowed in the partition. A sequence of numerical examples are carried out to show the results including incompressible flows are shown to demonstrate the theoretical results.

Feedback Boundary Control of Hyperbolic Systems with Stiff Source Term

Hui Yu
RWTH Aachen University, Germany
Michael Herty

We consider the feedback boundary control of hyperbolic systems with stiff source term. Such equations appear in many applications, such as gas dynamics in pipes and water flow in canals. By combining weighted Lyapunov functions, the structure is used to derive new stabilization results. The result is illustrated with the numerical analysis on the decay rate of the Lyapunov function in terms of the stiff parameter and an application to boundary stabilization of gas dynamics in pipes.

Compact WENO Limiters for Discontinuous Galerkin Methods

Xinghui Zhong
University of Utah, USA
Chi-Wang Shu

We design compact limiters using WENO methodology for discontinuous Galerkin methods for solving hyperbolic conservation laws, with the goal of obtaining a robust and high order limiting procedure to simultaneously achieve uniform high order accuracy and sharp, non-oscillatory shock transitions. The main advantage of these compact limiters is their simplicity in implementation, especially on multidimensional unstructured meshes.

Special Session 31: Celestial Mechanics and Beyond

Zhifu Xie, Virginia State University, USA
Ernesto Perez-Chavela, Ernesto Perez-Chavela, México
Alessandro Portaluri, Università degli Studi di Torino, Italy

This special session will concentrate on the latest developments in the field of celestial mechanics which laid the foundations for the birth of dynamical systems. The study of the N-body problem continues to attract researchers in a wide range of fields including dynamical systems, topology, variational methods, algebraic geometry, numerical methods and KAM theory. This special session provides a marketplace for ideas, and helps identify trends and areas of new opportunity in the field. This session brings established researchers and recent Ph.D.s together, some of whom are women or from groups underrepresented in mathematics. Some specific topics to be covered include Hamiltonian system, Ergodic theory, variational methods, central configurations, the N-body problem in spaces of constant curvature, discovery of new periodic solutions, regularization of collisions, stability of periodic solutions, spacecraft orbital design and applications of Morse index and Maslov index to the N-body problem. If the schedule permits, we anticipate ending the session with a discussion on open problems.

Lie Commutativity of Polynomial Vector Fields

John Alexander Arredondo
Konrad Lorenz University, Colombia
Jesus Mucino Raymundo

Let X, Y be polynomial vector fields on R^2 of degree at most d. If $[X, Y] = 0$ then X is integrable in the sense of Lie. Y is a symmetry of X, i.e., the local flow of Y sends local trajectories of X to local trajectories of X, preserving the parametrization along them. In this case X, Y have first integrals (real analytic and probably multivalued) tf, tg on $R^2 - det(X, Y) = 0$. Our goal in this talk is the study of the implications of the number of common zeros of X and Y for the Hamiltonian and non Hamiltonian case.

Singular Periodic Brake Orbits in the Planar Pairwise Symmetric Four Body Problem

Lennard Bakker
Brigham Young University, USA
Ammon Lam

We investigate the existence and stability of symmetric singular periodic brake orbits in the equal mass, fully symmetric planar four body problem. Using regularized coordinates, we remove the singularity of binary collision at the origin for each symmetric pair. We use topological and symmetry tools in our investigation.

Exchange Orbits in the 2n+1 Body Problem

Abimael Bengochea
Universidad Autonoma Metropolitana Iztapalapa, Mexico
Jorge Galan Vioque, Ernesto Perez Chavela

The horseshoe (exchange) orbits were first observed in the system conformed by Saturn, Janus and Epimetheus. In this talk we present the generalization of this kind of orbits to the $2n + 1$ body problem. We discuss some properties of these orbits, for instance, they are closely related with homographic solutions. It is expected that for some exchange orbits the variation of the moment of Inertia decreases as n increases. We show some numerical results for the case $n = 2$.

The Restricted Four Body Problem in the Solar System and Beyond.

Jaime Burgos-Garcia
Instituto Tecnologico Autonomo de Mexico, Mexico

The restricted four body problem (r4bp) studies the dynamics of a massless particle under the gravitational influence of three point masses that lie in an equilateral configuration provided by the well-known homographic solution of the general three body problem. This model has become relevant because of the observations of such configurations in our solar system, the most famous case is the configuration formed by the so called Trojan asteroids, Jupiter and the Sun although other Trojan asteroids have been detected for other planets and even between the moons of Saturn, however, recent studies show that is very likely to find this kind of configuration in some observed extrasolar systems. In this talk we will show how the r4bp and some of its modifications could be implemented to model the dynamics of small bodies interacting with these systems.

Convex Central Configurations of the Five-Body Problem

Kuo-Chang Chen
National Tsing Hua University, Taiwan
Jun-Shian Hsiao

In this talk we consider convex central configurations of the five-body problem and prove some necessary conditions for such configurations. In particular, for a planar central configuration with multiplier λ and total mass M, we show that all exterior edges are less than $r_0 = (M/\lambda)^{1/3}$, at most two interior edges are less than or equal to r_0, and its subsystem with four masses cannot be a central configuration. We also obtain some other necessary conditions for strictly convex central configurations with five bodies, and show examples with five bodies that have either one or two interior edges less than or equal to r_0. We develop some formulae by W. L. Williams in 1938, in the meanwhile we rectify some unsupported assumption in there.

A Property for Four-Body Isosceles Trapezoid Central Configurations

Yiyang Deng
SiChuan University, Peoples Rep of China
Bingyu Li, Shiqing Zhang

The four-body co-circural central configurations have two symmetric families,the kite and isosceles trapezoid.Using mutual distances as coordinates,we prove that if the four-body central configurations is isosceles trapezoid,the diagonals of the isosceles trapezoid can't be vertical.

About the Existence of Symmetry in Trapezoid Central Configurations.

Luis Franco Perez
Universidad Autonoma Metropolitana Cuajimalpa, Mexico

In the four body problem, non-collinear convex central configurations have an axis of symmetry provided that two bodies with equal mass are on the opposite vertices in the equilateral. In this case, the configuration must be of kite type. But if the pair of equal masses are located at adjacent vertices of a trapezoid, conforming a central cofiguration, the existence of a symmetry axis is not clear, unless the remaining pair of masses were equal too. In this talk we give some advances to show the existence of symmetry in trapezoid central configurations.

Melnikov Potential and Homoclinic Intersections in Higher Dimensional Hamiltonian Systems

Marian Gidea
Yeshiva University, USA
Rafael de la Llave

One basic method to establish the existence of hyperbolic horseshoes in Hamiltonian Systems (including models from Celestial Mechanics), is the Melnikov method. We present a version of the Melnikov method for higher dimensional Hamiltonian Systems, consisting of an unperturbed part, which can be described by a normally hyperbolic invariant manifold whose stable and unstable manifolds coincide, and a time-dependent perturbation. We define a Melnikov potential given in terms of convergent integrals of the perturbation along homoclinic orbits of the unperturbed system. We use the Melnikov potential to measure the splitting of the stable and unstable manifolds due to the perturbation. Unlike previous works, the homoclinic orbits are not asymptotic to periodic or quasi-periodic orbits, but to arbitrary orbits. We also show that for generic perturbations we obtain horseshoe-type dynamics.

Invariant Manifolds of the Parabolic Infinity in the Restricted Spatial Three Body Problem

Pau Martin
Universitat Politecnica de Catalunya, Spain
Inmaculada Baldoma, Ernest Fontich

In the spatial restricted three body problem, *infinity* is foliated by parabolic periodic orbits. It is known that this parabolic periodic orbits have two dimensional parabolic stable invariant manifolds. We prove that these invariant manifolds have polynomial approximation up to any order at the periodic orbit. Such result is rather surprising, since, as we will show, in general, invariant manifolds of parabolic points with dimension larger than one do not admit polynomial approximations and, in fact, may have rather low regularity at the fixed point. Our result follows from a more general result on parabolic fixed points. Our prove provides an algorithm that can be implemented to actually compute the approximations.

N-Body Problem of Celestial Mechanics and Numeric Simulation

Tiancheng Ouyang
Brigham Young University, USA
Zhifu Xie, Duokui Yan

In this talk, I will give a brief introduction of this variational methods of N-body Problem from 2001–2016 with emphasize the work of our collaborate research group. According to Newton's Second Law, the motion of N point bodies with positive masses $m_1, m_2, \ldots, m_N$ located at positions $x_1, x_2, \ldots, x_N \in \mathbb{R}^3$ is governed by the system of second-order nonlinear vector differential equations

$$m_i\ddot{x}_i = \sum_{\substack{j=1,\\ j\neq i}}^{N} \frac{Gm_im_j(x_j - x_i)}{\|x_i - x_j\|^3},$$

where the derivative is with respect to the time variable t, and G is the universal gravitational constant. By using variational methods and numerical computation, some new discovered periodic orbits in 2D and and 3D will be present.

Stability of Relative Equilibria in Curved Spaces

Ernesto Perez-Chavela
ITAM-Mexico, Mexico
Juan M. Sanchez-Cerritos

We consider N–point positive masses moving on a two dimensional space of constant curvature K. Using the cotangent potential as a generalization of the Newtonian one on these spaces, we describe some especial kind of periodic orbits where the mutual distances among the particles remain constant for all time. We classify these orbits called relative equilibria for the case $N = 2, 3$ and study the linear stability of them.

N-Body Problems on Surfaces of Revolution

Cristina Stoica
Wilfrid Laurier University, Waterloo, Canada

In this talk I will present some dynamical aspects of the N-body problems on a two dimensional surface of revolution. I will discuss symmetries, relative equilibria, and some properties related to bifurcations and stability of relative equilibria.

Periodic Orbits from Broucke-Henon Orbit to Figure Eight in Three-Body Problem

Zhifu Xie
Virginia State University, USA
Tiancheng Ouyang, Duokui Yan

In this talk, we discuss the possible connections of periodic solutions in two paradigmatic examples of three body problem: Broucke-Henon orbit and the figure eight orbit. Broucke-Henon orbit was numerically discovered by Broucke and Henon (1975) independently and their existence are recently proved via variational methods with Structural Prescribed Boundary Conditions (SPBC) by Ouyang-Xie-Yan (2015). Figure eight orbit was first numerically discovered by Moore (1993) and its existence was proved by Chenciner and Montgomery (2000).

Existence of the Broucke-Henon Orbit

Duokui Yan
Beihang University, Peoples Rep of China
Tiancheng Ouyang, Zhifu Xie

In 2003, Venturelli proposed an open problem on the existence of the Broucke-Henon orbit during a workshop at American Institute of Mathematics. In this talk, we introduce our variational method and show the existence of this orbit.

Special Session 32: Global or/and Blowup Solutions for Nonlinear Evolution Equations and Their Applications

Shaohua Chen, Cape Breton University, Canada
Runzhang Xu, Harbin Engineering University, Peoples Rep of China

This session is devoted to the recent developments in global or/and blowup solutions for nonlinear evolution equations and their applications, include reaction diffusion equations, fluid dynamics, delay, localized, nonlocal, degenerate evolution equations, travelling waves, steady states and their properties.

Blow-Up for the Wave Equation with Nonlinear Boundary Damping and Source Term

Md Salik Ahmed
College of Science, Harbin Engineering University, Bangladesh
Qiu Xiaotong

In this paper, we mainly study the initial boundary value problem of wave equations with nonlinear source and boundary damping terms under critical energy case.As we know, the dynamical boundary conditions make the space nature of solutions varying so much, as well as the invariant sets. The classical methods used to study qualitative studies are no longer entirely applicable. Therefore we first introduce the previous research results with local existence with the low initial energy. Then we concern with the global existence and blow-up in the case of critical energy. We prove that all the results still holds in further assumptions by potential well and Sobolev embedding theory.
This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

Global Well-Posedness for the Quasilinear Hyperbolic Equation with Nonlinear Damping and Source Terms

Tianlong Chen
College of Automation, Harbin Engineering University, Peoples Rep of China

In this paper we study the Cauchy problem at three different initial energy levels for the quasilinear hyperbolic equation with nonlinear damping and source terms. In the frame of potential well theory, we investigate the blowup of solutions with concavity method at both low and supercritical initial energy level. Moreover the global existence and asymptotic behavior are concerned.
This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

Global Existence for a Singular Activator-Inhibitor Model

Shaohua Chen
Cape Breton University, Canada

We discuss existence results for a singular Gierer-Meinhardt system with zero Dirichlet boundary conditions, which originally arose in studies of pattern-formation in biology and has interesting and challenging mathematical properties. The mathematical difficulties are that the system becomes singular near the boundary and it is non-quasimonotone. We show the existence of positive solutions for the general activator-inhibitor model.

Weighted Gagliardo-Nirenberg Inequalities Involving BMO Norms and Solvability of Strongly Coupled Parabolic Systems

Dung Le
University of Texas at San Antonio, USA

We will discuss new weighted Gagliardo-Nirenberg inequalities with applications to the local/global existence of solutions to nonlinear strongly coupled and uniform parabolic systems. Much weaker sufficient conditions than those existing in literature for solvability of these systems will be established.

Non-Simultaneous Blow-Up Solutions to Parabolic Equations with N Components

Bingchen Liu
China University of Petroleum, Peoples Rep of China
Fengjie Li

In this talk, we present the non-simultaneous blow-up solutions and their blow-up rates for the parabolic equations with n components. Firstly, we show some criteria for the existence and nonexistence of non-simultaneous blow-up solutions, classified by the constant or variable exponents. Secondly, blow-up rates and blow-up sets of the solutions are considered. It is interesting that the different blow-up mechanism, the coupled relationships, and the properties of initial data, etc. might lead to different blow-up phenomena of solutions.

Blow-Up Phenomena in Nonlinear Parabolic and Pseudoparabolic Problems

Monica Marras
University of Cagliari, Italy
G. Viglialoro

We investigate the question of blow-up for nonnegative classical solutions of some nonlinear parabolic problems defined in a bounded domain. Under conditions on data and geometry of the spatial domain, explicit lower bounds for the blow-up time are derived. Moreover we extend our results to a class of nonlinear pseudo-parabolic problems.

Traveling Wave Solutions to the Burgers-$\alpha\beta$ Equations

Byungsoo Moon
Incheon National University, Korea

The Burgers-$\alpha\beta$ equation, which was first introduced by D.D. Holm and M.F. Staley, is considered in the special case where $\nu = 0$ and $b = 3$. Traveling wave solutions are classified to the Burgers-$\alpha\beta$ equation containing four parameters b, α, ν, and β, which is a nonintegrable nonlinear partial differential equation that coincides with the usual Burgers equation and viscous b-family of peakon equation, respectively, for two specific choices of the parameter $\beta = 0$ and $\beta = 1$. Under the decay condition, it is shown that there are smooth, peaked and cusped traveling wave solutions for the Burgers-$\alpha\beta$ equation with $\nu = 0$ and $b = 3$ depending on the parameter β. Moreover, all traveling wave solutions without the decay condition are parametrized by the integration constant $k_1 \in \mathbb{R}$. In an appropriate limit $\beta = 1$, the previously known traveling wave solutions of the Degasperis-Procesi equation are recovered.

Degenerate Boundary Layer Solution to a System of Viscous Conservation Laws

Tohru Nakamura
Kumamoto University, Japan

We consider the existence and asymptotic stability of the stationary solution for system of viscous conservation laws in one-dimensional half space. We especially consider the degenerate stationary solution which verifies algebraic convergence as the space variable tends to infinity. With the aid of the center manifold theory, the existence of the degenerate stationary solution is proved under the situation that one characteristic is zero and the other characteristics are negative. Asymptotic stability of the degenerate stationary solution is also proved in an algebraically weighted Sobolev space provided that the weight exponent is less than 5. The stability analysis is based on deriving the a priori estimate by using the weighted energy method combined with the Hardy type inequality with the best possible constant.

Global Well-Posedness of Damped Multidimensional Generalized Boussinesq Equations

Yi Niu
College of Automation, Harbin Engineering University, Peoples Rep of China
Xiuyan Peng

We study the Cauchy problem for a class of sixth order Boussinesq equations with the generalized source term and damping term. By using Galerkin approximations and potential well methods, we prove the existence of global weak solution. Furthermore, we talk about the condition of damped coefficient k and obtain the finite time blow up solution.
This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

Time Behavior and Uniform Bounds for Solutions of Evolution Problems

Maria Michaela Porzio
Sapienza University of Rome, Italy

We will describe the behavior in time of the solutions of some nonlinear parabolic problems by means of a new method which allows to study regularity, uniqueness and asymptotic behavior in a very simple way and to obtain also uniform estimates for the solutions.

On a Pseudoparabolic Regularization of a Forward-Backward-Forward Parabolic Equation

Flavia Smarrazzo
University Campus Bio-Medico di Roma, Italy
Michiel Bertsch, Alberto Tesei

In this talk I will consider an initial-boundary value problem for a degenerate pseudoparabolic regularization of a nonlinear forward-backward-forward parabolic equation, with a bounded nonlinearity which is increasing at infinity. I will discuss existence of suitably defined nonnegative solutions of the problem in a space of Radon measures. The constructed solutions satisfy several monotonicity and regularization properties; in particular, their singular part is nonincreasing and may disappear in finite time. Joint work with M. Bertsch and A. Tesei.

Cauchy Problem of the Singularly Perturbed Sixth-Order Boussinesq-Type Equation

Changming Song
Zhongyuan University of Technology, Peoples Rep of China

In this talk, we will study the existence and uniqueness of the global generalized solution and the global classical solution for the Cauchy problem of the singularly perturbed sixth-order Boussinesq-type equation:

$$u_{tt} = u_{xx} + \sigma(u)_{xx} + \alpha u_{x^4} + \beta u_{x^6}.$$

Time-Periodic Solutions to the Drift-Diffusion Model for Semiconductors

Masahiro Suzuki
Nagoya Institute of Technology, Japan

We study the existence and asymptotic stability of time-periodic solutions to the drift-diffusion model for semiconductors. If alternating-current voltage is applied to PN-junction diodes, a time-periodic current flow is observed. The main purpose of this talk is mathematical analysis on this periodic flow. We construct a time-periodic solution by utilizing the Galerkin method. The solution is unique in a neighborhood of a thermal equilibrium, and it is globally stable. Proofs of the uniqueness and the stability are based on the energy method employing an energy form.

Behaviour in Time of Solutions to a Class of Fourth Order Evolution Equations

Stella Vernier-Piro
UNICA, Italy
G.A.Philippin

We consider some initial-boundary value problems for a class of nonlinear parabolic equations of the fourth order, whose solution $u(x,t)$ may or may not blow up in finite time. Under suitable conditions on data, a lower bound for t^* is derived, where $[0,t^*)$ is the time interval of existence of $u(x,t)$. Under appropriate assumptions on the data, a criterion which ensures that u cannot exist for all time is given, and an upper bound for t^* is derived. Some extensions for a class of nonlinear fourth order parabolic systems are indicated.

On Love Wave Dispersion in an Orthotropic Substratum Double-Layered Over Anisotropic Porous Mantle: an Analytic Approach

Sumit Vishwakarma
Birla Institute of Technology and Science, Pilani, India
Xu Runzhang

The present study investigate the propagation character of Love wave in an orthotropic substratum being layered doubly over anisotropic porous mantle. Separate displacements have been derived for each of the three layer and suitable boundary condition have been imposed to derive the dispersion equation of the wave in a closed form in terms of Whitaker function. It has been found that inhomogeneous parameter associated with the rigidity and the density of the layered medium has a special bearing on the Love wave propagation. Orthotropic properties and initial stresses of the media also affects the velocity of the wave to a great extent has been shown graphically.

Global Existence and Blowup of Solutions for the Multidimensional Sixth-Order Good Boussinesq Equation

Runzhang Xu
Harbin Engineering University, Peoples Rep of China

This paper is concerned with the Cauchy problem of solutions for some nonlinear multidimensional good Boussinesq equation of sixth order at three different initial energy levels. In the framework of potential well, the global existence and blowup of solutions are obtained together with the concavity method at both low and critical initial energy level. Moreover by introducing a new stable set, we present some sufficient conditions on initial data such that the weak solution exists globally at sup-critical initial energy level.

Finite Time Blow Up for the Nonlinear Fourth-Order Dispersive-Dissipative Wave Equation at High Energy Level

Yanbing Yang
College of Science, Harbin Engineering University, Peoples Rep of China
Xu Runzhang

This present investigates the initial boundary value problem of the nonlinear fourth-order dispersive-dissipative wave equation at sup-critical initial energy level. By using the concavity method, we establish a blow up result for certain solutions with arbitrary positive initial energy, which gives an answer to the global nonexistence of solutions for the nonlinear wave equations with dispersive term and dissipative term at sup-critical initial energy level.

A Diffusion Problem of Kirchhoff Type Involving the Nonlocal Fractional P-Laplacian

Binlin Zhang
Heilongjiang Institute of Technology, Peoples Rep of China
Patrizia Pucci, Mingqi Xiang

In this talk, we present an anomalous diffusion model of Kirchhoff type driven by a nonlocal integro–differential operator. As a particular case, we are concerned with an initial–boundary value problem involving the fractional p–Laplacian. Under some appropriate conditions, the well–posedness of solutions for the above problem is studied by employing the sub–differential approach. Furthermore, the asymptotic behavior and extinction of solutions are also investigated.

Special Session 33: Nonlinear Waves in Dispersive Equations

Francois Genoud, TU Delft, Netherlands
Stefan Le Coz, Paul Sabatier University (Toulouse 3), France

This session will be devoted to analytical and numerical properties of nonlinear waves in dispersive equations such as, for instance, nonlinear Schrödinger equations and Korteweg-de Vries equations. Particular topics of interest will include space-periodic solutions and equations from which some usual symmetries (e.g. space-translation or scaling invariance) are absent.

Ground States for the NLS on Graphs

Riccardo Adami
Politecnico di Torino, Italy
Enrico Serra, Paolo Tilli

The issue of finding the state of a Bose-Einstein condensate in a branched trap, can be modelled mathematically as the problem of minimizing a nonlinear (quartic) energy functional among functions defined on a graph. We give results on the existence of ground states for the cases of subcritical and critical nonlinearity power. The problem proves different and richer than the analogous problem on the line.

Nonlinear Schrodinger Equation with Double Delta-Interaction Wells

Jaime Angulo Pava
State University of Sao Paulo, Brazil
Luis Andres Rosso Ceron

We study analytically the existence and orbital stability of the peak-standing-wave solutions for the cubic nonlinear Schrodinger equation with two interactions points determined by Dirac's deltas. This equation admits at least two smooth curve of positive solutions with a profile given by the Jacobi elliptic function of dnoidal and cnoidal type between the defects point. Via Floquet theory and analytic perturbation, we obtain that in the case of an attractive defect (dnoidal case) the standing wave solutions are unstable and in the case of an repulsive defect (cnoidal case) the standing wave solutions are stable in H^1_{per}.

Modulations of Dispersive Periodic Waves

Sylvie Benzoni-Gavage
University of Lyon, France
C. Mietka, L.M. Rodrigues

One may consider as a whole a large class of nonlinear dispersive PDEs that includes NonLinear Schrödinger equations, generalized Korteweg-de Vries equations, and dispersive perturbations of the Euler equations for compressible fluids. These equations admit families of periodic waves, whose modulations are expected to be governed by averaged equations "á la Whitham". Whitham equations are still poorly understood. However, they are endowed with nice structures that can be investigated at least numerically.

On the Multiplier Method for Variable Coefficients Dispersive Equations.

Federico Cacciafesta
Univ. Milano Bicocca, Italy
P. D'Ancona, A.S. de Suzzoni, R. Luca

We show how the multiplier method can be developed in the framework of variable coefficients dispersive equations to prove local smoothing estimates. In particular we discuss the cases of the Schroedinger (via Kato smoothing theory) and the Dirac equations.

Modulation Equations Via Normal Forms

Martina Chirilus-Bruckner
University of Leiden, Netherlands

In this talk we will briefly survey the method of derivation and justification of modulation equations (such as the Korteweg-de Vries or Nonlinear Schrodinger equation) in dispersive systems, to then present a novel approach using normal forms, which allows for a more transparent derivation procedure and a different viewpoint on higher order correction terms.

A New Model Describing Tidle Bore on River

Mathieu Colin
IPB, INRIA CARDAMOM, France
David Lannes

The free surface Euler equations are a set of evolution equations on a three-dimensional, free-boundary domain. The aim of this talk is to present asymptotic models for river flows that approximate these equations in relevant physical configurations and that are much more simple. For a canal with rectangular section, we show that there exists at leading order solutions that are independent of the transverse variable

y. Their evolution is given by a set of two evolution equations on the surface elevation and on the longitudinal component of the vertically averaged velocity. We will then discuss the case of a river with lateral shorelines.

Dirac Points in PT Symmetric Optical Lattices

Christopher Curtis
San Diego State University, USA
Mark Ablowitz, Yi Zhu

Optical graphene, or an optical honeycomb waveguide, has become a material of much interest and excitement in the optics community. This is due to the presence of Dirac points in the dispersion relationship which are a result of the symmetry of the lattice. Also of interest in optics are so called parity-time (PT) symmetric perturbations, representing the careful introduction of gain and loss terms. In this talk, we examine the impact of introducing PT symmetritc perturbations into honeycomb lattices and their impact on Dirac points. We categorize two types of PT perturbations one of which we rigorously show prevents the formation of complex dispersion relationships. We then track how Dirac points separate, and thus we show how PT perturbations can be used to introduce band gaps. We also present numerical results which show how PT perturbations could be used to engineer dispersion relationships which induce wave motion in particular directions. The impact of nonlinearity in these systems will also be addressed.

Orbital Stability in Infinite Dimension: Geometric Theory

Stephan de Bievre
University of Lille I - INRIA Lille-Nord Europe, France
F. Genoud, S. Rota Nodari

In this talk, we (re)consider the energy-momentum method for proving the orbital stability of relative equilibria of Hamiltonian dynamical systems on Banach spaces, in the presence of higher dimensional symmetry groups. We will highlight the interplay that is at work in this method between (symplectic) geometry and (functional) analysis.

Existence of Peaked Waves in a Full-Dispersion Bi-Directional Shallow Water Model

Mats Ehrnstrom
NTNU Norwegian University of Science and Technology, Norway
M.A. Johnson

We consider the existence of periodic traveling waves in a bidirectional Whitham equation, combining the full two-way dispersion relation from the incompressible Euler equations with a canonical shallow water nonlinearity. Of particular interest is the existence of a highest, singular, traveling wave solution, which we obtain as a limiting case at the end of the main bifurcation branch of 2π-periodic traveling wave solutions. Unlike the uni-directional Whitham equation, containing only one branch of the full Euler dispersion relation, where such a highest wave behaves like $|x|^{1/2}$ near its crest, the waves obtained here behave like $|x \log |x||$ at their crest.

On the Wave Length of Smooth Periodic Traveling Waves of the Camassa-Holm Equation

Anna Geyer
University of Vienna, Austria
Jordi Villadelprat

In this talk we are concerned with the wave length λ of smooth periodic traveling wave solutions of the Camassa-Holm equation. The set of these solutions can be parametrized using the wave height a (or peak-to-peak amplitude). Our main result establishes monotonicity propoerties of the map $\lambda(a)$ i.e. the wave length as a function of the wave height. We obtain the explicit bifurcation values, in terms of the parameters associated with the equation, which distinguish between the two possible qualitative behaviours of $\lambda(a)$, namely monotonicity and unimodality. The key point is to relate $\lambda(a)$ to the period function of a planar differential system with a quadratic-like first integral, and to apply a criterion which bounds the number of critical periods for this type of systems.

The Energy-Critical Quintic NLS on Perturbations of Euclidean Space.

Casey Jao
UCLA, USA

Consider the defocusing quintic nonlinear Schrödinger equation on $\mathbf{R}^3$ with initial data in the energy space. This problem is energy-critical in view of a certain scaling invariance, which is a main source of difficulty in the analysis of this equation. It is a nontrivial fact that all finite-energy solutions scatter to linear solutions. We show that this remains true under small compact deformations of the Euclidean metric.

Asmptotics Behavior of Solutions for a Class of Nonlinear Schrödinger Equations

Hiroaki Kikuchi
Tsuda college, Japan

In this talk, we consider the following nonlinear Schrödinger equation:

$$i\frac{\partial\psi}{\partial t}(x,t) + \Delta\psi(x,t) + f(\psi(x,t)) = 0, \qquad (x,t) \in \mathbb{R}^d \times \mathbb{R},$$

where $i := \sqrt{-1}, d \geq 1, \psi$ is a complex-valued function on $\mathbb{R}^d \times \mathbb{R}, \Delta$ is the Laplace operator on $\mathbb{R}^d$ and $f : \mathbb{C} \to \mathbb{C}$ is a continuously differentiable function in $\mathbb{R}^2$-sense to be specified later. We will study the effects of the nonlinearity f to the dynamics of solutions (scattering/blowup/existence and stability of the standing waves) to (). If time permits, I'd like to mention about the global dynamics of solutions whose energy is greater than that of the ground state.

Analysis of Convergence of Schwarz Waveform Relaxation Domain Decomposition Methods for the Schroedinger equation

Emmanuel Lorin
Carleton University, Canada
X. Antoine

The aim of this talk is to derive and numerically validate some asymptotic estimates of the convergence rate of the Classical and Optimized Schwarz Waveform Relaxation domain decomposition methods applied to the computation of the stationary states to the one- and two-dimensional linear and nonlinear Schroedinger equations. The approach combines some methods developed by Gander and Halpern, and the theory of inhomogeneous pseudodifferential operators in conjunction with the associated symbolical asymptotic expansions.

Quasilinear Schrodinger Equations

Jeremy Marzuola
UNC Chapel Hill, USA
Jason Metcalfe, Daniel Tataru

We discuss results, applications and open problems in the study of quasilinear Schrödinger equations. Recently, with J. Metcalfe and D. Tataru, we have been studying local in time well-posedness for small and large data. I will recall the main ideas of the proofs in such a case and discuss future directions. I will also discuss ongoing work with Jianfeng Lu and Ludwig Gauckler relating to extensions and phenomenological studies.

Collision and Blow Up in NLS Equations

Claudio Munoz
University of Chile, Chile
Frank Merle

The purpose of this talk is to describe how collision between two NLS solitons in the unstable regime can lead to different results, which include blow up in finite time, or decay in time. This is a joint work with F. Merle.

The Fourth-Order Dispersive Nonlinear Schrodinger Equation: Orbital Stability of a Standing Wave

Fabio Natali
State University of Maringa, Brazil
Ademir Pastor

Considered in this talk is the one-dimensional fourth-order dispersive cubic nonlinear Schrödinger equation with mixed dispersion. Orbital stability, in the energy space, of a particular standing-wave solution is proved in the context of Hamiltonian systems. The main result is established by constructing a suitable Lyapunov function.

Nonlinear Schroedinger Equation with a Point Nonlinearity in Three Dimensions: Scaling Limit from a Mean Field theory

Diego Noja
University of Milano Bicocca, Italy
Claudio Cacciapuoti, Domenico Finco, Alessandro Teta

In this talk a nonlinear Schrödinger dynamics with a nonlinearity concentrated at a point will be defined and constructed. In particular it will be shown that such a model can be recovered as a scaling limit of a regularized nonlocal dynamics through a suitable renormalization procedure. The regularized dynamics is described by the equation

$$i\frac{\partial}{\partial t}\psi^{\varepsilon}(t) = -\Delta\psi^{\varepsilon}(t) + g(\varepsilon, \mu, |(\rho^{\varepsilon}, \psi^{\varepsilon}(t))|^{2\mu})(\rho^{\varepsilon}, \psi^{\varepsilon}(t))\rho^{\varepsilon}$$

where $\rho^{\varepsilon} \to \delta_0$ weakly and the function g embodies the nonlinearity and the scaling and has to be fine tuned in order to have a nontrivial limit dynamics. The generator of the limit dynamics is a nonlinear version of a delta interaction and it has previously studied as regards well posedness, blow up and existence and stability of standing waves.

Space-Modulated Stability and Periodic Waves of Dispersive Equations

Miguel Rodrigues
Université de Rennes 1, France

Recently, partly motivated by applications to surface waves, rapid progresses on the stability theory of periodic waves have been obtained. In particular, for parabolic systems — including those encoding the shallow water description of viscous roll-waves — an essentially complete theory is now available. We shall expound here some first contributions to a dispersive theory, still to come.

Orbital Stability in Infinite Dimension: Proving Coercivity

Simona Rota Nodari
IMB, Université de Bourgogne, France
Stephan De Bièvre

The proof of the orbital stability of relative equilibria of Hamiltonian dynamical systems on Banach spaces can be reduced to a “coercivity estimate” on an appropriately constructed Lyapunov function. In this talk we show how this estimate can be obtained in the general case of higher dimensional symmetry groups.

Numerical Methods and Orbital Stability of Solitary-Wave Solutions for Higher-Order BBM Equations

Juan-Ming Yuan
Providence University, Taiwan
Hongqiu Chen, Shuming Sun

We study the existence and stability of solitary-wave solutions of a general higher-order Benjamin-Bona-Mahony (BBM) equation, which involves pseudo-differential operators for the linear part. One of such equations can be derived from water-wave problems as the second-order approximate equations from fully nonlinear governing equations. Under certain conditions on the symbols of pseudo-differential operators and the nonlinear terms, it is shown that the general higher-order BBM equation has solitary-wave solutions. Moreover, under slightly more restrictive conditions, the set of solitary-wave solutions is orbitally stable. Here, the equation has a nonlinear part involving polynomials of the solution and its derivatives with different degrees (i.e., not homogenous), which has not been discussed before. We also develop numerical methods for the fifth-order BBM-type equations.

Special Session 34: Differential Equations and Applications to Biological Models

Nemanja Kosovalic, University of South Alabama, USA
Xiang-Sheng Wang, Southeast Missouri State University, USA

It is well known that ordinary differential equations, partial differential equations, and functional differential equations provide a suitable framework for the modelling of various biological phenomena. Moreover, many biological problems inspire the development of new mathematical ideas to draw qualitative and quantitative conclusions concerning the underlying models. It is the goal of this session to bring together researchers interested in either the dynamical modelling of biological phenomena, or the theoretical aspects concerning the underlying models, to discuss and exchange ideas.

Building a Model of Delay-Differential Equations with State-Dependent Delay for Megakaryopoiesis

Lois Boullu
Universite de Montreal, Canada
Nemenja Kosovalic, Laurent Pujo-Menjouet, Jianhong Wu, Jacques Belair

Since the early work of M. C. Mackey and J. Belair in the 80s [1], the intricate multiple feedbacks of megakaryopoiesis (the process along which the platelets are produced) have been the source of many investigations in the field of delay differential equations. In this talk I describe how an emphasis on TPO regulation and progenitor cells proliferation can lead to a model describing megakaryopoiesis with a system of delay differential equations, the delay being state-dependant and defined by threshold: using a tool presented by L H Smith in 1992 [2], we transform this system into a functional differential equation with fixed delay, allowing us to explore well-posedness and fixed point stability.

References

[1] Belair J, Mackey M C (1987) A model for the Regulation of Mammalian Platelet Production, Annals New York Academy of Sciences

[2] Smith L H (1992) Reduction of structured population models to threshold-type Differential equations and funtionnal differential equation : a case study Mathematical Biosciences 113:1-23

Contributions of the Latent Reservoir and of the Pool of Long-Lived Chronically Infected $CD4^+$ T Cells in HIV dynamics

Ana Carvalho
University of Porto, Portugal
Carla M.A. Pinto

In this paper, we study the effect of the size of the latent reservoir and of the pool of long-lived chronically infected $CD4^+$ T cells in a model for HIV dynamics with drug-resistance. We calculate the reproduction number and study the local stability of the disease-free equilibrium. The effects of the sizes of the latent reservoir and of the pool of long-lived chronically infected $CD4^+$ T cells were analyzed numerically. Our results are biologically reasonable. We found that the latent reservoir in resting $CD4^+$ T cells appears to be sufficient to the persistence of plasma viral load in patients under HAART. Moreover, the pool of long-lived chronically infected cells promotes an increase in drug-resistant virus, that escape treatment, which turns the eradication of the plasma virus an impossible goal.

Impulsive Fractional Integro-Differential Equations of Order $(1, 2)$ with Antiperiodic Boundary Conditions

Jaydev Dabas
Indian Institute of Technology Roorkee, India
Vidushi Gupta

In the present work our aim is to study the fractional order boundary value problem with jump impulsive conditions. These type of jump conditions represents a sudden change of values of state variable and its derivative at impulsive points. The impulsive effect considered in the system has memory of the past states. We establish the existence and uniqueness of solution for a new class of impulsive fractional integro-differential equation with separated boundary conditions by applying the classical fixed point theorems. At last an application is presented to verify our results.

Nonlinear Fractional Boundary Value Problem with Not Instantaneous Impulse

Vidushi Gupta
IIT Roorkee, India
Jaydev Dabas

This present work deals with the following boundary value problem of a fractional differential equation with non-instantaneous impulse

$$
\begin{gathered}
{}^cD^{\alpha}_{0,t}y(t) + f(t, y(t)) = 0, \\
t \in (s_i, t_{i+1}],\ i = 0, 1, 2, \ldots, m, \alpha\ \in (0, 1], \\
y(t) = H_i(t, y(t)),\ t \in (t_i, s_i], \\
y(0) = \mu {}^cD^q y(\psi),\ 0
\end{gathered}
$$

Dynamics of Mosquito Population Models with Or Without Stage Structure

Jia Li
University of Alabama in Huntsville, USA
Liming Cai, Yang Li

Mosquitoes undergo complete metamorphosis going through four distinct stages of development during a lifetime. To study the impact of the sterile insect technique on the transmission dynamics of mosquito-borne diseases, we formulate stage-structured continuous-time mathematical models for the interactive dynamics of the wild and sterile mosquitoes. We incorporate different strategies for the releases of sterile mosquitoes in the models and investigate the model dynamics, including the existence of positive equilibria and their stability. We compare the model dynamics with those of simplified models without stage structure and discuss what we have learned from this study.

Speed Selection and Stability of Traveling Waves to Reaction Diffusion Equations with Nonlinear Convection

Chunhua Ou
Memorial University, Canada

In this talk, we study the existence of traveling waves to reaction diffusion equation with nonlinear convection term. We also establish a mechanism on the linear or nonlinear determinacy of the minimal speed. Stability of these waves is also discussed. Finally we will present some applications including the Burgers-KPP-Fisher equation.

A Model of Platelet Production: Stability Analysis and Oscillations

Laurent Pujo-Menjouet
University of Lyon, France
M. Adimy, L. Boullu, F. Crauste

We propose here a new model of platelet production and regulation taking into ac- count the recent biological discoveries related to this topic, including the role played by thrombopoietin (TPO), a platelet regulation cytokine. We consider four different cell compartments corresponding to different cell maturity levels : the stem cell, megaka- ryocytic progenitors, megakaryocytes and platelets compartments. To the best of our knowledge, the progenitor compartment has never been taken into account in previous platelet production models. We consider also the quantity of circulating TPO that in- fluences the dynamics of each cell populations.
Our model consists in a non linear age structured partial differential equation system, where each equation corresponds to a compartment. This system can be reduced to a single non linear delay differential equation describing the dynamics the platelet population.
After a brief introduction of the model, we prove the existence of an unique steady state for the delay differential equation. We set up then conditions to get local and global asymptotic stability of this steady state. We determine then necessary and sufficient conditions for the existence of oscillating solutions.

A Model for Bacteria-Grazers Interactions in a Chemostat

Paul Salceanu
University of Louisiana at Lafayette, USA
J. Kong, H. Wang

We formulate a model of differential equations for the study of bacteria-grazers interactions in a carbon and nitrogen limited environment, in a chemostat. We provide sharp conditions that differentiate in between persistence and elimination of both bacteria and grazers from the system. For the case when bacteria and grazers natural death rates are assumed to be negligible, as compared to the dilution rate, we also provide sufficient conditions for global convergence to an interior equilibrium.

Bounded Global Hopf Branches for Stage-Structured Differential Equations

Hongying Shu
Tongji University, Peoples Rep of China
Lin Wang, Jianhong Wu

In this talk, we show that a class of stage-structured differential equations with unimodal feedback admit three types of global dynamic properties: a globally stable trivial equilibrium, or a globally stable positive equilibrium, or globally sustained oscillations. Using the delay as a bifurcation parameter, we analytically prove that there are finite number of neatly paired Hopf bifurcation values. Continuation of periodic solutions bifurcated from each Hopf bifurcation value forms a global Hopf branch, which is shown to be bounded and joint each Hopf bifurcation value to the corresponding paired one.

A Hybrid Mathematical Model for Cell Motility in Angiogenesis

Nicoleta Tarfulea
Purdue University Northwest, USA

The process of angiogenesis is regulated by the interactions between various cell types such as endothelial cells and macrophages, and by biochemical factors. In this talk, we present a hybrid mathematical model in which cells are treated as discrete units in a continuum field of a chemoattractant that evolves according to a system of reaction-diffusion equations, whereas the discrete cells serve as sources/sinks in this continuum field. It incorporates a realistic model for signal transduction and VEGF production and release, and gives insights into the aggregation patterns and the factors that influence stream formation. The model allows us to explore how changes in the microscopic rules by which cells determine their direction and duration of movement affect macroscopic formations. In particular, it serves as a tool for investigating tumor-vessel signaling and the role of mechano-chemical interactions of the cells with the substratum.

Modeling Optimal Control Treatment Strategies for HIV-TB Co-Infected Individuals

Naveen Vaidya
University of Missouri - Kansas City, USA

Co-infection of HIV and TB is one of the biggest problems for properly managing infections of these diseases. Particularly, individuals co-infected with HIV and TB are often in the situation of making critical decision on whether to begin treatments for both diseases simultaneously or wait to begin HIV-treatment until the completion of TB-treatment. In this talk, I will present a mathematical model that helps evaluate various treatment programs for HIV-TB co-infected individuals suggesting these individuals to make better treatment decisions. I will also present an optimal control problem formulated based on our model and how it can be used to identify the treatment strategies that result in the minimum burdens from this co-infection. This is a joint work with Abhishek Mallela and Suzanne Lenhart.

How Latency and Nonlocality Will Impact the Spread of Some Diseases.

Yanyu Xiao
University of Cincinnati, USA

The purpose of this work is to study the spatial dynamics of some delayed nonlocal reaction-diffusion systems. We will mathematically examine the effects of the delay and nonlocality on the spreading speed of the non-quasi-monotone systems.

A Nonlocal Spatial Model for Lyme Disease

Xiao Yu
Memorial Univeristy of Newfoundland, Canada
Xiao-Qiang Zhao

In this talk, I will report our recent research on a nonlocal and time-delayed reaction-diffusion model for Lyme disease with a spatially heterogeneous structure. In the case of a bounded domain, we first prove the existence of the positive steady state and a threshold type result for the disease-free system, and then establish the global dynamics of the model system in terms of the basic reproduction number. In the case of an unbound domain, we obtain the existence of the disease spreading speed and its coincidence with the minimal wave speed. At last, we use numerical simulations to verify our analytic results and investigate the influence of model parameters and spatial heterogeneity on the disease infection risk.

Ion Size Effects on Ionic Flows Via Steady-State Poisson-Nernst-Planck Models

Mingji Zhang
New Mexico Tech, USA
Peter W. Bates, Guojian Lin, Weishi Liu, Hong Lu, Yingfei Yi

We study boundary value problems of a quasi-one-dimensional steady-state Poisson-Nernst-Planck model with a local hard-sphere potential for ionic flows of two oppositely charged ion species through an ion channel, focusing on effects of ion sizes. The flow properties of interest, *individual fluxes and total flow rates of the mixture*, depend on multiple physical parameters such as boundary conditions (boundary concentrations and boundary potentials) and diffusion coefficients, in addition to ion sizes and valence. For the relatively simple setting and assumptions of the model, we are able to characterize, almost completely, the distinct effects of the nonlinear interplay between these physical parameters. The boundaries of different parameter regions are identified through a number of critical values that are explicitly expressed in terms of the physical parameters. We believe our results will provide useful insights for numerical and even experimental studies of ionic flows through membrane channels.

Special Session 35: Control and Optimization Theory for Partial Differential Equations

Lorena Bociu, North Carolina State University, USA
Daniel Toundykov, University of Nebraska-Lincoln, USA

The session will serve to promote and disseminate recent developments in the mathematical control theory for partial differential equations (PDEs). The discussion will address the following questions in the context of distributed-parameter systems modeled by evolution PDEs: controllability/observability results, inverse problems, stability and blow-up, optimization and sensitivity analysis, as well as pertinent qualitative properties of such systems, e.g., Hadamard well-posedness, regularity of solutions, or structure of global attractors.

Wellposedness Analysis of a Non-Linear Fluid-Structure PDE Model

George Avalos
University of Nebraska-Lincoln, USA
Pelin G. Geredeli

In this talk we shall focus on current work involving the wellposedness for a coupled partial differential equation model which governs a certain fluid-structure interaction. The basis of our approach is an argumentation thematically similar to that of Z. Yosida and Y. Giga, which was originally invoked for the (uncoupled) Navier-Stokes equations.

Stabilization of Two Coupled Wave Equations on a Compact Manifold with Boundary

Moez Daoulatli
University of Dammam, Saudi Arabia
Lassad Aloui

In this talk we present a result on stabilization of coupled wave equations by an order one term on a compact manifold with boundary. Only one of the two equations is directly damped by a localized damping term. Under natural hypotheses, we show that the energy of smooth solutions of the system decays polynomially.

Shape Differentials and Topological Semi-Differentials

Michel Delfour
Centre de recherches mathematiques, Universite de Montreal, Canada

Complete metric spaces of shapes and geometries are available without appealing to the notions of atlases or smooth manifolds. Since, at best, they are groups, the issue of characterizing their tangent spaces and differentials naturally arises. The Hadamard definition of a differentiable function is especially pertinent since it involves the construction of paths and tangent vectors to paths within the space. In 1937 Frèchet dropped the requirement that the differential be linear with respect to the direction while preserving two important properties of the differential calculus: continuity of the function and chain rule. His definition can be further relaxed to the one of semidifferential which handles convex and semiconvex functions while preserving the two properties. Since semidifferentials of functions $f : A \to B$ are not required to be linear, it relaxes the requirement that the tangent spaces to the sets A and B be linear spaces. It is sufficient to work with the Bouligand tangent cones to make sense of semidifferentials, shortcircuiting the requirement of a smooth manifold. It is shown that the topological derivative of Sokolowski and Zochowski is in fact a semidifferential on the space of characteristic functions and that the tangent space contains not only elements that create holes but also "currents of Federer and Fleming and H^d-rectifiable sets
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The Free Boundary Euler Equations in 3D

Marcelo Disconzi
Vanderbilt University, USA
David G. Ebin

We study the incompressible free boundary Euler equations with surface tension in three spatial dimensions. After establishing local well-posedness of the equations, we show that, under natural hypotheses, solutions are near those of the Euler equations in a fixed domain if the surface tension is sufficiently large.

Long Time Behavior of Berger Plate PDE Model Under Partial Nonlinear Boundary Dissipation

Pelin Guven Geredeli
Hacettepe University, Turkey
George Avalos, Justin T. Webster

We consider a (nonlinear) Berger plate in the absence of rotational inertia acted upon by nonlinear boundary dissipation. We assume the boundary to have two disjoint components: a clamped (inactive) portion and a controlled portion where the feedback is active via hinged-type condition. We emphasize the damping acts only in *one boundary condition* on a portion of the boundary. In this work we address the technical issues arising from damping active only on a portion of the boundary, including deriving a necessary trace estimate for $(\Delta u)\big|_{\Gamma_0}$. Additionally, we use recent techniques in the asymptotic behavior of

hyperbolic-like dynamical systems involving a stabilizability estimate to show that the compact global attractor has finite fractal dimension and exhibits additional regularity beyond that of the state space for finite energy solutions.

Analytical Problems for Free Liquid Films

Thomas Hagen
University of Memphis, USA

The theory of thin free films provides averaged models for the evolution of viscous fluid sheets and films forming in the ambient air. Models are given in the form of nonlinear transport equations for mass conservation and heat transfer, coupled with elliptic momentum balances. This presentation will focus on central analytical problems pertaining to the solvability of the governing equations and asymptotic regularity of the linearized semigroups. Of particular mathematical interest is the case of a temperature-induced frost line along the film. To tackle these problems, we make use of methods from nonlinear analysis, spectral and operator theory as well as pde specific techniques. Part of this work was done jointly with S. Ceci.

Analysis of a Fluid-Structure Interaction Problem with the Slip Boundary Condition

Boris Muha
University of Zagreb, Croatia
Suncica Canic

We study a nonlinear, moving boundary fluid-structure interaction (FSI) problem between an incompressible, viscous Newtonian fluid, modeled by the 2D Navier-Stokes equations, and an elastic structure modeled by the shell or plate equations. The fluid and structure are coupled via the *Navier slip boundary condition* and balance of contact forces at the fluid-structure interface. The slip boundary condition might be more realistic than the classical no-slip boundary condition in situations, e.g., when the structure is "rough", and in modeling FSI dynamics near, or at a contact. Cardiovascular tissue and cell-seeded tissue constructs, which consist of grooves in tissue scaffolds that are lined with cells, are examples of "rough" elastic interfaces interacting with an incompressible, viscous fluid. The problem of heart valve closure is an example of a FSI problem with a contact involving elastic interfaces. We design a stable partitioned numerical scheme for the considered problem and prove the existence of a weak solution to this class of problems by proving that the proposed scheme is convergent.

Regularity and Convergence of Solutions in Nonlocal Models

Petronela Radu
University of Nebraska-Lincoln, USA

The introduction of nonlocal theories in elasticity, dynamic fracture, biology has been successful in capturing real-world phenomena which can not be described through local models. At the theoretical level, many interesting questions emerge: what regularity should be expected of nonlocal solutions? What is the connection between nonlocal models and their local counterparts? In this talk I will present recent research summarizing some of these results and also their role in energy minimization.

Local and Global Solutions for Wave Equations of p-Laplacian Type with Generalized Robin Boundary Conditions

Mohammad Rammaha
University of Nebraska-Lincoln, USA
Nick Kass

In this talk we will discuss a wave equation with a p-Laplacian

$$u_{tt} - \Delta_p u = 0$$

subject to generalized Robin boundary conditions with damping and source terms:

$$|\nabla u|^{p-2}\partial_\nu u + |u|^{p-2}u + g(u_t) = f(u) \quad \text{on} \quad \partial\Omega \times (0,T).$$

The source feedback f is supercritical, in the sense that it is not locally Lipschitz $W^{1,p}(\Omega) \to L^2(\partial\Omega)$. Under suitable assumptions, we prove the existence of local weak solutions, which can be extended globally in time, provided the damping term dominates the source in some sense.

Implicit Parametrization and Fictitious Domain Approach in Shape Optimization

Dan Tiba
Romanian Academy, Romania

Fixed domain methods play an important role in variable/unknown domains problems like free boundary problems or optimal design problems. We use implicit representations of domains and recent constructive progress in implcit functions and implicit parametrizations theory, to develop a new approach based on general functional variations of open sets. The extension to the critical case will be discussed as well. Such variations may involve both boundary and topological variations simultaneously and may be very complex and rich from the geometric point of view. From the analytic point of view, they may be interpreted as directional derivatives in the class of implicit representations and enjoy certain advantages both theoretically and computationally (no remeshing, no recomputing of the mass matrices in each iteration, no Hamilton-Jacobi equation, etc.). We underline in this respect that, although shape functions are used, our method is different from the level set method. The approximating problem is an optimal control problem. Approximation results and computational examples, will be presented.

Modeling and Control of Wing and Flap Flutter: a Free-Clamped Plate in a Potential Flow

Justin Webster
College of Charleston, USA
I. Chueshov, E. Dowell, P.G. Geredeli, I. Lasiecka

We describe the difficult problem of modeling the flutter phenomenon for plates (or beams), where a portion of the structure's boundary is free, and the subsequent (open) issue of stabilizing—in some sense—the structure asymptotically in time. We discuss normal and axial flows. Much can be said at the qualitative level about flag, flap, and wing flutter; these phenomena are of great interest in the engineering literature, where finite dimensional studies for PDE models are compared with known experiments for specific configurations. Mathematically, there is a lack of any analysis. We begin by discussing the recent well-posedness and stabilization results for mathematical models of panel flutter: a nonlinear (von Karman or Berger plate) coupled to a perturbed wave equation (where the entire structural boundary is clamped). We then discuss the ways in which this analysis breaks down when a portion of the structure's boundary is free (e.g., the lack of an explicit formula for the flow Neumann map, and problematic interactive terms near the free boundary). Many of the peculiarities can be viewed as model deficiencies. Various partial results for this open model configuration will be presented, as well as many novel and challenging problems in mathematical aeroelasticity.

Special Session 36: New Trends in Nonlinear Partial Differential Equations and Applications

Julian Lopez-Gomez, Complutense University of Madrid, Spain
Santiago Cano-Casanova, Pontificia Comillas University, Spain
Marcela Molina-Meyer, Carlos III University, Spain

Since the emergence of the metasolutions to describe the dynamics of the most paradigmatic classes of parabolic equations in population dynamics in the presence of spatial heterogeneities took place, there have emerged a number of concepts and techniques, both theoretical and numerical, that are provoking a revolution in the theory of Nonlinear Partial Differential Equations whose consequences are far from understood yet. This session will try to give a unified version of some of the most significative recent advances from a number of different perspectives.

Traveling Waves in Parabolic Operators with a Flux-Limited Operator.

Juan Campos
Granada University, Spain

We are going to analize the travelling wave problem of

$$u_t = (c(u, u_x)u)_x + f(u)$$

where $c(u, u_x) = u^{p-1}\phi(u^{m-1}u_x)$ for a bounded increasing function $\phi : \mathbb{R} \to \mathbb{R}$ and f and look like $f(u) = u(1-u)$ of Fisher type. There is a regular travelling wave solution for $\sigma > \sigma_{smooth}$ with for some value $\sigma_{smooth} > 0$. This is the analogous to the minimal speed of propagation with regular waves and it is not always a minimum. For

Metasolutions in Logistic Problems with Spatial Heterogeneities and Nonlinear Mixed Boundary Conditions

Santiago Cano-Casanova
Comillas Pontifical University, Spain

In this talk will be analyzed the existence of metasolutons in a very general class of logistic problems with nonlinear mixed boundary conditions and spatial heterogeneities in the PDE and on the boundary conditions. All the results will be given in terms of the nodal behavior of the potentials appearing in the PDE and on the boundary conditions. The main tecniques used to achieve the results are bifurcation, monotonicity, continuation and blow up methods. The results obtained are strongly based in the previous results in the field due to R. Gòmez-Reñasco and J. Lòpez-Gòmez and they complement them.

Fitness Based Prey Dispersal and Prey Persistence in Intraguild Predation Systems

Robert Stephen Cantrell
University of Miami, USA
King-Yeung Lam, Tian Xiang, Xinru Cao

We establish prey persistence in intraguild persistence systems in bounded habitats under mild conditions when the prey disperses using its fitness as a surrogate for the balance between resource acquisition and predator avoidance. The model is realized as a quasilinear parabolic system where the dimension of the underlying spatial habitat is arbitrary.

A Free Boundary Approach to Spreading Problems

Yihong Du
University of New England, Australia

In this talk I'll describe some recent results on the spreading of species based on the equation

$$u_t - d\Delta u = f(u),\ x \in \Omega(t),\ t > 0,$$

where $\Omega(t)$ is a varying domain in R^N, whose boundary evolves according to a Stefan type free boundary condition. The results will be compared with classical results for the corresponding problem without a free boundary, namely

$$u_t - d\Delta u = f(u),\ x \in R^N,\ t > 0.$$

A Class of Degenerate Parabolic Equations

Patrick Guidotti
University of California, Irvine, USA
Yuanzhen Zhao

In this talk a class of degenerate parabolic equations will be introduced which is motivated by applications to Image Processing. It will be shown how lack of uniqueness affects the linear equations and their numerical approximations. Uniqueness in a special class of functions will be shown for the nonlinear equations.

A Two-Species Competition System with Slow Diffusion

Georg Hetzer
Auburn University, USA
Lourdes Tello

Two-species competition is considered in an isolated habitat (homogeneous Neumann conditions) in case that dispersal is modeled by a p-Laplacian with $p > 2$ (slow diffusion). Slow diffusion accounts, e.g., for effects of filtration. If none of the species has adapted to spatial conditions, the convergence to positive equilibria without persistence is established.

Nodal Solutions for a Class of Degenerate Boundary Value Problems

Julian Lopez-Gomez
Complutense University of Madrid, Spain
M. Molina-Meyer, P. H. Rabinowitz

In non-degenerate one-D sublinear boundary value problems the structure of the set of solutions where the problem admits a solution with n nodes is well known since the pioneering findings of P. H. Rabinowitz in the early seventies. Actually, these results tremendously facilitated the development of global bifurcation theory as used today. However, this paradigm changes dramatically in the context of singular boundary value problems, where the solution set can admit a large number of components making the analysis of these problems a real challenge.

Entire Solutions to Generalized Parabolic k-Hessian Equations

Kazuhiro Takimoto
Hiroshima University, Japan
Saori Nakamori

About a hundred years ago, Bernstein proved that if $f = f(x, y) \in C^2(\mathbb{R}^2)$ and the graph of $z = f(x, y)$ is a minimal surface in $\mathbb{R}^3$, then f is necessarily an affine function of x and y. This theorem gives the characterization of entire solutions to the minimal surface equation in $\mathbb{R}^2$. For Monge-Ampère equation, the following result is known; if $u \in C^4(\mathbb{R}^n)$ is a convex solution to $\det D^2u = 1$ in $\mathbb{R}^n$, then u is a quadratic polynomial. In this talk, we shall obtain this type of theorem for entire solutions to parabolic k-Hessian equation of the form

$$u_t = \mu\left(F_k(D^2u)^{\frac{1}{k}}\right) \quad \text{in } \mathbb{R}^n \times (-\infty, 0],$$

where $\mu : (0, \infty) \to \mathbb{R}$ is a function, under some assumptions.

Acceleration of Fisher-KPP Propagation in Presence of Reaction and Diffusion Heterogeneities

Andrea Tellini
EHESS, Paris, France
L. Rossi, E. Valdinoci

In this talk I will present some new propagation phenomena in the context of reaction-diffusion systems of Fisher-KPP type, which model the dynamics of a population in a 2 dimensional environment with one or more lines of fast diffusion. I will also consider the effect of reaction heterogeneities, again in $1D - 2D$ cases, but also in $2D - 2D$ ones.

Multiplicity of Stationary Solutions for a Nonlinear Climate Model.

Lourdes Tello
Universidad Politecnica de Madrid, Spain

We study a nonlinear energy balance model arising in Climatology. Time and space scales are relatively large. The space domain is a 2D-manifold. The main difficulties in the mathematical analysis of the model have been the nonlinear diffusion and the coalbedo term which is represented by a bounded maximal monotone graph. One of the characteristic of this kind of global climate model is the sensitivity to the terrestrial and solar parameters. We analyze the number of stationary solutions according those parameters.

A Loop Type Subcontinuum of Positive Solutions of an Indefinite Concave-Convex Problem with the Neumann Boundary condition

Kenichiro Umezu
Ibaraki University, Japan
Humberto Ramos Quoirin

In this talk we consider the concave-convex elliptic equation $-\Delta u = \lambda b(x)|u|^{q-2}u + a(x)|u|^{p-2}u$ in Ω coupled with the Neumann boundary condition $\frac{\partial u}{\partial \mathbf{n}} = 0$ on $\partial\Omega$, where Ω is a smooth bounded domain of $\mathbb{R}^N$, $N \geq 2$, λ is a real parameter, $1 < q < 2 < p$, and $a, b \in C^\alpha(\overline{\Omega})$, $\alpha \in (0, 1)$, change sign. We use variational and bifurcation combined arguments to discuss existence of a loop type subcontinuum of positive solutions and investigate its properties when λ varies. The loop type subcontinuum means a bounded one joining $(\lambda, u) = (0, 0)$ to itself and never meeting any $(\lambda, 0)$ with $\lambda \neq 0$.

Patch Effects on the Pattern Formation in a Population Model with Cross Diffusion

Zhifu Xie
Virginia State University, USA

My talk is concerned with the patch effects on the pattern formation in a population model with cross-diffusion. In previous paper without the patch, cross diffusion induces the existence of various patterns such as stripe pattern, spot pattern, and non-steady patterns. In particular, we are interested in how the size of the patch changes the dynamical properties of the positive stationary solutions. We also numerically illustrate how the patch affects the pattern formations due to the presence of the cross diffusion and the patch.

Multiple Spreading Phenomena for a Free Boundary Problem for Diffusion Equations with Bistable Nonlinearity

Yoshio Yamada
Waseda University, Japan

This talk is concerned with a free boundary problem for diffusion equations with bistable nonlinearity which allows two positive equilibrium states as an ODE model. The problem models the invasion or migration of a biological species and the free boundary represents the spreading front of the habitat. Our main interest is to study large time behaviors of solutions of the free boundary problem. It will be shown that our problem exhibits two different types of spreading behaviors of solutions. Moreover, we will give precise information on these two types of spreading behaviors such as asymptotic profiles of solutions and asymptotic speeds of free boundaries when time goes to infinity.

Sharp Asymptotic Profiles of Solutions to a Simplified Attraction-Repulsion Chemotaxis Model in the Whole Space

Tetsuya Yamada
National Institute of Technology, Fukui College, Japan

In this talk we consider the Cauchy problem for a simplified version of attraction-repulsion chemotaxis model in high dimensions which was introduced in Luca et al to describe the aggregation of microglia in Alzheimer's disease. Recently, Shi and Wang proved the existence of global classical solution and moreover gave the boundedness of solution and its decay estimate in L^p-norm if the repulsion prevails over the attraction. On the other hand, the solution may blow up in finite time in two dimensions under suitable conditions if the attraction prevails over the repulsion. We shall focus on the case where the repulsion prevails over the attraction and give more precise asymptotic profiles of the decaying solution, introducing a spatial shift and correction term.

Special Session 37: Recent Advances in Dynamical Systems with Applications to Ecology and Epidemiology

Hongying Shu, Tongji University, Peoples Rep of China
Yanyu Xiao, University of Cincinnati, USA
Guihong Fan, Columbus State University, USA

Dynamical systems have been playing an important role in modeling biological phenomena in ecology and epidemiology. Over the past decades, this research area has gained an increasing attention and received a growing momentum in mathematical biology. This special session will focus on recent advances in dynamical systems with applications to ecology and epidemiology. We will invite researchers from various backgrounds to work together and contribute to the study of dynamical systems. Speakers and talks are carefully selected to make the session attractive to a diverse audience.

Mathematical Assessment of the Role of Temperature and Rainfall on Mosquito Population Dynamics

Ahmed Abdelrazec
Arizona State University, USA
Abba B. Gumel

A new stage-structured model for the population dynamics of the mosquito (a major vector for numerous vector-borne diseases), which takes the form of a deterministic system of non-autonomous nonlinear differential equations, is designed and used to study the effect of variability in temperature and rainfall on mosquito abundance. Three functional forms of eggs oviposition rate, namely a constant, logistic and Maynard-Smith-Slatkin functions, are used. Rigorous analysis of the autonomous version of the model shows that, for any of the three oviposition functions considered, the trivial equilibrium of the model is locally- and globally-asymptotically stable if a certain vectorial threshold is less than unity. Conditions for the existence and globally-asymptotic stability of non-trivial equilibrium solutions of the model are also derived. The model, subject to the logistic and Maynard-Smith-Slatkin oviposition functions, is shown to undergo a Hopf bifurcation under certain conditions.
The analyses reveal that the Maynard-Smith-Slatkin oviposition function sustains more oscillations than the other two oviposition functions considered (hence, it is more suited, from ecological viewpoint, for modeling the oviposition process).
The non-autonomous model is shown to have a globally-asymptotic stabile trivial periodic solution, for all forms of the oviposition functions, when the associated reproduction threshold ($\mathbb{R}_0^*$) is less than unity. Furthermore, the model, with the logistic and Maynard-Smith-Slatkin oviposition functions, has a unique and globally-asymptotic stable periodic solution under certain conditions. Numerical simulations of the non-autonomous model, using mosquito surveillance and weather (temperature and rainfall) data from the Peel region of Ontario, Canada, show a peak mosquito abundance for temperature and rainfall in the ranges $[20-25]^oC$ and $[15-35]$ mm, respectively. These ranges are recorded in the Peel region between July and August (hence, this study suggests that anti-mosquito control effects should be intensified during this period).

Global Threshold Dynamics of a Heroin Epidemic Model with Age of Infection and Nonlinear Incidence

Yuming Chen
Wilfrid Laurier University, Canada
Junyuan Yang, Zhen Jin

In this talk, we propose and analyze a heroin model with a general nonlinear incidence rate and age of infection. Whether the use of heroin spread or not is determined by the basic reproduction number R_0. If $R_0 1$ then the drug-user steady state is globally asymptotically stable. This threshold dynamics is established by employing the fluctuation lemma and the Lyapunov functionals. Our results imply that improving the detection rate and drawing up efficient prevention ways play an more important role than just increasing the treatment rate for drug users. This is joint work with Professors Junyuan Yand and Zhen Jin.

Avian Species-Diversity and Dilution and The Transmission Dynamics of West Nile Virus

Guihong Fan
Columbus State University (Georgia), USA
Huaiping Zhu

West Nile virus is a typical mosquito-borne disease which vector mosquitoes play a critical rule in the transmission and spreading of the diseases. There are many compartmental models in literature about West Nile virus (WNv), but most of them ignore the impact of bird species diversity and impact of species dilution. In this work, we formulate a system of differential equations to model the transmission of WNv with an emphasis on the impact of bird species diversity. We classify birds into n species and for each species, we define a competent index $D(\mathcal{R}_j)$ which is a function of species specific parameters and related basic reproduction number $\mathcal{R}_j$. We also find the basic reproduction number $\mathcal{R}_0$ for the model with n species of birds. Study shows that if one more species of birds with the lowest competent index $D(\mathcal{R}_j)$ is added to the model, then the basic reproduction number $\mathcal{R}_0$ decreases. Analytical analysis and numerical simulations show that conservation of avian species diversity can help reduce human exposure to the virus.

But on the contrary, bird species diversity also increases the chance for virus to survive and may be responsible for the repeated outbreak of the disease from year to year. This is a joint work with Prof. Huaiping Zhu (York University).

Recovery Rates in Epidemiological and Ecological Models

Scott Greenhalgh
Queen's University, Canada
Troy Day

Differential equation models of infectious disease have undergone many theoretical extensions that have proved invaluable for the evaluation of disease spread. For instance, while one traditionally uses a bilinear term to describe the incidence rate of infection, physically more realistic nonlinear generalizations exist. However, such theoretical extensions of recovery rates have yet to be developed. This is despite the fact that a constant recovery rate does not perfectly describe the dynamics of recovery, and that the recovery rate is arguably as important as any incidence rate in governing the dynamics of a system. In this talk, a derivation of state dependent and time varying recovery rates in differential equation models of infectious disease is provided. We justify our derivation through an intimate connection between integral equations, differential equations, and stochastic processes. Finally, we apply our derivation to a obtain a novel recovery rate where infected individuals can only contribute to disease spread for a finite amount of time, which is in contrast to the behavior under constant recovery rates.

Bifurcation Behaviours in Convalescent Blood Transfusion System

Xi Huo
Ryerson University, Canada
Xiaodan Sun, Yanni Xiao, Kunquan Lan, Jianhong Wu

Convalescent blood transfusion refers to the therapy that transfuses blood donations from survivors of an infectious disease to the ills. This treatment strategy was historically practiced in the pre-antibiotic era, and is recommended for investigation to become an empiric treatment in many emerging infectious diseases - such as Ebola, H1N1, and MERS. We establish a differential equation system with single delay to describe a population level blood transfusion service and study the bifurcation behaviours of the three dimensional system. We will present the asymptotic stability of equilibria, challenges in analyzing the bifurcation behaviour of the system, and public health implications of such phenomenon.

Stability and Bifurcation in a Stoichiometric Producer-Grazer Model with Knife Edge

Yang Kuang
Arizona State University, USA
Xianshan Yang, Xiong Li, Hao Wang

All organisms are composed of multiple chemical elements such as nitrogen (N), phosphorus (P), and carbon (C). P is essential to build nucleic acids (DNA and RNA) and N is needed for protein production. To keep track of the mismatch between P requirement in the grazer and the P content in the producer, stoichiometric models have been constructed to explicitly incorporate food quality and quantity. Most stoichiometric models have suggested that the grazer dynamics heavily depend on P content in the producer when the producer has low nutrient content (low P:C ratio). Motivated by recent lab experiments, researchers explored the effect of excess producer nutrient content (extremely high P:C ratio) on the grazer dynamics. This phenomenon is called the stoichiometric knife edge. However, the global analysis of this knife-edge model has is challenging because the phase plane is separated into many different regions in which the governing equations are different. The aim of this paper is to present a sample of a complete mathematical analysis of the dynamics of this model and to perform a bifurcation analysis for the model with Holling type II functional response.

Hopf Bifurcation in Reaction-Diffusion Population Model with Spatial-Temporal Nonlocal Delayed Growth Rate

Junping Shi
College of William and Mary, USA
Wenjie Zuo

We consider the existence of spatially inhomogeneous time-periodic orbit in a reaction-diffusion population model with spatial-temporal nonlocal delayed growth rate and Dirichlet boundary condition. When the dispersal kernel is of strong or weak type, the scalar reaction-diffusion equation with distributed delay is converted into a system of two or three reaction-diffusion equations without delay. We prove the existence of periodic orbits for the system which are equivalent to periodic orbits for the original scalar model.

Turing-Hopf Bifurcation in the Reaction-Diffusion Equations and Its Applications

Yongli Song
Tongji University, Peoples Rep of China
T.Zhang, Y.Peng

In this talk, we consider the Turing-Hopf bifurcation arising from the reaction-diffusion equations. It is a degenerate case and where the characteristic equation has a pair of simple purely imaginary roots and a simple zero root. First, the normal form theory for partial differential equations (PDEs) with delays developed by Faria is adopted to this degenerate case so that it can be easily applied to Turing-Hopf bifurcation. Then, we present a rigorous procedure for calculating the the normal form associated with the Turing-Hopf bifurcation of PDEs. We show that the reduced dynamics associated with Turing-Hopf bifurcation is exactly the dynamics of codimension-two ordinary differential equations (ODE), which implies the ODE techniques can be employed to classify the reduced dynamics by the unfolding parameters. Finally, we apply our theoretical results to an autocatalysis model governed by reaction-diffusion equations; for such model, the dynamics in the neighbourhood of this bifurcation point can be divided into six categories, each of which is exactly demonstrated by the numerical simulations; and then according to this dynamical classification, a stable spatially inhomogeneous periodic solution has been found.

Explicitly Separating Growth and Motility in a Glioblastoma Tumor Model

Tracy Stepien
Arizona State University, USA
Erica Rutter, Meng Fan, Yang Kuang

Glioblastoma multiforme is an aggressive brain cancer that is extremely fatal. Gliomas are characterized by highly diffusive growth patterns, which makes them impossible to remove with surgery alone. To gain insight on the mechanisms most responsible for tumor growth and the difficult task of forecasting future tumor behavior, we investigate a mathematical model that is inspired by a bacteriophage infection-spreading model in which tumor cell motility and cell proliferation are considered as separate processes. Numerical and analytical results are compared to experimental data.

Mathematical Model of Pathogen Persistence in a Water Distribution Network

Benjamin Vaughan
University of Cincinnati, USA
Benjamin L. Vaughan, Jr.

In industrialized nations, the availability of clean water is provided through sophisticated water distribution systems that consist of a network of pipes, pumping stations, reservoirs, and other components. Biological contamination of the water distribution system, due to factors such as a breach in a pipe or other causes, can have an adverse effect on water quality. In this talk, we will discuss the development and analysis of a mathematical model pertaining to the release of pathogens into water distribution systems. Bacteria within a water distribution system frequently form biofilms on the interior surfaces of pipes that pathogens can then attach to and grow. Our model investigates the interaction between planktonic and biofilm-bound pathogens under time-constant and time-periodic flow regimes and is analyzed using Lyapunov stability and Floquet theory as well as numerical simulations. Often, water distribution networks have a significant number of connections, making direct calculations expensive. So, we have developed and analyzed an efficient approach for predicting the long-time behavior of the pathogen populations for large water-distribution networks. The analytical results are validated using numerical simulations of the full non-linear system on a range of water distribution network sizes.

Competition for Two Essential Resources with Internal Storage and Periodic Input

Feng-Bin Wang
Department of Natural Science, Center for General Education, Chang Gung University, Taiwan
Sze-Bi Hsu, Xiao-Qiang Zhao

We study a mathematical model of two species competing in a chemostat for two internally stored essential nutrients, where the nutrients are added to the culture vessel by way of periodic forcing functions. Persistence of a single species happens if the nutrient supply is sufficient to allow it to acquire a threshold of average stored nutrient quota required for growth to balance dilution. More precisely, the population is washed out if a sub-threshold criterion holds, while there is a globally stable positive periodic solution if a super-threshold criterion holds. When there is mutual invasibility of both semitrivial periodic solution of the two-species model, both uniform persistence and the existence of periodic coexistence state are established.

Sensitivity of the General Rosenzweig–MacArthur Model to the Mathematical Form of the Functional Response: a Bifurcation Theory Approach

Gail Wolkowicz
McMaster University, Canada
Gunog Seo

The Rosenzweig–MacArthur predator-prey model has been shown to be sensitive to the mathematical form used to model the predator response function even if the forms have the same basic shape: zero at zero, monotone increasing, concave down, and saturating. We revisit this model to help explain this sensitivity in the case of Holling type II Monod , Ivlev, and hyperbolic trigonometric response functions. We consider the local and global dynamics and determine the possible bifurcations with respect to variation of the carrying capacity of the prey, a measure of the enrichment of the environment. We give analytic expressions that determine the criticality of the Andronov-Hopf bifurcation, and prove that although all three forms can have supercritical Hopf bifurcation, only the hyperbolic trigonometric form can give rise to subcritical Hopf bifurcation, saddle-node bifurcation of periodic orbits, and multiple limit cycles, providing a counterexample to a conjecture of Kooij and Zegeling (1996) and a result in a paper by Attili and Mallak (2006). We revisit the ranking of responses, according to their potential to destabilize dynamics, and show that given data, not only the choice of functional form, but the choice of number and position of data points influences predicted dynamics.

Bridging the Behaviors of Intestinal Stem Cells and Their Molecular Control Networks with Mathematical Modeling

Tongli Zhang
University of Cincinnati, USA
Richard Ballweg, Emma Teal, Toru Matsuura, Benjamin Vidourek, Meredith Barone, Christian Hong

Intestinal stem cells continuously differentiate into mature functional cells (e.g. Enterocytes, Goblet cells, Enteroendocrine cells and Paneth cells). This essential process is tightly regulated in healthy organisms and its deregulation leads to diseases. As current molecular and cellular biology reveals the regulatory details of the pathways (e.g. Wnt, Notch, MAPK, BMP) controlling this process, the accumulated information proposes the challenge of integrating these details together into a logical and dynamical framework. In order to cope with this challenge, we have converted several key pathways into computational models. These model have been constrained with available data reported in the literature, and the novel predictions generated by these models are tested in intestinal enteroids.

Special Session 38: Evolution Equations and Integrable Systems

Alex Himonas, University of Notre Dame, USA
Dionyssis Mantzavinos, SUNY at Buffalo, USA

The theme of this session is nonlinear evolution equations and integrable systems including the NLS equation, the KdV equation, the Camassa-Holm equation, and the Euler equations of hydrodynamics. Topics covered for these equations include, among others, local and global well-posedness, scattering and stability issues, integrability and solitary waves.

Wellposedness of the Fully Non-Linear KdV Equation

Timur Akhunov
University of Rochester, USA
David Ambrose, Doug Wright

Dispersion is a phenomenon that describes waves of different frequency traveling at different speed. This effect is related to a rainbow forming with light passing through a prism and in uncertainty principle in quantum mechanics. One of the most famous dispersive equations is the Korteweg-de Vries (KdV) equation, which was originally proposed for the propagation of long waves in a shallow canal. Several generalizations of KdV, like K(m,n) model with compact solitary waves do not have linear dispersion and are much less understood. In this talk I would discuss new wellposedness and illposedness results.

Ill-Posedness of Some Water Wave Models

Jerry Bona
University of Illinois at Chicago, USA
David Ambrose, Mimi Dai, Timur Milgrom

We discuss ill-posedness of some water wave models. These models are members of a class of Boussinesq-type systems that arise in approximating surface waves in relatively shallow water. They include some well known systems of equations. The ill-posedness does not stem from attempting to work in a very large function class, but rather is intrinsic to the model.

Semiclassical Dynamics of the Three-Wave Resonant Interaction Equations.

Robert Buckingham
University of Cincinnati, USA
Robert Jenkins, Peter Miller

The three-wave resonant interaction equations describe the evolution of three electrical pulses in a dispersive medium with quadratic linearity. We use the inverse-scattering method to analyze the small-dispersion (or semiclassical) behavior. We present analytic results on the WKB approximation of the scattering data, as well as a numerical study of exact solutions that suggests semiclassical behavior (i.e. approximation of solutions by modulated elliptic functions) similar to that seen in other nonlinear wave equations such as the KdV, NLS, and sine-Gordon equations. This work is joint with Robert Jenkins and Peter Miller.

An Inverse Problem for the Modified Camassa-Holm (or called FORQ) Equation and Multi-Point Padé Approximants

Xiangke Chang
University of Saskatchewan, Canada
Jacek Szmigielski

An spectral and inverse spectral problem associated to multipeakons for the modified Camassa-Holm (or called FORQ) equation are studied. It is shown that the inverse problem is solvable in terms of multi-point Padè approximations, which leads to an explicit construction of peakon solutions.

Existence and Stability of Solitary Wave Solutions to a Coupled BBM System

Hongqiu Chen
University of Memphis, USA
Xiaojun Wang

Considered here is a class of system of BBM-type nonlinear dispersive equations

$$U_t + c_0 U_x - AU_{xxt} + (\nabla H(u))_x = 0,$$

where $U = U(x,t)$ is an $\mathcal{R}^2$-valued function, c_0 is a positive constant, A is a 2×2 real positive definite matrix, and ∇H is the gradient of a homogeneous polynomial function $H : \mathcal{R}^2 \to \mathcal{R}$ of degree p. In this work, we present existence and stability criteria for the solitary wave solutions to the system that contains coupled nonlinear terms. Using the idea by Bona, Chen and Karakashian
and exploiting the accurate point spectrum information of the associated Schrödinger operator, we found that the stability of solitary wave solution is determined by the degree p in a simple algebraic relation.

The Geometry of Fourier-Mellin Transforms, and How to Use It

Darren Crowdy
Imperial College London, England

The construction of novel Fourier/Mellin-type transform pairs that are tailor-made for given planar regions within the special class of circular domains is described. Circular domains are those having boundary components that are either circular arcs or straight lines. The new transform pairs generalize the classical Fourier and Mellin transforms. These geometry-fitted transform pairs can be used to great advantage in solving boundary value problems defined in these domains.

The Unified Transform Method for Systems

Bernard Deconinck
University of Washington, USA

The Unified Transform Method (UTM) due to Fokas has been successfully applied to a great number of very different problems. Most of these problems have been scalar problems. When dealing with systems of equations, the dispersion relation of the problem is of the same order as the system, which leads to branched frequency functions appearing in the global relation. Most systems that have been approached using the UTM avoid have dispersion relation solutions that are not branched, and as such their solution is not typical. A few systems with branched solutions have been considered, but on an ad hoc basis. I will outline how the UTM can systematically deal with systems of equations without much added effort, compared to the scalar case.

A Priori Symmetry and Decay Properties of a Nonlocal Shallow Water Wave Equation

Mats Ehrnstrom
NTNU Norwegian University of Science and Technology, Norway
G. Brüll, L. Pei

We prove the following interesting connection: that traveling solitary waves of a nonlocal wave equation are necessarily symmetric, monotone on a half-line, and of exponential decay rate; and that symmetric solutions of the initial-value problem for the same equation are necessarily traveling. Whereas the second proof is based on a quite general structural property (which we extend here to a nonlocal setting), the proof of the first three facts relies on a detailed analysis of the Fourier transform of $m(\xi) = \sqrt{\frac{\tanh \xi}{\xi}}$, which we prove is completely monotone. More precisely, we study the Whitham equation

$$u_t + uu_x + \int K(x-y)u_x \, dx = 0,$$

where the integral kernel K has the symbol $m(\xi)$, arising naturally in the study of water waves.
The talk is based on recent results joint with Gabriele Brüll and Long Pei; Anna Geyer; and Erik Wahlèn.

Scattering for a 3D Coupled Nonlinear Schrödinger System

Luiz gustavo Farah
Universidade Federal de Minas Gerais (UFMG), Brazil
Ademir Pastor

We consider the three-dimensional cubic nonlinear Schrödinger system.

$$\begin{cases} i\partial_t u + \Delta u + (|u|^2 + \beta |v|^2)u = 0, \\ i\partial_t v + \Delta v + (|v|^2 + \beta |u|^2)v = 0. \end{cases}$$

Let (P, Q) be any ground state solution of the above Schrödinger system. We show that for any initial data (u_0, v_0) in $H^1(\mathbb{R}^3) \times H^1(\mathbb{R}^3)$ satisfying $M(u_0, v_0)A(u_0, v_0) < M(P, Q)A(P, Q)$ and $M(u_0, v_0)E(u_0, v_0) < M(P, Q)E(P, Q)$, where $M(u, v)$ and $E(u, v)$ are the mass and energy (invariant quantities) associated to the system, the corresponding solution is global in $H^1(\mathbb{R}^3) \times H^1(\mathbb{R}^3)$ and scatters. Our approach is in the same spirit of Duyckaerts-Holmer-Roudenko, where the authors considered the 3D cubic nonlinear Schrödinger equation. The author was partially supported by CNPq/Brazil and FAPEMIG/Brazil.

Regularity Results for Generalizations of the Wave Maps Equation

Dan Geba
University of Rochester, USA

In this talk, we will present an overview of recent results for semilinear and quasilinear generalizations of the wave maps equation, which have a strong physical motivation and present many challenges from a mathematical point of view. The emphasis will be on local and global regularity theories for these problems and the new analytic tools developed in connection to these theories.

The Nonlinear Schrödinger Equation on the Half-Line

Alex Himonas
University of Notre Dame, USA
Athanassios S. Fokas, Dionyssios Mantzavinos

The initial-boundary value problem (ibvp) for the nonlinear Schrödinger (NLS) equation on the half-line with data in Sobolev spaces is analyzed via the formula obtained through the unified transform method, and a contraction mapping approach. First, the linear Schrödinger (LS) ibvp with data in Sobolev spaces is solved and the basic space and time estimates of the solution are derived. Then, using these estimates well-posedness of the corresponding nonlinear ibvp is proved for data belonging in Sobolev spaces with appropriate exponents.

An Ill-Posedness Result for Novikov's Equation

Curtis Holliman
The Catholic University of America, USA
Alex Himonas

Novikov produced a new integrable equation in 2009 that has many similarities to the Camassa-Holm equation. Well-posedness for this equation has been established in Sobolev spaces with exponents $s > 3/2$, and we now show that this result is sharp.

The Hunter-Saxton Equation in Besov Spaces on the Circle

John Holmes
The Ohio State University, USA
Feride Tiglay

We show that the Hunter-Saxton equation is well-posed (existence, uniqueness and continuous dependence) in Besov spaces $B^s_{2,r}$ on the circle when $s > 3/2$ and $r > 1$ as well as the case $s = 3/2$ and $r = 1$. Furthermore, we construct an example which demonstrates that the continuity of the data-to-solution map is not uniformly continuous from any bounded subset of $B^s_{2,r}$ to $B^s_{2,r}$. Our results are improvements upon earlier results in Sobolev spaces, and take advantage of some properties of functions in Besov spaces.

Stability of Closed Solutions to the VFE Hierarchy

Thomas Ivey
College of Charleston, USA
Stephane Lafortune

The Vortex Filament Equation (VFE) is part of an integrable hierarchy of geometric evolution equations for space curves, several of which have been shown to describe vortex filament motion in various situations. We develop a general framework for studying the existence and linear stability of closed solutions of the VFE hierarchy, based on the correspondence between the VFE and the nonlinear Schrodinger (NLS) hierarchies. Our results show that it is possible to establish a connection between the AKNS Floquet spectrum and the stability properties of the solutions of the filament equations. We apply our machinery to solutions of the filament equation associated to the Hirota equation.

Local Energy Conservation and Spectral Projection for an Integrable Long Wave Model

Henrik Kalisch
University of Bergen, Norway

The Kaup-Boussinesq system is a coupled system of nonlinear partial differential equations which has been derived as a model for surface waves in the context of the Boussinesq scaling. The system is known to be linearly ill-posed, but it is also known to be completely integrable, so that solutions of the system may be constructed more or less explicitly. This work presents a derivation of the energy density and energy flux of the Kaup-Boussinesq system. It is also shown that the total energy of the wave system is equal to the Hamiltonian function found by Craig and Groves. In addition, a spectral method for the numerical discretization of the Kaup-Boussinesq system is put forward, and shown to be convergent and stable for certain classes of solutions.

The Korteweg-De Vries Equation on the Half-Line

Dionyssios Mantzavinos
SUNY Buffalo, USA
Athanassios S. Fokas, A. Alexandrou Himonas

A new method for the well-posedness of initial-boundary value problems for nonlinear dispersive evolution equations was recently introduced via the analysis of the nonlinear Schrödinger (NLS) and the "good" Boussinesq (GB) equations on the half-line. This talk will be concerned with the further extension and implementation of this method for the Korteweg-de Vries (KdV) equation on the half-line with data in Sobolev spaces. As already known from the analysis of the Cauchy problem, the KdV nonlinearity requires special attention and, in particular, introduces the need for additional linear estimates that emanate from the relevant bilinear estimate. Moreover, the unified transform method solution formula, which assumes the role held by Duhamel's formula in the Cauchy problem, involves genuinely complex contours of integration that stay away from the real and imaginary axes of the Fourier plane. Consequently, several modifications and generalizations of the techniques previously employed for NLS and GB will now be necessary.

Two-Component Generalizations of the Camassa-Holm Equation

Vladimir Novikov
Loughborough University, England
Andrew N.W. Hone, Jing Ping Wang

We present a classification of integrable two-component systems of non-evolutionary partial differential equations that are analogous to the Camassa-Holm equation via the perturbative symmetry approach. Independently, we perform a clas-

sification of compatible pairs of Hamiltonian operators, which leads to bi-Hamiltonian structures for the same systems of equations. We also consider exact solutions and Lax representations for the obtained integrable systems.

Global Analyticity Well-Posedness for a Generalized Camassa-Holm Equation

Gerson Petronilho
Federal University of Sao Carlos, Brazil
Rafael F. Barostichi, Alex A. Himonas, Gerson Petronilho

We prove that the Cauchy problem for a generalized Camassa-Holm equation with initial data in an analytic space has a unique global analytic solution $u \in C^{\omega}(\mathbb{R}_x \times [0, \infty)_t)$ provided that $(1 - \partial_x^2)u_0$ does not change sign.

Negative Integrable Hierarchy Related to Peakon Equation

Zhijun (George) Qiao
University of Texas - Rio Grande Valley, USA
Baoqiang Xia

Integrable peakon equations are derived from the negative order hierarchy through Lax formulation. Some multi-peakon soltions are given through some feasible procedure.

Initial-To-Interface Maps for the Heat Equation on Composite Domains

Natalie Sheils
University of Minnesota, USA
Bernard Deconinck

A map from the initial conditions to the values of the function and its first spatial derivative evaluated at the interface is constructed for the heat equation on finite and infinite domains with n interfaces. The existence of this map allows changing the problem at hand from an interface problem to a boundary value problem which allows for an alternative to the approach of finding a closed-form solution to the interface problem.

Classical Solutions of the Generalized Camassa-Holm Equation

Ryan Thompson
University of North Georgia, USA
John Holmes

In this presentation, well-posedness in $C^1(\mathbb{R})$ (a.k.a. classical solutions) is investigated for a generalized Camassa-Holm equation (g-kbCH) having $(k+1)$-degree nonlinearities and containing as its integrable members the Camassa-Holm, the Degasperi-Procesi and the Novikov equations.

Nonuniform Dependence on Initial Data for Compressible Gas Dynamics

Feride Tiglay
Ohio State University, Newark, USA
Barbara Keyfitz

Once the well posedness of a Cauchy problem is established, a natural question to ask is whether the data to solution map is better than continuous. We study the data to solution map of the Cauchy problem for compressible gas dynamics.

Special Session 40: Polymer Dynamics Models and Applications to Neurodegenerative Disease

Leon Matar Tine, Universite de Lyon, France
Laurent Pujo-Menjouet, Universite de Lyon, France

This session is dedicated to the study of models dealing with polymer dynamics (coagulation / fragmentation / polymerization) such as the Smoluchowski equation, Becker-Döring model for instance but not only. Biological applications will mainly be focused on neurodegenerative diseases such as prion disease, Alzheimer, Parkinson but not only.

Asymptotic Behaviour of the Becker-Döring Equations

Jose Cañizo
University of Granada, Spain
Bertrand Lods, Amit Einav

The Becker-Döring equations appear often in models of polymerisation but their asymptotic behaviour is still not well understood. We present some recent advances in the study of the asymptotic behaviour of subcritical solutions, where entropy methods can be applied both in the linearised regime and in the nonlinear one. We obtain explicit exponential rates of relaxation to equilibrium for rapidly decaying initial conditions, and algebraic rates for initial conditions with a larger tail.

Modelling Interactions Between Alzheimer's Disease and Prions

Pauline Mazzocco
ICJ University Lyon 1, France
Matthieu Dumont, Abdelkader Lakmeche, Laurent Pujo-Menjouet, Human Rezaei, Leon Matar Tine

Alzheimer's disease (AD) is a fatal incurable disease leading to progressive neuron destruction. AD is caused by the presence of Aβ peptides inside the brain, especially Aβ-40 and Aβ-42 monomers. They have the faculty to aggregate into oligomers and fibrils, which eventually form amyloid plaques. These latter were thought to cause neuron loss for years. However it seems that oligomers are the most toxic structures as they can interact with neurons through membrane receptors, including PrP^C proteins. This interaction may lead to the misconformation of PrP^C into pathogenic prions, PrP^{SC}.
In this work, we developed a model describing Aβ polymerization process into oligomers and fibrils. We included the interactions between oligomers and PrP^C. The model consists of 13 equations, including size structured transport equations, ordinary differential equations and delayed differential equations. We provide theoretical results regarding existence of solutions, and numerical simulations of the model.

Dynamics of Intermediate Filaments

Stephanie Portet
University of Manitoba, Canada

Intermediate filaments are one of the cytoskeleton components. The cytoskeleton is made of structural proteins polymerized in filaments that are organized as networks in the cytoplasm. The organization of a cytoskeletal network is the main determinant of its function in cells. The cytological signature of some human diseases is the misorganization of cytoskeletal networks. Here, models based on experimental data combining assembly/disassembly processes and transport of intermediate filaments are presented to investigate the major contributors to their organization.

A Micellar On-Pathway Intermediate Step Explains the Kinetics of Prion Amyloid Formation

Laurent Pujo-Menjouet
University of Lyon, France
Erwan Hingant, Pascaline Fontes, Maria Teresa Alvarez-Martinez, Jacques-Damien Arnaud, Jean- Pierre Liautard

In a previous work by Alvarez-Martinez et al. in 2011, the authors pointed out some fallacies in the mainstream interpretation of the prion amyloid formation. It appeared necessary to propose an original hypothesis able to reconcile the in vitro data with the predictions of a mathematical model describing the problem. Here, a model is developed accordingly with the hypothesis that an intermediate on-pathway leads to the conformation of the prion protein into an amyloid competent isoform thanks to a structure, called micelles, formed from hydrodynamic interaction. The authors also compare data to the prediction of their model and propose a new hypothesis for the formation of infectious prion amyloids.

Prion Quasi-Species and Molecular Basis of Auto-Perpetuation of Prion Structural Information

Human Rezaei
Institut National de la Recherche Agronomique, France
Davy Martin, Joan Torrent i Mas, Stephanie Prigent, Vincent Beringue

The prion phenomenon is based on autonomous structural information propagation towards single or multiple protein conformation changes. Since this last decade, the prion concept referring to the transmission of structural information has been extended to several regulation systems and pathologies including Alzheimer and Parkinson's diseases. The unified theory in Prion replication implies structural information transference (SIT) from the prion to a non-prion conformer through a mechanism also called improperly, with regards to biophysical considerations "seeding" phenomenon. Therefore considering prion replication as a structural information transduction from a donor (*i.e.* template) to an acceptor (*i.e.* substrate) through a transduction interface a new questioning arises: what are molecular mechanisms of the auto-perpetuation of the Prion structural information and its faithfulness?

Considering the Prion propagation as more or less faithful perpetuation of structura information, in the present work, we explored the concept of prion quasi-species (*i.e.* existence of prion network heterogeneous assemblies) and highlighted the existence of prion network, which has an autopoietic behaviour (auto-replicative). Our observations strongly suggest that specific criteria in term of: protein structure, delay-process and thermo-kinetics should be collated before a system become dissipative and autopoietic.

Modeling in Vivo Prion Aggregate Dynamics in Yeast

Suzanne Sindi
University of California, Merced, USA
Jason K. Davis

Prion proteins are responsible for a variety of neurodegenerative diseases in mammals such as Creutzfeldt-Jakob disease in humans and "mad-cow" disease in cattle. While these diseases are fatal to mammals, a host of harmless phenotypes have been associated with prion proteins in *S. cerevisiae*, making yeast an ideal model organism for prion diseases. Most mathematical approaches to modeling prion dynamics have focused on either the protein dynamics in isolation, absent from a changing cellular environment, or modeling prion dynamics in a population of cells by considering the "average" behavior. However, such models have been unable to recapitulate *in vivo* properties of yeast prion strains including experimentally observed rates of prion loss. We have developed physiologically relevant models by considering both the prions and their yeast host. We first generalize a previously developed nucleated polymerization model for aggregate dynamics. We next discuss a stochastic model of prion protein dynamics in the context of a growing yeast culture. Our model is based on a stochastic chemical reaction network within a cell and a Crump–Mode–Jagers branching process model of population growth. In order to simultaneously conform to observations of two distinct prion strains we uncovered several novel aspects of prion biology.

Analysis and 3D Numerical Simulation of a Polymerization Model with Aggregation

Leon Matar Tine
Universite Lyon 1, France
Leye Babacar

In this talk we present an analytical and numerical modeling of a general polymerization process with possible lengthening by coagulation mechanism. The proposed model take into account the 3D spatial diffusion for the mass transfer between monomers and polymers. We will discuss about the well-posedness of this general polymerization model and we will propose for the simulations, a 3D numerical scheme based on a generalization of the anti-dissipative strategy method for the flux approximation.

Optimal Growth for Linear Growth-Fragmentation Processes

Leon Matar Tine
Universite Lyon 1, France
Vincent Calvez, Pierre Gabriel, Stèphane Gaubert

I will present recent advances in the optimization of linear, finite dimensional models of growth- fragmentation processes. Consider a linear differential inclusion which preserves positivity. When the matrices are uniformly irreducible and bounded, there exists a unique Lyapunov exponent which characterizes the infinite horizon optimal growth of the linear system. Moreover this exponent is related to the critical viscosity solution of a Hamilton-Jacobi-Bellman equation. Existence of such a critical viscosity solution is known as Fathi's weak-KAM theorem in Lagrangian dynamics. The corresponding Aubry set informs us about the optimal trajectories of the linear differential inclusion. The talk will be illustrated with numerical simulations of three dimensional systems. This is a joint work with Pierre Gabriel (University of Versailles, France) and Stèphane Gaubert (Inria, Saclay, France).

Recovering the History of a Fibril Population Undergoing Fragmentation from Its Asymptotic Profile

Magali Tournus
Aix-Marseille Universite, France
Marie Doumic, Miguel Escobedo, Wei-Feng Xue

Using measurements of the size distribution in a population in order to infer the characteristics of their growth is a field of growing interest in population dynamics. Such techniques allow one to assess on a solid ground an empirical model without a priori information on the microscopic laws for each individuals growth and division. Linked to recent developments in experimental biology, which gives access to the size distribution of amyloid fibrils, we focus here on a pure fragmentation process. The quantity of interest is the density $f(t,x) \geq 0$ of particles of size $x \in \mathbb{R}^+$ at time $t \geq 0$, expressed as the solution of a continuous fragmentation equation. Based on the knowledge of the system at equilibrium, we present a methodology to estimate the parameters. The mathematical tools used are the Mellin transform and functional equations.

Nucleation Time in Stochastic Becker-Döring Model

Romain Yvinec
INRA Tours, France
Samuel Bernard, Tom Chou, Julien Deschamps, Maria R. D'Orsogna Erwan Hingant, Laurent Pujo-Menjouet

This work is motivated by protein aggregation phenomena in neurodegenerative diseases. A key observation of *in-vitro* polymerization experiments of prion protein is the large variability of the so-called "nucleation time", which is experimentally defined as the lag time before the polymerization of proteins trully stars. In this context, we study a stochastic version of a well-known nucleation model in physics, namely the Becker-Döring model. In this model, aggregates may increase or decrease their size one-by-one, by capturing or shedding a single particle. We will present numerical and analytical investigation of the nucleation time as a first passage time problem [1,2].

Finally, we will present limit theorem techniques to study the link from the discrete size Becker-Döring model to a continuous size version (the Lifshitz-Slyozov model). For general coefficients and initial data, we introduce a scaling parameter and show that the empirical measure associated to the Becker-Döring system converges in some sense to the Lifshitz-Slyozov equation when the parameter goes to 0 [3].

Contrary to previous studies, we use a weak topology that includes the boundary of the state space, allowing us to rigorously derive a boundary value for the Lifshitz-Slyozov model. It is the main novelty of this work and it answers to a question that has been conjectured or suggested by both mathematicians and physicists. We emphasize that the boundary value depends on a particular scaling (as opposed to a modeling choice) and is the result of a separation of time scale and an averaging of fast (fluctuating) variables.

References

[1] R. Y., M. R. D'Orsogna, and T. Chou. First passage times in homogeneous nucleation and self-assembly. Journal of Chemical Physics, 137:244107, (2012).

[2] http://arxiv.org/abs/1510.04730

[3] http://arxiv.org/abs/1412.5025

Special Session 41: Stochastic Partial Differential Equations

Benjamin Gess, University of Bielefeld, Germany
Michael Röckner, University of Bielefeld, Germany

Stochastic Partial Differential Equations (SPDE) and their applications is a relatively young field of mathematics. In the past two decades and it has, however, become one of the main research directions of Probability Theory, with rising activity across its entire spectrum. In particular, modern SPDE techniques and their combination with ideas from rough path theory led to Martin Hairer's theory of regularity structures for which he was awarded the Fields medal in 2014. The field of SPDE combines the classical area of partial differential equations with modern branches of probability theory, in particular, stochastic analysis, and thus constitutes one of the most prominent contact points between analysis and stochastics. Besides various other connections to pure mathematics (e.g. Differential Geometry, Dynamical Systems) one main focus of SPDE are its applications to the Sciences, in particular Physics, but also Biology and Chemistry. Another main area of applications is Economics, in particular Mathematical Finance. The aim of the session is to give an update on recent developments on SPDE and at the same time identify new frontiers with challenging open problems for the field, with emphasis on both theory and applications.

Burgers Equation with Random Forcing.

Yuri Bakhtin
Courant Institute, NYU, USA

Burgers equation with random forcing
I will talk about the ergodic theory of randomly forced Burgers equation (a basic nonlinear evolution PDE related to fluid dynamics and growth models) in the noncompact setting. The basic objects are one-sided infinite minimizers of random action (in the inviscid case) and polymer measures on one-sided infinite trajectories (in the positive viscosity case). Joint work with Eric Cator, Kostya Khanin, Liying Li.

Regularization by Noise for Some Stochastic Differential Equations.

Rèmi Catellier
University of Rennes 1, France

We first study the linear transport equation

$$\frac{\partial}{\partial t}u(t,x) + b(t,x)\cdot\nabla u(t,x) + \nabla u(t,x)\cdot\frac{\partial}{\partial t}X(t) = 0, \qquad u(0,x) = u_0(x)$$

where b is a vectorfield of limited regularity and X a vector-valued Hölder continuous driving term. Using the theory of controlled rough paths we give a meaning to the weak formulation of the PDE and solve that equation for smooth vectorfields b. In the case of the fractional Brownian motion a phenomenon of regularization by noise is displayed.
By using the same techniques, and thanks to the scalar conservation laws formulation for equations of the form

$$\partial_t u(t,x) + \mathrm{div}_x\left(A\big(x,u(t,x)\big)\right) + \nabla u(t,x)\cdot\frac{\partial}{\partial_t}X(t) = 0, \quad u(0,\cdot) = u_0,$$

the same kind of regularization by noise phenomenon is shown in the latter case.

Singular SPDEs on Manifolds

Joscha Diehl
TU Berlin, Germany
Antoine Dahlqvist, Bruce Driver

We show how the theories of paracontrolled distributions and regularity structures can be implemented on manifolds, to solve singular SPDEs like the parabolic Anderson model. This is ongoing work with Bruce Driver (UCSD) and Antoine Dahlqvist (Cambridge)

Malliavin Calculus and Regularity Structures

Paul Gassiat
University Paris-Dauphine, France
G. Cannizzaro, P. Friz

Many nonlinear stochastic PDEs of interest are ill-posed in the sense that one cannot give a canonical meaning to the nonlinearity. Hairer's theory of regularity structures allows to give a good notion of solution for a large class of such equations. In this talk I will explain how this can be combined with classical tools from Malliavin calculus, which allows in particular to obtain information on the densities of the marginal laws of the solutions.

A Stochastic Equation of Ginzberg-Landau Type

Wilfried Grecksch
Martin-Luther-University Halle-Wittenberg, Germany

We introduce for a complex valued process $(X(t,x))_{t\in[0,T],x\in G}$ the following Itô equation in the sense of the variational solution

$$\begin{aligned} dX(t,x) &= (a_1+ia_2)\Delta_x X(t,x)+ \\ &+(b_1+ib_2)f(X(t,x),\overline{X(t.x)})X(t,x) \\ &+g(t,x(t,x))dw(t)\ X(0,x)=X_0(x). \end{aligned}$$

The Laplacian operatorΔ_x is defined on a bounded set $G\subset R^m$ with homogeneous Neumann boundary conditions and a W denotes a cylindrical Wiener process and $a_1\geq 0$, a_2, b_1, b_2, c_1, $c_2\in R^1$.
Concrete examples are discussed and the existence and uniqueness of a variational solution are proved. The solution process of this problem is approximated by solutions of linear problems.

Hypocoercivity for SPDEs

Martin Grothaus
University of Kaiserslautern, Germany

Recently we provided hypocoercivity concepts for degenerate Kolmogorov (backward) equations. In this talk we plan to apply them to stochastic partial differential equations.

Rough Gronwall Lemma and Weak Solutions to RPDEs

Martina Hofmanova
Technical University Berlin, Germany

We put forward a general framework for the study of a wide class of rough path driven PDEs. To be more precise, we introduce an intrinsic notion of distributional solution, i.e. weak solution in the PDE sense, and develop new a priori estimates based on a rough Gronwall lemma argument leading to the proof of existence and uniqueness. Our approach does not rely on any sort of transformation formula (flow transformation, Feynman-Kac representation formula etc.) and is therefore rather flexible and applicable in various contexts, also when such a transformation is no longer available. The talk is based on a joint work with Aurelien Deya, Massimiliano Gubinelli and Samy Tindel.

Multiplicative Stochastic Heat Equations on the Whole Space

Cyril Labbe
University Paris Dauphine, France
Martin Hairer

The theory of regularity structures allows one to give a meaning to some singular stochastic PDEs such as the Kardar Parisi Zhang (KPZ) equation or the parabolic Anderson model (PAM) in dimension 2 or 3. However, the original framework of the theory deals with equations on bounded domains while it is very natural to consider these equations on the whole space. Actually, to measure the regularity of the white noise on the whole space, one needs to add weights in the spaces of Holder distributions. These weights do not behave well under multiplication: this induces some serious difficulties for solving these stochastic PDEs. In collaboration with Martin Hairer, we have adapted the theory of regularity structures to construct the solutions of some multiplicative stochastic heat equations on the whole space, such as (PAM) in dimension 3. Moreover, our result allows one to consider irregular initial conditions: in the case of (PAM), we can start the equation from a Dirac mass, which is a natural initial condition for this model.

Perturbation and Homogenization

Xue-Mei Li
The University of Warwick, England

I will discuss stochastic homogenization motivated by geometry and mathematical physics. In particular we consider the family of operators of the form $(1/\epsilon^2)L_1+(1/\epsilon)L_0$ with a parameter ϵ, where L_0 is a first order differential operator, L_1 a sum of squares Hormander type second order operator satisfying Hormander's conditions. These can be considered as random perturbation to a conserved quantity taking value in a non-linear space. Examples fall into this category include: random perturbation to geodesic equations, approximation of Brownian motion by a random Hamiltonian system, and collapsing of Riemannian manifolds.

Linking Regularity Structures and Paracontrolled Analysis

Joerg Martin
HU Berlin, Germany
Nicolas Perkowski

Quite recently the study of singular SPDEs has produced remarkable new theories such as Hairer's regularity structures and the usage of paracontrolled distributions by Gubinelli, Imkeller and Perkowski. We demonstrate how both theories can be combined and show a universal link between them, thereby enabling the usage of classical, analytical tools in Hairer's algebraic framework.

Regularization by Noise for Stochastic Scalar Conservation Laws

Mario Maurelli
WIAS Berlin & TU Berlin, Italy
Benjamin Gess

We say that a regularization by noise phenomenon occurs for a possibly ill-posed differential equation if this equation becomes well-posed (in a pathwise sense) under addition of noise. Most of the results in this direction are limited to SDEs and associated linear SPDEs.
In this talk, we show a regularization by noise result for a nonlinear SPDE, namely a stochastic scalar conservation law on $\mathbb{R}^d$ with a space-irregular flux:

$$\partial_t v + b \cdot \nabla[v^2] + \nabla v \circ \dot{W} = 0,$$

where $b = b(x)$ is a given deterministic, possibly irregular vector field, W is a d-dimensional Brownian motion ($\circ$ denotes Stratonovich integration) and $v = v(t, x,\omega)$ is the scalar solution. More precisely we prove that, under suitable Sobolev assumptions on b and integrability assumptions on its divergence, the equation admits a unique entropy solution. The result is false without noise.
The proof of uniqueness is based on a careful combination of arguments used in the linear case: first we show the renormalization property for the kinetic formulation of the equation, then we use second order PDE estimates and a duality argument to conclude. If time permits, we will discuss also some open questions.

The Enskog Process

Barbara Ruediger
University Wuppertal, Germany
S. Albeverio, P. Sundar

The existence of a weak solution to a McKean-Vlasov type stochastic differential system corresponding to the Enskog equation of the kinetic theory of gases is established under natural conditions. The distribution of any solution to the system at each fixed time is shown to be unique. The existence of a probability density for the time-marginals of the velocity is verified in the case where the initial condition is Gaussian, and is shown to be the density of an invariant measure.

Generalized Couplings and Convergence of Transition Probabilities

Michael Scheutzow
TU Berlin, Germany
Alexei Kulik

We provide sufficient conditions for the uniqueness of an invariant measure of a Markov process as well as for the weak convergence of transition probabilities to the invariant measure. Our conditions are formulated in terms of generalized couplings.

Exponential Stability of SPDE Driven by a Fractional Brownian Motion

Björn Schmalfuß
Friedrich Schiller University of Jena, Germany

We consider the S(P)DE on some Hilbert space V

$$du = Audt + F(u)dt + G(u)d\omega, \quad u(0) = u_0 \in V$$

where $u \equiv 0$ solves this equation. A, F, G satisfy particular regularity conditions. ω is a fractional Brownian motion with a special Hurst parameter. We discuss conditions ensuring the asymptotic stability of the trivial solution.

Vector Analysis for Dirichlet Forms and Related Questions

Alexander Teplyaev
University of Connecticut, USA

The talk will give an overview of several questions related to the recent progress in vector analysis for Dirichlet forms. One of the applications of such analysis is the existence and uniqueness of solutions of the non-linear heat equation that appears in the hydrodynamic limit of weakly non-symmetric simple exclusion processes on non-smooth spaces. In fact, a large class of nonlinear vector PDEs can be defined and studied on spaces that have no differential structure but only a Dirichlet form. If time permits, Hodge theorem, Dirac semigroup, magnetic Schrodinger semigroup and related stochastic analysis, and some estimates for SPDEs and infinite dimensional SDEs will be discussed. This work has been done in collaboration with Joe Chen, Masha Gordina, Michael Hinz, Dan Kelleher, Michael Roeckner.

Stability of Solutions and Ergodicity for Stochastic Local and Nonlocal p-Laplace Equations

Jonas Tölle
Aalto University, Finland
Benjamin Gess, Jonas M. Tölle

We provide a general framework for the stability of solutions to stochastic partial differential equations with respect to perturbations of the drift. More precisely, we consider stochastic partial differential equations with drift given as the subdifferential of a convex function and prove continuous dependence of the solutions with regard to random Mosco convergence of the convex potentials. To this aim, we identify the concept of stochastic variational inequalities (SVI) as a well-suited framework to study such stability properties. In particular, we provide an SVI treatment for stochastic singular nonlocal p-Laplace equations and prove their convergence to the respective local models. Furthermore, ergodicity for local and nonlocal stochastic singular p-Laplace equations is proved, without restriction on the spatial dimension and for all $p \in [1, 2)$. This generalizes previous results from [Benjamin Gess, J. M. T.; JMPA (2014)], [Wei Liu, J. M. T.; ECP (2011)], [Wei Liu; JEE (2009)]. In particular, the results include the multivalued case of the stochastic nonlocal and local total variation flows. Under appropriate rescaling, the convergence of the unique invariant measure for the nonlocal stochastic p-Laplace equation to the unique invariant measure of the local stochastic p-Laplace equation is proved.
The presentation is based on the following works:
1. Benjamin Gess, J. M. T., Stability of solutions to stochastic partial differential equations, Journal of Differential Equations, Volume 260, Issue 6 (2016), pp. 4973–5025.
2. Benjamin Gess, J. M. T., Ergodicity and local limits for stochastic local and nonlocal p-Laplace equations, preprint (2015), 28 pp., http://arxiv.org/abs/1507.04545

Stochastic Maximal Regularity for Equations with Adapted Drift

Mark Veraar
TU Delft, Netherlands

In this talk we present recent results on optimal regularity estimates and existence and uniqueness results for parabolic SPDEs with multiplicative noise. The results are a far reaching extension of previous results of Krylov and collaborators and results of van Neerven and Weis and the speaker.

Special Session 42: Dynamics of Evolution Equations in Viscoelasticity and Thermoelasticity

To Fu Ma, University of Sao Paulo, Brazil
Marcelo Moreira Cavalcanti, State University of Maringa, Brazil
Yuming Qin, Donghua University, Peoples Rep of China

This section is devoted to the dynamics of classical and new systems in viscoelasticity and thermoelasticity. They include models with, for instance, fading memory, heat waves, localized damping, delay, nonautonomous forcing and nonlocal terms. On these models we discuss qualitative properties as, for instance, exponential stability, analyticity of linear system, global attractor and singular limits. Such problems are important to the real world applications and represent a major research subject in PDEs.

Exponential Decay Estimates for the Damped Defocusing Schrödinger Equation in Exterior Domains

Wellington Correa
Federal Technological University of Parana, Brazil
Nicolas Burq, Marcelo Moreira Cavalcanti, Valèria Neves Domingos Cavalcanti

In this work, we study the well-posedness as well as the exponential stability in H^1-level for the damped defocusing Schrödinger equation posed in a two dimensional exterior domain Ω with smooth boundary $\partial\Omega$. The proofs of the well-posedness are based on properties of pseudo-differential operators introduced in Dehman, Gèrard and Lebeau and Brèzis – Gallouet's inequality, while the exponential stability is achieved combining arguments firstly considered by Zuazua for the wave equation adapted to the present context and a global uniqueness theorem.

New Decay Rates for the Magneto-Thermo-Elastic System

Cleverson da Luz
Federal University of Santa Catarina, Brazil
Jauber Cavalcante de Oliveira

In this work we study the asymptotic behavior of solutions for a Cauchy problem associated to a magneto-thermo-elastic system. We improve results on decay rates considering weaker regularity on the initial data when compared to previous works in the literature. We also improve the method developed by R. C. Charao, C. R. da Luz and R. Ikehata (2013) in works for the wave equation and plates, extending it for this coupled system of mixed hyperbolic-parabolic partial differential equations.

Stability for a Transmission Problem in Thermoelasticity with Second Sound.

Hugo Fernandez Sare
Univ. Federal do Rio de Janeiro, Brazil
Reinhard Racke, Jaime E. Munoz Rivera

We consider a semilinear transmission problem for a coupling of an elastic and a thermoelastic material. The heat conduction is modeled by the Cattaneo law removing the physical paradox of infinite propagation speed of signals. The damped, totally hyperbolic system is shown to be exponentially stable, and the existence of a global attractor is shown.

Attractors for a Class of Extensible Beam Models with Nonlocal Nonlinear Damping

Marcio Jorge Silva
State University of Londrina, Brazil
Vando Narciso

In this talk we consider some results on well-posedness and long-time dynamics for a class of extensible beam models with nonlocal nonlinear damping posed on bounded domains. This kind of nonlinear damping given by the product of two nonlinear terms constitutes a mathematical generalization of nonlocal frictional dissipations first proposed for plate models. Our main results encompass the existence of attractors to the dynamical system associated with the model. Moreover, we also analyze the qualitative properties of such attractors.

Vanishing Viscosity Limit of the Compressible Isentropic Navier-Stokes Equations with Degenerate Viscosities

Yachun Li
Shanghai Jiao Tong University, Peoples Rep of China

In this talk I will first establish the local-in-time well-posedness of the unique regular solution to the compressible isentropic Navier-Stokes equations with density-dependent viscosities in a power law and with vacuum appearing in some open set or at the far field, then after establishing uniform energy-type estimates

with respect to the viscosity coefficients for the regular solutions we prove the convergence of the solution of the Navier-Stokes equations to that of the Euler equations with arbitrarily large data containing vacuum. This is a joint work with Yongcai Geng and Shengguo Zhu.

Dynamics of Wave Equations with Degenerate Memory

To Fu Ma
University of Sao Paulo, Brazil
M. M. Cavalcanti, L. H. Fatori

This talk is concerned with the long-time dynamics of viscoelastic wave equations with some mild degeneracy in the memory kernel. The existence of a global attractor is obtained by adding locally complementary frictional damping.

Uniform Analyticity and Exponential Decay of the Semigroup Associated with a Thermoelastic Plate Equation with Perturbed boundary conditions

Louis Tebou
Florida International University, USA

In a bounded domain, we consider an Euler-Bernoulli type thermoelastic plate equation with perturbed boundary conditions. The boundary conditions are such that when the perturbation parameter goes to infinity, we recover the hinged boundary conditions, while one recovers the clamped boundary conditions when the perturbation parameter goes to zero. Relying on resolvent estimates, we show that the underlying semigroup is uniformly, with respect to the perturbation parameter, analytic and exponentially stable. The main features of our proof are: appropriate decompositions of the components of the system and the use of Lions‘ interpolation inequalities.

Large Time Behavior for the Three-Dimensional Generalized Benjamin-Bona-Mahony Equation with Large Initial Data

Yutong Wang
Shanghai Jiao Tong University, Peoples Rep of China
Weike,Wang

In this talk, we consider the global existence as well as the optimal decay estimates of the Cauchy problem for the 3-dimensional Benjamin-Bona-Mahony equation with large initial data. The results are obtained by Green's function method, Fourier analysis method and energy method combined with the time-frequency decomposition method. We mainly focus on some difficulties in our problem. First of all, we lose a good term- a dissipation term$\Delta^2 u$ in our equation, which tend to make the decay better, so we have to divide it into two parts to consider them separately. What's more, there is no maximum principle like the viscous Burgers equation, which increases the difficulty. And even worse, we even can not simply use the energy method to get the L^∞ bounded estimate. To overcome the difficulties, we first prove the convolution property of the Green's function G, then by combining it with variable substitution and Green's function method, we obtain the L^1 and L^∞ bounded estimate. Now since we get the L^p bounded estimate, the optimal decay can be obtained by the time-frequency decomposition method.

Zero Dielectric Constant Limit for Systems of an Electromagnetic Fluid

Xin Xu
Shanghai Jiao Tong University, Peoples Rep of China

The magnetohydrodynamic equations were obtained as the singular limit of the complete equations for an electrically conducting compressible fluid at the vanishing of the dielectric constant. We want to give a rigorous justification to this singular limit.

Pullback Attractors for 2D Navier-Stokes Equations with Inhomogeneous Boundary Conditions Or Delay on Lipschitz Domain

Xinguang Yang
Henan Normal University, Peoples Rep of China

The Navier-Stokes equations give the essential law of the fluid flow. Based on the global wellposedness, we derive the pullback attractors for incompressible 2D Navier-Stokes equations with nonhomogeneous boundary conditions or multi-delays on Lipschitz domain.

Special Session 43: Long Time Dynamics, Numerical Analysis and Control of Evolutionary Systems

Louis Tebou, Florida International University, USA
Ciprian Gal, Florida International University, USA
Theodore Tachim, Florida International University, USA

Recent developments in the mathematical analysis of distributed parameter systems will be explored. A special attention will be paid to physically relevant models. Problems to be addressed include, but are not limited to, well-posedness, global behavior, numerical analysis, and control of such models. Possible areas of application encompass fluid dynamics, heat transfer, acoustics,...

On the Upper Semicontinuity of the Global Attractor for Nonlinear Parabolic Equations with Large Diffusion

Maria Astudillo
State University of Maringa, Brazil
Marcelo M. Cavalcanti

In this talk we discuss the asymptotic behavior (in terms of attractors) of a class of nonlinear parabolic problems involving porous medium type equations as the diffusion coefficient becomes large. We prove the convergence of the solutions of a homogeneous Neumann problem associated to a class of porous medium type equations with relative general conditions on the reaction term, as the diffusion coefficient goes to infinity. Moreover, we prove the upper semicontinuity of the associated global attractor.

Analysis and Sensitivity in the Lamina Cribrosa

Lorena Bociu
NC State University, USA
Giovanna Guidoboni, Riccardo Sacco, Justin Webster

We consider a nonlinear system of PDEs that models fluid flow through poro-visco-elastic material. The ability of the fluid to flow within the solid is described by the permeability tensor, which is assumed to vary nonlinearly with the volumetric solid strain. We study the problem of existence of weak solutions in bounded domains with mixed boundary conditions, accounting for non-zero volumetric and boundary forcing terms. Moreover, we investigate the influence of viscoelasticity on the smoothness of the solution and on the regularity requirements for the forcing terms. The theoretical results are complemented with numerical simulations, and interpreted in the context of the lamina cribrosa in the eye and the connection between its biomechanics and the development of glaucoma.

Nonlinear Dynamics and Stability in an Asset Flow Model

Gunduz Caginalp
Univ of Pittsburgh, USA

Modeling traders who focus on price trend as well as fundamentals yields a curve of equilibria, rather than a single point that one has in classical economics. With different groups assessing value and using different strategies, there is a curve of equilibria that contains both stable and unstable points. Instability is fostered by momentum traders focused on a small time scale. Stability and bifurcation properties are characterized by the strategies of the traders.
Most of this work is in collaboration with M. DeSantis and D. Swigon.

Unilateral Problem for the Wave Equation with Spatial-Time Degenerate Nonlinear Damping: Well-Posedness and exponential stability

Marcelo Cavalcanti
State University of Maringa, Brazil
Valeria Domingos Cavalcanti, Marcio A. Jorge Silva, Luci H. Fatori

A unilateral problem related to a wave model with a spatial-time degenerate damping on a compact Riemannian manifold is considered. Our results are new and concern two main issues: (a) to establish the well-posedness of the variational problem; (b) to show that the corresponding energy decays exponentially to zero under sharp conditions of zone for the effect of dissipativity. These conditions are used by means of an observability condition related to the sharp zone where the localized damping is acting.

Well-Posedness and Uniform Stability for Nonlinear Schrodinger Equations with Dynamic/Wentzell Boundary Conditions

Wellington Correa
Federal Technological University of Parana, Brazil
M. M. Cavalcanti, I. Lasiecka, C. Lefler

Schrodinger equation with a defocusing nonlinear term and dynamic boundary conditions defined on a smooth boundary of a bounded domain in dimensions $N = 2, N = 3$ is considered. Local well-posedness of strong H^2 solutions is established. In the case $N = 2$ local solutions are shown to be global. In addition, existence of weak H^1 solutions in dimensions $N = 2, N = 3$ is also shown. The energy corresponding to weak solution is shown to satisfy uniform decay rates under appropriate monotonicity conditions imposed on the nonlinear terms appearing in the dynamic boundary conditions. The proof of well-posedness is critically based on converting the equation into Wentzell boundary value problem associated with Schrodinger dynamics. The analysis of this later problem with nonhomogeneous boundary data allows to build a theory suitable for the treatment of the dynamic boundary conditions.

New Phenomena for the Null Controllability of Parabolic Systems: Minimal Time and Geometrical Dependence.

Luz de Teresa
UNAM, Mexico

We consider the distributed null controllability problem for two coupled parabolic equations with a space-depending coupling term. We exhibit a minimal time of $T_0 \in [0, \infty)$ such that the corresponding system is null controllable at any time $T > T_0$ and is not null controllable if T¡T_0 . This minimal time depends on the relative position of the control interval and the support of the coupling term. We also prove that, for a fixed control interval and a time $\tau_0 \in [0, \infty)$, there exist coupling terms such that the associated minimal time is τ_0

Exponential Stability for the Wave Equation with Degenerate Nonlocal Weak Damping

Valeria Domingos Cavalcanti
State University of Maringa, Brazil
M. M. Cavalcanti, M. A. Jorge Silva, C. M. Webler

A damped nonlinear wave equation with a degenerate and nonlocal damping term is considered. Well posedness results are discussed, as well as the exponential stability of the solutions. The degeneracy of the damping term is the novelty of this stability approach.

Exact Boundary Control for 1-D Wave and Schrodinger Equations

Julian Edward
Florida International University, USA
Sergei Avdonin

We consider the problem of boundary control for a one dimensional wave equation with N interior point masses. We assume the control is at the left end, and the string is fixed at the right end. Singularities in waves are smoothed out to one order as they cross a point mass. We show that the reachable set for a L^2 control equals $(L^2 \times H^{-1}) \times (H^1 \times L^2) \times \ldots \times (H^N \times H^{N-1})$ plus some compatibility conditions. We prove exact controllability results. The proof reduces the control problem to a moment problem, which is then solved using the theory of exponential divided differences in tandem with a unique shape controllability result. The methods are then extended to Schrodinger-wave type equations with strong potential singularities. This is work done in collaboration with Sergei Avdonin.

Semi-Lagrangian Forward Methods for Time-Dependent Partial Differential Equations

Daniel Guo
University of North Carolina Wilmington, USA

One-step semi-Lagrangian forward method is investigated for computing the numerical solutions of time-dependent partial differential equations with initial and boundary conditions. This method is based on Lagrangian trajectory or the integration from the departure points (regular nodes) to the arrival points. The arrival points are traced forward from the departure points along the trajectory of the path. Most likely the arrival points are not on the regular grid nodes. The convergence and stability are studied for the explicit methods. The numerical examples show that those methods work very efficient for the time-dependent partial differential equations.

Null Boundary Controllability of a Heat Equation with an Internal Point Mass

Scott Hansen
Iowa State University, USA
Jose de Jesus Martinez

We consider a one dimensional heat equation with an internal point mass. The dynamics at the point mass are obtained as a limit of a sequence of heat equations with densities that tend to a delta function. We show that the system is null controllable when a control acts at one end. We also describe some related results for the analogous Schrödinger system.

Suspension Bridge: a Semi Linear Model

Salim Messaoudi
King Fahd University of Petroleum and Minerals, Saudi Arabia
Soh Edwin

In a statistics appeared in the University of Cambridge 2004, an estimate of 400 suspension bridges have collapsed after construction. Some of these failures are due to human error while others from natural disaster such as overloading of building materials, tsunami and earthquake. Thus, there is a necessity for reliable mathematical models to give a precise description of its instability and structural behavior. A first attempt to model a suspension bridge through a plate is due to Ferrero-Gazzola, who introduced recently the hyperbolic equation with some unique and novel boundary conditions and proved some existence and regularity results. In this talk, we discuss two variants of this model and establish some existence and stability results.

Cahn-Hilliard Inpainting

Alain Miranville
University of Poitiers, France

Our aim in this talk is to discuss variants of the Cahn-Hilliard equation in view of applications to image inpainting. We will present theoretical results as well as numerical simulations.

The Inverse Spectral Problem for the Wave Equation on Finite Trees

Gulden Murzabekova
Kazakh Agrotechnical University, Kazakhstan
Sergei Avdonin

We consider the inverse spectral problem for differential equations on graphs. Leaf-peeling method allows recalculate the inverse data from the original tree to smaller tree, and so on to the roots edge. We describe the main step of the spectral version of peeling algorithm. This is work done in collaboration with Sergei Avdonin.

The Problem of Recovering Function with Leaf-Peeling Method

Karlygash Nurtazina
Eurasian National University, Kazakhstan
Sergei Avdonin, Jonathan Bell

In this talk we discuss how unknown coefficients and source terms for a parabolic equation can be recovered from the dynamical Neumann-to-Dirichlet map associated with the boundary vertex. We show that with a companion wave equation problem the topology of the graph and lengths of the edges can be recovered from the same dynamical Neumann-to-Dirichlet map. This is work done in collaboration with Sergei Avdonin and Jonathan Bell.

Long Time Dynamics of Autonomous and Non-Autonomous Reaction- Diffusion Equations with Robin Boundary Condition

Eylem Ozturk
Hacettepe University, Turkey

We investigated the long-time behavior and solvability of the reaction-diffusion equation, which has a polynomial growth nonlinearity of arbitrary order, with Robin boundary condition on the bounded domain.
The problem that we investigate as the following:

$$\begin{cases} u_t - \Delta u + a(x)|u|^\rho u - b(x)|u|^\nu u = h(x,t),\ (x,t) \in Q_T & (1) \\ (\frac{\partial u}{\partial \eta} + k(x')u)|_{\partial\Omega} = 0,\ (x',t) \in \Sigma_T & (2) \\ u(x,0) = u_0(x),\ x \in \Omega & (3) \end{cases}$$

here$\Omega \subset \mathbb{R}^n (n \geq 3)$, is a bounded domain such that $\partial\Omega$ sufficiently smooth boundary, $T > 0$, $Q_T = \Omega \times (0,T)$ and $\Sigma_T = \partial\Omega \times [0,T]$. In super linear case, for the existence and uniqueness of the generalized solution of problem (1)-(3) in corresponding spaces, we obtain sufficient conditions for functions a, b and k and relations between ρ and ν.For the long-time behavior, firstly we prove that solutions has an absorbing set in $L_2(\Omega)$. Also in autonomous case, we prove some asymptotic regularity of solutions and the existence of global attractor in $W_2^1(\Omega) \cap L_{\rho+2}(\Omega)$ by using Moser's iteration technique and corresponding stationary problem. In non-autonomous case, the existence of uniform attractor is obtained in $W_2^1(\Omega) \cap L_{\rho+2}(\Omega)$.

Longtime Behavior of Solutions for Nonlocal Wave Equations with Damping

Petronela Radu
University of Nebraska-Lincoln, USA
Grozdena Todorova, Borislav Yordanov

Nonlocal wave equations with damping have only recently started to be explored in the context of peridynamics and other theories that allow solutions to be discontinuous. In this talk I will focus on results that connect the asymptotic behavior of solutions to dissipative wave equations to solutions of the corresponding diffusion equations, more precisely, show that the abstract diffusion phenomenon takes place. The results hold true in fact for systems that involve two non-commuting self-adjoint operators in a Hilbert space. When the diffusion semigroup has the Markov property and satisfies a Nash-type inequality, we obtain precise estimates for the consecutive diffusion approximations and remainder. Also, I will present some applications including sharp decay estimates for dissipative hyperbolic equations with variable coefficients on an exterior domain. To our knowledge we have obtained the first decay estimates for nonlocal wave equations with damping terms; the decay rates are sharp. Some of these results have been obtained in collaboration with Grozdena Tododrova and Boris Yordanov.

Some Robust Control Problems Associated with the Cahn-Navier-Stokes System

Theodore Tachim Medjo
Florida International University, USA

In this work we study a class of robust control problems associated with a coupled Cahn-Hilliard-Navier-Stokes model in a two dimensional bounded domain. The model consists of the Navier-Stokes equations for the velocity, coupled with the Cahn-Hilliard model for the order (phase) parameter. We prove the existence and uniqueness of solutions and we derive a first-order necessary optimality condition for these robust control problems. We also present an adjoint-based iterative method for the numerical approximation of these control problems.

Long Time Stability of the Implicit Euler Scheme for an Incompressible Two-Phase Flow Model

Florentina Tone
University of West Florida, USA
Theodore Tachim Medjo

In this talk we present results on the stability for all positive time of the fully implicit Euler scheme for an incompressible two-phase flow model. More precisely, we consider the time discretisation scheme and with the aid of the discrete Gronwall lemma and of the discrete uniform Gronwall lemma we prove that the numerical scheme is stable.

The Discrete Inf-Sup Inequality for a Finite Element Hydro-Elastic Model

Daniel Toundykov
University of Nebraska-Lincoln, USA
George Avalos

A seminal result concerning mixed finite element approximations of the Stokes equation was the discrete inf-sup inequality, uniform with respect to the (small) discretization mesh parameter h. This inequality ultimately leads to error estimates for the convergence of the FEM scheme. An essential requisite ingredient for this result was the no-slip condition imposed on the entire boundary of the fluid domain. On the other hand, in fluid-structure interaction problems, the interface between the fluid and the solid is subject to velocity and stress matching constraints which do not enforce the no-slip condition. Accordingly, we establish the discrete inf-sup estimate for the case when the no-slip condition holds only on a portion of the boundary, thus paving a way to FEM convergence estimates for fluid-structure interaction problems.

Special Session 44: Fractal Geometry, Dynamical Systems, and Their Applications

Michael Barnsley, Australian National University, Australia
James Keesling, University of Florida, USA
Mrinal Kanti Roychowdhury, University of Texas Rio Grande Valley, USA

The aim of this session is to bring together scientists including the young researchers to discuss and exchange ideas in the areas of fractal geometry, dynamical systems, and their recent advances and emerging applications.

Interleaving of Path Sets

William Abram
Hillsdale College, USA
Artem Bolshakov, Jeffrey Lagarias, Daniel Slonim

We study an interleaving operation on path sets, which are spaces of one-sided infinite symbol sequences corresponding to the one-sided infinite walks beginning at a fixed initial vertex in a labeled graph $\mathcal{G}$. Path sets are useful for the study of intersections of fractals, and have been used to study intersections of multiplicative translates of 3-adic Cantor sets. They have also been used by Ban and Chang to study the mosaic solutions to the initial value problem of multi-layer cellular neural networks. Interleaving is a kind of multiplication for path sets that has proved useful in computations. We study its dynamical and algebraic properties and provide several examples.

Some Complexity Results in the Theory of Normal Numbers

Dylan Airey
University of Texas at Austin, USA
Stephen Jackson, Bill Mance

Let $\mathcal{N}(b)$ be the set of real numbers that are normal in base b. G. Rauzy characterized the set $\mathcal{N}^{\perp}(b) = \{r : \text{ for all } x \in \mathcal{N}(b), r + x \in \mathcal{N}(b)\}$ in terms of an entropy condition. Using this characterization we determine the descriptive set theoretic complexity of $\mathcal{N}^{\perp}(b)$ and related sets.

Topological Speedups of Odometers and Substitutions

Lori Alvin
Bradley University, USA
Drew Ash, Nic Ormes

Given a minimal Cantor system (X, T), one can define a topological speedup of (X, T) as a system (X, S) where $S = T^{p(\cdot)}$ is a minimal homeomorphism for some $p : X \to \mathbb{Z}^+$. We investigate when the minimal Cantor system (X, T) is topologically conjugate to a non-trivial speedup of itself. We show that when (X, T) is an odometer and (X, S) is a bounded speedup of (X, T), then (X, S) is topologically conjugate to (X, T). On the other hand, when (X, T) is a substitutive system and (X, S) is a bounded speedup of (X, T), then (X, S) is a substitutive system that is not topologically conjugate to (X, T).

Approximation of Fractal Functions and Fractal Flows

Michael Barnsley
Australian National University, Australia
C. Bandt, A. Vince

I will describe a constellation of results and ideas concerning the approximation of rough objects and flows on fractals.

Nonequilibrium Stationary State in a Weakly Driven System

Federico Bonetto
Georgia Tech, USA
Joel Lebowitz

We study a system of one or more particles moving in a chaotic billiard under the influence of an external electric field. A friction term, in the form of a Gaussian thermostat, is added to keep the total kinetic energy exactly constant. We study the invariant measure of this system both analytically and numerically. We also introduce a simplified stochastic model to better understand the long time evolution of the system.

On Dynamics of the Sierpinski Carpet

Jan Boronski
IT4 Innovations, Ostrava, Czech Rep
P. Oprocha

In 1993, Aarts and Oversteegen proved that the Sierpinski carpet S admits a transitive homeomorphism, answering a question of Gottschalk. They also showed that it does not admit a minimal one. Earlier, in 1991 Kato proved that S does not admit expansive homeomorphisms. In 2007 Bis, Nakayama and Walczak proved that S admits a homeomorphism with positive entropy, and that it admits a minimal group action. We show that S admits homeomorphisms with strong mixing properties. Namely, there is a homeomorphism $H : S \to S$ that has a fully supported measure m, such that (H, m) is Bernoulli, H

has a dense set of periodic points, and H does not have Bowen's specification property. In particular, S admits a topologically mixing homeomorphism. The starting point of our construction is Arnold's cat map.

Ramsey-Type Results for Dynamical Systems

Will Brian
Baylor University, USA

In certain dynamical systems, arbitrary sequences of points can exhibit surprising amounts of structure. In other words, it seems that complete disorder is impossible inside some dynamical systems, even highly chaotic ones. We will discuss several results of this type.

Non-Linear Analysis of Logistic Maps

Renu Chugh
Maharishi Dayanand University, Rohtak, India

The logistic map describes all possible behaviors of a nonlinear system. The idea of logistic map $rx(1-x)$ was given by the Belgian mathematician Pierre Francois Verhulst around 1845 and worked as basic model to study the discrete dynamical system. It is a model of population growth that exhibits different types of behavior depending on the value of a few parameters. Above a certain parameter value, the logistic map shows the chaotic behavior. For choosing x between 0 and 1 and $0 < r \leq 4$, the logistic map has found a celebrated place in chaos, fractal and discrete dynamics. The aim of this talk is to study the periodicity and chaotic behavior of logistic map using Mann iterative process (a two-step feedback method). We see that the nature of chaotic behavior of logistic maps increases drastically as compared to Picard iterative process and the chaotic behavior disappears for some ranges of parameter values.

Orbit Portraits for Non-Autonomous Iteration

Mark Comerford
University of Rhode Island, USA

The combinatorics associated with angles of external rays has been one of the most important tools used to understand parameter spaces associated with families of rational functions, most particularly quadratic polynomials. We show how the notion of an orbit portrait as first introduced by Milnor can be generalized to the setting of non-autonomous polynomial iteration where one studies iterates which are compositions of a sequence of polynomials with suitably bounded degrees and coefficients. In the case of sequences of constant degree, all portraits are eventually periodic which is similar to, though not exactly the same as, the classical case. On the other hand, if the degrees of the polynomials in the sequence are allowed to vary, we show that one can obtain portraits with complementary intervals whose angles are irrational multiples of 2π which are fundamentally different from the classical ones.

How Sticky Is the Chaos/order Boundary?

Carl Dettmann
University of Bristol, England

In dynamical systems with divided phase space, the vicinity of the boundary between regular and chaotic regions is often "sticky," that is, trapping orbits from the chaotic region for long times. Here, we investigate the stickiness in the simplest mushroom billiard, which has a smooth such boundary, but surprisingly subtle behaviour. As a measure of stickiness, we investigate $P(t)$, the probability of remaining in the mushroom cap for at least time t given uniform initial conditions in the chaotic part of the cap. The stickiness is sensitively dependent on the radius of the stem r via the Diophantine properties of $\rho = (2/\pi)\arccos r$. Almost all ρ give rise to families of marginally unstable periodic orbits (MUPOs) where $P(t) \sim C/t$, dominating the stickiness of the boundary. Here we consider the case where ρ is MUPO-free and has continued fraction expansion with bounded partial quotients. We show that $t^2P(t)$ is bounded, varying infinitely often between values whose ratio is at least 32/27. When ρ has an eventually periodic continued fraction expansion, that is, a quadratic irrational, $t^2P(t)$ converges to a log-periodic function. In general, we expect less regular behaviour, with upper and lower exponents lying between 1 and 2. The results may shed light on the parameter dependence of boundary stickiness in annular billiards and generic area preserving maps.

Algebro-Geometric Solutions of the Schlesinger Systems

Vladimir Dragovic
The University of Texas at Dallas, USA
Vasilisa Shramchenko

A new method to construct algebro-geometric solutions of rank two Schlesinger systems is presented. For an elliptic curve represented as a ramified double covering of CP^1, a meromorphic differential is constructed with the following property: the common projection of its two zeros on the base of the covering, regarded as a function of the only moving branch point of the covering, is a solution of a Painlevè VI equation. This differential provides an invariant formulation of a classical Okamoto transformation for the Painlevè VI equations. A generalization of this differential to hyperelliptic curves is also constructed. The corresponding solutions of the rank two Schlesinger systems associated with elliptic and hyperelliptic curves are constructed in terms of this differential. The initial data for construction of the meromorphic differential include a point in the

Jacobian of the curve, under the assumption that this point has nonvariable coordinates with respect to the lattice of the Jacobian while the branch points vary. The research has been partially supported by the NSF grant 1444147.

Cantor Sets Within Cantor Sets: Hausdorff Dimensions of P-Adic Julia Sets

Joanna Furno
IUPUI, USA

Haar measure and Hausdorff dimension are two possible methods of measuring size in the set of p-adic numbers and its finite extensions. We use these two tools to measure the size of the Julia set for some p-adic polynomials.

Climate Change and The Fractal Geometry of Arctic Melt Ponds

Kenneth Golden
Univ Utah, USA

During the Arctic melt season, the sea ice surface undergoes a remarkable transformation from vast expanses of snow covered ice to complex mosaics of ice and melt ponds. Sea ice reflectance or albedo, a key parameter in climate modeling, is largely determined by the complex evolution of melt pond configurations. In fact, ice-albedo feedback has played a major role in the recent declines of the summer Arctic sea ice pack. However, understanding melt pond evolution remains a significant challenge to improving climate projections. I will discuss recent findings on the evolution of melt pond geometry. In particular, as the ponds grow and coalesce, their fractal dimension undergoes a transition from 1 to about 2, around a critical length scale of 100 square meters in area. As the ponds evolve they take complex, self-similar shapes with boundaries resembling space-filling curves. Moreover, I will outline how mathematical models of composite materials and statistical physics, such as percolation and Ising models, are being used to describe this evolution and make predictions of key geometrical parameters that agree very closely with observations.

Fractal Curves Arising from Cutting and Resewing Pillow Cases

Patrick Hooper
City College of New York, USA

I will discuss the dynamics of a fairly simple piecewise isometry of a square pillowcase. We cut the pillowcase along two horizontal edges we obtain a cylinder, which we can rotate and then sew back together. We can then do the same in the vertical direction. The composition of these two cutting and resewing operations yields a piecewise isometry of the pillowcase with interesting dynamics. We will describe how in some cases the collection of aperiodic points forms a fractal curve, and the dynamics on this curve is topologically conjugate to a rotation (modulo concerns related to discontinuities). Properties of this map such as the existence of this curve depend on the even continued fraction expansions of the parameters.

Mappings with a Single Critical Point and Applications to Rational Difference Equations

Sue Huang
Pace University, USA
Peter Knopf

Convergence properties of maps $xn + 1 = f(xn)$ are established for a general class of mappings f; where f has at most one critical point. Using these results, necessary and sufficient conditions are obtained for the convergence of the solutions for a very general class of rational quadratic difference equations.

Wavelets on Fractals

Palle Jorgensen
University of Iowa, USA
Dorin Dutkay

Palle Jorgensen. The class of fractals referred to are those which may be specified by a finite system of affine transformations, assuming contractive scaling; and their corresponding selfsimilar measures, mu. They include standard Cantor spaces such as the middle third, and the planar Sierpinski caskets in various forms, and their corresponding selfsimilar measures, but the class is more general than this; including fractals realized in R^d, for $d > 2$. In part 1, we motivate the need for wavelets in the harmonic analysis of these selfsimilar measures mu. While classes of the Hilbert spaces $L^2(\mu)$ have Fourier bases, it is known (the speaker and Pedersen) that many do not, for example the middle third Cantor can have no more than two orthogonal Fourier frequencies. In part 2 of the talk, we outline a construction by the speaker and Dutkay to the effect that all the affine systems do have wavelet bases; this entails what we call thin Cantor spaces.

Postcritically Finite Maps, Lattès Maps, and Symmetrization in $\mathbb{CP}^k$

Scott Kaschner
Butler University, USA
Thomas Gauthier

We discuss various types of postcritically finite maps of $\mathbb{CP}^k$ and present examples of each type. Symmetric products have been used to produce examples of endomorphisms of $\mathbb{CP}^k$ $(k \geq 2)$ with certain characteristics. We use them in this talk to produce endomorphisms of $\mathbb{CP}^k$ that are strongly postcritically finite. We also use them to characterize families of Lattès maps.

Little's Law Analysis of a Stochastic Network

James Keesling
University of Florida, USA
Celeste Vallejo

Little's Law is a principle that is used in the analysis of stochastic networks. Little's Law states that if α is the arrival rate at a facility, W is the average waiting time of an individual in the facility, and $\overline{n}$ is the average number in the facility, then $\alpha \cdot W = \overline{n}$ provided that the numbers are finite. There are no assumptions about the distributions giving rise to these numbers or even if any such distributions exist. The only assumption is that the limits exist. Little's Law is used largely in making some difficult calculation in a network. In this talk we suggest that the analysis of the flow through a network can be analyzed starting with Little's Law rather than starting with various stochastic assumptions. The motivation for this approach was the analysis of the flow of patients through an Emergency Department and through an entire hospital. We give other applications as well.

Topological Entropy in Generalized Inverse Limits

Judy Kennedy
Lamar University, USA
Goran Erceg

We generalize the definition of topological entropy due to Adler, Konheim, and McAndrew to set-valued functions from a closed subset A of the interval to closed subsets of the interval. We view these set-valued functions, via their graphs, as closed subsets of $[0, 1]^2$. We show that many of the topological entropy properties of continuous functions of a compact topological space to itself hold in our new setting, but not all. We also compute the topological entropy of some examples, and give an idealized application to impulse functions, as these are not functions in the usual sense, but do occur in models coming from other branches of science.

Ground States on the Boundary of Rotation Sets

Tamara Kucherenko
The City College of New York, USA
Christian Wolf

Ground states are accumulation points of equilibrium states when the temperature goes to zero. They play a fundamental role in statistical physics. We consider rotation sets associated with a continuous dynamical system on a compact metric space and a multi-dimensional continuous potential. We study the question for which boundary vectors of the rotation set one can realize an entropy maximizing measure as a ground state associated with a certain linear combination of the potential. We show that at an exposed point there always exists a ground state that maximizes entropy in its rotation class. We also construct examples of rotation sets (in any dimension) that have exposed boundary points without a ground state in its rotation class. Finally, we consider non-exposed points and show that the following two phenomena exist: a) boundary points without an associated ground state; b) boundary points with a unique ground state that is not ergodic.

Estimating the Hausdorff Dimension of Minimal Sets in Smooth Counterexamples to the Seifert Conjecture

Krystyna Kuperberg
Auburn University, USA

The smooth counterexamples to the Seifert Conjecture posses a large minimal set. We describe a method of estimating Hausdorff dimension of this minimal set using flow boxes in a similar fashion as in the box-counting method of Minkowski and Bouligand.

Fractals: from Laminations to Julia Sets

John Mayer
UA-Birmingham, USA

Julia sets of complex analytic functions are well-known examples of (almost) self-similar fractals in the complex plane or Riemann sphere. Laminations of the unit disk were introduced by William Thurston as a topological/combinatorial model for understanding the (connected) Julia sets of polynomials. This understanding can be extended, at least partially, to laminations corresponding to connected Julia sets for degree $d \geq 2$ polynomials; the Julia set is the monotone image of the lamination and with semiconjugate dynamics. Laminations whose critical chords are not in the lamination are, for that very reason, self-similar. The corresponding Julia set is thus also self-similar; the polynomial on its Julia set is a covering map. We focus on laminations of degree 2 and 3, and in particular on degree 3 laminations that contain an identity return leaf or triangle. The topological correspondence between laminations and polynomial Julia sets is thus a correspondence of fractals.

Topological Transitivity of Extensions of Hyperbolic Systems with Infinite Dimensional Fiber.

Viorel Nitica
West Chester University, USA

We will discuss topological transitivity of extensions of hyperbolic systems with infinite dimensional fiber.

The Effect of Projections on Fractal Sets and Measures in Banach Spaces

William Ott
University of Houston, USA
Zijie Zhou

Many infinite-dimensional dynamical systems of interest in the physical sciences admit finite-dimensional invariant attracting sets. This motivates the following: Given a compact subset A of a Banach space $\mathcal{X}$ and a typical C^1 map $\varphi : \mathcal{X} \to \mathbb{R}^n$, what is the relationship between the Hausdorff dimension of A and that of $\varphi(A)$? Do A and $\varphi(A)$ have the same Hausdorff dimension (the ideal situation)? If not, what determines the gap between the two values? Ott, Hunt, and Kaloshin answered these questions when the ambient space $\mathcal{X}$ is a Hilbert space. In particular, they showed that Hausdorff dimension is typically preserved up to a factor involving the thickness exponent $\tau(A)$. In this talk we provide answers in the general Banach space case. We formulate two results - one involving the thickness exponent and one involving the dual thickness exponent. The latter result answers a question posed by Robinson.

A Class of Cubic Rauzy Fractal

Tatiana Rodrigues
UNESP, Brazil
Jefferson Bastos

In this work we study arithmetical and topological properties for a class of Rauzy fractals $\mathcal{R}_a$ given by the polynomial x^3-ax^2+x-1 where $a \geq 2$ is an integer. In particular, we prove the number of neighbors of $\mathcal{R}_a$ in the periodic tiling is equal to 8. We also give explicitly an automaton that generates the boundary of $\mathcal{R}_a$, for $a = 2$. In this case we calculate the Hausdorff dimension. We also give explicitly an automaton that generates the boundary of $\mathcal{R}_a$ for $a > 2$ and using an exotic numeration system we prove that $\mathcal{R}_a$ is homeomorphic to a topological disk.

Quantization

Mrinal Roychowdhury
University of Texas Rio Grande Valley, USA

Quantization for probability distributions refers to the idea of estimating a given probability by a discrete probability with finite support. Quantization dimension gives the speed how fast the specified measure of the error goes to zero as the number of points in the underlying support goes to infinity. Recently, several works have been done in this direction. I will talk about it.

Dimension Calculation for an Invariant Measure Supported on a Subfractal

Elizabeth Sattler
North Dakota State University, USA

In this talk, we will examine properties, including fractal dimensions, of a subfractal induced by a subshift of finite type or sofic subshift. We will construct an invariant measure supported on a subfractal induced by a subshift of finite type and discuss a method for calculating the Hausdorff dimension of such a measure.

Topological Measures of Order for Pattern-Forming Systems

Patrick Shipman
Colorado State University, USA
R. M. Bradley, F. C. Motta, R. Neville, D. Pearson

Exploiting theory and methods from computational topology, we introduce several measures which quantify the order of nearly hexagonal, planar lattices produced by dynamical systems or systems of PDEs. For example, when the surface of a nominally flat binary material is bombarded with a broad, normally incident ion beam, disordered hexagonal arrays of nanodots can form. Defects, such as dislocations in ripple patterns or penta-hepta pairs in hexagonal arrays, limit the utility of patterns produced by ion bombardment. To evaluate the efficacy of this method, a means of quantifying the degree of order is needed. We compare time-honored methods, such as the autocorrelation function, with the new topological methods.

Hereditarily Non Uniformly Perfect Sets

Rich Stankewitz
Ball State University, USA
Toshiyuki Sugawa, Hiroki Sumi

We introduce the concept of hereditarily non uniformly perfect sets, compact sets for which no compact subset is uniformly perfect, and compare the following properties: hereditarily non uniformly perfect sets, Hausdorff dimension zero sets, logarithmic capacity zero sets, Lebesgue 2-dimensional measure zero sets, and porous sets. In particular, we give an example of a compact set of positive Hausdorff dimension and positive logarithmic capacity which is hereditarily non uniformly perfect.

Hausdorff Dimension of the Julia Sets of Postcritically Bounded Polynomial Semigroups and The Transversality Condition

Hiroki Sumi
Osaka University, Japan

We consider the dynamics of the semigroups generated by two polynomial maps $\{f_1, f_2\}$ of degree two or more on the Riemann sphere. Let B be the set of (f_1, f_2) for which the planar postcritical set of the semigroup $\langle f_1, f_2\rangle$ generated by $\{f_1, f_2\}$ is bounded. Let C be the set of (f_1, f_2) for which the Julia set of $\langle f_1, f_2\rangle$ is connected. Let H be the set of (f_1, f_2) for which $\langle f_1, f_2\rangle$ is hyperbolic. Let I be the set of (f_1, f_2) for which the Julia sets of f_1, f_2 intersect. We show that there exists an open dense subset A of $((\partial C) \cap B \cap H) \setminus I$ such that for every $(f_1, f_2) \in A$, there exists an open neighborhood U of (f_1, f_2) such that for almost every $(g_1, g_2) \in U$ with respect to the Lebesgue measure on U, the Hausdorff dimension of the Julia set of the semigroup generated by $\{g_1, g_2\}$ is equal to the Bowen parameter of (g_1, g_2) (i.e. Bowen's formula holds). Note that we cannot expect the open set condition in U. The key idea in the proof is to show that the transversality condition holds in U.

S-Adic Dynamical Systems and Rauzy Fractals

Jorg Thuswaldner
Leoben University, Austria
P. Arnoux, V. Berthè, M. Minervino, W. Steiner

We study dynamical systems that are defined in terms of a sequence of substitutions. We relate Rauzy Fractals to these dynamical systems. Tiling properties of these fractals allow to conclude that they are conjugate to rotations on the torus. Moreover, these dynamical systems have relations to generalized continued fraction algorithms like Brun's algorithm.

Rational Maps from Polynomial Matings Using Combinatorial Methods and Thurston's Algorithm

Mary Wilkerson
Coastal Carolina University, USA

'Topological mating' describes an operation that combines two complex polynomials to obtain a new dynamical system on the quotient of a 2-sphere. The dynamics of the mating are then dependent upon the two polynomials that are mated. While in many cases, we wouldn't expect this process to yield a map or quotient space that is 'nice', the topological mating of two postcritically finite polynomials is frequently Thurston equivalent to a rational map on the Riemann sphere. Given a postcritically finite quadratic polynomial pair, there is a simple parameter test to determine whether such a rational map F exists—but this test is not constructive of F. We present an iterative method that uses combinatorial techniques and Thurston's algorithm to construct an approximation to F, and follow with the discussion of an example using this method.

Hitting Time Distribution and Extreme Value Law for Flows

Fan Yang
UFRJ, Brazil
Maria Jose Pacifico

For flows whose return map on a cross section has sufficient mixing property, we show that the hitting time distribution of the flow to balls is exponential in limit. We also establish a link between the extreme value distribution of the flow and its hitting time distribution, generalizing a previous work by Freitas et al in the discrete time case. Finally we show that for maps that can be modeled by Young's tower with polynomial tail, the extreme value law holds. As examples we consider the classic Lorenz attractor and the Rovella attractor. This is a joint work with Maria Jose Pacifico.

Multi-Chaos

James Yorke
University of Maryland, USA
S. Das, Y. Saiki

We investigate systems on a torus that have a dense set of periodic saddles and a dense set of periodic repellers. Such systems may be a prototype of systems with higher dimensional chaos.

Special Session 45: Nonlinear Waves and Singularities in Optical and Hydrodynamic Systems

Sergey Dyachenko, University of Illinois, USA
Pavel Lushnikov, University of New Mexico, USA

The session is devoted to recent advances in nonlinear optics, free surface hydrodynamics and others nonlinear systems. The emphasis is given to the works on singularity formation in Hamiltonian and non-Hamiltonian systems described by partial differential equations. The examples include but are not restricted to new advances in free surface hydrodynamics, self-focusing of laser beam, filamentation and laser-plasma interaction.

Analyzing the Stability Spectrum for Elliptic Solutions to the Focusing NLS Equation

Bernard Deconinck
University of Washington, USA

The one-dimensional focusing cubic nonlinear Schroedinger (NLS) equation is one of the most important integrable equations, arising in a multitude of applications. The stability of the stationary periodic solutions of NLS is well studied, leading to, for instance, the iconic figure-eight spectrum for its cnoidal wave solutions. We present an explicit expression for the linear stability spectrum of both the trivial- and nontrivial-phase solutions. We use this expression to generate many explicit results about the spectrum.

Finding the Stokes Wave: from Small Steepness to the Wave of Greatest Height

Sergey Dyachenko
University of Illinois, USA
Pavel Lushnikov, Alexander Korotkevich

A Stokes wave is a fully nonlinear wave that travels over the surface of deep water. We solve Euler equations with free surface in the framework of conformal variables via Newton Conjugate Gradient method and find Stokes waves in regimes dominated by nonlinearity. By investigating Stokes waves with increasing steepness we observe peculiar oscillations occur as we approach Stokes limiting wave. Finally by analysing Pade approximation of Stokes waves we infer that analytic structure associated with those waves has branch cut nature.

Numerical Methods for the Chemotaxis Models

Yekaterina Epshteyn
University of Utah, USA

In this talk, I will first introduce and review several chemotaxis models including the classical Patlak-Keller-Segel model. Chemotaxis is the phenomenon in which cells, bacteria, and other single-cell or multicellular organisms direct their movements according to certain chemicals (chemoattractants) in their environment. Chemotaxis is an important process in many medical and biological applications including bacteria/cell aggregation, pattern formation mechanisms, and tumor growth. The mathematical models of chemotaxis are usually described by highly nonlinear time dependent systems of partial differential equations (PDEs). Therefore, accurate and efficient numerical methods are very important for the validation and analysis of these systems. Furthermore, a known property of the existing chemotaxis models is their ability to describe a concentration phenomenon that mathematically results in solutions rapidly growing in small regions of concentration points or curves. The solutions can blow up in finite-time or can develop a singular, spiky behavior. This blow-up represents a mathematical description of a cell concentration phenomenon that happens in real biological systems. Hence, capturing such solutions numerically is a very challenging problem. In this presentation, we will introduce and discuss several recently developed numerical methods for the approximation and simulation of the chemotaxis and related models. Numerical experiments to demonstrate the stability and accuracy of the proposed methods for chemotaxis models will be presented. Ongoing research projects will be discussed as well.

Stable Solitons of the Cubic-Quintic NLS with a Delta-Function Potential

Francois Genoud
TU Delft, Netherlands
Boris Malomed, Rada Weishaupl

This talk is about the one-dimensional nonlinear Schrodinger equation with a combination of cubic focusing and quintic defocusing nonlinearities, and an attractive delta-function potential. This model comes from nonlinear optics, and the delta-function potential describes the interaction of a broad solitonic beam with a narrow defect. I will show that all spatial solitons can be determined explicitly in terms of elementary functions. Using bifurcation and spectral-theoretic arguments, I will then prove that they are all (nonlinearly) stable. A noteworthy feature of the model is a regime of 'bistability', where two solitons with same propagation constant coexist.

Circular Instability of a Standing Gravity-Capillary Wave: Theory and Wave Tank Experiment

Alexander Korotkevich
University of New Mexico, USA
Lukaschuk S.

We provide a theory and compare numerical simulation of instability of weakly nonlinear standing waves on the surface of deep fluid in the framework of the primordial dynamical equations and in a laboratory wave tank experiment. The instability offers a new approach for generation of nearly isotropic spectrum using parametric excitation. Direct measurements of spacial Fourier spectrum confirm existence of the instability in a real life conditions for gravity-capillary waves.

Nonlinear Schrodinger Systems with Non-Zero Boundary Conditions

Gregor Kovacic
Rensselaer Polytechnic Institute, USA
Gino Biondini, Daniel Kraus

The study of scalar and vector nonlinear Schrodinger (NLS) systems with non-zero boundary conditions at infinity has received renewed interest recently. This talk will report on recent results on focusing scalar and vector NLS equations with non-zero boundary conditions. It will be shown how the inverse scattering transform can be constructed in both cases, and a number of explicit soliton solutions will be discussed.

Studies of (in)stability of the Numerical Method of Characteristics Applied to Conservative Hyperbolic PDEs

Taras Lakoba
University of Vermont, USA
Z. Deng

The numerical Method of Characteristics (MoC) is widely used to solve hyperbolic evolution equations. This method reduces the solution of the partial differential equation (PDE) to that of a small system of ordinary differential equations (ODEs), where the solution of each ODE is obtained along "its" characteristic direction. The ODEs are solved by an ODE numerical solver, such as, e.g., a simple Euler or a Runge–Kutta method. This study was motivated by our numerical solution of a set of PDEs describing propagation of light in birefringent optical fibers:

$$\mathbf{S}_t^+ + \mathbf{S}_x^+ = \mathbf{S}^+ \times \mathbf{J}\,\mathbf{S}^-, \qquad \mathbf{S}_t^- - \mathbf{S}_x^- = \mathbf{S}^- \times \mathbf{J}\,\mathbf{S}^+. \quad (1)$$

Here $\mathbf{S}^\pm$ are Stokes vectors of the fields propagating along each of the principal polarization axes of the fiber and $\mathbf{J}$ is some diagonal matrix.These equations support propagation of solutions of the domain wall type. It is important to mention that the $S_2^\pm$ components of these solutions asymptotically tend to *nonzero* constants, while the other components asymptotically vanish:

$$S_2^+(x \to \pm\infty) \to \pm a, \quad S_2^-(x \to \pm\infty) \to \mp a; \quad S_{1,3}^\pm(|x| \to \infty) \to 0. \quad (2)$$

It can be shown that Eqs. (1) conserve the norm, $|\mathbf{S}^\pm|^2$, of each field. We numerically solved Eqs. (1), supplemented with *non-reflecting* boundary conditions (BCs), by the MoC. We used the Leapfrog method as the ODE solver, because it is known to nearly conserve energy in Hamiltonian ODEs and thus is expected to be a "good" method for Eqs. (1). However, we observed a conspicuous numerical instability in our simulations. This is the **first unexpected result** of our study. When we replaced the Leapfrog method with the modified Euler method, which is known *not* to conserve energy in ODEs and hence was expected to be a "bad" method for this problem, the numerical instability of the MoC was strongly reduced. This is the **second unexpected result** of our study. Before we had attempted to analyze this behavior, we repeated simulations with the same two numerical ODE solvers in the MoC but used *periodic* BCs. The reason was that for periodic BCs, one could use the von Neumann analysis to infer stability of the numerical scheme. On the other hand, as written in many textbooks, we expected the BCs to have little to no effect on the numerical (in)stability. Here we observed our **third unexpected result** : while the instability of the Leapfrog method remained the same as for non-reflecting BCs, the near stability of the modified Euler method was replaced with an instability about as strong as that for the Leapfrog method. We have found analytical explanations to all three unexpected results. Since the results reported above occurred not only for the domain-wall solutions but also for the constant solutions (2), we carried out our analyses for those simpler solutions, for the sake of clarity and simplicity. Thus, first, by means of the von Neumann analysis, we explained why an energy-preserving ODE solver, such as Leapfrog, when combined with the MoC, can produce a strong instability for periodic BCs. Second, we explained why changing BCs from periodic to non-reflecting suppresses a relatively strong numerical instability of the modified Euler method.

Formation of Limiting Stokes Wave from Non-Limiting Stokes Wave: Merging of Square Root Branch Points from the Infinite Set of sheets of Riemann surface to form 2/3 singularity of limiting wave

Pavel Lushnikov
University of New Mexico, USA

Stokes wave is the fully nonlinear periodic gravity wave parameterized by its height. Wave of greatest height has the limiting form with 120 degrees angle on the crest. Assume $z(\zeta)$ provides a conformal map of a free fluid surface of Stokes wave into the real line with fluid domain mapped into the lower complex half-plane of ζ. Then Stokes wave is fully characterized by the complex singularities in the upper complex half-plane. The only singularity in the physical sheet of Riemann surface of non-limiting wave is the square-root branch point located at $\zeta = i\zeta_c$. Corresponding branch cut defines the second sheet of the Riemann surface if we cross the branch cut. We found the infinite number of square root singularities in infinite number of non-physical sheets of Riemann surface. These singularities located both symmetrically $\zeta = \pm i\zeta_c$ and on diagonals (with respect to vertical axis) corresponding to different non-physical sheets of Riemann surface. Increase of the height of the Stokes wave means that all these singularities simultaneously approach the real line from different sheets of Riemann surface and merge together forming 2/3 power law singularity of the limiting wave. It was conjectured (P.M. Lushnikov, ArXiv:1507.02784) that non-limiting Stokes wave $z(\zeta)$ at the leading order consists of the infinite product of nested square root singularities which form the infinite number of sheets of Riemann surface.

Polarization Switching in a Resonant Optical Medium

Katie Newhall
UNC Chapel Hill, USA

Optical-pulse polarization switching along an active optical medium in the Λ-configuration is described using the idealized integrable Maxwell-Bloch model. Analytical results for the final limiting polarization show the dependence on the initial preparation of the medium. Right- and left-circularly polarized light are stable even in the case of spatially disordered occupation numbers in the lower energy sub level pair. However, arbitrary elliptical polarization can be made stable by modifying the initial virtual polarization between the two degenerate lower energy states.

Obtaining Stokes Wave with High-Precision Using Conformal Maps and Spectral Methods on Non-Uniform Grids

Denis Silantyev
University of New Mexico, USA
Pavel Lushnikov, Sergey Dyachenko

Two-dimensional potential flow of the ideal incompressible fluid with free surface and infinite depth has a class of solutions called Stokes waves which is fully nonlinear periodic gravity waves propagating with the constant velocity. The increase of the scaled wave height H/L, where H is the wave height and L is the wavelength, from H/L= 0 to the critical value Hmax/L marks the transition from almost linear wave to a strongly nonlinear limiting Stokes wave. Fully nonlinear Euler equations describing the flow can be reformulated in terms of conformal map of the fluid domain into the complex lower half-plane, with fluid free surface mapped into the real line. This description is convenient for analysis and numerical simulations since the whole problem is now reduced to a single equation on the real line. We use spectral method together with an iterative scheme to obtain solutions. Extending solutions to the rest of the complex plane one can see that the distance Vc from the closest singularity in the upper half-plane to the real line goes to zero as we approach the limiting Stokes wave, which is the reason for the widening of the solution's spectrum. This makes us seek a new approach that allows one to overcome this difficulty. We improve performance of our numerical method drastically by introducing second conformal map that pushes the singularity higher into the upper half-plane and correspondingly shrinks the spectrum of the solution.

Modeling of Novel Parity-Time Symmetric Systems.

Alexey Sukhinin
Southern Methodist University, USA

Rapid emergence of PT-symmetric systems is a new trend in nonlinear optics. In this talk I will describe modeling techniques in non-Hermitian systems with balanced gain and loss and discuss the nonlinear governing equations that are used for the analysis.

Cascades in Wave Turbulence

Natalia Vladimirova
University of New Mexico, USA
Gregory Falkovich

We consider developed turbulence in the 2D Gross-Pitaevsky model, which describes wide classes of phenomena from atomic and optical physics to condensed matter, fluids and plasma. The well-known difficulty of the problem is that the hypothetical local spectra of both inverse and direct cascades in the weak-turbulence approximation carry fluxes which are either zero or have the wrong sign; such spectra cannot be realized. We analytically derive the exact flux constancy laws (analogs of Kolmogorov's 4/5-law for incompressible fluid turbulence), expressed via the fourth-order moment and valid for any nonlinearity. We confirm the flux laws in direct numerical simulations. We show that a constant flux is realized by non-local wave interaction in both the direct and inverse cascades. Wave spectra (second-order moments) are close to slightly (logarithmically) distorted thermal equilibrium in both cascades.

Special Session 47: Mathematical Contribution Towards the Understanding of the Dynamics of the 2014 Ebola Epidemic in West Africa

Miranda Teboh-Ewungkem, Lehigh University, USA
Abba Gumel, Arizona State University, USA
Folashade Agusto, Austin Peay State University, USA

The 2014 West African Ebola epidemic riveted the world, and is the worst yet on record. Even though there has been a slowdown of the epidemic, there is a risk of resurgence in the near future or sometime later. Thus research to fully understand factors that contributed to its spread and control efforts that were crucial in mitigating its spread need to continue. Mathematical models were crucial in some of these regards. The aim of this minisymposium is to highlight the mathematical contributions towards the understanding of the dynamics, spread and control of the 2014 Ebola epidemic.

Mathematical Assessment of the Role of Traditional Belief Systems and Customs and Health-Care Settings on the Transmission Dynamics of the 2014 Ebola Outbreaks

F Agusto
University of Kansas, USA
Miranda Teboh-Ewungkem, Abba Gumel

A mathematical model is designed and used to assess population-level impact of basic non-pharmaceutical control measures on the 2014 Ebola outbreaks. The model incorporates the effects of traditional belief systems and customs, disease transmission within health-care settings and by Ebola-deceased individuals. A sensitivity analysis is performed to determine model parameters that most affect disease transmission. The model is parameterized using data from Guinea, one of the three Ebola-stricken countries. Three effectiveness levels of basic public health control measures are considered. The distribution of the basic reproduction number ($\mathcal{R}_0$) for Guinea (in the absence of basic control measures) is such that $\mathcal{R}_0 \in [0.77, 1.35]$, for the case when the belief systems do not result in more unreported Ebola cases. When such systems inhibit control efforts, the distribution increases to $\mathcal{R}_0 \in [1.15, 2.05]$. The total Ebola cases are contributed by Ebola-deceased individuals (22%), symptomatic individuals in the early (33%) and latter (45%) infection stages. The 2014 outbreaks are controllable using a moderately-effective basic public health intervention strategy alone. A much higher ($> 50\%$) disease burden would have been recorded in the absence of such intervention.

Modeling the Ebola Outbreak and Contact Tracing

Cameron Browne
University of Louisiana at Lafayette, USA
Hayriye Gulbudak, Glenn Webb

The 2014-2015 Ebola outbreak in West Africa resulted in over 28,000 cases. Previous Ebola outbreaks had been rapidly controlled with contact tracing and isolation strategies. However, the failure of initial containment and the subsequent unprecedented scale of the epidemic challenged public health authorities to employ effective control measures. Mathematical modeling can be an important tool to evaluate the efficacy and implementation of interventions, along with generally providing insights into the dynamics of Ebola or other emerging pathogens. In this talk, I discuss recent work in which we develop a mechanistic differential equation model incorporating the key features of contact tracing during a disease outbreak. We characterize the impact of contact tracing on the effective reproduction number $\mathcal{R}_e$, and formulate $\mathcal{R}_e$ completely in terms of reported case and contact tracing observables. Data from the West Africa Ebola outbreak is utilized to form real-time estimates of $\mathcal{R}_e$ and evaluate the impact of contact tracing on the epidemic. Taken together, our model and results quantify contact tracing as both a dynamic intervention strategy impacting disease spread and a probe into the epidemic status at the population level.

Transmission Dynamics and Final Epidemic Size of Ebola Virus Disease Outbreaks with Varying Interventions

Attila Denes
Bolyai Institute, University of Szeged, Hungary
Maria Vittoria Barbarossa, Gábor Kiss, Yukihiko Nakata, Gergely Röst, Zsolt Vizi

The 2014 Ebola Virus Disease outbreak in West Africa was the largest and longest ever reported since the first identification of this disease. We propose a compartmental model for Ebola dynamics, including virus transmission in the community, at hospitals and at funerals. Using time-dependent parameters, we incorporate the increasing intensity of intervention efforts. Estimating the parameter values by generating sample sets from previously proposed parameter ranges, we fit the system to the early phase of the 2014 West Africa Ebola outbreak, and estimate the basic reproduction number as 1.44. By PRCC analysis, we find that the most important factor for the spread of the epidemic is virus transmission during traditional burial practices. We derive a final size relation which allows us to forecast the total number of cases during the outbreak when effective interventions are in place. Our model predictions show that, as long as cases are reported in any country, intervention strategies cannot be dismissed. Since the main driver in the slowdown of the epidemic is not the depletion of susceptibles, future waves of infection might be possible, if control measures or population behaviour are relaxed. We also show the importance of timely intervention, showing that few weeks delay can result into twice as large total number of cases.

Period-Doubling Bifurcation and Chaos in an Autonomous Model for Malaria

Calistus Ngonghala
Harvard Medical School, USA
Miranda I. Teboh-Ewungkem, Gideon A. Ngwa

An autonomous model for the dynamics of malaria transmission is presented. The model differs from standard malaria transmission models in that mosquito demography, feeding and reproduction patterns are modeled explicitly. The effects of various mosquito birth functions on the dynamics of the system are examined. It is demonstrated that the system transitions from a stable disease-free equilibrium to a subcritical bifurcation when the basic reproduction number is less than unity and then to a stable endemic equilibrium when the basic reproduction number is bigger than unity. For one of the birth functions, it is shown that further increases in the basic reproduction number drives the system into period-doubling bifurcations, closely followed by chaotic dynamics and then period-halving bifurcations. The occurence of a subcritical bifurcation indicates that malaria intervention strategies have to be applied adequately long to ensure that the basic reproduction number falls below the saddle-node bifurcation point, where the subcritical bifurcation occurs. On the other hand, the chaotic dynamics indicates that modeling mosquito demography, feeding and reproduction patterns explicitly might be important in understanding the complexity involved the dynamics of malaria. We conclude that malaria data may require more careful examination for complex dynamics.

A Mathematical Model with Quarantine States for the Dynamics of Ebola Virus Disease in Human Populations

Miranda Teboh-Ewungkem
Lehigh University, USA
Gideon Akumah Ngwa

A deterministic ordinary differential equation model for the dynamics and spread of Ebola Virus Disease is derived and studied. The model contains quarantine and non-quarantine states and can be used to evaluate transmission both in treatment centers and in the community. Possible sources of exposure to infection, including cadavers of ebola virus victims, are included in the model derivation and analysis. Our model's results show that there exist a threshold parameter, R_0, with the property that when its value is above unity, an endemic equilibrium exists whose value and size is determined by the size of this threshold parameter, and when its value is less than unity, the infection does not spread into the community. The equilibrium state, when it exists is locally and asymptotically stable with oscillatory returns to the equilibrium point. The basic reproduction number, R_0 is shown to be strongly dependent on the initial response of the emergency services to suspected cases of ebola infection. When intervention measures such as quarantining are instituted fully at the beginning, the value of the reproduction number reduces and any further infections can only occur at the treatment centers. Effective control measures, to reduce R_0 to values below unity, are discussed.

Special Session 48: Uncertainty Quantification in Dynamical Systems

Marcos A. Capistran, CIMAT A.C., Mexico

This session aims is to bring together researchers of applied mathematics to show advances on uncertainty quantification of dynamical systems. Uncertainty quantification encompasses methods to account for a cascade of errors arising while observing, modeling, discretizing, etc., physical models of reality. We are interested on both, forward propagation of uncertainty as well as inverse assessment of model and parameter uncertainty.

A Randomized Misfit Approach for Big Data in Large-Scale Bayesian Inverse Problems

Tan Bui-Thanh
The University of Texas at Austin, USA
Ellen Le, Aaron Myers

We present a randomized misfit approach (RMA) for efficient data reduction in large-scale inverse problems. The method is a random transformation approach that generates reduced data by randomly combining the original ones. The main idea is to first randomize the misfit and then use the sample average approximation to solve the resulting stochastic optimization problem. At the heart of our approach is the blending of the stochastic programming and the random projection theories, which brings together the advances from both sides and exploits opportunities at their interfaces. This allows us to conduct a more complete analysis of the RMA method, which is unlikely possible using sole theory from either of the communities separately. One of the main results of the paper is the interplay between the Johnson-Lindenstrauss lemma and large deviation theory. In particular, the former provides sharp bounds on the reduced data dimensions for a large class of interesting sparse random transformations, while the latter introduces a new look and proof of the former. To justify the RMA approach, a detailed theoretical analysis is carried out for both linear and nonlinear inverse problems. A tight connection between the Morozov discrepancy principle and the Johnson-Lindenstrauss lemma is presented. It is this connection that allows us to explain the ability of the RMA method in significantly reducing observation data with acceptable accuracy lost for the solution of inverse problems. Various numerical results to motivate and to verify our theoretical findings are presented for inverse problems governed by elliptic partial differential equations in one, two, and three dimensions.

Numerical Posterior Distribution Error Control and Expected Bayes Factors in the Bayesian Uncertainty Quantification of Dynamical Systems

Andres Christen
CIMAT, Mexico
Marcos Capistrán, MIguel Ángel Moreles

In the bayesian analysis of ODEs Inverse Problems most relevant cases have intractable analytical solutions. These necessarily involve a numerical method to find approximate versions of such solutions and lead to a numerical/approximate posterior distribution. Recently several results have been published on the regularity conditions required on such numerical methods to ensure converge of the numerical to the theoretical posterior. However, more practical guidelines are needed to ensure a suitable working numerical posterior. Capistrán, Christen and Donnett (2013) (arXiv:1311.2281) prove for ODEs that the Bayes Factor of the approximate vs the theoretical model tends to 1 in the same order as the numerical method order. In this work we generalize the latter paper in that we consider 1) correlated observations, 2) practical guidelines in a multidimensional setting and 3) explore the use of expected Bayes Factors. This permits us to obtain bounds on the absolute global errors to be tolerated by the numerical solver, which we illustrate with some examples. Since the Bayes Factor is kept above 0.95 we expect that the resulting numerical posterior is basically indistinguishable from the theoretical posterior. The method is illustrated with some examples using synthetic data.

Characterizing Marginal Distributions Using MCMC with Local Polynomial Approximations

Andrew Davis
MIT, USA
Youssef Marzouk, Patrick Heimbach

We develop methods to characterize selected marginal distributions of high-dimensional probability distributions in a setting where evaluating the joint (high-dimensional) density is computationally expensive. We use importance sampling to estimate the selected marginal density, and combine the resulting noisy estimates with local polynomial regression to approximate the smooth underlying marginal density. The approximation is continually and infinitely

refined in conjunction with a Markov chain Monte Carlo (MCMC) algorithm that enables asymptotically exact sampling from the marginal distribution of interest. By exploiting regularity of the low-dimensional marginal, the overall scheme significantly reduces computational expense relative to both regular MCMC and pseudo-marginal MCMC. We use our approach to solve a Bayesian inverse problem in a goal-oriented setting. In particular, we examine a dynamical model of an ice stream in West Antarctica, where significant uncertainty is associated with high-dimensional parameter fields (e.g., basal topography) which translates into uncertainty in future evolution of ice stream volume above floatation. The targeted local approximation scheme allows us to directly and efficiently characterize the posterior distribution of this ice volume.

Iterative Updating of Model Error in Bayesian Inversion

Matthew Dunlop
University of Warwick, England
Daniela Calvetti, Errki Somersalo, Andrew Stuart

One of the computational challenges associated with large-scale inverse problems is the cost of forward model evaluations. Often a compromise must be made between accuracy (fine models) and speed (coarse models). We outline an algorithm that iteratively estimates the distribution of the model error arising from using a coarse model, allowing for more accurate sampling of the posterior distribution. The convergence of the algorithm in the linear Gaussian case is analyzed, wherein the posterior remains Gaussian and can be characterized by the evolution of its mean and covariance. We show that these both converge exponentially fast, with the limiting covariance being non-degenerate. In the non-linear case, numerically all measures will be approximated by ensembles of particles; we show convergence of some different particle approximations in the large particle limit. Finally we present some numerical results to illustrate the behavior of the algorithm.

A Bayesian Approach for Parameter Identification in Aquifers

Miguel Moreles
CIMAT, Mexico
Liliana Guadalupe

An inverse problem of interest in Geohydrology is to estimate phenomenological parameters in aquifers from hydraulic potential data. We regard the parameters as functions in a Hilbert space, and explore bayesian estimation for gaussian priors as well as gaussian noise. First we consider the elliptic case, and show estimation of transmissivity in an isotropic confined aquifer for which Darcy's law nad the two dimensional approximation hold. Then we discuss the joint estimation of transmissivity and storativity in the underlying parabolic partial differential equation.

Special Session 49: Recent Advances of Differential Equations with Applications in Life Sciences

Ping Liu, Harbin Normal University, Peoples Rep of China
Ying Su, Harbin Institute of Technology, Peoples Rep of China
Fengqi Yi, Harbin Engineering University, Peoples Rep of China

Differential equations have been playing important roles in explaining the rich phenomena arising from life sciences. This special session is to concentrate on the recent advances of differential equations of various types (with or without delay) with applications in life sciences. We will invite researchers in this field from around the world to Orlando, Florida for the purpose of providing an excellent forum to exchange ideas, create new research collaborations, and rekindle old connections. Speakers and talks are carefully selected to make the session attractive to a diverse audience

Hopf Bifurcation and Optimal Control in a Diffusive Predator-Prey System with Time Delay and Prey Harvesting

Xiaoyuan Chang
Harbin University of Science and Technology, Peoples Rep of China
Junjie Wei

In this paper, we investigated the dynamics of a diffusive delayed predator-prey system with Holling type II functional response and nozero constant prey harvesting on no-flux boundary condition. At first, we obtain the existence and the stability of the equilibria by analyzing the distribution of the roots of associated characteristic equation. Using the time delay as the bifurcation parameter and the harvesting term as the control parameter, we get the existence and the stability of Hopf bifurcation at the positive constant steady state. Applying the normal form theory and the center manifold argument for partial functional differential equations, we derive an explicit formula for determining the direction and the stability of Hopf bifurcation. Finally, an optimal control problem has been considered.

A Spatial SIS Model in Advective Heterogeneous Environments

Renhao Cui
Renmin University, Peoples Rep of China
Yuan Lou

We study the effects of diffusion and advection for a susceptible-infected-susceptible epidemic reaction-diffusion model in heterogeneous environments. The definition of the basic reproduction number $\mathcal{R}_0$ is given. If $\mathcal{R}_0 1$ is studied. The effects of diffusion and advection rates on the stability of the DFE are further investigated. Among other things, we find that if the habitat is a low-risk domain, there may exist one critical value for the advection rate, under which the DFE changes its stability at least twice as d_I varies from zero to infinity, while the DFE is unstable for any d_I when the advection rate is larger than the critical value. These results are in strong contrast with the case of no advection, where the DFE changes its stability at most once as d_I varies from zero to infinity.

Bifurcation Analysis and Chaos Switchover Phenomenon in a Nonlinear Financial System with Delay Feedback

Yuting Ding
Northeast Forestry University, Peoples Rep of China
Jun Cao

In this talk, we study dynamics in delayed nonlinear financial system, with particular attention focused on Hopf and double Hopf bifurcations. Firstly, we identify the critical values for stability switches, Hopf and double Hopf bifurcations. We show how the parameters effect the dynamical behavior of the system. Secondly, the normal forms near the Hopf and double Hopf bifurcations, as well as classifications of local dynamics are analyzed. These bifurcations lead a chaotic system to be stable states, such as the coexistence of a pair of stable equilibria or a pair of stable periodic oscillations, and the chaos disappears. Numerical simulations are presented to verify the analytical predictions. Furthermore, detailed numerical analysis using MATLAB extends the local bifurcation analysis to a global picture, namely, a family of stable periodic solutions exist in a large region of delay and "chaos switchover" phenomenon appears. Therefore, in accordance with above theoretical analysis, reasonable parameters can be designed in order to achieve various applications.

Pattern Formation in a Cross-Diffusive Schnakenberg System

Gaetana Gambino
University of Palermo, Italy
Maria Carmela Lombardo, Salvatore Lupo, Marco Sammartino

In this talk the Turing pattern formation mechanism of a two components reaction-diffusion system modeling the Schnakenberg chemical reaction is considered. Linear and nonlinear cross-diffusion terms, characterised by a gradient in the concentration of one species inducing a flux of the other chemical species, are introduced in the system. Cross-diffusion leads to the destabilization of the constant steady state and is responsible for the initiation of spatial patterns even if the diffusion constant of the inhibitor is smaller or equal to the diffusion constant of the activator.

The Turing and the Hopf instability boundaries are also determined, showing that the presence of cross-diffusion extends the range of diffusion coefficients over which Turing patterns can occur. The process of pattern formation is studied both in 1D and 2D spatial domains. Through a weakly nonlinear multiple scales analysis the equations for the amplitude of the stationary patterns are derived. The analysis of the amplitude equations shows the occurrence of a number of different phenomena, including traveling patterning waves, stable subcritical Turing patterns or multiple branches of stable solutions leading to hysteresis.

Amplitude Death and Spatiotemporal Bifurcations in a Nonlocally Delay-Coupled System

Yuxiao Guo
Harbin Institute of Technology, Peoples Rep of China

Amplitude death and spatiotemporal oscillations are remarkable patterns in coupled systems arising from biology, neuroscience, etc. We consider a ring of n identical oscillators with distance dependent couplings and time delay. The amplitude death region is the intersection of three stable regions. Employing the method of multiple scales and normal form theory, the stability and criticality of spatiotemporal oscillations are determined. Around the amplitude death boundary there exist one branch of synchronized oscillations, n-3 branches of co-existing phase-locked oscillations, n branches of mirror-reflecting oscillations, n branches of standing-wave oscillations, one branch of quasiperiodic oscillations and two branches of co-existing synchronized oscillations. It is proved that amplitude death is robust to small inhomogeneity of couplings, and the stability of synchronized or phase-locked oscillations inherits that of the individual decoupled oscillator. For the arbitrary form of coupling functions, some general results are also obtained for the thermodynamic limit. Finally, two examples are given to support the main results.

Hopf Bifurcation and Turing Instability in the Reaction-Diffusion Holling-Tanner Predator-Prey Model

Weihua Jiang
Harbin Institute of Technology, Peoples Rep of China
Xin Li, Junping Shi

In this talk, we will report our research results on Hopf bifurcation and Turing instability in the reaction - diffusion Holling-Tanner predator-prey model with Neumann boundary condition. We perform a detailed stability and Hopf bifurcation analysis and derive conditions for determining the direction of bifurcation and the stability of the bifurcating periodic solution. For partial differential equation, we consider the Turing instability of the equilibrium solutions and the bifurcating periodic solutions. Through both theoretical analysis and numerical simulations, we show the bistability of a stable equilibrium solution and a stable periodic solution for ordinary differential equation and the phenomenon that a periodic solution becomes Turing unstable for partial differential equation. We also will introduce our recent research on on codimension-two bifurcations.

Pattern Formation Close and Off Equilibrium Driven by Nonlinear Diffusion

Maria Carmela Lombardo
University of Palermo, Italy
Gaetana Gambino, Marco Sammartino

In this talk we will explore the pattern forming properties of some classical reaction-diffusion system in the presence of nonlinear diffusion and cross diffusion terms. Weakly nonlinear analysis is employed to distinguish between super- and sub-critical bifurcations and to construct solutions close to equilibrium. In subcritical regimes we shall also show the emergence of far-from-equilibrium oscillating patterns whose existence cannot be not predicted by the weakly nonlinear theory. In the final part of the talk we shall discuss, using classical methods of asymptotic analysis, how it is possible to construct significant far-from-equilibrium solution such as spots or mesa-patterns.

Two-Parameter Bifurcations in a Neutral Functional Differential Equation

Ben Niu
Harbin Institute of Technology, Peoples Rep of China

Neutral functional differential equations are often used for describing evolutionary systems, which are widely applied to lossless transmission line, ecosystem and control theory. In this talk, we mainly study two-parameter bifurcations in neutral functional differential equation, by extending the center manifold reduction method and the method of multiple scales, including Bogdanov-Takens bifurcation, Hopf-zero bifurcation and double Hopf bifurcation. We investigate van der Pol's equation with extended delay feedback as a second order neutral equation. Bifurcation sets are drawn by analyzing the roots' distribution of the characteristic equation, thus the stability domain is obtained. Universal unfoldings near Bogdanov-Takens bifurcation are obtained, thus homoclinic bifurcation and three coexisted periodic solutions are found by analyzing the normal forms. Around Hopf-pitchfork bifurcation, we find two stable coexisted quasi-periodic solutions. Near the double-Hopf bifurcation, invariant two-torus, three-torus and an attractor of Ruelle-Takens-Newhouse type are found.

The Delay Effect on Some Diffusive Population Models

Ying Su
Harbin Institute of Technology, Peoples Rep of China
Junping Shi, Junjie Wei

In this talk, we will show the existence and stability of the spatially inhomogeneous periodic solutions for some diffusive population models subject to Dirichlet boundary condition. We demonstrate that the spatially inhomogeneous periodic solutions can be bifurcated from the positive steady state for both Logistic and weak Allee type population models. For a special Logistic type model, such bifurcated periodic solutions are shown to be persistent when the parameter is far away from the bifurcation values.

Existence of Anti-Periodic Solutions for Sturm-Liouville Equations

Jiebao Sun
Harbin Institute of Technology, Peoples Rep of China
Wenjuan Yao, Shengzhu Shi, Boying Wu

This article is devoted to the anti-periodic boundary value problem for fractional Sturm-Liouville equations. By applying Schaefer's fixed point theorem, we establish the existence of anti-periodic solutions under certain nonlinear growth conditions of the nonlinearity term. Finally, we give an example to illustrate our result.

Coexistence and Competitive Exclusion in an SIS Model with Standard Incidence and Diffusion

Necibe Tuncer
Florida Atlantic University, USA
Maia Martcheva, Yixiang Wu

In this talk, we introduce a two-strain spatially explicit SIS epidemic model with space-dependent transmission parameters. We define reproduction numbers of the two strains, and show that the disease-free equilibrium will be globally stable if both reproduction numbers are below one. We also introduce the invasion numbers of the two strains which determine the ability of each strain to invade the single-strain equilibrium of the other strain. The main question that we address is whether the presence of spatial structure would allow the two strains to coexist, as the corresponding spatially homogeneous model leads to competitive exclusion.We show analytically that if both invasion numbers are larger than one, then there is a coexistence equilibrium. We devise a finite element numerical method to numerically confirm the stability of the coexistence equilibrium and investigate various competition scenarios between the strains. Finally, we show that the numerical scheme preserves the positive cone and converges of first order in the time variable and second order in the space variables.

Periodic Systems with Time Delay and Avian Influenza Dynamics

Xiang-Sheng Wang
Southeast Missouri State University, USA
Jianghong Wu

Modeling the spread of avian influenza by migratory birds between the winter refuge ground and the summer breeding site gives rise to a periodic system of delay differential equations exhibiting both the cooperative dynamics (transition between patches) and predator-prey interaction (disease transmission within a patch). Such a system has two important basic reproductive ratios, each of which being the spectral radius of a monodromy operator associated with the linearized sub-system (at a certain trivial equilibrium): the (ecological) reproduction ratio for the birds to survive in the competition of birth and natural death, and the (epidemiological) reproduction ratio for the disease to persistent. We calculate these two ratios by our recently developed finite dimensional reduction and asymptotic techniques, and we show how these two ratios characterize the nonlinear dynamics of the full system.

Hopf - Transcritical Bifurcation in Toxic Phytoplankton-Zooplankton Model with Delay

Hongbin Wang
Harbin Institute of Technology, Peoples Rep of China
Yong Wang, Weihua Jiang

In this talk, we will report our research on Hopf - transcritical bifurcation in a phytoplankton-zooplankton model with toxic liberation delay. Firstly, the critical values of Hopf bifurcation, transcritical bifurcation and Hopf-transcritical bifurcation are given, and to give more detailed information about the periodic oscillations, the direction and stability of Hopf bifurcation is studied by using the normal-form theory and center manifold theorem. Then, we give the detailed bifurcation set by calculating the universal unfoldings near the Hopf-transcritical bifurcation point. Finally, we show that the plankton system may exhibit quasi-periodic oscillations, which are verified both theoretically and numerically, and explain the experimental observed fluctuation phenomenon of plankton
population.

Hopf Bifurcation for a Partial Neutral Functional Differential Equation

Chuncheng Wang
Harbin Institute of Technology, Peoples Rep of China

A partial neutral functional differential equation involving a stable operator D is considered. Semigroup properties for the linear equation and decompostion theorem of solution opertors for the nonlinear equation are establised for this equation. The decomposition of phase space using an alternative theorem is also proved. Based on these properties, Hopf bifurcation theorem for nonlinear equation and an algorithm for computing the bifurcation properties are provided.

A PDE System Modeling the Growth of Phytoplankton with Internal Storage in a Water Column

Feng-Bin Wang
Chang Gung University, Taiwan
Sze-Bi Hsu, Linfeng Mei

In this talk, we will analyze a PDE system describing the growth of phytoplankton in a water column, where growth of species increases monotonically with the nutrient quota stored within individuals. We establish a threshold result on the global extinction and persistence of phytoplankton. Basically, condition for extinction/persistence is shown to depend on the principal eigenvalue of an eigenvalue problem, which is related to the diffusivity, the sinking speed, nutrient uptake rate, and growth rate.

Attractor of Two-Patch Brusellator Model

Yan Wang
College of William and Mary, USA
Junping Shi

Many dynamic problems in physics, chemistry, biology and other fields interest in modeling individual movement between coupled, nonlinear, discrete cells. The dynamic behavior of two coupled Brusselator reaction is described. The reactors are coupled through a permeable wall through which all species can diffuse. The initial conditions and the refuelling concentration of all reactants in each reactor are identical. The Brusselator model is the minimal mathematical model possess nonlinear oscillation. The coupled system maintain the nonlinear oscillation either in a synchronize way or asynchronize way. After the diffusion is coupled to the Brusselator chemical reactions, it can destabilize the stable symmetric equilibrium, generating four more asymmetric equilibria through two saddle-node bifurcation. Hopf bifurcation also can occurs not only on the symmetric eqilibrium but on the asymmetric ones which lead to rich bifurcation structure. The numerical simulation is carried out for various parameter values to show that diffusion-induced instability can lead to multiple steady states and oscillatory states.

Global Stability of the Predator-Prey System with Prey-Taxis

Zhian Wang
Hong Kong Polytechnic University, Hong Kong
Haiyang Jin

We consider the global boundedness and stability to the predator-prey system with prey-taxis. By Moser iteration and L^p-estimates, we show that the intrinsic interaction between predators and preys in the predator-prey system is sufficient to prevent the population overcrowding without any technical assumption on prey-taxis imposed in the existing results. Furthermore, by constructing appropriate Lyapunoval functional, we show that prey-only steady state is globally asymptotically stable if the predation is weak, and the co-existence steady state is globally asymptotically stable under some conditions (like the prey-taxis is weak or prey diffuse fast) if the predation is strong. The exponential decay rates of solutions to the steady states are also derived in our work.

Spatiotemporal Patterns of a Homogeneous Diffusive Predator-Prey System with Holling Type III Functional Response

Jinfeng Wang
Harbin Normal University, Peoples Rep of China

The dynamics of a diffusive predator-prey system with Holling type-III functional response subject to Neumann boundary conditions is investigated. The parameter region for the stability and instability of the unique constant steady state solution is derived, and the existence of time-periodic orbits and nonconstant steady state solutions are proved by bifurcation method and Leray-Schauder degree theory. The effect of various parameters on the existence and nonexistence of spatiotemporal patterns is analyzed. These results show that the impact of Holling type-III response essentially increases the system spatiotemporal complexity.

The Numerical Method for Time-Fractional Convection-Diffusion Problems with High-Order Accuracy

Boying Wu
Harbin Institute of Technology, Peoples Rep of China
Wenjuan Yao, Jiebao Sun

In this paper, we consider the numerical method for solving the two-dimensional fractional convection-diffusion equation with a time fractional derivative of high order accuracy.

Global Existence of Solutions and Uniform Persistence of a Diffusive Predator-Prey Model with Prey-Taxis

Sainan Wu
Harbin Institute of Technology, Peoples Rep of China
Junping Shi, Boying Wu

We prove the global existence and boundedness of solutions to a general reaction-diffusion predator-prey system with prey-taxis defined on a smooth bounded domain with no-flux boundary condition. The result holds for domains in arbitrary spatial dimension and small prey-taxis sensitivity coefficient. We also prove the existence of a global attractor and the uniform persistence of the system under some additional conditions. Applications to models from ecology and chemotaxis are discussed.

Traveling Pulses in a Lateral Inhibition Neural Network

Aijun Zhang
Drexel University, USA
Yixin Guo

We study the spatial propagating dynamics in a neural network of excitatory and inhibitory populations. Our study demonstrates the existence and nonexistence of traveling pulse solutions with a nonsaturating piecewise linear gain function. We prove that traveling pulse solutions do not exist for such neural field models with even(symmetric) couplings. The neural field models only support traveling pulse solutions with asymmetric couplings. We also show that such neural field models with asymmetric couplings will lead to a system of delay differential equations. We further compute traveling 1-bump solutions using the system of delay differential equations. Finally, we develop Evans functions to assess the stability of 1-bump traveling pulse solutions.

Invariant Manifolds for Impulsive Equations and Nonuniform Polynomial Dichotomies

Jimin Zhang
Heilongjiang University, Peoples Rep of China
Luis Barreira, Meng Fan, Claudia Valls

For impulsive differential equations in Banach spaces, we construct stable and unstable invariant manifolds for sufficiently small perturbations of a polynomial dichotomy. We also consider the general case of nonuniform polynomial dichotomies. Moreover, we introduce the notions of polynomial Lyapunov exponent and of regularity coefficient for a linear impulsive differential equation, and we show that when the Lyapunov exponent never vanishes the linear equation admits a nonuniform polynomial dichotomy.

Stability, Bifurcation and Pattern Formation of a Diffusive Population Model for Pioneer-Climax Species

Xingfu Zou
University of Western Ontario, Canada
Ying Su

We consider a general diffusive model that describes the interaction of pioneer and climax species in a bounded domain subject to the zero-flux boundary condition is considered. Among the results are local and global steady state bifurcations, Hopf bifurcations, as well as conditions for Turing instability leading to the formation of spatial patters. We will also present some numerical simulation results that can visualize and extend the analytic results. These results suggest that diffusion may make the climax species more dominant.

Traveling Wave Solutions of a Diffusive Ratio-Dependent Holling-Tanner System with Distributed Delay

Wenjie Zuo
China University of Petroleum, Peoples Rep of China
Junping Shi

The paper is devoted to the existence of traveling wave solutions of a diffusive ratio-dependent predator-prey system with distributed delay. For the case without distributed delay, the existence of the minimal wave speed and traveling wave front solutions are established by using the comparison principle and upper/lower solutions method. Secondly the existence of periodic traveling wave train solutions are shown using Hopf bifurcation theory with the diffusive coefficient as a bifurcation parameter. For the case with distributed delay, the existences of traveling wave solutions and traveling wave trains are proved via geometric singular perturbation theory when the mean delay is sufficiently small.

Special Session 50: Transition Dynamics of Parabolic Type Equations

Wan-Tong Li, Lanzhou University, Peoples Rep of China
Guo Lin, Lanzhou University, Peoples Rep of China
Zhi-Cheng Wang, Lanzhou University, Peoples Rep of China

This special session focuses on the transition dynamics of parabolic type equations including reaction-diffusion equations, lattice differential equations, integrodifference equations, nonlocal dispersal equations, etc. Due to the backgrounds of these equations in life sciences and physics as well as the importance in mathematical literature, we pay attention to the dynamics formulated by traveling wave solutions, entire solutions, asymptotic spreading and some generalized transition modes. This session will try to present as much information as possible and offer opportunities for potential cooperation.

Solvability and Numerical Schemes for Non-Local Kolmogorov Forward Equations in L1 Space

Linghua Chen
Norwegian University of Science and Technology, Norway
Espen Robstad Jakobsen, Arvid Naess

In this talk we present new results for non-local Kolmogorov Forward equations with unbounded coefficients: solvability and the strong convergence of numerical schemes in L1 space. The solution of such equations corresponds to the evolution of probability density functions of non-linear stochastic differential equations (SDE) driven by non-Gaussian noise. The latter has a large number of applications in various areas - including physics, economics, and finance. Existence and uniqueness of a mild solution is derived. On the numerical side, we study the so-called discrete path integration method which produces approximate probability density functions for the solutions of corresponding SDEs. We prove that this scheme strongly converges in L1 norm, uniformly in any finite time horizon. Specifically, we use the concept of dissipative operators, combined techniques from semigroup and PDE theory, as well as methods from stochastic analysis.

Entire Solutions in a Nonlocal Dispersal Epidemic Model

Wan-Tong Li
Labzhou University, Peoples Rep of China

This talk is concerned with entire solutions of a nonlocal dispersal epidemic model. Unlike local dispersal problems, a nonlocal dispersal operator is not compact and the solutions of nonlocal dispersal system studied here lack regularity in suitable spaces, which affects the uniform convergence of the solution sequences. A key idea is to characterize the asymptotic behaviors of the traveling wave solutions at infinite. This is the joint work with Li Zhang and Shi-Liang Wu.

Persistence and Spreading Speeds of a Spatial Model with an Expanding Or Contracting Habitat

Bingtuan Li
University of Louisville, USA
Sharon Bewic, Michael R. Barnard, William F. Fagan

We discuss a spatial model that describes the spatial dynamics of a species in an expanding or contracting habitat. We give conditions under which the species disperses to a region of poor quality where the species eventually becomes extinct. We show that when the species persists in the habitat, the rightward spreading speed and leftward spreading speeds are determined by c, the speed at which the habitat quality increases or decreases in time, as well as the dispersal kernel and species growth rates in both directions. We demonstrate how c affects the spreading speeds. We also show that it is possible for a solution to form a two-layer wave, with the propagation speeds of the two layers analytically determined.

Persistence and Failure of Complete Spreading in Delayed Reaction-Diffusion Equations

Guo Lin
Lanzhou University, Peoples Rep of China
Shigui Ruan

This talk deals with the long time behavior in terms of complete spreading for a population model described by a reaction-diffusion equation with delay, of which the corresponding reaction equation is bistable. When a complete spreading occurs in the corresponding undelayed equation with initial value admitting compact support, it is proved that the invasion can also be successful in the delayed equation if the time delay is small. To spur on a complete spreading, the choice of the initial value would be very technical due to the combination of delay and Allee effects. In addition, we show the possible failure of complete spreading in a quasimonotone delayed equation to illustrate the complexity of the problem.

Existence and Continuous Dependence of Random Impulsive Neutral Stochastic Functional Integro Differential Evolution systems

Tongyi Ma
Hexi University, Zhangye, Gansu, Peoples Rep of China

In this talk, we establish the results by using a fixed point analysis approach. The results improve some recent results.

Traveling Wave Solutions of Nonmonotone Delayed Systems with Dispersal

Shuxia Pan
Lanzhou University of Technology, Peoples Rep of China

In this talk, I shall present some results of traveling wave solutions for nonlocal dispersal systems with time delay. When the system is not monotone and time delay is not small, we investigate the existence of traveling wave solutions by generalized upper and lower solutions. To establish the asymptotic behavior, we combine the contracting rectangle with the asymptotic spreading. These results are applied to competitive and predator-prey systems to obtain the minimal wave speed.

Time Periodic Traveling Waves for a Periodic and Diffusive SIR Epidemic Model

Zhi-Cheng Wang
Lanzhou University, Peoples Rep of China

In this paper, we study the time periodic traveling wave solutions for a periodic SIR epidemic model with diffusion and standard incidence. We establish the existence of periodic traveling waves by investigating the fixed points of a nonlinear operator defined on an appropriate set of periodic functions. Then we prove the nonexistence of periodic traveling via the comparison arguments combined with the properties of the spreading speed of an associated subsystem. This is a joint work with Dr. Liang Zhang and Professor Xiao-Qiang Zhao.

Special Session 51: Advances in Population Dynamics and Epidemiology

Necibe Tuncer, Florida Atlantic University, USA
Maia Martcheva, University of Florida, USA
Hayriye Gulbudak, Georgia Institute of Technology, USA

This special session will bring together researchers, who are experts and at the heart of the areas of mathematical biology, and students to share ideas and methods for mathematical modeling, analysis, and simulations related to biological systems. It will allow for discussion of pressing topics and exchange of novel ideas. It is expected that the session will lead to the development of mathematical theory for biological dynamics. Potential topics of interest include population dynamics, pathogen-immune system dynamics, and infectious disease epidemiology. The purpose of the special session is to address recent advances of mathematical methods used to study these focal areas and discuss a wide range of topics: computational, mathematical and statistical.

Modelling Insect Population Dynamics: the Physiologically-Structured Approach Applied to Laboratory Data on Drosophila Suzukii

Souvik Bhattacharya
University of Florida, India

Developmental and demographic rates of an Italian population of Drosophila suzukii were measured in the laboratory in three separate experiments: under optimal temperature and humidity conditions; at low temperatures, following or not a freeze exposure; at low humidity levels. The results were similar but not identical to those already obtained for Oregon populations (Dalton et al., 2011, Tochen et al., 2014, 2015). On the basis of these data as well of those already published, we developed a physiologically structured population model; parameter estimation yielded a most parsimonious model where all traits are affected by temperature according to simple polynomial functions, while humidity affects only adult mortality and fruit-diet only fecundity. The model produces a reasonably good fit of available laboratory data, while differences among populations and experiments prevent a perfect fit. The model has also been tested with actual field temperatures in two different years in three nearby sites; this test highlights the relevance of small temperature differences for the potential population growth. Use of the model with field data will require adequate modelling of resource availability, and of insect movement, in order to build a useful tool for predicting and managing D. suzukii infestations

Virus-Immune Dynamics in Age-Structured HIV Model

Cameron Browne
University of Louisiana at Lafayette, USA

Mathematical modeling of viruses, such as HIV, has been an extensive area of research over the past two decades. For HIV, some important factors that affect within-host dynamics include: the CTL (Cytotoxic T Lymphocyte) immune response, intra-host diversity, and heterogeneities of the infected cell lifecycle. Motivated by these factors, I consider extensions of a standard within-host virus model. In a cell infection-age structured PDE model with multiple virus strains, I show that competitive exclusion occurs. I also investigate the effect of CTL immune response acting at different times in the infected-cell lifecycle based on recent studies demonstrating superior viral clearance efficacy of certain CTL clones that recognize infected cells early in their lifecycle. Interestingly, explicit inclusion of early recognition CTLs can induce oscillatory dynamics and promote coexistence of multiple distinct CTL populations. Finally, I discuss several directions of ongoing modeling work attempting to capture complex HIV-immune system interactions suggested by experimental data.

Containing Emerging Epidemics: a Quantitative Comparison of Quarantine and Symptom Monitoring

Lauren Childs
Harvard Chan School of Public Health, Virginia Tech, USA
Corey Peak, Yonatan Grad, Caroline Buckee

The containment of emerging epidemics of infectious diseases like Ebola or SARS requires the rapid deployment of interventions, which may require nonpharmaceutical strategies such as isolation because drugs or vaccines do not yet exist. Two strategies have historically been considered to accelerate the isolation of a potentially infected contact: complete quarantine of potentially infected contacts, or intermittent monitoring of contacts, followed by quarantine if and when symptoms occur. Complete quarantine is a highly conservative approach to epidemic containment, but there are diverse associated costs, ranging from implementation expenses and restriction of personal liberties, to less obvious impacts like the stigmatization of health workers, a reluctance to comply with health orders, and interruption of financial and trade markets. Here, we develop a mathematical model to compare the performance of symptom monitoring and quarantine containing an emerging infectious disease. We consider case studies of seven known pathogens with a wide range of natural histories that have the potential for causing sudden, severe epidemics and even potentially pandemics. We

identify which disease characteristics and intervention attributes are most critical for judging between quarantine and symptom monitoring, and provide a clear, general framework for understanding the consequences of isolation policies during an epidemic.

Global Stability of Age-Structured Population Models

Daniel Franco
Univ. Nacional de Educaci'on a Distancia (UNED), Spain
Juan Peran

During this talk, we will discuss recent sufficient conditions for the existence of a globally asymptotically stable non-zero equilibrium of certain nonlinear discrete-time population models, describing the dynamics of age-structured species.

From Ecology to Evolution of Host Ajd Vector-Bornepathogen in a Structured Immuno-Epidemiological Model

Hayriye Gulbudak
Georgia Institute of Technology, USA
Vincent Cannataro, Necibe Tuncer, Maia Martcheva

The interaction of a pathogen and a host's immune system governs the pathogen's transmission potential between hosts. After a host is infected, the pathogen population grows inside the host, triggering an immune response of pathogen-specific antibodies, which help clear the infection. Pathogen and antibody dynamics are often monitored in laboratory experiments and modeling their interaction may inform our understanding of disease spread. In this study, we formulate a novel immunological model to capture the within-host dynamics of vector-borne diseases and link it to a vector-host age-since-infection structured epidemiological model. By incorporating within-host pathogen and immune response dynamics, we are able to capture the het erogeneity in infectivity of infected individuals and analyze how this heterogeneity scales up and influences population-level dynamics. On the evolutionary scale, we analytically derive host and parasite fitness functions depending on intra-host pathogen-immune response antibody dynamics. By using numerical schemes, we study host and pathogen evolutionary trajectory, along with the effect of tradeoff functions and vector inoculum load on the coevolutionary attractor. This nested modeling approach provides a tool to study the effect of immune-pathogen interactions on the epidemiological threshold and disease prevalence, which may give important insights on control strategies such as drug treatment and vaccination. In addition, it offers a compelling method for investigating virus-host evolution, which is an important topic in emergence of new viruses, virulence and emerging outbreaks.

Backward Bifurcations in Models for Disease Dynamics

Abba Gumel
Arizona State University, USA

Backward bifurcation, a dynamic phenomenon associated with the coexistence of a stable disease-free equilibrium and a stable endemic equilibrium when the reproduction threshold of the model is less than unity, has been shown to arise in a number of epidemiological settings. The presence of such bifurcation in the transmission dynamics of a pathogen in a community makes its effective community-wide control difficult (since, in such a backward bifurcation situation, bringing the reproduction number of the model to a value less than unity, while necessary, is no longer sufficient for such effective control). This presentation discusses some of the common and new causes of backward bifurcations, as well as their governing mechanisms, in disease transmission dynamics.

Dynamical Models of Task Organization in Social Insect Colonies

Yun Kang
Arizona State University, USA
Guy Theraulaz

The organizations of insect societies, such as division of labor, task allocation, collective regulation, mass action responses, have been considered as main reasons for the ecological success. In this article, we propose and study a general modeling framework that includes the following three features: (a) the average internal response threshold for each task (the internal factor); (b) social network communications that could lead to task switching (the environmental factor); and (c) dynamical changes of task demands (the external factor). Since workers in many social insect species exhibit age polyethism, we also extend our model to incorporate age polyethism in which worker task preferences change with age. We apply our general modeling framework to the cases of two task groups: the inside colony task versus the outside colony task. Our analytical study of the models provides important insights and predictions on the effects of colony size, social communication, and age related task preferences on task allocation and division of labor in the adaptive dynamical environment. Our study implies that the smaller size colony invests its resource for the colony growth and allocates more workers in the risky tasks such as foraging while the larger colony shifts more workers to perform the safer tasks inside the colony. Social interactions among different task groups play an important role in shaping task allocation depending on the relative cost and demands of the tasks.

Structural and Practical Identifiability Issues of Outbreak Models

Trang Le
University of Tulsa, USA
Necibe Tuncer

Structural identifiability in epidemic models is a crucial issue because unreliable estimates of parameters could result in inaccurate estimates of important epidemiological values such as the basic reproduction number. We performed structural identifiability analysis on various epidemic models using a differential algebra approach to investigate the characteristics of these models. It is necessary to note that a model which is structurally identifiable may not be practically so. Furthermore, we carried out practical identifiability analysis on these models using Monte Carlo simulations and sensitivity based analysis.

Estimating the Risk of Disease Outbreak

Evan Milliken
University of Florida, USA
Sergei S. Pilyugin

Infectious Salmon Anemia (ISA) virus (ISAV) is an Orthomyxovirus which causes ISA, a disease which has caused increased mortality in Atlantic salmon, Salmon salar L. ISA has had a substantial impact on the aquaculture industry in all major salmon producing countries. The disease can be transmitted directly from fish to fish and indirectly from contaminated sea water. In a salmon farm, salmon movement is restricted within individual net cages, but contaminated sea water can move from cage to cage. Deterministic and stochastic models are presented for the disease in a single patch, in two patches when fish occupy only one patch and in two patches when fish occupy both patches. Analytical and numerical methods are combined to approximate the risk of outbreak in the stochastic models. Results of approximation techniques are compared to numerical simulations. In each case, the threshold for the extinction of the virus in the stochastic model is compared to the threshold for the associated deterministic model.

Generation of Plasmodium Falciparum Parasite Diversity Within Mosquito

Olivia Prosper
University of Kentucky, USA
Lauren Childs

Plasmodium falciparum, the malaria parasite causing the most severe disease in humans, undergoes an asexual stage within the human host, and a sexual stage within the vector host, *Anopheles* mosquitoes. Because mosquitoes may be superinfected with parasites of different genotypes, this sexual stage of the parasite life-cycle presents the opportunity to create genetically novel parasites. To investigate the role that mosquitoes' biology plays on the generation of parasite diversity, which introduces bottlenecks in the parasites' development, we first constructed a stochastic model of parasite development within-mosquito, generating a distribution of parasite densities at five parasite life-cycle stages: gamete, zygote, ookinete, oocyst, and sporozoite, over the lifespan of a mosquito. We then coupled a model of sequence diversity generation via recombination between genotypes to the stochastic parasite population model. With this modeling framework, we demonstrate that parasite diversity increases at the population level as a consequence of the stage transitions within-mosquito, which has important implications for disease-transmission and control.

Computing Human to Human Avian Influenza $\mathcal{R}_0$ Via Transmission Chains and Parameter Estimation

Omar Saucedo
University of Florida, USA
Maia Martcheva, Juliet Pulliam

The transmission of avian influenza between humans is extremely rare, and it mostly affects individuals who are in contact with infected poultry. Although this scenario is uncommon, there have been multiple outbreaks that occur in small infection clusters in Asia with relatively low transmissibility, and thus are too weak to cause an epidemic. Still, subcritical transmission from chain data is vital for determining whether avian influenza is close to the threshold of $\mathcal{R}_0 > 1$. In this talk, we will explore two methods of computing $\mathcal{R}_0$ using transmission chains and parameter estimation via data fitting.

Coupled Infectious Disease Model Via Human Movement

Zhisheng Shuai
University of Central Florida, USA

Many recent outbreaks and spatial spread of infectious diseases have been influenced by human movement over air, sea and land transport networks. Mathematical models can be used to investigate the effects of human movement on disease dynamics. Models that explicitly incorporate human movement are commonly called multi-patch models or Lagrangian models. A classic multi-patch SIR model will be revisited incorporating nonlinear incidence transmission which can be used to model large classes of infectious diseases. Dynamics of the model system will be investigated both analytically and numerically, providing interesting dynamic phenomena and biological insights.

Structural and Practical Identifiability Issues of Immuno-Epidemiological Vector-Host Models with Application to Rift Valley Fever

Necibe Tuncer
Florida Atlantic University, USA
Maia Martcheva, Hayriye Gulbudak, Vincent Cannataro

In this talk, I will discuss the structural and practical identifiability of a nested immuno-epidemiological model of arbovirus diseases, where host-vector transmission rate, host recovery, and disease induced death rates are governed by the within-host immune system. We fit multi-scale models to multi-scale data. For an immunological model we use Rift Valley Fever Virus (RVFV) time-series data obtained from livestock under laboratory experiments and for an epidemiological model we incorporate the human compartment to the nested model and use the number of human RVF cases reported by the CDC during the 2006 − 2007 Kenya outbreak. We show that the immunological model is not structurally identifiable for the measurements of time-series viremia concentrations in the host. Thus, we study the nondimensionalized and scaled versions of the immunological model and prove that both are structurally globally identifiable. After fixing estimated parameter values for the immunological model derived from the scaled model, we develop numerical methods to fit observable RVF epidemiological data to the nested model for the remaining parameter values of the multi-scale system. For the given (CDC) data set, numerical studies indicate that only two parameters of the epidemiological model are identifiable when the immune model parameters are fixed. Alternatively, we fit multi-scale data to multi-scale model simultaneously. Numerical results for simultaneous fitting suggest that the parameters of the immunological model and the parameters of the immuno-epidemiological model are not identifiable. We suggest that analytic approaches for studying the structural identifiability of nested models is a necessity, so that identifiable parameter combinations of the parameters can be derived to reparameterize the nested model to obtain an identifiable one. This is a crucial step in developing multi-scale models which explain multi-scale data.

Coexistence and Competitive Exclusion in an SIS Model with Standard Incidence and Diffusion

Yixiang Wu
University of Western Ontario, Canada
Necibe Tuncer, Maia Martcheva

In this talk, we present a two strain SIS model with diffusion, spatially heterogeneous coefficients of the reaction part and distinct diffusion rates of the separate epidemiological classes. First, it is shown that the model has bounded classical solutions. Next, it is established that the model with spatially homogeneous coefficients leads to competitive exclusion and no coexistence is possible in this case. Furthermore, it is proved that if the invasion number of strain j is larger than one, then the equilibrium of strain i is unstable; if, on the other hand, the invasion number of strain j is smaller than one, then the equilibrium of strain i is neutrally stable. In the case when all diffusion rates are equal, global results on competitive exclusion and coexistence of the strains are established. Finally, evolution of dispersal scenario is considered and it is shown that the equilibrium of the strain with the larger diffusion rate is unstable. Simulations suggest that in this case the equilibrium of the strain with the smaller diffusion rate is stable.

Special Session 52: Function Spaces and Inequalities

Annamaria Barbagallo, Naples University "Federico II", Italy
Sorina Barza, Karlstad University, Sweden
Maria Alessandra Ragusa, University of Catania, Italy

The aim of this Section is to study the relations between some of the main used spaces in Mathematical Analysis: Lebesgue Spaces, Orlicz Spaces, Lorentz Spaces, Sobolev Spaces, Morrey Spaces. These classes and the their relations can be used to study regularity properties of solutions of partial differential equations and variational inequalities.

Semigroup C* - Crossed Products of Lattice-Orderedgroups and Their Ideal Structure

Mamoon Ahmed
Princess Sumaya University for Technology, Jordan

Let (G, G_+) be a quasi-lattice-ordered group with positive cone G_+ and H_+ a hereditary subsemigroup of G_+. Adji and Raeburn have shown a result about the structure of the primitive ideal space of the C^*-algebra $B_{G_+} \times_\alpha G_+$ for a totally ordered abelian group. In this paper we extend their result to this more general setting. In particular we show that if $\sum(G)$ is the set of subgroups $H := H_+ - H_+$ partially ordered by inclusion, then there exists a well-defined map F from the disjoint union $\bigsqcup\{\widehat{H} : H \in \sum(G)\}$ to the primitive ideals of the Toeplitz algebra $B_{G_+} \times_\alpha G_+$. This allows us to deduce information about the irreducible representations of the C^*-crossed product $B_{G_+} \times_\alpha G_+$.

A General Quasi-Variational Inequality for Cournot-Nash Principle and Inverse Formulation

Annamaria Barbagallo
Unuversity of Naples Federico II, Italy
Paolo Mauro

The talk deals with the study of an evolutionary quasi-variational inequality, which expresses the equilibrium conditions of a general oligopolistic market equilibrium model, and to present its inverse formulation. More precisely, the model allows the presence of capacity constraints, production and demand excesses and, moreover, both the production and demand functions are considered depending on the forecasted equilibrium distribution. As a consequence, the capacity constraint set is defined by a multivalued function. The equivalence between the dynamic elastic Cournot-Nash equilibrium principle and an evolutionary quasi-variational inequality is proved. Furthermore, we provide the analysis of existence, regularity and sensitivity of solution. Finally, the behavior of control policies for our problem, whose aim is to regulate the exportation through the adjustment of taxes on the firms, is analyzed.

Factorizations of Weighted Cesaro and Copson Spaces

Sorina Barza
Karlstad University, Sweden

Any inequality expressing the boundedness of a linear operator on a Banach space may be interpreted as an inclusion between two spaces. One way of improving the inequality is to replace the smaller space by a larger space such that the best constant in the original inequality remains the same. The product between the original space and the space of all multipliers from the smaller space to the bigger one will satisfy the above requirements. In some concrete cases the space of multipliers can be described and the inclusion becomes actually an indentity. The first inequality, produced in this way, is an enhancement of the original operator inequality, while the reversed inclusion replaces the operator inequality by an equality. Based upon this factorization we get also a renorming of the bigger space. We present such factorizations for some weighted spaces of functions, namely Cesaro and Copson, generalizing in this way previous results proved by Graham Bennet for classical spaces of sequences.

Gradient Regularity for Solutions of a Class of Nonlinear Elliptic Equations with a Degenerating Lower Order Term.

Salvatore D'Asero
Università di Catania, Italy
P. Cianci, G.R. Cirmi, S. Leonardi

We consider the following prototype problem:

$$\begin{cases} -\Delta u + M\dfrac{|\nabla u|^2}{|u|^\theta} = f & \text{in}\Omega \\ u = 0 & \text{on } \partial\Omega \end{cases},$$

and we study the regularity of the gradient of a solution both in Morrey spaces and in fractional Sobolev spaces, in correspondence of the regularity of the right-hand side.

Duality for A^∞ Weights on the Real Line

Luigi D'Onofrio
Napoli Parthenope University, Italy

The aim of the talk is to describe the relation between the notion of Bi-Sobolev function of the real line and the Gehring and Muckenhoupt classes of weight. To do this we give explicit formulas for the sharp transition exponents and sharp improvement exponents of these classes

The Brezis-Nirenberg Problem: Recent Results on Sign-Changing Solutions

Alessandro Iacopetti
University of Torino, Italy

In this talk we show some recent results about sign-changing solutions for the Brezis-Nirenberg problem, which is the following semilinear elliptic problem:

$$\begin{cases} -\Delta u = \lambda u + |u|^{p-1}u & \text{in}\Omega \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded smooth domain of $\mathbb{R}^N$, $N \geq 3$, λ is a positive parameter, while $p+1 = \frac{2N}{N-2}$ is the critical Sobolev exponent for the embedding of $H_0^1(\Omega)$ into $L^{p+1}(\Omega)$. In 1990 Atkinson-Brezis-Peletier pointed out some interesting phenomena related to the existence or nonexistence of radial sign-changing solutions for the Brezis-Nirenberg problem in the ball, in the low dimensions $N = 4, 5, 6$. From their results several questions arose, in particular concerning the presence of analogous situations in other domains. A related question about the existence or not, in general domains, of sign-changing solutions whose positive and negative part concentrate at the same point, raised from later papers of Ben Ayed-El Mehdi-Pacella. We will show recent contributions on these questions, proving, in particular, some existence and nonexistence results in general bounded domains, possibly with some symmetry.

Pointwise Convergence of Fourier Series Near L^1

Victor Lie
Purdue University, USA

In our talk we will discuss the old and celebrated question regarding the pointwise behavior of Fourier Series near L^1. This presentation will include: 1) the resolution of Konyagin's conjecture (ICM, Madrid 2006) on the pointwise convergence of Fourier Series along lacunary subsequences; 2) the L^1-strong convergence of Fourier Series along lacunary subsequences. We end with several considerations on the relevance/impact of the above two items on the subject of the pointwise convergence of Fourier Series.

Sublinear Operators in Weighted Grand Morrey Spaces

Alexander Meskhi
A. Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, Rep of Georgia

The boundedness of sublinear integral operators in grand Morrey spaces defined by means of measures generated by the Muckenhoupt weights is derived. The operators under consideration involve operators of Harmonic Analysis such as Hardy-Littlewood and fractional maximal operators, Calderòn-Zygmund operators, potential operators etc. The same problem for commutators of singular and fractional integral operators is also studied. This research is a continuation of the investigation carried out by the speaker jointly with V. Kokilashvili and H. Rafeiro in the unweighted case (see e.g., Chapter 16 of the forthcoming monograph V. Kokilashvili, A. Meskhi, S. Samko, and H. Rafeiro, Integral operators in non-standard function spaces. Volumes I and II, Birkhäuser, 2016).

Differentiability Results Concerning the Minima of Nondifferentiable Functionals

Maria Alessandra Ragusa
Catania University, Italy

The author shows some results obtained in cooperation with Atsushi Tachikawa. We study Hölder regularity for minimizers of functionals having more than quadratic growth and discontinuous coefficients. Starting with the well-known results by Giaquinta, Giusti and Modica , the direct approach was introduced. Later the study made by Giaquinta and Giusti and through the work of many authors, among others Huang, Danecek and Viszus was developed a theory of partial regularity of solutions of minimizers of variational integrals in vector valued case, using the direct method.

A Regularity Result for Solutions of Some P.D.E. with Discontinuous Coefficients

Andrea Scapellato
University of Catania, Italy
Maria Alessandra Ragusa

The author shows some conditions to obtain estimates for singular and fractional integral operators on Generalized Morrey Spaces. The boundedness of the operators in this kind of spaces is finalized to obtain regularity results for solutions of partial differential equations of elliptic type.

Lusin (N) Condition and The Distributional Determinant

Roberta Schiattarella
Università degli Studi di Napoli Federico II, Italy
Luigi D'Onofrio- Stanislav Hencl-Jan Maly

Let$\Omega \subset \mathbb{R}^n$ be an open set. We show that for a continuous mapping $f \in W^{1,n-1}(\Omega, \mathbb{R}^n)$ with $J_f \in L^1(\Omega)$ the validity of the Lusin (N) condition implies that the distributional Jacobian equals to the pointwise Jacobian.

Approximation of Smooth Functions on $[1, \infty)$ and $\mathbb{R} \setminus (-1, 1)$ by Entire Functions of Exponential Type

Alexander Tovstolis
University of Central Florida, USA

In 1946, Sergey M. Nikolskĭi discovered an effect of better pointwise approximation of a smooth function by algebraic polynomials. Namely, for a function from Sobolev class $W^1[-1, 1]$, there is a sequence of algebraic polynomials $\{p_n\}_{n=0}^{\infty}$, $p_n \in \mathbb{R}_n[x]$, such that

$$|f(x) - p_n(x)| \leq \frac{\pi}{2} \frac{\sqrt{1-x^2}}{n+1} + O\left(\frac{\ln(n+2)}{(n+1)^2}\right).$$

The constant $\pi/2$ in the first term cannot be improved.
There are several generalizations of this result. The most recent one is due to Roald M. Trigub (1993), where the asymptotically sharp estimate was obtained for the $W^r[-1, 1]$ class. We will focus on pointwise approximation of a function from the Sobolev class $W^r(\mathbb{R} \setminus (-1, 1))$ by entire functions of exponential type at most σ. Known estimates of such approximation is due to Ju. A. Brudnyĭ (1959). Our goal is to obtain an asymptotically sharp estimate. As in the Trigub's article, we deduce our estimates from corresponding result on the uniform approximation. In our case, this is the Akhiezer's theorem on uniform approximation (on $\mathbb{R}$) of a function from $W^r(\mathbb{R})$ class by entire functions of exponential type σ.

Mean Field Equations with Probability Measure in 2D-Turbulence

Gabriella Zecca
University of Naples Federico II, Italy

We study the mean field equation derived by Neri [C. Neri, Ann. Inst. H. Poincarè Anal. Non Linèaire (2004)] in the context of the statistical mechanics description of 2D-turbulence, under a stochastic assumption on the vortex circulations. The corresponding mathematical problem is a nonlocal semilinear elliptic equation with exponential type nonlinearity, containing a probability measure $\mathcal{P} \in \mathcal{M}([-1, 1])$ which describes the distribution of the vortex circulations. Unlike the more investigated "deterministic version, we prove that Neri's equation may be viewed as a perturbation of the widely analyzed standard mean field equation, obtained by taking $\mathcal{P} = \delta_1$. In particular, in the physically relevant case where $\mathcal{P}$ is non-negatively supported and $\mathcal{P}(\{1\}) > 0$, we prove the mass quantization for blow-up sequences. We apply this result to construct minimax type solutions on bounded domains in $\mathbb{R}^d$ and on compact 2-manifolds without boundary. Those results are obtained in collaboration with T. Ricciardi

Special Session 53: Interface Dynamics and Transport Phenomena

Tatiana Savina, Ohio University, USA
Alexander Nepomnyashchy, Technion - Israel Institute of Technology, Israel

The goal of this session is to bring together mathematicians and physicists studying different processes related to the surface dynamics and transport phenomena, including free boundary problems and anomalous diffusion.

Non-Analytic Reconstruction of the Vortex Core in Bosonic Superfluids, and Its Implications on the Vortex Dynamics and the instability of Abrikosov's lattice

Oded Agam
The Hebrew University, Israel

We analyze the motion of quantum vortices in a two-dimensional spinless superfluid within Popov's hydrodynamic description. In the long healing length limit (where a large number of particles are inside the vortex core) the superfluid dynamics is shown to be described by weak solutions of the Gross-Pitaevskii equation. We solve the resulting equations of motion for a vortex moving with respect to the superfluid and find the reconstruction of the vortex core to be a non-analytic function of the force applied on the vortex. As a result, the spectrum associated with the vortex motion exhibits narrow resonances lying within the phonon part of the spectrum, contrary to traditional view. The implication of this nonanalytic core reconstruction to the disordered limited motion of vortices and the instability of Abrikosov's lattice is discussed.

Free Boundary Problems in Compressible Vortex Flows

Darren Crowdy
Imperial College London, England
Vikas Krishnamurthy

The usefulness of conformal mapping techniques in solving Laplacian-growth problems (e.g. the Hele-Shaw problem) where the governing field equation is harmonic is well-known. It has led to the discovery of broad classes of analytical solutions to such free boundary problems. But conformal mapping methods can also be useful in solving free boundary problems that are not conformally invariant, or where the governing field is not harmonic. These mathematical ideas will be showcased in the application to vortex dynamics for weakly compressible flows where new analytical solutions for physically important flow scenarios, such as compressible von Karman vortex streets, have recently been uncovered.

Gibbs-Thomson Equation for the Rapidly Moving Interfaces

Peter Galenko
University of Jena, Germany

Using a phase-field model for fast phase transformations an interface condition for the rapidly moving solid-liquid interface is obtained. The model is described by equations for the hyperbolic transport and fast interface dynamics, which are reduced to a sole equation of the phase field with the driving force given by deviations of temperature and concentration from their equilibrium values within the diffuse interface. It is shown that the obtained interface condition presents the acceleration- and velocity-dependent Gibbs-Thomson interfacial condition. This condition is identical to the advanced Born-Infeld equation for the hyperbolic motion by mean curvature with the driving force. As a limiting case, the interface condition presents "velocity-driving force relationships found earlier as traveling wave solutions for slow and fast phase field profiles. Predictions of the analytical solutions are qualitatively compared with literature data of atomistic simulations on crystal growth kinetics.

Continuum Mean-Field Theory for Transport in Highly Concentrated Electrolytes

Nir Gavish
Technion - IIT, Israel
Doron Elad, Arik Yochelis

The Poisson-Nernst-Planck (PNP) theory is one of the most widely used analytical methods to describe electrokinetic phenomena for electrolytes. The model, however, considers isolated charges and thus is valid only for dilute ion concentrations. The key importance of concentrated electrolytes in applications has led to the development of a large family of generalized PNP models. In particular, the Bikerman model that takes into account the finite size of the ions has been one of the most commonly used extension to PNP. In this talk, we derive a thermodynamically consistent mean-field model for concentrated solutions. Our model recovers the Bikerman term, but shows that it is inconsistent in the sense that additional terms of equal magnitude should be taken into account. Furthermore, our study shows that the Bikerman approach inherently fails to describe finite-size effects at the highly concentrated regime, and presents a supplementary approach in this regime. The result is a modelling framework that is valid over the whole range of concentrations - from dilute electrolyte solutions to highly concentrated solution, such as ionic liquids. Importantly, the new model predicts distinct transport properties which are not governed by Einstein-Stokes relations, but are rather effected by inter-diffusion and emergence of nano-structure.

Global Aspects of a Free Boundary Problem for the Laplace Operator

Lavi Karp
ORT Braude College, Israel

The talk will consider free boundary problems that arise from Hele-Shaw flows and unbounded quadrature domains. We will show how to construct solutions with arbitrary growth near infinity and discuss conditions which guarantee a quadratic growth. The last one is crucial to the instigation of the geometric properties of the free boundaries. Another aspect is the asymptotic behavior of the free boundaries, while in the two dimensional case there is a satisfactory description of it, in higher dimensions the problem is wide open.

Anomalous Diffusion in a System with a Thin Membrane

Tadeusz Kosztolowicz
Jan Kochanowski University, Kielce, Poland
Katarzyna D. Lewandowska

We consider anomalous diffusion in a system which consists of two media separated by a thin partially permeable membrane. In each part of the system, bounded by the membrane, there may be different kinds of diffusion (normal diffusion, subdiffusion, slow subdiffusion). We derive the Green's functions using a simple random walk model with both discrete time and spatial variables. Next we move to the continuous variables. The obtained Green's functions are used to derive a boundary condition at the membrane. It is shown that the boundary condition contains a specific term which can be interpreted as a 'memory term' depending on kinds of diffusion occurring in the system.

Discontinuity in the Behavior of Mother Body and 2D Orthogonal Polynomial

Seung-Yeop Lee
University of South Florida, USA
Meng Yang

We consider the two-dimensional orthogonal polynomials whose zeros are conjectured to converge on the mother body, a certain potential theoretic skeleton. We take the simplest case where the mother body behaves discontinuously over the continuous variation of the underlying domain. We studied the behavior of the zeros in detail at the discontinuity.

Anomalous Diffusion and Ergodic Violation in Intermembrane and Membrane-Mediated Transport

Ralf Metzler
Potsdam University, Germany

It will be shown that diffusion of lipid molecules and embedded proteins in bilayer membranes is transiently anomalous. The temporal range of the anomalous regime is significantly extended when disorder in the form of cholesterol or proteins is added. In protein crowded membranes long range anomalous diffusion is observed, combining antipersistent long-ranged noise with non-Gaussian fluctuations. Passing to in vivo membranes, the transport is again anomalous, but dominated rather by jump like diffusion with scale free, power-law waiting time density, causing the diffusion to be non-ergodic in the Boltzmann sense that time and ensemble averages of physical observables no longer converge to each other. Adding another spatial dimension, it will be discussed how surface-bulk exchange due to reactive boundary conditions leads to effective, Levy flight-type anomalous diffusion on the membrane surface mediated by transient bulk excursions. Again, this process is non-ergodic in the above sense.

Interfacial Instability During Phase Change

Ranga Narayanan
University of Florida, USA
K.E. Uguz

The physics of evaporative convection for binary systems is presented. Two results are of importance. The first is that a binary system, in the absence of gravity, can generate an instability only when heated from the vapor side. This is to be contrasted with the case of a single component where instability can occur only when heated from the liquid side. The second result is that a binary system, in the presence of gravity, will generate an instability when heated from either the vapor or the liquid side provided the heating is strong enough. In addition to these results we show the conditions at which interfacial patterns can occur.

Dynamics of Subdiffusion-Reaction Fronts

Alexander Nepomnyashchy
Technion, Israel
Mohammad Abu Hamed, Vladimir Volpert

The dynamics of a reaction front is considered in the framework of the Seki-Lindenberg subdiffusion-reaction model, where the time derivative in the diffusion-reaction equation is replaced by a Caputo fractional derivative. Using a piecewise linear reaction function, we obtain exact analytical solutions for plane fronts (i) between two stable uniform states and (ii) between a stable state and an unstable state

('pulled' and 'pushed' fronts). In the case of equal potentials of stable phases, where the plane front is motionless, the dynamics of fronts is determined (i) by the front curvature and (ii) by interaction of multiple fronts. We obtain a closed equation governing a slow motion of a small-curvature front and find its approximate solutions. Also, we derive and solve, analytically and numerically, integro-differential equations that describe the interaction of distant fronts.

On a Muskat Problem with Line Distributions of Sinks and Sources

Tatiana Savina
Ohio University, USA
A.A. Nepomnyashchy, L. Akinyemi

We consider a Muskat problem with zero surface tension. We give examples of exact solutions when the distribution of sinks and sources is defined by the initial shape of the interface separating two fluids.

Stationary Boundary Points for a Laplacian Growth Problem in Higher Dimensions

Tomas Sjoedin
Linkoeping University, Sweden
Stephen J. Gardiner

This talk will concern the behaviour of corners for certain Laplacian growth processes driven by source terms in higher dimensions. In two dimensions this process corresponds to Hele-Shaw flow, and it is known that corners of interior angle less than $\pi/2$ in the boundary of a plane domain are initially stationary for such growth processes. The aim here is to present analogous results in higher dimensions.

Weak Resolution of Singularities in Integrable Free-Boundary Dynamics

Razvan Teodorescu
University of South Florida, USA

In two dimensions, free-boundary dynamics featuring an infinite set of conserved quantities is an universal model rich in applications and also in mathematical properties. In many cases, such models are characterized by the formation of boundary singularities, which sometimes does not allow continuation of strong solutions past a finite evolution time. The weak resolution of singularities is obtained by a deformation of the original dynamical system, which introduces an associated class of weak solutions. We prove that the deformation is universal and that it leads to weak solutions in the family of modulated elliptic functions. For the case of generic singularities, we provide explicit weak solutions and we indicate an interesting connection to complex Burgers-type equations and convex nonlinear optimization.

FRAP Dynamics in the Random Comb Model

Santos Yuste
Dpt. Fisica. U.Extremadura, Spain
E. Abad, A. Baumgaertner

The problem of computing concentration recovery curves mimicking FRAP experiments in comb-like geometries is addressed. Our approach relies on the effective mean-field CTRW description of the random comb model, which means that the diffusion of particles can be described by means of a fractional diffusion equation. We first obtain analytically the value of the anomalous diffusion coefficient and, then, we solve analytically the fractional diffusion equation in the Laplace space with boundary conditions typical of FRAP experiments. We show that, although the recovery curves cannot be fitted by a standard diffusion equation with a time-dependent diffusion coefficient (scaled Brownian motion model), the differences between the exact curves and such fits are small. This provides support for the generalized practical use of scaled Brownian motion for describing diffusion experiments in comb-like systems. Our theoretical results are confirmed by numerical simulations.

Special Session 54: Nonlinear PDEs and Variational Methods

Olimpio Hiroshi Miyagaki, Universidade Federal de Juiz de Fora-UFJF, Brazil
Djairo Guedes de Figueiredo, IME-UNICAMP, Brazil
Joao Marcos Bezerra do O, Universidade Federal da Paraiba-UFPB, Brazil

This session will focus on some recent developments in the theory of Nonlinear Partial Differential Equations of the elliptic type. Questions like uniqueness, multiplicity, geometric properties and behaviour of solutions will be discussed. Nonlinear elliptic problems have been studied in recent years,also motivated by their applicability to other areas of PDE, as well as in many areas of Physics, Biology, Economy.

On the Persistence of the Eigenvalues of a Perturbed Fredholm Operator

Pierluigi Benevieri
University of Sao Paulo, Brazil
Alessandro Calamai, Massimo Furi, Maria Patrizia Pera

Let H be a real Hilbert space and denote by S its unit sphere. Consider the nonlinear eigenvalue problem $Lx + \varepsilon N(x) = \lambda x$, where ε,$\lambda \in \mathbb{R}$, $L : H \to H$ is a bounded self-adjoint, linear operator with nontrivial kernel and closed image, and $N : H \to H$ is a nonlinear perturbation term. A unit eigenvector $\vec{x} \in S \cap mKer, L$ of L (corresponding to the eigenvalue $\lambda = 0$) is said to be *persistent* if it is close to solutions $x \in S$ of the above equation for small values of the parameters $\varepsilon \geq 0$ and λ. We give an affirmative answer to a conjecture formulated by R. Chiappinelli, M. Furi and M.P. Pera in 2008. Namely, we prove that, if N is Lipschitz continuous and the eigenvalue $\lambda = 0$ has odd multiplicity, then the sphere $S \cap mKer, L$ contains at least one persistent eigenvector. We provide examples in which our results apply, as well as examples showing that if the dimension of $mKer, L$ is even, then the persistence phenomenon may not occur.

On a Class of Superlinear Elliptic Problems

David Costa
University of Nevada Las Vegas, USA
Siegfried Carl, Hossein Tehrani

We consider a class of superlinear slliptic problems in $\mathbb{R}^N$ (($N \geq 3$). By means of variational methods, we present a new approach to finding multiple solutions.

Some Classical Inequalities Revisited for the Fractional Laplacian

Olivaine de Queiroz
UNICAMP, Brazil

We are interested in the study, in the context of the fractional Laplacian in bounded domains, of some classical inequalities such as Sobolev-Trudinger-Moser and also the Faber-Khran. We apply our results in the study of some free boundary problems and also in some nonlinear PDEs from Conformal Geometry.

An Inverse Iteration Method for Obtaining Q-Eigenpairs of the P-Laplacian

Grey Ercole
Universidade Federal de Minas Gerais, Brazil

We consider a Lane-Emden type problem for the Dirichlet p-Laplacian operator in an N-dimensional Lipschitz domain. The nonlinearity contains a nonlocal factor that makes the problem (p-1)-homogeneous, so that it can be considered as an eigenvalue problem. We show that eigenpairs can be obtained as limits of sequences suitably built from an inverse iteration scheme.

Ground States of Elliptic Problems Involving Non Homogeneous Operators

Giovany Figueiredo
Universidade Federal do Para, Brazil
Humberto Ramos

We investigate the existence of ground states for functionals with nonhomogenous principal part. Roughly speaking, we show that the Nehari manifold method requires no homogeinity on the principal part of a functional. This result is motivated by some elliptic problems involving nonhomogeneous operators. As an application, we prove the existence of a ground state and infinitely many solutions for three classes of boundary value problems.

Self-Similar Solutions for the Heat Equation

Marcelo Furtado
Univeristy of Brasilia, Brazil

We are concerned with the existence of solutions for the equation

$$-\Delta u - \frac{1}{2}(x \cdot \nabla u) = f(u), \qquad x \in \Omega,$$

where $\Omega = \mathbb{R}^N$ or $\Omega = \mathbb{R}^N_+$, with suitable nonlinear boundary condition on the second case. It can be showed that this problem arises in finding self-similar solutions for the nonlinear heat equation. By using variational methods we obtain solutions which rapidly decay to zero at infinity. We consider several cases depending on the special profile of the function f and the dimension of the space.

Convex Solutions of the Fractional Heat Equation

Antonio Iannizzotto
University of Cagliari, Italy
Antonio Greco

We prove several properties, involving sign, convexity and uniqueness results, for unbounded solutions of stationary problems driven by the fractional Laplacian on the whole space. Exploiting such properties, we prove an existence/uniqueness result for the fractional heat equation provided the (possibly unbounded) initial datum has a moderate growth, and moreover that the fractional heat flow preserves convexity of the initial datum. Our result complements the Widder's type theorem of Barrios, Peral, Soria and Valdinoci (Arch. Rational Mech. Anal. 213 (2014) 629-650).

Nonradial Positive Solutions of the P-Laplace Emden-Fowler Equation

Ryuji Kajikiya
Saga University, Japan

In this lecture, we study the p-Laplace Emden-Fowler equation with a radial and sign-changing weight under the Dirichlet boundary condition. We show that if the weight function is negative in the unit ball except for a small neighborhood of the boundary and positive at somewhere in this neighborhood, then no least energy solution is radially symmetric. Moreover, in the one dimensional case, we prove that if the neighborhood is large, then a positive solution is unique.

Singularly Perturbed PDEs and Patterns with Periodic Profiles

Fethi Mahmoudi
Universidad de Chile, Chile

We consider a class of singularly perturbed equations in planar domains: as the singular perturbation parameter tends to zero, we exhibit a family of solutions concentrating at the boundary with asymptotically periodic profile. As solutions with uniform profile at the boundary were known to exist, the result here reflects the phenomenon of Turing's instability, which triggers formation of inhomogeneous structures from more homogeneous ones.

A Critical Nonlinear Fractional Elliptic Equation with Saddle-Like Potentical in $\mathbb{R}^N$

Olimpio Miyagaki
UFJF-Universidade Federal de Juiz de Fora, Brazil
Claudianor O. Alves

In this work, we study the existence of positive solution for the following class of fractional elliptic equation

$$\epsilon^{2s}(-\Delta)^s u + V(z)u = \lambda |u|^{q-2}u + |u|^{2^*_s-2}u, , , \text{in}, , , \mathbb{R}^N,$$

where $\epsilon, \lambda > 0$ are positive parameters, $q \in (2, 2^*_s), 2^*_s = \frac{2N}{N-2s}$, $N > 2s$, $s \in (0,1)$, $(-\Delta)^s u$ is the fractional laplacian, and V is a saddle-like potential. The result is proved by using minimizing method constrained to the Nehari manifold. A special minimax level is obtained by using an argument made by Benci and Cerami.

Some Results about Singular Elliptic Problems in Orlicz-Sobolev Spaces

Carlos Santos
University of Brasilia, Brazil
Claudianor Alves, Jefferson Santos, Jose Valdo Goncalves, Marcos Leandro Carvalho, Jiazheng Zhou

In this talk, I will present some new results about singular elliptic problems. We will do this for different settings of function spaces and non-linearities.

Special Session 56: Junior Session on Nonlinear Hyperbolic Equations and Related Topics

Laura Spinolo, IMATI-CNR, Pavia, Italy
Fabio Cavalletti, University of Pavia, Italy

Hyperbolic systems of conservation laws are a class of nonlinear partial differential equations with several applications coming from both physics and engineering. In particular, the archetype are the Euler equations of fluid dynamics. The mathematical understanding of this class of equations is complicated by the presence of highly nonlinear phenomena, like the fact that, in general, classical solution can breakdown in finite time owing to the formation of shocks. In recent years, the analysis of system of conservation laws has taken great advantage from the interplay with different but closely related fields, like geometric measure theory, convex integration, and others. This session aims at gathering young mathematicians working on nonlinear hyperbolic equations and related topics, in order to present some of the most recent developments of the theory and favour scientific interactions.

Finite Energy Weak Solutions of the Quantum Navier-Stokes Equations - Part II

Paolo Antonelli
Gran Sasso Science Institute, Italy
Stefano Spirito

In this talk I will present an existence result for finite energy weak solutions for the quantum Navier-Stokes system. By considering a suitable regularization of the system, which maintains the compactness properties of the original system, I will show the existence of a sequence of regular approximate solutions. The a priori bounds obtained will then guarantee the convergence of the sequence to a finite energy weak solution to the quantum Navier-Stokes system. This is a joint work with S. Spirito from Gran Sasso Science Institute, L'Aquila.

Surprise and Predictability in Bounded Sources of Non-Convex Balance Laws

Laura Caravenna
Università di Padova, Italy

In the talk I will show surprising and predictable aspects of bounded source terms in a non-convex balance law, with smooth flux, when it admits a continuous solution. Namely, I will discuss to what extent the conservation law can be reduced to an (infinitely dimensional) system of ODEs along the characteristic curves. This correspondence is evident in the classical setting but it is surprising in this context with lack of regularity. Part of the correspondence just requires suitable definitions and smart technicality, but concerning part of it new odd unexpected behaviors show up. The presentation is mostly based on a joint work with G. Alberti (Pisa) and S. Bianchini (SISSA) , and it extends previous works by several authors relative to the case of the quadratic flux.

Recent Progress on the Large Solution of Compressible Euler Equations

Geng Chen
Georgia Institute of Technology, USA

Compressible Euler equations (introduced by Euler in 1757) model the motion of compressible inviscid fluids such as gases. It is well-known that solutions of compressible Euler equations often develop discontinuities, i.e. shock waves. Successful theories have been established in the past 150 years for small solutions in one space dimension. The theory on large solutions is widely open for a long time, even in one space dimension. In this talk, I will discuss some recent exciting progresses in this direction. In the first part of this talk, I will discuss our complete resolution of shock formation problem, which extends the celebrated work of Peter Lax on small solutions in 1964. Our breakthrough relies on the discovery of a sharp time-dependent lower bound on density, when solutions approach vacuum in infinite time. In the second part, I will show our recent negative example concerning the failure of current available frameworks on approximate solutions in order to establish large BV (bounded total variation) theory. The talk is based on my joint works with A. Bressan, H.K. Jenssen, R. Pan, R. Young, Q. Zhang, and S. Zhu.

Surprising Solutions to the Isentropic System of Gas Dynamics

Elisabetta Chiodaroli
EPFL, Lausanne, Switzerland
C. De Lellis, O. Kreml

In this talk we discuss some applications of the method of convex integration to the compressible Euler system of gas dynamics in several space dimensions. This leads to the construction of non-standard solutions which disprove the efficiency of different admissibility criteria proposed in the literature.

On the Global Structure of the Set of 2D Stationary Euler Flows

Antoine Choffrut
University of Warwick, England
Herbert Koch

The incompressible Euler equations govern the evolution of an ideal fluid. They enjoy a very elegant geometric Hamiltonian formulation and as such their steady-states are of particular interest. Being an infinite-dimensional system, the analysis becomes considerably more challenging and the usual toolbox is not adequate for the Euler equations. In this talk I will present some recent work with Herbert Koch on the existence of steady-states with prescribed vorticity distribution. It is a global version of previous work in collaboration with Vladimir Sverak. One crucial ingredient is to derive sufficiently strong a priori estimates in order to repeat the local, perturbative result. I will also discuss other interesting aspects of the proof.

Finite Time Singularities of a Hyperbolic System of Elasticity

Tao Huang
New York University Shanghai, Peoples Rep of China
Geng Chen, Chun Liu

We study the formation of finite time singularities in the form of super norm blowup for a spatially hyperbolic system modeling inhomogeneous elasticity, which is related to the variational wave equations. The system possesses a unique C^1 solution before the emergence of vacuum in finite time, for given initial data that are smooth enough, bounded and uniformly away from vacuum. At the occurrence of blowup, the density becomes zero, while the momentum stays finite, however the velocity and the density of the energy are both infinite.

Holder Continuous Euler Flows

Philip Isett
MIT, USA

Motivated by the theory of hydrodynamic turbulence, L. Onsager conjectured in 1949 that solutions to the incompressible Euler equations with Holder regularity less than 1/3 may fail to conserve energy. C. De Lellis and L. Szèkelyhidi, Jr. have pioneered an approach to constructing such irregular flows based on an iteration scheme known as convex integration. This approach involves correcting "approximate solutions" by adding rapid oscillations that are designed to reduce the error term in solving the equation. In this talk, I will discuss an improved convex integration framework, which yields solutions with Holder regularity as much as 1/5-.

On Measure Valued Solutions to the Compressible Euler Equations

Ondrej Kreml
Czech Academy of Sciences, Czech Rep
Elisabetta Chiodaroli, Eduard Feireisl, Emil Wiedemann

In a very interesting paper, Szekelyhidi and Wiedemann (2012) proved that every measure valued solution to the incompressible Euler equations can be approximated by a sequence of weak solutions, implying that there is no significant difference between weak and measure valued solutions to the incompressible Euler system. In this talk we prove that such a property does not hold for the compressible case and we show the construction of a measure valued solution which can not be generated by weak solutions. Moreover we show an abstract neccesary condition for measure valued solutions to be generated by sequences of weak solutions. The proof is based on a generalization of a rigidity result by Ball and James, the necessary condition is obtained as a consequence of the works of Fonseca and Muller.

Effective Monte Carlo Methods for Computing Measure Valued Solutions and Statistical Solutions

Kjetil Olsen Lye
ETH Zurich, Switzerland
Siddhartha Mishra

Recent developments have indicated the correct notion of solution for system of non-linear conservation laws, especially in the presence of turbulence, is the notion measure valued solutions. We briefly review the theory of entropy measure valued solutions and statistical solutions and show how we can compute statistics of multidimensional systems of conservation laws using entropy preserving schemes and stochastic sampling. We assess the applicability of Multi-level Monte Carlo methods for both scalar conservation laws and system of equations.

Lower Bound on the Temperature Background Field Method for Rayleigh-Bénard Convection

Camilla Nobili
University of Basel, Switzerland

By the background field method, finding bounds on the average upward heat transport for the Rayleigh-Bénard convection reduces to a variational problem: minimize over background profiles that satisfy a certain stability condition. After recalling the method (introduced in 1996 by Doering and Constantin in this context) we characterize the admissible profiles and then we argue the non-optimality of this method, by exhibiting a lower bound on its solution. This is a joint work with Felix Otto.

A Time Discretization Scheme for the Multi-Dimensional Compressible Euler Equations.

Marc Sedjro
KAUST, Saudi Arabia
Fabio Cavalletti, Michael Westdickenberg

We construct a time discretization scheme for the multi-dimensional compressible Euler equations, inspired by minimizing movements for curves of maximal slope. Each timestep requires the minimization of a functional measuring the acceleration of fluid elements, over the cone of monotone transport maps. We prove convergence to measure-valued solutions.

Finite Energy Weak Solutions of the Quantum Navier-Stokes Equations

Stefano Spirito
Gran Sasso Science Institute, Italy
Paolo Antonelli

In this talk we focus on a new compactness result about finite energy weak solutions of the quantum Navier-Stokes equations. The novelty of the result is that we are able to consider the vacuum in the definition of weak solutions. The main tool is a new formulation of the equations which allows us to get additional a priori estimates to prove compactness. Some remarks concerning the choice of the approximation system to get global existence will be made. This is a joint work with Paolo Antonelli (GSSI - Gran Sasso Science Institute).

Special Session 57: Lie Symmetries, Conservation Laws and Other Approaches in Solving Nonlinear Differential Equations

Chaudry Masood Khalique, North-West University, So Africa
Mufid Abudiab, Texas A&M University-Corpus Christi, USA
Maria Gandarias, Universidad de Cádiz, Spain

This session is devoted to research areas that are related to nonlinear differential equations and their applications in science and engineering. The main focus of this special session is on the Lie symmetry analysis, conservation laws and their applications to ordinary and partial differential equations. Other approaches in finding exact solutions to nonlinear differential equations will also be discussed. This includes, but not limited to, asymptotic analysis methodologies, the simplest equation method, the multiple exp-function method, inverse scattering transform techniques, the upper-lower solutions method, the Hirota method, and others.

Solutions and Conservation Laws for a Two Plus One Dimensional Modified Korteweg De Vries

Mufid Abudaib
Texas A&M University-Corpus Christi, USA
Chaudry Masood Khalique

In this talk we study a two plus one dimensional modified Korteweg de Vries equation which has applications in many scientific fields. We obtain exact solutions and conservation laws for this equation.

Symbolic Computation of Conservation Laws and Exact Solutions of a Coupled Variable-Coefficient Modified Korteweg-De Vries System

Abdullahi Adem
North-West University, So Africa

In this talk we study a generalized coupled variable-coefficient modified Korteweg-de Vries (CVCmKdV) system that models a two-layer fluid, which is applied to investigate the atmospheric and oceanic phenomena such as the atmospheric blockings, interactions between the atmosphere and ocean, oceanic circulations and hurricanes. The conservation laws of the CVCmKdV system are derived using the multiplier approach and a new conservation theorem. In addition to this, a similarity reduction and exact solutions with the aid of symbolic computation are computed.

A Predator-Prey Model with Starvation Driven Diffusion in Heterogeneous Environment

Inkyung Ahn
Korea University, Korea
Wonhyung Choi, Kwangjoong Kim

Recently, population models with starvation driven diffusions is introduced to represent the fitness property of certain species, which describes a random dispersal strategy with a motility increase on starvation. In this talk, we discuss predator-prey models with starvation driven diffusion(SDD) in a heterogeneous environment. The Lotka-Volterra type is considered to understand the effect of SDD for the stability of semi-trivial solutions of models with SDD under no-flux boundary conditions. In addition, the positive coexistence is investigated in terms of the eigenvalue analysis for the linearized operator derived from the model. The methods employed is the fixed-point theory applied to a positive cone on a Banach space. We present the biological interpretation and simulations based on the result.

Cattanneo Christov Heat Flux Model Study for Water Based CNT Suspended Nanofluid Past a Stretching Sheet

Noreen Akbar
National University of Sciences and Technology, Pakistan

In this talk I will discussed the magnetic field effects on the flow of Cattanneo–Christov heat flux model for water based CNT suspended nanofluid over stretching sheet. Cattanneo–Christov heat flux model for water based CNT suspended nanofluid is not explored so far for stretching sheet. The flow equations will be presented first time in literature transformed into ordinary differential equations using similarity transformations. The numerical solutions using shooting technique will be presented and Comparison will be presented with existing literature. Graphical results will be presented to illustrate the effects of various fluid flow parameters on velocity, heat transfer, Nusselt number, Sherwood number and skin friction coefficient for different type of nanoparticles.

Existence Result for a Class of Strongly Nonlinear Stochastic Parabolic Initial Boundary Value Problem

Zakaria Ali
University of Pretoria, So Africa
Ali Zakaria Idriss, Mamadou Sango

This paper treats a very important class of stochastic partial differential equations. our main purpose in this paper is to prove an existence result for such type of problems under Dirichlet boundary condi-

tions. The main obstacle in solving the present problem is that the existence result cannot be easily retrieved from the well known results of Krylov - Rozovskii and Etienne Pardoux illustrating the extension of many papers including some in the references therein. Strong solutions are obtained for stochastic evolution equations driven by Wiener processes under Lipchitz assumptions of the random forces $G(t, u)$.

Computation of Conservation Laws

Stephen Anco
Brock University, Canada

Some recent work on general methods for computation of conservation laws of differential equations will be reviewed.

Exact Solutions for Stokes Flow Model of an Incompressible Third-Grade Nanofluid

Asim Aziz
National University of Sciences and Technology, Pakistan
Taha Aziz

The fully developed time-dependent flow and heat transfer of an incompressible, thermodynamically compatible non-Newtonian third-grade nanofluid is investigated. Classical Stokes model is considered in which the flow is generated due to the impulsive motion of the plate in its own plane. Lie symmetry approach is utilized to convert the governing nonlinear partial differential equation to a system of linear and nonlinear ordinary differential equations. Exact solutions for the model equation are deduced in the form of closed-form exponential functions which are not available in the literature before. The physical features of the pertinent parameters are discussed in detail through several graphs

Group Theoretical Analysis and Invariant Solutions for Time-Dependent Flow Model of a Non-Newtonian Fluid

Taha Aziz
DST/NRF Centre of Excellence in Mathematical and Statistical Sciences, University of the Witwatersrand, So Africa
Fazal M. Mahomed

The present work deals with the modelling and solution of the unsteady flow of an incompressible third grade fluid over a porous plate within a porous medium. The flow is generated due to an arbitrary velocity of the porous plate. The fluid is electrically conducting in the presence of a uniform magnetic field applied transversely to the flow. Lie group theory is employed to find symmetries of the model equation and these symmetries are used to transform the original third order partial differential equation to a third order ordinary differential equations. The third order ordinary differential equations are then solved analytically and numerically. The manner in which various emerging parameters have an effect on the structure of the velocity is discussed with the help of several graphs.

Nonclassical Symmetries and Conservation Laws for a Generalization of the Korteweg-De Vries Equation

Maria Bruzon
University of Cadiz, Spain
Tamara Garrido, Rafael de la Rosa

In this work, a class of sixth-order nonlinear wave equations, which leads to the well-known Sawada-Kotera-Caudrey-Dodd-Gibbons, Kaup-Kupershmidt, and Drinfeld-Sokolov-Satsuma-Hirota system of coupled Korteweg-de Vries equations, has been considered to determine nonclassical symmetries. Furthermore, nonlinear self-adjointness is proved and conservation laws are obtained.

Survival Strategy on a Predator-Prey Interacting System with SDD

Wonhyung Choi
Korea University, Korea
Inkyung Ahn, Seunghyeon Baek

In this talk, we present a Lotka-Volterra type predator-prey model with starvation driven diffusion under Neumann boundary condition. A starvation driven diffusion(SDD) is a dispersal strategy that increase their motility when food or other resource is not sufficient. We investigate whether the starvation driven diffusion is a better survival strategy than constant diffusion by using stability analysis for semi-trivial solutions to the system. Furthermore, the extinction and coexistence of predator and prey are discussed.

On a Homogeneous Evolution Equation: Lax Pair and Peakon Solutions

Priscila da Silva
Universidade Federal do ABC, Brazil
Igor Leite Freire

In this talk we discuss integrability and peakon solutions of a family of homogeneous evolution equations. Recursion operators are obtained and two members related to KdV-type equations are shown to be completely integrable using a Lax representation and the existence of an infinite number of conserved quantities. Conditions for the existence of peakon solutions are then given.

Conditional Symmetries for Ordinary Differential Equations and Applications

Aeeman Fatima
DST/NRF Centre of Excellence in Mathematical and Statistical Sciences, University of the Witwatersrand, So Africa
Fazal M. Mahomed

We refine the definition of conditional symmetries of ordinary differential equations and provide an algorithm to compute such symmetries. A proposition is proved which provides criteria as to when the symmetries of the root system of ODEs are inherited by the derived higher-order system.We provide examples and then investigate the conditional symmetry properties of linear nth-order equations subject to root linear second-order equations. First this is considered for simple linear equations and then for arbitrary linear systems. We prove criteria when the symmetries of the root linear ODEs are inherited by the derived scalar linear ODEs and even order linear system of ODEs. Furthermore, we show that if a system of ODEs has exact solutions, then it admits a conditional symmetry subject to the first order ODEs related to the invariant curve conditions which arises from the known solution curves. Moreover, we give examples of the conditional symmetries of nonlinear third-order equations which are linearizable by second-order Lie linearizable equations. Applications to classical and fluid mechanics are presented.

A Multicomponent System of Camassa-Holm-Novikov Equations

Igor Freire
UFABC, Brazil
Diego Catalano Ferraioli

In this talk we discuss a system of equations generalising the Camassa-Holm and Novikov equations. Point symmetries are shown, as well as some classical solutions. Peakons and multipeakon solutions are also considered.

On Conservation Laws for a Boussinesq Equation with a Damping Term

Maria Luz Gandarias
University of Cadiz, Spain
Maria Rosa

In this talk we present the classical Lie symmetries admitted by a Boussinesq equation with a damping term as well as the reduced ordinary differential equations. We find a classification of the low-order conservation laws for this equation. By using some symmetry-invariant conservation laws we apply the double reduction method.

Interconnectedness of Group Theory and Dynamical Systems Analysis

Kesh Govinder
UKZN, So Africa

There are a number of different approaches to finding analytic solutions to, as well as determining properties of, solutions of differential equations. Group analysis (which exploits the invariance of equations under transformations) has been very successfully applied in the classification and explicit solution of differential equations. Singularity analysis (which requires solutions of equations to have, at worst, moveable poles) has also been used to classify equations. The long term evolution of systems of equations can be determined by application of dynamical systems analysis (where the behaviour of equilibrium points is paramount).
In this talk we will show how, in some cases, the results obtained via these methods coincide (especially in parameter determination) while in other cases, the different approaches are complementary. We will also comment on the role of transformations in these synchronous/complementary results and show that well–known results may not be as accurate as previously thought. Examples from a variety of applications will be used to illustrate our findings.

Noether and Lie Group Classification of a Radial Form of a Coupled Hyperbolic System

Chaudry Masood Khalique
North-West University, So Africa

In this talk we present a complete Noether symmetry classification of the radial form of a coupled system of hyperbolic equations and construct conservation laws corresponding to the Noether operators. Lie group classification of the system is then performed.

On a Diffusive Predator-Prey System with an Infected Prey with Ratio-Dependent Functional Response

Kwangjoong Kim
Kookmin University, Korea
Inkyung Ahn

We examine a diffusive ratio-dependent predator-prey system with disease in the prey under homogeneous Dirichlet boundary conditions with a hostile environment at its boundary. We investigate the positive coexistence of three interacting species (susceptible prey, infected prey, and predator) and pro-

vide the asymptotical behavior of semi-trivial solution with disease free. The methods are employed from a comparison argument for the elliptic problem as well as the fixed-point theory as applied to a positive cone on a Banach space.

Differentiation of Solutions to a Parameter Dependent BVP with Integral Boundary Conditions

Jeffrey Lyons
Nova Southeastern University, USA
Kaitlyn Seabrook

We discuss derivatives of the solution of a parameter dependent boundary value problem with an integral boundary condition with certain assumptions and its relationship to a nonhomogeneous differential equation closely associated with the traditional variational equation.

Lambda-Symmetries and Mu-Symmetries by Integrable Couplings

Wen-Xiu Ma
University of South Florida, USA

We will talk about connections of lambda-symmetries for ODEs and mu-symmetries for PDEs with Lie-Baecklund symmetries of integrable couplings. Special integrable couplings are used to construct novel lambda-symmetries and mu-symmetries. The classification problem of integrable couplings of soliton equations and symmetries of integro-differential equations will be discussed.

Relativistic Stars and Symmetries

Sunil Maharaj
KwaZulu-Natal University, So Africa

A systematic analysis of the stellar surface junction condition is undertaken. This is a highly nonlinear partial differential equation in general. We obtain the Lie point symmetries that leave the boundary condition invariant. Using a linear combination of the symmetries, we transform the junction condition into simpler form. We present several new exact solutions to the junction condition. In each case we can identify the exact solution with a Lie point generator. Some of the solutions obtained satisfy a barotropic equation of state. As a special case we regain previously known models. Our analysis highlights the interplay between Lie algebras, nonlinear differential equations and application to relativistic astrophysics.

Hamiltonians: Symmetries and Integrals

Fazal Mahomed
Wits University, So Africa

We give explicit criteria when symmetries for a Hamiltonian system correspond to their first integrals. It is shown what conditions need to be imposed on the Hamiltonian symmetry so that it constructively yields a first integral. We show that both the classical and recent Noether based approaches are in fact equivalent. It is proved that when the Hamiltonian symmetries provide first integrals they form a Lie algebra. We prove that the Hamilton first integral is invariant under the Hamilton symmetry. Examples are provided.

Lie Symmetry Classification of a Chemotaxis Model

Motlatsi Molati
National University of Lesotho, Lesotho

We perform Lie symmetry classification of a chemotaxis model. The model comprises a system of nonlinear differential equation with an arbitrary function of the dependent variable whose functional form is specified via the direct method of group classification.

Symmetry Analysis of a Potential Kadomtsev-Petviashvili Equation with Power Nonlinearity

Dimpho Mothibi
North-West University, Mafikeng Campus, So Africa

This talk aims to study the potential Kadomtsev-Petviashvili equation with power nonlinearity, which arises in many nonlinear problems of mathematical physics. We perform Noether symmetry classification and determine exact solutions for this equation.

Conservaton Laws for a Generalized Hyperbolic Lane-Emden System

Ben Muatjetjeja
North-West University, So Africa

In this talk, we aim to perform Noether symmetry classification of a coupled (1+ 1)-dimensional hyperbolic Lane-Emden system. Several cases arise for which Noether symmetries exist. In addtion, conservation laws are constructed for each case.

Lie Ymmetries of Functional Differential Equations of Hopf Type

Martin Oberlack
TU Darmstadt, Germany
Daniel Janocha, Marta Waclawczyk

Functional differential equations are partial differential equations with an infinite set of independent variables in a continuous sense. They appear in quantum mechanics and turbulence theory, where it is called Hopf functional equation. In turbulence theory the Hopf functional describes the full turbulence statistics as its n-th functional derivative defines the n-th statistical moment. The Hopf equation contains both functional as well as regular derivatives. As they are essentially no methods to derive solutions for this type of equations we have extended Lie symmetry methods. In a first example we show how to derive Lie symmetries of the Hopf functional equations derive from the Burgers equation.

Non-Modal Hydrodynamic Stability Theory Based on an Extended Set of Lie Symmetries

Martin Oberlack
TU Darmstadt, Germany
Andreas Nold, Alexei Cheviakov, Cedric Sanjon

Classical hydrodynamic stability is, to a large part, based on the modal ansatz, which, implemented into the linearized Navier-Stokes equation, leads to the famous Orr-Sommerfeld equation for the eigenfunctions. Analyzing linearized Navier-Stokes equation with respect to its Lie symmetries it turned out, that the modal ansatz is based on translation in space and time as well as on scaling of the dependent variable - independent of the base flow, which is analysed on its stability. However, various canonical flows such as linear shear, Couette, Poiseuille, pipe or Taylor-Couette flows exhibit one or more additional symmetries, which in turn lead to invariant solutions that are very different from the modal ansatz. Hence, it allows for a new and, in fact, non-modal type of hydrodynamic stability theory. In turn, the resulting equation for the respective eigenfunctions are derived.

On the General Solution Based on the Time-Independent Integral, the Lagrangian and The Hamiltonian Functions for Fin Equation

Ozlem Orhan
Istanbul Technical University, Turkey
Teoman Ozer

We derive the time-independent integral for a nonlinear equation, namely fin equation in which the thermal conductivity and heat transfer coefficient are assumed to be functions of the temperature. Using the modified Prelle-Singer approach, we point out that explicit the time-independent first integral and general solution of the equation corresponding to these integrals can be identified for the fin equation for different thermal conductivity and heat transfer coefficient functions. Then using this approach, an appropriate the Lagrangian and the Hamiltonian forms are obtained. Finally, we discuss on these solutions by their graphics.

A Novel Method for Solving a Class of Nonlinear Integro-Differential Equations

Suares Clovis Oukouomi Noutchie
North-West University, So Africa
Richard Guiem

The global solvability of a class of nonlinear nonautonomous integro- differential equations describing coagulation-fragmentation processes with growth is investigated using a modified monotone method. Existence and uniqueness results are obtained thanks to Gronwall's inequality. In particular, a new concept of upper-lower solution is introduced and the comparison principle established.

Design, Analysis and Simulation of a Robust Numerical Technique to Solve Fractional Partial Differential Equations Arising in Computational Finance

Kailash C. Patidar
University of the Western Cape, So Africa
F. Gideon, S.M. Nuugulu

Conventional partial differential equation approaches have been explored extensively over past three decades to solve option pricing problems. With the progress in computational methodologies, it is now possible to consider solving many complex problems, for example, fractional partial differential equations (FPDEs) that are used to model more realistic phenomena. In this talk, we will discuss pricing of options using such FPDEs. Apart from the construction of the numerical method, we will also discuss its thorough analysis. Finally, we will present some reliable numerical results for a class of problems for which the proposed approach is applicable.

Conservation Laws and Exact Solutions for a Nonlinear p-Laplacian Evolution Equation

Elena Recio
Brock University, Canada
Stephen Anco, Maria S Bruzon, Maria Luz Gandarias

In this work, we consider a nonlinear p-Laplacian evolution equation that in the initial value problem, admits solutions exhibiting an interesting extinction behavior. A complete classification of point and contact symmetries and conservation laws is presented for the

corresponding n-dimensional radial p-Laplacian diffusion equation. The structure of a gradient flow of this equation is studied, leading to an useful energy identity. In addition, exact group-invariant solutions are obtained.

Exact Lie Symmetric Solutions of Nonlinear Partial Differential Equations

Barbara Shrauner
Washington University, USA

Barbara Abraham-Shrauner, Washington University, St. Louis, MO A balance of nonlinearity of nonlinear partial differential equations (NLPDEs) terms identifies if Jacobian elliptic functions are probably traveling wave solutions. Another approach of identification is to use NLPDEs solved by hyperbolic functions. Search for superposition of Jacobian elliptic function solutions of NLPDEs is simplified by a change of basis of sn, cn and dn functions. Examples include generalized KdV equations, nonlinear Schrodinger equations and Zakharov equations. The extended tanh method for the Zakharov equations is contrasted to the direct method discussed here.

Nonlinear Differential Equations in Exterior Domains from General Relativity

Nicolae Tarfulea
Purdue University Northwest, USA

We study a class of nonlinear elliptic equations with nonlinear Neumann boundary conditions in exterior domains. Problems of this type arise in many diverse contexts. An application related to the initial data problem in general relativity is presented.

On Kolmogorov's Energy Spectrum for MHD Turbulence

Tesfalem Abate Tegegn
University of Pretoria, So Africa
Mamadou Sango

A Fourier Analysis method has been used rigorously to investigate the spectral behaviour Magnetohydrodynamics flow governed by:

$$\begin{cases} \partial_t u + (u\cdot\nabla)u + \nabla\Pi - (b\cdot\nabla)b - \nu\Delta u = f_1 & \text{in } \mathbb{R}^3\times(0,T) \\ \partial_t b + (u\cdot\nabla)b - (b\cdot\nabla)u - \eta\Delta b = f_2 & \text{in } \mathbb{R}^3\times(0,T) \\ \nabla\cdot u = \nabla\cdot b = \nabla\cdot f_1 = \nabla\cdot f_2 = 0 & \text{in } \mathbb{R}^3\times(0,T) \\ u|_{t=0} = u_0, \qquad b|_{t=0} = b_0 & \text{in } \mathbb{R}^3 \end{cases}$$

where $u(x,t)$ is the flow velocity, $b(x,t)$ is the magnetic field, $\Pi = p + \frac{1}{2}|b|^2$ is the total pressure, $\nu > 0$ is the kinetic viscosity of the fluid, $\eta > 0$ is the resistivity of the fluid, and f_1 and f_2 are external forces on the system. Denoting a Fourier transforms of u and b by $\hat{u}(\xi,t)$ and $\hat{u}(\xi,t)$ respectively, we define the energy spectral function by the surface integral

$$E(k,t) = \int_{|\xi|=k} |\hat{u}(\xi,t)|^2 + |\hat{b}(\xi,t)|^2 dS(\xi),$$

where $\xi \in \mathbb{R}^3$ is a Fourier space variable. The celebrated 1941 works of Kolmogorov predict that the Navier-Stokes equation in three dimensions with large Reynolds number should obey

$$E(k) \sim C_0 \epsilon^{3/2} k^{-5/3},$$

for an inertial range $k_1 \leq k \leq k_2$ in some sense. Motivated by the 2012 work of Andrei Biryuk and Walter Craig, we have investigated bounds on the inertial range for the weak solution of the MHD equation by using Fourier transform, rate of energy dissipation.

Lie Symmetry Classification of a Quantum Hydrodynamical Model

Rita Tracina
University of Catania, Italy

In this talk we present the symmetry classification obtained by the infinitesimal method of a quantum hydrodynamical model. The model comprises a system of partial differential equations with arbitrary elements.

Bifurcations in Traveling Wave Solutions of the Kuramoto Sivashinsky Type Perturbed Forced Equation

Muhammad Usman
University of Dayton, USA
Chi Zhang

In this work a perturbation analysis is used to study the Kuramoto Sivashinsky type perturbed forced equation. Using traveling coordinates the PDE is converted into an ODE. We drive the operating curve for the relation between the bifurcation parameters. A condition is derived for the stability of the solutions using eigenvalue analysis. This model also includes the delays.

Symmetry Reductions, Exact Solutions and Conservation Laws for a New (3+1)-Dimensional Nonlinear Evolution Equation with MKdV Equation Constituting Its Main Part

Emrullah Yasar
Uludag University, Turkey

In this talk, we perform the Lie group method to a new (3+1)-dimensional nonlinear evolution equation with mKdV equation constituting its main part. This new (3+1)-dimensional equation models shallow-water waves and short waves in nonlinear dispersive models with a more realistic way. The symmetry reductions and exact solutions which include the 1-soliton and 2-soliton solutions are obtained. In addition, the conservation laws of the equation are also derived using the invariance-multiplier approach.

Local and Nonlocal Constants of Motion in Conservative and Dissipative Dynamics

Gaetano Zampieri
University of Verona, Italy
Gianluca Gorni

We give a recipe to generate nonlocal constants of motion for ODE Lagrangian systems and we apply the method to find useful constants of motion which permit to prove global existence and estimates of solutions to dissipative mechanical systems. We show examples where our recipe can be used to find genuine first integrals too. Our applications are the mechanical systems with homogeneous potential of degree -2, and the conservative Maxwell-Bloch system with RWA, where in particular we can separate one of the variables. Finally, we show a generalization of our recipe to some non-variational systems. As an application we find another (time-dependent) first integral for the dissipative Maxwell-Bloch equations for some special values of the parameters.

Traveling Wave Solutions and Infinite-Dimensional Linear Spaces of Multiwave Solutions to Jimbo-Miwa Equation

Lijun Zhang
Zhejiang Sci-Tech University, Peoples Rep of China

In this talk, the traveling wave solutions and multiwave solutions to $(3 + 1)$-dimensional Jimbo-Miwa equation are investigated. We obtain the existence of two families of bounded periodic traveling wave solutions and their implicit formulas by analysis of phase portrait of the corresponding traveling wave system.We derive the exact 2-wave solutions and two families of arbitrary finite N-wave solutions by studying the linear space of its Hirota bilinear equation, which confirms that the $(3 + 1)$-dimensional Jimbo-Miwa equation admits multiwave solutions of any order and is completely integrable.

Special Session 58: Qualitative Properties of Nonlinear Differential Equations of Elliptic and Parabolic Type

Raul Manasevich, University of Chile, Chile
Marta Garcia Huidobro, Catholic University, Chile

The aim of this session is to discuss new properties of solutions to the various types of nonlinear differential equations of elliptic and parabolic PDEs and systems as well as applications to importante applied problems. Topics will include nonlinear analysis methods, existence, uniqueness and stability of solutions, Liouville-type theorems, bifurcation of solutions, pattern recognition, blow up of solutions, and related methods.

Precompactness of Minimizing Sequences Under Multiple Independent Constraints

Santosh Bhattarai
Trocaire College, USA

The variational technique based on the concentration compactness principle has been adapted by many different authors to study existence and stability of solitary waves for a variety of nonlinear dispersive equations. A key to proving the existence of minimizers (and hence the stability of solitary waves) using this method is to exclude possibilities of vanishing and dichotomy alternatives. The more delicate part is to prove that the dichotomy alternative cannot occur. A crucial technical obstacle in order to rule out the dichotomy alternative is to establish the strict subadditivity condition. For one constraint problems, several techniques have been developed to establish the strict subadditivity condition. To prove the strict subadditivity condition for multiconstraint problems, however, seems to be more delicate. As a matter of fact only a few papers address the issue of compactness of minimizing sequences for coupled systems of dispersive equations. Moreover, in most of these papers, the variational problem has consisted of either only one constraint or two constraints were not independently chosen. In this talk we consider some coupled nonlinear Schrodinger systems and prove the precompactness of the minimizing sequences under multiple independent constraints. To exclude the dichotomy alternative, we show the subadditivity condition using a technique based on certain rearrangement inequality. As a consequence of the concentration compactness, we also obtain the stability of solitary waves associated to the set of minimizers.

Multiplicity Results for Bound State Solutions of a Semilinear Equation

Marta Garcia-Huidobro
Pontificia Universidad Catolica de Chile, Chile
Carmen Cortazar, Pilar Herreros

In this talk we give conditions on the nonlineary f so that the problem

$$\Delta u + f(u) = 0, \quad x \in \mathbb{R}^N, N \geq 2, \quad \lim_{|x|\to\infty} u(x) = 0,$$

has at least two solutions having a prescribed number of nodal regions and for which $u(0) > 0$. Any nonconstant solution to this problem is called a bound state solution. Bound state solutions that are nontrivial and nonnegative for all $x \in \mathbb{R}^N$, are referred to as a first bound state solution, or a ground state solution.

Multiplicity of Solutions for an Elliptic Equation with a Singular Nonlinearity and A Gradient Term

Ignacio Guerra
Universidad de Santiago de Chile, Chile

We consider the problem

$$-\Delta u = \lambda(1 + |\nabla u|^q)/(1 - u), \quad u \in (0,1) \quad \text{in} \quad B,$$
$$u = 0 \quad \text{on} \quad \partial B,$$

where B is the unit ball in $\mathbb{R}^N$, $q \geq 0$ and $\lambda \geq 0$. The problem with $q = 0$ is well known. In fact, Joseph & Lundgren found that for $2 < N < 4 + 2\sqrt{2}$ there are infinitely many solutions for some $\lambda = \lambda_* > 0$. On the other hand, they also found that for $N > 4 + 2\sqrt{2}$ there exists λ^* such that there exists a unique solution for each $0 < \lambda < \lambda^*$.
Here we study the existence of solutions for this problem when $q > 0$. In particular, we found a range of q and N where there exists $\lambda_* > 0$ such that there are infinitely many solutions for $\lambda = \lambda_*$.

Klein-Gordon-Maxwell Systems in Bounded Spatial Domains

Monica Lazzo
University of Bari, Italy
Lorenzo Pisani

I will talk about a system of elliptic equations, obtained in the framework of Klein-Gordon-Maxwell systems when looking for standing waves in equilibrium with a purely electrostatic field. I will present some results about nonexistence, existence, and multiplicity of solutions, under different boundary conditions and assumptions on the data. In view of the variational structure of the system, most results are based on global variational methods.

Obtaining Solutions with Prescribed Number of Zeros for a Nonlinear Equation Containing the P-Laplace Operator with Weights

Raul Manasevich
University of Chile, Chile
Marta Garcia-Huidobro, Carmen Cortazar, Jean Dolbeault

We consider radial sign-changing solutions of

$$\mathrm{div}\big(\mathsf{a}\,|\nabla u|^{p-2}\nabla u\big)+\mathsf{b}\,f(u)=0\,,\qquad \lim_{|x|\to+\infty}u(x)=0,$$

where a and b are two positive, radial, smooth functions defined on $\mathbb{R}^d\backslash\{0\}$, $f\in C(\mathbb{R})$ satisfying some additional conditions, and $\mathrm{div}\big(\mathsf{a}\,|\nabla u|^{p-2}\nabla u\big)$ denotes a $p-$Laplace operator with weight, $p>1$.
For radial solutions this problem becomes

$$\big(a(r)\,\phi_p(u')\big)'+b(r)\,f(u)=0\,,\quad r>0,$$
$$\lim_{r\to 0_+}a(r)\,\phi_p(u'(r))=0\,,\quad \lim_{r\to+\infty}u(r)=0$$

where $r=|x|$, and $a(r)=r^{d-1}\,\mathsf{a}(x)$, $b(r)=r^{d-1}\,\mathsf{b}(x)$.
Our problem is to find solutions of this last problem changing sign exactly k times, for any arbitrary $k\in\mathbb{N}$.

Interior $W^{1,\infty}$ Estimates for Solutions of Nonlinear Degenerate Parabolic Systems

Truyen Nguyen
University of Akron, USA

We study interior $W^{1,\infty}$ regularity for weak solutions of nonlinear degenerate parabolic systems of the form $u_t=\mathrm{div}\,\mathbf{A}(x,t,u,\nabla u)+\mathbf{B}(x,t,u,\nabla u)$, which include those of p-Laplacian type. We derive interior L^∞ estimate for u by employing Moser's iteration. Using some recent regularity results together with a suitable modification of the arguments by DiBenedetto and Friedman, we also establish the boundedness of ∇u.

Moderate Solutions of Semilinear Elliptic Equations with Hardy Potential

Phuoc Tai Nguyen
PUC, Chile
Moshe Marcus

Let Ω be a bounded smooth domain in $\mathbb{R}^N$. We study positive solutions of equation (E) $-L_\mu u+u^q=0$ in Ω where $L_\mu=\Delta+\frac{\mu}{\delta^2}$, $\mu>0$, $q>1$ and $\delta(x)=\mathrm{dist}\,(x,\partial\,\Omega)$. A positive solution of (E) is moderate if it is dominated by an L_μ-harmonic function. If μ is small then every positive L_μ- harmonic functions can be represented in terms of a finite measure on $\partial\Omega$ via the Martin representation theorem. However the classical measure boundary trace of any such solution is zero. We introduce a notion of normalized boundary trace by which we obtain a complete classification of the positive moderate solutions of (E).

Travelling Waves for an Internal Water Waves Model of the Benjamin-Ono Type

Jose Quintero
Universidad del Valle, Colombia
Juan C. Muñoz

In this talk we discuss the existence of solitary wave solutions for the regularized Benjamin-Ono system

$$\begin{cases} \zeta_t-((1-\alpha\xi)u)_x &= \frac{\epsilon^2}{6}\zeta_{xxt}, \\ u_t+\alpha uu_x+\left(1-\frac{\rho_2}{\rho_1}\right)\zeta_x &= \frac{\rho_2}{\rho_1}\epsilon\mathcal{H}(u_{xt})+\frac{\epsilon^2}{6}u_{xxt}. \end{cases} \tag{1}$$

System (1) describes the propagation of a weakly nonlinear internal wave propagating at the interface of two immiscible fluids with constant densities ρ_1, ρ_2 with $\rho_2/\rho_1>1$, which are contained at rest in a long channel with a horizontal rigid top and bottom, and the thickness of the lower layer is assumed to be effectively infinite (deep water limit). Constants a,ϵ are small positive numbers such that $\alpha=O(\epsilon^2)$ defined as $\alpha=\frac{a}{h_1}$ and $\epsilon=\frac{h_1}{L}$ that measure the intensity of nonlinear and the dispersive effects, respectively. h_1 denotes the thickness of the upper fluid layer and the parameters L and a correspond to the characteristic wavelength and characteristic wave amplitude, respectively. The function $u=u(x,t)$ is the velocity monitored at the normalized depth $z=1-\sqrt{\frac{2}{3}}$, and $\xi=\xi(x,t)$ is the wave amplitude, measured with respect to the rest level of the two-fluid interface.
The existence of travelling waves is a consequence of some results for positive operators in a cone due to T. Benjamin, J. Bona and D. Bose , which is useful in the case of not having an appropriate variational structure, as is the case

Boundary Blow-Up in Polyharmonic Equations with Power Nonlinearities

Paul Schmidt
Auburn University, USA

We study explosive radial solutions of semilinear elliptic PDEs involving an integer power of the Laplacian and a power-type nonlinearity. Depending on the sign and monotonicity of the nonlinearity, two very different types of blow-up behavior are observed. Explosive solutions of the first type diverge to infinity or negative-infinity, and their blow-up profile is by now fairly well understood. Explosive solutions of the second type, which do not occur in the classical second-order case, are unbounded from above and from below, oscillating wildly with increasing amplitude and frequency; details of their asymptotic behavior are only beginning to emerge. Our analysis employs dynamical-systems methods, applied to an associated system of asymptotically autonomous ODEs.

Boundedness of Weak Solutions of a Class of Non-Linear Elliptic Systems with Morrey Data

Lyoubomira Softova
Second University of Naples, Italy

Let $\mathbf{u} : \Omega \subset \mathbb{R}^n \to \mathbb{R}^N$, $n, N \geq 2$, be a *weak solution* of the non-linear elliptic system

$$\operatorname{div} \mathbf{A}(x, \mathbf{u}(x), D\mathbf{u}(x)) = \mathbf{b}(x, \mathbf{u}(x), D\mathbf{u}(x)), \quad x \in \Omega \tag{2}$$

where Ω is a *Reifenberg-flat domain* and $\mathbf{A}(x, \mathbf{z}, \Xi)$ and $\mathbf{b}(x, \mathbf{z}, \Xi)$ are *Carathèodory maps* satisfying *controlled growth conditions*. Precisely, for $n > 2$ there exist a constant$\Lambda > 0$ such that for $i = 1, \ldots, n$ and $j = 1, \ldots, N$ it holds

$$\begin{cases} |a_i^j(x, \mathbf{z}, \Xi)| \leq \Lambda\big(\varphi(x) + |\mathbf{z}|^{\frac{n}{n-2}} + |\Xi|\big) \\ |b_i(x, \mathbf{z}, \Xi)| \leq \Lambda\big(\psi(x) + |\mathbf{z}|^{\frac{n+2}{n-2}} + |\Xi|^{\frac{n+2}{n}}\big) \end{cases}$$

for a.a. $x \in \Omega$, all $(\mathbf{z}, \Xi) \in \mathbb{R}^N \times \mathbb{R}^{N \times n}$ and $\varphi \in L^{p,\lambda}(\Omega)$, $\psi \in L^{q,\mu}(\Omega)$ being suitable Morrey functions.

We cannot expect boundedness of the solution $\mathbf{u}$ of (2) unless we impose some restrictions on the structure of the operator $\mathbf{A}$. For this goal, we suppose that for large values of u^j, the operator $\mathbf{A}$ is *component-wise coercive*.

We show that if $\mathbf{u} \in W_0^{1,2}(\Omega, \mathbb{R}^N)$ is a *weak* solution of the system (2), than $\mathbf{u}$ is *essentially bounded* inΩ in terms of known quantities.

Bifurcation of Positive Solutions for the One-Dimensional (p, q)-Laplace Equation

Satoshi Tanaka
Okayama University of Science, Japan

In this talk, we study the bifurcation of positive solutions for the one-dimensional (p, q)-Laplace equation $(\varphi_p(u'))' + (\varphi_q(u'))' + \lambda(\varphi_p(u) + \varphi_q(u)) = 0$ under the Dirichlet boundary condition $u(-L) = u(L) = 0$, where $\varphi_p(u) = |u|^{p-2}u$, $p > 1$. We investigate the shape of the bifurcation diagram and prove that there exist five different types of bifurcation diagrams. As a consequence, we prove the existence of multiple positive solutions and show the uniqueness of positive solutions for a bifurcation parameter in a certain range. This is a joint work with Ryuji Kajikiya and Mieko Tanaka.

Global Bifurcation Diagrams for Problems with Sign-Changing Nonlinearities with High Multiplicity of Solutions

Andrea Tellini
EHESS, Paris, France
Juliàn Lòpez-Gòmez, Marcela Molina-Meyer, Fabio Zanolin

In this talk I will present some results of high multiplicity of positive steady states for the one dimensional logistic equation with diffusion and a weight in front of the nonlinearity that changes sign. Moreover, I will show the structure of the bifurcation diagrams of such positive steady states, obtained by varying a parameter which modulates the positive part of the weight. In particular, I will focus on situations where such diagrams are connected or present an arbitrarily high number of isolated components, according to the properties of the weight. The results have been obtained in collaboration with Juliàn Lòpez-Gòmez (Univ. Complutense de Madrid, Spain), Marcela Molina-Meyer (Univ. Carlos III, Madrid, Spain) and Fabio Zanolin (Univ. Udine, Italy).

Special Session 59: Mathematical Models of Cell Motility and Cancer Progression in Microenvironment: Design, Experiments, Mathematical Framework, and Hypothesis Test

Yangjin Kim, Konkuk University, Korea
Yi Jiang, Georgia State University, USA

Cancer is a complex, multi-scale process, in which genetic mutations occurring at a sub-cellular level manifest themselves as functional changes at the cellular and tissue scale. The main aim of this session is to discuss current stages and challenges in modeling tumor growth and developing therapeutic strategies. Specific goals of the session include: (i) to analyze both computational and analytical solutions to mathematical models from tumor modeling (ii) to discuss creative ways of laboratory experimentation for better clinical diagnosis (iii) to improve our biochemical/biomechanical understanding of fundamental mechanism of tumor growth such as analysis of signaling pathways in relative balances between oncogenes and suppressors. Both the immediate microenvironment (cell-cell or cell-matrix interactions) and the extended microenvironment (e.g. vascular bed) are considered to play crucial roles in tumour progression as well as suppression. Microenvironment is known to control tumor growth and cancer cell invasion to surrounding stromal tissue. However, it also prohibits therapeutics from accessing the tumor cells, thus causing drug resistance. Therefore, a thorough understanding of the microenvironment would provide a foundation to generate new strategies in therapeutic drug development. At the cellular level, cancer cell migration is a main step for metastasis and further progression of cancer and metastasis in a given microenvironment. Thus, understanding of cell motility under the control of signal transduction pathways would improve technical and specific advances in cancer therapy by targeting the specific pathways that are associated with the diseases. Analysis of mathematical models would identify fundamental (abstract) structure of the model system and shed a light on our understanding of tumor growth in the specific host tissue environment and biochemical and biomechanical interactions between players in cancer progression. More comprehensive multi-scale (hybrid) models can be used to meet the needs of designing patient-specific agents. The focus of this session is threefold: (a) to present mathematical models of tumor growth and analysis of the models (b) to discuss therapeutic strategies for curing the disease and to showcase mathematical models incorporating mechanical aspects of cancer cell movement and clinical implications (c) see recent development of cell-mechanical aspect of cell-ECM interactions.

Exploiting Evolutionary Trade-Offs As a Novel Cancer Therapy

Alexander Anderson
IMO, Moffitt Cancer Centre, USA
Arig Ibrahim Hashim, Mark Robertson-Tessi, Robert Gillies, Robert Gatenby

Solid tumours export metabolically derived acid into surrounding stroma. We view this microenvironmental acidosis as a niche construction evolutionary strategy in which acid-producing/acid-adapted cancer cell phenotypes benefit by decreasing the fitness of non-adapted stromal competitors, promoting local invasion. These phenotypic properties, in turn, promote transition from in-situ to invasive cancer and a progressive expansion of primary or metastatic tumours. However, there is a significant cost for maintaining an acid-producing/adapted phenotype due to reduced efficiency in energy production and increased energy demand for adaptations to the acidic environment. We hypothesize that this cost may be cancers Achilles heel and a novel route for therapeutic intervention. We have generated a multiscale mathematical model that predicts that even subtle perturbations in pH can dramatically alter the progression of invasive and non-invasive tumour populations. We investigated these model predictions in the TRAMP mouse model, our experiments demonstrated that an increase of intratumoural pHe by only 0.2 pH units prevented transition from in situ to invasive cancer during carcinogenesis and significantly reduced growth and invasion in primary and metastatic cancers. Taken together, our results demonstrate a novel strategy that exploits evolutionary trade-offs to steer the tumour population towards less invasive phenotypes.

A Model for Direction Sensing in Dictyostelium Discoideum: Ras Activity and Symmetry Breaking Driven by a Gbetagamma- Mediated, Galpha2-Ric8 – Dependent Signal Transduction Network

Yougan Cheng
University of Minnesota, USA
Hans Othmer

Many eukaryotic cells, including Dictyostelium discoideum (Dicty), neutrophils and other cells of the immune system, can detect and reliably orient themselves in chemoattractant gradients. In Dicty, signal detection and transduction involves a G-protein-coupled receptor (GPCR) through which extracellular cAMP signals are transduced into Ras activation via an intermediate heterotrimeric G-protein (G2). Ras activation is the first polarized response to cAMP gradients in Dicty. Recent work has revealed mutiple new characteristics of Ras activation in Dicty, thereby providing new insights into direction sensing mechanisms and pointing to the need for new models of chemotaxis. Here we propose a novel reaction-diffusion model of Ras activation based on three major components: one involving the GPCR, one centered on G2, and one involving the monomeric G protein Ras. In contrast to existing local excitation, global inhibition (LEGI) models of direction sensing, in which a fast-responding but slowly-diffusing activator and a slow-acting rapidly diffusing inhibitor set up an internal gradient of activity, our model is based on equal diffusion coefficients for all cytosolic species, and the unbalanced local sequestration of some species leads to gradient sensing and amplification. We show that Ric8-modulated G2 cycling between the cytosol and membrane can account for many of the observed responses in Dicty. including imperfect adaptation, multiple phases of Ras activity in a cAMP gradient, rectified directional sensing, and cellular memory.

Identifiability of Multistage Clonal Expansion Models in Cancer

Marisa Eisenberg
University of Michigan, Ann Arbor, USA
Andrew Brouwer, Rafael Meza

Multistage clonal expansion (MSCE) models of carcinogenesis and other continuous-time Markov process models are often used to relate cancer and other disease incidence to biological mechanism. However, successful parameter estimation depends on the identifiability of the underlying models. In this talk I will discuss recent work using a differential algebra-based approach to examine the identifiability of a class of clonal expansion models for cancer, determining identifiable combinations that can be successfully estimated from data and evaluating how different mechanisms can affect parameter estimation.

Biomarker for Cancer

Avner Friedman
Ohio State University, USA

Tumor microenvironment includes proteins, mRNAs, and microRNAs (miRs) expressed at abnormal levels. Some of these concentration levels are highly positively correlated to their concentration in blood serum, and could therefore potentially be used as non-invasive biomarkers for early diagnosis of cancer, as well as for monitoring the stage of cancer. In this talk I give two examples of potential biomarkers. The first example is uPAR as a biomarker for early detection of breast cancer recurrence. The second example involved expression of miRs in pancreatic cancer. The corresponding mathematical models are represented by systems of PDEs in a growing tumor. Assuming radial symmetry, the expression level of the potential biomarkers are quantitatively associated with the growing radius of the tumor. This is joint work with Wenrui Hao.

Modeling Cell-ECM Interactions in Cancer Invasion

Yi Jiang
Georgia Sate University, USA

The extracellular matrix (ECM) is important in many cellular processes, from development to wound healing and cancer invasion. While in vivo and in vitro data have established the strong correlation between the extracellular matrix remodeling and cancer invasion, it is not clear how much biomechanics matter in cancer invasion or it is all genetics and biochemistry. We have designed a series of mathematical models to understand how structure, density, alignment and mechanics of the ECM in regulating cell migration. I will present our latest ongoing work on quantifying ECM structure during cancer invasion and on modeling of biomechanical cell-ECM interactions.

Mathematical Modeling of Enzyme Cluster Formation in Cancer

Hye-Won Kang
University of Maryland, Baltimore County, USA

Glycolysis is a metabolic pathway involving enzyme-catalyzed reactions. Previous studies have observed large clusters of the enzymes involved in glycolysis and increased production of serine in cancer cells. We hypothesize that the production of large clusters of the enzymes will change the glucose flux into serine production in cancer cells, and constructed a mathematical model to describe enzyme clustering in normal and cancer cells using a system of ordinary differential equations. Using the method of the partial rank correlation coefficient, a subset of sensitive modeling parameters is identified. Enzyme turnover numbers, Michaelis constant, and concentrations of enzymes and metabolites were estimated based on the literature. Using this model, we analyzed various cases with different-sized enzyme clusters in both cancer and normal cells. We simulated these cases and were able to see the increased serine production in cancer cells when there are large-sized enzyme clusters. The simulation result is qualitatively consistent with the experimental result. This is joint work with Songon An and Jane Pan.

Cancer-Immune Dynamics of Oncolytic Virotherapy and Dendritic Cell Vaccines

Peter Kim
University of Sydney, Australia
Adrianne Jenner, Il-Kyu Choi, Jana Gevertz, Joanna Wares, Arum Yoon, Chae-Ok Yun

Recent experiments with engineered oncolytic adenovirus have caused substantial reduction in growth rates of tumors in mice. We develop ordinary differential equation (ODE) models based on the data from five different treatments. By fitting time series data of tumor growth to our ODE models, we attempt to elucidate the underlying cancer-virus and cancer-immune dynamics to clarify the strengths and limitations of oncolytic virotherapy combined with dendritic cell vaccines. Using modeling, we consider how different treatment strategies can be used to (1) rapidly kill the tumor with a goal of complete elimination or (2) maintain the tumor long-term at low levels. We also describe the problem of improving the delivery of oncolytic virus into tumors. Images show that viruses seem to penetrate hardly more than a few millimeters from the site of injection and only infect isolated and sparse clusters of cells, rather than dispersing comprehensively throughout the tumor. Understanding the kinetics of virus delivery into a tissue and the extracellular matrix poses a useful problem that could require the formulation of partial differential equation or other spatial models.

Androgen Resistance Prediction in Prostate Cancer Patients Under Androgen Suppression Therapy

Yang Kuang
Arizona State University, USA
Javier Baez

Predicting castrate resistant prostate cancer is critical to optimize treatment and improve the quality of life of advanced prostate cancer patients. We compare and analyze two plausible mathematical models that aim to forecast future levels of prostate-specific antigen (PSA) with the help of clinical data of locally advanced prostate cancer patients undergoing androgen deprivation therapy (ADT). While these models are simplifications of a previously published model, they fit data with the same level of accuracy and improve forecasting results. Both models can predict the progression of resistance. Model 1 models cancer cells in a single population while model 2 divides cancer cells into two types. Model 1 is simpler but can fit clinical data at the same or greater precision. However, we found the biologically more plausible model 2 can forecast future PSA levels more accurately. These findings suggest that including more realistic mechanisms of resistance may help predict long-term androgen resistance.

Computational Simulations of Glioma Invasion Using the Immersed Boundary Method

Wanho Lee
National Institute for Mathematical Sciences, Korea
Sookkyung Lim, Yangjin Kim

Gliomas are malignant tumors that are commonly observed in primary brain cancer. Glioma cells migrate through a dense network of normal cells in microenvironment and spread long distances within brain. In this poster we present a two-dimensional hybrid model in which a glioma cell is surrounded by normal cells and its migration is controlled by cell-mechanical components in the harsh microenvironment via the regulation of myosin II in response to chemoattractants. Our simulations reveal that the myosin II plays a key role in deformation of the cell nucleus as the glioma cell passes through the narrow intercellular space smaller than its nuclear diameter. In addition, our results suggest that in the presence of myosin II the strong signal of chemoattractants may retract invasive glioma cells back to the resection site so that they can be removed completely. This study sheds lights on the understanding of glioma infiltration through the narrow intercellular spaces and a potential approach for the development of anti-cancer invasion strategies.

Image-Guided Genomics Reveals Phenotypic Heterogeneity Supporting a Symbiotic Model of Collective Cancer Invasion

Adam Marcus
Emory University, USA
J. Konen, E. Summerbell, B. Dwivedi, K. Galior, Y. Hou, L. Rusnak, A. Chen, J. Saltz, P. Vertino, L. Cooper, K. Salaita, J. Kowalski, A.I. Marcus

To probe the phenotypic heterogeneity found in cell populations, we developed an image-guided genomics technique termed spatiotemporal genomic and cellular analysis (SaGA) that allows for precise selection and amplification of living and rare cells. SaGA was used on collectively invading 3-D cancer cell packs to create unique purified leader and follower cell cultures. The leader cell cultures are phenotypically stable and highly invasive in contrast to follower cultures, which show phenotypic plasticity over time and do not invade. Genomic and molecular interrogation reveals a VEGF-based vascular sprouting mimicry that facilitates recruitment of follower cells but not for leader cell motility itself, which instead utilizes focal adhesion kinase- fibronectin signaling. While leader cells provide an escape mechanism for followers, follower cells in turn provide leaders with increased proliferation and survival. These data support a symbiotic model of collective invasion where different phenotypic cell types cooperate to promote their escape.

Decoding of Intracellular Signal Transfer from FRET Imaging: Distinct Functions of Rac1 and Cdc42 in Cell Migration

Honda Naoki
Kyoto Univ., Japan
Yamao Masataka, Shin Ishii

We propose a computational approach for decoding how intracellular molecular signals are transferred during cell migration. In this approach, we performed FRET time-lapse imaging of Rho GTPases, Rac1 and Cdc42, and quantitatively identified the response functions that describe how molecular signals are transferred into the motile morphodynamics. Based on the identified response functions, we found that Rac1 and Cdc42 activation triggers laterally propagating membrane protrusion, and that the membrane protrusion is driven by temporal derivatives of Rac1 and Cdc42 activities. Using the response function, we could predict the morphological change from molecular activity, and its predictive performance provides a new quantitative measure of how much the Rho GTPases participate in the cell migration. Interestingly, we discovered distinct predictive performance of Rac1 and Cdc42 depending on the migration modes, indicating that Rac1 and Cdc42 distinctly contribute to persistent and random migration, respectively. Thus, our approach enabled us to uncover the hidden information processing rules of Rho GTPases in the cell migration.

Role of Cellular and Microenviromental Heterogeneities in Multidrug Resistance in Cancer: a Multiscale Approach

Gibin Powathil
Swansea University, Wales
Mark Chaplain

Although multidrug combination chemotherapy has been widely used in cancer treatment, the development of drug resistance by cancer cells continues to be a major impediment to the successful delivery of these multi-drug therapies. Studies have indicated that intra-tumoural heterogeneities have a significant role in driving resistance to chemotherapy in many human malignancies. Multiple factors including the internal cellular phenotypes, cell-cycle dynamics and the external microenvironment contribute to the intra-tumoural heterogeneity. In this talk, I will present a hybrid, multiscale, individual-based mathematical model, incorporating intracellular dynamics and changes in oxygen concentration, to study the effects of multidrug chemotherapy on a heterogeneous tumour. We will analyse various factors that may contribute to the potential multidrug resistance within the heterogeneous cancer cell mass.

A Systems Biology Approach to Understanding Cell-Substrate Interaction During Cell Spreading

Magdalena Stolarska
University of St. Thomas, USA
Aravind Rammohan

The mechanical interaction of a cell with its environment affects various intracellular processes. For example, stem cell fate is determined by substrate stiffness, and extracellular matrix stiffness determines malignant phenotypes in cancer cells. We develop a two-dimensional mathematical model and computational tool that allows us to investigate how various mechanical processes interact during cell spreading over a deformable substrate. The cell is modeled as a deformable hypoelastic continuum, and the substrate is treated as linear elastic. Focal adhesions, the macromolecular assemblies through which a cell attaches to the substrate, are modeled as collections of linear springs that can dynamically grow and shrink in a stress-dependent manner. The complex biochemical interactions governing the formation of the focal adhesions are reduced to a single reaction, and actin polymerization and actomyosin contractions are incorporated into the model via an active rate of deformation tensor. We use this model of the cell-substrate system to explain the mechanical mechanisms required to obtain increased cell spread areas on stiffer substrates and to investigate the effects of focal adhesion size on intracellular forces that develop during spreading.

Multiscale Computational Modelling of Cancer Growth and Spread: a Novel Three-Scale Mathematical Approach

Dumitru Trucu
University of Dundee, Scotland

Cancer invasion is a complex multi-scale phenomenon involving many inter-related genetic, biochemical, and cellular processes at many different spatial and temporal scales that play a crucial role in the overall cancer development. The process of invasion consists of cancer cells secreting various matrix-degrading enzymes, which degrade the surrounding tissue or the extracellular matrix. Combining abnormal proliferation with favourable migratory conditions enabled by altered cell adhesion characteristics, the cancer cells actively spread locally into the surrounding tissue. As these multiscale phenomena lead naturally to a question concerning the establishment of a fundamental framework that would enable a rigorous analysis and modelling of cancer invasion, in this talk we will present a new integrated multiscale modelling framework involving two biological scales, cell and tissue, as well as the links between these cell- and tissue-scale processes. This novel two-scale approach will explore the dynamics of cell adhesion inside the tumour in conjunction with the activity of various proteolytic processes occurring along the invasive edge of the tumour. Finally, we will present computational simulations of the resulting multiscale moving boundary model and discuss a number of important fundamental properties that follow.

Quantitative Studies Reveal a Relay Mechanism for TGF-Beta Signaling

Jianhua Xing
University of Pittsburgh, USA
Jingyu Zhang, Xiao-Jun Tian, YiJiun Chen, Weikang Wang

Epithelial to mesenchymal transition (EMT), a process of transforming polygon-shaped epithelial cells with tight cell-to-cell attachment to spindle-like mesenchymal cells with loose or rare cell-to-cell attach-

ment, has been suggested to play a key role in many pathological processes such as fibrosis and cancer metastasis. Previous studies showed that exogenesis signals such as TGF-β can induce EMT in many mammalian epithelial cell lines. In the canonical TGF-β signal transduction pathway, transmembrane TGF-β receptors (TGFBR) receive the extracellular signal, pass downstream via the Smad transcription factor family, and activate multiple genes such as Snail1, a key regulator of EMT. However, our measurements reveal a temporal gap between the early pulsed upregulation of Smads and late sustained Snail1 expression. After careful examination of the Smad dependent and independent TGF-β pathways, we hypothesize that sustained Snail1 activation is achieved through a network motif composed of Smads, Gli1/2 (a main component of the SHH pathway), and $GSK3\beta/\beta - catenin$ (main components of the WNT pathway). Our combined mathematical modeling and quantitative measurements confirmed this hypothesis. Currently we are using the CRISPR-Cas9 technique to fuse fluorescence proteins to selected players in the network, and will use live cell imaging to monitor the cell dynamics in real time.

Controlling the Heterogeneous Cellular Quiescent State by an Rb-E2F Network Switch

Guang Yao
University of Arizona, USA
Sarah Jungeun Kwon, Xia Wang, Nick Everetts, Kimiko Della Croce

Quiescence is a critical non-proliferative cellular state. Reactivating quiescent cells (e.g., fibroblasts, lymphocytes, and stem cells) to proliferation is fundamental to tissue repair and regeneration. Often described as the “G0 phase“, quiescence is in fact not a homogeneous state. As cells remain quiescent for longer durations, they move progressively “deeper“ into quiescence, exiting from which requires prolonged and stronger growth stimulation. Nevertheless, underlying mechanisms of deep vs. shallow quiescence remain an enigma, and represent a currently underappreciated layer of complexity in growth control. Previously, we have shown that the retinoblastoma (Rb)-E2F gene network functions as a bistable switch, converting graded and transient growth signals into an all-or-none E2F activity, which underlies the all-or-none transition from quiescence to proliferation. Here by coupling modeling and single-cell measurements, we show that quiescent depth is controlled by the activation threshold of the Rb-E2F switch, and that different network components have different efficacies in modulating quiescence depth. We also show that by affecting the Rb-E2F activation threshold, Notch pathway, circadian clock, as well as metabolic state modulate quiescence depth. Further elucidating the control of quiescence depth may help develop novel strategies to correct abnormal quiescent states of diseased cells.

Special Session 60: Infinite-dimensional Dynamical Systems from Differential Equations under Singular Perturbations

Alexandre Nolasco de Carvalho, Universidade de Sao Paulo, Brazil
Marcone Corra Pereira, Universidade de Sao Paulo, Brazil

This special session will discuss recent results concerned with qualitative behavior of infinite-dimensional dynamical systems subjected to singular perturbations which are originated by approximations in the parameters of the model. Such dynamical systems are often associated to autonomous and non-autonomous differential equations which possess a maximal compact and invariant set that attracts bounded subsets under the action of the semi-flow and contains all relevant information about the asymptotics of the system and is an important object of study (the global attractors). There are many researchers working to understand its existence, characterisation, robustness, as well as upper and lower semicontinuity under perturbations. Observe that the study of elementary invariant sets such as steady state solutions, periodic orbits and their unstable (stable) manifolds and their behavior under singular perturbation play an important role in the study of the characterisation and robustness of attractors.

Transverse Stability of Nonlinear Klein-Gordon Periodic Wavetrains

Jaime Angulo Pava
State University of Sao Paulo, Brazil
Ramon G. Plaza

In this talk we present the orbital stability of a class of spatially periodic wavetrain solutions to multi-dimensional nonlinear Klein-Gordon equations with periodic potential. We show that the orbit generated by the one-dimensional wavetrain is stable under the flow of the multi-dimensional equation under perturbations which are, on one hand, co-periodic with respect to the translation (or Galilean) variable of propagation, and, on the other hand, periodic with respect to the transverse directions. That is, we show their transverse orbital stability. The class of periodic wavetrains under consideration is the family of subluminal rotational waves, which are periodic in the momentum but unbounded in their position.

Limit of Nonlinear Elliptic Equations with Concentrated Terms and Varying Domains

Simone Bruschi
University of Brasilia, Brazil
Gleiciane S. Aragao

In this talk we discuss the limit of the solutions of a semilinear elliptic equations with nonlinear Neumann boundary conditions when the boundary presents a highly oscillatory behavior and the nonlinear term is concentrated in a region which neighbors the boundary of the domain. We consider two cases: the uniformly and the non uniformly Lipschitz oscillatory behavior in the boundary.

Non-Autonomous Dynamical Systems and Their Attractors Under Perturbations

Alexandre Carvalho
Universidade de Sao Paulo, Brazil
E. R. Aragao-Costa, M. C. Bortolan, T. Caraballo, J. A. Langa

We present a careful description of the relationship between pullback and uniform attractors, leading to a detailed description of the uniform attractor and providing the understanding of its dynamical structures. That will enable us to talk about upper and lower semicontinuity, topological and geometrical structural stability of uniform attractors, at least for a non-autonomous perturbation of a semigroup.

A Diffusion Equation with Localized Chemical Reactions

Cesar Hernandez Melo
Marina State University, Brazil
Edgar Mayorga

In this work, we study the existence and stability of equilibrium solutions of a reaction-diffusion equation with a Dirac delta distribution, Namely

$$u_t = u_{xx} + Z\delta(x)u + wu + au^p + bu^{2p-1},$$

where $u : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ denotes the unknown, $\delta : H^1(\mathbb{R}) \to \mathbb{R}$ defined by $\delta(g) = g(0)$ is the Dirac delta distribution localized at zero, and a, b, p, Z denote real numbers with $p > 1$.

The equilibrium solutions are obtained explicitly by direct integration of the equation after removing the distribution, as well as, from certain conditions which appear as an effect of the Dirac delta distribution. The stability/unstability of these solutions is obtained in the classical way: by analysing the spectrum of the linear operator that results of the linear

approximation of the vector field associated to the equation at the equilibrium solution. In this part, the spectral analysis of these operators is based on the analytic perturbation theory of linear operators.

Periodic Orbits of Strongly Indefinite Ill-Posed PDEs Via Rigorous Numerics

Jean-Philippe Lessard
Laval University, Canada
Marcio Gameiro,Roberto Castelli

In this talk, we introduce a computer-assisted technique for the analysis of periodic orbits of ill-posed PDEs. As a case study, our proposed method is applied to the Boussinesq equation, which has been investigated extensively because of its role in the theory of shallow water waves. The idea is to use the symmetry of the solutions and a Newton-Kantorovich type argument (the radii polynomial approach), to obtain rigorous proofs of existence of the periodic orbits in a weighted ell-one Banach space of space-time Fourier coefficients with geometric decay. We present several computer-assisted proofs of existence of periodic orbits at different parameter values.

Stability of Partially Damped Nonuniform Timoshenko System

To Fu Ma
University of Sao Paulo, Brazil
M. A. Jorge Silva, J. E. Munoz Rivera

This talk is concerned with the Timoshenko system with non-homogeneous coefficients. The main result shows the exponential stability of the partially damped system by requiring only locally, the well-known equal wave speeds assumption. Otherwise, we prove that the system is polynomially stable.

Bifurcation and Stability of Steady States of Parabolic Equations Under Indefinite Logistic Flux Boundary conditions

Gustavo Madeira
Federal University of Sao Carlos, Brazil
A. S. do Nascimento

We consider the heat equation on a bounded smooth domain supplied with a flux boundary condition driven by a logistic function, an indefinite weight and a positive parameter. Our aim is to completely describe the bifurcation and stability properties of the steady states for the dynamical system associated to the problem and draw the corresponding diagrams. The techniques for establishing our results are demanded according to the vanishing or not of the average of the weight over the boundary of the domain.

Orbital Stability of Periodic Traveling Wave Solution for a Kawahara-Type Equation

Fabio Natali
State University of Maringa, Brazil

In this talk, we investigate the orbital stability of periodic traveling waves for a Kawahara-Type equation. We prove that the periodic traveling wave, under certain conditions, minimizes a convenient functional by using an adaptation of the well known method developed by Grillakis, Shatah e Strauss. The required spectral property for the orbital stability was obtained by knowing the positiveness of Fourier transform associated with the periodic wave.

On the A Priori Bounds and The Existence of Positive Solutions for Hamiltonian Elliptic Systems.

Rosa Pardo
Universidad Complutense de Madrid, Spain
Nsoki Mavinga

In [1] the authors prove the existence of a-priori bounds for positive solutions of elliptic equations $-\Delta u = f(u)$ with Dirichlet homogeneous boundary conditions, when $f(u) = u^{\frac{N+2}{N-2}}/\ln(e+u)^{\alpha}$, with $\alpha > 2/(N-2)$, see [1, Corollary 2.2]. Previous result do not cover this nonlinearity. In view of this recent advance, it is natural to ask whether it is possible to obtain the corresponding results for systems. We prove a priori L^{∞}-bounds for classical positive solutions of Hamiltonian elliptic systems in bounded convex domains$\Omega \subset \mathbb{R}^N$, with a type of subcritical nonlinearity. By subcritical we understand a nonlinearity, say $f = f(s,t)$, such that $f(s,t)/|(s,t)|^{\frac{N+2}{N-2}} \to 0$ as $|(s,t)| \to \infty$. Our analysis provides a new class of nonliterary depending on both components, for which classical positive solutions of Hamiltonian elliptic systems are a priori bounded. We use a version for systems of moving planes arguments and an extension of Rellich-Pohozaev type identity, combined with some estimates lying on Morrey's Theorem.

References

[1] A. Castro and R. Pardo, A priori bounds for Positive Solutions of Subcritical Elliptic Equations, Rev. Mat. Complut. **28** (2015), 715-731.

[2] N. Mavinga and R. Pardo, A priori bounds for positive solutions of Hamiltonian elliptic systems, Preprint.

Parabolic Problems in Oscillating Thin Domains

Marcone Pereira
Universidade de Sao Paulo, Brazil

In this talk we discuss some recent results obtained about the asymptotic behavior of the solutions of semilinear parabolic problems with homogeneous Neumann boundary conditions posed on two dimensional thin domains with locally periodic structure on the boundary. We first obtain the limit problem assuming the thin domain degenerates to the unit interval also analyzing its dependence with respect to the geometry of the thin channel. Next we study the convergence of the nonlinear semigroup investigating the upper and lower semicontinuity of the family of global attractors taking dissipative assumptions to the system.

Unbounded Attractors for Parabolic Equations

Juliana Pimentel
Universidade Federal do ABC, Brazil
A. Carvalho, C. Rocha

We consider non-dissipative semilinear parabolic equations on a bounded interval. We review the recently developed theory for the autonomous version of this class of problems and present a characterization for the associated noncompact global attractor. We also examine a non-autonomous variation and take into account distinct regimes for the non-autonomous linear term; in this setting we investigate the dynamics on the related unbounded pullback attractor.

On Quasiconvergence of Solutions of Parabolic Equations on the Real Line

Peter Polacik
University of Minnesota, USA

We examine bounded solutions of semilinear parabolic equations on the real line. Such a solution is quasiconvergent, if it approaches a set of steady states in a localized topology. We start by showing examples of bounded solutions which are not quasiconvergent. Then we identify classes of initial data which give quasiconvergent solutions. Some of these results are based on joint papers with Hiroshi Matano.

Dynamic and Continuity for a Non-Classical Non-Autonomous Diffusion Problem

Felipe Rivero
Universidade Federal Fluminense, Brazil
Matheus C. Bortolan

In this talk I will show our study of the existence and continuity of four different notions of *non-autonomous attractors* for a family of non-autonomous non-classical parabolic equations given by

$$\begin{cases} u_t - \gamma(t)\Delta u_t - \Delta u = g_\epsilon(t,u), \text{ in}\Omega \\ u = 0, \text{ on } \partial\Omega. \end{cases}$$

in a smooth bounded domain$\Omega \subset \mathbb{R}^n$, $n \geqslant 3$, where the terms g_ϵ are a small perturbation, in some sense, of a function f that depends only on u.

Behavior of the P-Laplacian on Thin Domains

Ricardo Silva
University of Brasilia, Brazil
Marcone C. Pereira

In this talk we present the limiting behavior of solutions of quasilinear elliptic equations on thin domains. As we will see the boundary conditions play an important role. If one considers homogeneous Dirichlet boundary conditions the sequence of solutions will converges to the null function, whereas, if one considers Neumann boundary conditions there is a non trivial equation which determines the limiting behavior.

Reaction-Diffusion Problems with Large-Diffusion Up to the Boundary

Alejandro Vidal-Lopez
Xi‘an Jiaotong-Liverpool University, Peoples Rep of China
Anibal Rodriguez-Bernal

We will discuss some results regarding the continuity of reaction-diffusion problems when the diffusion in some region of the domain becomes larger and larger. This induces a homogenisation process which determines the type of the limit problem. We will consider the case in which the large-diffusion area is either a neighbourhood of the boundary or it has non-empty contact with parts of the boundary having Robin and Dirichlet conditions.

Special Session 61: Recent Trends in Navier-Stokes Equations and Related Problems

Sarka Necasova, Institute of Mathematics, Academy of Sciences of the Czech Republic, Czech Rep
Reimund Rautmann, University of Paderborn, Germany
Werner Varnhorn, University of Kassel, Germany

Due to the active research on this field in many places around the world, there are important new results in diverse directions: e.g. concerning local smoothness criteria for 3D-solutions, regularity by properties of the vorticity, singular solutions, lower bounds to blowing up solutions, norm inflation, analyticity in general spaces, Lagrangean representation of flows, fluid flow with chemotaxis. The aim of our special session is to bring together reseachers working in different directions and to initiate fruitfull discussions.

Global Well-Posedness of the Axisymmetric Navier-Stokes Equations in the Exterior of an Infinite Cylinder

Ken Abe
Kyoto University, Japan

We consider the initial-boundary value problem of the Navier-Stokes equations for axisymmetric initial data with swirl in the exterior of an infinite cylinder, subject to the slip boundary condition. We construct global solutions and estimate potential singularities of axisymmetric flows in the whole space by using the size of the cylinder. The proof is based on the Boussinesq system. We show that the system is globally well-posed in the exterior domain for axisymmetric initial data without swirl.

Local Regularity Condition Involving Two Velocity Components for the Navier-Stokes Equations

Hyeong-Ohk Bae
Ajou University, Korea
Jorg Wolf

The present paper deals with the problem of local regularity of weak solutions to the Navier-Stokes equation in$\Omega \times (0,T)$ with forcing term $\mathbf{f}$ in L^2. We prove that $\mathbf{u}$ is strong in a sub-cylinder $Q_r \subset \Omega \times (0,T)$ if two velocity components u^1, u^2 satisfying a Serrin-type condition.

Regularity of the 3D Navier-Stokes Equations and Related Problems

Mimi Dai
University of Illinois Chicago, USA
Alexey Cheskidov

As one of the most significant problems in the study of partial differential equations arising in fluid dynamics, Leray's conjecture in 1930s regarding the appearance of singularities for the 3-dimensional (3D) Navier-Stokes equations (NSE) has been neither proved nor disproved. The problems of blow-up have been extensively studied for decades using different techniques. By using a method of wavenumber splitting which originated from Kolmogorov's theory of turbulence, we obtained a new regularity criterion for the 3D NSE. The new criterion improves the classical Prodi-Serrin, Beale-Kato-Majda criteria and their extensions. Related problems, such as the well/ill-posedness, will be discussed as well.

On Measure Valued Solutions to the Euler and Navier-Stokes System

Eduard Feireisl
Czech Academy of Sciences, Prague, Czech Rep
E. Chiodaroli, P. Gwiazda, O. Kreml, A. Swierczewska-Gwiazda, E. Wiedemann

We introduce a new concept of dissipative measure valued solution to the Euler and Navier-Stokes system. We show several properties of these solutions including the weak-strong uniqueness principle with applications to certain stability problems. We also show the existence of a measure-valued solution of the compressible Euler system that cannot be obtained as a limit of a sequence of weak solutions. This is in sharp contrast with the measure-valued solutions of the incompressible Euler system when this is always possible.

Onsager's Conjecture for the Compressible Models

Piotr Gwiazda
Polish Academy of Sciences, Poland
Eduard Feireisl, Agnieszka Ŝwierczewska-Gwiazda, Emil Wiedemann

We give sufficient conditions on the regularity of solutions to the inhomogeneous incompressible Euler and the compressible isentropic Euler systems in order for the energy to be conserved. Our strategy relies on commutator estimates similar to those employed by P. Constantin W. E and E. Titi for the homogeneous incompressible Euler equations.

Robustness of Strong Solutions of the Navier-Stokes Equations with Various Types of Boundary Conditions

Petr Kucera
Czech Technical University, Czech Rep

We study qualitative properties of solutions of a system of the Navier-Stokes equations with various types of boundary conditions. Some properties of these solutions, e.g. local in time existence of strong solutions, are presented. Further, consequences of perturbations of initial conditions of strong solutions are investigated. We prove that the corresponding solutions are also strong for sufficiently small perturbations in some norms.

Existence Results for Equations Describing Multicomponent Reactive Flows

Martine Marion
Ecole Centrale de Lyon, France
R. Temam

We investigate some mathematical issues arising in the context of the coupling of multi-species exothermic chemical reactions to fluid motion. The incompressible Navier-Stokes equations are coupled with equations for the temperature and the concentrations of the chemical species. The equations for the chemistry involve non-linear diffusion coefficients that are obtained by resolution of the so-called Stefan-Maxwell equations.

The Motion of Incompressible Viscous Fluid Around a Moving Rigid Body

Sarka Necasova
Academy of Sciences, Institute of Math., Czech Rep
S.Kracmar, P. Deuring

The dynamics of fluids, i.e. liquids and gases, is an important part of the continuum mechanics. This lecture is devoted to the qualitative analysis of mathematical models of motion of a viscous incompressible fluid around a compact body $\mathcal{B}$, translating and rotating in the fluid with given time–independent translational and angular velocities u_∞ and ω. The translation can be considered, without the loss of generality, to be parallel to the x_3–axis. We shall discuss the fundamental solution of the Oseen rotating system and the asymptotic decay for the Oseen case and also for nonlinear case,

Stability of a Quasi-Steady Flow of a Viscous Incompressible Fluid Past a Rotating Obstacle

Jiri Neustupa
Czech Academy of Sciences, Czech Rep

We deal with stability of a solution of the mathematical model, describing the flow of a viscous incompressible fluid past a rotating body. The considered solution is steady in the body-fixed frame. We show that the stability follows from a "sufficiently fast" decay (in time) of a finite number of suitable functions in a norm restricted to a "sufficiently large" bounded region around the body. We also discuss the relation to the spectrum of an associated linearized operator. No assumption on the smallness of the steady solution is required.

Navier-Stokes Flow in the Weighted Hardy Spaces

Takahiro Okabe
Hirosaki University, Japan
Yohei Tsutsui

The asymptotic expansions of the Navier-Stokes flow in the whole spaces and the rates of decay are discussed with aid of weighted Hardy spaces. Fujigaki and Miyakawa, Miyakawa proved the n-th order asymptotic expansion of the Navier-Stokes flow if initial data has suitable pointwise decay in space and n-th moment is finite. In this presentation, it is clarified that the moment condition for initial data is essential in order to obtain higher order asymptotic expansion of the flow and to consider the rapid time decay problem. Firstly we derived the existence theorem in the weighted Hardy spaces with smallness only in L^n as a refinement of the previous work of the second author, Tsutsui. As an application, the rapid time decay of the flow are investigated with aid of asymptotic expansions and of the symmetry conditions introduced by Brandolese.

Heat-Conducting, Compressible Mixtures with Multicomponent Diffusion

Milan Pokorny
Charles University in Prague, Czech Rep
Vincent Giovangigli, Piotr B. Mucha, Tomasz Piasecki, Ewelina Zatorska

We study a model for heat conducting compressible chemically reacting gaseous mixture, based on the coupling between the compressible Navier–Stokes–Fourier system and the full Maxwell–Stefan equations. The viscosity coefficients are density-dependent functions vanishing on vacuum and the internal pressure depends on species concentrations. We consider the question of existence of a solution to this system and based on several levels of approximations we construct a weak solution without any restriction on the size of the data. Furthermore, we also consider (for a mixture of isomers) the steady version of the system above. The viscosity coefficients are considered to be temperature-dependent. We construct weak solutions for different formulations of the problem (weak solution or variational entropy solution).

Monotonicity in Singular Volterra Integral Equations and Application.

Reimund Rautmann
Paderborn University, Germany

The wellknown bounds to the resolvent of linear singular Volterra integral equations imply a useful monotonicity in the prescibed data. The application to a (seemingly new) approximation scheme to the 3D- initial-boundary value problem of the Navier-Stokes equations leads to the proof of convergence and error estimates in a scale of Banach spaces on each compact time interval in case of sufficiently small initial values.

News on the Helmholtz Decomposition and Very Weak Solutions

Juergen Saal
University Düsseldorf, Germany

The aim of the talk is to construct very weak solutions to the Navier-Stokes equations for a general class of boundary conditions including partial slip type. We essentially follow the approach given by Herbert Amann which relies on interpolation-extrapolation scales generated by the corresponding Stokes operator. In this regard also a suitable extension of the Helmholtz projector to function spaces of negative order plays a crucial role. Related to this fact, Amann's approach includes a misinterpretation of the properties of this Helmholtz extension operator. A second aim of the talk therefore is to clarify this misinterpretation and to demonstrate that it has no influence on the correctness of Amann's results on the existence of very weak solutions.

Optimal Control in Blood Flow Simulations

Adelia Sequeira
University of Lisbon, Portugal
Jorge Tiago, Telma Guerra

Blood flow simulations can be improved by integrating known data into the numerical simulations. Data Assimilation techniques based on a variational approach play an important role in this issue. We propose a non-linear optimal control problem to reconstruct the blood flow profile from partial observations of known data in different geometries. To simplify, blood flow is assumed to behave as a Navier-Stokes fluid. Using a Discretize then Optimize (DO) approach, we solve a non-linear optimal control problem and present numerical results that indicate its robustness with respect to different idealized geometries and measured data. Blood flow in real vessels will also be considered, including the discussion of a particular clinical case.

The Regularity of Solutions to the Navier-Stokes Solutions Based on Items of the Velocity Gradient

Zdenek Skalak
Czech Technical University, Czech Rep

We will present a survey of recent criteria for the regularity of the solutions to the 3D Navier-Stokes equations based on information on several items of the velocity gradient.

About Questions of Stability for Multiple Gasballs

Gerhard Strohmer
University of Iowa, USA

We are considering barotropic gasballs in space. We show that an energy stability condition implies the linear stability of such balls and discuss the consequences for non-linear stability.

On Ill-Posedness of Euler System with Non-Local Terms

Agnieszka Swierczewska-Gwiazda
University of Warsaw, Poland
J. A. Carrillo, E. Feireisl, P. Gwiazda

The talk will concern the issue of existence of weak solutions to the Euler equations with pairwise attractive or repulsive interaction forces and non-local alignment forces in velocity appearing in collective behavior patterns. We consider several modifications of the Euler system of fluid dynamics including its pressureless variant driven by non-local interaction repulsive-attractive and alignment forces in the space dimension $N = 2, 3$. These models arise in the study of self-organisation in collective behavior modeling of animals and crowds. We adapt the method of convex integration, adapted to the incompressible Euler system by De Lellis and Szèkelyhidi, to show the existence of infinitely many global-in-time weak solutions for any bounded initial data. Then we consider the class of dissipative solutions satisfying, in addition, the associated global energy balance (inequality). The discussed result is in a certain sense negative result concerning stability of particular solutions. It turns out that the solutions must be sought in a stronger class than that of weak and/or dissipative solutions. We essentially show that there are infinitely many weak solutions for any initial data and that there is a vast class of velocity fields that gives rise to infinitely many admissible (dissipative) weak solutions.

Local in Time Regularity Region for General Weak Solutions of the Navier-Stokes Equations

Werner Varnhorn
Kassel University, Germany
Reinhard Farwig, Hermann Sohr

We consider a general weak solution of the Navier-Stokes equations concerning the unsteady motion of a viscous incompressible fluid. Our main result concerns Leray's structure theorem (Leray 1934) , extending some well-known results (Galdi 2000) to several directions. In particular, we do not assume the validity of the strong energy inequality for the underlying weak solution.

Special Session 62: Imaging Methods in Coupled Physics Models

Alexandru C. Tamasan, University of Central Florida, USA

A current trend in Inverse Problems assume models where interacting fields are model by two or more physical phenomena. Examples include but are not llimited to Elastography, Thermo/Photoacoustic, AcoustoOptics, Magnetic Resonance Electrical Impedance Tomography (MREIT), Current Density Impedance Imaging, Lorentz force based Electrical Impedance Tomography. This minisymposium will bring together researchers in the above mentioned areas to present their advances made both in the mathematics, and the modeling employed in acquiring the data in these imaging methods. The organizers believe that time passed since the last minisymposium within the AIMS meeting (the 9th) in these areas is optimal for reporting in the new findings.

Photoacoustic Imaging and Thermodynamic Attenuation

Sebastian Acosta
Baylor College of Medicine, USA
Carlos Montalto

We consider a mathematical model for photoacoustic imaging to take into account attenuation due to thermodynamic dissipation. The propagation of acoustic waves is governed by a scalar wave equation coupled to the heat equation for the excess temperature. We seek to recover the initial acoustic profile from knowledge of boundary measurements. This inverse problem is a special case of boundary observability for a thermoelastic system. This leads to the use of control/observability tools to prove the unique and stable recovery of the initial acoustic profile in the weak thermoelastic coupling regime. We propose and implement (numerically) a reconstruction algorithm.

Generalized Radon Transforms Arising in Single Scattering Optical Tomography

Gaik Ambartsoumian
University of Texas at Arlington, USA

Single scattering optical tomography (SSOT) uses light photons that scatter once in the body to recover its interior features. The mathematical model of 2D SSOT is based on the broken ray (or V-line Radon) transform, which puts into correspondence to an image function its integrals along V-shaped piecewise linear trajectories in a plane. The related conical Radon transform appears in some 3D imaging techniques based on Compton scattering effect. The process of image reconstruction in these modalities requires inversion of the corresponding transforms. The talk will discuss the known results and recent developments in the study of these integrals transforms, their applications and the open problems.

On the Stability of Reconstructing Convection Terms from Boundary Measurements

Toufic el Arwadi
Beirut Arab University, Lebanon
Hoda Malak, Alexandru Tamasan

This talk concerns the inverse convection problem. A. Tamasan introduced a direct method to reconstruct the convection terms. However, the inverse convection problem is ill posed. In this talk, a regularization technique is presented. Some technical lemmas and estimates for the forward and inverse problem are presented, moreover, error estimates are obtained from this regularization will be discussed .

Direct Computation of the Radiative Transfer Equation for Near-Infrared Light Propagation in Biological Tissue

Hiroshi Fujiwara
Kyoto University, Japan
Naoya Oishi

Near-infrared light is a new modality to measure brain activities noninvasively. To realize it, the analysis of the radiative transfer equation (RTE), which is a mathematical model of light propagation in tissue, is required. Since the discretization of RTE is a large-scale problem, Monte Carlo method has been conventionally used. In this talk we show some numerical examples using the direct discretization of the stationary RTE in the three dimensions for accurate and reliable imaging based on the recent progress of computer resources.

Fourier Expansion of Disk Automorphisms and Scattering in Layered Media

Peter Gibson
York University, Canada

An exotic family of orthogonal polynomials on the disk serves to formulate a remarkable Fourier expansion of the composition of a sequence of Poincarè disk automorphisms. The resulting identity is inti-

mately connected with the scattering of plane waves in piecewise constant layered media. Indeed, a recently established combinatorial analysis of scattering sequences provides a key ingredient of the proof. At the same time, the polynomial obtained by truncation of the Fourier expansion elegantly encodes the structure of the nonlinear measurement operator associated with the finite time duration scattering experiment.

Solvability of Discrete Inverse Problems on Networks

Fernando Guevara Vasquez
University of Utah, USA

The complex geometric optics approach has been used to show uniqueness for several inverse problems, including the electrical impedance tomography problem. We show how to use ideas from complex geometric optics to show uniqueness for inverse problems on networks, where the goal is to recover possibly complex node or edge based quantities. Our results show that if the linearized problem is injective then the non-linear problem admits a unique solution, except on a zero measure set.

Photoacoustic Tomography with Circular Integrating Detectors

Yulia Hristova
University of Michigan - Dearborn, USA
Sunghwan Moon, Dustin Steinhauer

We propose a detector geometry for photoacoustic tomography that offers the practical advantage that it could be implemented with a single rotating circular detector. The Radon type transform that arises can be decomposed into the spherical Radon transform and the Funk transform. An inversion formula and numerical simulations demonstrating the proposed algorithm will be presented.

Schwartz Theorem for the Radon Transform and Metal Artifact Reduction

Sungwhan Kim
Hanbat University, Korea
Chi Young Ahn, Kiwan Jeon

In computed tomography (CT), X-ray photons passing through metallic objects in a X-ray scanned object are highly attenuated and the number of photons detected at the detector pixels behind them is actually zero. This photon starvation causes the metal shadow area in the sinogram domain in which the projection data are considered to be missing. CT images reconstructed by the filtered back projection (FBP) from such sinogrms suffer from severe streak artifacts which deteriorate the image quality and make it difficult to interpret valuable details close to metallic objects for medical diagnosis. During the last couple of decades, it has become a challenging problem to reduce streak artifacts caused by metallic objects. One of important methods for metal artifact reduction is the projection completion approach. In this method, the missing projections in the sonogram domain are synthesized from metal-free projections neighboring to the metal shadow area using various inpainting techniques such as polynomial interpolation, wavelet interpolation, adaptive filtering, total variation, etc. In this talk, we show that if a sinogram synthesized by the projection completion technique is in the range of the X-ray Radon transform, its reconstructed CT image is same to the true image outside the convex hull of metal regions implanted in the scanned object and introduce a novel projection completion algorithm which exploits the Schwartz theorem for the X-ray Radon transform in order to reduce metal artifacts in CT.

Nonlinear Imaging with Waves Via Reduced Order Model Backprojection

Alexander Mamonov
University of Houston, USA
Vladimir Druskin, Andrew Thaler, Mikhail Zaslavsky

We introduce a novel nonlinear method for imaging reflectors with waves. The method is based on reduced order models (ROMs). The ROMs are projections of the wave equation propagator on the subspace of certain time domain wavefield snapshots. These projections can be computed entirely from the time domain data measured on the boundary. The image is a backprojection of the ROM using the subspace basis for a known kinematic velocity model. Nonlinear implicit orthogonalization of wavefield snapshots differentiates our approach from conventional linear methods. It automatically removes the multiple reflection artifacts. Moreover, it doubles the resolution in range direction compared to conventional time reversal. Both seismic exploration and medical ultrasound imaging applications are considered.

Multiwave Imaging in an Enclosure with Variable Sound Speed

Carlos Montalto Cruz
University of Washington, USA
Sebastian Acosta

This talk considers the problem of photoacoustic and thermoacoustic tomography in the presence of physical boundaries such as reflectors or interfaces, which reflect some wave energy back into the domain. To model the physical boundaries we consider the wave problem where an impedance Robin boundary condition is imposed. We relate the inverse problem with a statement in boundary observability and stabilization of waves. We present uniqueness and stability of the inverse problem and propose two different reconstruction methods. In both cases, if well-known geometrical conditions are satisfied, the approaches are naturally suited for variable wave speed and for measurements on a subset of the boundary.

Photoacoustic Tomography Model with Varying Material Density and Variable Bulk Modulus.

Kamran Sadiq
The Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austria
A. Beigl, P. Elbau , O. Scherzer

We study the Photoacoustic tomography model taking into consideration that the material has variable density and bulk modulus is spatially varying. We propose an approach to simultaneously reconstruct absorption density, bulk modulus and material density from photoacoustic measurements using photoacoustic sectional imaging.

Acoustic CT for Concrete Structures

Takashi Takiguchi
National Defense Academy of Japan, Japan

For the time being, there exist no non-destructive inspection method for concrete structures which enables us to reconstruct their interior structure, concretely and completely. In this talk, we shall discuss how to develop such a non-destructive inspection method for concrete structures, for which we shall apply ultrasonic waves. We shall pose a mathematical problem in order to establish a tomographic non-destructive inspection method and discuss how to solve this problem from the viewpoint of practical applications.

Special Session 63: Topological Methods for Nonlinear Boundary and Initial Value Problems

John R. Graef, University of Tennessee at Chattanooga, USA
Miroslawa Zima, University of Rzeszow, Poland

Topological methods have proved to be an important technique in the study of boundary and initial value problems and related topics for ordinary and partial differential equations. There has been a rapidly growing interest in applying these techniques to such problems in recent years, and this session is devoted to the use of such methods in the study of boundary and initial value problems including singular problems and those with multi-point conditions.

On the Solution to the Conserved Kuramoto-Sivashinski Equation

Gabriella Bognar
University of Miskolc, Hungary

The study of the growth processes of surfaces has turned out to be source of a variety of nonlinear dynamics. The growth is intrinsically two-dimensional and the variable describing the growth dynamics is the local height $h(x,y,t)$ of the surface. Our aim is to examine the solutions to the conserved Kuramoto-Sivashinsky equation (CKS)

$$h_t + \Delta\left(h + \Delta h + \gamma\left|\nabla h\right|^2\right) = 0, \quad (x,y) \in (a,b)^2, \quad t > 0$$

and γ is a positive parameter. The CKS equation displays coarsening in the growth processes of surfaces.

Asymptotically Linear System of Three Equations Near Resonance

Maya Chhetri
UNC Greensboro, USA
Petr Girg

We will consider an asymptotically linear system of three semilinear equations satisfying Dirichlet boundary conditions. The nonlinear perturbations are Carathèodory functions that are bounded by some appropriate nonnegtaive function. There are only two simple eigenvalues, associated to the linear part, whose corresponding eigenfunctions are componentwise nonnegative. We will discuss bifurcation of positive solutions from infinity from these simple eigenvalues. In particular, we will provide sufficient conditions under which the system has bifurcation of positive solutions (from infinity) from both, one, or none of the simple eigenvalues.

A Survey on Recent Results Related to the Pumping Effect

Jose Angel Cid
University of Vigo, Spain

We shall review some recent results about the existence of positive periodic solutions for a singular second order equation. This boundary value problem was presented in 2006 by G. Propst as a simple model for the "pumping effect".

Generalized Third Order Coupled Systems with Full Nonlinearities

Infeliz Coxe
ESP- Malanje, Angola
Feliz Minhos

This work studies the solvability of a nonlinear third order coupled system composed by coupled third order fully differential equations with L^1-Carathèodory nonlinearities, and two-point boundary conditions. An adequate truncature together with Nagumo-type conditions allow the dependence of the nonlinearities on the second derivatives. By lower and upper solutions method is obtained some strips where the unknown functions and their derivatives must lie, which provides some qualitative data on the solutions.

On the Existence of Three Positive Solutions for a Two-Point Second-Order BVP

Abdulkadir Dogan
Abdullah Gul University, Turkey

In this paper, we consider the existence of three positive solutions for the boundary value problem, with using the new fixed-point theorem and imposing growth conditions on the nonlinearity. The interesting point is that the nonlinear term f explicitly involves a first-order derivative.

Green's Functions of Differential Systems with Reflection

F. Adrián F. Tojo
University of Santiago de Compostela, Spain
Alberto Cabada

In this work we generalize the previous theory of differential equations with reflection to the case of systems. This is achieved by remaking the classical approach to ordinary differential systems through fundamental matrices. In this process we can appreciate some important similarities and differences between both cases.

Modelling Idiopathic Scoliosis - a Test Case Using ATT(Antalgic-Trak Technology) Device

Joao Fialho
American University of the Middle East, Kuwait

In this paper the author studies and develops the mathematical model for idiopathic scoliosis in human spine and its therapy using the ATT (Antalgic Track Technology) device. The therapy for idiopathic scoliosis using this device relies on the application of vertical traction to the spine. It is a relatively new technique that differs from the commonly used orthotic stabilization, making the existent models inaccurate or insufficient.
The model is composed by the fourth order differential equation

$$EIy_1^{(iv)}(x) + Py_1''(x) = -Py_0''(x) + Py_0(x) \qquad (1)$$

for $x \in \left[\frac{L}{2}, -\frac{L}{2}\right]$ and P, EI, constants to be defined, along with the boundary conditions

$$y_1\left(-\frac{L}{2}\right) = y_1\left(\frac{L}{2}\right) = 0, \qquad (2)$$

$$y_1''\left(-\frac{L}{2}\right) = 0, \qquad (3)$$

$$-EIy_1''\left(\frac{L}{2}\right) + k_i y_1\left(\frac{L}{2}\right) = 0. \qquad (4)$$

The arguments applied are based on the upper and lower solution method and a test case is studied as the main result. In this test case, based on the information provided by the upper and lower solution, a therapy plan using the ATT device is outlined.

Existence Results for Initial Value Problems of First-Order Systems of Stieltjes Differential Equations

Marlene Frigon
University of Montreal, Canada
R. López Pouso

We present the basic theory of existence and uniqueness of solutions for systems of differential equations with the usual derivative replaced by a Stieltjes derivative. This derivative, called g-derivative, was introduced by Lòpez Pouso and Rodrìguez [1]. The problems that we consider contain as particular cases dynamic equations on time scales and impulsive ordinary differential equations.

References

[1] R. Lòpez Pouso and A. Rodrìguez, A new unification of continuous, discrete, and impulsive calculus through Stieltjes derivatives, Real Anal. Exchange **40** (2014/15), no. 2, 1–35.

[2] M. Frigon and R. Lòpez Pouso, Theory and applications of first–order systems of Stieltjes differential equations, Adv. Nonlinear Anal. (to appear).

An Application of a New Cone to BVPs with Nonlocal, Nonlinear Boundary Conditions

Christopher Goodrich
Creighton Preparatory School, USA

We demonstrate that by using a new order cone, existence results for boundary value problems with nonlocal, nonlinear boundary conditions can be improved. In addition, the cone is flexible enough to be useful in the case of vanishing or nonpositive Green's functions. Finally, together with the use of a particular open set, existence results utilizing only pointwise (rather than limit- or interval-type) conditions can be given.

A Nonlinear Fractional Boundary Value Problem Involving Riemann-Liouville Fractional Derivatives of Two Different Orders

John Graef
University of Tennessee at Chattanooga, USA
Min Wang, Grant Yost

The authors consider the nonlinear boundary value problem consisting of the fractional differential equation

$$-D_{0+}^{\alpha} z + aD_{0+}^{\beta} z = w(t)f(t, z), \quad 0 < t < 1,$$

and the boundary conditions

$$z(0) = z'(0) = \cdots = z^{(n-2)}(0) = 0, \quad z(1) = k,$$

where $k \in \mathbb{R}$, $a \in \mathbb{R}$, $n \in \mathbb{N}$, $0 \leq \beta \leq n-2$, $n-1 < \alpha < n$, $w \in L[0,1]$ with $w(t) \geq 0$, $f \in C([0,1]\times\mathbb{R}, \mathbb{R})$, and D^{γ}_{0+} is the γ-th left Riemann-Liouville fractional derivative. They give sufficient conditions for the existence and uniqueness of solutions to this problem. In so doing, they introduce a new approach that may be useful in analyzing such problems with other types of boundary conditions.

Impulsive Boundary Value Problems on Unbounded Domains

Nickolai Kosmatov
University of Arkansas at Little Rock, USA

We apply cone-theoretic methods to study positive solutions of impulsive differential equations of second order on an unbounded domain. The formulation of the problem involves infinitely many impulsive conditions.

Positive Solutions of Some Boundary Value Problems Via the Fixed Point Index Theory for Nowhere Normal-Outward Compact Maps

Kunquan Lan
Ryerson University, Canada

In this talk, I shall briefly present recent joint work on the fixed point index theory for nowhere normal-outward compact maps. Applications to the existence of positive solutions for some boundary value problems will be given.
This is joint work with Professor Guangchong Yang

Existence of Solutions to a Discrete Fourth Order Periodic Boundary Value Problem

Xueyan Liu
University of Tennessee at Chattanooga, USA
John Graef, Lingju Kong

We consider periodic boundary value problems for a fourth order nonlinear difference equations. We use variational methods and critical point theory to get sufficient conditions for the existence of at least one solution, two solutions, and nonexistence of solutions. The advantage that primitive function of the nonlinear term is not involved in the conditions of our main results make it much easier to apply our theorems in specific problems.

Existence of Solutions of Singular Fractional Boundary Value Problem

Jeffrey Lyons
Nova Southeastern University, USA
Christina A. Hollon, Jeffrey T. Neugebauer

We use iterative and fixed point methods to show the existence of a positive solution of a fractional boundary value problem $D^{\alpha}_{0+}x + f(t, x, D^{\mu}_{0+}) = 0$, 0

Green's Functions for 2n-Order Boundary Value Problems

Lucía López Somoza
University of Santiago de Compostela, Spain

We express the Green's functions of the 2n-order linear differential equation coupled with different boundary value conditions as a linear combination of Green's functions of Periodic problems. From that expressions we are able to deduce some results in spectral theory and to compare the solutions of the considered problems.

Heteroclinic Solutions for ϕ-Laplacian Equations

Feliz Minhos
Evora University, Portugal

In this paper we consider a second order discontinuous equation with a ϕ -Laplacian, in the real line, with ϕ an increasing homeomorphism such that $\phi(0) = 0$ and $\phi(\mathbb{R}) = \mathbb{R}$. We remark that the existence of heteroclinic solutions is obtained without asymptotic or growth assumptions on the nonlinearities ϕ and f. Moreover, as far as we know, our main result is even new when $\phi(y) = y$.

Existence and Comparison Results for Eigenvalues of a Higher Order Fractional Boundary Value Problem with a Fractional Boundary Condition

Jeffrey Neugebauer
Eastern Kentucky University, USA
Angela M. Koester

Let $\alpha \in (n-1, n]$. We show the existence of and then compare smallest eigenvalues of the fractional boundary value problems $D^{\alpha}_{0+}u + \lambda_1 p(t)u = 0$, $D^{\alpha}_{0+}u + \lambda_2 q(t)u = 0$, $t \in (0,1)$, satisfying boundary conditions $u^{(i)}(0) = 0$, $i = 0, 1, \ldots, n-2$, $D^{\beta}_{0+}u(1) = 0$, $\beta \in [0, n-1]$, where p and q are nonnegative continuous functions on $[0,1]$ which do not vanish identically on any nondegenerate compact subinterval of $[0,1]$. Here D^{α}_{0+} and D^{β}_{0+} denote the standard Riemann-Liouville fractional derivatives. The cases where $\beta = 0$ and $\beta > 0$ are treated separately and then compared.

Positive Solutions for First-Order Differential Equations with Riemann-Stieltjes Integral Boundary Conditions

Seshadev Padhi
Birla Institute of Technology, India

This article concerns the existence of positive solutions of the first-order differential equation

$$x'(t) = r(t)x(t) + f(t, x(t)) \quad t \in [0, 1],$$
$$x(0) = \int_0^1 h(s, x(s))\, d\alpha(s),$$

where the nonlinear boundary condition is a Riemann-Stieltjes integral. We use the Leray-Schauder and the Leggett-Williams fixed point theorems to obtain positive solutions.

Oscillation Criteria for Second Order Nonlinearneutral Differential Equations with Deviating Argument

Saroj Panigrahi
University of Hyderabad, India

With a geometric idea, we obtain a set of new oscillation criteria for the forced second order neutral delay differential equation with deviating argument of the form

$$(x(t) + p(t)x(\sigma(t)))'' + q(t)f(x(\tau(t))) = g(t).$$

This criteria improves the results obtained by Q. Kong, M. Wang, *Oscillation criteria for a forced second order differential equations with deviating arguments*, Commu. App. Anal. **16** (2012), 459-470.

Recent Developments in Delay Differential Equations on Time Scales

Gnana Bhaskar Tenali
Florida Institute of Technology, USA
T. Gnana Bhaskar

Time scale, arbitrary nonempty closed subset of $\mathbb{R}$ (with the topology and ordering inherited from $\mathbb{R}$), is an efficient and general framework to study different types of problem to discover the commonalities and highlight the essential differences. We discuss some recent developments in the existence theory of functional Dynamic Equations including a few (counter) examples. In particular, we discuss first order functional dynamic equations with delay, namely, $x^{\Delta}(t) = f(t, x_t)$ on a time scale. Here $x_t \in C([-\tau, 0], \mathbb{R}^n)$ and is given by $x_t(s) = x(t + s), \quad -\tau \le s \le 0$. We consider an appropriate timescale on which such delay equations can be studied meaningfully. We establish an existence result for the solutions of such problems. We also present a few examples.

On a Fractional Boundary Value Problem with a Positive Green's Function

Min Wang
Equifax, USA
John R. Graef, Lingju Kong, Qingkai Kong

In this paper, the authors study a nonlinear fractional boundary value problem. The associated Green's function is derived as a series of functions. Criteria for the existence and uniqueness of positive solutions are then established based on it.

Existence of Positive Solutions for First-Order Resonant Nonlocal Problem

Miroslawa Zima
University of Rzeszow, Poland

We discuss the existence of positive solutions for a first-order equation subject to a nonlocal condition formulated in the terms of the Riemann-Stieltjes integral. We are interested in the resonant case. Our approach relies on the application of the theorem on positive solutions for semi-linear equations due to O'Regan and Zima (2006).

Special Session 64: Dynamics of Evolutionary Equations in the Applied Sciences

Tomas Caraballo, Universidad de Sevilla, Spain
Xiaoying Han, Auburn University, USA
Felipe Rivero, Universidade Federal Fluminense, Brazil

Evolutionary differential equations are primary mathematical tools used to model various problems from a large variety of subjects such as biology, physics, engineering, material sciences, neurosciences, etc. In additional to fundamental questions such as the existence and well-posedness of evolution systems, more efforts in recent years have been devoted to the study of dynamics and qualitative/quantitative properties of the systems, e.g., asymptotic behaviors, special structures of solutions and stability/instability of special structures, travelling waves, temporal/spatial chaos, etc. Results in these areas have shown significant impact on various interdisciplinary areas, such as atmosphere and ocean studies, macro-micro mechanics of materials, cancer research, etc. The main goal of this special session is to present most recent results on various dynamical aspects of evolutionary systems, with an emphasis on those arising from the applied sciences.

On the Rate of Convergence of the 2d Stochastic Alpha Model

Hakima Bessaih
University of Wyoming, USA
Paul Razafimandimby

We study the convergence of the solution of the two dimensional stochastic Leray-α model to the solution of the 2-D stochastic Navier-Stokes equations. We are mainly interested in the rate, as $\alpha \to 0$, of the following error function

$$\epsilon_\alpha(t) = \sup_{s\in[0,t]} \|u^\alpha(s) - u(s)\|_{L^2}$$

where u^α and u are the solution of stochastic Leray-α model and the stochastic Navier-Stokes equations, respectively. We show that when properly localized the error function ϵ_α converges in mean square as $\alpha \to 0$ and the convergence is of order $O(\alpha)$. We also prove that ϵ_α converges in probability to zero with order at most $O(\alpha)$.

Rumour Flow As a Dynamical System on a Social Network

Bernard Brooks
Rochester Institute of Technology, USA

Mathematical models can provide some insight into how rumours flow on a social network such as Facebook. Understanding rumour and information flow in situations of high anxiety and uncertainty is critical. Disaster rumours such as occurred in the aftermath of Katrina in New Orleans hampered rescue efforts and threatened public safety. Health rumours encourage African Americans in Chicago to avoid cancer screening. Rumours that American soldiers distribute pornography to Iraqi children solidify the mistrust and hatred that decimates any feelings of safety or wellbeing both in Iraq and the world over. Our interdisciplinary team of mathematicians, social psychologists and computer scientists embarked on a multi-year effort to understand how beliefs in rumours propagate across social networks. The diverse nature of our research group resulted in mathematical models with empirically calibrated parameters. A summary of our models will be presented. These models include a generalized SIR style model of rumour as epidemic, rumour as a dynamical system flowing over various network topologies and computer assisted panel studies (CAPS) which are used to calibrate the models‘ parameters.

Construction of Quasi-Periodic Response Solutions for Forced Systems with Strong Damping

Renato Calleja
IIMAS-UNAM, Mexico
Livia Corsi, Alessandra Celletti, Rafael de la Llave

I will present a method for constructing quasi-periodic response solutions (i.e. quasi-periodic solutions with the same frequency as the forcing) for over-damped systems. Our method applies to nonlinear wave equations subject to very strong damping and quasi-periodic external forcing and to the varactor equation in electronic engineering. The strong damping leads to very few small divisors which allows to prove the existence by using a contraction mapping argument requiring very weak non-resonance conditions on the frequency. This is joint work with A. Celletti, L. Corsi, and R. de la Llave.

Impulsive Nonautonomous Dynamical Systems

Rodolfo Collegari
USP, Brazil
E. M. Bonotto, M. Bortolan, T. Caraballo

In this work we define the notions of impulsive nonautonomous dynamical systems“ and impulsive cocycle attractors“. We also establish conditions to ensure the existence of an impulsive cocycle attractor for a given impulsive non-autonomous dynamical system, which are analogous to the continuous case.

Stochastic Shell Models of Turbulence with Fractional Noise

Maria Garrido-Atienza
University of Seville, Spain
Hakima Bessaih, Björn Schmalfuß

We consider some shell models of turbulence in a very general form. These are phenomenological approximations of the Navier-Stokes equations, with a viscous linear part that is dissipative and a nonlinear part that is not globally Lipschitz. We assume that this model is perturbed by a multiplicative fractional Brownian noise with Hurst parameter $H \in (1/2, 1)$. We will prove the existence and uniqueness of a pathwise mild solution, and the proof will be achieved in two steps. In the first step we shall prove the existence and uniqueness of variational solutions to the Shell model but driven by smooth paths, for which we are able to get some important uniform estimates in appropriate functional spaces. In a second step, by using these estimates and a compactness argument, we are able to pass to the limit, showing that this limit is a pathwise mild solution for the Shell model driven by a fractional Brownian motion.

Epsilon Optimal Controls for Control of Stochastic Nonlinear Schrødinger Equations

Wilfried Grecksch
Martin-Luther-University Halle-Wittenberg, Germany

We introduce a stochastic Schrødinger equation in the sense of variational solutions over rigged Hibert spaces (V, H, V^*) driven by a cylindrical Wiener process W for certain admissible controls U

$$dX(t) = -iAX(t)dt + f_1(t, X(t))dt + f_2(t)U(t)dt + \\ + g(t, X(t))dW(t),\ X(0) = X_0 \in V. \qquad (1)$$

The optimal control problem consists of the minimizing of a integral functional

$$J(U, X) = \mathbf{E} \int_0^T (J_1(X(t)) + J_2(U(t)))\, dt \qquad (2)$$

with respect to U. We approximate (1) by the solutions of linearized problems (see [1]). Then we calculate optimal controls by using of a maximum principle for the linearized problem (1) and the corresponding goal functional (2). These controls are ε-optimal for the original problem.

References

[1] Grecksch, W., Lisei, H., Linear Approximation of nonlinear Schrødinger Equations Driven by Cylindrical Wiener Process. To appear.

Periodic Structure of Some Zero Entropy Diffeomorphisms: the Sphere and The Torus Case

Juan L.G. Guirao
Technical University of Cartagena, Spain
Jaume Llibre

We consider discrete dynamical systems given by a self–diffeomorphism f defined on a given compact manifold $\mathbb{M}$. In this setting usually the periodic orbits play an important role. In dynamical systems often the topological information can be used to study qualitative and quantitative properties of the system. Perhaps the best known example in this direction are the results contained in the paper entitle *Period three implies chaos* for continuous self–maps on the interval. For continuous self–maps on compact manifolds one of the most useful tools for proving the existence of fixed points, or more generally of periodic points, is the Lefschetz Fixed Point Theorem and its improvements. The Lefschetz zeta function $\mathcal{Z}_f(t)$ simplifies the study of the periodic points of f. This is a generating function for the Lefschetz numbers of all iterates of f. In talk we put our attention in the class of discrete smooth dynamical systems defined by the *Morse–Smale diffeomorphisms* on the tori and the sphere.

Demographic Stochasticity and Evolution of Dispersion

Hyejin Kim
University of Michigan-Dearborn, USA
Yen Ting Lin, Charles Doering

The selection of dispersion is a classical problem in ecology and evolutionary biology. Deterministic dynamical models of two competing species differing only in their passive dispersal rates suggest that the lower mobility species has a competitive advantage in inhomogeneous environments, and that dispersion is a neutral characteristic in homogeneous environments. We consider models including local population fluctuations due to both individual movement and random birth and death events to investigate the effect of demographic stochasticity on the competition between species with different dispersal rates. For homogeneous environments where deterministic models predict degenerate dynamics in the sense that there are many (marginally) stable equilibria with the species‘ coexistence ratio depending only on initial data, demographic stochasticity breaks the degeneracy. A novel large carrying capacity asymptotic analysis, confirmed by direct numerical simulations, shows that a preference for faster dispersers emerges on a precisely defined time scale. While there is no evolutionarily stable rate for competitors to choose in these spatially homogeneous models, the stochastic selection mechanism is the essential counterbalance in spatially inhomogeneous models including demographic fluctuations which do display an evolutionarily stable dispersal rate.

Robustness of Attractors for a Family of Parabolic Equations with Nonlocal Diffusion

Pedro Marin-Rubio
Universidad de Sevilla, Spain
Tomas Caraballo, Marta Herrera-Cobos

The existence of weak solutions to a family of reaction-diffusion problems with nonlocal viscosity term is established. However, uniqueness of solution is not guaranteed. Then, some continuity properties and the long-time behavior of the solutions are analyzed in the framework of the theory of attractors.

This is a joint work with T. Caraballo and Marta Herrera-Cobos, from Universidad de Sevilla (SPAIN). It has been partially supported by FEDER and Ministerio de Economìa y Competitividad (Spain) Grant MTM2011-22411 and by Junta de Andalucia Grant P12-FQM-1492. M.H.-C. is a fellow of Programa de FPI del Ministerio de Economìa y Competitividad, reference BES-2012-053398.

On the Spectral Stability of Traveling Fronts for Reaction Diffusion Degenerate Equations

Ramon Plaza
Universidad Nacional Autonoma de Mexico, Mexico
Juan Francisco Leyva

Motivated by biological applications (e.g. spatial ecology, bacterial aggregation), several reaction-diffusion models consider density-dependent diffusion coefficients which, in addition, are degenerate in one (or more) equilibrium points of the reaction. These degenerate diffusions describe, for example, the avoidance of crowded areas by individuals of certain biological populations. In this talk I present new results and techniques in the study of spectral stability of traveling fronts for reaction-diffusion equations with degenerate diffusion. I will explain the two main ideas to control, on one hand, the essential spectrum and, on the other hand, the point spectrum of the linearized operator around the wave. Both techniques are designed to overcome the degeneracy of the diffusion at the end point. This is joint work with J. Francisco Leyva.

Colony and Evolutionary Dynamics of a Two-Stage Model with Brood Cannibalism and Division of Labor in Social Insects

Marisabel Rodriguez
Arizona State University, USA
Yun Kang

Division of labor (DOL) is a major factor for the great success of social insects because it increases the efficiency of a social group because different individuals perform different tasks repeatedly and presumably with increased performance. Cannibalism plays an important role in regulating colony growth and development by regulating the number of individuals in a colony and increasing survival by providing access to essential nutrients and minimizing competition among colony mates. To understand the synergy effects of division of labor and cannibalistic behavior on colony dynamic outcomes, we propose and study a compartmental two-stage model using evolutionary game theory settings. Our analytical results of the evolutionary models suggest that: (1) Division of labor can prevent colony death and natural selection can preserve *strong Allee effects* by selecting the traits with the largest investment on brood care and the lowest cannibalism rate. (2) Natural selection may increase the fitness of the colony, i.e. the successful production of workforce which results in the increase of total worker population size, colony survival and reproduction. Our results suggest both cannibalism and division of labors are adaptive strategies that increase the size of the worker population, and therefore, persistence of the colony.

Topology of Foliations and Decomposition of Stochastic Flows of Diffeomorphisms

Paulo Ruffino
University of Campinas, Brazil
Alison Melo, Leandro Morgado

Let M be a compact manifold equipped with a pair of complementary foliations, say horizontal and vertical. In Catuogno, Silva and Ruffino (*Stoch. Dyn.* 2013) it is shown that, up to a stopping time τ, a stochastic flow of local diffeomorphisms φ_t in M can be written as a Markovian process in the subgroup of diffeomorphisms which preserve the horizontal foliation composed with a process in the subgroup of diffeomorphisms which preserve the vertical foliation. Here, we discuss topological aspects of this decomposition. The main result guarantees the global decomposition of a flow if it preserves the orientation of a transversely orientable foliation. In the last section, we present an Itô-Liouville formula for subdeterminants of linearised flows. We use this formula to obtain sufficient conditions for the existence of the decomposition for all $t \geq 0$.

Initial and Boundary Value Problems for the Deterministic and Stochastic Zakharov-Kuznetsov Equation in a Bounded Domain

Chuntian Wang
University of California, Los Angeles, USA
Roger Temam, Jean-Claude Saut

In this talk I will focus on the well-posedness and regularity of the Zakharov-Kuznetsov (ZK) equation in the deterministic and stochastic cases, subjected to a rectangular domain in space dimensions 2 and 3. Mainly we have established the existence, in 3D, and uniqueness, in 2D, of the weak solutions, and the local and global existence of strong solutions in 3D. Then we extend the results to the stochastic case and obtain in 3D the existence of martingale solutions, and in 2D the pathwise uniqueness and existence of pathwise solutions. The main focus is on the mixed features of the partial hyperbolicity, nonlinearity, nonconventional boundary conditions, anisotropicity and stochasticity, which requires methods quite different than those of the classical models in fluid dynamics, such as the Navier-Stokes equation, Primitive Equation and related equations.

Random Dynamics and Averaging for Non-Autonomous Stochastic Wave Equations

Yuncheng You
University of South Florida, USA
Hongyan Li

The existence of a random attractor for a non-autonomous stochastic wave equation with nonlinear damping and multiplicative noise on an unbounded domain will be presented. The Hausdorff distance between the random attractors of the original equation with rapidly oscillating external force and the averaged equation is estimated.

Special Session 65: On Singular Problems Related to Distance Functions and Very Weak Solutions

Rakotoson, University Poitiers LMA, France

The aim of this session is to present the recent developments concerning singular problems related to distance functions and the notion of very weak solutions. Some new notions like the Uniform Hopf inequality and Ultra-weak solution should be presented.

Uniform Gradient Estimates for Viscosity Solutions to the Normalized p-Poisson Problem

Amal Attouchi
University of Jyvaskyla, Finland
Eero Ruosteenoja, Mikko Parviainen

This talk is concerned with regularity and uniform estimates of the gradient of viscosity solutions to the normalized p-Laplacian equation:

$$-\Delta_p^N u = f \qquad \text{in}\Omega \quad .$$

We present some recent results on the $C^{1,\alpha}$ regularity of the solutions when $f \in L^q(\Omega) \cap C(\Omega)$ and $f > 0$. The main idea to derive a gradient estimate relies on the study of an approximate problem, potential theory tools, some known regularity results of weak solutions of the classical p-Laplacian equation (with divergence structure) and the uniqueness of viscosity solutions.

The Uniform Hopf Inequality for Discontinuous Coefficients and Optimal Regularity in BMO for Singular Problems

Nada el Berdan
LMA- Universite de Poitiers, France
J.M Rakotoson, J.I Diaz

We discuss the existence and non existence of the so called Hopf uniform Inequality (variant of a maximum principle) for the linear equation $Lv = f$ with measurable coefficients and under the homogeneous Dirichlet Boundary condition. Then we apply such inequality to prove the $W_0^{1,p}$-regularity of a semi linear singular problem $Lu = F(u)$, $F(0)$ is infinite (under a precise growth) and with the coefficients of the main operator of L in the space of vanishing mean oscillation. Moreover , when those coefficients are Lipschitz , we show that the gradient of the solution is at most in the space of bounded mean oscillation: bmo_r.
This work is in collaboration with Professor J.I Diaz (Universidad Complutense de Madrid) and Professor J.M Rakotoson , and the detailed paper The Uniform Hopf Inequality for discontinuous coefficients and optimal regularity in BMO for singular problems, will appear in Journal of Mathematical analysis and its applications).

Pointwise Estimates for G-Gamma Functions and Applications

Maria Rosaria Formica
University of Naples Parthenope, Italy
A. Fiorenza, J.M. Rakotoson

We present some new results, based on a joint work with A. Fiorenza and J. M. Rakotoson, on the regularity of some linear PDEs, expressed by means of certain nonstandard spaces, namely, the small Lebesgue spaces and their generalization, the G-Gamma spaces. We deal with the relative rearrangement technics introduced by Mossino and Temam, we use some borderline Sobolev embeddings related to the Grand Lebesgue spaces (which generalize the Fusco-Lions-Sbordone results) and we get either new estimates for the gradient of weak solutions in the small Lebesgue spaces, either new estimates of very weak solutions in the spaces G-Gamma.

Uniqueness of Very Weak Solutions of Linear Elliptic Equations with Singular Absorption Potentials Without Boundary Conditions

David Gomez-Castro
Universidad Complutense de Madrid, Spain
J.I. Diaz

A question related to the famous 1928 study by Gamow of the Schroedinger equation for the infinite well potential is the consideration of solutions of the following equation:

$$-\Delta u + V(x)u = f \tag{1}$$

where $V(x) \sim d(x,\partial\Omega)^{-\alpha}$ for some $\alpha > 0$. From here on $\delta(x) = d(x,\partial\Omega)$. It is natural to assume $Vu \in L^1(\Omega, \delta)$ so that the solutions must be searched for in the class of local very weak solutions of the form

$$-\int_\Omega u\Delta\varphi + \int_\Omega Vu\,\varphi = \int_\Omega f\varphi \tag{2}$$

where $\varphi \in W_c^{2,\infty}(\Omega)$ in the case that $V \in L^1_{loc}(\Omega)$, $V \geq 0$. Existence and uniqueness in the case $V \in L^1(\Omega, \delta)$ with Dirichlet boundary conditions requires considering test functions $\varphi \in W^{2,\infty}(\Omega) \cap W_0^{1,\inf}(\Omega)$ (notice how the choice of test functions is adapted to the boundary condition). For $V = \delta^{-r}$ this condition means that $\int |u|\delta^{1-r}$

Some Nonlinear Elliptic Systems with Right-Hand Side Integrable Data with Respect to the Distance to the Boundary

Dalian University of Techology, Peoples Rep of China

In this talk, we will discuss the very weak solutions to some nonlinear elliptic systems with right-hand side integrable data with respect to the distance to the boundary. By using an approximate method and a priori estimates technology in the framework of weighted spaces, we obtain the existence, uniqueness and regularity of very weak solutions.

Optimal Control of Very Weak Solutions to Elliptic Systems with Singular Data Related to Neumann Boundary Conditions

Jochen Merker
HTWK Leipzig University of Applied Sciences, Germany

In this talk, we discuss optimal L^1-control of very weak solutions $u : \Omega \to \mathbb{R}^M$ to elliptic systems with singular data on a smooth bounded domain$\Omega \subset \mathbb{R}^N$. More precisely, we consider the linear optimization problem

$$\langle g, u\rangle = \text{max! subject to } Au \leq f \text{ and } u \geq 0\,, \quad (*)$$

where $g \in L^\infty(\Omega, \mathbb{R}^M)$, the operator $A : L^{p'}(\Omega) \to W^{2,p}(\Omega, \mathbb{R}^M)^*$, $1 < p < \infty$, is a very weak realization of a vector-valued elliptic differential operator $-\operatorname{div}(\sum_{j=1}^M a_{ij}\nabla u_j) + \sum_{j=1}^M b_{ij}u_j$, and the right hand side $f \in W^{2,p}(\Omega, \mathbb{R}^M)^*$ is a singular integral like $\langle f, v\rangle := \int_\Omega \int_{\partial\Omega} \tilde{f}(x, y)(v(x) - v(y))\, dy\, dx$ with $\tilde{f}(x, y)|x - y|^\alpha \in L^1(\Omega \times \partial\Omega)$ for $\alpha \in (0, 1]$, but not neccessarily $\tilde{f} \in L^1(\Omega \times \partial\Omega)$. As was shown in [1] for Poisson's equation in the scalar case, such singular integrals as right hand sides allow a rigorous discussion of singular Neumann boundary conditions $\left(\sum_{j=1}^M a_{ij}\nabla u_j\right) \cdot \nu = -\infty$ on parts of $\partial\Omega$, which model an explosive flow into the domain. We formulate corresponding results for elliptic systems (see [2]) prove existence of optimizers for (*) and provide some numerical examples with an explosive inflow over the inner boundary of an annular domain [3].

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Special Session 66: Mathematical Oncology

Alexander R. A. Anderson, Integrated Mathematical Oncology, Moffitt Cancer Center, Tampa, FL, USA

Cancer is a complex, multiscale process, in which genetic mutations occurring at a subcellular level manifest themselves as functional changes at the cellular and tissue scale. We believe that such a complex system can only truly be understood via the integration of theory and experiments. Therefore mathematical oncology is focused on integrating mathematical and computational modeling approaches with experimental and clinical data to better understand cancer growth and development and to translate this understanding into novel therapies. Since there is no single right model of cancer, in fact by definition all cancer models are wrong, we utilize a diverse portfolio of mathematical and computational approaches that cover the gamut of biological scales. The aim of this session will be to showcase this diversity, with a clear focus on data driven models that have direct translational potential.

Modelling the Vicious Cycle in Bone Metastases

David Basanta
H. Lee Moffitt Cancer Center, USA
Arturo Araujo, Leah Cook, Pranav Warman, Conor Lynch

Prostate cancer is one of the most commonly diagnosed cancers in men. Every year more than 30 thousand men in the US get diagnosed with cancer but of those, only 10cancer becomes lethal when it metastasizes to other organs and, in 90of the cases, patients show evidence of metastases to the bone. Understanding prostate cancer metastasis to the bone is thus key if we want to find ways to improve treatment and decrease mortality. Successful metastases come from prostate cancer cells that can interact with the resident stromal cells and take advantage of the host physical microenvironment. These interactions are key but difficult to model experimentally due to the complexity of the factors and actors involved. Mathematical models can help by integrating experimental and clinical data, biological insights and by balancing the need to capture enough detail to model biological reality with the need for simplicity. In this talk I will describe two different mathematical models, one agent-based and one game theoretical, that aim to synergistically bring new understanding on the evolutionary dynamics of metastatic success as well as treatment resistance.

Interacting Scales in Modeling HPV and Oropharyngeal Cancer

Marisa Eisenberg
University of Michigan, Ann Arbor, USA
Andrew Brouwer, Rafael Meza

Human papillomavirus (HPV) is a sexually transmitted infection which is associated with several forms of cancer, including cervical and oropharyngeal cancer. HPV-related oropharyngeal cancers have recently overtaken cervical as the most common HPV-related cancer in the US. Interactions between infectious diseases and cancer form an inherently multiscale problem, with population-level disease transmission driving within-host carcinogenesis, yielding overall population-level cancer trends. In this talk, I will discuss recent work examining the dynamics of HPV and oropharyngeal cancer, at both the cellular and human population scales.

Abscopal Benefits of Localized Radiotherapy Depend on Activated T Cell Trafficking and Distribution Between Metastatic Lesions

Heiko Enderling
Moffitt Cancer Center & Research Institute, USA
Jan Poleszczuk

It remains unclear how localized radiotherapy for cancer metastases can occasionally elicit a systemic antitumor effect, known as the abscopal effect, but historically it has been speculated to reflect the generation of a host immunotherapeutic response. The ability to purposefully and reliably induce abscopal effects in metastatic tumors could meet many unmet clinical needs. Here, we describe a mathematical model that incorporates physiological information about T cell trafficking to estimate the distribution of focal therapy-activated T cells between metastatic lesions. We integrated a dynamic model of tumor-immune interactions with systemic T cell trafficking patterns to simulate the development of metastases. In virtual case studies, we found that the dissemination of activated T cells among multiple metastatic sites is complex and not intuitively predictable. Furthermore, we show that not all metastatic sites participate in systemic immune surveillance equally, and therefore the success in triggering the abscopal effect depends, at least in part, on which metastatic site is selected for localized therapy. Moreover, simulations revealed that seeding new metastatic sites may accelerate the growth of the primary tumor because T cell responses are partially diverted to the developing metastases, but the removal of the primary tumor can also favor the rapid growth of pre-existing metastatic lesions. Collectively, our work provides the framework to prospectively identify anatomically-defined focal therapy targets that are most likely to trigger an immune-mediated abscopal response, and therefore may inform personalized treatment strategies in patients with metastatic disease.

Targeting the Phenotype: Treatment Strategies for Heterogeneous Cancer

Jill Gallaher
Moffitt Cancer Center, USA
Alexander R. A. Anderson

Targeted cancer drugs attack pathway specific phenotypes and can lead to very positive outcomes when a particular phenotype dominates the population of a specific tumor. However, these drugs often fail because not all cells express the targeted phenotype to the same degree. This leads to a heterogeneous response to treatment, and ultimate recurrence of the cancer as sensitive cells die off and resistant cells take over. We explore how treatment strategies informed by a tumor's phenotypic mix, can help slow the emergence of resistance and stave off tumor recurrence. We use an off-lattice agent-based model that incorporates inheritance of two phenotypes - proliferation rate and migration speed - and is modulated by a space limiting selection force. We find how and when distinct distributions of phenotypes require different treatment strategies.

Exploitng Evolutonary Dynamics to Improve Cancer Therapy

Robert Gatenby
Moffitt Cancer Center, USA
Pedro Enriquez-Navas, Ariosto Silva, Joel Brown, Jessica Reynolds, Jingsong Zhang

A number of successful systemic therapies are available for treatment of disseminated cancers. However, tumor response to these treatments is almost invariably transient and therapy fails due to emergence of resistant populations. The latter reflects the temporal and spatial heterogeneity of the tumor microenvironment as well as the evolutionary capacity of cancer phenotypes to adapt to therapeutic perturbations. Interestingly, although cancers are highly dynamic systems, cancer therapy is typically administered according to a fixed, linear protocol. Treatment is changed only when the tumor progresses but successful tumor adaptation begins immediately upon administration of the first dose. Applying evolutionary models to cancer therapy demonstrate the potential advantage of using more dynamic, strategic approaches that focus not just on the initial cytotoxic effects of treatment but also on the evolved mechanisms of cancer cell resistance and the associated phenotypic costs. The goal of evolutionary therapy is to prevent or exploit emergence of adaptive tumor strategies. Examples of this approach include adaptive therapy and double bind therapy. The former continuously alters therapy to maintain a stable tumor volume using a persistent population of therapy-sensitive cells to suppress proliferation of resistant phenotypes. The latter uses the cytotoxic effects of an initial therapy to promote phenotypic adaptations that are then exploited using follow-on treatment. In pre-clinical models, application of adaptive therapy permits indefinite tumor control with a single cytotoxic drug. Clinical results from studies using adaptive therapy and double bind therapy will be presented.

Identifiability in Compartmental Models of Cancer Chemotherapy

Harsh Jain
Florida State University, USA

In this talk, we will present ordinary differential equation models of chemotherapy targeting solid tumors. The drugs considered include: platinum based compounds that induce cell death by inflicting DNA damage; and taxols that target cells undergoing mitosis. Although such models are relatively simple in formulation, it is still not clear whether model parameters are identifiable from available experimental data. Here, we show that while nonlinearities in model terms generally result in structural identifiability, the increased numbers of parameters lead to issues of practical identifiability (due to imperfect or incomplete data). We discuss methods to estimate practically identifiable combinations of parameters in such cases. An important result of our analysis is predicting what sort of additional experimental data would render the model completely identifiable.

Tailoring Delivery of Targeted Therapy to the Tumor: a Study in a Pancreatic Cancer and TLR2

Aleksandra Karolak
Moffitt Cancer Center, USA
Veronica Estrella, Tingan Chen, Amanda Huynh, David Morse, Katarzyna Rejniak

The pancreatic adenocarcinomas are particularly difficult to detect and treat, and the 6% 5-year survival rate for this cancer has not been improved in the last twenty years. Recently, our experimental collaborators reported the toll-like receptor 2 (TLR2) as a bona fide cell-surface marker for targeting pancreatic cancer. In order to provide a quantitative assessment of tumor penetration and uptake, we combined computational modeling with the real time in vivo imaging of the TLR2 nanoligand (L) conjugated to near-infrared fluorescent dye, Cyanine-5 (Cy5), in pancreatic adenocarcinoma tumor xenografts in mice.
The integration of intravital fluorescence microscopy with cell-level modeling allowed us to quantify the spatio-temporal dynamics of TLR2L-Cy5 complex. Our studies also led to evaluation of the nanoparticle intratumoral transport including extravasation, interstitial diffusion and intracellular accumulation in relation to explicitly defined tumor tissue architecture. The computational model offers cost-effective method for determining dynamics of agent binding, internalization and intracellular uptake, and serves as a potential tool for future predictions of the appropriate nanoparticle schedules and dosage based on patient-specific data.

The Role of M1/M2 Microglia in Regulation of Cell Infiltration in Glioblastoma

Yangjin Kim
Konkuk University, Korea
Hyejin Jeon, Hans Othmer

Malignant gliomas are the most common type of brain cancer, which arise from glial cells, and in their most aggressive form are called GBMs. GBMs are highly invasive and difficult to treat because cells migrate into surrounding healthy brain tissue rapidly, and thus these tumors are difficult to completely remove surgically. GIMs, which can comprise up to one third of the total tumor mass, are present in both intact glioma tissue and necrotic areas. They apparently originate from both resident brain macrophages (microglia) and newly recruited monocyte-derived macrophages from the circulation. Activated GIMs exhibit several phenotypes: one called M1 for classically activated, tumor suppressive, and another called M2 for alternatively activated, tumor promoting, and immunosuppressive. Within a tumor the balance between these phenotypes is typically shifted to the M2 form. Numerous factors secreted by glioma cells can influence GIM recruitment and phenotypic switching, including growth factors, chemokines, cytokines and matrix proteins. In this work, we focus on mutual interaction between a glioma and M1/M2 microglia mediated by CSF-1, TGFbeta, and EGF. Up-regulated TGFbeta leads to up-regulation of Smad within the tumor cells and secretion of MMPs, leading to proteolysis for EMT process and cell infiltration. The mathematical model consists of densities of glioma cells, M1 type cells, M2 type cells, and concentrations of CSF-1, EGF, TGFbeta, Extracellular matrix, and MMPs. We developed the model to investigate the mutual interactions between tumor cells in the upper chamber and microglia in the lower chamber. In the experiments, Boyden invasion assay was used to show that this mutual interaction induces glioma infiltration in vitro and in vivo. We show that our simulation results are in good agreement with the experimental data and we generate several hypotheses that should be tested in future experiments in vivo.

Phase I Trials in Melanoma: a Framework to Translate Preclinical Findings to the Clinic

Eunjung Kim
H.Lee Moffitt Cancer Center and Research Institute, USA
Vito Rebecca, Keiran Smalley, Alexander Anderson

We present a mathematical model driven framework, phase i (virtual/imaginary) trials, that integrates the heterogeneity of actual patient responses and preclinical studies through a cohort of virtual patients. The framework includes an experimentally calibrated mathematical model, a cohort of heterogeneous virtual patients, an assessment of stratification factors, and treatment optimization. We show the detailed process using both preclinical and clinical data of melanoma combination therapy (chemotherapy and an AKT inhibitor). The mathematical model composed of ordinary differential equations predicts melanoma treatment response and resistance to mono and combination therapies. Model parameters were estimated by utilizing an optimization algorithm to identify parameters that minimized the difference between predicted cell populations and experimentally measured cell numbers. The model was then validated with in vitro experimental data. The validated model and a genetic algorithm were used to generate virtual patients whose tumor volume responses to the combination therapy matched statistically the actual heterogeneous patient responses in the clinical trial. Analyses on simulated cohorts revealed key model parameters such as a tumor volume doubling rate and a therapy-induced phenotypic switch rate that stratified simulated virtual clinical trial outcomes. Finally, our approach predicts optimal AKT inhibitor scheduling suggesting more effective but lower cumulative doses of AKT inhibitor treatment strategies.

Drug-Resistance Strategies: Evolution of Heterogeneity Over Space and Time

Mark Robertson-Tessi
Moffitt Cancer Center, USA
Dan Nichol, Alexander Anderson

Drug resistance is implicated in the majority deaths due to cancer. The failure of drugs to produce complete regression leads to tumor recurrence. The mechanisms by which a tumor escapes a given therapy are many and varied, including genotypic and phenotypic heterogeneity, variegation in drug delivery, and numerous other physiological barriers. In this talk, phenotypic heterogeneity will be surveyed from a theoretical perspective.

A cellular automaton is used to probe the short- and long-term effects of different mechanisms that generate phenotypic heterogeneity in a population of cells, including combinations of: genetic mutation; environmentally-driven plasticity; phenotypic bet-hedging; and probabilistic resistance. The cytotoxic drug is applied to a simulation of cell culture. The success of a given strategy depends on the drug regimen, the re-plating protocol, and the size of the dish. Sequential versus combination therapies with two drugs alter the evolutionary dynamics and select for different drug-resistance strategies. Combinations of mechanisms produce exceedingly robust cell populations which are essentially impossible to completely destroy with any drug regimen, suggesting that disabling the underlying mechanisms that generate the heterogeneity may be more successful than modifying drug dosing and treatment schedules.

Steering Evolution to Prevent the Emergence of Resistance

Jacob Scott
Department of Integrated Mathematical Oncology, Moffitt Cancer Center and Research Institute, USA
Alexander Anderson, Daniel Nichol, Quan Tran

To better understand, and prevent the emergence of resistance, we construct a simple markov model of evolution on fitness landscapes. We utilize an empirically derived dataset of E. coli resistance to beta-lactam antibiotics to derive experimental designs for lung cancer resistance. We then attack the problem of collateral sensitivity and resistance, and show that the experiments done to date can be misleading or even dangerous and suggest an alternate formulation. To address overly stringent assumptions needed for the Markov model (strong selection weak mutation) we formulate a stochastic model which we use to consider the issue of evolutionary convergence times and drug ordering. We Finally, I will present early results from evolution experiments done in ALK mutated non-small cell lung cancer and discuss theoretical extensions and ongoing experimental work

Special Session 67: Applications of Mathematical Modeling in Developmental and Cell Biology

Jia Zhao, University of South Carolina, USA
Xinfeng Liu, University of South Carolina, USA
Qi Wang, University of South Carolina, USA

This minisymposium aims to bring researchers in applied mathematics with diverse background to address recent advances in mathematical modeling and computational technologies for complex systems in developmental and cell biology that include (but not limited to) cell signaling pathways, cell oscillation and polarization, cancer stem cells, cancer tumor growth and cell mitosis etc. Such systems usually consist of multiple interacting components that exhibit complicated temporal and spatial dynamics with multiple time and length scales, which are extremely difficult to model and make faithful predictions. In this mini symposium, the challenges of modeling these complex systems will be discussed and, moreover, the new analytical and computational techniques to simulate these models will also be presented.

Modeling Dorsal Closure

Andreas Aristotelous
West Chester University, USA
Stephanos Venakides

Dorsal closure is a part of the Drosophila embryogenesis. It constitutes a model for cell sheet morphogenesis during development and wound healing. For this reason the study of the various mechanisms governing dorsal closure is of great importance. Here we devise a flexible modeling platform employing individual based techniques that allows us to incorporate experimental data and gives us the freedom to test different kinds of modeling equations on the various parts of the simulated Drosophila embryo. Our model simulates the behavior of not only the entire amnioserosa, but its constituent cells, as well. The model considers the combination of elastic and contractile properties of the cells in the tissue on a robust geometric platform, in order to capture various experimentally observed phenomena.

Robust Dynamics in Tissue Growth and Developmental Patterning

Weitao Chen
University of California, Irvine, USA
Arthur Lander, Qing Nie

Robustness is observed widely in biological systems and the related study is essential in mathematical modeling. In particular, size control and pattern formation, both displaying strong robustness, can serve as good models to investigate the related mechanisms. Tissue and organ size is genetically specified with remarkable precision, independent of growth rate, cell size, only weakly sensitive to initial conditions and relatively resistant to a variety of external perturbations. The patterning of many developing tissues is organized by morphogens and its formation is often quite resistant to embryonic difference, intrinsic or extrinsic noises. The robustness of different systems can be enhanced by particular mechanisms. In this talk, I will use a multi-stage cell lineage model to discuss general strategies that may contribute in achieving large tissue size robustly. I will also present two particular systems, the papillae formation on a mouse tongue or the scaling behavior during the growth of a wing disc in drosophila, to reveal the mechanisms for obtaining specific patterns with robustness.

Moment Stability for Nonlinear Stochastic Growth Kinetics of Breast Cancer Stem Cells with Time-Delays

Xinfeng Liu
University of South Carolina, USA

Solid tumors are heterogeneous in composition. Cancer stem cells (CSCs) are a highly tumorigenic cell type found in developmentally diverse tumors that are believed to be resistant to standard chemotherapeutic drugs and responsible for tumor recurrence. Thus understanding the tumor growth kinetics is critical for development of novel strategies for cancer treatment. In this paper, the moment stability of nonlinear stochastic systems of breast cancer stem cells with time-delays has been investigated. First, based on the technique of the variation- of-constants formula, we obtain the second order moment equations for the nonlinear stochastic systems of breast cancer stem cells with time-delays. By the comparison principle along with the established moment equations, we can get the comparative systems of the nonlinear stochastic systems of breast cancer stem cells with time-delays. Then moment stability theorems have been established for the systems with the stability properties for the comparative systems. Based on the linear matrix inequality (LMI) technique, we next obtain a criteria for the exponential stability in mean square of the nonlinear stochastic systems for the dynamics of breast cancer stem cells with time-delays. Finally, some numerical examples are presented to illustrate the efficiency of the results.

Kinetic Monte Carlo Simulations of Multicellular Aggregate Self-Assembly in Biofabrication

Yi Sun
University of South Carolina, USA
Xiaofeng Yang, Qi Wang

We present a three-dimensional lattice model to study self-assembly and fusion of multicellular aggregate systems by using kinetic Monte Carlo (KMC) simulations. This model is developed to describe and predict the time evolution of postprinting morphological structure formation during tissue or organ maturation in a novel biofabrication process (or technology) known as bioprinting. In this new technology, live multicellular aggregates as bio-ink are used to make tissue or organ constructs via the layer-by-layer deposition technique in biocompatible hydrogels; the printed bio-constructs embedded in the hydrogels are then placed in bioreactors to undergo the self-assembly process to form the desired functional tissue or organ products. Here we implement our model with an effcient KMC algorithm to simulate the making of a set of tissues/organs in several designer's geometries like a ring, a sheet and a tube, which can involve a large number of cells and various other support materials like agarose constructs etc. We also study the process of cell sorting/migration within the cellular aggregates formed by multiple types of cells with different adhesivities.

Macroscopic Limits of Pathway-Based Kinetic-Transport Models in the Exponential Large Gradient Environment

Min Tang
Shanghai Jiao Tong University, Peoples Rep of China
Weiran Sun

It is possible to develop predictive agent-based models thanks to the understanding of the intracellular signaling pathway. It is of great biological interest to understand the molecular origins of chemotactic behavior of E. coli by deriving population-level model based on the underlying signaling pathway dynamics. We derive macroscopic models for E.coli chemotaxis using kinetic equations that incorporate intracellular chemosensory system. These macroscopic models are shown to be in good agreement with individual based simulations. They match the average speed for the whole range of the exterior gradient. This in particular gives an answer to the question about the average speed for large gradients and suggest that the bacteria have a maximum drift velocity at their favorite gradient.

Modeling the Dynamical Organization of the Genome in Live Yeast Cells

Paula Vasquez
University of South Carolina, USA
Greg Forest, Caitlin Hult, David Adalsteinsson, Josh Lawrimore, Kerry Bloom

The genome comprises the entire genetic information that makes up an organism. This information is encoded in DNA and stored in the nucleus of every cell in that organism in a dynamical manner. Understanding the three-dimensional, dynamic, structure of the genome is a crucial step in characterizing how cellular DNA adopts and transitions between different entropic functional states over the course of the cell cycle, facilitating vital functions such as gene expression, DNA replication, recombination, and repair. In this talk we discuss our existing models of chromosomes and protein function in live yeast cells. Our mathematical models show that enzymes do not create the topological and energetic landscapes in the nucleus; rather they bias the entropy-dominated stochastic dynamics into cycle-specific states. In this paradigm, entropy and confinement dictate the leading order structure and dynamics of the genome, and the role of enzymes is to guide, stabilize, and sustain cycle-specific genome states.

Two Phase Flow Models of Cancer Growth and Polymer Solvent Interactions

Steven Wise
The University of Tennessee, USA

In this talk I will describe and analyze a model for polymer-solvent interaction in an application in the production of organic photovoltaics. I will show that this model is closely related to another describing cancer growth in the presence of interstitial fluid flow.

A New Nonlocal Poisson-Fermi Model for Ion Channel Studies

Dexuan Xie
University of Wisconsin-Milwaukee, USA

Ion channels are the valves of cells and the main controllers of many biological functions. While advances have been made in their mathematical modeling, one substantial challenge we face to address is how to incorporate the effects of ion sizes and polarization correlations among water molecules into a dielectric continuum model in partial differential equations. In this talk, I will review some progresses we made on this important research issue. I then will introduce a new nonlocal Poisson-Fermi model (NPF), which is a fourth order elliptic partial differential equation subjected to a system of nonlinear algebraic equations. Here, each ion is treated as a hard sphere with different radii for different ionic species, and the solution of NPF leads to the ionic concentration functions and the electrostatic potential. Furthermore, I will show that NPF includes the previous Poisson-Fermi models as special cases, and its solution is the convolution of a solution of the corresponding nonlocal Poisson dielectric model with a Yukawa-like kernel function. Finally, some simulation results will be discussed to illustrate the potential application of our new NPF model in ion channel studies. This project was partially supported by the National Science Foundation, USA, through grant DMS-1226259.

Multiscale Modeling of Axonal Cytoskeleton Dynamics in Disease

Chuan Xue
Ohio State University, USA
Blerta Shytulla, Anthony Brown, Jonathan Toy, Wenrui Hao

The shape and function of an axon is dependent on its cytoskeleton, including microtubules, neurofilaments and actin. Neurofilaments accumulate abnormally in axons in many neurological disorders. An early event of such accumulation is a striking radial segregation of microtubules and neurofilaments. This segregation phenomenon has been observed for over 30 years now, but the underlying mechanism is still poorly understood. I will present a stochastic multiscale model that explained these phenomena and generated testable predictions. I will also present our progress in the derivation and analysis of a continuum PDE model which yielded further insights into the problem.

CQLM—A Novel Numerical Approach to Solve the Gradient Flow Problem

Xiaofeng Yang
Department of math, U of South Carolina, USA

There are two commonly used numerical approaches to solve the gradient flow problem while preserving the desired energy stability: the Convex Splitting approach and the Stabilized approach. The Convex Splitting approach is energy stable, however, it produces a nonlinear scheme at most cases, thus the implementation is complicated and the computational cost is high. Stabilized approach generates linear scheme that is extremely easy to implement, however, the magnitude of the stabilizing term depends on the upper bound of the second order derivative of the nonlinear potential. Therefore, such method is particularly reliable for those models with maximum principle. For many nonlinear models, both of the two methods are not optimal choices. We introduce a novel, so called Compulsory-Quasi-Lagrange-Multiplier approach, that can possess the advantages of both the convex splitting approach and the stabilized approach, but avoid their imperfections mentioned above. More precisely, the schemes (i) are accurate (up to second order in time); (ii) are stable (unconditional energy dissipation law holds); and (iii) are efficient and easy to implement (only need to solve some linear equations at each time step.

A Mechanochemical Model for Cell Polarity

Lei Zhang
Peking Universtiy, Peoples Rep of China

Cell polarization toward the attractant is related to both physical and chemical factors. Most existing mathematical models are based on reaction diffusion systems and only focus on the chemical process during cell polarization. However, experiments reveal that membrane tension may act as a long-range inhibitor for cell polarization. Here we present a mathematical model that incorporates the interplays between Rac, filamentous actin (F-actin), and cell membrane tension for the formation of cell polarity. We also test the predictions of this model with singe cell measurements on the spontaneous cell polarization of cancer stem cells (CSC) and non-cancer stem cells (NCSS) as the former have smaller cell membrane tension. Both our model and experimental results show that the cell polarization is more sensitive to stimuli under low membrane tension, and high membrane tension improves the robustness and stability of cell polarization so that polarization is persistent under random perturbations. Furthermore, our simulations for the first time reproduce the results from the aspiration-release experiment and the pseudopod-neck-cell body morphology severing experiment, demonstrating that aspiration (elevation of tension) and release (reduction of tension) result in decrease and recover of the activity of Rac-GTP, respectively, and relaxation of tension leads to the formation of new polarity of the cell body when the cell with morphology of pseudopod-neck-body is severed. The joint work with Weikang Wang (PKU), Feng Liu (PKU).

A Multiphasic Complex Fluids Model for Cytokinesis of Eukaryotes

Jia Zhao
University of South Carolina, USA
Qi Wang

In this presentation, we develop a full 3D multiphase hydrodynamic model to study the fundamental mitotic mechanism in cytokinesis, the final stage of mitosis. The model describes the cortical layer, a cytoplasmic layer next to the cell membrane rich in F-actins and myosins, as an active liquid crystal system and integrate the extra cellular matrix material and the nucleus into a multiphase complex fluid mixture. With the novel active matter model built in the system, our 3D simulations show very good qualitative agreement with the experimental obtained images. The hydrodynamical model together with the GPU based numerical solver provides an effective tool for studying cell mitosis theoretically and computationally

Special Session 68: Rate-Dependent and Rate-Independent Evolution Problems in Continuum Mechanics: Analytical and Numerical Aspects

Giuliano Lazzaroni, SISSA, Trieste, Italy
Marita Thomas, Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany

In recent years, energetic criteria have been extensively used to study processes of both rate-dependent and -independent type. This session will illustrate different variational approaches to evolution problems arising mainly from continuum mechanics. Such problems lead to systems of nonlinear partial differential equations, often involving non-smooth constraints. Special focus will be given to different notions of quasistatic, viscous, gradient flows as well as inertial dynamic evolutions, and to their relationships. Analytical and numerical results will be discussed and compared for these different notions. Several fields of applications will be taken into account, such as damage, fracture, dislocations, plasticity, phase separation. Moreover, models arising in other fields of application, which can be treated with comparable techniques, will also be addressed.

Thin-Film Limits in Elastic Solids: Issues Related to Thermomechanical-Coupling and Non-Interpenetration

Barbora Benevsovà
University of Würzburg, Germany
Martin Kružìk, Gabriel Pathò

The talk will be concerned with a thin-film passage and will be divided into two parts: In the first part we will consider a full thermo-mechanical model of the bulk with a non-simple elastic energy, i.e., with one that includes a higher-order-gradient surface-energy part. Such energy is typical if we consider microstructure-forming materials on a scale on which the microstructure can be fully resolved. We will then perform a passage to a model for thin films large in area in the following way: We will first perform the dimension reduction and in a second step scale the surface energy part to zero.
In the second part on the talk we have a closer look on the dimension reduction in the elastic part of the energy. Here, we will consider the so-called membrane regime in which the procedure has become standard by now; however, not if combined to non-interpenetration conditions. We will concentrate on exactly this case in the talk. Thus, we will need to find a definition of non-interpenetration in the thin film and show that it is fulfilled if the bulk deformations in the thin film passage have been non-interpenetrative, too.

Gradient Damage Models for Elastoplastic Materials

Vito Crismale
SISSA, Italy

The talk concerns gradient damage models for elastoplastic materials. These are a generalization both of the coupling between damage and elasticity, and of elastoplasticity models without damage. The coupling I consider has been studied in many engineering and numerical works. Starting from the perfect plasticity in a linearized setting, I point out the importance of the damage in the elastic and plastic responses, and how some fatigue phenomena can be included in the formulation. Afterwards, I introduce the mathematical treatment of the model, based on the two approaches of energetic solutions and of evolutions via vanishing viscosity. The work is in collaboration with G. Lazzaroni.

Dynamics of Discrete Screw Dislocations Along Glide Directions

Lucia de Luca
Technical University of Munich, Germany
Roberto Alicandro, Adriana Garroni, Marcello Ponsiglione

We consider a zero temperature model for screw dislocations in discrete lattices. Using a discrete-in-time variational scheme, we study the motion of dislocations in the dilute regime, towards low energy configurations. Letting the spacing and time parameters go to zero, we deduce an effective fully overdamped dynamics predicting motion along the glide directions of the crystal.

On the Genesis of Directional Dry Friction Through Bristle-Like Mediating Elements

Paolo Gidoni
SISSA (Trieste), Italy
Antonio DeSimone

We propose an explanation of the genesis of directional dry friction, as emergent property of the oscillations produced in a bristle-like mediating element by the interaction with microscale fluctuations on the surface. Mathematically, we extend a convergence result by Mielke for Prandtl–Tomlinson-like systems, showing the rate-independent nature of the limit of a family of systems characterized by a vanishing viscosity and a wiggly perturbation in the energy, that scales non-homothetically to zero. We then apply the result to some simple mechanical models, that exemplify the interaction of a bristle with a surface having small fluctuations. We find that the resulting friction is the product of two factors: a geometric one, depending on the bristle angle and on the fluctuation profile, and a energetic one, proportional to the normal force exchanged between the bristle-like element and the surface. Finally, we explain the "with the nap/against the nap" asymmetry in terms of our result.

Phase Field Modeling of Fracture - Quasi-Static Vs. Dynamic Formulations

Charlotte Kuhn
University of Kaiserslautern, Germany
Alexander Schlueter, Ralf Mueller

In phase field models of brittle fracture cracks are represented by means of a continuous scalar field, which resembles a damage variable. The evolution equation of this fracture field is coupled to the mechanical field equations in order to model the mutual interaction of fracture propagation and the mechanical loading. The extension of the well-known quasi-static phase field fracture model to a dynamic setting is straightforward. Concerning numerical simulations of fracture with the finite element method, the phase field modeling technique is beneficial since there are neither displacement jumps across the crack ligaments nor stress singularities at crack tips. However, simulations with the quasi-static formulation are faced with certain difficulties concerning numerical stability especially in situations when cracks do not propagate smoothly but extend by finite increments. In this case special numerical techniques or a regularization of the model are required to stabilize the simulations. On the other hand such events of finite crack extension do not occur in the dynamic phase field fracture model since the velocity of crack extension is naturally bounded in this setting. The observed crack speeds are in good agreement with experimental data and analytic considerations from classical fracture mechanics.

Quasistatic and Dynamic Evolution of Materials with Defects

Giuliano Lazzaroni
SISSA, Trieste, Italy

I will present some recent results on quasistatic and dynamic evolution of materials affected by fracture, damage, and plasticity, studied with variational methods.

Gradient Flow of Fractional Interaction Equations

Edoardo Mainini
Vienna University, Italy
Stefano Lisini, Antonio Segatti

We prove existence of a global weak solution for a fractional interaction equation. The equation is interpreted as the gradient flow of the square norm of the Sobolev space H^{-s}, $s \in (0,1)$. We provide dissipation estimates and decay rates of L^p norms as well. Finally, we show the convergence of the constructed solutions to the solution of the standard porous media equation, as $s \to 0$.

Quasistatic Evolution for a Cohesive Fracture Model with Fatigue: a Proof of Global Stability by Means of Optimal Transport of Young measures

Gianluca Orlando
SISSA, Italy
Vito Crismale, Giuliano Lazzaroni

We introduce a model for the evolution of a material which may present a cohesive fracture on a prescribed crack path. The main feature of this model is that some energy is dissipated both when the crack opening increases and when it decreases. This peculiar response leads to a fatigue behaviour, so that a complete fracture may occur even after small oscillations of the jump. We prove the existence of a globally stable quasistatic evolution in terms of Young measures: the proof of the global stability is carried out by employing Optimal Transport of Young measures.

Crystal Dislocations with Different Orientation and Collisions

Stefania Patrizi
UT Austin, USA
Enrico Valdinoci

Dislocations are moving defects in crystals that can be described at several scales by different models. We consider a 1D evolution equation arising in the Peierls-Nabarro model, which is a phase field model describing dislocation dynamics at a microscopic scale. Differently from the previous literature, we treat the case in which dislocations do not occur all with the same orientations (i.e. opposite orientations are allowed as well). We show that, at a long time scale, and at a mesoscopic space scale, the dislocations have the tendency to concentrate as pure jumps at points which evolve in time, driven by the external stress and by a singular potential. Due to differences in the dislocations orientation, these points may collide in finite time. We provide an estimates on the relaxation times of the system after collision. The results that will be presented have been obtained in some papers in collaboration with E. Valdinoci.

Stability of a Three Dimensional Variational Model of Superelastic Shape-Memory Alloys

Kim Pham
ENSTA ParisTech, France
Roberto Alessi

We present a variational framework for the three-dimensional macroscopic modeling of superelastic shape-memory alloys in an isothermal setting. Phase transformation is accounted through a unique second order tensorial internal variable, acting as the transformation strain. Postulating the total strain energy density as the sum of a free energy and a dissipated energy, the model depends on two material scalar functions of the norm of the transformation strain and a material scalar constant. The quasi-static evolution problem of a domain is formulated in terms of two physical principles based on the total energy of the system: a stability criterion which selects the local minima of the total energy and an energy balance condition which ensures the consistency of the evolution of the total energy with respect to the external loadings. The local phase transformation laws in terms of Kuhn-Tucker relations are deduced from the first-order stability condition and the energy balance condition. Based on this variational framework, stability of homogeneous states under for proportional and non-proportional loadings is provided by means of second-order stability conditions.

Slow Motion for the Nonlocal Allen-Cahn Equation in N Dimensions

Matteo Rinaldi
Carnegie Mellon University, USA
Ryan Murray

Slow motion of solutions of the nonlocal Allen–Cahn equation in a bounded domain$\Omega \subset \mathbb{R}^n$, for $n > 1$, is studied. The initial data is assumed to be close to a configuration whose interface separating the states minimizes the surface area (or perimeter); both local and global perimeter minimizers are taken into account. The evolution of interfaces on a time scale ϵ^{-1} is deduced, where ϵ is the interaction length parameter. The key tool is a second-order Γ–convergence analysis of the energy functional, which provides sharp energy estimates. New regularity results are derived for the isoperimetric function of a domain. Slow motion of solutions for the Cahn–Hilliard equation starting close to global perimeter minimizers is proved as well.

Discretization of Evolutions of Critical Points and Applications in Fracture Simulation

Francesco Solombrino
TU München, Germany
Massimo Fornasier, Filippo Cagnetti, Marco Artina

We introduce a novel constructive approach to define time evolution of critical points of an energy functional. Our procedure is prone to efficient and consistent numerical implementations, and allows for an existence proof under very general assumptions. We consider in particular rather nonsmooth and nonconvex energy functionals, provided the domain of the energy is finite dimensional. Nevertheless, in the infinite dimensional case study of a cohesive fracture model, we prove a consistency theorem of a discrete-to-continuum limit. We show that a quasistatic evolution can be indeed recovered as limit of evolutions of critical points of finite dimensional discretizations of the energy, constructed according to our scheme. To illustrate the results, we provide several numerical experiments both in one and two dimensions. These agree with the crack initiation criterion, which states that a fracture appears only when the stress overcomes a certain threshold, depending on the material.

Capabilities and Numerical Implementation of a Variational Gradient-Damage Plasticity Model for Cohesive and Ductile fracture

Erwan Tanne
Ecole Polytechnique, France
R. Alessi, B. Bourdin, J.J. Marigo

It is now well established that gradient damage models are very efficient to account for the behavior of brittle and quasi-brittle materials. However such models cannot describe the nucleation of cohesive cracks, i.e. the existence of surface of discontinuity of the displacement field with a non vanishing interfacial stress. Moreover, these models do not account for residual strains, typically observed during ductile fracture. A natural way to include such effects is to introduce plastic strains into the model and to couple their evolution with damage. In this talk, I will extend a coupled damage and plasticity model owing these features [R. Alessi, J.J Marigo, S. Vidoli, Springer 2014], by implementing general perfect plasticity criteria, and present some novel numerical simulations. Specifically, the capability and versatility of this extended model to describe ductile and cohesive fracture behaviours is shown on a traction test in both one- and multi-dimensional settings by means of parametric analyses. Also crack path and cohesive effects comparisons between numerical simulations and experiments will be shown.

From Adhesive Contact to Brittle Delamination in Visco-Elastodynamics

Marita Thomas
Weierstrass Institute Berlin, Germany
Riccarda Rossi

This contribution addresses two models describing the rate-independent fracture of a material compound along a prescribed interface in a visco-elastic material. This unidirectional process is modeled in the framework of Generalized Standard Materials with the aid of an internal delamination parameter. In the context of (fully) rate-independent systems within the energetic formulation it has become a well-established procedure to obtain solutions of the brittle model via an adhesive-contact approximation based on tools of evolutionary Gamma-convergence. This means that the non-smooth, local brittle constraint, confining displacement jumps to the null set of the delamination parameter, is approximated by a smooth, non-local surface energy term. Here, we discuss the extention of this approach for systems that couple the rate-independent evolution of the delamination parameter with a viscous and dynamic evolution of the displacements in the bulk.

Evolutionary Convergence of Discrete Dislocation Dynamics in 2D

Patrick van Meurs
Kanazawa University, Japan
A. Garroni, M. A. Peletier, L. Scardia

Plasticity of metals is facilitated by the collective motion of many dislocations, which we model as point particles in two-dimensions. The starting point is a gradient flow description for the dislocation positions, which is driven by an interaction energy. Our main result is the evolutionary convergence of these gradient flows as the number of dislocations tends to infinity. Our proof strategy relies on variational tools such as Gamma-convergence and the framework for evolutionary convergence developed by Sandier and Serfaty in 2004. The main challenge in the proof is to control the logarithmic nature of the interaction potential, which is unbounded and non-local.

Numerical Approximation of a Mesoscale Approach for the Dislocation Density Evolution in Single Crystal Plasticity

Christian Wieners
KIT, Germany

Classical macroscopic approaches in continuum mechanics for single crystal plasticity fail to describe the physical mechanisms induced by dislocation motion. On the other hand, small-scale approaches directly simulating single dislocations require extremely high computational costs. Here, we consider a mesoscale model for the evolution of dislocation densities based on small-strain single-crystal plasticity combined with Orowan's relation for the plastic shear strain in every slip system of the crystal. The system is closed by an evolution system for averaged dislocation densities and dislocation curvature densities (derived from the higher order continuum dislocation theory developed by Hochrainer et al.) and a constitutive law for the dislocation velocity. We introduce a fully coupled numerical method combining a conforming finite element approximation of elasto-plasticity with an implicit Runge-Kutta discontinuous Galerkin discretization of the dislocation system. The mesoscale method is evaluated for several 3D benchmark configurations and compared with results obtained from simulations on smaller scales.

Special Session 69: Dispersive Effects in Nonlinear PDEs

Paolo Antonelli, Gran Sasso Science Institute - INFN, Italy

The aim of this session is to discuss the analysis of dispersive behavior emerging in the study of some nonlinear PDEs. Those equations arise in the description of various phenomena in physics, from nonlinear optics to low temperature physics, to acoustic waves in compressible fluid dynamics. Particular attention will be given to gather young mathematicians working in this active research area.

Novel Schrodinger-Strichartz Estimates for Liouville Equations, and Well-Posedness for Rough and Nonlinear Transport

Agissilaos Athanassoulis
University of Leicester, England

Strichartz estimates for Schrödinger equations have been used to construct a powerful well-posedness theory for a very large class of nonlinear problems and problems with very general space-time dependent potentials. Several attempts to replicate this approach for different PDEs have appeared in recent years, with various degrees of success.
Kinetic transport equations are a natural candidate for a Strichartz-type-estimates theory, since in the simplest case they describe the same dynamics as the Schrödinger equation, simply unfolded in phase-space. Indeed, there exist "kinetic Strichartz estimates", e.g.

$$\|f\|_{L^q_t L^p_x L^r_k} \leq \|f_0\|_{L^a_{x,k}},$$
$$\frac{2}{q} = n\left(\frac{1}{r} - \frac{1}{p}\right),$$
$$\frac{1}{a} = \frac{1}{2}\left(\frac{1}{r} + \frac{1}{p}\right),$$

but they do not seem to drive many of the nonlinear or rough problems that are currently under investigation.
We present a new class of dispersive estimates for kinetic equations, in direct analogy with the Schrödinger-Strichartz estimates on $\mathbb{R}^{2n}$, e.g.

$$\|f\|_{L^q_t L^r_{x,k}} \leq \|f_0\|_{L^2}, \qquad \frac{1}{q} = n\left(\frac{1}{2} - \frac{1}{r}\right),$$

along with applications in the well-posedness of rough and nonlinear kinetic problems. Using these estimates, a large class class of nonlinear Liouville equations is essentially transformed to derivative-NLS equations.

Invariant Measure for Cubic NLS on the Real Line

Federico Cacciafesta
Univ. Milano Bicocca, Italy
Anne-Sophie de Suzzoni

The existence of invariant measures has proven to yield significant improvements in the deterministic analysis of nonlinear dispersive equations. In this talk I will try to explain the general ideas of the theory, focusing on some recent results we obtained in collaboration with A. S. de Suzzoni concerning the cubic nonlinear NLS on the real line.

Modulational Instability in Dispersion-Kicked Optical Fibers

Guillaume Dujardin
Inria, France
S. Rota-Nodari, M. Conforti, A. Kudlinski, A. Mussot, S. Trillo, S. De Bievre

We present a theoretical, numerical and experimental study of modulational instability in dispersion-kicked optical fibers. In our setting, the evolution along the fiber of a weakly perturbed continuous wave is described by a cubic nonlinear Schrödinger equation with periodically-modulated dispersion. In the context of normal averaged dispersion, using Floquet theory, we provide simple analytical estimates on the positions of the gain bands. In particular, we show that their positions do not depend on the form of the modulation of the dispersion. Moreover, when the dispersion takes the form of a Dirac comb, we provide analytical estimates on the bandwiths of the gain bands. We conclude by illustrating our theoretical analysis with numerical experiments as well as actual physical experiments that are in good agreement of the analysis.

Growth of Sobolev Norms for the Non-Linear Schrödinger Equation on the Two-Dimensional Torus

Emanuele Haus
University of Naples "Federico II", Italy
Marcel Guardia, Michela Procesi

We study the non-linear Schrödinger equation (with analytic nonlinearity of any order) on the two-dimensional torus and exhibit orbits whose Sobolev norms grow with time. The main point is to make use of an accurate combinatorial analysis in order to reduce to a sufficiently simple toy model, which generalizes the one discussed in the paper by J. Colliander, M. Keel, G. Staffilani, H. Takaoka and T. Tao for the case of the cubic NLS. We also give estimates of the time needed to obtain such growth, by refining and adapting to this more general case the techniques used for the cubic case in the work by M. Guardia and V. Kaloshin.

On a Singularly Perturbed Gross-Pitaevskii Equation

Stefan le Coz
University of Toulouse 3, France
Isabella Ianni, Julien Royer

We consider the 1D Gross-Pitaevskii equation perturbed by a Dirac potential. Using a fine analysis of the properties of the linear propagator, we study the well-posedness of the Cauchy Problem in the energy space of functions with modulus 1 at infinity. Then we study existence and stability of the black solitons with a combination of variational and perturbative arguments. This is a joint work with Isabella Ianni and Julien Royer.

Freezing of Energy of a Soliton in an External Potential

Alberto Maspero
University of Nantes, France
Dario Bambusi

We study the dynamics of a soliton in the generalized NLS with a small external potential ϵV of Schwartz class. We prove that there exists an effective mechanical system describing the dynamics of the soliton and that, for any positive integer r, the energy of such a mechanical system is almost conserved up to times of order ϵ^{-r}. In the rotational invariant case we deduce that the true orbit of the soliton remains close to the mechanical one up to times of order ϵ^{-r}.

Quasi-Periodic Standing Wave Solutions for Gravity Capillary Water Waves

Riccardo Montalto
University of Zurich, Switzerland
Massimiliano Berti

I will present a recent result concerning the existence and the stability of small-amplitude quasi-periodic solutions for the water waves equations with surface tension. The core of the proof is the reduction of the linearized equation (at any approximate solutions) to constant coefficients. Such a reduction procedure is achieved by using Pseudo differential operators theory and a KAM reducibility scheme.

Dispersive Analysis of Kink Solutions

Claudio Munoz
University of Chile, Chile
Michal Kowalczyk and Yvan Martel

The purpose of this talk is to review some recent results about the asymptotic dynamics of kink solutions for the well-known physical model called ϕ^4 in 1 1 dimensions. We will show that orbital stability leads to a particular case of asymptotic stability under suitable assumptions on the initial data.

Stein-Tomas and Strichartz Estimates for Orthonormal Functions

Julien Sabin
Univ. Paris-Sud Orsay, France
Rupert L. Frank

We generalize the Stein-Tomas and Strichartz inequalities to systems of orthonormal functions, with an optimal dependence on the number of such functions. These can be used to study dispersive properties in infinite quantum systems.

Schroedinger Regularization of a Van Der Waals Gas

Marta Strani
Université Paris Diderot, IMJ-PRG, France
Benjamin Texier

We consider a dispersive regularization of the compressible Euler equations in Lagrangian coordinates. We assume a Van der Waals pressure law, which presents both hyperbolic and elliptic zones. The dispersive regularization is of Schroedinger type. This implies in particular that the regularized system satisfies the same conservation law as the original, physical system. For this system, we prove local-in-time existence of solutions generated by initial data of amplitude one, over time intervals that depend on the regularization parameter.

Bound on the Slope of Steady Water Waves with Vorticity

Miles Wheeler
NYU Courant Institute, USA
Walter Strauss

We consider the maximum angle between the free surface of a steady two-dimensional water wave and the horizontal. In the absence of vorticity, McLeod proved that this angle exceeds 30 degrees for very large waves, while Amick proved that it can never exceed 31.15 degrees. With adverse vorticity, on the other hand, there is numerical evidence that steady waves can become much steeper and even overturn. We prove an upper bound of 45 degrees for a large class of waves with favorable vorticity, in particular for constant vorticity of the appropriate sign.

Special Session 70: Vortex Dynamics and Geometry: Analysis, Computations and Applications

Takashi Sakajo, Kyoto University, Japan
Bartosz Protas, McMaster University, Canada

This session will survey recent developments in vortex dynamics focusing on problems in which geometry, understood in a broad sense, plays an important role. Examples of such problems include flows defined on various manifolds as well as free-boundary problems arising in the dynamics of finite-area vortices. Geometric aspects are also important for applications such as superfluid turbulence and vortex-dominated flows in domains with complex boundaries. The topics of the session will be selected to emphasize the interaction between mathematical analysis and computations.

Existence and Regularity of V-States for the Euler and SQG Patch Equations

Javier Gomez-Serrano
Princeton University, USA
Angel Castro, Diego Cordoba

The evolution of the motion of a patch is completely determined by the evolution of the boundary, allowing the problem to be treated as a non-local one dimensional equation for the contour. In this talk we will discuss the existence and regularity of uniformly rotating solutions - also known as V-states - for the vortex patch and generalized surface quasi-geostrophic (gSQG) patch equation.

New Solutions for Hollow Vortices in an Infinite Channel.

Christopher Green
Queensland University of Technology, Australia

We will present new analytical solutions for a co-travelling hollow vortex pair and a single row of hollow vortices in an infinite channel. These new solutions generalise several known classical solutions for hollow vortices. The mathematical problems to be solved are particular types of free boundary problem over a multiply connected domain. We have found concise formulae for the conformal mapping determining the shape of the boundaries of the hollow vortices in both channel geometries by employing free streamline theory in combination with the function theory of the Schottky-Klein prime function. Various properties of the solutions will also be presented.

Axisymmetrization of a Non-Uniform Elliptic Vortex and Inverse Energy Cascade in 2D Turbulence

Yoshifumi Kimura
Nagoya University, Japan

A non-uniform elliptic vortex tends to shed filaments of vorticity while rotating, and reshapes it into a circular vortex. This process is often called the axisymmetrization process. This process is investigated by using two different methods, pseudo-spectral DNS (Eulerian) and a system of point vortices (Lagrangian). In particular, the physical cascade mechanisms in 2D turbulence, i.e. inverse energy cascade and forward enstrophy cascade, are studied concerning with the axi-symmetrization process. With the DNS, it will be shown that the squared vorticity gradient called palinstrophy grows along the filament ejection, and the development of palinstrophy is analyzed. With a system of point vortices, it can be demonstrated that an averaged wave number decreases during the process, which indicates that the distribution of energy tends towards a lager scale.

A General Theory of Viscous Selection in Two-Dimensional Vortical Flows

Paolo Luzzatto-Fegiz
UC Santa Barbara, USA

Vortical equilibria prove useful in modeling problems including aircraft wakes, biolocomotion, and geophysical flows. A common approximation is to neglect viscosity; while this enables steady vortices (whose shape must be found), it does not predict the vorticity distribution (which must be assumed). Unfortunately, model results can be very sensitive to the choice of distribution.

Interestingly, simulations have shown that viscous vortices (such as vortex pairs) can relax to nearly self-similar states. A similarity ansatz decouples the vorticity equation into a steady advection equation, together with a diffusion equation. Aside from the special cases of axisymmetric or parallel flow (where advective terms vanish), no other solution is known, to the best of our knowledge.

To address these issues, we discretize the flow by superposing uniform-vorticity regions, yielding a weak formulation. This reveals more equations than unknowns, such that an exact solution is not possible. We circumvent this by retaining the advection equation, while enforcing an averaged version of the diffusion equation, such that the distribution is selected by viscosity. This revised problem is solved with low computational effort, finding flows in good agreement with full DNS results, thereby yielding a general approach to find viscously selected equilibria.

Linear Stability of Hill's Vortex to Axisymmetric Perturbations

Bartosz Protas
McMaster University, Canada
Alan Elcrat

We consider the linear stability of Hill's vortex with respect to axisymmetric perturbations. Given that Hill's vortex is a solution of a free-boundary problem, this stability analysis is performed by applying methods of shape differentiation to the contour dynamics formulation of the problem in a 3D axisymmetric geometry. This approach allows us to systematically account for the effect of boundary deformations on the linearized evolution of the vortex under the constraint of constant circulation. The resulting singular integro-differential operator defined on the vortex boundary is discretized with a highly accurate spectral approach. This operator has two unstable and two stable eigenvalues complemented by a continuous spectrum of neutrally-stable eigenvalues. By considering a family of suitably regularized (smoothed) eigenvalue problems solved with a range of numerical resolutions we demonstrate that the corresponding eigenfunctions are in fact singular objects in the form of infinitely sharp peaks localized at the front and rear stagnation points. These findings thus refine the results of the classical analysis by Moffatt & Moore (1978). It is also shown that the magnitude of the eigenvalues associated with the unstable and stable eigenmodes is proportional to the translation velocity of the vortex.

Vortex Knots Cascade by HOMFLYPT Polynomial

Renzo Ricca
U. Milano-Bicocca, Italy
Xin Liu

Since Moffatt's original work of 1969, it is well-known that the kinetic helicity of a vortex filament admits topological interpretation in terms of Călugăreanu-White self linking number, and geometric decomposition in terms of writhe and twist. By applying knot theoretical techniques Liu & Ricca (2012; 2015) derived well-known knot polynomials, most notably the HOMFLYPT polynomial, from the helicity of fluid systems, hence showing that these are new invariants of ideal fluid mechanics. In the case of HOMFLYPT the two polynomial variables are shown to be related to the writhe and twist of the vortex knot.
In presence of dissipation topology is bound to change by the continuous interaction and reconnection of fluid strands. A prototype reconnection mechanism is based on the anti-parallel recombination of the interacting strands. During such an event writhe helicity remains conserved, thus showing that any change in helicity is due to a change in intrinsic twist of the interacting strands. We show that the natural vortex cascade process seen in real experiments can be detected by a monotonically decreasing sequence of HOMFLYPT values.

Words and Trees: Symbolic Classifications of Streamline Topologies for 2D Incompressible Vortex Flows

Takashi Sakajo
Kyoto University, Japan
Tomoo Yokoyama

I will provide a mathematical theory of topological pattern characterizations for 2D incompressible flows that has recently been developed in our research project. It enables us to identify topological flow patterns, i.e. streamline patterns, with simple symbolic expressions called maximal words and regular expressions with graph (tree) representations. The symbolic descriptions bring us new qualitative information on the correspondence between flow patterns and their functions/evolutions, that are applicable to many flow phenomena appearing in material and life sciences.

Drift Due to a Viscous Vortex Ring

Jean-Luc Thiffeault
University of Wisconsin – Madison, USA
Thomas Morrell, Saverio Spagnolie

Biomixing is the study of fluid mixing caused by swimming organisms. The swimming of large organisms can lead to mixing by the turbulent flows in their wakes, but the wakes created by small swimming organisms are not turbulent. Instead, the main mechanism of mixing by smaller organisms is the net particle displacement (drift) induced by the swimmer. Several experiments have been performed to examine this drift for small jellyfish; these produce vortex rings which trap and transport a fair amount of fluid. However, since inviscid theory implies infinite particle displacements, the effects of viscosity must be included to understand the damping of real vortex motion. We use a model viscous vortex to compute particle displacements and other relevant quantities, such as the integrated moments of the displacement. Fluid entrainment at the tail end of a growing vortex 'envelope' is found to play an important role in the total fluid transport and drift.

Self-Similar Vortex Filament Motion Under the Non-Local Biot-Savart Model

Robert van Gorder
University of Oxford, England

The self-induced motion of a thin vortex filament is governed by the Biot-Savart model which, due to inherent non-locality and nonlinearity, is often approximated by the local induction approximation (LIA). While very regular filaments, such as those exhibiting helical or planar geometries, have been studied analytically under both formulations, more complicated filament structures are often only studied under LIA. One type of vortex filament structure to have attracted interest in recent years is that which obeys a self-similar scaling. Among various applications, these filaments have been used to model the motion of quantized vortex filaments in superfluid Helium after reconnection events. While similarity solutions have been described analytically and numerically using the LIA, they have not been studied (or even shown to exist) under the non-local Biot-Savart model. We will show not only that self-similar vortex filament solutions exist for the non-local Biot-Savart model, but that such solutions are qualitatively similar to their LIA counterparts. This suggests that the various LIA similarity solutions found previously should be valid physically (at least when they are of sufficient bounded variation), since they agree well with the dynamics from the Biot-Savart model. Extensions to more complicated models involving mutual friction and an imposed normal fluid, which are useful for describing vortex filament dynamics in superfluid Helium above the zero-temperature limit, may be discussed.

The Interplay of Curvature and Vortices in Flow on Curved Surfaces

Axel Voigt
Technische Universität Dresden, Germany
Sebastian Reuther

Incompressible fluids on curved surfaces are considered with respect to the interplay between topology, geometry and fluid properties using a surface vorticity-stream function formulation, which is solved using parametric finite elements. Motivated by designed examples for superfluids, we numerically consider the influence of a geometric potential on vortices for fluids with finite viscosity and show examples in which a change in the geometry is used to manipulate the flow field.

Special Session 71: Elliptic Equations and Systems, and Concentration Phenomena

Cyril Tintarev, Uppsala University, Sweden
Kanishka Perera, Florida Institute of Technology, USA
Marco Squassina, University of Verona, Italy

Existence Results for Nonlinear Elliptic Problems

Pasquale Candito
Mediterranean University of Reggio Calabria, Italy
G. Bonanno, R. Livrea, D. Motreanu

The aim of this talk is to present a recent coincidence point theorem for sequentially weakly continuous maps defined on Banach spaces. As a consequence, some existence results for nonlinear boundary value problems are showed.

Radial Solutions for Elliptic Problems with Symmetry

Florin Catrina
St. John's University, USA

We discuss the existence of radial solutions for certain second order semilinear elliptic PDEs. An energy balance identity is employed to prove nonexistence of such solutions. These nonexistence cases are due to the loss of compactness in embeddings of the appropriate functional spaces. The loss of compactness is manifested in the concentration of minimizing sequences at singularities of the potential.

Two Non-Zero Solutions to Elliptic Dirichlet Problems

Giuseppina Dagui
University of Messina, Italy
Gabriele Bonanno

This talk deals with the existence of two non-zero critical points for an appropriate class of differentiable functionals. Our main tools are a recent local minimum theorem and the classical Ambrosetti-Rabinowitz theorem. Our main result is an appropriate combination of such results in order to obtain two non-zero critical points. In fact, once obtained the first non-zero critical point by the local minimum theorem, a direct application of the mountain pass theorem allows to get the second critical point that in general can be zero. Instead, we verify that a local minimum actually is a global minimum for a suitable restriction of the functional and, hence, we prove that all the paths starting from it have a high level greater than zero, and this guarantees the second critical point must be non-zero. As an application, we get two non-zero weak solutions to nonlinear elliptic Dirichlet problems.

Three Nontrivial Solutions for Nonlinear Fractional Laplacian Equations

Antonio Iannizzotto
University of Cagliari, Italy
F. Gamze Duzgun

We study a Dirichlet-type boundary value problem for a pseudodifferential equation driven by the fractional Laplacian, proving the existence of three nonzero solutions. When the reaction term is sublinear at infinity, we apply the second deformation theorem and a recent characterization of the second eigenvalue of the fractional Laplacian. When the reaction term is superlinear at infinity, we apply the mountain pass theorem and Morse theory.

Second Order Derivatives of Solutions of Uniformly Elliptic Isaacs Equations

Jay Kovats
Florida Institute of Technology, USA

In this talk, we discuss continuity and integrability properties of second order derivatives of viscosity solutions of uniformly elliptic Isaacs equations of the form $F(D^2u) := \max_{y\in Y}\min_{z\in Z} tr[A(y,z)D^2u] = 0$ in a domain $D \subset \mathbb{R}^d$, where Y, Z are finite sets. Here, $\forall\, y \in Y$, $z \in Z$, $A(y,z)$ is a symmetric $d \times d$ matrix, satisfying $\lambda I_d \leq A(y,z) \leq \Lambda I_d$, for some constants 0

On a Dirichlet Problem with the p-Laplacian

Roberto Livrea
University of Reggio Calabria, Italy
S. Carl, P. Candito

The aim of the talk is to show some existence and multiplicity results for a class of parameter-dependent Dirichlet problems involving the p-Laplacian. By using different approaches based on variational and topological methods, possible different intervals of parameters for which the problem under examination admits solutions are detected.

On a Robin Problem with Indefinite Weight and Asymmetric Reaction

Salvatore Angelo Marano
University of Catania, Italy
Nikolaos S. Papageorgiou

The existence of multiple smooth solutions to a semilinear Robin problem with indefinite unbounded potential and asymmetric nonlinearity is established. Both crossing and resonance are allowed. Proofs exploit variational methods, truncation techniques, and Morse theory.

The Brezis-Nirenberg Problem for the Fractional p-Laplacian

Kanishka Perera
Florida Institute of Technology, USA
Sunra Mosconi, Marco Squassina, Yang Yang

We obtain nontrivial solutions to the Brezis-Nirenberg problem for the fractional p-Laplacian operator, extending some results in the literature for the fractional Laplacian. The quasilinear case presents two serious new difficulties. First an explicit formula for a minimizer in the fractional Sobolev inequality is not available when $p \neq 2$. We get around this difficulty by working with certain asymptotic estimates for minimizers. The second difficulty is the lack of a direct sum decomposition suitable for applying the classical linking theorem. We use an abstract linking theorem based on the cohomological index to overcome this difficulty.

A Survey of Cocompactness Results and Uses

Ian Schindler
University of Toulouse 1 Capitole, France

Cocompact embeddings and profile decompositions are two fundamental tools in concentration analysis on a functional analytic level . Cocompact embeddings (relative to a non-compact group) are similar but weaker than compact embeddings. In addition to the classical cases of compact embeddings of Sobolev spaces relative to the non-compact groups of translations and dilations, cocompact embeddings have been established in a wide range of spaces. We survey some of these embeddings and give their applications to PDE.

Uniqueness of Positive Solutions of Brezis-Nirenberg Problems on the Hyperbolic Space

Naoki Shioji
Yokohama National University, Japan
Kohtaro Watanabe

Uniqueness of positive solutions for the equations like the scalar field equation have been studied by many researchers. Recently, the author and Prof. Kohtaro Watanabe introduced a generalized Pohožaev identity and gave uniqueness results which are applicable to various equations including the scalar field equation; see JDE 252 (2012) and CVPDE 55 (2016). In this talk, we study the uniqueness of positive solutions of

$$\Delta_{\mathbb{H}^n}\varphi + \lambda\varphi + \varphi^p = 0$$

on the n-dimensional hyperbolic space $\mathbb{H}^n$, where $n \in \mathbb{N}$ with $n \geq 2$,$\Delta_{\mathbb{H}^n}$ is the Laplace-Beltrami operator on $\mathbb{H}^n$, $\lambda \leq (n-1)^2/4$, and p is subcritical or critical. In particular, in the case $n = 2$, we improve Mancini and Sandeep's uniqueness result in ASNSP 7 (2008).

On Degenerate P(x)-Laplace Equations Involving Critical Growth with Two Parameters

Inbo Sim
University of Ulsan, Korea
Ky Ho

After establishing a concentration-compactness principle for weighted variable exponent spaces, we show the existence of a nonnegative solution and infinitely many solutions for degenerate p(x)-Laplace equations involving critical growth with two parameters using various techniques in Calculus of Variations.

Special Session 72: Optimal Control and its Applications

Alexander J. Zaslavski, The Technion - Israel Institute of Technology, Israel
Monica Motta, University of Padua, Italy
Franco Rampazzo, University of Padua, Italy

A special session on "Optimal Control and its applications" will bring together a selected group of experts in this area. The growing importance of optimal control has been realized in recent years. This is due not only to theoretical developments in this area, but also because of numerous applications to engineering, economics and life sciences. Approximately 25 participants from Australia, Brazil, France, Germany, Israel, Italy, Russian and USA will participate in the session. The topics which will be discussed include infinite horizon control problems, turnpike phenomenon, averaging in optimal control, optimality conditions in control problems, qualitative and quantitative aspects of optimal control, control and stabilization of PDEs.

Optimal Feedback Control Law for Uncertain Dynamic Systems on Banach Spaces

Nasiruddin Ahmed
University of Ottawa, Canada

In this talk we consider a class of partially observed dynamic systems on infinite dimensional Banach spaces subject to dynamic and measurement uncertainty. The problem is to find an output feedback control law, an operator valued function, that minimizes the maximum risk. This involves optimization on the space of operator valued functions. We present a recent result on existence of optimal feedback law. Inspired by the existence result, we consider the question of characterization of the optimal feedback operator. We develop the necessary conditions of optimality which an optimal feedback law must satisfy. A conceptual algorithm for computation of the optimal operator valued function is also presented.

Second Order Analysis of Control-Affine Problems with Scalar State Constraint

Maria Soledad Aronna
School of Applied Mathematics, FGV/Rio, Brazil
J. Frèdèric Bonnans, Bean-San Goh

In this article we establish new second order necessary and sufficient optimality conditions for a class of control-affine problems with a scalar control and a scalar state constraint. These optimality conditions extend to the constrained state framework the Goh transform, which is the classical tool for obtaining an extension of the Legendre condition. We provide examples to illustrate the theory.

Useful Conditions for Non Trivial Pontryagin's Adjoint Variable in Infinite Dimension

Mohammed Bachir
University Paris 1 Panthéon-Sorbonne, France
J. Blot

We give some usable conditions on a sequence of norm one in a dual Banach space under which the sequence does not converges to the origin in the w^*-topology. These requirements help to ensure that the Lagrange multipliers are nontrivial, when we are interested for example on the infinite dimensional infinite-horizon Pontryagin principles for discrete-time problem.

Pontryagin Principles for Systems Governed by Delay Functional Differential Equations

Joel Blot
University Paris 1 Panthéon-Sorbonne, France
J. Blot, M.I. Koné

We provide new results on Pontryagin principles when the differential equation which governs the system is a delay functional differential equation. The new results are based on the study of precise properties of a linearization of the differential equation to obtain results under lightest possible assumptions.

Minimizers for Nonconvex Variational Problems in the Plane Via Convex/Concave Rearrangements

Dean Carlson
Mathematical Reviews, American Mathematical Society, USA

Recently, A. Greco utilized convex rearrangements to present some new and interesting existence results for noncoercive functionals in the calculus of variations. Moreover, the integrands were not necessarily convex. In particular, using convex rearrangements permitted him to establish the existence of convex minimizers essentially considering the uniform convergence of the minimizing sequence of trajectories and the pointwise convergence of their derivatives. The desired lower semicontinuity property is now a

consequence of Fatou's lemma. In this paper we point out that such an approach was considered in the late 1930's in a series of papers by E. J. McShane for problems satisfying the usual coercivity condition. Our goal is to survey some of McShane's results and compare them with Greco's work. In addition, we will update some hypotheses that McShane made by making use of a result due to T. S. Angell on the avoidance of the Lavrentiev phenomenon.

A New Sufficient Optimality Condition for Infinite Horizon Optimal Control Problems

Valeriano de Oliveira
UNESP - Univ. Estadual Paulista, Brazil

The work is devoted to introduce a new sufficient optimality condition for infinite horizon optimal control problems. It is showed that normal extremal processes are optimal under this new condition, termed as MP-pseudoinvexity. Moreover, problems in which every normal extremal process is optimal necessarily obey the definition of MP-pseudoinvexity.

An Optimal Control Problem for Hybrid Dynamical Systems with Polynomial Impulses

Elena Goncharova
Institute for System Dynamics and Control Theory of the Siberian Branch of the Russian Academy of Sciences, Russia
Maxim Staritsyn

We consider an optimal control problem for a measure driven hybrid dynamical system. The dynamics is a BV-relaxation of a system of a polynomial structure. The relaxed system is described by a measure differential equation subject to nonstandard mixed constraints imposed on a state and a control measure. The main results concern a special space-time transformation technique aimed at reducing the optimal impulsive control problem to an equivalent conventional one, and nesessary conditions for optimality.

Estimating the Number of Zeros of the Switching Functions in Optimal Control Problems for Compartment Models

Ellina Grigorieva
Texas Woman's University, USA

Estimation of the number of switchings of the optimal controls under assumption of the absence of singular arcs is a major challenge. Such estimates allow us to reduce the initial complex optimal control problem to a simpler problem of finite-constrained optimization. In turn, the number of the switchings of optimal controls is associated with the estimation of the number of zeros of the corresponding switching functions, which completely determine the behavior of these controls. Currently there are two approaches actively used for estimating the number of zeros of the switching functions. The first approach is to convert a linear non-autonomous system of differential equations for the switching function and related auxiliary functions to the linear non-autonomous differential equation with bounded coefficients with subsequent application of the Vallee-Poussin's theorem to such equation. The second approach is to reduce the matrix of the mentioned non-autonomous linear system to an upper-triangular form on a given time interval and then apply the generalized Rolle's Theorem to the transformed linear system. In this report, we propose our new results associated with estimating the number of zeros of the switching functions in optimal control problems for various compartment models. These results are compared with the results obtained earlier by the authors for SIR and SEIR models.

The Relationship Between Revealed Preference and The Slutsky Matrix

Yuhki Hosoya
Kanto-Gakuin University, Japan

This study provides a calculation method for utility function from a smooth demand function whose Slutsky matrix is negative semi-definite and symmetric. Moreover, this study presents an axiom of demand functions, and show that under the strong axiom, this axiom is equivalent to the existence of the corresponding continuous preference relation. If the demand function obeys this axiom, then such a preference relation is unique, and our calculating utility function is continuous and represents its preference relation. These results are obtained even if the demand function is not income-Lipschitzian. Further, this study shows that the mapping from demand function into continuous preference relation is continuous, which assures the applicability of our results for econometrics. Moreover, this study shows that if this demand function satisfies the rank condition, then our utility function is smooth. Lastly, this study shows that under an additional axiom, the above results hold even if the demand function has a corner solution.

Equilibrium Locus of the Flow on Circular Networks of Cells

Yirmeyahu Kaminski
Holon Institute of Technology, Israel

We perform a geometric study of the equilibrium locus of the flow that models the diffusion process over a circular network of cells. We prove that when considering the set of all possible values of the parameters, the equilibrium locus is a smooth manifold with boundary, while for a given value of the parameters, it is an embedded smooth and connected curve. For different values of the parameters, the curves are all isomorphic.

Moreover, we show how to build a homotopy between different curves obtained for different values of the parameter set. This procedure allows the efficient computation of the equilibrium point for each value of some first integral of the system. This point would have been otherwise difficult to be computed for higher dimensions. We illustrate this construction by some numerical experiments.
Eventually, we show that when considering the parameters as inputs, one can easily bring the system asymptotically to any equilibrium point in the reachable set, which we also easily characterise.

Case Studies for Optimal Control of Coupled ODE-PDE Systems

Sven-Joachim Kimmerle
Universitaet der Bundeswehr Muenchen, Germany
Matthias Gerdts, Roland Herzog

We consider several case studies for optimal control problems subject to coupled systems of ordinary and partial differential equations. As a key example we focus on a crane transporting a load.

The crane beam is considered as elastic and the load is modelled by a pendulum that is moved by a trolley running along the crane beam. The goal is to consider time-optimal control subject to initial and terminal conditions. Other examples are a truck with a fluid container where the fluid is subject to the shallow water equations and an elastic bridge under moving loads.

We derive mathematical models, formulate the optimal control problems and discuss numerical and analytic approaches for optimal control. We consider first-discretize-then-optimize methods as well as first-optimize-then-discretize methods.

Approximations with Bounded Functions in Scalar Multidimensional Variational Problems

Carlo Mariconda
UNIPD, Italy

We consider an integral functional $I(u) = \int_\Omega L(x, u(x), \nabla u(x))\, dx$ (the "energy") defined in the space of Sobolev functions on an open and bounded set, and that agree with a prescribed Lipschitz function ϕ on the boundary of the domain Ω, a bounded open subset of $\mathbb{R}^n$. We deal with the problem of approximating a given Sobolev function both in energy and in the strong topology with bounded functions that agree with ϕ on the boundary of Ω. As a consequence we prove the non occurrence of the Lavrentiev phenomenon for a class of non convex problems.

Small-Time Local Attainability for a Class of Control Systems with State Constraints

Antonio Marigonda
University of Verona, Italy
Thuy Thi Le

We consider the problem of small time local attainability (STLA) for nonlinear finite-dimensional time-continuous control systems in presence of state constraints. More precisely, given a nonlinear control system subjected to state constraints and a closed set S, we provide sufficient conditions to steer to S every point of a suitable neighborhood of S along admissible trajectories of the system, respecting the constraints, and giving also an upper estimate of the minimum time needed for each point to reach the target. The results presented generalize previous result obtained by Krastanov and Quincampoix with different techniques. Special emphasis is given to the control-affine systems, in which more explicit conditions can be given exploiting the natural Lie algebraic structure associated with the system. Methods of nonsmooth analysis are used.

Minimum Time and Unbounded Variation

Monica Motta
Padua University, Italy
Caterina Sartori

Given a control system with dynamics affine in the (unbounded) derivative of the control u and a closed target set, we study the minimum time problem without constraints on the total variation of u. Together with inputs u in AC, we introduce piecewise, locally AC inputs (with possibly unbounded variation) and consider the corresponding Charateodory solutions and extended Charateodory solutions, respectively. We embed such controls and solutions into a set of space-time solutions with unbounded variation and prove that they all belong to the larger class of (simple) limit solutions, recently introduced by M.S. Aronna and F. Rampazzo. Then we compare the minimum times over limit solutions and space-time solutions, with the infima over either AC or piecewise, locally AC inputs, respectively. We introduce sufficient conditions to have either that all these values coincide or that the minimum times over limit solutions and space–time solutions are equal just to the infimum over piecewise, locally AC controls. We conclude with a regularization result, where the (unique) minimum time function is approximated by penalized minimum time functions with minimizing inputs of bounded variation.

Maharam-Types and Lyapunov's Theorem for Vector Measures on Locally Convex Spaces Without Control Measures

Nobusumi Sagara
Hosei University, Japan
M. Ali Khan

We formulate the saturation property for vector measures in locally convex Hausdorff spaces as a nonseparability condition on the derived Boolean *sigma*-algebras by drawing on the topological structure of vector measure algebras. We exploit a Pettis-like notion of vector integration in locally convex Hausdorff spaces, the Bourbaki–Kluvanek–Lewis integral, to derive an exact version of the Lyapunov convexity theorem in locally convex Hausdorff spaces without the Bartle–Dunford–Schwartz property. We apply our Lyapunov convexity theorem to the bang-bang principle in Lyapunov control systems in locally convex Hausdorff spaces to provide a further characterization of the saturation property.

On Second Order Necessary Conditions in Optimal Control

Daniela Tonon
CEREMADE Universite Paris Dauphine, France
Helene Frankowska

This talk is devoted to second order necessary optimality conditions for the Mayer optimal control problem when the control set is a closed subset of $\mathbb{R}^n$ and endpoint constraints are present. Admissible controls are supposed to be just measurable. We show how to exploit the properties of particular second order variations, obtained using the adjacent tangent cone and the second order adjacent tangent subset to the control constraint, in order to obtain several different formulations of second order necessary conditions in integral form. These second order necessary conditions are then applied to obtain Goh condition, a pointwise second order necessary optimality condition which turns out to be useful when the optimal control for the Mayer problem is singular, i.e. when the classical Legendre-Clebsch condition is no more of use.

Linear Control Systems with Periodic Convex Integrands

Alexander Zaslavski
The Technion - Israel Institute of Technology, Israel

We study the structure of approximate optimal trajectories of linear control systems with periodic convex integrands and show that these systems possess a turnpike property. To have this property means, roughly speaking, that the approximate optimal trajectories are determined mainly by the integrand, and are essentially independent of the choice of time interval and data, except in regions close to the endpoints of the time interval. We also show the stability of the turnpike phenomenon under small perturbations of integrands and study the structure of approximate optimal trajectories in regions close to the endpoints of the time intervals.

Injectivity Properties of Pole Placement Maps of Linear Control Systems

Igor Zelenko
Texas A&M University, USA
Yanhe Huang, Frank Sottile

Our original question is for which linear control systems generically among all configurations of poles that can be realized by a static output feedback there is another static feedback that realizes the same configuration? For this goal we introduce the notion of generalized Wronski maps (GWM) on Grassmanians and study its degree. We distinguish a subclass of these maps, called self-adjoint GWM, for which the degree of the map is at least 2. This class generalizes Wronski maps on the Grasmannians of the space of solutions of self-adjoint differential operators and pole placement maps of symmetric linear control systems. The main question: are there non-selfadjoint GWM with degree greater than 1? We give a negative answer for small-dimensional Grassmannians of half-dimensional subspaces and in this way answer our original question in the case when the input and output spaces have the same small dimension.

Special Session 73: Mathematical Modeling and Computations

Hi Jun Choe, Yonsei University, Korea
Seick Kim, Yonsei University, Korea

Considering nonlinear phenomena like fluid flow, we make mathematical model by partial differential equations. As Hilbert announced in his 6th problem of Paris ICM, foundation of modeling and related computations are main themes. Hamiltonian dynamics, dynamical system and continuum mechanics are discussed to understand their relations. Our scope includes particle transport, nonlinear flow, turbulence, mathematical questions and their computations.

Asymptotic Self-Similarity of Singular Solutions for Quasilinear Lane-Emden Equations

Soohyun Bae
Hanbat National University, Korea

We consider the asymptotic self-similarity of singular solutions for quasilinear Lane-Emden equations. The main result is described for $p-$ Laplace Lane-Emden equation.

Effects of White Noise in Multistable Dynamics

Carey Caginalp
Brown University, USA
Xinfu Chen, Jianghao Hao, Yajing Zhang

The concept of stable equilibrium plays a key role in the theory of ordinary differential equations. A given initial condition uniquely determines how the system evolves to a particular stable equilibrium point. An important question that one can ask involves introducing white noise into the problem, and how even a perturbation by a very small amount of noise can influence which particular equilibrium point to which the system will evolve. Multiscale dynamics are well-known to describe practical examples such as patterns in physics, chemistry, and biology. This talk will address a prototypical multistep dynamics with multiple equilibrium points where the probabilities of ending up change depending on initial conditions.

Initial and Boundary Values for $L^q_\alpha(L^p)$ Solution of the Navier-Stokes Equations in the Half-Space

Tongkeun Chang
Yonsei University, Korea
Bum Ja Jin

In this paper, we study the initial and boundary value problem of the Navier-Stokes equations in the half-space.

Incompressible Flow Computations by VIP(virtual Interpolation Point Collocation)

Hi Jun Choe
CMAC, Yonsei University, Korea
Gahyung Jo, Seongkwan Park

In this talk, we introduce meshfree method for fluid computations. Developing virtual interpolation technique, we formulate Navier-Stokes and Stokes equations in virtual nodes and prove the inf-sup condition for virtual interpolation scheme. The stationary Stokes flow and Navier-Stokes flow can be computed easily in complicated geometries. If time is allowed we continue discussing projection method and stability for time dependent flows.

Holder Continuity of a Bounded Weak Solution for Degenerate and Singular Equations.

Sukjung Hwang
Yonsei University, Korea
Gary Lieberman

We generalise quasilinear parabolic p-Laplace type equations to obtain the prototype equation

$$u_t - \operatorname{div}\left(\frac{g(|Du|)}{|Du|}Du\right) = 0$$

where g is a nonnegative, increasing, and continuous function trapped in between two power functions $|Du|^{g_0-1}$ and $|Du|^{g_1-1}$ with either $2 \le g_0 \le g_1 < \infty$ or $1 < g_0 \le g_1 \le 2$. We provide a proof for the Höolder continuity of such solutions emphasising on geometrical properties, so called, the expansion of positivity. Also we discuss the same regularity for the porous medium type equations.

Staggered Discontinous Galerkin Methods for Flow Problems

Hyea Hyun Kim
Kyung Hee University, Korea
Eric T. Chung, Chak Shing Lee, Siu Wun Cheung, Yue Qian

Staggered discontinuous Galerkin methods are developed for flow problems. By introducing additional unknowns, a first order system for the Stokes problem is obtained and then discreteized by using staggered

discontinuous finite element functions. The staggered discontinuous finite element functions give a flux condition on the element boundaries without the need of artificial flux or tuning of penalty parameters in contrast to the standard discontinous finite element functions. Optimal order of errors can be obtained for a given polynomial degree k for all the unknowns. In addition, the method is applied to Navier-Stokes equations. Numerical results are presented to show the capability of the proposed method for various test examples.

Convergence Proof for Augmented Hodge Projection of Fluid-Solid Interaction

Chohong Min
Ewha Womans University, Korea
Gangjoon Yoon, Seick Kim

In this work, we study a Fluid-Solid interaction based on the Helmholtz-Hodge decomposition. Fluid and solid dynamics interact intimately during the movement and the interaction rather makes then a new physical object combined than a simple union of two separated objects. For this reason, we take the monolithic treatment on fluid-solid interaction that the two-way couplings are imposed at the same time. We introduce a novel decomposition of the state variable into two orthogonal components, which is a variation of the Hodge decomposition. We refer to the decomposition as an augmented Hodge decomposition. The decomposition enables us to decouple the computations of the velocity and the pressure in the incompressible Navier-Stokes equation, giving an elliptic equation for the pressure non-local Robin type boundary condition, Then, the decomposition is fulfilled by solving the equation. We show the existence, uniqueness and the regularity of the solution to the equation. The monolithic treatment leads to the stability that the kinetic energy does not increase in the projection step by taking one of two components of the composition. Using an Heaviside function, we express the boundary condition independent of the interfaces of the fluid and the solid. Also, we proposes a numerical method and shows that the unique decomposition and orthogonality also hold in the discrete setting. Also, we show that the numerical method produces the numerical solution at least with first order accuracy. We carry out numerical experiments that validate our analysis and arguments.

Special Session 74: Infinite Dimensional Stochastic Systems and Applications

Wilfried Grecksch, Martin Luther University Halle-Wittenberg, Germany

This special session should give a general overview as well about new tendencies in the field of infinite dimensional stochastic systems as applications to sciences, economic sciences and problems of optimal control. The invited speakers will discuss especially issues related to stochastic partial differential equations, stochastic differential equations with memory, stochastic integral equations, stochastic integrodifferential equations and stochastic difference equations in infinite dimensional spaces.

A Stochastic Optimal Control Problem in UMD-Banach Space L^q by Using Maximum Principl

Mahdi Azimi
Martin Luther University Halle-Wittenberg, Germany

We consider an optimal control problem in Banach space E, where $E = L^q(S, \mathcal{S}, \mu)$, μ is σ-finte measure. The state process will be defined as Ito Volterra stochastic integral equation and the stochastic integral is defined with respect to a cylindrical Wiener process. The concept of L^p-stochastically integrability in Banach spaces will be used. By using appropriate assumptions, corresponding backward stochastic integral equations will be derived and a duality principle between forward and backward stochastic integral equations will be calculated. Then we use maximum principle method to solve the optimal control problem.

Well-Posedness and Regularization by Noise for Nonlinear PDE

Benjamin Gess
Max Planck Institute for Mathematics in the Sciences, Germany

In this talk we will revisit regularizing effects of noise for nonlinear SPDE. In this regard we are interested in phenomena where the inclusion of stochastic perturbations leads to increased regularity of solutions as compared to the unperturbed, deterministic cases. Closely related, we study effects of production of uniqueness of solutions by noise, i.e. instances of nonlinear SPDE having a unique solution, while non-uniqueness holds for the deterministic counterparts. The talk will concentrate on these effects in the case of nonlinear scalar conservation laws.

A Control Theory Approach to the Schrödinger Equation

Jeanette Koeppe
MLU Halle-Wittenberg, Germany
Wolfgang Paul

In 1966, E. Nelson [1] established a new interpretation of quantum mechanics, whereby the particles follow some conservative diffusion process, i.e. forward-backward stochastic differential equations (FBSDEs), which are equivalent to the Schrödinger equation. Until now, this equivalence has been applied in such a way that a known solution to the Schrödinger equation is used to integrate the stochastic differential equations numerically and analyze the statistical properties of the sample paths.

However, in analogy to classical mechanics, the stochastic equations of motion can be derived from an optimal control problem, whereby the variation of stochastic action functionals leads to the Schrödinger equation as the Hamilton-Jacobi-Bellman equation of the problem. We show that the stochastic equations of motion, i.e. the stochastic Hamilton equations, can be determined from optimal control problems by using the maximum principle, which leads to equations for the adjoint processes. The equations for the diffusion process as well as the equations of the corresponding conjugated momentum constitute the Hamilton equations of motion in the stochastic case. Solving these coupled FBSDEs numerically, we determined the probability distribution of the process, i.e. the square of the absolute value of the wave function.

References

[1] *E. Derivations to the Schrödinger Equation from Newtonian Mechanics*, Phys. Rev., **1966**, 150, 1079-1085

Using Stochastic Differential Equations to Model the Cell Cycle

Rachel Leander
Middle Tennessee State University, USA
Edward Allen, Darren Tyson, Shawn Garbett, Vito Quaranta, Zack Jones

Given that division is one of the most important tasks a cell performs, it is surprising that, even within a clonal population, there is considerable variability in the time it takes a cell to divide. Furthermore, indi-

vidual responses to a disturbance, such as drug treatment, are dynamic and heterogeneous. A better characterization of the distribution of division times can help researchers compare populations of proliferating cells, describe growth at the population level, and investigate cell cycle control mechanisms. In this talk, I will discuss the development, parameterization, and implications of a simple stochastic differential equation model of the cell cycle.

Parameter Estimation for a Mean Reversion Model Driven by Fractional Processes

Jens Lueddeckens
MLU Halle-Wittenberg / Institute of Mathematics, Germany

The aim of this talk is to estimate the mean reversion level μ of an inhomogeneous linear fractional stochastic differential equation with fractional Brownian motion $B^H(t)$ and two fractional compensated Poisson processes $\tilde{N}^H_{\lambda_1}(t)$ and $\tilde{N}^H_{\lambda_2}(t)$. $B^H(t)$, $\tilde{N}^H_{\lambda_1}(t)$ and $\tilde{N}^H_{\lambda_2}(t)$ are stochastically independent. The estimation of μ, based on observed data and on the condition of a positive constant mean reversion rate K, is done using a generalization of the least square method. Finally the unbiasedness and consistency of the estimator for μ is proved.

Abstract Fractional Stochastic Evolution Equations with Applications to Nonlinear Beam Dynamics and Stochastic Wave Equations

Mark Mckibben
West Chester University, USA

A class of abstract stochastic evolution equations driven by a fractional Brownian motion process with Poisson jumps will be the focus of this talk. Results concerning the existence and uniqueness of solutions corresponding to different types of nonlinear forcing terms and under various growth conditions are established. Results concerning stochastic stability and various convergence schemes will also be discussed. The motivation of the general theory established is rooted in the study of nonlinear beam dynamics and stochastic wave equations. The applicability of the general theory to such applications will be illustrated.

Random Attractors for Stochastic Differential Inclusions in Banach Spaces

Alexandra Neamtu
Friedrich-Schiller University Jena, Germany

We consider on separable Banach spaces X stochastic differential inclusions constituted by

$$\begin{cases} dx(t) \in Ax(t)dt + F(t, x(t))dt + \sigma(t, x(t))dW_H(t) \\ x(0) = x_0, \ t \in [0, T]. \end{cases}$$

Here A is the generator of a compact analytic C_0-semigroup, the nonlinear term $F : [0, T] \times X \to 2^X \setminus \{\emptyset\}$ is a set-valued map, $\sigma : [0, T] \times X \to \mathcal{L}(H; X)$ and W_H is an H-cylindrical Wiener process. By using fixed-point theorems for set-valued mappings, we establish the existence of at least one solution for the given evolution inclusion. Furthermore, under suitable assumptions we derive a multivalued random dynamical system and show that it has a random attractor.

Asymptotic Growth and Fluctuation for a Class of Nonlinear Stochastic Volterra Equations

Denis Patterson
Dublin City University, Ireland
John Appleby

Motivated by economic applications, we develop precise bounds on the growth rates and fluctuation sizes of unbounded solutions of forced nonlinear Volterra equations. The nonlinearity is assumed sublinear for large values of the state and we prove general results applicable to a range of state–independent perturbations, random or deterministic. If an appropriate functional of the forcing term has a limit L at infinity, solutions of the forced equation behave asymptotically like the unforced equation when $L = 0$, like the forcing term when $L = +\infty$, and inherit properties of both the forcing term and unperturbed equation for $L \in (0, \infty)$. In the special case of Brownian driven noise, for L sufficiently large, solutions are recurrent on the real line and obey a type of time–changed iterated logarithm law.
This talk is based on joint work with Prof. John Appleby (DCU) and is supported by the Irish Research Council under the project GOIPG/2013/402.

On a Rescaling Transformation for Stochastic Partial Differential Equations

Michael Röckner
Bielefeld University, Germany
Viorel Barbu

In the talk we shall give a survey on the rescaling transformation to stochastic partial differential equations. We shall also present some recent new applications.

4th Order Stochastic PDEs with Q-Regular Space-Time Noise

Henri Schurz
SIU, USA
Boris Belinskiy

Some stochastic PDEs with Q-regular space-time noise are discussed (with applications to Mechanical Engineering). Existence and uniqueness of approximate strong solutions, some stability and energy estimates are presented under Dirichlet-type boundary conditions and with L^2-integrable initial data on bounded rectangular domains D. The results are connected to a joint work with Boris Belinskiy (UTC).

Invariance of Closed Convex Cones for Stochastic Partial Differential Equations

Stefan Tappe
University of Hannover, Germany

The goal of this talk is to clarify when a closed convex cone is invariant for a stochastic partial differential equation (SPDE) driven by a Wiener process and a Poisson random measure, and to provide conditions on the parameters of the SPDE, which are necessary and sufficient. Our result is accompanied by several examples arising in natural sciences and economics.

Optimal Control of a Stochastic Heat Equation Driven by Q-Wiener Processes

Christoph Trautwein
Max Planck Institute Magdeburg, Germany
Peter Benner

We consider a linear quadratic control problem of the stochastic heat equation with Neumann boundary condition, where controls and noise terms are defined inside the domain as well as on the boundary. The noise terms are given by independent Q-Wiener processes. The control problem is given by a so called tracking problem such that we can utilize the specific cost functional to derive necessary and sufficient conditions stochastic optimal controls have to satisfy. Using these optimality conditions, we deduce explicit formulas to obtain that stochastic optimal controls are given by feedback controls. Therefore, we conclude that the optimal controls are adapted to a certain filtration and we ensure that the state is an adapted process as well.

Cylindrical Continuous Martingales and Stochastic Integration in Infinite Dimensions

Mark Veraar
TU Delft, Netherlands
Ivan Yaroslavtsev

In this talk we define a new type of quadratic variation for cylindrical continuous local martingales on an infinite dimensional spaces. It is shown that a large class of cylindrical continuous local martingales has such a quadratic variation. For this new class of cylindrical continuous local martingales we develop a stochastic integration theory for operator valued processes under the condition that the range space is a UMD Banach space. We obtain two-sided estimates for the stochastic integral in terms of the gamma-norm. In the scalar or Hilbert case this reduces to the Burkholder-Davis-Gundy inequalities. An application to a class of stochastic evolution equations is given at the end of the paper.

On the Stochastic Heat Equation with Sticky Reflected Boundary Condition

Robert Vosshall
University of Kaiserslautern, Germany
Martin Grothaus, Torben Fattler

In this talk we study the stochastic heat equation with sticky reflected boundary condition. Dirichlet form techniques are used to construct and characterize its solution. The obtained process already for some time is conjectured to be the scaling limit of the dynamical wetting model, also known as Ginzburg-Landau dynamics with pinning and reflection competing on the boundary. For the second part of this talk it is planned to discuss the progress on this problem.

Minimization of Distance Measures to Efficiently Capture the Macroscale Behavior of Stochastic Systems.

Przemyslaw Zielinski
KU Leuven, Belgium
Kristian Debrabant, Tony Lelievre, Giovanni Samaey

I will present a method to accelerate the simulation of observables of processes determined by stochastic ODEs, motivated by the recent development of generic multiscale frameworks. The technique exploits the separation between the fast time scale on which we compute trajectories and the slow evolution of observables. The algorithm combines short bursts of paths simulation with extrapolation of a number of moments forward in time.

I will concentrate on the crucial step – how to obtain, after extrapolation, a new ensemble of particles/replicas compatible with given moments. To deal with this inference problem, we introduce the matching operator based on the minimization of suitable distance between probability distributions. I am going to discuss the generic properties of this operator that allow to establish the theorem on the order of convergence of our method. As a particular example, I will present the matching based on the logarithmic entropy that also provides a convenient numerical approach.

Special Session 75: Recent Trends on PDEs Driven by Gaussian Processes with Applications

Hakima Bessaih, University of Wyoming, USA
Maria J. Garrido-Atienza, University of Seville, Spain

We are interested in the theory and applications of stochastic systems driven by Gaussian noise. Of particular importance are systems driven by fractional Brownian motion (fBm), and other long-memory and/or self-similar noise models. Classical probabilistic tools cannot be used to analyze fBm, because it fails to have the martingale and Markov property. In recent years, there have been major breakthroughs in integration theory for fBm, particularly in rough path, fractional calculus, and random dynamical systems. This special session will be devoted to have a discussion about these various techniques and to what extent they can be used jointly. Employing these theories to solve dependent-noise-driven SDEs and SPDEs will be the first main goal of this session, as well as the study of stability properties of the solutions by using the theory of random dynamical systems. The second goal will be to bring together the theoretical and applied community of researchers in order to bridge the gap between theory and applications. On the applied side, a particular attention will be devoted to porous media and data assimilation.

Reynolds Number Dependence in Homogeneous Turbulence

Bjorn Birnir
UC Santa Barbara, USA
Gregory Bewley, John Kaminsky, Shahab Karmini

We prove the existence of (local) solutions to a stochastic Navier-Stokes equation, in Sobolev spaces corresponding to the Kolmogorov-Obukhov-She-Leveque scaling. This permit a computation of the structure function of turbulence including their dependance on the Reynolds-Taylor number. The theory is applied to the data obtained in a variable density wind-tunnel in the Max-Planck Institute for Dynamics and Self-Organization in Gottingen, Germany, that is a modern version of the classical Prandtl wind-tunnel experiment. A good agreement is found for a range of Reynolds-Taylor numbers, and some surprising result on the smoothness of the solutions in turbulent flow.

Intermittency Properties for a Class of SPDEs Driven by Fractional Noise.

Daniel Conus
Lehigh University, USA

A space-time random-field is called physically intermittent if it develops high-valued peaks concentrated on small spatial islands as time gets large. In this talk, we will illustrate this notion by discussing several examples of intermittent random fields given by solutions to a class of parabolic and hyperbolic SPDEs. In particular, we will present some results related to equations driven by fractional noise.

Stochastic Navier-Stokes Equations in R^d with Not Regular Multiplicative Noise

Benedetta Ferrario
University of Pavia, Italy
Z. Brzezniak

We consider the Navier-Stokes equation in R^d ($d = 2, 3$) with multiplicative Gaussian white noise of very low space regularity. We present results on solutions and invariant measures/stationary solutions. This is a joint work with Z. Brzeżniak.

Accuracy of Filters for Quadratic Dissipative Systems

Kody Law
ORNL, USA

The solution to the problem of nonlinear filtering may be given either as an estimate of the signal (and ideally some measure of concentration), or as a full posterior distribution. Similarly, one may evaluate the fidelity of the filter either by its ability to track the signal or its proximity to the posterior filtering distribution. Hence, the field enjoys a lively symbiosis between probability and control theory, and there are many applications which benefit from algorithmic advances. This talk will survey some recent theoretical results involving accurate signal tracking for quadratic dissipative deterministic PDE (as well as some related ODE). In the limit of continuous-time observations, the equation for the filter is a stochastic PDE (SPDE).

Dynamics of Non-Densely Defined Stochastic Evolution Equations

Alexandra Neamtu
Friedrich-Schiller University Jena, Germany

We consider on separable Banach spaces a class of stochastic evolution equations with a non-densely defined linear part. In this case, the C_0-semigroup theory can no longer be applied. Such situations occur when certain additional restrictions are incorporated in the domain of a linear operator. The noise term is constituted by a Banach space-valued Brownian motion. A suitable transformation allows us to reduce the stochastic equation into a random one, from which we can derive a random dynamical system and investigate the existence of random attractors. Our theory is based on the integrated semigroup approach, considered in the deterministic case by P. Magal and S. Ruan (2009). As applications, we discuss parabolic equations with nonlinear boundary conditions.

On the Sochastic Transport Equation

Christian Olivera
IMECC UNICAMP, Brazil
E. Fedrizzi, W. Neves, C. Tudor

We study the existence and uniqueness of solutions to the stochastic transport equations with irregular coefficients driven by standard Brownian motion and Fractional Brownian motion. Also we study the regularity in law of the solutions using Malliavin Calculus.

Data Assimilation Using Stochastically Noisy Data

Eric Olson
University of Nevada, USA
Hakima Bessaih, Edriss Titi

We analyze the performance of a data-assimilation algorithm based on a linear feedback control when used with observational data that contains measurement errors. Our model problem consists of dynamics governed by the two-dimension incompressible Navier-Stokes equations, observational measurements given by finite volume elements or nodal points of the velocity field and measurement errors which are represented by stochastic noise.

Sample Path Properties for SDEs Driven by Fractional Brownian Motions

Cheng Ouyang
University of Illinois at Chicago, USA
Fabrice Baudoin, Shuwen Lou, Eulalia Nualart, Samy Tindel

In this talk, we present some of our recent results on sample path properties for SDEs driven by fractional Brownian motions, including hitting probability, Hausdorff dimension of sample paths and level sets, and existence and regularity of local times.

Fractional White-Noise Limit and Paraxial Approximation for Waves in Random Media

Olivier Pinaud
Colorado State University, USA
Christophe Gomez

This talk is devoted to the asymptotic analysis of high frequency wave propagation in random media with long-range dependence. We are interested in two asymptotic regimes, that we investigate simultaneously: the paraxial approximation, where the wave is collimated and propagates along a privileged direction of propagation, and the white-noise limit, where random fluctuations in the background are well approximated in a statistical sense by a fractional white noise. The fractional nature of the fluctuations is reminiscent of the long-range correlations in the underlying random medium. A typical physical setting is laser beam propagation in turbulent atmosphere. Starting from the high frequency wave equation with fast non-Gaussian random oscillations in the velocity field, we derive the fractional Itô-Schrödinger equation, that is a Schrödinger equation with potential equal to a fractional white noise. The proof involves a fine analysis of the backscattering and of the coupling between the propagating and evanescent modes. Because of the long-range dependence, classical diffusion-approximation theorems for equations with random coefficients do not apply, and we therefore use moment techniques to study the convergence.

Irreducbility and Ergodicity of Some Stochastic Hydrodynamical Systems Driven by Tempered Stable Processes

Paul Razafimandimby
Montanuniversitaet Leoben, Austria

In this talk, we present several results related to the long-time behavior of a class of stochastic semilinear evolution equations in a separable Hilbert space H:

$$d\mathrm{u}(t)+[\mathrm{A}\mathrm{u}(t)+\mathbf{B}(\mathrm{u}(t),\mathrm{u}(t))]dt = dL(t), \mathrm{u}(0) = x \in \mathrm{H}.$$

The driving noise L is basically a general tempered stable process satisfying several technical assumptions, A is a positive self-adjoint operator and **B** is a bilinear map. By using a density transformation theorem type for Lèvy measure, we first prove a support theorem and an irreducibility property of the Ornstein-Uhlenbeck processes associated to the nonlinear stochastic problem. Second, by exploiting the previous results we establish the irreducibility of nonlinear problem provided that for a certain $\gamma \in [0, 1/4]$ **B** is continuous on $D(\mathrm{A}^\gamma) \times D(\mathrm{A}^\gamma)$ with values in $D(\mathrm{A}^{-1/2})$. Proving a Bismut-Elworthy-Li type lemma for SDEs driven by pure jump noise and using a method from the 1995 paper of Flandoli and Maslowski, the uniqueness of invariant measure is also proved under the assumption, which is a much stronger than the previous one, that **B** is continuous on $\mathrm{H} \times \mathrm{H}$. While the latter condition is only satisfied by the nonlinearities of GOY and Sabra Shell models, the assumption under which irreducibility property holds is verified by several hydrodynamical systems such as the 2D Navier-Stokes, Magnetohydrodynamic equations, the 3D Leray-α model, the GOY and Sabra shell models.
The talk is based on joint works with H. Bessaih, E. Hausenblas and P. Fernando.

Dynamics of SPDE Driven by a Fractional Brownian Motion

Björn Schmalfuß
Friedrich Schiller University of Jena, Germany

We consider the stochastic evolution equation on a Hilbert-space V

$$du + Audt = G(u)d\omega, \quad u(0) = u_0 \in V$$

where ω is a fractional Brwonian motion with Hurst-parameter $H \in (1/2, 1)$, A generates an exponentially stable analytic semigroup and G is sufficiently smooth. We discuss several aspects of the dynamics of such an equation, like the existence of a random attractor.

Exit Times for Solutions of Stochastic Navier-Stokes Equations

Padmanabhan Sundar
Louisiana State University, USA

Exponential estimates for exit times are obtained for solutions of the stochastic Navier-Stokes system in 2D and for related equations in hydrodynamics. The study involves the asymptotic behavior of solutions in time as well as in a parameter. Equations perturbed by fractional Brownian noise will also be discussed.

Filtering Problems in Stochastic Tomography

Jason Swanson
University of Central Florida, USA
Tyler Gomez, Alexandru Tamasan

We consider the X-ray transform of a function-valued random variable, and study the conditional distribution of the inversion in the presence of noise. Due to its medical applications, the mathematical theory of the X-ray transform is rich with numerous results spanning analytical and numerical aspects of inversion. However, these results assume a deterministic approach. Here, we introduce a stochastic approach to the tomography problem. This is joint work with Tyler Gomez and Alexandru Tamasan.

Quadratic Variations for the Fractional-Colored Stochastic Heat Equation

Frederi Viens
Purdue University, USA
Soledad Torres, Ciprian Tudor

Using multiple stochastic integrals and Malliavin calculus, we analyze the quadratic variations of a class of Gaussian processes that contains the linear stochastic heat equation on $\mathbf{R}^d$ driven by a non-white noise which is fractional Gaussian with respect to the time variable (Hurst parameter H) and has colored spatial covariance of α-Riesz-kernel type. The processes in this class are self-similar in time with a parameter K distinct from H, and have path regularity properties which are very close to those of fractional Brownian motion (fBm) with Hurst parameter K (in the heat equation case, $K = H-(d-\alpha)/4$). However the processes exhibit marked inhomogeneities which cause naive heuristic renormalization arguments based on K to fail, and require delicate computations to establish the asymptotic behavior of the quadratic variation. A phase transition between normal and non-normal asymptotics appears, which does not correspond to the familiar threshold $K = 3/4$ known in the case of fBm. We apply our results to construct an estimator for H and to study its asymptotic behavior.

Special Session 76: Advances in the Numerical Solution of Nonlinear Evolution Equations

Mechthild Thalhammer, Universität Innsbruck, Austria
Winfried Auzinger, Technische Universität Wien, Austria

The intention of this special session is to bring together mathematicians and theoretical physicists, interconnected through their field of application, relevant analytical tools, or the numerical methods used. The scope of topics includes, but is not limited to, evolutionary Schrödinger type equations, highly oscillatory equations, and adaptive integration methods for partial differential equations.

A Hybrid WKB-Based Method for the Stationary Schroedinger Equation in the Semi-Classical Limit

Anton Arnold
Vienna Univ. of Technology / Analysis & Scientific Comp., Austria
Claudia Negulescu

We are concerned with the efficient numerical integration of ODEs of the form $\epsilon^2 u" + a(x)u = 0$ for $0 < \epsilon 0$) with evanescent regions (i.e. for $a(x) < 0$). In the oscillatory case we use a marching method that is based on an analytic WKB-preprocessing of the equation. And in the evanescent case we use a FEM with WKB-ansatz functions. We present a full convergence analysis of the coupled method, showing that the error is uniform in epsilon and second order w.r.t. h. We illustrate the results with numerical examples for scattering problems for a quantum-tunnelling structure.

Local Error Estimation and Adaptive Splitting in Time

Winfried Auzinger
Technische Universitaet Wien, Austria
Harald Hofstaetter, David Ketcheson Othmar Koch, Mechthild Thalhammer

We consider the integration of evolution equations by adaptive time-splitting methods, where the right-hand side is split into two or more components. Adaptive integrators are based on different techniques for estimating local error, including embedded or adjoint pairs of schemes or defect-based estimators. Theoretical as well as implementation aspects are discussed. Several numerical examples are presented, in particular equations of Schroedinger type and parabolic equations.

Geometric Integration of a Damped Driven Nonlinear Schrodinger Equation

Ashish Bhatt
University of Central Florida, USA
Brian E. Moore

A damped driven nonlinear Schrodinger equation (NLSE) is known to possess rich dynamics including solitary and shock wave solutions, periodic solutions, bifurcation and chaos. Numerical methods aim to reduce quantitative errors whereas geometric integrators aim to reduce or eliminate qualitative errors (e.g. error in preservation of a conserved quantity) as well in order to improve accuracy. Geometric integrators have attracted a lot of attention in the past several decades. They have been shown to be advantageous as compared to non-structure preserving methods in resolving the dynamics of a problem. The purpose of this talk is to present application of a geometric integrator to a damped driven NLSE and discuss numerical results.

Exponential Asymptotic Splitting for the Linear Schroedinger Equation

Karolina Kropielnicka
University of Gdansk, Poland
Philipp Bader, Arieh Iserles, Pranav Singh

The discretization of a linear Schroedinger equation is difficult due to the presence of a small parameter which induces high oscillations. A standard approach consists of a spectral semidiscretization, followed by an exponential splitting. This, however, is sub-optimal, because the exceedingly high precision in space discretization is married by low order of the time solver. In this talk we sketch an alternative approach. Our analysis commences not with semi-discretisation, but with the investigation of the free Lie algebra generated by differentiation and by multiplication with the interaction potential: it turns out that this algebra possesses a structure which renders it amenable to a very effective form of exponential asymptotic splitting: exponential splitting where consecutive terms are scaled by increasing powers of the small parameter. The semi-discretisation in deferred to the very end of computations. We will focus on the method for the time dependant linear

Schroedinger equation with potential non-depending on time, however we will also discuss the difficulties that appear with time dependant potential and will briefly propose the remedy to that stage of an affair.

Stochastic Homogenization of a Visco-Elastic Model for Strain-Stress Hysteresis

Frederic Legoll
Ecole des Ponts, France
Thomas Hudson, Tony Lelievre

Motivated by the modelization of hysteresis in filled rubber, we consider a time-dependent viscoelastic model in which the constitutive law for the solid varies randomly on a small lengthscale. At the fine scale, the model includes an elastic energy, viscous friction and solid friction, all with random highly oscillatory coefficients. It exhibits hysteretic behaviour which persists under slow loading. We identify the homogenized limit of this model, and demonstrate that, at the coarse scale, the model again exhibits hysteretic behaviour which persists under slow loading.

Structure-Preserving Algorithms for Perturbed Nonlinear Schrodinger Equations

Brian Moore
University of Central Florida, USA
Laura Norena, Constance Schober

Some nonlinear Schrodinger equations with added linear damping and/or convection terms have a special structure, which is known in the ODE literature as conformal symplectic. This special structure leads to several conserved quantities that can be preserved exactly by various discretizations. Presentation of methods of this type will be followed by a brief explanation of the benefits of using such discretizations, including both theoretical and numerical results.

Comparison of Convergence of Fully Discrete Splitting Schemes for the Nonlinear Schrödinger Equation Using Fourier Transform and Iterative Linear Solvers

Benson Muite
University of Tartu, Estonia
E. Vainikko

Convergence of splitting schemes for the nonlinear Schrödinger equation are examined. Rates of convergence and accuracy are measured for several serial and parallel algorithms based on Fourier pseudospectral discretizations and on finite difference discretizations.

Non-Local NLS of Derivative Type for Modeling Highly Nonlocal Optical Nonlinearities

Hans Peter Stimming
Univ. of Vienna, WPI, Austria
Igor Mazets, Ephraim Shamoon, Gershon Kurizki, Pjotrs Grisins

A new NLS type equation is employed for modeling long-range interactions in nonlinear optics, in a collaboration with experimental physicists. It is of quasilinear type, and models fluctuations around a ‘continuous-wave polariton‘ which are chosen according to Bogoliubov excitation theory. Mathematical theory / analysis for this equation is work in progress, results exist however for other quasilinear NLS at the same order of derivatives. A similar model was used for numerical simulation of time-dependent decoherence in split BECs.

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Special Session 77: Delay Differential Equations with State-Dependent Delays and their Applications

Qingwen Hu, The University of Texas at Dallas, USA
Bernhard Lani-Wayda, University of Giessen, Germany
Eugen Stumpf, University of Hamburg, Germany

Delay differential equations with state-dependent delays have become an active research field of mathematics. More and more models in biology, machining dynamics, physiology, traffic control and laser dynamics have been reported. The special session aims at bringing together researchers in this field and interested scholars in mathematics and/or applied sciences for communications on the current state of this subject.

On Oscillation, Stability, Boundedness and Persistence of Nonlinear Equations with Several Delays

Elena Braverman
University of Calgary, Canada
Leonid Berezansky

In the first part of the talk, we explore nonlinear equations and systems with a delayed positive feedback, where the delay, is, generally, distributed. Such equations are globally asymptotically stable (and intrinsically non-oscillatory), under some natural conditions on the unique positive equilibrium, when the delay is bounded. If there are several equilibrium points, multistability is observed. In the case of the unique positive equilibrium and monotonicity, similar results are obtained for a system of two equations.
In the second part, nonlinear equations with more than one delay

$$\dot{x}(t) = \sum_{k=1}^{m} f_k(t, x(h_1(t)), \ldots, x(h_l(t))) - g(t, x(t)),$$

where the functions f_k increase in some variables and decrease in the others, we obtain conditions when a positive solution exists on $[0, \infty)$, as well as explore boundedness and persistence of solutions. Examples include the Mackey-Glass equation with non-monotone feedback and two variable delays; its solutions can be neither persistent nor bounded, unlike the well studied case when these two delays coincide.

A PDE of Variational Electrodynamics

Jayme de Luca
UFSCar, Brazil

The electromagnetic two-body problem is an infinite-dimensional problem with four state-dependent delays of neutral type, which system can have periodic orbits with discontinuous velocities. Con- tinuous maps of light-cone type from R4 to points along such periodic orbits naturally inherit the derivative discontinuities and our distributional construct to avoid these discontinuities involves a second order PDE. We will discuss the distributional construction and the importance of such PDE to variational electrodynamics.

Stability of Linear Discrete Systems with Delays

Josef Diblik
CEITEC, Brno University of Technology, Czech Rep

In the talk we give sufficient conditions for the exponential stability of linear difference systems with delays

$$x(k+1) = Ax(k) + \sum_{i=1}^{s} B_i(k)x(k - m_i(k)), \; k = 0, 1, \ldots$$

where A is an $n \times n$ constant matrix, $B_i(k)$ are $n \times n$ matrices, $m_i(k) \in \mathbb{N}$, $m_i(k) \leq m$ for an $m \in \mathbb{N}$, $s \in \mathbb{N}$ and $x = (x_1, \ldots, x_n)^T : \{-m, -m+1, \ldots\} \to \mathbb{R}^n$. The results are compared with some previously published results. The exponential stability is studied by the second Lyapunov method.

On Neutral Differential Equations with State-Dependent Delays

Ferenc Hartung
University of Pannonia, Hungary

In this talk we consider a class of nonlinear neutral differential equations with state-dependent delays in both the neutral and the retarded terms. We study well-posedness and continuous dependence issues and differentiability of the parameter map with respect to the initial function and other possibly infinite dimensional parameters.

Quasi-Periodic Solutions for State-Dependent Delay Differential Equations

Xiaolong He
Georgia Institute of Technology, USA
Rafael de la Llave

The existence of quasi-periodic solutions for state-dependent delay differential equations is investigated by using the parameterization method, which is different from the usual way-working on the solution manifold. Under the assumption of finite-times differentiability of functions and exponential dichotomy, the existence and smoothness of quasi-periodic solutions are investigated by using contraction arguments. Meanwhile, we show that there are Lindstedt series under some non-degeneracy conditions for the analytic case. In particular, a KAM theory is developed to seek analytic quasi-periodic solutions, which gets involving the theory of foliation preserving torus mapping. Moreover, we prove that the set of parameters which guarantee the existence of analytic quasi-periodic solutions is of positive measure. All of these results are given in an a-posterior form. Namely, given an approximate solution satisfying some non-degeneracy conditions, there is a true solution nearby.

On Diffusion Processes with Threshold-Type State-Dependent Delay

Qingwen Hu
The University of Texas at Dallas, USA

We develop a Hopf bifurcation theory of diffusion processes with threshold-type delay and investigate the effect of the state-dependent diffusion time on intracellular regulatory dynamics. A general model which is an extension of the classic differential models with constant or zero time delays is developed to study the stability of steady state and the occurrence and stability of oscillations in regulatory dynamics. Using the multiple time scale method, we compute the normal form of the general model and show that the state-dependent diffusion time may stabilize or destabilize the bifurcated periodic solutions of the corresponding models which do not incorporate the state-dependent diffusion time.

Lyapunov-Razumikhin Techniques for State-Dependent DDEs

Tony Humphries
McGill University, Canada
Felicia Magpantay

We present theorems for the Lyapunov and asymptotic stability of the steady state solutions to general state-dependent delay differential equations (DDEs) using Lyapunov-Razumikhin methods. The Lyapunov stability result applies to nonautonomous DDEs with multiple discrete state-dependent delays of the form

$$\begin{cases} \dot{u}(t) = f\big(t, u(t), u(t-\tau_1(t,u(t))), \ldots, u(t-\tau_N(t,u(t)))\big), & t \geqslant t_0, \\ u(t) = \varphi(t), & t \leqslant t_0, \end{cases}$$

and is proved by a contradiction argument which is adapted from a previous result of Barnea for retarded functional differential equations (RFDEs).

Our asymptotic stability result applies to autonomous DDEs with multiple state-dependent discrete delays. Its proof is entirely new, and is based on a contradiction argument together with the Arzelà-Ascoli theorem. This alleviates the need for an auxiliary function to ensure the asymptotic contraction, which is a feature of the other Lyapunov-Razumikhin asymptotic stability results of which we are aware. We apply our results to the state-dependent model equation

$$\begin{array}{ll} \dot{u}(t) = \mu u(t) + \sigma u(t-a-cu(t)), & t \geq 0, \\ u(t) = \varphi(t), & t \leqslant 0, \end{array}$$

which includes Hayes equation as a special case (when $c = 0$), to directly establish asymptotic stability in parts of the stability domain along with lower bounds for the basin of attraction. We also generalise our techniques to derive a condition for global asymptotic stability of the zero solution to the model problem, and also to find bounds on the periodic solutions when the steady-state solution is unstable.

Dynamical Behavior in Some Low-Dimensional Delay Systems

Anatoli Ivanov
Pennsylvania State University, USA

The system of delay differential equations of the form

$$x_i'(t) = -\lambda_i\, x_i(t) + \sum_{j=1}^{n} w_{ij} f_i(x_j(t-\tau_{ij})), \quad (*)$$

for $i = 1, 2, \ldots, n$ appears as a mathematical model of numerous real world phenomena. It exhibits rich dynamical behavior such as global asymptotic stability of equilibria, existence and stability of periodic solutions, and existence of complex/chaotic solutions among others. We discuss some known results, derive several new results, and propose several conjectures for the cases of low-dimensional system $(*)$, $n = 3, n = 2$, and $n = 1$.

Oscillation Speed and Slowly Oscillating Solutions for a Class of Equations with State-Dependent Delay

Benjamin Kennedy
Gettysburg College, USA

We consider a class of differential delay equations with state-dependent delay and negative feedback that includes the equation

$$x`(t + g(x(t))) = f(x(t))$$

introduced by Walther in 2008. We define a non-increasing oscillation speed for this class of equations. Motivated primarily by known results in the constant-delay case, we discuss the behavior of slowly-oscillating solutions in the case that f is monotonic. This work is in a similar spirit to work by Krisztin and Arino from the early 2000s on the equation $x`(t) = -\mu x(t) + f(x(t - d(x(t))))$.

Lyapunov Method for Stability of FDEs with State-Dependent Delay

Xiaodi Li
Shandong Normal University, Peoples Rep of China
Jianhong Wu

This talk will address the stability problem for functional differential systems with state-dependent delay, which is not assumed the be a priori bounded. Using some Lyapunov functions coupled with differential inequality techniques tailed at state-depedent delay, we shall present some sufficient conditions for exponential stability and Lipschitz stability. Then we shall introduce some specific examples to illustrate the effectiveness of the approach and our general results.

Intra-Specific Competition and Insect Larval Development: a Model with Time-Dependent Delay

Yijun Lou
The Hong Kong Polytechnic University, Hong Kong
Stephen A. Gourley, Rongsong Liu

This talk presents a stage-structured model for an insect population in which a larva matures on reaching a certain size, and in which there is intra-specific competition among larvae which hinders their development thereby prolonging the larval phase. The model, a system of delay differential equations for the total numbers of adults and larvae, assumes two forms. One of these is a system with a variable state-dependent time delay determined by a threshold condition, the other has constant and distributed delays, a size-like independent variable and no threshold condition. We prove theorems on boundedness and on the linear stability of equilibria.

State Dependent Delays in Infectious Disease Modeling

Gergely Röst
University of Szeged, Hungary

We discuss two recent examples from mathematical epidemiology where state dependent delays arise naturally. The first is an SIRS type model with waning and boosting of immunity, the second is malaria dynamics with long incubation periods.

An Inverse Problem for Delay Differential Equations: Parameter Estimation and Sensitivity Analysis

Fathalla Rihan
United Arab Emirates University, United Arab Emirates

This contribution presents a theoretical framework to solve inverse problems for Delay Differential Equations (DDEs). Given a parametrized DDE and experimental data, we estimate the parameters appear in the model, using Least Squares approach. We also investigate the sensitivity and robustness of the models to small perturbations in the parameters, using Variational approach. The results may provide guidance for the modelers to determine the most informative data for a specific parameter, and select the best fit model. The nonlinearity may make the problem ill-posed. Discontinuity and noisy data are also challenges. The consistency of neutral delay differential equations with bacterial cell growth is shown, as a numerical example, by fitting the model to real observations.

Existence of Unbounded Solutions of Delayed Linear Differential Equations.

Zdenek Svoboda
CEITEC, Brno University of Technology, Czech Rep

Recently, much attention was paid to a new formalization of the well-known method of steps in the theory of linear differential equations with constant coefficients and single delay. In the first part of the talk we will study solutions of linear differential systems of the first and second orders with constant matrices and constant delays. Solutions of such systems can be represented by special delayed matrix functions defined on intervals $(k-1)\tau \leq t0$ is a delay, as matrix polynomials continuous at nodes $t = k\tau$. In the second part of the talk we will discuss problems of asymptotic analysis of solutions, represented by delayed matrix functions. We show that that the sequence of values of solutions at nodes $t = k\tau$, $k = 0, 1, \ldots,$ can be approximately represented by a geometric-type sequence. Its "quotient" factor is defined by the principal branch of the Lambert function. Obtained asymptotic properties of the delayed exponential of a matrix are modified (utilizing an Euler identity for delayed matrix functions) for the cases of delayed matrix sine and delayed matrix cosine.

Stability and Stabilization in Functional Differential Equations with State-Dependent Delays

Janos Turi
UTD, USA

In this talk we show that a constant steady-state of an autonomous state-dependent delay differential equation is stable (stabilizable) if the trivial solution of a corresponding linear autonomous equation is stable (stabilizable). The result is applied to a state-dependent distributed-delay model of turning operations.

Hydra Effect for Delay Differential Equations

Gabriella Vas
University of Szeged, Hungary
Tibor Krisztin, Mònika Polner

We consider a delay differential equation modeling production and destruction: $\dot{y}(t) = -\mu y(t) + f(y(t-1))$ with $\mu > 0$ and nonincreasing $f : \mathbb{R} \to (0, \infty)$. We prove the presence of the paradoxical hydra effect, namely, we show that increasing the value of the destruction parameter μ can result in an increase of the mean value of certain solutions. The nonlinearity f in the equation is a step function or a smooth function close to a step function. This particular form of f allows us to construct periodic solutions, and to evaluate the mean values of the periodic solutions. Our result explains how the global form of the production term f induces the appearance of the hydra effect.

Existence and Uniqueness of Mild and Strict Solutions for Abstract Differential Equations with State Dependent Delay

Jianhong Wu
York, Canada
Michelle Pierri, Eduardo Hernandez

In this talk, we present some results related the existence and uniqueness of mild and strict solutions for abstract differential equations with state dependent delay, with particular focus on alpha-Holder strict solutions.

Special Session 78: Advances in Analysis of Mathematical Problems arising from Materials and Biological Science

Toyohiko Aiki, Japan Women's University, Japan
Adrian Muntean, Karlstad University, Sweden
Tomomi Yokota, Tokyo University of Science, Japan

This special session focuses on technologically important and mathematically interesting problems arising from material and biological sciences. Besides well-posedness and asymptotic studies of nonlinear PDEs and related particle systems, the session also includes presentations on averaging processes in porous media, non-periodic homogenization, statistical mechanics of particle systems, and multiscale dynamics

Recent Results Related to the Two-Scale Model for Concrete Carbonation

Toyohiko Aiki
Japan Women's University, Japan
Yusuke Murase, Naoki Sato

From civil engineering point of view there exists several kinds of serious issues on concrete, for example, carbonation, cracks, corrosion and etc. In our recent works we have proposed and studied the mathematical model for concrete carbonation. The model consists of two conservation laws for moisture and carbon dioxide. Here, we discuss only the moisture transport equation. On modeling for the moisture transport process it is a crucial step how to represent the relationship between the relative humidity and the degree of saturation. For this difficulty we already gave the two-scale model containing a one-dimensional free boundary problem and shown the well-posedness of the initial boundary value problem for the model. The main aim of this talk is to establish results on large time behavior of the solution of the free boundary problem.

Numerical Simulation of Waves Propagation in Materials with Microstructure

Mihhail Berezovski
Embry-Riddle Aeronautical University, USA

Results of numerical experiments are presented in order to compare direct numerical calculations of wave propagation in a laminate with prescribed properties and corresponding results obtained for an effective medium with the microstructure modelling. These numerical experiments allowed us to analyse the advantages and weaknesses of the microstructure model.

Global Existence and Boundedness in a Mathematical Model of Urban Crime Model

Kentarou Fujie
Tokyo University of Science, Japan

In this talk we consider a mathematical model describing burglary in residential areas. Based on two sociological effects the broken window effect and the repeat near-repeat effect, this model was proposed in Short et al.(2008). From a mathematical view point, the system is similar as the chemotaxis system and involves a logarithmic sensitivity function and specific interaction and relaxation terms. Under some condition, global existence and boundedness will be shown.

Equation and Dynamic Boundary Condition of Cahn-Hilliard Type with Related Topics

Takeshi Fukao
Kyoto University of Education, Japan

In this talk, the perspective for some equation and dynamic boundary condition of Cahn-Hilliard type is treated:

$$\frac{\partial u}{\partial t} - \Delta\mu = 0 \quad \text{in } Q := (0,T)\times\Omega,$$
$$\mu = -\Delta u + W'(u) - f \quad \text{in } Q,$$
$$u_\Gamma = u_{|\Gamma}, \quad \mu_\Gamma = \mu_{|\Gamma}, \quad \frac{\partial u_\Gamma}{\partial t} + \partial_\nu \mu - \Delta_\Gamma \mu_\Gamma = 0$$
$$\text{on } \Sigma := (0,T)\times\Gamma,$$
$$\mu_\Gamma = \partial_\nu u - \Delta_\Gamma u_\Gamma + W'_\Gamma(u_\Gamma) - f_\Gamma \quad \text{on}\Sigma\ ,$$
$$u(0) = u_0 \quad \text{in}\Omega\ , \quad u_\Gamma(0) = u_{0\Gamma} \quad \text{on}\Gamma\ .$$

Firstly, based on the result of the well-posedness by Colli-Fukao (2015), the asymptotic limit of Cahn-Hilliard systems to the degenerate parabolic equations is discussed under the dynamic boundary condition. Secondly, the boundary control problem is discussed. Both of them, the structure of the total mass conservation, namely the volume in the bulk plus the boundary is the key of the problem.

The Solvability of the Perfect Plasticity Model with Time Dependent Constraints

Risei Kano
Kochi University, Japan
Takeshi Fukao

In this talk, We discuss problems with the plastic deformation of perfect materials. This problem has been discussed by many scholars. In particular, G.Duvaut and J.L.Lions showed the solutions of the evolution problem that has the constraints on threshold of stress, in 1976. We think about the solvability of the extended problem having the function f(t,x) of the threshold. Here follows the way of Duvaut-Lions, we talk about the existence of solutions of the parabolic approximation problem.

Continuous Models of Graph-Like Matter

Ivan Kryven
University of Amsterdam, Netherlands

The duality of concepts that emerges from two seemingly contradicting descriptions of matter: discrete versus continuous, is commonly accepted by viewing each of the concepts at its own "spatial scale". Nonetheless, substances consisting of randomly interconnected network of small units (e.g. polymer materials) are hard to analyse from exclusively discrete or continuous point of view. This is mainly due to the fact that the graph topology that stands behind the network plays definitive role in shaping the properties of continuum. I shall focus on models that measure important characteristics of such networks with distributions and predict their evolution employing continuous balance equations. The talk will be fortified with examples of analytical, numerical, and stochastic nature.

On a Multiscale Model for Moisture Transport Appearing Concrete Carbonation Process

Kota Kumazaki
Tomakomai National College of Technology, Japan
Toyohiko Aiki, Naoki Sato, Yusuke Murase

In this talk, we consider a mathematical model for moisture transport in concrete carbonation process. In this process, it is known that the relationship between the relative humidity and the degree of saturation is a hysteresis, and this is caused by a wetting and drying phenomenon which occurs in each pore inside of concrete (Microscopic level). Recently, a free boundary problem describing the wetting and drying process is proposed, and is shown the existence of a solution of this problem. In this talk, we propose a mathematical model for moisture transport consisting of a diffusion equation of the relative humidity and the above free boundary problem, and discuss the existence of a time local solution of this problem.

Chemotaxis with Logistic Source

Johannes Lankeit
Paderborn University, Germany

Chemotaxis is the directed movement of e.g. cells or bacteria in response to concentration differences of a chemical in their neighbourhood and plays an important role in a multitude of biological contexts, ranging not only from aggregation and pattern formation in colonies of bacteria or slime mold to cancer invasion of a tumour into surrounding healthy tissue. In this talk we will deal with a cross-diffusive system of PDEs that combines the effects of chemotaxis with those of (logistic) population growth. We will consider qualitative properties of solutions, in particular their asymptotic behaviour and transient growth phenomena.

Mathematical Modeling of Growth Patterns and Consecutive Bone Destruction in Multiple Myeloma

Anna Marciniak-Czochra
University of Heidelberg, Germany
Flip Klawe, Andro Mikelic

The talk is devoted to mathematical modeling of growth and transport dynamics in a heterogenous evolving structure. The model is motivated by spatiotemporal dynamics of multiple myeloma, a malignant disease of plasma cells causing a variety of clinical signs and symptoms including loss of bone mass and mineral density and osteolytic lesions. The process is related not only to growth and spread of cancer cells but also to changes of bone microstructure. We present derivation and well-posedness of a multiscale model coupling surface and volume reactions. It leads to a moving boundary problem with density dependent velocity. Coupling volume processes with reaction-diffusion dynamics on moving surfaces provides challenges for mathematical modeling, analysis and simulation.

Asymptotic Stability in a Two-Species Chemotaxis System with Logistic Source

Masaaki Mizukami
Tokyo University of Science, Japan

This talk is concerned with asymptotic bahavior of solutions to a two-species chemotaxis system with logistic source. This system describes a situation in which multi populations react on single chemoattractant. Moreover we assume that both populations reproduce themselves, and mutually compete with the other, according to the classical Lotka-Volterra kinetics. Zhang and Li proved existence of bounded global-in-time solutions to the system. The main result asserts asymptotic stability in the system.

Periodic Homogenisation of Filtering Combustion in Fast Drift and Fast Reaction Regimes

Adrian Muntean
Karlstad University, Sweden

We discuss a basic homogenisation scenario that refers to periodic approximations of porous media hosting combustion. The special feature of our setting is the presence of both fast drifts and fast reactions. Using suitable two-scale convergence-like concepts, we obtain effective model equations and transport coefficients. Additionally, we briefly discuss the numerical simulation of the limit combustion problem. This is a joint work with Ekeoma Ijioma (Univ. of Meiji, Tokyo, Japan).

Mathematical Model for Brewing Japanese Sake with Stirring Effect and Its Analysis

Yusuke Murase
Meijo University, Japan

Our main theme is a solvability of mathematical model for brewing process of Japanese Sake with stirring effect. In this model, staring effect is represented by advection terms. Our full model is formulated by 15 advection-diffusion equations and one constraint condition, but we take up a simple model (corresponding to 1st brewing stage) made by 6 equations and a constraint condition, in this talk. The solution of this model is written in variational inequality form. Moreover, the solution depends upon the solution self in consequence of the constraint condition. It means that the model has a character which is so-called quasi-variational inequality.

On the Existence Theorem of Periodic Solution to One-Dimensional Free Boundary Problem for Adsorption Phenomena in a Two-Scale Model for Concrete Carbonation Process

Naoki Sato
Nagaoka National College of Technology, Japan
Toyohiko Aiki

Our main interest is to study the moisture transport of two-scale model for concrete carbonation process in a three dimensional domain from a non-liner analytic point of view. In this model we consider a bounded macro domain occupied with concrete. We suppose that for any point of this domain one pore is corresponded and regards the pore as a micro domain interval decomposed to water domain and the air region. The humidity in the macro domain controls the boundary between the water domain and the air region in the micro domain interval. We can express the change as a one-dimensional free boundary problem by law of conservation of mass and Fick's laws. In the free boundary condition, the value of free boundary depends on the value of the time derivative of the free boundary. In this talk, we are going to talk about the existence theorem of periodic solution to this free boundary problem. We show that from Schauder's fixed point theorem, there is at least one fixed point. It is a periodic solution to our problem. Moreover, we are going to introduce physical background of our model in detail.

Lateral Self-Contacts of Slender Viscoelastic Rods

T Seidman
UMBC, USA

We consider the dynamics of a slender viscoelastic Cosserat rod admitting strong enough geometric variation to create lateral self-contact while prohibiting self-penetratration. This is developed from collaborations with S.Antman and K.Hoffman.

Energy Dissipation in an Anisotropic Kobayashi–Warren–Carter Type Model of Grain Boundary Motion

Ken Shirakawa
Chiba University, Japan
Salvador Moll, Hiroshi Watanabe

In this talk, a type of Kobayashi–Warren–Carter model of grain boundary motion is considered with the anisotropic effect in a polycrystal. The mathematical model is formulated as a gradient system of a governing free-energy, consisting of a parabolic variational inequality, and a type of quasi-variational inequality. Here, it is particularly worth noting that the quasi-variational situation enables to reproduce not only the shapes of stable structures of grains, but also the dynamic changes of the structures working with the rotations of crystalline orientations.
Recently, the mathematical validity of our anisotropic model is verified as the existence result of time-global solutions. On this basis, we set the objective of this talk to discuss about the continuing topics, concerned with energy-dissipation and large-time behavior for the anisotropic model.

Diffusion Phenomena for the Wave Equation with Space-Dependent Damping in an Exterior Domain

Motohiro Sobajima
Tokyo University of Science, Japan
Yuta Wakasugi

In this talk we consider the initial-boundary value problem to the N-dimensional wave equation with space-dependent damping in an exterior domain. The coefficient of the damping term is radially symmetric, strictly positive and behaves like $|x|^{-\alpha}$ with $\alpha \in [0, 1)$ in a neighborhood of spatial infinity. The purpose of this talk is to give weighted energy estimates and precise asymptotic behavior of solutions by using corresponding semigroup generated by an elliptic operator with an unbounded diffusion coefficient. This is a joint work with Dr. Yuta Wakasugi (Nagoya University).

Nonlinear Evolution Equations with Some Singular Potential Ralated to the Hardy Inequality

Toshiyuki Suzuki
Kanagawa University, Japan

We consider the nonlinear evolution equations with inverse-square potentials $a|x|^{-2}$:

$$\frac{\partial u}{\partial t} + e^{i\theta}(P_a u + f(u)) = 0 \quad \text{in } \mathbb{R} \times \mathbb{R}^N,$$

where $i = \sqrt{-1}$, $N \geq 3$, $\theta \in [0, \pi/2]$, $P_a = -\Delta + a|x|^{-2}$ and $a = -(N-2)^2/4$. We see that $-\Delta$ and the strongly singular potential $a|x|^{-2}$ are the same scale symmetry and hence scaling argument can not be applied to P_a. $a = -(N-2)^2/4$ is the threshold of the selfadjointness of P_a in the sense of formsum in $L^2(\mathbb{R}^N)$. Here we note that $D((1+P_a)^{1/2})$ does not coincide with $H^1(\mathbb{R}^N)$. In this talk we show the global existence in the energy space and related topics in a view of $\theta = \pi/2$ (nonlinear Schrödinger equations).

Existence of Solutions to Problems for Charged Particles in Plasmas

Yutaka Tsuzuki
Tokyo University of Science, Japan
Alexander Leonidovich Skubachevskii

We consider local existence of solutions to the Vlasov-Poisson system in a half space, which is a model of charged ions and elements in plasmas. Local existence of solutions to this system has been established by A. L. Skubachevskii (2012), where the existence time T is small even if some conditions for given data are strict. In view of physics, the existence time T should not be small. This talk will give a result which enables us to admit large T under some conditions.

Weak Solutions to a Nonlinear Parabolic-Psedoparabolic System Modeling Gypsum Growth in Concrete

Arthur Vromans
Eindhoven University of Technology, Netherlands
Adrian Muntean, Fons van de Ven, Jan Zeman

In this talk, I will use mixture theory from classical continuum mechanics to derive the structure of a coupled parabolic-pseudoparabolic PDEs system describing the growth of a gypsum layer out of a cement mixture exposed to acid attack. The system is special in the sense that it captures, for each component of the mixture, the simultaneous evolution of diffusion, bulk and surface chemical reactions, mechanical displacements, as well as acid flow velocity. Using a variant of Rothe's method, we show the existence of weak solutions to our system.
This represents joint work with Adrian Muntean (Karlstad, Sweden), Fons van de Ven (Eindhoven, The Netherlands) and Jan Zeman (Prague, Czech Republic).

Parabolic-Hyperbolic Conservation Laws with Variable and Nonlocal Type Coefficients

Hiroshi Watanabe
Oita University, Japan

Parabolic-hyperbolic conservation laws are regarded as a linear combination of the time-dependent conservation laws (quasilinear hyperbolic equations) and the porous medium type equations (nonlinear degenerate parabolic equations). Thus, these equations have both properties of hyperbolic equations and those of parabolic equations and describe various nonlinear convective diffusion phenomena such as filtration problems, Stefan problems and so on. In this talk we consider parabolic-hyperbolic conservation laws with variable and nonlocal type coefficients. In particular, we show the existence and uniqueness of generalized solutions to the equations.

Stability Criteria for Numerical Simulations of Allen–Cahn Equation with Double-Obstacle Constraint Via Yosida Approximation

Noriaki Yamazaki
Kanagawa University, Japan
Tomoyuki Suzuki, Keisuke Takasao

We consider a two-dimensional Allen–Cahn equation with double-obstacle constraint, numerically. The constraint is provided by the subdifferential of the indicator function on the closed interval, which is the multivalued function. Therefore, it is very hard to perform a numerical simulation of our problem. In this talk we approximate the constraint by the Yosida approximation. Then, we give the stability criteria for the standard forward Euler method to provide the stable numerical experiments of the approximating equation.

An Embedding Estimate for the Repulsive Hamiltonian

Kentarou Yoshii
Tokyo University of Science, Japan
Motohiro Sobajima

We consider the repulsive Hamiltonian $H_0 := -\Delta - |x|^2$, which describes the quantum particle affected by a strong repulsive force from the origin. In this talk, we prove an embedding estimate for the functions in the domain $D(H_0)$. For simplicity we focus our attentions to the one-dimensional case.

Nonlinear M-Accretive Operator Theoretic Approach to Parabolic-Elliptic Keller-Segel Systems with Nonlinear Diffusion

Noriaki Yoshino
Tokyo University of Science, Japan

In this talk, we deal with parabolic-elliptic chemotaxis systems with nonlinear diffusion by using the theory for nonlinear m-accretive operator. Marinoschi (2013) established m-accretive theoretic approach to the solvability of chemotaxis systems with Lipschitz diffusion. However the method did not work when the diffusion coefficient is not Lipschitz continuous, such as $D(r) = r^m$ $(m > 1)$. We develop the method and show existence of solutions to the systems in the case of non Lipschitz diffusion.

Special Session 79: Nonlinear Differential Dynamic Systems in Fluid Dynamics

Zheng Ran, Shanghai University, Shanghai Institute of Applied Mathematics and Mechanics, Peoples Rep of China
Jiezhi Wu, Peking University, Peoples Rep of China

There are many challenges in fluid mechanics, including vortex dynamics, unsteady flow separation, turbulence and aeroacoustics, even a great progress have been achieved in the last thirty years due to the fast development of numerical method and parallel computing. The studies demonstrated that many phenomena in fluid mechanics can be described by differential dynamic system and offer us better understanding. The main objective of this special session is to bring together the experts who apply differential dynamic system to fluid mechanics to present their latest results and discuss future directions.

The Reynolds Number Dependance of the Townsend-Perry Constants in Turbulent Boundary Layers

Bjorn Birnir
UC Santa Barbara, USA
Xi Chen, Can Li, John Kaminsky, Shahab Karmini

The existence of (local) solutions to a stochastic Navier-Stokes equation, in Sobolev spaces corresponding to the Kolmogorov-Obukhov-She-Leveque scaling, allows us to develop a theory to explain the sub-Gaussian behavior of the Townsend-Perry constants recently measured for high-order fluctuation moments in turbulent boundary layers. It yields the generalized (Prandtl-von Karman) logarithmic law for the high-order moments. The Reynolds-Taylor number depencence of the Townsend-Perry constants is computed and compared with experimental results and simulations at different Reynolds-Taylor numbers. The predicted constants are in good agreement with experimental and simulation data.

On the Steady-State Nearly-Resonant Water Waves with Time-Independent Spectrum

Shijun Liao
Shanghai Jiaotong University, Peoples Rep of China
Dali Xu, M. Stiassnie

The steady-state nearly-resonant water waves are gained by means of the homotopy analysis method (HAM), a analytic approximation method for highly nonlinear equations. Our strategy is to mathematically transfer the steady-state nearly resonant wave problem into the steady-state exactly resonant ones. By means of choosing a generalized auxiliary linear operator that is a little different from the linear part of the original wave equations, the small divisor, which is unavoidable for nearly resonant waves in the frame of perturbation theory, is avoided, or moved far away from low wave frequency to rather high wave frequency with physically negligible wave energy. It is found that the steady-state nearly resonant waves have nothing fundamentally different from the steady-state exactly resonant ones, from physical and numerical viewpoints.

Nature of Vortex Bifurcation and Cascade in Isotropic Turbulence

Zheng Ran
Shanghai University, Peoples Rep of China

The central problem of fully developed turbulence is understanding the energy cascading process and multiscale interaction. Update, there is no deductive theory which leads to a full physical understanding or mathematical formulation. The definition, development, challenge and the corresponding status of turbulence cascade were briefly reviewed. The limitation of present methods were emphasized. Based on the Karman-Howarth equation in 3D incompressible fluid, a new isotropic turbulence scale evolution equation and its related theory progress, the existence of nonlinear dynamic system measured by turbulence Taylor microscale was proven. The present results indicate that the energy cascading process has remarkable similarities with the deterministic construction rules of the logistic map. The cascade appears as an infinite sequence of period-doubling vortex bifurcations.

Lie-Group Symmetry Analysis of Wall-Bounded Turbulent Flow

Zhen-Su She
Peking University, Peoples Rep of China
Xi Chen, Fazle Hussain

Canonical wall-bounded turbulent flows include channel, pipe and turbulent boundary layer (TBL), the latter may extend to realistic situations involving effects of pressure gradient, compressibility, buoyancy, etc. Searching for universal principle governing the mean flow (velocity and fluctuation intensity) distribution has been one of the central goals for turbulence studies over one hundred years since the pioneer work of Prandtl in 1904. We report a recent attempt to formulate a Lie-group analysis of Reynolds averaged Navier-Stokes (RANS) equation in the presence of an infinite wall, where an invariant dilation group in the direction normal to the wall is identified and a generalized dilation invariant solution is postulated to describe a multi-layer structure which is generic to all wall-bounded flows. Evidence for the validity of this description will be reported, from detailed comparison with experimental and numerical data, involving all canonical flows mentioned above. Furthermore, it will be shown that the physical and mathematical understanding of fluid turbulence derived from this symmetry-based approach helps to improve engineering models for aerodynamic simulation of turbulent flows.

Convection Onset of Two-Dimensional Rayleigh-Benard System with Non-Oberbeck-Boussinesq Effects

De-Jun Sun
University of Science and Technology of China, Peoples Rep of China
Shuang Liu, Shu-Ning Xia, Zhen-Hua Wan

Convection onset of two-dimensional Rayleigh-Benard system with perfectly conducting horizontal walls and adiabatic sidewalls is studied by means of direct numerical simulation of low-Mach-number equations near a codimension-2 point where single-vortex and double-vortex modes become unstable simultaneously. The Prandtl number is fixed at 0.71. Particular attention is paid to the effects of breaking of up-down symmetry due to non-Oberbeck-Boussinesq effects on mode interaction of single-vortex and double vortex modes. Under Oberbeck-Boussinesq approximation bifurcations to single-vortex and double-vortex patterns are supercritical and mode interaction is weak with various solution branches uncorrelated. When non-Oberbeck-Boussinesq effects are taken into account, bifurcation giving rise to one double-vortex pattern becomes subcritical. For strong non-Oberbeck-Boussinesq effects the single-vortex solution branch is disrupted due to mode interaction. Various flow patterns are identified and their hydrodynamic characteristics are analyzed. Multiple-solution and hysteresis phenomena are observed. Bifurcation diagrams corresponding to varying degrees of symmetry breaking are presented. Furthermore, convection in the weakly nonlinear regimes can be qualitatively described by amplitude equations constructed by symmetry considerations.

Theoretical and Numerical Studies on Vorticity Dynamics of Two-Dimensional Flows

Xilin Xie
Fudan University, Peoples Rep of China
Qian Shi, Yu Chen

Two dimensional incompressible and compressible flows on fixed and moving smooth surfaces have been studied in the point of view of vorticity dynamics. The unified vorticity and dilatation algorithm for general two dimensional flows have be attained through the decompositions of vorticity and dilatation. Based on the Levi-Civita connection operator for surface tensor fields, the deformation kinematics of continuous mediums with surface configurations can be presented in an intrinsic way. On the other hand, the conservation laws can be obtained based on the surface gradient operator and the generalized intrinsic Stokes formula. Some primary conclusions in classical vorticity dynamics have been extended to the case of two dimensional flows, including the governing equations of vorticity and dilatation, Lagrange theorem on vorticity and representation of rate-of-strain tensor on solid boundaries. The newly developed theories are characterized by the appearance of surface curvatures in some primary relations and governing equations. Some numerical results of two-dimensional flows based on our theories will be presented in order to reveal the intrinsic features of two-dimensional flows.

Multi-Vortices in Separation Surface

Shuhai Zhang
China Aerodynamics Research and Development Center, Peoples Rep of China

In this talk, I will introduce our recent study on unsteady flow separation. Based on nonlinear dynamic analysis and numerical simulation, we studied the flow structure of unsteady flow separation. The trajectories of fluid particles are computed based on the unsteady flow velocity to study Lagrangian structure. It is found that there are many vortices on the separation line on the body surface, which will developed to vortex tubes in the separation surfaces. Secondly, One or more limit cycle will appear on body surface at certain condition, which becomes a new type of flow separation. It will develop into a tornado-like structure.

Special Session 80: Inverse Limits in Dynamical Systems

Krystyna Kuperberg, Auburn University, USA
Judy Kennedy, Lamar University, USA

The principal topic of the session is the role of inverse limits and generalized inverse limits in discrete dynamical system. We will consider topological group actions, laminations, attractors, rotation number, inverse limit of a branched coverings, McCord solenoids, and algebraic properties of generalized inverse limits. Talks on related research in continuous dynamics and complex dynamics may be included as well.

Homeomorphic Restrictions of Unimodal Maps

Lori Alvin
Bradley University, USA

In this talk we provide a symbolic characterization of a class of unimodal maps whose restriction to the omega-limit set of the turning point is a minimal homeomorphism on a Cantor set. This characterization is given in terms of the shift space generated by the kneading sequence of the unimodal map. If time provides, we will discuss how these results can be used to better understand the topological structure of the associated inverse limit space.

Golden Bee Tilings from the Point of View of Inverse Limits

Michael Barnsley
Australian National University, Australia
Andrew Vince

Recent results of the authors concerning dynamics and inverse limits associated with certain tilings of the plane by will be presented.

New Homogeneous Continua and Semi-Conjugacies of Interval Maps

Jan Boronski
IT4 Innovations, Ostrava, Czech Rep
M. Smith

In 1985 M. Smith constructed a nonmetric pseudo-arc; i.e. a Hausdorff homogeneous, hereditary equivalent and hereditary indecomposable continuum. Taking advantage of a decomposition theorem of W. Lewis, he obtained it as a long inverse limit of metric pseudo-arcs with monotone bonding maps. Extending his approach, and the results of Lewis on lifting homeomorphisms, we construct a nonmetric pseudo-circle, and new examples of homogeneous 1-dimensional continua: a circle and solenoids of nonmetric pseudo-arcs. Among many corollaries we also obtain an analogue of another theorem of Lewis from 1984: any interval map is semi-conjugate to a homeomorphism of the nonmetric pseudo-arc.

Fundamental Groups and Generalized Covering Spaces

James Keesling
University of Florida, USA
Jerzy Dydak, Joanna Furno

Classical covering spaces have played an invaluable role in the study of spaces that are locally connected and locally simply connected. There is good reason to generalize these spaces and considerable work has been done in this direction. We contribute to this effort in two different directions. We give examples of what we believe are the ideal examples of generalized covering spaces. That is, these have almost all of the properties one works with in covering spaces except the locally simply connected property. The examples that we give have a topological group structure on $\pi_1(X)$ rather than being discrete groups. The worst examples for generalized covering spaces are those spaces X that are arcwise connected and locally arcwise connected with non-trivial fundamental group, but which have no covering spaces. We develop tools to analyze such spaces and show that such spaces can also have a rich structure.

Horseshoes and Lambda-Dendroids in Generalized Inverse Limits

Judy Kennedy
Lamar University, USA
Van Nall

Set-valued functions from an interval into the closed subsets of an interval arise in various areas of science and mathematical modeling. When studying the dynamical properties of a set-valued function, the problem is that if one iterates in the standard way, the orbit of a point is not well defined. We study instead the dynamical system of the shift map defined on the inverse of the set-valued function. The inverse limit is the space of all possible inverse orbits, and the shift map is a continuous function from this compact subset of the Hilbert cube onto itself. The inverse limit can be very complicated topologically, and here again, as in so many other settings, complicated topology and complicated dynamics go hand-in-hand. We explore the relationship between the topological structure of the inverse limit space and the dynamics of the shift map.

Vietoris Theorems for Generalized Inverse Limits

Krystyna Kuperberg
Auburn University, USA

In 1927, L. Vietoris introduced homology groups for general metric spaces and proved a his famous isomorphism theorem. Since then, many formulations of the theorem appeared in the literature. We will discuss Vietoris theorems for generalized inverse limits.

Inverse Limits of Forward Invariant Laminations of the Unit Disk

John Mayer
UA-Birmingham, USA

Laminations of the unit disk were introduced by William Thurston as a topological/combinatorial model for understanding the (connected) Julia sets of polynomials. To each locally connected Julia set of a degree d polynomial there corresponds a lamination with a dynamical "angle-d-tupling" map σ_d on it semiconjugate to the polynomial on its Julia set. In most cases, a finite amount of information (a forward invariant finite lamination) allows for construction by "pullback" of the entire lamination corresponding to the Julia set. The pullback lamination is a direct limit of a sequence of finite laminations in the unit disk, and the limit lamination carries the semiconjugate σ_d dynamics. In contrast, we investigate the inverse limit of the finite laminations in the pullback sequence with bonding map the σ_d map, restricted to the finite laminations, corresponding to the degree d polynomial on its Julia set. Each point of the inverse limit corresponds to an inverse orbit of the dynamics. We seek to determine how much information about orbits of the polynomial dynamics on the Julia set can be thus obtained. We consider polynomial Julia sets of degree $d = 2$ and $d = 3$.

Inverse Limits and Shadowing

Jonathan Meddaugh
University of Birmingham, England

Inverse limits are excellent models for the structure of the orbit space for certain dynamical systems. In this talk we will discuss some connections between inverse limit spaces with various topologies and shadowing properties of dynamical systems.

Euclidean Fractals

Magdalena Nowak
Jan Kochanowski University in Kielce, Poland
Taras Banakh

An Iterated Function System (IFS) on the metric space X is a finite family $\mathcal{F}$ of contractive self-maps on X. Consider the space $\mathcal{H}(X)$ of nonempty, compact subsets of X. We can define a dynamical system $(\mathcal{H}(X), \mathcal{F})$ such that $\mathcal{F}(K) = \bigcup_{f\in\mathcal{F}} f(K)$ for each $K \in \mathcal{H}(X)$.
By the attractor of Iterated Function System (IFS-attractor) we understand a nonempty compact set $A \subset X$ such that

$$A = \mathcal{F}(A) = \bigcup_{f\in\mathcal{F}} f(A)$$

and for every compact set $K \in \mathcal{H}(X)$ the sequence $\big(\mathcal{F}^n(K)\big)_{n=1}^{\infty}$ converges to A with respect to the Hausdorff metric on $\mathcal{H}(X)$.
We deal with the question of which compact metrizable spaces are homeomorphic to attractors of Iterated Function Systems in the Euclidean space. We called such spaces Euclidean fractals. We extend the result obtained by Duvall and Husch, and proved that given any compact metrizable finite-dimensional doubling space X that contains an open zero-dimensional uncountable subset, we can topologically transform the space X such that the image is the attractor of some IFS in $\mathbb{R}^n$. The proof bases on the representation of the space X as the inverse limit of some fractal structure and on the morphisms of the normed colored trees.

On Almost Specification and Entropy

Piotr Oprocha
AGH University, Poland

Almost specification property is a generalization of Bowens specification property, which is present in a wider class of systems. For example, all beta-shifts have this property while it is know that some of them do not satisfy specification property. In this talk we will present some relations between almost specification property and topological entropy, and its consequences to other notions from topological dynamics.

Invariant Measures for Set-Valued Maps

Brian Raines
Baylor, USA
Jonathan Meddaugh, Tim Tennant

We consider the problem of constructing invariant measures for multi-valued dynamical systems, in particular we are interested in non-atomic invariant measures with full support. We examine multi-valued dynamical systems, (X, F), with the specification property. The associated inverse limit space and forward orbit space are useful for studying multi-valued dynamical systems because they have an associated shift map that is a continuous function rather than an upper-semi-continuous relation. These associated spaces also have the specification property, and, thus, on these spaces the set of invariant measures contains a dense G_δ-set of non-atomic invariant measures with full support. Unfortunately these measures do not always induce a measure on the original system that is non-atomic. One of the main obstructions is the presence of multiperiodic points for the original multi-valued dynamical system.

Quantization for Probability Distributions

Mrinal Roychowdhury
University of Texas Rio Grande Valley, USA

Quantization for probability distributions refers to the idea of estimating a given probability by a discrete probability with finite support. Recently, several works have been done in this direction. I will talk about it.

About the Cohomology of Attractors

Francisco Ruiz del Portal
Universidad Complutense de Madrid, Spain
Jaime J. Sanchez Gabites

Suppose K is an asymptotically stable attractor for a dynamical system on $\mathbb{R}^m$. In the continuous case (a flow) it is well known that the inclusion of K in its basin of attraction $A(K)$ induces isomorphisms in Cech cohomology. We discuss whether the same holds true in the discrete case (a homeomorphism). We show that (i) it is true if coefficients are taken in $\mathbb{Q}$ or $\mathbb{Z}_p$ (p prime) and (ii) it is true for integral cohomology if and only if the Cech cohomology of K or $A(K)$are finitely generated.

Three Classic Theorems

David Ryden
Baylor University, USA

In this talk we will consider three properties from the classical theory of inverse limits that do not hold for inverse limits with upper semicontinuous set-valued bonding maps, and we provide necessary and sufficient conditions for them to hold in the more general setting. The properties in question are (1) the closed set theorem, (2) the full projection property (and weak full projection property), and (3) a version of the subsequence theorem.

Some Constraints on the Topology of Isolated Invariant Sets and How They Sit in Phase Space

Jaime J. Sánchez-Gabites
Universidad Autonoma de Madrid, Spain

Let $K \subseteq \mathbb{R}^3$ be an isolated invariant set (in the sense of Conley) for a flow. Suppose K has finitely generated Cech cohomology with $\mathbb{Z}_2$–coefficients. Then it is possible to construct isolating neighbourhoods N of K that are compact 3–manifolds and such that the inclusion $K \subseteq N$ is a shape equivalence. Using this and other related results we shall obtain some constraints on the topology of K and how it sits in $\mathbb{R}^3$. As an application, we generalize a result by E. S. Thomas (One-dimensional minimal sets. Topology, 12:233-242, 1973) concerning isolated generalized solenoids.

Shape, Bifurcations and Splittings of Some Isolated Invariant Sets

Jose Sanjurjo
Universidad Complutense de Madrid, Spain
Hector Barge

We study the dynamics and topology of the isolated invariant sets created by bifurcations consisting in implosions of the region of attraction of asymptotically stable equilibria of flows. The topological properties are formulated in terms of Borsuk's shape theory. We also study attractor-repeller splittings of non-saddle sets.

Decompositions of Non-Metric Continua

Michel Smith
Auburn University, USA

Hernandez-Gutierrez (2014) showed that there is a generalized Whitney map from the non-metric pseudo-arc M of Smith (1985) onto the long arc. Using this map a continuous decomposition of M may be constructed. We show that any continuous decomposition of M into non-degenerate continua is a metric pseudo-arc. A technique is presented using inverse limits for producing a class of non-metric continua so that if X is one of them and G is a continuous decomposition of X into non-degenerate subcontinua, then the decomposition space X/G is metric. Upper semi-continuous decompositions are also considered.

Chaotic Dynamics in the Twisted Horseshoe Map Via a Topological Approach

Elisa Sovrano
University of Udine, Italy

In recent years, several different approaches have been proposed in order to extend, in a topological direction, the classical geometry of the Smale horseshoe. These generalizations have been motivated by the investigation of some new problems in chaotic dynamics. The concept of topological horseshoe, developed in the works of Burns and Weiss, Kennedy and Yorke, and other authors, lead to a powerful theory useful to detect chaos in dynamical systems. In this framework, we show, in an analytical way, the presence of complex behaviors for a discrete dynamical system arising form the study of the twisted horseshoe map considered by Guckenheimer, Oster and Ipaktchi in 1977.

When the Topology of the Map Determines the Dynamics

James Yorke
University of Maryland, USA
S. Das, Y. Saiki

Let $F : T^d \to T^d$ where T^d is a d-dimensional torus. The mapping always can be written as linear term Mx, where M is a $d \times d$ matrix with integer entries, plus a bounded term$G$ that is periodic with period 1 in each variable, so that $F(x) = Mx + G(x) \mod 1$ in each variable. We report on how M can determine much of the dynamics even when G is very large. We can think of M as determining the topology of the map since it is the homology matrix. It can also determine some of the dynamics.

Special Session 81: Advances in Computer Assisted Proofs for Dynamical Systems and Differential Equations

Gianni Arioli, Politecnico di Milano, Italy
Jason Mireles-James, Florida Atlantic University, USA

The last 20 years have seen a significant development of techniques for computer assisted proofs in dynamical systems theory. Such techniques are used to prove the existence of solutions of ordinary and partial differential equations, and to study invariant sets in a mathematically rigorous way using the computer. These techniques use two main fundamental approaches: one is based on topological tools, the other on functional analysis. The first approach, when applied to differential equations, relies on the development of efficient rigorous integrators and studies how sets in phase space map across one another. The second approach relies on transforming the equations into fixed points problems in some functional space. These approaches are somehow complementary, and the aim of this session is to discuss recent advances in order to learn how to exploit the best features of both.

Continuation of Connecting Orbits for Maps

Ronald Adams
Embry Riddle Aeronautical University, USA
Jason Mireless James

I will discuss a validated continuation method for connecting orbits between fixed points for finite dimensional maps. The argument exploits high order parameterizations of one parameter families of local stable/unstable manifolds. The argument can also be modified to compute saddle node bifurcations for connection orbits (i.e. prove the existence of collisions of the stable/unstable manifolds giving rise to connections). I will illustrate the utility of this method by applying it to a one parameter family of Henon maps which contains the classic parameters.

On a Nonlinear Nonlocal Hyperbolic System Modeling Suspension Bridges

Gianni Arioli
Politecnico di Milano, Italy
Filippo Gazzola

This talk does not concern (yet) computer assisted proofs; instead, it presents some open problems that we hope to be able to address with computer assistance. We suggest a new nonlinear model for a suspension bridge and we perform numerical experiments with the parameters corresponding to the collapsed Tacoma Narrows Bridge. We show that the thresholds of instability are in line with those observed the day of the collapse. Our analysis enables us to give a new explanation for the torsional instability, only based on the nonlinear behavior of the structure.

Recent Results Verifying Stability of Traveling Waves with Computer Assisted Proof

Blake Barker
Brown University, USA
Bjorn Sandstede, Kevin Zumbrun

We describe recent results in which we rigorously verify spectral stability of travelling waves, a sufficient condition to imply non linear stability in the systems examined. We outline our general approach and the solutions we used to overcome such challenges as the rapping effect and computational cost of the study. In particular, we approach these studies with an eye toward a general strategy for rigorously verifying stability of travelling waves.

Efficient Methods for Long Term Verified Integration in Hyperbolic Systems

Martin Berz
Michigan State University, USA
Kyoko Makino

Hyperbolic systems, including those of the non-uniform kind, represent one of the most important class of problems encountered in the study of chaotic dynamical systems. For rigorous self-verified study, the practically appearing elongation in one of the directions typically represents a formidable challenge, since solution sets even under the best of circumstances will necessarily elongate in a similar way. We present a Taylor model based method to overcome this problem for the computation of orbits in autonomous hyperbolic systems. As an example, the approach allows the rigorous self-verified computation of stable and unstable manifolds of systems for in principle often unlimited times, and in practice for far longer times than conventional verified orbit integration is possible. We show practical results and their use for some typical systems including the Henon and Lorenz families, and compare the resulting possible integration lengths with those of other state of the art methods.

Computer Proof of Heteroclinic Connection in 1D Diblock Copolymers PDE Model

Jacek Cyranka
Rutgers University, USA

We present a computer proof of a heteroclinic connection in the one dimensional diblock copolymers PDE model (Cahn-Hillard type) with periodic boundary conditions

$$u_t = -\Delta(\varepsilon^2 \Delta u + f(u)) - \sigma(u - \mu) \text{ in } \Omega.$$

For some (σ,ε)'s the equation exhibit rich attractor structure. We establish existence of a heteroclinic connection between the homogeneous state $u \equiv 0$ representing a perfect mixture of copolymers and a local energy minimizers. This is a nongeneric phenomenon in Cahn-Hillard PDE models. It has an intriguing physical interpretation – given a perfect mixture of copolymers, shake it a bit, and the stable state resulting from the time evolution of this model cannot be anymore predicted. Due to the complicated structure of the global attractor the system exhibit several stable states, and which one is the result of time evolution of an initial condition close to homogeneous is highly dependent on the initial condition. The proof is conceptually simple, first we calculate a interval bounds contained in a fixed point's basin of attraction, and a piece of unstable manifold of the homogeneous state, and then we integrate in time the bounds for unstable manifold. If the bounds are mapped into the basin of attraction our procedure is successful in proving the existence of the particular heteroclinic connection. We combine several techniques from some of the authors' and P. Zgliczyǹski works. We obtain a bounded region within the basin of attraction by using a logarithmic norm calculation, an explicit bound for an unstable manifold by using so-called cone conditions. We propagate in time an interval box containing a piece of unstable manifold by using our efficient algorithm for rigorous integration of PDEs forward in time, which is able to handle large integration times within a reasonable time. Moreover, we present how to use the PDE scaling invariance to extend those results for arbitrary large ε parameter values.

Explicit Error Estimates for a Hopf Bifurcation in Wright's Equation

Jonathan Jaquette
Rutgers University, USA
Jan Bouwe van den Berg

Wright's equation $y'(t) = -\alpha y(t-1)\{1 + y(t)\}$ has a supercritical Hopf bifurcation at $\alpha = \pi/2$, and a long-standing conjecture claims that there are no periodic orbits when $\alpha < \pi/2$ and there is an unique slowly oscillating periodic orbit when $\alpha > \pi/2$. Recent works have used computer assisted proofs to make progress on this conjecture, however these techniques break down near the critical value $\alpha = \pi/2$. In this talk, we use a Newton-Kantorovich type argument to develop explicit error estimates on the branch of periodic orbits described by the Hopf bifurcation. This bifurcation analysis produces a fundamentally local result; we can prove the uniqueness of periodic solutions with small amplitude when $\alpha - \pi/2$ is small and the frequency is close to $\pi/2$. Nevertheless, for a small neighborhood about $\alpha = \pi/2$, we are able to extend our bifurcation analysis into global result by way of computer assisted proofs.

Set-Based Methods for Global Dynamics

William Kalies
Florida Atlantic University, USA
Sarah Day, Vincent Naudot, Marcus Fontaine

In this talk we will give a brief overview of set-based methods for computing global dynamics using Conley theory and the current state of algorithms and software. We will illustrate the techniques with some examples. First we obtain computable, rigorous cover of the global invariant set and an outer approximation of the dynamics for a class of integro-difference equations with smooth nonlinearities that are well-approximated by polynomials. The dynamics of the Kot-Schaffer map with Ricker nonlinearity is used to illustrate the computational method. Next we compute the dynamics of an attractor which includes a "re-injected" cuspidal horseshoe, as well as other examples as time permits.

Periodic Solutions for Some Hyperbolic-Type PDEs

Hans Koch
The University of Texas at Austin, USA
Gianni Arioli

We consider the wave equation and the beam equation, with cubic nonlinearity, on the unit interval. These are infinite dimensional Hamiltonian systems, and there are small denominator issues for periodic orbits with irrational frequencies. Nevertheless, solutions for rational frequencies appear to lie on "branches", and these branches appear to undergo bifurcations. We describe some numerical observations and some rigorous results. The latter include the existence of non-small periodic solutions for the beam equation for positive-measure sets of frequencies.

Computer-Assisted Proofs for Long Periodic Orbits of Parabolic PDEs and Their Stability

Jean-Philippe Lessard
Laval University, Canada
Jordi-Lluis Figueras, Marcio Gameiro, Rafael de la Llave

In this talk, we introduce a computer-assisted technique for the analysis of periodic orbits of parabolic PDEs. The idea is to use a Newton-Kantorovich type argument to obtain rigorous proofs of existence of the periodic orbits in a weighted ell-infinity Banach space of space-time Fourier coefficients with algebraic decay. As an application, our proposed method is applied to prove existence of several periodic orbits in the Kuramoto-Sivashinsky PDE, which is a popular model to study spatiotemporal chaos. We prove the existence of some periodic orbits with large period. We also present rigorous results about the stability of the orbits by solving an associated eigenvalue problem.

Verified Propagation of Large Sets of Initial Conditions and Parameter Ranges and Applications for Proofs of chaoticity

Kyoko Makino
Michigan State University, USA
Martin Berz

Rigorous high-order methods including the Taylor model approach allow the propagation of initial conditions with an error that scales as a high power of the diameter. This allows for propagation of large ranges of domains and parameter ranges by virtue of automatic domain decomposition tools, not unlike methods for mesh generation in non-verified PDE solvers. Using these methods, it is often possible to propagate very large domains in an economical manner. We show various applications of the method, including proofs of chaoticity of the well-known Lorenz system for large ranges of parameters, as well as applications for the commonly used method of discretizing the motion into a finite but large graph and studying global dynamics based on that representation.

Rigorous Numerics for Fast-Slow Systems with an Explicit Multi-Scale Parameter Range Via the Covering-Exchange

Kaname Matsue
The Institute of Statistical Mathematics, Japan

We provide a rigorous numerical computation method to validate periodic, homoclinic and heteroclinic orbits as the continuation of singular limit orbits for the fast-slow system $\dot{x} = f(x,y,\epsilon)$, $\dot{y} = \epsilon g(x,y,\epsilon)$. Our validation procedure is based on topological tools called isolating blocks, cone condition and covering relations. Such tools provide us with existence theorems of global orbits which shadow singular orbits in terms of a new concept, the covering-exchange. The covering-exchange consists of three parts; slow shadowing, drop and jump, which give us not only generalized topological verification theorems, but also easy implementations for validating trajectories near slow manifolds in a wide range, via rigorous numerics. Our procedure is available to validate global orbits not only for sufficiently small $\epsilon > 0$ but all ϵ in a given half-open interval $(0, \epsilon_0]$. Several sample verification examples are shown as a demonstration of applicability.

Parameterization Method for Invariant Manifolds: Efficient Numerics and Validated Computatoin

Jason Mireles James
Florida Atlantic University, USA

I will discuss some aspects of the parameterization method for invariant manifolds, focusing on numerical computations with mathematically rigorous a-posteriori error bounds. The parameterization method is a functional analytic framework for studying invariant manifolds such as invariant circles and tori, and also stable/unstable manifolds for equilibria and periodic orbits. One feature of the method is that it provides a natural notion of a-posteriori error which can be exploited in computer assisted proofs. Parameterized manifolds are also useful for studying connecting orbits in dynamical systems and I will mention some applications.

Validated Computation of Unstable Manifolds for Parabolic PDEs

Christian Reinhardt
VU Amsterdam, Netherlands
Jason Mireles James

This talk presents a method for computing polynomial approximations together with a-posteriori error bounds of unstable manifolds associated with equilibrium solutions to parabolic PDEs posed on compact domains with suitable boundary conditions. These polynomials have a finite number of variables, even though they map into an infinite dimensional state space. The approach is based on the parametrization method and builds on explicit knowledge of the spectral stability data at the equilibrium solution that we also obtain via validated numerical methods. We implement the method numerically, and develop explicit a-posteriori error bounds using the notion of radii polynomials. By combining the a-posteriori error estimates with careful management of floating point round-off errors we obtain mathematically rigorous bounds on the truncation and discretization errors associated with our polynomial approximation. One of the motivations for this method is the validated computation of connecting orbits for PDEs. We illustrate the method with applications to Fisher's equation. This is joint work with Jason Mireles-James.

Validation of the Bifurcation Diagram in the 2D Ohta-Kawasaki Problem

Jan Bouwe van den Berg
VU Amsterdam, Netherlands
JF Williams

In this talk we discuss a rigorous numerical method to compare local minimizers of the Ohta-Kawasaki functional in two dimensions. In particular, we validate the phase diagram identifying regions of parameter space where rolls are favorable, where hexagonally packed spots have lowest energy and finally where the constant mixed state does. More generally, we present a computational method to rigorously determine such features in problems where optimal domain sizes are not known a priori. In terms of the practical realities of applying the computer assisted theorems ideas, this work represents a step forward past clean-cut test problems to more elaborate variational problems in pattern formation.

Special Session 82: Numerical Simulations and Computations for Stochastic Dynamics

Yao Li, University of Massachusetts Amherst, USA
Molei Tao, Georgia Institute of Technology, USA

Numerical methods for stochastic dynamics have a wide range of applications in many disciplines, such as chemical physics, statistical mechanics, biological sciences, and fluid dynamics. In this special session, we focus on recent developments of numerical simulations and computations for stochastic dynamics arising from various applications. Problems to be addressed include both improvements of algorithms and novel applications to other scientific fields.

Markov Chain Approximation Methods for the Numerical Solution of Stochastic Differential Equations

Nawaf Bou-Rabee
Rutgers University – Camden, USA

Stochastic differential equations (SDEs) model random fluctuations in applications as diverse as: molecular dynamics, mathematical finance, population dynamics, epidemiology, laser dynamics and atmosphere/ocean sciences. For the most part, these SDEs cannot be solved exactly and numerical methods are used to approximate their solution. The main goal of these approximations is to estimate statistics associated to the SDE solution like mean first passage times, exit probabilities, and multi-time expectations of observables. This talk demonstrates that the go-to method for numerically solving SDEs – Euler-Maruyama – is impractical to use for: SDEs with numerical stiffness, long-time simulation of ergodic SDEs, SDEs with unattainable boundaries, SDEs with internal discontinuities, SDEs with boundary conditions (e.g. Dirichlet, Neumann, and oblique derivative boundary conditions), and SDEs with interface conditions (for these SDEs existence/uniqueness of weak solutions are active areas of research). These issues motivate a shift in perspective that naturally leads to Markov Chain Approximation Methods. We will show how to use numerical PDE methods to construct these types of approximations, and stochastic methods to simulate/analyze them.

Analysis of Long-Time Behavior of Large Continuous-Time Markov Chains with Exponentially Small Transition rates

Maria Cameron
University of Maryland, USA
Tingyue Gan

Large stochastic networks (continuous-time Markov chains) with exponentially small transition rates arise in modeling of complex physical, chemical, and biological systems. Time-reversible networks of this kind represent, e.g., energy landscapes of atomic or molecular clusters, while time-irreversible ones can model the dynamics of e.g. biological cell cycle or walks of molecular motors. I will present a novel algorithm for the analysis of long-time behavior of such networks consisting of two steps. It is a greedy graph algorithm that builds the hierarchies of optimal W-graphs and Freidlin's cycles. The output of this algorithm allows us to find asymptotic approximations for eigenvalues, extract Freidlin's cycles, identify metastable and quasi-invariant subsets of states, and offer several ways to cluster the network. The proposed methodology will be illustrated on examples coming from natural sciences.

Uncertainty Quantification for Stochastic Systems with Memory

Eric Hall
University of Massachusetts Amherst, USA
Markos Katsoulakis, Luc Rey-Bellet

We give a minimal variance method for estimating parametric sensitivities of observables of stochastic systems with memory.

Numerical Dynamics of Random Ordinary Differential Equations

Xiaoying Han
Auburn University, USA
Peter E. Kloeden

Numerical dynamics is concerned with the relationship between the dynamical behaviour of the solutions of a random ordinary differential equation (RODE) and that of the solutions of a numerical scheme. Two major issues, the preservation of an attractor and a hyperbolic neighborhood under discretization will be considered in the context of RODEs and the random dynamical systems.

Capturing Rare Events with the Heterogeneous Multiscale Method

David Kelly
New York University, USA
Eric Vanden-Eijnden

We discuss heterogeneous multiscale methods (HMM) and their ability to capture fluctuations acting on the slow variables in fast-slow systems. In particular, it is shown via analysis of central limit theorems (CLT) and large deviation principles (LDP) that the standard version of HMM artificially amplifies these fluctuations. A simple modification of HMM, termed parallel HMM, is introduced and is shown to remedy this problem, capturing fluctuations correctly both at the level of the CLT and the LDP.

Coarse-Graining of Stochastic Dynamics

Frederic Legoll
Ecole des Ponts, France

The question of coarse-graining is ubiquitous in many applied sciences, including molecular dynamics. In this work, we are interested in deriving effective properties for the dynamics of a coarse-grained variable $\xi(X)$, where X describes the configuration of the system, and ξ is a smooth scalar function. Typically, X is a high-dimensional variable (representing for instance the positions of all the particles in the system), whereas $\xi(X)$ is a coarse-grained information (e.g. a particular angle between some atoms of the molecule).
We assume that the configuration X_t of the complete system evolves according to the overdamped Langevin stochastic differential equation, and we propose an effective closed dynamics that approximates (under time-scale separation assumptions) the evolution of $\xi(X_t)$. Such an effective dynamics may be useful to compute more efficiently e.g. transition rates from one configuration of the system to another one. Several estimations of the accuracy of the effective dynamics will be given, using various mathematical tools.
Based on joint works with T. Lelievre and S. Olla.

A Fast Exact Simulation Method for Markov Jump Processes

Yao Li
University of Massachusetts Amherst, USA

In this talk I will present a new method for the simulation of Markov jump processes, called the Hashing-Leaping method (HLM). As a novel method of the stochastic simulation algorithm(SSA), the HLM has conditional constant computational cost per event, which is independent of the number of exponential clocks in the Markov process. In particular, it is compatible with parallel machines in many ways. In the talk, I will introduce the method, demonstrate its implementation and parameter tuning, and discuss its parallelization.

A New Model for Realistic Random Perturbations of Stochastic Oscillators

Wuchen Li
Georgia Institute of Technology, USA

Classical theories predict that solutions of differential equations will leave any neighborhood of a stable limit cycle, if white noise is added to the system. In reality, many engineering systems modeled by second order differential equations, like the van der Pol oscillator, show incredible robustness against noise perturbations, and the perturbed trajectories remain in the neighborhood of a stable limit cycle for all times of practical interest. In this talk, we propose a new model of noise to bridge this apparent discrepancy between theory and practice. Restricting to perturbations from within this new class of noise, we consider stochastic perturbations of second order differential systems that -in the unperturbed case- admit asymptotically stable limit cycles. We show that the perturbed solutions are globally bounded and remain in a tubular neighborhood of the underlying deterministic periodic orbit. This is a joint work with Prof Luca Dieci and Prof Haomin Zhou in Georgia tech.

An Analysis of Implicit Sampling in the Small-Noise Limit

Kevin Lin
University of Arizona, USA
Jonathan Goodman, Matthias Morzfeld

Weighted direct samplers, also known as importance samplers, are Monte Carlo algorithms for generating independent, weighted samples from a given target probability distribution. Such algorithms have a variety of applications in, e.g., data assimilation and state estimation problems involving stochastic and chaotic dynamics. One challenge in designing and implementing weighted samplers is to ensure the variance of the weights, and that of the resulting estimator, are well-behaved. In recent work, Chorin, Tu, Morzfeld, and coworkers have introduced a class of novel weighted samplers called implcit samplers, which have been shown to possess a number of desirable properties. In this talk, I will report on an analysis of the variance of implicit samplers in the small-noise limit, and describe a simple method suggested by the analysis for obtaining higher-order implicit samplers. The algorithms are compared on concrete test problems.

Analysis and Simulation of Ntracellular Bio-Chemical Reacting Networks with Multiple Time Scales

Di Liu
Michigan State University, USA

Intracellular reacting networks involving gene regulation often exhibits multiscale properties. That includes multiple reacting rates, multiple population magnitudes and multi-stability. Direct Stochastic Simulation Algorithm (SSA) would turn out to be inefficient dealing with such systems. Schemes such as Nested SSA and Tau-leaping method have proved to be effective for certain asymptotic regimes. I will present recent results on the convergence analysis and applications of the algorithms.

Path-Space Information Metrics and Coarse-Graining of Non-Equilibrium Stochastic Systems

Petr Plechac
University of Delaware, USA

We discuss information-theoretic tools for obtaining optimized coarse-grained molecular models for both equilibrium and non-equilibrium molecular dynamics. The presented approach compares microscopic behavior of molecular systems to parametric or non-parametric coarse-grained systems using the relative entropy between distributions on the path space. It allows us to formulate a corresponding path space variational inference problem. The methods become entirely data-driven when the microscopic dynamics are replaced with corresponding correlated data in the form of time series.

Some Numerical Methods for Hyperbolic Periodic Orbits and Rare Events in Nongradient Systems

Molei Tao
Georgia Tech, USA

We consider differential equations perturbed by small noises. The goal is to quantify what noises can do and possibly also utilize them. Noise-induced transitions are understood by optimizing probabilities characterized by Freidlin-Wentzell large deviation theory. In gradient systems, metastable transitions were known to cross separatrices at saddle points. We investigate nongradient systems, and show a very different type of transitions that cross hyperbolic periodic orbits instead. Numerical tools for both identifying such periodic orbits and computing transition paths are described. If time permits, I will also discuss how these results may help design control strategies. Joint work with Eric Vanden-Eijnden.

Special Session 83: Dynamical Systems and their Applications

Maria Correia, University of Evora, Portugal
Scott Cook, Tarleton State University, USA

This special session will cover a variety of applications of dynamical systems, with special focus on billiard systems and application to statistical mechanics. However, we invite talks from all areas of dynamical systems. The aim of this session is to bring together researchers who work in dynamical systems to exchange and discuss recent developments in this field. Young researchers working in dynamical systems are especially welcome to present their recent results.

Entropy and Its Variational Principle for Locally Compact Metrizable Systems

Andre Caldas
University of Brasilia (UnB), Brazil
Mauro Patrao

For a given topological dynamical system (X, T) over a compact set X with a metric d, the variational principle states that

$$\sup_{\mu} h_{\mu}(T) = h(T) = h_d(T),$$

where $h_{\mu}(T)$ is the Kolmogorov-Sinai entropy, with the supremum taken over every T-invariant probability measure, $h_d(T)$ is the Bowen entropy, and $h(T)$ is the topological entropy as defined by Adler, Konheim and McAndrew.
In the present work, we extend the definition of topological entropy $h(T)$ for the non compact case in a useful manner. Instead of arbitrary open covers, we restrict our attention to what we have called "admissible covers". Then, we extend the above result to include any continuous map $T : X \to X$, when X is locally compact separable and metrizable. In this case, the variational principle reads

$$\sup_{\mu} h_{\mu}(T) = h(T) = \min_{d} h_d(T),$$

where the minimum is taken over every distance compatible with the topology of X, and is attained when d comes from the one-point compactification of X.

The Periodic Lorentz Gas with Random Billiard Microstructure

Timothy Chumley
Iowa State University, USA

The periodic Lorentz gas model of mathematical physics consists of a point particle moving in space and colliding with a periodic arrangement of fixed rigid scatterers. In this talk we introduce a variation on the periodic, finite horizon Lorentz gas where collisions are not modeled by the usual law of specular reflection, but rather by a novel class of Markov operators P derived to model a microscopic structure on the surface of the scatterers. The resulting Markov chain on the state space of positions and velocities, which we call the periodic Lorentz gas with random billiard microstructure, is shown to be geometrically ergodic with a rate that depends on parameters specified by the microscopic structure. We also show that a similar result holds in the presence of a weak homogeneous electric field, and prove that the resulting steady state velocity of the particle is proportional to the strength of the field in agreement with the classical Ohm's law. Our proof of geometric ergodicity requires the construction of a Lyapunov function, which we accomplish by approximating P by a differential operator whose spectral theory is well understood.

Heat Flow, Entropy Production, and Thermophoresis in Random Billiard Systems

Scott Cook
Texas A&M - Tarleton State University, USA
Renato Feres

We discuss a class of random billiard systems with small scale structure as a framework to study thermodynamics in probabilistic, microscopic models. We first introduce a notion of temperature via a probabilistic reflection law. This allows for heat reservoirs along the boundary and the study of thermodynamical properties like heat flow and entropy production. Though we begin with a simple one dimensional, single particle system, we will focus on three dimensional systems with many interacting particles. Motivated by the study of thermophoresis in aerosols, we distinguish one large Brownian particle and study its drift velocity under various conditions. If time permits, we will discuss techniques for simulating such systems using parallel algorithms on a GPU.

Elliptic Periodic Orbits and Ergodicity of Moon Billiards

Maria Correia
University of Massachusetts Amherst, USA
Hongkun Zhang

We construct a two-parameter family of moon-shaped billiard tables with boundary made of two circular arcs. We analytically study the stability of some periodic orbits and prove there is a class of billiards in this family with elliptic periodic orbits. These moon billiards can be viewed as generalization of annular billiards which all have KAM islands. We also numerically observe a subclass of moon-shaped billiards with a single ergodic component.

The Dynamics of No-Slip Billiards

Chris Cox
Washington University in St. Louis, USA

No-slip billiards are based on a collision model in which small rotating disks may exchange linear and angular momentum at collisions with the boundary, unlike the standard billiard model. We present results on periodicity and boundedness of orbits which demonstrate the marked difference in the dynamics from those of standard billiards. Computer generated phase portraits demonstrate non-ergodic features, suggesting chaotic no-slip billiards cannot readily be constructed using the common techniques for generating chaos in standard billiards.

Parametric Study of Novel Model of Friction

Krzysztof Jankowski
Lodz University of Technology, Poland
Andrzej Stefanski

Friction is one of the most essential factors influencing dynamical response of engineering system. This mechanical phenomenon shows strongly non-linear and complex behavior, taking into consideration interactions on the surfaces and in their strict vicinity. Due to this, friction stands a vital problem for tribologists and engineers, as it leads to the energy loss, wear and deterioration of materials. Much has been done to reduce and overcome its detrimental effects, yet to date no generic solution has been found. The first step, leading to deeper understanding of friction, is modeling. In this paper, we introduce a novel dynamic friction model and perform a detailed parametric study to analyze the influence of friction parameters on the system responses and friction force relations.

General KAM Theorems and Their Applications to Invariant Tori with Prescribed Frequencies

Xu Junxiang
Southeast University, Peoples Rep of China
Xuezhu Lu

In this paper we develop some new KAM technique to prove two general KAM theorems for nearly integrable Hamiltonian systems without assuming any non-degeneracy condition. Many of KAM-type results (including the classical KAM theorem) are special cases of our theorems under some non-degeneracy condition and some smoothness condition. Moreover, we can obtain some interesting results about KAM tori with prescribed frequencies.

A Trichotomy of the Singularities of 2-Dimensional Bounded Invertible Piecewise Isometric Dynamics

Byungik Kahng
University of North Texas at Dallas, USA

The iterative dynamics of planar piecewise isometries is a 2-dimensional analogue of the interval exchange dynamics in 1-dimensional space. Its applications include billiard and dual billiard dynamics, digital signal processing in electric engineering and kicked oscillators in nonlinear physics. The complexity of 2-dimensional piecewise isometric dynamics comes exclusively from the singularity, and therefore, the characterization of the singularity is an important step toward better understanding of the system. We begin our talk with some known results on the classification of the singularities. However, the aforementioned classification is somewhat incomplete in that clear distinctions between some types of the singularities and practical criteria to test them are unavailable. Through this talk, we aim to resolve this difficulty and complete the trichotomy. We also discuss some of the dynamical properties that appear to be related to this trichotomy.

Dynamics of Quasiperiodic Cocycles in $T \times SU(2)$

Nikolaos Karaliolios
UFF, Brazil

We present a full classification of the dynamics of quasiperiodic cocycles in $T \times SU(2)$, under a full measure Diophantine assumption on the frequency of the cocycle. The phenomena encountered comprise unique ergodicity in the space of Distributions, weak mixing, preservation of a, measurable or smoother, foliation in invariant tori, countable Lebesgue spectrum. The proof of the classification uses Renormalization of the Dynamics, methods inspired from K.A.M. theory, and an Anosov-Katok-like construction. Our work follows that of H. Eliasson and R. Krikorian, among others.

On the Reducibility of a Nonlinear Periodic System with Degenerate Equilibrium

Jianli Liang
Huaqiao University, Peoples Rep of China

In this paper, we prove the reducibility of a class of nonlinear periodic differential equation with degenerate equilibrium point under small perturbation, and obtain a periodic solution near the equilibrium point. The result is new and more general, including that of the other paper.

Lyapunov Functions and Strongly Homogeneous Sets for Singular Flows

Luciana Salgado
University of Bahia, Brazil

Consider a compact finite dimensional riemannian manifold M. A set Λ is said to be strongly homogeneous of index I for a C^1 vector field X over M if there exist neighborhoods U of $\Lambda$and $\mathcal{V}$ of X such that all (hyperbolic) periodic orbits in U with respect to any vector field in $\mathcal{V}$ have the same index I. We relate the notions of infinitesimal Lyapunov functions (J-algebra of Potapov) and strongly homogeneous sets for singular flows, and we present some applications of this theory to singular hyperbolicity.

On Reducibility of 2-Dimensional Linear Quasi-Periodic System with Small Parameter

Kun Wang
Southeast University, Peoples Rep of China
Junxiang Xu, Min Zhu

In this paper we consider a real linear analytic quasi-periodic system of 2-dimension, whose coefficient matrix depends on a small parameter C^m-smoothly and closes to constant. Under some non-resonance conditions about the basic frequencies and the eigenvalues of the constant matrix and without any non-degeneracy assumption with respect to the small parameter, we prove that the system is reducible for many of the sufficiently small parameters.

Some Topological Properties and Entropy for Partially Hyperbolic Diffeomorphisms

Yujun Zhu
Hebei Normal University, Peoples Rep of China
Huyi Hu, Lin Wang, Yunhua Zhou

In this talk, some topological properties, such as quasi-stability, quasi-shadowing property, center spectral decomposition and center specification, for partially hyperbolic diffeomorphisms are investigated. As applications, some results on the entropy are obtained for partially hyperbolic diffeomorphisms. The results are from the works joint with Huyi Hu, Lin Wang and Yunhua Zhou.

Special Session 84: Recent Advances and Challenges in Coastal Dynamics

Antoine Rousseau, INRIA, France
Emmanuel Frenod, Université de Bretagne-Sud, France

Coastal areas are more and more threatened by the sea level rise caused by global warning, and yet 60% of the world population lives in a 100km wide coastal strip (80% within 30km in French Brittany). This is why coastlines are concerned with many issues, of various types: economical, ecological, social, political, etc. To address some of these very important questions, this session will cover various modeling issues related to coastal environments, such as coastal oceanography (including wave modeling), morphodynamics or biology of coastal ecosystems. This session will be an opportunity to present a state of the art of various mathematical fields applied to coastal environment (modeling, statistics, scientific computing, numerical analysis, etc.) and to introduce challenges brought by natural and social sciences.

Numerical Simulation for a Two Dimensional Dispersive Shallow Water System.

Nora Aissiouene
Inria Paris, France
Marie-Odile Bristeau, Edwige Godlewski, Jacques Sainte-Marie

We propose a numerical method for a two dimensional dispersive shallow water system with topography. This model is derived from a depth averaged Euler incompressible system with free surface and takes into account a non-hydrostatic pressure which implies to solve an elliptic equation. The contribution of the dispersive terms leads to having a difficult problem which requires to develop new numerical methods. The approach is based on an operator splitting procedure in the spirit of the prediction-correction method initially introduced by Chorin-Temam. The prediction part leads to solving a shallow water system for which we use finite volume methods, while the correction part leads to solving a mixed problem in velocity and pressure. Then, from the variational formulation of the mixed problem proposed, we apply a finite element method with compatible spaces to the two dimensional problem on unstructured grids. Comparisons with analytical solutions and classical test cases are performed to evaluate the efficiency of our method.

Discrete Asymptotic Equations for Long-Wave Propagation

Mathieu Colin
IPB, INRIA CARDAMOM, France
S. Bellec, M. Ricchuito

In this talk, we present a new systematic method to obtain some discrete numerical models for incompressible free-surface flows. The method consists in first discretizing the Euler equations with respect to one variable, keeping the other ones unchanged and then performing an asymptotic analysis on the resulting system. For the sake of simplicity, we choose to illustrate this method in the context of the Peregrine asymptotic regime, that is we propose an alternative numerical scheme for the so-called Peregrine equations. We then study the linear dispersion characteristics of our new scheme and present several numerical experiments.

Multi Scale Approach for Coastal Phenomena

Emmanuel Frenod
Universite Bretagne Sud, France

In this talk, we will present situations in which asymptotic analysis allows us to tackle phenomena arising in coastal areas. In particular, we will consider dune dynamics for which tide is a high frequency phenomenon and confinement in which there are interlocked areas with various scales.

Recent Tsunamis in Chile: Learning from Observations, Hydrodynamics, and Numerical Modeling

Pontificia Universidad Católica de Chile, Chile

Since 2010, three destructive tsunamis have been generated by large subduction earthquakes causing casualties and important economic damages in coastal areas. We have characterized the impact of these events through post tsunami surveys, tidal gage records analysis, and numerical modeling. The application of Nonlinear Shallow Water Equations (NSWE) to model tsunami hydrodynamics and inundation processes has been a valuable source of information, which in combination with field observations and water wave hydrodynamic theories, allowed us to characterize the physics of these events and to understand the importance of the bathymetric control on wave amplification and runups. In this talk, we will summarize this new knowledge, highlighting the usefulness of state-of-the art NSWE models for interpreting tsunami hydrodynamics, and particularly, some results with respect to their validity and performance. The experience gained in recent years through the study of these events has also enabled us to transfer this knowledge into Decision Support Systems (DSS) for tsunami Early Warning Systems (EWS) that are now been tested in the national service in charge of tsunamis in Chile.

A Numerical Study of Nonlinear Wave-Currents Interactions in Shallow Water

Fabien Marche
University of Montpellier and Inria, France

In this talk, we focus on the recently introduced Green-Naghdi equations with vorticity. In a first part, after describing this new set of equations, we study the existence of solitary waves which result from a complex balance between non-linear effects, dispersion and vortic- ity. We highlight some interesting behaviors, like a dissymmetry in the waves'profiles and the occurrence of a critical wave height ruling the existence of the right going wave and of special peaked waves with a singularity at the crest. In a second part, we introduce a new discrete formulation for these equations, based on a hybrid Finite-Volume/Finite-Difference strategy. A new Finite-Volume scheme is developed, based on an approximate Riemann solver of the HLLC type, and a discrete preservation of the set of admissible states is ensured. Some numerical validations are performed, highlight- ing in the meantime some of the special solitary waves exhibited in the first part. In a last part, we describe a new numerical strategy allowing to compute the full velocity field, including its vertical variation, based on an asymptotic description and the introduc- tion of level lines. This is a joined work with D.Lannes.

Towards Coupling Coastal and Large Ocean Models

Antoine Rousseau
Inria Chile, France
Eric Blayo

In this work we are interested in the search of interface conditions to couple hydrostatic and nonhydrostatic ocean models. To this aim, we consider simplified systems and use a time discretization to handle linear equations. We recall the links between the two models (with the particular role of the aspect ratio $\delta = H/L \ll 1$) and introduce an iterative method based on the Schwarz algorithm (widely used in domain decomposition methods).

Special Session 85: Differential Equation Modeling and Analysis for Brain and Other Complex Bio-systems

Jianzhong Su, University of Texas at Arlington, USA
Lixia Duan, North China University of Technology, Peoples Rep of China
Pengcheng Xiao, University of Evansville, USA

Many biological systems, such as neuronal systems, genomic systems, and immune systems, are featured by nonlinear and complex patterns in spatial and temporal dimensions. These phenomena carry significant biological information and regulate down-stream biological mechanisms. Understanding the mechanisms underlying such events by quantitative modeling represents a mathematical challenge of current interest. Yet all these systems share the similar dynamical system issues in ordinary/partial different equation such as bifurcation, stability, oscillations, stochastic noise as well as issues in determining model parameters from experimental data sets and computational errors of the models. This special session offers a forum to exchange the state of the art theoretical advances related to this promising area as well as computational tools. It will foster and encourage communication and interaction between researchers in these directions. The common themes include mathematical models and data analysis, theoretical analysis, computational and statistical methods of dynamical systems and differential equations for the bio-system based models, as well as applications in brain research. The topics may include but not restrict to:

1. Dynamics and computation of neuronal systems
– Modeling and dynamical analysis of biological neurons and neuronal networks, – Generation, encoding and transduction of neuronal signals and patterns. – Modeling and analysis of cognitive information processing mechanisms – Dynamic abnormality in neruronal systems due to diseases.

2. Dynamics of immune systems
– Modeling biomedical processes, including tumor growth, cardio-vascular diseases, infection, and healing, mediated by immunologic mechanisms. – Analysis of mathematical models for dynamics features such as instabilities, bifurcations that provide insight into the nature of the underlying bio-physical mechanisms. – Modeling wound healing and inflammatory responses, including cell to cell interactions, foreign body reactions and quantitative as well as qualitative comparison with experimental data.

3. Data analysis and modeling of brain activities
– Complexity theory applied to brain – Perception, learning and memory functions in brain. – Computational evolutionary biology. – Models, analysis and algorithms in Bioinformatics.

Stability and Bifurcations of Rhythms in Neuronal Circuits

Deniz Alacam
Georgia State University, USA
Jarod Collens, Aaron Kelley, Krishna Pusuluri, Drake Knapper, Justus Schawabedal

We are interested in exploring repetitive dynamics generated by constituent building blocks, or “motifs“ that make up more complex central pattern generator (CPG) circuits, and the dynamic principles underlying more general multi-stable rhythmic patterns. We consider basic 3-cell motifs, as well as biologically plausible circuits determining locomotion behaviors in sea slugs. The roles of asymmetric and unique connections, and the intrinsic properties of their associated cells in generating a set of coexisting synchronous patterns of bursting are studied. The particular kinds of network structures reflecting the known physiology of various CPG networks in real animals are described. We characterize how observed multi-stable states arise from coupling, and how real circuits may take advantage of multi-stability to dynamically switch between the corresponding polyrhythmic outputs.

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Using a Mathematical Model to Assess the Roles of T Cells and Cytokines in Transplant Rejection

Julia Arciero
Indiana University-Purdue University Indianapolis, USA
Anirudh Arun, Andrew Maturo, Giorgio Raimondi

Organ transplantation is a life-saving surgical procedure through which the functionality of a failing organ system can be restored. However, without the life-long administration of immunosuppressive drugs, the recipient's immune system will launch a massive immune attack that will ultimately destroy the graft. Long-term use of immunosuppressive drugs leads to an increased risk of infection, cardiovascular disease, and cancer, and thus there is currently a great medical need to identify new strategies of intervention to induce transplant acceptance while preserving the functionality of the immune system. This study introduces an experimentally-based mathematical model to examine the complexities of the dynamic interactions between key elements of the immune system and the transplant and to predict how alterations in the immune response influence the rejection of the transplant. The assumptions of the ODE model are based on mouse models of heart transplant rejection. The model predicts that decreasing the translocation rate of effector cells from the lymph node to the graft generally delays transplant rejection. Adoptive transfer of naive regulatory T cells also delays rejection. Ultimately, this model will be used to identify new strategies that preserve the protective role of the immune response while maintaining the functional role of the transplant.

Chemical and Mechanical Mechanisms Making Arterial Plaques Vulnerable to Rupture

Jonathan Bell
UMBC, USA
Animikh Biswas

Atherosclerosis is an inflammatory disease leading to high risk arterial plaques. A high risk plaque is vulnerable to rupture, causing a stroke, a heart attack, or liver damage, depending on the plaque's location. Most arterial plaques are stable, that is, are not high risk. Our interest is why, mechanistically, some plaques become vulnerable, while others remain stable. We first model the cellular and chemical dynamics in a maturing plaque, where a fibrous cap is developing and chemotaxis plays a significant role. We explain cross-chemotaxis, presenting some theory and simulations. As time permits, we then briefly discuss the role of blood shear stress on the endothelial cell layer, its effects on the chemical pathways within the endothelial cells, and how to incorporate this mechanism into our plaque model.

Volume Transmission and Homeostasis of Neurotransmitters

Janet Best
Ohio State University, USA
Mike Reed, Fred Nijhout, Sean Lawley

In volume transmission, neurons in one brain nucleus send their axons to a second nucleus where neurotransmitter is released into the extracellular space, modulating the activity of the electrophysiological circuitry. Here we present a number of mathematical issues that arise in calculating the concentration of neurotransmitter in the extracellular space and its homeostatic regulation.

The Effects of Sodium on the Bursting Transitions in the Pre-Botzinger Complex

Lixia Duan
North China University of Technology, Peoples Rep of China
Xi Chen, Qishao Lu, Mingjun Ji

Activity of neurons in the pre-Bötzinger complex within the mammalian brain stem has an important role in the generation of respiratory rhythms. Neurons within the pre-Bötzinger complex have been found experimentally to yields typical bursting activities. Previous experimental results have shown that the dynamics of sodium and calcium within each cell may be responsible for various bursting mechanisms. In this talk, we study the bursting activities related to the respiratory rhythms in the pre-Bötzinger complex based on a mathematical model proposed by Butera. Using the one-dimensional first recurrence map induced by dynamics, we investigate the different bursting patterns and their transition of the pre-Bötzinger complex neurons based on the Butera model. After we derived a one-dimensional map from the dynamical characters of the differential equations, and we obtained analytical conditions for the transition of different bursting patterns. These analytical results were verified through numerically simulations. We conclude that the one-dimensional map contains similar rhythmic patterns as the Butera model and can be used as a simpler modeling tool to study fast-slow models like pre-Bötzinger complex neural circuit.

Rhythmic Oscillations of Unweighted and Weighted Bursting Hodkin-Huxley Neuronal Networks

Fang Han
Donghua University, Peoples Rep of China
Qi Shi, Zhijie Wang

Rhythmic oscillations of neuronal networks are actually synchronous behaviors, which play an important role in neural systems. In this paper, the properties of oscillation frequency of unweighted and weighted bursting Hodkin-Huxley neuronal networks are stud-

ied, respectively. For the unweighted neuronal network, neurons are coupled by inhibitory synapses and it is found that the oscillation frequency of the whole network is sensitive to the parameter values of synaptic delay and synaptic decay time, but is less sensitive to the parameter value of synaptic conductance. For the weighted network, neurons are coupled by excitatory synapses with a synaptic learning rule applied, and it is found that the oscillation frequency of the network decreases monotonically along with the increase of the synaptic learning rate, the coupling strength, the delay time and the decay time constant.

Evaluation of Bifurcation Phenomena in a Modified Shen-Larter Model for Intracellular Ca2+ Bursting Oscillations

Quanbao Ji
Huainan Normal University, Peoples Rep of China
Yi Zhou, Zhuoqin Yang, Xiangying Meng

The present work describes an evaluation of the bifurcation phenomena in a modified Shen-Larter model based on calcium-induced calcium release and inositol triphosphate crosscoupling for calcium ion (Ca^{2+}) bursting oscillations. A time delay for negative Ca^{2+} feedback on the inositol triphosphate (IP_3) receptor is added to the original Shen-Larter model, by introducing the proportion of receptors not inactivated by Ca^{2+} as a new variable. Compared with the original model, the number of chaotic regions for a stimulation level r is significantly reduced, and regions of Ca^{2+} oscillations (particularly bursting) appear to become slightly enlarged. In addition to the free Ca^{2+} concentration in the endoplasmic reticulum, the IP_3 concentration in the cytosol is also considered as a slow variable. Different topological types of bursting oscillations in this modified model are presented and classified, based on fast/slow dynamical analysis and codimension-2 bifurcations. Furthermore, classification and transition mechanisms of Ca^{2+} oscillations could help to understand or detect more distinctive oscillatory behaviors of real cells in response to different levels of stimulation.

Spatiotemporal Patterns in Energy-Efficient Cortical Hodgkin-Huxley Network Model

Jiajia Li
Xi'an Jiaotong University, Peoples Rep of China
Ying Wu

Studies of electrical activities in neurons got a breakthrough when the Hodgkin-Huxley neuron model was established in 1952. However, the model used squid giant axon, which cannot adapt to the mammal cortical neuron with stable temperature about 37 celsius degree. In recent studies, there has been a cortical Hodgkin-Huxley model of homeothermal mammals established, which successfully includes mammal's energy-efficient properties when single cortical neuron cell was considered. But the population electrical activities of this type neuron model have been studied sparsely. Here, based on this model of a two-dimensional regular neuron network, we studied the formation of special spatiotemporal patterns in the network, target and spiral waves, and their interconversion due to temperature changes.

A Concise and Exhaustive ODE Model for Characterizing the Ca^{2+} Channel/BKCa Channel Complex at Local and Whole-Cell Levels

Francesco Montefusco
University of Padova, Italy
Morten G. Pedersen

Large-conductance Ca^{2+} -activated K^+ channels (BKCa channels) are co-localized with voltage-gated Ca^{2+} channels (CaV), and are thus regulated by local Ca^{2+} levels. First, we show that an ODE model of single-channel gating with two states (closed and open) is able to reproduce the BKCa channel characteristics and dynamics by fitting experimental data. Then, we propose a concise ODE model of the CaV/BKCa channel complex by coupling the two state model for the BKCa channel with a three state (closed, open and inactivated) model for the Ca^{2+} channel. We also perform Monte Carlo simulations for the devised complex in order to model its stochastic gating; our ODE model is able to estimate the number of open BKCa channels as function of the amplitude and duration of the action potential, thus providing an analytical explanation for the Monte Carlo results. Finally, we show that it is possible to reduce the ODE model of the developed CaV/BKCa complex by exploiting an approximated one state model for the activation of the CaV/BKCa complex and coupling it with the inactivation of the Ca^{2+} channel. This approximated model can provide insights into the main features of the complex and is easily used for whole-cell models.

Bursting and Synchronization in a Two-Compartment Model with Current Feedback Control

Meng Pan
Guangdong Pharmaceutical University, Peoples Rep of China
Quanbao Ji, Haixia Wang, Qishao Lu

We investigate bursting patterns in the single modified two-compartment model and synchronization transition path of two coupled bursters. It is found that with proper parameters, the single cell can produce two type of bursting, that is, "sub-Hopf/homoclinic" via "fold/homoclinic" hysteresis loop and "circle/fold cycle" via "circle/subHopf" hysteresis loop. By coupling two bursters electrically, a transition process from non-synchronization to burst synchronization, then to spike synchronization, and finally toward nearly complete synchronization is ob-

served with the increase of the coupling strength. Moreover, when two neurons achieve nearly complete synchronization, it is discovered that the fast variables are easier to get synchronization than the slow variables, which is opposite to the ordinary situation. The reason is worthy of further research. In addition, the generation mechanism of nearly complete synchronization phenomenon is given by extending fast-slow analysis from the single to the coupled system. The results can help us better understand the synchronization dynamics in the coupled system with multi-time-scale. It is known that synchronization plays an important role in the transfer of information, thus our results have theoretical and physiological significance.

A Mathematical Model of Hematopoietic Stem Cell Treatments in Patients with Lymphoma

Angela Reynolds
Virginia Commonwealth University, USA
Racheal Cooper, AllisonScalora, Elaine Wang, John McCarty, Jennifer Anderson, Catherine Roberts, Troy Lund, Amir Toor

Hematopoietic stem cells (HSCs) present in the bone marrow are responsible for maintaining peripheral blood cell counts. Patients may need HSCs mobilized and collected for transplantation after high dose chemotherapy. Additionally, healthy individuals can donate HSCs. There is considerable variability in the yield in patients and normal donors despite using the same collection protocol. Understanding the mechanisms governing HSC growth will allow optimization of this treatment. To explore these mechanisms, we created a mathematical model, which accounts for HSC proliferation and most of the cell types that differentiated from HSCs in both the bone marrow and peripheral blood. Also, we model treatment with Filgrastim and HSC collection. The effect of the drug Plerixafor was also simulated in individuals with poor collection.

Using Optimal Control Theory to Analyze the Treatment of a Bacterial Infection in a Wound with Oxygen Therapy

Richard Schugart
Western Kentucky University, USA
Suzanne Lenhart, K. Renee Fister, Stephen Guffey

A mathematical model describing the interactions of bacteria, inflammatory cells, and oxygen in a wound describing the treatment of a bacterial infection using oxygen therapy will be presented and analyzed. A second variation of the model will be presented in an optimal control setting with the control variable being the input of supplemental oxygen. Numerical results of the optimal control model will be presented and future directions will be discussed.

Interaction Function of Coupled Bursting Neurons

Xia Shi
Beijing University of Posts and Telecommunications, Peoples Rep of China
Jiadong Zhang

The interaction functions of electrically coupled Hindmarsh-Rose (HR) neurons for different firing patterns are investigated in this paper. By applying the phase reduction technique, the phase response curve (PRC) of the spiking neuron and burst phase response curve (BPRC) of the bursting neuron are derived. Then the interaction function of two coupled neurons can be calculated numerically according to the PRC (or BPRC) and the voltage time course of the neuron. Results show that the BPRC is more and more complicated with the increase of the spiking number within a burst, and the curve of the interaction function oscillates more and more frequently with it. However, two certain things are unchanged: $\phi = 0$, which corresponds to the in-phase synchronization state, is always the stable equilibrium, while the anti-phase synchronization state with $\phi = 0.5$ is an unstable equilibrium.

Influence of Coupling on Oscillatory Properties of Bursting Solutions in Neuron Models

Jianzhong Su
University of Texas - Arlington, USA
Alice Lubbe

Neurons often exhibit bursting oscillations, as a mechanism to modulate and set pace for other brain functionalities. These bursting oscillations are distinctly characterized by a silent phase of slowly evolving steady states and an active phase of rapid firing oscillations. These bursting neurons are modeled by fast-slow systems consisting of several ordinary differential equations. In a network of neurons, their collective oscillatory behavior may differ from individual neurons due to coupling and inputs from other neurons. We analyze the transition mechanisms between periodic and chaotic/random behavior in a coupled system of neurons. Using geometric and bifurcation analysis, we provide insight how coupling can regularize chaotic trajectories using the flow induced Poincare map.

Fast Regular Firing Induced by Inter and Intra Time Delays in Two Clustered Neuronal Networks

Xiaojuan Sun
Beijing University of Posts and Communications, Peoples Rep of China

In this paper, we consider two clustered neuronal networks with dense intra synaptic links within each cluster and sparse inter synaptic links between them. We focus on the effects of intra and inter time delays on the spiking regularity and timing in the two clusters. With the aid of simulation results, we show that intermediate intra and inter time delays are able to separately induce fast regular firing spiking activity with a high firing rate as well as a high spiking regularity. Moreover, when both intra and inter time delays are present, we find that fast regular firings are induced much more frequently than if only a single type of delay is present in the system. The presented results indicate that appropriately adjusted inter and intra time delays can significantly facilitate fast regular firing in neuronal networks. Based on a detailed analysis, we conjecture that this is most likely when the greatest common divisor of the intra and inter time delay falls into a range where fast regular firings are induced by suitable intra or inter time delays alone.

Modeling of Immune Response to Viral and Bacterial Infections

David Swigon
University of Pittsburgh, USA
E. Mochan, B. Ermentrout, G. Clermont

Immune response to responds to infection begins by initiating a spectrum of immune responses regulated by an intricate network of signaling interactions that have not yet been completely characterized. In the talk I will outline a series of models that provide qualitative and quantitative prediction of the time course of a disease, aid in understanding of the mechanisms of the immune response, and can be utilized in the study the effects of an anti-viral or anti-bacterial drug treatment. Our latest effort has been focused on ensemble models that reflect the uncertainty about parameter values, data sparseness, and the likely variation of the disease outcome across a population. The technique is useful when the model contains many unknown parameters, such as reaction rate constants of biochemical processes, which are poorly constrained and their direct measurement in vivo is not feasible.

Map Reduction of a 3D Hodgkin-Huxley Model

Kelly Toppin
Drexel University, USA
Dennis Yang, Yixin Guo

We study the dynamics of a 3D Hodgkin-Huxley model for neurons in the basal ganglia. The model switches between two different states that represent neurons with or without external input. To investigate how these neurons respond to input signals, we develop a projection method to reduce the 3D Hodgkin-Huxley model to a 2D map. Since each of the two different states has a stable limit cycle with weak and strong stable bundles, we project trajectories onto a weak stable manifold of the limit cycle along an invariant fiber bundle over the manifold, reducing the dynamics to a rotation along the limit cycle and an exponential decay toward the limit cycle. By this projection, we reduce the 3D Poincarѐ map of the Hodgkin-Huxley system to a 2D map. Then we analyze the existence and bifurcations of periodic orbits of the 2D map as parameters vary.

An Immuno-Chemotherapy Against Colorectal Cancer: Modeling and Analysis

Qing Wang
Shepherd University, USA
Zhijun Wang, David J. Klinke

Recently, a chemotherapy agent oxaliplatin (OXP) in combination with interleukin-12 (IL-12) was used to eliminate pre-existing liver metastatic colorectal cancer in a murine model. We developed a multi-scale impulsive ODE model to describe the interaction between the immune system and tumor in response to the combined IL-12 and OXP therapy. Model parameters were calibrated to published experimental data using a genetic algorithm. Sensitivity of parameters, local stability analysis, and treatment strategies to control tumor growth were discussed. This research was supported by the NIGMS of the NIH grant as part of the WV-INBRE (P20GM103434).

Applications of Ordinal Longitudinal Data Analysis with Multiple Predictors

Xiaohui Wang
University of Texas-Rio Grande Valley, USA

Ordinal longitudinal data is often seen in behavioral sciences, public health and medical studies. For example, in Alliance for a Healthy Border program (2006-2008), a chronic disease prevention program through twelve federally qualified community health centers serving primarily Hispanics in communities along the U.S.-Mexico border, survey and health measurements were obtained at three time points according to the pre-post-post study design. Successes of the program were evaluated with dichotomous or trichotomous ordinal outcomes of weight reduction, glycemic control, and physical activity improvement. In this study, we apply several modeling methods to evaluate program successes that are ordinal longitudinal data in nature. The ordinal response modeling methods include generalized estimating equation (GEE) approach for cumulative logit models and transitional ordinal modeling with multiple predictors.

Extending Levelt's Propositions to Perceptual Multistability Involving Interocular Grouping

Yunjiao Wang
Texas Southern University, USA
Alain Jacot-Guillarmod, Yunjiao Wang, Claudia Pedroza, Haluk Ogmen, Kresimir Josic, Zachary Kilpatrick

Levelt's Propositions have been a touchstone for experimental and modeling studies of perceptual multistability. We asked whether Levelt's Propositions extend to perceptual multistability involving interocular grouping. To address this question we used split-grating stimuli with complementary halves of the same color. As in previous studies, subjects reported four percepts in alternation: the two stimuli presented to each eye, as well as two interocularly grouped, single color percepts. Most subjects responded to increased color saturation by more frequently reporting a single color image, thus increasing the predominance of grouped percepts (Levelt's Proposition I). In these subjects increased predominance was due to a decrease in the average dominance duration of single-eye percepts, while that of grouped percepts remained largely unaffected. This is in accordance with Levelt's Proposition II, which posits that the average dominance duration of the stronger (in this case single-eye) percept is primarily affected by changes in stimulus strength. To explain these observations, we introduced a hierarchical model consisting of four low-level neural populations responding to input to each visual hemifield, and higher-level populations representing the four percepts. The model explains perceptual multistability involving interocular grouping via competition across multiple visual system layers.

Bifurcation Analysis in the Hypothalamic-Pituitary-Adrenal Axis Including Gulcocorticoid Receptor Complex

Pengcheng Xiao
University of Evansville, USA
Jianzhong Su

The hypothalamic-pituiary-adrenal (HPA) axis plays an important role in the regulation of neuroendocrine and sympathetic nervous systems. Emerging evidence has shown that glucocorticoid act on glutamate neuro-transmission system and consequently influences neuronal activities cognitive function. In this paper, we numerically derive the two parameter bifurcation analysis on one HPA model including Gulcocorticoid Receptor (GR) to explore the glucocorticoid bistability.

Moment-Flux Models for Bacterial Chemotaxis in Large Signal Gradients

Chuan Xue
Ohio State University, USA
Xige Yang

Chemotaxis is a fundamental process in the life of many prokaryotic and eukaryotic cells. Chemotaxis of bacterial populations has been modeled by both individual-based stochastic models that take into account the biochemistry of intracellular signaling, and continuum PDE models that track the evolution of the cell density in space and time. Continuum models have been derived from individual-based models that describe intracellular signaling by a system of ODEs. The derivations rely on quasi-steady state approximations of the internal ODE system. While this assumption is valid if cell movement is subject to slowly changing signals, it is violated if cells are exposed to rapidly changing signals. In the latter case current continuum models break down and do not match the underlying individual-based model quantitatively. We derive new PDE models for bacterial chemotaxis in large signal gradients that involve not only the cell density and flux, but also moments of the intracellular signals as a measure of the deviation of cell's internal state from its steady state. The derivation is based on a new moment closure method without calling the quasi-steady state assumption of intracellular signaling. Numerical simulations suggest that the resulting model matches the population dynamics quantitatively for a much larger range of signals.

Stochasticity and Bifurcations in a Reduced Model with Interlinked Positive and Negative Feedback Loops of CREB1 and CREB2 Stimulated by 5-HT

Zhuoqin Yang
Beihang University, Peoples Rep of China

The cyclic AMP (cAMP) response element binding protein (CREB) family of transcription factors is crucial in regulating gene expression required for long-term memory (LTM) formation. Song et al. proposed a minimal model with only interlinked positive and negative feedback loops of transcriptional regulation by the activator CREB1 and the repressor CREB2. Without considering feedbacks between the CREB proteins, Pettigrew et al. developed a computational model characterizing complex dynamics of biochemical pathways downstream of 5HT receptors. In this work, to describe more simply the biochemical pathways and gene regulation underlying

5HT induced LTM, we add the important extracellular sensitizing stimulus 5HT as well as the product Apuch into the minimal model. Different dynamics including monostability, bistability and multistability is investigated by means of codimension-2 bifurcation analysis. Comparative analysis of deterministic and stochastic dynamics reveals diverse stochastic behaviors resulted from the finite number of molecules.

Modeling and Analysis of Neonatal Seizures with Synaptic Plasticity for Real Infants' EEG Signals

Honghui Zhang
Northwestern Polytechenical University, Peoples Rep of China
Jianzhong Su

This paper aims to explore the internal dynamical mechanisms of seizure through modeling and analysis of EEG signals, which can help to do early seizure detection and treatment for epileptic patients. We studied the nonlinear dynamics of neonatal seizures by a mathematical model established with synaptic plasticity considered. First, according to several neonatal electroencephalogram with epilepsy, we present a dynamics seizure model that accounts for the basic experimental observations of neonatal seizure motivated by previous model. The great ability of the proposed model to produce qualitatively relevant behaviors was shown by numerical simulations. Meanwhile, the rationality of the connecting structure hypothesis in the modeling process was verified. Further, through adjusting the threshold condition and excitation strength of synaptic plasticity, we elucidate the effect of synaptic plasticity on neonatal seizure. Our results show that synaptic plasticity has great effect on the duration of seizure activities, which can support the hormonal therapy of synaptic plasticity for seizure control.

Special Session 86: Pattern Formation and Recognition in Structured Information and Biological Systems with External Forcing

Jianhong Wu, York University, Canada
Kunquan Lan, Ryerson University, Canada
Xi Huo, Ryerson University, Canada

This session will bring together participants working on nonlinear dynamics using differential equations and real data to detect and describe spatio-temporal patterns in dynamical systems from life sciences and information management. Emphasis will be on how structural complexity and external forcing including harvesting impact system dynamics, and how complex data and parameters can be summarized via relative simple indices, and how these are relevant to bifurcation and scenario analysis and optimal control.

Absenteeism, Presenteeism and Infectious Diseases in a Local Economy

Monica Cojocaru
University of Guelph, Guelph ON, Canada
E. Thommes, S. Athar

In this paper we study the cost of absenteeism and presenteeism (going to work while sick) during a pandemic in a local economy with several geographically distinct locations, and with work force populations consisting of individuals who live and work in the same city, and individuals who live and work in different locations (daily commuters). We run simulations to study the effects of the fear factor and of the severity of disease on the number of missed work days in the region, which we translate into loss of productivity costs. We find that higher values of the fear parameter lead to high absenteeism and lower infection levels. However, , we also show that for severe pandemics (such as when the number of secondary infections is higher) there are scenarios where there exists a unique value of the fear parameter which leads to minimum economic costs for the regional economy. This indicates that "staying at home" policies during an epidemic could be implemented for the work force, without reaching a state of emergency.

Instability and Non-Synchronization in Delayed Differential Equations Induced by Using Fast and Random Switching

Yao Guo
York University, Canada
Wei Lin, Yuming Chen, Jianhong Wu

This talk first presents a counterintuitive example where even though all the time-delayed subsystems are exponentially stable, the behaviours of the randomly switched system change from stable dynamics to unstable dynamics with a decrease of the dwell time. Then by using the theories of stochastic processes and delay differential equations, we present a general theoretical result on when the fast and random switching induced instability should occur. And we extend this to the case of nonlinear time-delayed switched systems, which is eventually used to deal with the non-synchronization problem of switched networks. Many simulations are given to illustrate our theory as well. In addition, further numerical simulations also show that even switched systems without time-delays can be destabilized or desynchronized by using fast and random switching.

Linear Stability of Delayed Reaction-Diffusion Systems

Peter Hinow
University of Wisconsin - Milwaukee, USA
Maya Mincheva

A common feature of pattern formation in both space and time is the destabilization of a stable equilibrium solution of an ordinary differential equation by adding diffusion, delay, or both. Here we study linear stability of general reaction-diffusion systems with off-diagonal time delays. We show that a delay-stable system cannot be destabilized by diffusion, and that a diffusion stable system is also stable with respect to delay, if the diffusion is sufficiently fast. A system with direct negative feedback which is strongly stable with respect to diffusion can be destabilized by off-diagonal delay.

We acknowledge support from the Simons Foundation through the grant "Collaboration on Mathematical Biology" to Peter Hinow.

Application of Lasalle's Invariance Principle in a 3-Dimensional Delay Differential Equation System

Xi Huo
Ryerson University, Canada
Xiaodan Sun, Yanni Xiao, Kunquan Lan, Jianhong Wu

We will present our mathematical results, proofs, and open problems arise from the convalescent blood transfusion system. This system models the donation, treatment, and storage dynamics for large-scale blood therapy - an issue originated from the 2014-2015 West Africa Ebola outbreaks.

Identification of Time Delay and Its Role in Biological Dynamical Systems

Wei Lin
Fudan University, Peoples Rep of China

Time delays are omnipresently observed in many nature and artificial systems including physical, biological, and chemical systems. Naturally, two kinds of questions arise: "How to identify the time delays when a certain amount of datasets are obtained from the experiments or real world systems" and "How to characterize the intrinsic roles of time delays that are played in coupled network systems?" In this talk, we will introduce recent works that address the previous two questions, and show the significance of time delays in dealing with various representative systems of biological significance.

The Dynamics of HIV Spreading in the Network of Lymphocyte Recirculation in Vivo

Jie Lou
Shanghai University, Peoples Rep of China
Ying Huang

Although antiretroviral therapy (ART) can effectively inhibit replication of human immunodeficiency virus (HIV), the virus is able to persist in cellular and anatomical viral reservoirs. The aim of this model is to gain better understanding of HIV persistence, in regards to the lymphocyte recirculation network of immune system as well as the central nervous system. Our findings probably can reflect the inability of some drugs to penetrate the blood-brain (or blood-testis) barrier and emphasize the ability of an infected individual to transmit HIV even under the ART. We also find that level of HIV free virus in peripheral blood may not give a correct reflection of what is occurring within other organs in vivo.

Detecting Unknown Unstable States in High-Dimensional Nonlinear Dynamical Systems

Huanfei Ma
Soochow University, Peoples Rep of China

There are various kinds of unstable states in nonlinear dynamical systems, such as unstable periodic orbits (UPOs) and unstable steady states (USSs). Detecting such unstable states of a high-dimensional nonlinear dynamical systems is generally believed to be a challenging problem. Particularly, when the unstable states are unknown, the detecting or control methods should be designed in a reference-free way. We propose some non-invasive and adaptive methods to deal with such problems, without requiring any a priori information of the unstable states. Moreover, we will further discuss how to detect UPOs and USSs of a system only with the output time series.

Global Analysis on Viral Dynamics

Hongying Shu
Tongji University, Peoples Rep of China
Lin Wang, James Watmough

Determining sharp conditions for the global stability of equilibria remains one of the most challenging problems in the analysis of models for the management and control of biological systems. Yet such results are necessary for derivation of parameter thresholds for eradication of pests or clearing infections. This applies particularly to models involving nonlinearity and delays. In this talk, we provide some general results applicable to immune system dynamics. This general model admits three types of equilibria: infection-free equilibria, CTL-inactivated infection equilibria, and CTL-activated infection equilibria, and two critical values: the basic reproduction number for viral infection and the viral reproduction number at the CTL-inactivated infection equilibrium. Our results cover and improve many existing ones and include the case when the nonlinear functions are nonmonotone.

Using a System of Reaction-Diffusion Equations with Nonlocal Effects to Model Spatial Patterning in the York River Tidal marshes

Sofya Zaytseva
The College of William and Mary, USA
Leah Shaw, Junping Shi, Rom Lipcius

Spatial patterning in multi-species communities can be critical to ensuring their proper function and survival. Therefore, studying the formation of self-organized patterns in ecology is crucial for understanding the underlying interactions in the community and its ability to adapt to various environmental changes. A pattern of finger-like projections has recently been observed on the shore of the York River for salt marsh cordgrass, mussels and sediment. We propose a system of reaction-diffusion equations with nonlocal effects to explain the formation of this pattern through interactions between grass, mussels and sediment. The nonlocal effects are modeled through an influence kernel, specifying the strength of interactions between individuals as a function of the distance between them. We numerically integrate the PDE model in MATLAB and find that the model displays stable patterns reminiscent of those observed in the field. To achieve a better understanding of the underlying dynamics, we analyze the full system as well as the corresponding grass-sediment subsystem. We investigate the stability of various steady states, the bifurcations they undergo and their ecological interpretation.

Special Session 87: Direct, Inverse and Control Problems for Differential Equations

Angelo Favini, Università di Bologna, Italy
Davide Guidetti, Università di Bologna, Italy

New results on direct , inverse and control problems related to abstract differential equations and their applications to PDEs are concerned. Particular attention is devoted to recent developments on Sobolev type equations.

Inverse Problems for Neuronal Cable Models on Graphs

Jonathan Bell
UMBC, USA
Sergei Avdonin

For a parabolic equation defined on a tree graph domain, a dynamic Neumann-to-Dirichlet map associated with the boundary vertices can be used to recover the topology of the graph, length of the edges, and unknown coefficients and source terms in the equation. The motivation for this investigation is that the parabolic equation comes from a neuronal cable equation defined on the dendritic tree of a neuron, and the inverse problem concerns parameter identification of k unknown distributed conductance parameters.

Optimal Control Problems for Leontief Type Systems: Numerical Methods and Applications

Alevtina Keller
South Ural State University, Russia
Aleksandr Shestakov

The report provides an overview of results of a numerical research of optimal control problems for the Leontief type systems. Such systems arise in modelling of different processes and objects, for example measuring transducers, economic systems of an enterprise, and dynamics of a cell cycle. Leontief type system is a finite-dimensional analogue of the Sobolev type equation therefore our research is based on the methods of the theory of degenerate groups of operators. The report presents an algorithm for finding of approximate solutions to a variety of optimal control problems for Leontief type systems with the Showalter - Sidorov initial condition which is more convenient in numerical research. The proof of the convergence of the approximate solutions to the precise one is an important result. The issues of improving of the efficiency of numerical algorithms and their modifications in the numerical study of applications are discussed. Special attention is given to the numerical algorithms for solving of the optimal measurement problems which are the problems of restoration of signals dynamically distorted both by inertia of the measuring device, and resonances in its circuits. The results of computational experiments are presented.

Analytical and Numerical Investigations of the Optimal Control Problem for Semilinear Sobolev Type Equations

Natalia Manakova
South Ural State University, Russia

Generally, the processes applied in mechanics, engineering and production are controllable, therefore, within respective applied problems it is usually essential to control the external actions efficient enough to achieve required results in such processes. Despite the fact that the research field of optimal control problems for distributed systems is rather large, the solutions control matters for confluent semi-linear equations, unresolved for derivative with time, are studied insufficiently. Such equations are called semilinear Sobolev type equations. Author proposed sufficient conditions for the existence of a solution of the optimal control problems for semilinear Sobolev type equations with s-monotone and p-coercive operators. Theorems of existence and uniqueness of weak generalized solution to the Cauchy or the Showalter - Sidorov problem for a class of degenerate non-classical equations of mathematical physics are stated. The theory is based on the phase space and the Galerkin methods. The developed scheme of a numerical method allows one to find an approximate solution to optimal control problems for considered models. On the basis of abstract results the existence of optimal control of processes of filtration and deformation are obtained. The necessary conditions for optimal control are provided.

On the Solvability of Degenerate Operator-Differential Equations of the First Order in the Spaces of Differentiable Noises

Minzilia Sagadeeva
South Ural State University, Russia

The equations unsolved with respect to the derivative was began to study systematically in the middle of the last century. Often such equations are called the Sobolev type equations. The report examines the solvability of such equations in the spaces of "noises". These specific spaces different from the standard spaces of random processes by that the derivative of the process can be determined in these spaces. Namely, the derivative of the process is defined as a Nelson - Gliklih derivative.

The concepts previously introduced for the spaces of differentiable real-valued "noises" using the Nelson - Gliklikh derivative are carried over to the case of complex-valued "noises". We consider the Sobolev type equations in the spaces of "noises" on the condition that a degenerate resolving semigroup of class C0 exists. So we consider the relatively radial operators in spaces of complex-valued "noises". We construct a solution to the weakened Showalter - Sidorov problem for Sobolev type equation with relatively p-radial operator in a space of complex-valued processes. We study the solvability of Chen - Gurtin in spaces of complex-valued "noises" as the application of the main results of this report.

The Sobolev Type Equations and Degenerate Operator (Semi)groups. Theory and Applications

Georgy Sviridyuk
South Ural State University, Russia

Sobolev type equations were firstly studied in the works of A. Poincare. Then they appeared in the works of S.V. Oseen, J.V. Boussinesq, S.G. Rossby and other researchers, that were dedicated to the investigation of some hydrodynamics problems. Their systematical study started in the middle of the XX century with the works of S.L. Sobolev. The first monograph devoted to the study of such equations appeared in 1999. Nowadays the number of works devoted to such equations is increasing extensively. Sometimes such equations are called equations that are not of Cauchy - Kovalevskaya type, pseudoparabolic equations, degenerate equations or equations unsolvable with respect to the highest derivative. The term Sobolev type equations was firstly proposed in the works of R. Showalter. Nowadays Sobolev type equations constitute the vast area in nonclassical equations of mathematical physics. The proposed theory of degenerate semigroups of operators is a suitable mathematical tool for the study of such equations. The theory is developing in different directions: optimal control problems, initial-final value problems, equations of high order, and finds applications in elasticity theory, fluid dynamics, oil production, economics, biology, and in the solution of many technical problems, for example in the theory of dynamic measurements.

The Multipoint Initial-Final Value Problem for Sobolev-Type Equations

Sophiya Zagrebina
South Ural State University, Russia

The models of mathematical physics, whose representation in the form of equations or systems of partial differential equations do not fit one of the classical types such as elliptic, parabolic or hyperbolic, are called nonclassical and can be regarded as Sobolev-type equations. The report provides an overview of the author's results in the field of nonclassical determinate and stochastic equations of mathematical physics for which the initial-final value problems, generalizing the Cauchy and Showalter - Sidorov conditions, are considered. Basic method for the research is the Sviridyuk relative spectrum theory. In addition, we use a generalized theorem of splitting of the space and operators actions in the case of relatively bounded operator. Abstract results are illustrated by the specific initial-final value problem for the equations in partial derivatives occurring in applications, namely, the filtration theory, fluid dynamics theory and deformation theory, considered on the sets of different geometrical structure.

The Sobolev Type Equations of Higher Order

Alyona Zamyshlyaeva
South Ural State University, Russia

This report surveys the author's results concerning mathematical models based on Sobolev-type equations of higher order. The theory is constructed using the available facts on the solvability of initial (initial-final) problems for the first-order Sobolev-type equations. The main idea is a generalization of the theory of degenerate (semi)groups of operators to the case of higher-order equations: decomposition of spaces and actions of the operators, construction of the propagators and the phase space for the homogeneous equation, as well as the set of valid initial values for the inhomogeneous equation. We use the phase space method, which is quite useful for solving the Sobolev-type equations and consists in a reduction of a singular equation to a regular one defined on a certain subspace of the original space. We reduce mathematical models to initial (initial-final) problems for abstract Sobolev-type equations of higher order. The results may find further applications in the study of optimal control problems, nonlinear mathematical models, and in the construction of the theory of Sobolev-type equations of higher order in quasi-Banach spaces and stochastic spaces of noises.

Special Session 88: Data Assimilation and Nonlinear Filtering

Kody Law, Oak Ridge National Laboratory, USA
David Kelly, Courant Institute of Mathematical Sciences, USA

Filtering describes the solution of a sequence inverse problems, in which the data arrives in an online fashion. The subject of filtering has enjoyed a long standing symbiosis between classical and probabilistic approaches. Data assimilation can be viewed as a bridge between these approaches, built out of the necessity to obtain solutions to the filtering problem quickly for very high dimensional, turbulent, nonlinear forecast models, with notable applications in atmospheric and oceanographic science. This mini-symposium aims to bring together experts interested in nonlinear filtering, data assimilation and applications, to share their latest research.

A Basis for Improving Numerical Weather Prediction in the Gulf Area by Assimilating Doppler Radar Radial Winds

Mohamed-Naim Anwar
United Arab Emirates University, United Arab Emirates
Fathalla A. Rihan, Chris Collier

This contribution presents a theoretical framework to Data Assimilation of Doppler Radial Winds into a high resolution NWP model using 3D-Var system. NWP is considered as an initial-boundary value problem: given an estimate of the present state of the atmosphere, the model simulates (forecasts) its evolution. Specification of proper initial conditions and boundary conditions for numerical dynamical models is essential in order to have a well-posed problem and subsequently a good forecast model (A well-posed initial/boundary problem has a unique solution that depends continuously on the initial/boundary conditions). The goal of data assimilation is to construct the best possible initial and boundary conditions, known as the analysis, from which to integrate the NWP model forward in time. We discuss the types of errors that occur in radar radial winds. Some related problems such as nonlinearity and sensitivity of the forecast to possible small errors in initial conditions, random observation errors, and the background states are also considered. The technique can be used to improve the model forecasts, in the Gulf area, at the local scale and under high aerosol (dust/sand/pollution) conditions.

Nonlinear Filtering Without a Model and with a Partial Model

Tyrus Berry
George Mason University, USA
John Harlim, Franz Hamilton, Tim Sauer

We first introduce and compare two methods of filtering without a model, which we call nonparametric filters. The first method is based on the diffusion forecasting algorithm combined with a Bayesian update. The second is the Kalman-Takens approach which combines a Kalman update with a local linear forecast in the Takens embedding space. The limitation of these approaches is that the amount of historical data required for learning the model grows exponentially in the intrinsic dimension of the underlying dynamics. To overcome this limitation, we assume that an incomplete or imperfect parametric model is available, and we show how to use the nonparametric approach to correct the model error. This means that we only assume hat the model error is low dimensional. We demonstrate this semiparametric approach to data assimilation and forecasting on a Lorenz-96 system with model error governed by stochastic and chaotic dynamics.

Well-Posedness and Accuracy of a Class of Nonlinear Mini-Max Filters

Michal Branicki
Mathematics, University of Edinburgh, UK, Scotland
S. Zhuk

We consider the continuous and discrete time filtering problem for quasi-linear and quadratic truth dynamics with linear observations where the optimal estimate is given w.r.t. the quadratic mini-max cost function. The filter equations are derived by exploiting a certain duality between the filtering problem and the optimal stochastic control problem. In particular, this approach provides a framework for a systematic derivation of suboptimal Bayesian filters which, unlike approximate Gaussian filters including the Extended Kalman filter or the Ensemble Kalman filter, does not use ad hoc assumptions on the system covariance. For quadratic dissipative truth dynamics we show that a class of approximate filters derived within this framework is well-posed and accurate if the linear observation operator has a sufficiently high rank.

A Stable Particle Filter in High-Dimensions

Dan Crisan
Imperial College London, England
Alex Beskos, Ajay Jasra, Kengo Kamatani, Yan Zhou

We consider the numerical approximation of the filtering problem in high dimensions, that is, when the hidden state lies in R^d with d large. For low dimensional problems, one of the most popular numerical procedures for consistent inference is the class of approximations termed particle filters or sequential Monte Carlo methods. However, in high dimensions,

standard particle filters (e.g. the bootstrap particle filter) can have a cost that is exponential in d for the algorithm to be stable in an appropriate sense. We develop a new particle filter, called the *space-time particle filter*, for a specific family of state-space models in discrete time. This new class of particle filters provide consistent Monte Carlo estimates for any fixed d, as do standard particle filters. Moreover, we expect that the state-space particle filter will scale much better with d than the standard filter. We illustrate this analytically for a model of a simple i.i.d. structure and one of a Markovian structure in the d-dimensional space-direction, when we show that the algorithm exhibits certain stability properties as d increases at a cost $O(nNd^2)$, where n is the time parameter and N is the number of Monte Carlo samples, that are fixed and independent of d. Similar results are expected to hold, under a more general structure than the i.i.d. case, independently of the dimension. Our theoretical results are also supported by numerical simulations on practical models of complex structures. The results suggest that it is indeed possible to tackle some high dimensional filtering problems using the space-time particle filter that standard particle filters cannot handle.

Kernel Methods for Nonparametric Analog Forecasting

Dimitrios Giannakis
New York University, USA

Analog forecasting is a nonparametric technique introduced by Lorenz in 1969 which predicts the evolution of observables of dynamical systems by following the evolution of samples in a historical record of observations of the system which most closely resemble the observations at forecast initialization. In this talk, we discuss a family of forecasting methods which improve traditional analog forecasting by combining ideas from kernel methods for machine learning and state-space reconstruction for dynamical systems. A key ingredient of our approach is to replace single-analog forecasting with weighted ensembles of analogs constructed using local similarity kernels. The kernels used here employ a number of dynamics-dependent features designed to improve forecast skill, including Takens' delay-coordinate maps (to recover information in the initial data lost through partial observations) and a directional dependence on the dynamical vector field generating the data. Mathematically, the approach is closely related to kernel methods for out-of-sample extension of functions, and we discuss alternative strategies based on the Nystrom method and the multiscale Laplacian pyramids technique. We illustrate these techniques in forecasts of North Pacific sea surface temperature and arctic sea ice cover.

Stability of the Ensemble Kalman Filter

David Kelly
New York University, USA
Andrew Majda, Xin Tong

The Ensemble Kalman Filter (EnKF) is one of the cornerstone filtering methods in geoscience. It allows for computationally efficient assimilation of observational data with high dimensional physical models, such as those found in numerical weather prediction. Despite its success, the dynamical behavior of EnKF is poorly understood, particularly in the realistic regime of small ensemble sizes. In this talk we will address two key questions regarding the stability of EnKF : 1) Is EnKF stable to perturbations in ensemble initializations ? (The answer is no, in general) and 2) Are there simple modifications that can help enhance stability? (The answer is yes, thankfully). These questions are addressed in a simple framework of ergodicity for Markov processes.

Data Assimilation Algorithm for 3D Bènard Convection in Porous Media Employing Only Temperature Measurements

Evelyn Lunasin
United States Naval Academy, USA
Aseel Farhat, Edriss S. Titi

In this paper we propose a continuous data assimilation (downscaling) algorithm for the Bènard convection in porous media using only discrete spatial-mesh measurements of the temperature. In this algorithm, we incorporate the observables as a feedback (nudging) term in the evolution equation of the temperature. We show that under an appropriate choice of the nudging parameter and the size of the mesh, and under the assumption that the observed data is error free, the solution of the proposed algorithm converges at an exponential rate, asymptotically in time, to the unique exact unknown reference solution of the original system, associated with the observed (finite dimensional projection of) temperature data. Moreover, we note that in the case where the observational measurements are not error free, one can estimate the error between the solution of the algorithm and the exact reference solution of the system in terms of the error in the measurements.

Importance Sampling: Computational Complexity and Intrinsic Dimension

Daniel Sanz-Alonso
Brown University, Spain
Sergios Agapiou, Omiros Papaspiliopoulos, Andrew Stuart

We aim to give unity and provide new mathematical insight into the growing published literature addressing the curse of dimensionality of importance sampling. We focus on the use of importance sampling in Bayesian learning problems. We highlight, following the pioneering work of Bickel and co-authors, the importance of defining suitable notions of dimension for these problems. We establish precise connections between the intrinsic dimension used by Bickel, and the notion of effective number of parameters, used in statistics an machine learning. We show that both are finite as long as there is absolute continuity of posterior with respect to prior. This suggests a unifying idea: importance sampling degenerates as loss of absolute continuity is approached. We study the rates at which this degeneracy occurs as the dimension of the data and the unknown grow, but also as the prior becomes less informative or the noise in the observations smaller. The relevance of these ideas for data assimilation and filtering problems will become apparent.

A Reduced-Basis Approach for Parameterized Backward Stochastic Differential Equations

Guannan Zhang
Oak Ridge National Laboratory, USA
Weidong Zhao

This effort is motivated by the relationship between backward SDEs and a class of quasilinear PDEs, described by the nonlinear Feynman-Kac theory, such that our approach can be applied to solving quasilinear parabolic PDEs with deterministic or random parameters, where we aim at approximating the parameterized viscosity solution of the PDEs. In the SDE setting, the temporal-spatial differential operator in the PDE is described by the dynamics of the underlying stochastic process, and the key task in developing numerical schemes is to approximate the conditional mathematical expectation of the solution and the forcing term with respect to the stochastic process. In this effort, we utilize the empirical interpolation method (EIM) to approximate the involved expectation operator, such that we can obtain the off-line online decomposition of computational cost. The main feature of the SDE approach is that the approximate solution can be computed independently at each spatial grid point without solving linear systems when using implicit time-stepping schemes. This feature will lead to a significantly reduction by avoiding the off-line cost of approximating the Jacobian of the nonlinear operator using the EIM. In addition, our approach can provide a reduced-basis approximation to not only the solution of the PDE, but also its gradient. Various numerical examples on backward SDEs and the corresponding quasilinear PDEs with parameterized coefficients are presented to demonstrate effectiveness of our approach.

Kernel Methods for Nonparametric Analog Forecasting

Zhizhen Zhao
New York University, USA
Dimitrios Giannakis

Analog forecasting is a nonparametric technique introduced by Lorenz in 1969 which predicts the evolution of observables of dynamical systems by following the evolution of samples in a historical record of observations of the system which most closely resemble the observations at forecast initialization. We discuss a family of forecasting methods which improve traditional analog forecasting by combining ideas from kernel methods for machine learning and state-space reconstruction for dynamical systems. A key ingredient of our approach is to replace single-analog forecasting with weighted ensembles of analogs constructed using local similarity kernels. The kernels used here employ a number of dynamics-dependent features designed to improve forecast skill, including Takens‘ delay-coordinate maps (to recover information in the initial data lost through partial observations) and a directional dependence on the dynamical vector field generating the data. We illustrate these techniques in atmosphere ocean science applications.

Special Session 89: Dynamics and Computation

William D. Kalies, Florida Atlantic University, USA
Vincent Naudot, Florida Atlantic University, USA
Jason Mireles-James, Florida Atlantic University, USA

This session will focus on computational methods for analyzing dynamical systems, such as extracting invariant structures, symbolic dynamics, and bifurcations. The aim of this session is to bring together experienced and young researchers to exchange ideas and explore recent developments and applications.

Computing Invariant Circles of the Area Preserving Henon Map

David Blessing
Florida Atlantic University, USA
Jason Mireles-James

We examine the area preserving Hènon map, where numerical experiments suggest the existence of contractable invariant circles. We use a weighted averaging scheme due to Yorke, Das, Dock, Saiki, Wu, Flores, and Sander in order to approximate the rotation number and Fourier coefficients of the parameterization of the circle. Then we apply a Newton-like method due to Haro and de la Llave in order to refine the parameterization. This method is also adapted to study periodic circles of the area preserving Hènon map (invariant sets where orbits fill several disjoint contractible circles densely). In this case we adapt the techniques of Haro and de la Llave to simultaneously compute parameterizations all the circles in a way which does not require computing compositions of the map.

Computational Aspects in the Restricted Four Body Problem

Jaime Burgos-Garcia
Instituto Tecnologico Autonomo de Mexico, Mexico

The restricted four body problem (r4bp) studies the dynamics of a massless particle under the gravitational influence of three point masses that lie in an equilateral configuration provided by the well-known homographic solution of the general three body problem. Recently there have been several preliminary studies on the main objects of the dynamics: equilibrium points, periodic orbits, invariant manifolds etc. However, the structure of these invariant objects is still far to be well understood for all the admissible values of the masses. In this talk we will show some explorations of the above mentioned invariant objects and we will discuss some perspectives on further explorations and their possible applications for the solar system.

KAM Tori in Self-Consistent Map Models

Renato Calleja
IIMAS-UNAM, Mexico
David Martinez, Diego del Castillo, Arturo Olvera

I will present a Hamiltonian mean-field model. The model provides a simplified description of transport in marginally stable systems including vorticity mixing in strong shear flow and electron dynamics in plasmas. Self-consistency is incorporated through a mean-field that couples all the degrees of freedom. The model is formulated as a large set of N coupled standard-like twist maps. Invariant tori and their breakup play a central role in the study of global transport in these self-consistent map examples. I will present an algorithm to compute, continue and approximate the breakdown of analyticity of invariant tori in a simplified version of a self-consistent model. This is joint work with Diego del Castillo, David Martinez and Arturo Olvera.

Inertial Manifolds Computations

Yu-Min Chung
The College of William and Mary, USA

An inertial manifold, first introduced by Foias, Sell, and Temam in 1988, is a finite-dimensional, exponentially attracting, and positively invariant Lipschitz manifold. If a dynamical system possess an inertial manifold, it is known that all long time behaviors, such as fixed points, limit cycles, and more importantly, the global attractor, are contained in the inertial manifold. Moreover, when restricted dynamics on the inertial manifold, such system not only becomes finite dimensional but also shares the same long time behavior of the original system. Although its theory is well developed, the computation remains a challenge problem. In this talk, we present recent progress on inertial manifolds computations, including algorithms, convergent analysis, implementations, and some open questions.

Parametrization Method for Stable/Unstable Manifolds of Periodic Points for Maps

Jorge Gonzalez
Florida Atlantic University, USA
J.D Mireles James

The Parameterization Method is a general functional analytic framework for studying invariant manifolds of dynamical systems. We develop a version of the method for stable/unstable manifolds associated with periodic points of discrete time dynamical systems. The novelty of our approach is that by introducing new variables we are able to avoid computing compositions of the map. We describe the method in general and implement the method for some one and two dimensional manifolds in some two and three dimensional dynamical systems.
The rigorous validations of our numerical computations are established using functional analysis techniques and relying on the Radii Polynomial Theorem. This computer-assisted tool is a Newton-Kantorovich type argument tailored to the field.

CAP-KAM Part II: on the Application of an A Posteriori KAM Theorem

Alex Haro
Universitat de Barcelona, Spain
Jordi-Lluìs Figueras, Alejandro Luque

We present a methodology to rigorously validate a given approximation of a quasi-periodic Lagrangian torus of an exact symplectic map. The approach consists in verifying the hypotheses of an a-posteriori KAM theorem based of the parameterization method (following Rafael de la Llave and collaborators). A crucial point of our imprementation is an Approximation Lemma that allows us to control the norm of periodic functions using their discrete Fourier transform. This an other technical aspects that are of independent interest are presented by Alejandro Luque in this session. An outstanding consequence of this approach is that the computational cost of the validation is assymptotically equivalent of the cost of the numerical computation of invariant tori using the parameterization method. We illustrate the methodology with several examples, as the standard map, the nontwist standard map and the Froeschlè map.

Rabies SEIR with a Kick and Computing Chaos

Stephen Ippolito
ASTA, USA

In the field of epidemiology the standard dynamic techniques involve discrete maps, and continuous models such as ODEs. The intent of this work is to present the mathematics necessary to study hybrids of these two models in the cases where complexity is believed to exist. In particular we study the spread of rabies with the introduction of a birth pulse to the system. Believing the resulting model to exhibit complex dynamics, we then consider techniques for computing the stable and unstable manifold of a saddle point of this hybrid map resulting in a transverse intersection.

Use of Lattice Structures of Attractors and Other Techniques for an Efficient Computation of Lyapunov Functions for Morse Decompositions

Dinesh Kasti
Florida Atlantic University, USA
Arnaud Goullet, Shaun Harker, Konstantin Mischaikow, William D. Kalies

Some recently developed techniques that lead us to form an efficient algorithm to construct a piecewise constant Lyapunov functions for dynamics generated by a continuous nonlinear map will be discussed. The algorithm uses a memory efficient data structure for storing nonuniform grids. It utilizes dijkstra algorithm with a dikstra distance that approximates the manhattan distance to compute distance potential function, which is utilized to compute the Lyapunov function. We further prove that if the diameters of the grid elements go to zero, then the sequence of piecewise constant Lyapunov functions generated by our algorithm converge to a continuous Lyapunov function for the dynamics generated by the nonlinear map. We illustrate these techniques via the applications on two problems from population biology. Finally, we will elaborate the use and importance of lattice structures of attractors for these techniques.

Rigorous Integration of Material Surfaces

Shane Kepley
Florida Atlantic University, USA
William Kalies, Jay Mireles-James

The evolution of a particle advected by an analytic vector field can be expressed as a Taylor series on some interval in time. It is reasonable to expect that a higher dimensional smooth manifold of initial conditions should be analytic in both space and time. We show how the evolution of such a surface can be computed as a sequence of multi-variate Taylor coefficients embedded in an appropriate Banach algebra. The computation is made rigorous by defining a suitable Newton-like operator on this algebra and proving it is a contraction. We will illustrate the method with an example and discuss applications which emphasize two important features of our method: Integration of non-autonomous flows and computation of "Lagrangian" features such as material derivatives.

CAP-KAM Part I: Rigorous Computer-Assisted Estimates in Small Divisors Problems

Alejandro Luque
Instituto de Ciencias Matem'aticas, Spain
Jordi Lluis Figueras, Alex Haro

The aim of this talk is to discuss several problems that arise when applying an a posteriori KAM theorem in particular problems. We will consider the three following situations:
1) Obtaining rigorous and sharp estimates of analitic norms for functions depending on multiple angles. To this end, we resolt to an Approximation Lemma based on DFT.
2) Assigning Diophantine constants to a frequency vector. This vector may be given with finite precision.
3) Obtaining sharp Rüssmann's estimates that improve the applicability of the KAM theorem.
The above problems are part of a validation algorithm that will be presented and illustrated by Alex Haro in a subsequent talk of this session SS89. This is a joint work with Jordi-Lluis Figueras and Alex Haro.

Rigorous Numerics of Blow-Up Solutions for ODEs

Kaname Matsue
The Institute of Statistical Mathematics, Japan
Akitoshi Takayasu, Takiko Sasaki, Kazuaki Tanaka, Makoto Mizuguchi, Shin-ichi Oishi

This talk is concerned with blow-up solutions of an autonomous system of ordinary differential equations. We propose a numerical verification method for constructing blow-up solutions with their blow-up times on the basis of the compactification and the Lyapunov function validation. Blow-up solutions are regarded as connecting orbits for the desingularized dynamics on the compactified space via compactification. Under appropriate assumptions, blow-up time can be estimated by Lyapunov tracing, a re-parameterization technique of the time variable. The necessary criteria for the construction are ensured with verified numerical computations.

Connecting Orbits Between Periodic Orbits for the Lorenz Equation

Maxime Murray
Florida Atlantic University, USA
Jason Mireles-James, Jean-Philippe Lessard

In this talk I discuss the use Taylor/Fourier expansions to compute parametrizations of the stable and unstable manifolds for periodic orbits. Theses parametrisations are then used to recast the study of connecting orbits into boundary value problems that are solved with Chebyshev series. Finally, a-posteriori analysis is done to verify existence and transversality of the manifolds and their intersections. The Lorenz equation is used to give a concrete illustration of application for this method.

Invariant Manifold for Hybrid Maps

Vincent Naudot
Florida Atlantic University, USA
Jayson Mireles James, Qiuying Lu

We study, from a numerical point of view, the (un)stable manifolds for hybrid maps. This latter is the composition of a linear diffeomorphism and the time T of smooth vector field. Such systems model physical processes where a differential equation is occasionally kicked by a strong disturbance. We propose a numerical method for computing these invariant manifold which leads to high order polynomial parameterization of the immersion. We obtain a representation of the dynamics on the manifold in terms of a simple conjugacy relation. We illustrate the utility of the method by studying a planar example system, that is an Hamiltonian base hybrid map.

Computation of Lyapunov Constants in Switching Systems

Yun Tian
Shanghai Normal University, Peoples Rep of China
Pei Yu

In this talk, a new method with an efficient algorithm is developed for computing the Lyapunov constants of planar switching systems, and then applied to study bifurcation of limit cycles in a switching Bautin system. A complete classification on the conditions of a singular point being a center in this Bautin system is obtained. Further, an example of switching systems is constructed to show the existence of 10 small-amplitude limit cycles bifurcating from a center. This is a joint work with Pei Yu.

Periodically Perturbed Hamiltonian-Hopf Bifurcation.

Arturo Vieiro
Universitat de Barcelona, Spain
E. Fontich, C. Simò

We shall consider the effect of a periodic perturbation on a 2-dof autonomous Hamiltonian system undergoing a Hamiltonian-Hopf bifurcation. The system considered is obtained as a suitable truncation of the normal form plus a concrete perturbation. We will describe the asymptotic behaviour of the invariant 2-dimensional stable/unstable manifolds and their splitting. The theoretical results will be compared with direct computations of the invariant manifolds. A careful analysis of the associated Poincarè-Melnikov integral will provide a description of the sequence of parameters corresponding to changes on the dominant harmonics of the splitting function.

Special Session 91: Harmonic Analysis and Partial Differential Equations

William Bray, Missouri State University, USA
Mark A. Pinsky, Northwestern University, USA

The focus of this special session is to bring together experts to present and discuss the various deep facets of the connections between harmonic analysis and partial differential equations. Subtopics could include: asymptotics of Fourier transforms, Fourier integral operators, regularity theorems in PDE, and connections with geometric analysis.

Integrability of Fourier Integrals on Euclidean Space

William Bray
Missouri State University, USA

Under what conditions (smoothness, regularity, decay) on a function defined on Euclidean space of d dimension is it the Fourier transform of an integrable function? This problem cast in the realm of classical trigonometric series has a history spanning 100+ years beginning with the work of W. Young (1913). Results on this problem for Fourier integrals and $d = 1$ are of more recent origin (e.g. Moricz (1992), Liflyand (1993)). In this talk recent results in the case $d > 1$ will be discussed and related to other known results.

Fractional Operators with Singular Drift: Smoothing Properties and Morrey-Campanato Spaces

Diego Chamorro
Universite d'Evry, France
Stephane Menozzi

We investigate some smoothness properties for a transport-diffusion equation involving a class of non-degenerate Lèvy type operators with singular drift. Our main argument is based on a duality method using the molecular decomposition of Hardy spaces through which we derive some Hölder continuity for the associated parabolic PDE. This property will be fulfilled as far as the singular drift belongs to a suitable Morrey-Campanato space for which the regularizing properties of the Lèvy operator suffice to obtain global Hölder continuity.

L^p Bounds for Wave Operators for the Schrödinger Equation with Threshold Eigenvalue

William Green
Rose-Hulman Institute of Technology, USA
Michael Goldberg

The wave operators $W_{\pm}$ are a valuable tool for linking properties of the perturbed Schrödinger evolution $e^{itH}P_{ac}(H)$ to properties of the corresponding free evolution $e^{-it\Delta}$. We consider operators of the form $H = -\Delta + V$ on $\mathbb{R}^n$, $n \geq 5$ which have an eigenvalue at zero. It was recently proven by Yajima that, under these conditions, the wave operators are bounded on $L^p(\mathbb{R}^n)$ for all $p \in (1, \frac{n}{2})$. We recover this result, including the $p = 1$ endpoint, and should that the upper end of the range can be expanded if the eigenspace satisfies certain orthogonality conditions.

Fourier Transform Estimates for 2D Functions Defined Through Real-Analytic Functions and Associated PDE and PDE-Like Problems

Michael Greenblatt
University of Illinois at Chicago, USA

We describe some Fourier transform decay estimates for a reasonably general class of functions defined through real-analytic functions in two dimensions. These functions are allowed to have singularities. The estimates are proved with the help of an appropriate resolution of singularities algorithm. They improve on earlier work in that, among other things, they apply to globally defined functions and have a natural geometric interpretation which we will describe. The estimates are sharp for a certain range of indices in the theorems. We will also describe some applications to associated PDE and PDE-like problems.

A Remark on Geometric Separation with Shearlets

Kanghui Guo
Missouri State University, USA
Demetrio Labate

Shearlets may be viewed as directional wavelets. Shearlets is well known for its ability to represent optimally the cartoon-like images both in $\mathbb{R}^2$ and $\mathbb{R}^3$. In the literature, this fact helped to prove that Shearlets can separate geometric objects. In these results of geometric separation, one could only handle the images with piecewise linear edges. In this talk, we will show that in the setting of the above results, the geometric objects can be separated only if the the edges of the images are piecewise linear.

Deep Wavelet Scattering for Quantum Energy Regression

Matthew Hirn
Michigan State University, USA
Stephane Mallat, Nicolas Poilvert

Physical functionals are usually computed as solutions of variational problems or from solutions of partial differential equations, which may require huge computations for complex systems. Quantum chemistry calculations of molecular energies is such an example. Machine learning algorithms do not simulate the physical system but estimate solutions by interpolating values provided by a training set of known examples. However, precise interpolations may require a number of examples that is exponential in the system dimension, and are thus intractable. This curse of dimensionality may be avoided by computing interpolations in smaller approximation spaces, which take advantage of physical invariants. We present a novel approach for the regression of quantum mechanical energies based on the scattering transform of an intermediate electron density representation. The scattering transform is composed of iterated wavelet transforms and modulus operators, and possesses the appropriate invariant and stability properties for molecular energy regression. Numerical experiments give state of the art accuracy over data bases of organic molecules, while theoretical results guarantee performance for the component of the energy resulting from Coulombic interactions.

General Types of Spherical Mean Operator and K-Functionals of Fractional Orders

Thais Jordao
Universidade de Sao Paulo, Brazil
Xingping Sun

We design a general type of spherical mean operators, depending on a real number as parameter, and employ them to approximate L_p class functions. We show that optimal orders of approximation are achieved via appropriately defined K-functionals of fractional orders. Asymptotic relations between the rate of approximation of the new operator and the K-functional of fractional order were stablished. When the parameter we work with is taken as a natural number the general type of spherical mean operator, the K-funcional and also the result relating such objects turn out the same as in Dai & Ditzian (2004) which introduces a class of "multi-layered spherical mean operators. More details can be found in Jordão & Sun (2015).

*Universidade de São Paulo, Brasil. Partially supported by FAPESP #2014/06209-1.

Higher Dimensional Scattering Theory and Integral Representation Formulas

Dorina Mitrea
University of Missouri, USA
E. Marmolejo-Olea, I. Mitrea, M. Mitrea

In this talk I will answer the following basic question: What are the optimal assumptions, of geometric and analytic nature, which guarantee that a null-solution u of the Helmholtz operator $\Delta + k^2$ in an exterior domain Ω can be represented in terms of layer potentials naturally associated with the said Helmholtz operator and given domain? This work, at the interface between Geometric Measure Theory, Harmonic Analysis, Scattering Theory, and Clifford Analysis, generalizes and unifies classical results of Sommerfeld, Weyl, Müller, and Calderòn.

The Role of Infinitesimal Flatness in the Solvability of Elliptic Boundary Problems in Uniformly Rectifiable Domains

Marius Mitrea
University of Missouri, USA

The goal of this talk is to illustrate the phenomenon that uniform rectifiability together with infinitesimal flatness (understood as the demand that the outward unit normal is close to having vanishing mean oscillations) typically implies solvability results for elliptic boundary value problems formulated in such a geometric setting.

Action of a Scattering Map on Weighted Sobolev Spaces in the Plane

Katharine Ott
Bates College, USA
Russell Brown, Peter Perry

We consider a scattering map that arises in the $\bar{\partial}$ approach to the scattering theory for the Davey-Stewartson II equation and show that the map is an invertible map between certain weighted L^2 Sobolev spaces.

Well-Distributed Points on Annuli.

Steven Senger
Missouri State University, USA
Alex Iosevich, Xingping Sun, Shelby Kilmer

We consider several classes of well-distributed point sets in $\mathbb{R}^d$, including lattice points, and present estimates on how many points from such sets could be incident with annuli.

Quasi-Monte Carlo Approximaiton of Continuous Functions

Xingping Sun
Missouri State University, USA
Steven Senger, Zongmin Wu

We design a class of quasi-Monte Carlo operators, and employ them to approximate continuous functions defined on domains with the interior cone condition. We show that such an approximation scheme achieve optimal orders in terms of the modulus of continuity.

Special Session 92: Variational, Topological and Set-Valued Methods for Nonlinear Problems

Pasquale Candito, Università di Reggio Calabria, Italy
Giuseppina D'Agua, Università di Messina, Italy
Roberto Livrea, Università di Reggio Calabria, Italy
Salvatore A. Marano, Università di Catania, Italy

The aim of this session is to focus on the qualitative analysis of nonlinear problems, e.g., ordinary and partial differential equations, variational-hemivariational inequalities, difference and algebraic systems. Emphasis will be given to the results obtained by exploiting the synergy between the classical nonlinear analysis methods like the critical point theory, the fixed point theorems, the topological degree, the Morse theory, the set-valued analysis and so on. In particular, the existence, non-existence, multiplicity and sign information of the solutions of a wide range of nonlinear problems will be studied.

High Multiplicity of Positive Periodic Solutions for Super-Sublinear Indefinite Problems: a Topological Approach

Guglielmo Feltrin
SISSA (Trieste), Italy
Alberto Boscaggin, Fabio Zanolin

We study the periodic boundary value problem associated with the second order nonlinear differential equation

$$u'' + \big(\lambda a^{+}(t) - \mu a^{-}(t)\big)\, g(u) = 0,$$

where $g(u)$ has superlinear growth at zero and sublinear growth at infinity. For λ, μ positive and large, we prove the existence of $3^m - 1$ positive T-periodic solutions when the weight function $a(t)$ has m positive humps separated by m negative ones (in a T-periodicity interval). As a byproduct of our approach we also provide abundance of positive subharmonic solutions and symbolic dynamics. The proof is based on coincidence degree theory for locally compact operators on open unbounded sets and also applies to Neumann and Dirichlet boundary conditions.
This is a joint work with Alberto Boscaggin (University of Torino, Italy) and Fabio Zanolin (University of Udine, Italy).

On Pòlya's Inequality for Torsional Rigidity and First Dirichlet Eigenvalue

Vincenzo Ferone
Università di Napoli Federico II, Italy
Carlo Nitsch, Cristina Trombetti

An inequality proved by Pòlya establishes a bound from above of the product between the torsional rigidity and the first Dirichlet eigenvalue for the laplacian in un domain with fixed measure. We will discuss the sharpness of the bound and the possibility of improving it in suitable classes of domains.

Twist Conditions for a Higher Dimensional Poincarè–Birkhoff Theorem: an Avoiding Cones Formulation

Paolo Gidoni
SISSA (Trieste), Italy
Alessandro Fonda

In this talk we discuss how the concept of twist can be extended to higher dimensional systems, in particular when considering generalizations of the Poincarè–Birkhoff Theorem for $2N$-dimensional Hamiltonian systems. Following the spirit of similar results obtained for Poincarè–Miranda-like fixed point theorems, I present a new kind of boundary condition, called *avoiding cones condition*, that unifies and extend the higher dimensional twist conditions previously proposed for the Poincarè–Birkhoff Theorem.

Existence Theorems for Elliptic and Evolutionary Variational and Quasi-Variational Inequalities

Akhtar Khan
Rochester Institute of Technology, USA
Dumitru Motreanu

This talk gives new existence results for elliptic and evolutionary variational and quasi-variational inequalities. Specifically, we give an existence theorem for evolutionary variational inequalities involving different types of pseudo-monotone operators. Another existence result embarks on elliptic variational inequalities driven by maximal monotone operators. We propose a new recessivity assumption that extends all the classical coercivity conditions. We also obtain criteria for solvability of general quasi-variational inequalities treating in a unifying way elliptic and evolutionary problems.

Critical Groups Under Saddle Point Reduction and Applications to Elliptic Resonant Problems

Shibo Liu
Xiamen University, Peoples Rep of China

Infinite dimensional Morse theory is very useful in studying nonlinear equations. The basic concepts in the theory are critical groups at isolated critical points and critical groups at infinity. On the other hand, saddle point reduction is a powerful tool in critical point theory. With this reduction, the problem of finding critical points of a functional reduces to finding critical points for a reduced functional defining in s subspace. To combine the Morse theory and the reduction method, a natural problem is the relation between the critical groups of the original functional and the reduced functional. In this talk, we will present our results on this topic. It turns out that the critical groups are almost isomorphic. As application, we study some elliptic resonant problems with variable coefficients, where the energy functional may not satisfy the PS condition. We obtain multiple solutions for such problems.

Symmetry Breaking for a Problem in Optimal Insulation

Carlo Nitsch
University of Napoli Federico II, Italy
Dorin Bucur, Giuseppe Buttazzo

We consider the problem of optimally insulating a given domain; this amounts to solve a nonlinear variational problem, where the optimal thickness of the insulator is obtained as the boundary trace of the solution. We deal with two different criteria of optimization: the first one consists in the minimization of the total energy of the system, while the second one involves the first eigenvalue of the related differential operator. Surprisingly, the second optimization problem presents a symmetry breaking in the sense that for a ball the optimal thickness is nonsymmetric when the total amount of insulator is small enough.

N-Laplacian Problems with Critical Trudinger-Moser Nonlinearities

Kanishka Perera
Florida Institute of Technology, USA
Yang Yang

We prove existence and multiplicity results for a N-Laplacian problem with a critical exponential nonlinearity that is a natural analog of the Brezis-Nirenberg problem for the borderline case of the Sobolev inequality. This extends results in the literature for the semilinear case $N = 2$ to all $N \geq 2$. When $N > 2$ the nonlinear operator $-\Delta_N$ has no linear eigenspaces and hence this extension requires new abstract critical point theorems that are not based on linear subspaces. We prove new abstract results based on the $\mathbb{Z}_2$-cohomological index and a related pseudo-index that are applicable here.

Some Existence Results of Infinitely Many Solutions to Elliptic Problems with $p(x)-$Laplacian and Nonhomogeneous Neumann Conditions

Angela Sciammetta
Università degli Studi di Messina, Italy
Giuseppina D'Agui

The aim of this talk is to establish the existence of an unbounded sequence of weak solutions for a class of differential equations with $p(x)-$Laplacian and subject to small perturbations of nonhomogeneous Neumann conditions. The approach is based on variational methods.

Remarks on the Ambrosetti-Prodi Periodic Problem

Elisa Sovrano
University of Udine, Italy
Fabio Zanolin

In 2011 a very interesting note of Antonio Ambrosetti in honor of Giovanni Prodi appeared along with a list of open questions about global inversion theorems and their applications. One of these questions regards the study of the periodic Ambrosetti-Prodi problem for an ordinary differential second order equation. Our contribution to this problem concerns the study of the equation: $u'' + f(u) = p(t)$ where $p(t)$ is a T-periodic stepwise forcing term and the nonlinearity f is a locally Lipschitz continuous function such that $f(0) = 0$, $f(s) > 0$ for all $s \neq 0$, $\lim_{s\to\pm\infty} f(s) = +\infty$ and it is strictly decreasing for $s \leq 0$ and strictly increasing for $s \geq 0$. Assuming that the nonlinear term is a positive function with global minimum at zero which satisfies the previous growth conditions, we prove under suitable conditions on $p(t)$ the existence of infinitely many periodic solution. Moreover, we show the presence of chaotic-like dynamics via topological methods.

Fractional Inclusions with Impulses and Nonlocal Boundary Conditions in a Banach Space

Valentina Taddei
University of Modena and Reggio Emilia, Italy
I. Benedetti, V. Obukhovskii

As it is well known, fractional equations are used to describe anomalous diffusion with long-range effects as well as memory or hereditary properties of various processes. On the other hand, there are many phenomena characterized by parameters subject to short-term perturbations, which are represented through impulses. In this talk we give an existence result for fractional inclusions in abstract spaces with impulses and non-local boundary conditions. We apply a technique based on weak topology to avoid any compactness assumption, usually required to use topological tools in abstract spaces. Applications to integro-partial-differential inclusions coming from population dynamic are given.

The Neumann Eigenvalue Problem for the ∞-Laplacian

Cristina Trombetti
University of Napoli Federico II, Italy
Luca Esposito, Bernd Kawohl, Carlo Nitsch

The first nontrivial eigenfunction of the Neumann eigenvalue problem for the p-Laplacian converges, as p goes to ∞, to a viscosity solution of a suitable eigenvalue problem for the ∞-Laplacian. We show among other things that the limiting eigenvalue is in fact the first nonzero eigenvalue, and derive a number consequences, which are nonlinear analogues of well-known inequalities for the linear (2-)Laplacian.

Global a Priori Bounds for Weak Solutions to Quasilinear Parabolic Equations with Nonstandard Growth

Patrick Winkert
University of Technology Berlin, Germany
Rico Zacher

In this talk we study a rather wide class of quasilinear parabolic problems with nonlinear boundary condition and nonstandard growth terms. It includes the important case of equations with a $p(t,x)$-Laplacian. By means of the localization method and De Giorgi's iteration technique we derive global a priori bounds for weak solutions of such problems. Our results seem to be new even in the constant exponent case.

Special Session 93: Nonlinear Dispersive Equations and Integrable Systems

Joachim Escher, Leibniz Universität Hannover, Germany
Zhaoyang Yin, Sun Yat-Sen University and Macau University of Science and Technology, Peoples Rep of China
Zhijun Qiao, University of Texas - Rio Grande Valley, USA

The session is devoted to recent developments of the analysis of water waves, with the particular focus on oceanic waves, solitons, integrable systems and related PDEs. Topics include the qualitative mathematical analysis, such as local and global well-posedness, regularity, stability, asymptotic behavior of solutions, integrability and solitary waves.

Classification of Integrable 2-Component Peakon Equations from Lax Pairs

Stephen Anco
Brock University, Canada
Fatane Moberashermini

A classification is presented for integrable 2-component peakon equations arising from 2x2 Lax pairs

Multipeakons of a Two-Component Modified Camassa-Holm Equation and The Relation with the Finite Kac-Van Moerbeke Lattice

Xiangke Chang
University of Saskatchewan, Canada
Xingbiao Hu, Jacek Szmigielski

This talk is concerned about a two-component modified Camassa-Holm equation, which was proposed by Song, Qu and Qiao. A spectral and the inverse spectral problem are studied for the two-component modified Camassa-Holm type for measures associated to interlacing peaks. It is shown that the spectral problem is equivalent to an inhomogenous string problem with Dirichlet/Neumann boundary conditions. The inverse problem is solved by Stieltjes' continued fraction expansion, leading to an explicit construction of peakon solutions. Sufficient conditions for the global existence in t are given. The large time asymptotics reveals that, asymptotically, peakons break into two-peakon bound-states moving with constant speeds. The peakon flow is shown to project to one of the isospectral flows of the finite Kac-van Moerbeke lattice (or called the finite Lotka-Volterra lattice or the Langmuir lattice).

Analyticity, Gevrey Regularity and Unique Continuation for an Integrable Multi-Component Peakon System with an Arbitrary Polynomial Function

Qiaoyi Hu
South China Agricultural University, Peoples Rep of China
Zhijun Qiao

In this paper, we study the Cauchy problem for an integrable 2N-component peakon system which is involved in an arbitrary polynomial function. Based on a generalized Ovsyannikov type theorem, we first prove the existence and uniqueness of solutions for the system in the Gevrey-Sobolev spaces with the lower bound of the lifespan. Then we show the continuity of the data-to-solution map for the system. Furthermore, by introducing a family of continuous diffeomorphisms of a line and utilizing the fine structure of the system, we demonstrate the system exhibits unique continuation.

High-Order Approximations of Traveling Water Waves

Konstantinos Kalimeris
RICAM, Austrian Academy of Sciences, Austria

In this talk we consider the classical water wave problem described by the Euler equations with a free surface under the influence of gravity over a flat bottom. We restrict our attention to two-dimensional, finite-depth periodic water waves with general vorticity. We formulate this problem as nonlinear (fixed) boundary value problem, through a semi-hodograph transformation. An asymptotic technique is applied to approximate the solutions of this problem that correspond to non-laminar flows. We provide high-order approximations to periodic traveling wave profiles, depending on the total mechanical energy of the water wave. Moreover, we provide the velocity field and the pressure beneath the waves, in flows with constant vorticity over a flat bed.

Control Theory of Time-Varying Linear Systems in the Frame Work of Nest Algebra

Liu Liu
Dalian University of Technology, Peoples Rep of China
Yufeng Lu

As the development of H^∞ control theory, a lot of insight has been obtained by considering its time-varying analogue on an appropriate complex Hilbert space of input-output signals. In the context of operator theory, the algebra of stable, causal, discrete (continuous) time, time-varying linear systems is in fact the discrete (continuous) nest algebra. In this talk, we introduce the connection between control theory and nest algebra theory, some properties of the system representations, factorizations and give some new stabilizability criteria for the time-varying linear systems in the framework of nest algebra.

Well-Posedness and Global Solutions in Fluid Models with Vorticity

Tony Lyons
Waterford Institute of Technology, Ireland
Joachim Escher, David Henry, Boris Kolev

In this talk we present a two-component model for shallow-water waves incorporating vorticity. It will be outlined how the model may be interpreted as a geodesic flow on a right invariant metric of the group of smooth diffeomorphisms of the circle. The geometric interpretation of the fluid model also ensures the well-posedness of the model, while the existence of global solutions can be shown to follow from a priori estimates.

Extremal Norms of the Potentials Recovered from Inverse Dirichlet Problems

Jiangang Qi
Shandong University at Weihai, Peoples Rep of China
Shaozhu Chen

Consider the Sturm-Liouville eigenvalue problem $-y``(x) + q(x)y(x) =\lll y(x)$, $x \in [0,1]$, $y(0) = y(1) = 0$, where $q \in L^1[0,1]$, and denote its spectrum by $\sigma(q)$. For a real number λ, define $\Omega(\lambda) = \{q \in L^1[0,1] : \lambda \in \sigma(q)\}$ and $E(\lambda) = \inf\{\|q\| : q \in \Omega(\lambda)\}$. We will set up a formula for $E(\lambda)$ explicitly in terms of λ and specify where the infimum can be attained. As an application, we will give extremal values of the nth eigenvalue of the Dirichlet problem for potentials on a sphere in $L^1[0,1]$, $n \geq 1$. The proofs are based on a new Lyapunov-type inequality for Sturm-Liouville equations with potentials.

Steady Periodic Water Waves for Fixed-Depth Rotational Flows with Discontinuous Vorticity

Silvia Sastre-Gomez
University College Cork, Ireland
D. Henry

In this work we study steady two-dimensional periodic water waves problems over a fixed depth with the vorticity discontinuous. We consider a modified height function, which explicitly introduces the mean depth into the rotational water wave problem. Since the vorticity is discontinuous, the equations are expressed in a weak form and the solutions are considered in the sense of distributions. We use Crandall-Rabinowitz local bifurcation to prove the existence of weak solutions.

Blow-Up Results and Soliton Solutions for a Generalized Variable Coefficient Nonlinear Schroedinger Equation

Erwin Suazo
University of Texas, RGV, USA
Jose Escorcia

In this talk, by means of similarity transformations we study exact analytical solutions for a generalized nonlinear Schroedinger equation with variable coefficients. This equation appears in literature describing the evolution of coherent light in a nonlinear Kerr medium, Bose-Einstein condensates phenomena and high intensity pulse propagation in optical fibers. By restricting the coefficients to satisfy Ermakov-Riccati systems with multiparameter solutions, we present conditions for existence of explicit solutions with singularities and a family of oscillating periodic soliton-type solutions. Also, we show the existence of bright-, dark- and Peregrine-type soliton solutions, and by means of a computer algebra system we exemplify the nontrivial dynamics of the solitary wave center of these solutions produced by our multiparameter approach.

Spectra of a Class of Non-Symmetric Operators in Hilbert Spaces with Applications to Singular Differential Operators

Huaqing Sun
Shandong University at Weihai, Peoples Rep of China
Bing Xie

This paper is concerned with a class of non-symmetric operators. A sufficient condition for points of the essential spectrum of a $\mathcal{J}$-symmetric operator is given in terms of the numbers of linearly independent solutions of certain homogeneous equation, and a characterization for points of the essential spectrum plus the set of all eigenvalues of a $\mathcal{J}$-

symmetric operator is obtained in terms of the numbers of linearly independent solutions of certain inhomogeneous equation. These results are established in a purely operator-theoretic setting. Therefore, they can be used for any $\mathcal{J}$-symmetric differential expressions for whatever $\mathcal{J}$ is taken. They are applied to singular $\mathcal{J}$-symmetric Hamiltonian systems and its special form of singular Sturm-Liouville equations with complex-valued coefficients, where $\mathcal{J}$ is specified as the usual operation of complex conjugation in the corresponding spaces. Furthermore, three concrete examples are provided to illustrate these results.

Riemann-Hilbert Approach for the FQXL Model: a Generalized Camassa-Holm Equation with Cubic and Quadratic Nonlinearity

Zhen Wang
Dalian University of Technology, Peoples Rep of China
Qiao Zhijun

In this paper, the inverse scattering transform associated with a Riemann-Hilbert problem is formulated for the FQXL model: a generalized Camassa-Holm equation $m_t = \frac{1}{2}k_1[m(u^2-u_x^2)]_x + \frac{1}{2}k_2(2mu_x + m_x u)$, $m = u - u_{xx}$, which was originally included in the work of Fokas and recently shown integrable in the sense of Lax pair, bi-Hamilton structure, and conservation laws by Qiao, Xia, and Li. Moreover, the parametric multi-soliton solutions are presented for the case of reflectionless potentials.

Qualitative Analysis of a Time-Delayed Free Boundary Problem for Tumor Growth Under the Action of External Inhibitors

Shihe Xu
Zhaoqing University, Peoples Rep of China
Qinghua Zhou, Meng Bai

In this paper we consider a time delayed free boundary problem for tumor growth under the action of external inhibitors. It is assumed that the process of proliferation is delayed compared to apoptosis. The existence and uniqueness of a global solution is proved. Morever, the asymptotic behavior of the solution is studied.

Submodule of Hardy Space Over the Bidisc

Yixin Yang
Dalian University of Technology, Peoples Rep of China

Submodules of $H^2(\mathbb{D}^2)$ are well known to be very complicated. We can construct submodules from the point of view of operator model theory of B. Sz.-Nagy and C. Foias. For a $\mathcal{B}(H^2(w))$-valued inner function $\Theta(z)$, $M = \Theta(z)H^2_{H^2(w)}(\mathbb{D})$ is a T_z-invariant subspace of $H^2(\mathbb{D}^2)$, but in general M is not a submodule of $H^2(\mathbb{D}^2)$. For some$\Theta(z)$, it may produce some new submodules and have very interesting properties. By using this type of submodules, we show that multivariable analogue of the Berger-Shaw theorem is not likely to be possible.

Remarks on the Well-Posedness of Camassa-Holm Type Equations in Besov Spaces

Zhaoyang Yin
Macau University of Science and Technology, Macau
Jinlu Li

In this paper, we prove the solution map of the Cauchy problem of Camassa-Holm type equations depends continuously on the initial data in nonhomogeneous Besov spaces in the sense of Hadamard by using the Littlewood-Paley theory and the method introduced by Kato and Danchin.

From the Solutions to Construct the Schrodinger-Like Equation with Source Term and Its Numerical Simulations

Fajun Yu
School of Mathematics and Systematic Sciences, Shenyang Normal University, Peoples Rep of China

We present a brand new method to construct the Schrodinger-like equations from a solution in this paper. Some Schrodinger-like equations with sources are derived by using a generalized solution.And we prove that the equation with source term has a weak solution. At last, the numerical simulations on the evolution and solitons collision of rogue wave solutions are performed to verify the prediction of the analytical formulations.

Somes Explicit Solutions of a Finite-Dimensional Integrable System

Jinshun Zhang
Huaqiao University, Peoples Rep of China

A finite-dimensional Hamiltonian system associated with CKdV equation is considered. It's an integrable system in Liouville sense. We present an effective method to construct explicit solutions of the system. Some explicit solutions are obtained.

On the Cauchy Problem for a Generalized Cross-Coupled Camassa-Holm System with N-Peakons and Higher-Order Nonlinearities

Shouming Zhou
Chongqing Normal University, Peoples Rep of China
Zhijun Qiao and Chunlai Mu

In present talk, we study the Cauchy problem for a generalized cross-coupled Camassa-Holm system with peakons and higher-order nonlinearities. By the transport equations theory and the classical Friedrichs regularization method, the local well-posedness of solutions for this system in nonhomogeneous Besov spaces $B^s_{p,r} \times B^s_{p,r}$ with $1 \leq p, r \leq +\infty$ and $s > \max\{2+\frac{1}{p}, \frac{5}{2}\}$ is obtained. Moreover, the local well-posedness in critical Besov space $B^{5/2}_{2,1} \times B^{5/2}_{2,1}$ and the blow-up criteria are also established. Our other purpose is to consider the well-posedness in the sense of Hadamard, non-uniform dependence and Hölder continuity of the data-to-solution map for this system on both the periodic and the nonperiodic case. Using a Galerkin-type approximation scheme, it is shown that this equation is well-posed in Sobolev spaces $H^s \times H^s, s > 5/2$ in the sense of Hadamard, that is, the data-to-solution map is continuous. Furthermore, in conjunction with the well-posedness estimate, it is also proved that this dependence is sharp by showing that the solution map is not uniformly continuous. Finally, the Hölder continuous in the $H^r \times H^r$ topology when $0 \leq r < s$ with Hölder exponent α depending on both s and r are shown.

Global Existence of Solutions and Steady States of the Nonlinear Size-Structured Population Models with Distributed Delay in the Recruitment

Qinghua Zhou
Zhaoqing University, Peoples Rep of China
Meng Bai, Minhai Huang

We study the nonlinear size-structured population models with distributed delay in the recruitment. The global existence of solutions and the non-trivial steady states of the models are obtained by using the semigroup techniques in Banach spaces.

Special Session 94: Infinite Dimensional Dynamics in Analysis

Cho-Ho Chu, Queen Mary University of London, England

In recent years, there has been active research in complex and functional analysis relating to linear and non-linear dynamics, in particular, in infinite dimensional holomorphic dynamics and C*-dynamical systems. The aim of the special session is to bring together researchers in these fields and related areas, for instance, differential equations and operator theory, to inform recent progress and explore potential links between their research areas.

Viscous Dynamics in Relativistic Fluids

Shabnam Beheshti
Queen Mary University London, England

It is known that viscous effects lead to nontrivial dynamical behaviour in homogeneous cosmological models such as FLRW and Bianchi spacetimes. Recent progress by in well-posedness of certain Einstein-Navier-Stokes systems motivates revisiting models involving dynamic velocities, first proposed by Lichnerowicz in 1967. Using a dynamical systems approach, we investigate the role of dynamic velocity in a cosmological background; we demonstrate that the additional degree of freedom afforded by an associated index plays a key, geometric role in the evolution of relativistic fluids. An important open question involves finding admissibility conditions for the equation of state of the system, possibly via thermodynamic methods for holomorphic maps.

Dynamics of Maps

Cho-Ho Chu
Queen Mary, University of London, England

The dynamics of a map $f : X \to X$ studies the behaviour of its iterates (f^n). The main question is whether one can predict the fate of all orbits of f, where an orbit of a point p in X is the sequence $(f^n(p))$. This is a difficult but interesting question. We will discuss some examples in which f is a translation map or a smooth map.

Young Diagram Differential Operators

Arran Hamm
Winthrop University, USA
Shabnam Beheshti

A Young diagram is a collection of cells arranged in left-justified rows with row length weakly decreasing down the rows. In this talk I will discuss a class of differential operators defined in connection with Young diagrams. Using these operators, equations from the KP hierarchy take on surprisingly simple forms. Additional properties of these operators will be discussed including a “mixed partial derivative“ formula which, when applied repeatedly, gives rise to a natural combinatorial identity. Joint work with Shabnam Beheshti.

Holomorphic Mappings on a Complex Banach Space

Tatsuhiro Honda
Hiroshima Institute of Technology, Japan

Let X be a complex Banach space with the unit ball B. The family $\mathcal{M}$ is a natural generalization to complex Banach spaces of the well known Caratheòdory family of functions with positive real part on the unit disc U in $\mathbb{C}$. We consider subfamilies of $\mathcal{M}$, and discuss some properties for holomorphic mappings $f : B \to X$ which belongs to the subfamilies.

Extremal Problems, Loewner Chains and The Loewner Differential Equation in $\mathbb{C}^n$ and Complex Banach Spaces

Gabriela Kohr
Babes-Bolyai University, Cluj-Napoca, Romania
Ian Graham, Hidetaka Hamada, Mirela Kohr

In this talk we survey classical and also modern results in the theory of Loewner chains and the generalized Loewner differential equation on the unit ball in $\mathbb{C}^n$ and complex Banach spaces. We also present recent applications in the study of extremal problems associated with compact families of normalized biholomorphic mappings, which have parametric representation on the unit ball in $\mathbb{C}^n$. Finally, we point out certain open problems and conjectures related to Loewner chains, Herglotz vector fields, and the generalized Loewner differential equation in $\mathbb{C}^n$ and reflexive complex Banach spaces.

A Note on Kirwan's Conjecture

Yuk Leung
University of Delaware, USA

Using Schiffer's modification of Loewner's chain, Kirwan and Schober obtained a new set of extremal coefficient problems on the class Σ of functions

$$z + \sum_{n=0}^{\infty} \frac{b_n}{z^n}$$

of functions that are univalent and analytic in $|z| > 1$. Based on their results, William Kirwan proposed that the real part of $nb_1 - b_n \leq n$ must hold for every function in Σ with equality holding for the Koebe function $K(z) = z + 2 + 1/z$.

In this talk, we develop a second variational formula of the Koebe function using the classical Loewner's differential equation. Together with various known identities of the hyper-geometric polynomials $P_n^{(0,1)}(x)$, we prove that Kirwan's conjecture is true for functions in Σ that are close to the Koebe function.

Inverse of Disjointness Preserving Operators

Lei Li
Nankai University, Peoples Rep of China

A linear operator between function spaces is disjointness preserving if it maps disjoint functions to disjoint functions. Here, two functions are said to be disjoint if at least one of them vanishes at each point. I will talk about the linear disjointness preserving operators between various types of function spaces, including spaces of (little) Lipschitz functions, uniformly continuous functions and differentiable functions. It is shown that a disjointness preserving linear isomorphism whose domain is one of these types of spaces (scalar-valued) has a disjointness preserving inverse.

Continuous Orbit Equivalence

Xin Li
Queen Mary University of London, England

We discuss the notion of continuous orbit equivalence, and its relationship to Geometric Group Theory as well as Cartan subalgebras of C*-algebras.

Picard Type Theorems and Applications to Certain PDEs

Bao Qin Li
Florida International University, USA

A connection between Picard's theorem and entire solutions of certain functional equations will be given, from which Picard type theorems are then derived and applied to certain nonlinear partial differential equations.

Complex Oscillation and Explicit Determination of Semi-Finite Band Gap Solutions of the Whittaker-Hill Equation

Xudan Luo
The Hong Kong University of Science and Technology, Hong Kong
Yik-Man Chiang

In this talk, we discuss the general solution of Whittaker-Hill equation under confluent hypergeometric basis. According to certain initial conditions and boundary conditions, we get four special solutions which are either even or odd and either periodic or semi-periodic. We have extended Ince's work in 1923. Moreover, we show that the solutions of Whittaker-Hill equation with finite exponent of convergence of zero-sequence correspond to the terminating solutions of Whittaker-Hill equation. Finally, the semi-finite band gap problem of Whittaker-Hill potential is considered also.

Lie Structures in Operator Algebras

Lina Oliveira
Universidade de Lisboa, Portugal
M. Santos

In this talk we consider reflexive operator algebras having subspace lattices of fixed types and their Lie modules. It will be shown how the interplay between the subspace lattice of the algebra and its Lie algebraic structure leads to establishing a decomposition theorem for Lie modules along the lines of that obtained in the 90's by Hudson–Marcoux–Sourour for Lie ideals.

The Inverse Linearization Problem

Maxim Olshanii
UMass Boston, USA

We investigate the relationship between the nonlinear partial differential equations (PDEs) of mathematical physics and the their linearizations around stationary localized solutions. It turns out that for some classes of PDEs, it is possible to solve the Inverse Linearization Problem, i.e. given the linearization, to restore the original PDE. Of a particular interest are the instances of transparency of the former that are shown to hint on the possible integrability of the latter.

On the Structure of C_0-Semigroups of Holomorphic Carath'eodory Isometries

Laszlo Stacho
University of Szeged, Hungary

We extend Vesentini's description of the infinitesimal generators of strongly continuous one-parameter semigroups of holomorphic Caratòodory isometries of the unit ball of a complex Hilbert space to the setting of reflexive Cartan factors. Our treatment is based on intensive use of joint fixed points along with Kaup type ideas with partial vector fields of second degree. In particular we establish closed formulas for the Hilbert space case in terms of spectral resolutions of skew self-adjoint dilations related to the Reich-Shoikhet non-linear infinitesimal generator. We also provide partial results toward a Hille-Yosida type theory for holomorphic self-maps of bounded domains in Banach spaces.

Symmetries of the Darboux Equation

Chiu Yin Tsang
The Hong Kong University of Science and Technology, Hong Kong
Yik-Man Chiang, Avery Ching

The Darboux equation (1882) was a generalization of both Picard's and Hermite's equations. All these equations are generalizations of the well-known Lamè equation (1837). The equation was rediscovered by Treibich and Verdier in the 1980s concerning it having finite-gap property in an algebraic geometric characterization. The equation is a (doubly periodic) torus version of the Heun equation which lives on the Riemann sphere. It turns out that the Darboux equation has a better symmetry structure compared to that of the Heun equation.
In this talk, we will describe the symmetry of the Darboux equation via the study of the transformations which induce the automorphisms of the Darboux equation. We show how to apply the automorphisms to generate the 192 local solutions of the Darboux equation. This is a joint work with Yik-Man Chiang and Avery Ching.

Evolution Algebras of Arbitrary Dimension

M. Victoria Velasco
University of Granada, Spain

In this talk we discuss the structure of the evolution algebras or arbitrary dimension, a type of non-associative algebras (which are nor even power-associative) that, dynamically, represent discrete dynamical systems. These algebras have deep connections with the graph theory, the stochastic processes, the mathematical physics and some other branchs of the Science. They emerged to enlighten the study of non-Mendelian inheritance in genetics.

Shift Operators on Continuous Functions

Ngai-Ching Wong
National Sun Yat-sen University, Taiwan
Li-Shu Chen, Jyh-Shyang Jeang

We study (quasi-)n-shift operators on $C_0(X)$, where X is a locally compact and Hausdorff space. When $C_0(X)$ admits a disjointness preserving quasi-n-shift T, there is a countable subset of $X_\infty = X \cup \{\infty\}$ equipped with a tree-like structure, called φ-tree, with exactly n joints such that the action of T on $C_0(X)$ can be implemented as a shift on the φ-tree. If T is an n-shift, then the φ-tree is dense in X and thus X is separable. By analyzing the structure of the φ-tree, we show that every (quasi-)n-shift on c_0 can always be written as a product of n (quasi-)1-shifts. Although it is not the case for general $C_0(X)$ as shown by our counter examples, we can do so after dilation. We also study the case when the shift is an isometry instead. In this case, we will work on a shift on a tree structure of the dual space $M(X)$ of $C_0(X)$ instead.

On Hausdorff Dimension of the Julia Set of Entire Functions with Fast Growth

Zhuan Ye
Northern Illinois Univeristy, USA
Jie Ding, Jun Wang

We construct a family of transcendental entire functions which lie outside the Eremenko-Lyubich class in general and are of infinity growth order. Most importantly, we show that the intersection of Julia set and escaping set of these entire functions has full Hausdorff dimension. We also obtain some theorems on Hausdorff measures of the intersection of Julia set and escaping set.

A Galoisian Approach to Complex Oscillation Theory

Chiang Yik Man
Hong Kong University of Science and Technology, Hong Kong
Guofu Yu

We demonstrate that complex non-oscillatory solutions (in the sense of Nevanlinna theory) of certain class of Hill equations are among the Liouvillian solutions of associated differential equations. We shall establish a full equivalence between the two viewpoints when the Hill potential is a linear combination of four exponential functions. This equation is closely related to the classical Lame and Mathieu equations. We shall also discuss new orthogonality found for these non-oscillatory solutions.

Special Session 95: PDEs from Gauge Field Theories and Mathematical Physics

Jongmin Han, Kyung Hee University, Korea
Namkwon Kim, Chosun University, Korea

This special session deals with recent advances in PDEs from mathematical physics. Special focuses are brought into PDEs from (2+1) dimensional gauge field theories including Maxwell-Chern-Simons models, SU(2), SU(3), O(3) models, etc. Popular topics are existence and properties of solutions to self-dual equations, non-self-dual equations, and time-dependent problems. Related topics for self-dual equations include elliptic systems involving exponential nonlinearity in two dimension. This session also deals with other PDEs from various fields of mathematical physics such as fluid mechanics.

Short Time Regularity to the Equations of Unsteady Motion of Shear Thickening Incompressible Fluids

Hyeong-Ohk Bae
Ajou University, Korea
Jorg Wolf

We address the existence of strong solutions to a system of equations of motion of an incompressible non-Newtonian fluid. Our aim is to prove the short-time existence of strong solutions for the case of shear thickening viscosity, which corresponds to the power law $\nu(\mathbf{D}) = |\mathbf{D}|^{q-2}$ $(2 < q2.23\cdots$. The results are obtained by flattening the boundary and by using the difference quotient method. Near the boundary, we use weighted estimates in the normal direction.

Asymptotic Self-Similarity of Entire Solutions for Quasilinear Equations with Exponential Nonlinearity

Soohyun Bae
Hanbat National University, Korea

We consider the asymptotic self-similarity of entire solutions for quasilinear equations with exponential nonlinearity. The main result is described for $p-$Laplace equation with exponential nonlinearity.

Small Data Global Existence and Decay for Relativistic Chern–Simons Equations

Myeongju Chae
Hankyong Natinal University, Korea
Sung-Jin Oh

We establish a general small data global existence and decay theorem for Chern–Simons theories with a general gauge group, coupled with a massive relativistic field of spin 0 or 1/2. A key idea is to develop and employ a gauge invariant vector field method for relativistic Chern–Simons theories, which allows us to avoid the long range effect of charge.

Uniqueness of Positive Solution to a Coupled Cooperative Variational Elliptic System on an Interval

Jann-Long Chern
National Central University, Taiwan
Junping Shi, Yulian An

Oscillatory behavior of solutions of linearized equations for cooperative semilinear elliptic systems of two equations on one-dimensional domains are proved, and it is shown that the stability of the positive solutions for such semilinear system is closely related to the oscillatory behavior. These properties are used to prove the uniqueness of positive solutions to some semilinear elliptic systems with nonlinearities satisfying certain variational structure and growth conditions.
This talk is based on the joint work with Profs. Junping Shi and Yulian An.

Mixed Type Solutions in the Self-Dual SU(3) Chern-Simons Theory

Kwangseok Choe
Inha University, Korea
Namkwon Kim, Chang-Shou Lin

We consider an elliptic system arising from the nonabelian relativistic self-dual SU(3) Chern-Simons theory. This system is given on $\mathbb{R}^2$, and it consists of two elliptic equations with exponential nonlinearities. We review recent progress on the existence of radially symmetric solutions (u_1, u_2) satisfying the mixed-type boundary condition near ∞; u_1 converges while u_1 tends to $-\infty$ near ∞.

Radial Solutions for the Gravitational Maxwell Gauged O(3) Sigma Model

Na Ri Choi
Jongmin Han

In this paper, we study an elliptic equation arising from the self-dual equations for the Maxwell gauged $O(3)$ sigma model coupled with gravitation. We completely classify all radial solutions for single vortex case according to values of aN where a is a scaled gravitational constant and N is the total vortex number.

Existence for Mixed Type Solutions in Chern-Simons Theory

Namkwon Kim
Chosun University, Korea
Kwangseok Choe, C-S Lin

We present some of recent progress on the existence of mixed type solutions in the gauged Chern-Simons theories in the whole space. In the radial symmetric setting, mixed type solutions correspond to bounday points in the phase space and their information is closely related to the nontopological solutions. Hence, in the point of view of shooting, such solutions are very subtle and one needs different approach. We present degree theoretic and variational approach.

Dynamic Transitions of Generalized Burgers Equation

Kiah Wah Ong
Indiana University, USA
Limei Li

Phase transitions and bifurcations are of central importance in nonlinear sciences. Some typical examples include the solid/liquid/gas transitions, segregation of block copolymers in a polymer melt, the onset of Rayleigh-Benard convection, etc. In this talk, the dynamic transition of the one dimensional generalized Burgers equation with periodic boundary condition will be discussed.

Standing Waves for the Nonlinear Semi-Relativistic Schrödinger Equations

Jinmyoung Seok
Kyonggi University, Korea
Woocheol Choi

The nonlinear Schrödinger equations arises from the quatum physics to describe the dynamics of a large set of identical quantum particles interacting each other. During several decades, their standing wave solutions, one of their solitons, have been extensively studied under the aid of variational methods and, nowadays, we have fine knowledge about the existence, regularity and qualitative properties such as the radial symmetry or the decay rate at infinity.
In this talk, we are concerned with the nonlinear semi-relativistic Schrödinger equations, one of relativistic counterparts of nonilnear Schrödinger eqations. We introduce some results about the existence, nonexistence and qualitative properties of their standing waves on the entire space. We also deal with the Dirichlet problems on bounded domains. In this setting, we are mainly interested in the problems involving the nonlinearity with critical exponent, under which Brezis-Nirenberg type results are discussed.

Radial Solutions for the Einstein-Maxwell-Higgs Model

Ju Hee Son
Kyung Hee University, Korea
Jongmin Han

In this paper, we are concerned with an elliptic system arising from the Einstein-Maxwell-Higgs model which describes electromagnetic dynamics coupled with gravitational fields in space-time. Reducing this system to a single equation and setting up the radial ansatz, we prove the existence of the radial solutions of topological and nontopological boundary conditions of type I and II. There are two important positive constants: a representing the gravitational constant and N representing the total vortex number. We establish the solutions for any a and N and give a complete classification of all solutions, which improves previous known results.

On the Gravitational Maxwell Gauged O(3) Sigma Model

Kyungwoo Song
Kyung Hee University, Korea

We improve the existence result of
nite energy solutions to the self-dual equations of the gravitational Maxwell gauged O(3) sigma model in (2+1)-dimensional Minkowski space for arbitrary location of strings for a small gravitational constant using the standard super- and subsolution method. We construct an explicit supersolution and use the weighted Sobolev space for a subsolution.

Diffusions with Singular Drift: Convergence to Robin Boundary Condition

Minha Yoo
National Institute for Mathematical Sciences, Korea
Inwon Kim

In this talk, we consider parabolic equations with singular drift, where the drift penalizes diffusion outside of a given space-time domain. Using only PDE arguments we show that the corresponding solutions converge to solutions of boundary value problems. In the case of divergence-form equations we show, by explicit formula, that the limiting boundary condition depends not only on diffusion operator but also on the space-time geometry of the confining domain. In particular if the domain is time-dependent we show that robin boundary data appears in the limit even for the most generic choice of diffusion and drift.

Special Session 96: Complex Biological and Ecological Systems

Yun Kang, Arizona State University, USA
Komi Messan, Arizona State University, USA
Marisabel Rodriguez, Arizona State University, USA

The emergence and evolution of complex interaction among biological organisms have trigger researchers aiming to understand structural, functionality, and relational patterns that are imperative to species survivability. The interdisciplinary study of the dynamics of ecological and evolutionary processes in adaptive environments present both challenges and opportunities for research and education. We bring both biologists and mathematicians to study dynamics of complex biological systems including topics of disease, ecology, social insects, species interactions, and the link of genetics, environmental changes and social behavior. Theoretical frameworks will include agent-based modeling, network analysis, differential equations, stochastic processes, and optimization. The invited speakers will consist of both mathematician and biologists, both senior and junior researchers, especially minority population, to promote the maximum collaborations.

Weakly Nonlinear Analysis of Nonlocal Fisher Equation

Ozgur Aydogmus
Social Sciences University of Ankara, Turkey

We consider the Fisher equation with resource competition which is modeled by an asymmetric nonlocal convolution integral. We introduce two parameters for the convolution kernel describing the range of nonlocality and the extent of asymmetry. In previous studies it was shown that the spatially homogenous equilibrium of this model becomes unstable for sufficiently small diffusion rates and traveling and stationary wave type patterns was observed near the stability boundary. We further analyze the behavior of solutions to this model near the stability boundary using the techniques of weakly nonlinear analysis. We first obtain a cubic Stuart-Landau type equation and give its parameters in terms of Fourier transforms of the kernels. This analysis allows us to study the change in amplitudes of the solutions with respect to range of nonlocality and extent of asymmetry of the kernel function. We show that both continuous and discontinuous transitions from disordered behavior to ordered one are possible. When discontinuous transitions are observed, cubic equations are not enough to get quantitave behavior of the amplitudes of the patterns. We then obtain and study quartic Stuart-Landau equations to get a complete picture of the amplitudes near stability boundary. We also verify these results numerically.

Comparison of Pollination Networks

David Chan
VCU, USA
James Lee, Rodney Dyer

From the agent-based, correlated random walk model presented, we observe the effects of varying the maximum turning angle, $\delta_{\max}$, tree density, ω, pollen carryover, κ_{max}, and the probability of fertilization, P_κ, on the distribution of pollen within a tree population. We see that varying $\delta_{\max}$ and κ_{max} changes the dispersal distance of pollen, which greatly affects many measures of connectivity. The clustering coefficient of fathers is maximized when $\delta_{\max}$ is between 60° and 90°. Varying ω does not have a major effect on the clustering coefficient of fathers, but it does have a greater effect on other measures of genetic diversity. In particular we compare our simulations with randomly-placed trees with that of actual tree placement of *C. florida* at the VCU Rice Center, and show that knowing the tree locations is critical in the understanding how pollen is distributed within a specific ecosystem.

Modeling the Geographic Spread of Rabies in China

Jing Chen
University of Miami, USA
Lan Zou, Zhen Jin, Shigui Ruan

In the last 20 years or so, rural communities and areas in Mainland China invaded by rabies are gradually and significantly enlarged. Some provinces such as Shaanxi and Shanxi, used to be rabies free, have increasing numbers of human infections cases now. Recent phylogeographical analyses of rabies virus clades indicate that the human rabies cases in different and geographically unconnected provinces in China are epidemiologically related. In order to investigate how the movement of dogs changes the geographically inter-provincial spread of rabies in Mainland China, we propose a multi-patch model for the transmission dynamics of rabies between dogs and humans, in which each province is regarded as a patch. To investigate the rabies virus clades lineages observed in the phylogeographical analyses, the two-patch model will be used to simulate the human rabies data to study the inter-provincial spread of rabies between Guangxi and Guizhou, Fujian and Hebei and Sichuan and Guizhou, respectively. In order to reduce and prevent geographical spread of rabies in China, our results suggest that the management of dog market and trade need to be regulated and transportation of dogs need to be better monitored and under constant surveillance.

An Adaptive Feedback Methodology for Determining Information Content in Stable Population Studies

Rebecca Everett
North Carolina State University, USA
H. T. Banks, J. E. Banks, J. D. Stark

Entomologists use demographic data to estimate the effects of toxicants on populations. Although demographic data has been shown to give a more accurate estimate in many instances than other types of data, the collection of demographic data can be both time-consuming and costly. An important question is whether partial demographic data can replace full demographic data while still providing an accurate picture of the impact of a toxicant on a population. We develop statistical and mathematical based methodologies for determining (as the experiment progresses) the amount of information required to complete the estimation of stable population parameters with pre-specified levels of confidence. This will be discussed in the context of life table models and data for growth/death for three species of Daphniids. This represents joint work with H. T. Banks at North Carolina State University, J. E. Banks at California State University, Monterey Bay, and J. D. Stark at Washington State University, Puyallup.

Dynamics and Pattern Formation in a ModifiedLeslie-Gower Model with Allee Effect and Bazykin Functional Response

Peng Feng
Florida Gulf Coast University, USA

In this paper, we study the dynamics of a diffusive modified LeslieGower model with the multiplicative Allee effect and Bazykin functional response. We give detailed study on the stability of equilibria. Nonexistence of nonconstant positive steady state solutions are shown to identify the rage of parameters of spatial pattern formation. We also give the conditions of Turing instability and perform a series of numerical simulations and find that the model exhibits complex patterns.

Traveling Waves for a Biological System

Zhaosheng Feng
University of Texas-Rio Grande Valley, USA

In this talk, we study the case that some species migrate from densely populated areas into sparsely populated areas to avoid crowding, and investigate a more general reaction-diffusion system by considering density-dependent dispersion as a regulatory mechanism of the cyclic changes. Here the probability that an animal moves from the point x1 to x2 depends on the density at x1. Under certain conditions, we apply the higher terms in the Taylor series and the center manifold method to obtain the local behavior around a non-hyperbolic point of codimension one in the pase plane, and use the Lie symmetry reduction method to explore bounded traveling wave solutions.

A Biophysical Model of Contact-Mediated Dormancy of Archaea by Viruses

Hayriye Gulbudak
Georgia Institute of Technology, USA
Joshua S. Weitz

The canonical view of the interactions between viruses and their microbial hosts presumes that changes in host and virus fate require the initiation of infection of a host by a virus. That is, first virus particles diffuse randomly outside of host cells, then the virus genome enters the target host cell, and only then do intracellular dynamics and regulation of virus and host cell fate unfold. Intracellular dynamics may lead to the death of the host cell and release of viruses, to the elimination of the virus genome through cellular defense mechanisms, or the integration of the virus genome with the host as a chromosomal or extra-chromosomal element. Here we revisit this canonical view, inspired by recent experimental findings of Bautista and colleagues (mBio, 2015) in which the majority of target host cells can be induced into a dormant state when exposed to either active or de-activated viruses, even when viruses are present at low relative titer. We propose that both the qualitative phenomena and the quantitative time-scales of dormancy induction can be reconciled given the hypothesis that cellular physiology can be altered by contact on the surface of host cells rather than strictly by infection. In order to test this hypothesis, we develop and study a biophysical model of contact-mediated dynamics involving virus particles and target cells. We show how virus particles can catalyze cellular transformations amongst many cells, even if they ultimately infect only one (or none).

Semi-Kolmogorov's Predator-Prey Models in Varying Environments

Xiaoying Han
Auburn University, USA
Tomas Caraballo, Renato Colluci

In this talk I will introduce several semi-kolmogorov type of predation models with indirect effect, including a nonautonomous model, a random model with real bounded noise, a stochastic model with white noise, and a stochastic model with continuous-time Markov chain. In particular I will talk about the long term behavior of these systems.

Evolutionary Games on the Lattice: Death-Birth Updating Process

Nicolas Lanchier
Arizona State University, USA
Stephen Evilsizor

The death-birth updating process is an example of spatial evolutionary game where the players, located on the infinite square lattice and characterized by one of two possible strategies, update their strategy at rate one by mimicking one of their neighbors chosen at random with a probability proportional to their payoff. In this talk, the stochastic spatial model is compared with its deterministic non-spatial counterpart: the replicator equation. Though strategies can coexist in both models, the inclusion of space significantly reduces the coexistence parameter region. Moreover, for the prisoner's dilemma game, at least in the presence of one-dimensional nearest neighbor interactions, cooperators can out-compete defectors, which contrasts with the replicator equation in which the defectors always win. The main ingredients to prove these results are block constructions, coupling arguments and optional stopping for martingales.

Mathematical Assessment of Methamphetamine Epidemic Among Men Who Have Sex with Men

Aprillya Lanz
Norfolk State University, USA
Abba Gumel

Methamphetamine is an addictive stimulant that releases high levels of neurotransmitter dopamine. The use of methamphetamine has shown to increase libido and reduces inhibition. As a result, methamphetamine is commonly used among men who have sex with men to initiate, enhance, and prolong sexual encounters. This, in turns, promotes high risk sexual behavior in this community of methamphetamine users which increases the risk of acquiring an STD. Furthermore, studies have shown that the use of methamphetamine is associated with more frequent risky sexual behaviors among HIV positive men when compared with HIV negative men. In this presentation, we will present a compartmental model that represents the dynamics of methamphetamine abuse in the men who have sex with men community from a mathematical perspective. The model considers different stages of progression of meth use and individuals who are temporary or permanently quitters. The analysis of the model is presented in terms of the meth generation number, a threshold value typically known as $\mathbb{R}_0$. It is shown that the model has multiple equilibria for which their stabilities are determined. Furthermore, numerical simulations are performed along with sensitivity analysis to determine important parameters to the model.

Fighting Mosquito-Borne Diseases with Sterile Mosquitoes

Jia Li
University of Alabama in Huntsville, USA

To prevent the transmissions of mosquito-borne diseases, sterile mosquitoe become an effective weapon. To study the impact of releasing sterile mosquitoes into the field of wild mosquitoes, we formulate mathematical models of interactive wild and sterile mosquitoes, considering different strategies of releases. Density-dependent vital rates are included and Allee effects are incorporated in the functional mating rates. With fundamental analysis of the dynamics of the interactive mosquitoes, we then introduce them into simple compartmental malaria transmission models. We study the dynamics of the simple malaria model connecting with the mosquito models, and investigate the impact of the different strategies of releases on the disease transmissions.

Dynamics of Low and High Pathogenic Avian Influenza in Wild and Domestic Bird Populations

Maia Martcheva
University of Florida, USA
N. Tuncer, Juan Torres, M. Barfield, R.D. Holt

Avian influenza H5N1 is at present the most dangerous zoonotic disease infecting wild and domestic birds. Should the virus mutate and become efficiently human-to-human transmittable, a pandemic will occur with high mortality. Avian influenza H5N1 exists in two forms: Low pathogenic (LPAI) and high pathogenic (HPAI). In this talk we build a model of LPAI and HPAI in wild and domestic birds. Birds, wild and domestic, who have been priorly infected with LPAI are partially protected against HPAI. We compute the relevant reproduction numbers and invasion reproduction numbers. We find that the systems has a disease-free equilibrium, LPAI-only equilibrium, HPAI-only equilibrium and at least one coexistence equilibrium. Furthermore, the LPAI-only equilibrium and HPAI-only equilibrium are locally asymptotically stable under appropriate conditions on the reproduction numbers. In contrast, the coexistence equilibrium can lose stability and oscillations are possible. We show that the oscillations are caused by the cross-immunity and can exist in the wild bird system, separate from the domestic bird system. For a pathogen circulating in a multi-species system, species A is called a sink (source), if the pathogen cannot (can) sustain itself in species A without the inflow of infectives from other species. We investigate the sink/source status of LPAI and HPAI in wild and domestic birds.

Evolution of Biodiversity Patterns in Evolving Fluvial Landscapes

Rachata Muneepeerakul
University of Florida, USA
Enrico Bertuzzo, Andrea Rinaldo, Ignacio Rodriguez-Iturbe

Biodiversity patterns are governed by landscape structure and dispersal behaviors of organisms that live in it. Landscape, however, evolves, and organisms evolve their dispersal behaviors with it. How do biodiversity patterns change amid these geomorphological and biological evolutions? To address this question, we implement neutral metacommunity models in river network landscapes at different stages of the geomorphological evolution. Here, organisms compete for space according to neutral dynamics and at the same time adapt their dispersal behaviors. Preliminary results suggest that the landscape and dispersal evolutions affect various biodiversity patterns, namely relative species abundance, local and global richness, and spatial similarity of species compositions, in different ways. These points to interesting interplay between geomorphological and biological evolutions.

Seasonality in Disease Modeling

Jin Wang
University of Tennessee at Chattanooga, USA

The transmission and spread of many infectious diseases exhibit seasonal patterns, which can be mathematically described by non-autonomous differential equations with periodic coefficients. We compute the basic reproduction number, R_0, associated with such periodic epidemic models, using a non-trivial numerical procedure. We then analyze the threshold dynamics characterized by the basic reproduction number. Particularly, we establish the uniform persistence of the system when $R_0 > 1$. In addition, we present several examples to illuminate the results.

Special Session 97: Qualitative and Quantitative Techniques for Differential Equations arising in Economics, Finance and Natural Sciences

Rehana Naz, Centre for Mathematics and Statistical Sciences, Lahore School of Economics Lahore, Pakistan, Pakistan
Mariano Torrisi, Dipartimento di Matematica ed Informatica, Università di Catania, Italy
Imran Naeem, Lahore University of Management Sciences (LUMS), Pakistan
Celestin Wafo Soh, Mathematics Department, Jackson State University, USA

The differential equations play a vital role in many disciplines from natural to social sciences. Most of physical laws in natural sciences are expressed in terms of differential equations. In this session we try to integrate analysis, models and methods in the scope of natural sciences as well as social sciences framework. The Economists study dynamical systems for sustainable Economic growth. Stochastic differential equations are the standard models for financial quantities important in financial market. Biologists (Epidemiologists) investigate the determinants of health-related states (including disease) using mathematical tools. Differential equations are mathematically studied from several different perspectives; this session will focus on the Qualitative and Quantitative techniques (including numerical methods) for ordinary differential equations, partial differential equations, fractional differential equations, difference equations, stochastic differential equations, integro-differential equations. Potential topics, of this session, include but are not limited to:
– Economic growth theory – Optimal control – Differential equations modeling natural and economic models – Financial models e.g. Hamilton-Jacobi equation, Hamilton-Jacobi-Bellman equations, Option models, Black-Schole models – Equivalence transformations – Stability analysis – Numerical techniques for special problems in modeling – Symmetries, Differential Equations, and Applications – Modeling and Math Biology – Fluid Mechanics

Metachronal Beating of Cilia with the Effects of Different Shaped Nanoparticles in a Pump with Expanding Or Contracting Wall

Noreen Akbar
National University of Sciences and Technology, Pakistan

In this talk, metachronal beating of Cilia for Cu-water nanofluid will be considered with the effects of different shaped nanoparticles. For the analysis Hamilton-Crosser model will be used for the effective thermal conductivity of the nanofluids. In addition, heat transfer in human body will also be studied. Governing equations modification with long wavelength and low Reynold number approximation case will be presented. Exact solutions will be presented for the simplified equations. Behavior of different flow parameters for different shapes of nanosize particles will be discussed through graphs.

Flow and Heat Transfer Analysis for a Non-Newtonian Past Over a Porous Plate with Partial Slip Conditions

Yasir Ali
National University of Sciences and Technology, Pakistan

In this talk I will discuss a steady non-Newtonian slip boundary layer condition post over a porous plate embedded in a porous medium. The similarity transformations will be presented to transform the governing partial differential equations (PDEs) into a system of nonlinear ordinary differential equations (ODEs). The numerical solution will be discussed for the resulting ODEs. Physical interpretation for the velocity, temperature, skin friction coefficient, permeability, suction/injection parameter, Prandlt number and Nusselt number will be presented through graphs and tables.

Convergence of Aggregated Variability in Energy Consumption Data

Ai Ling Amy Poh
Okayama University, Japan
Evgeny Mozgunov, Tan Chin Woo

A variability of energy consumption is the total variance divided by total mean consumption. Real data shows convergence of aggregated variability with the number of customers. We investigate the mathematical reasons of this phenomenon, as well as the subtleties of convergence rate. We show that the results for convergence on real data are consistent with the prediction of a simple sum of random correlated variables.

Mixed Regimes of Fluids in Porous Media

Emine Celik
Texas Tech University, USA
Luan Hoang, Akif Ibragimov, Thinh Kieu

In porous media, there are three known regimes of fluid flows, namely, pre-Darcy, Darcy and Forchheimer/post-Darcy. Because of their different natures, these are usually treated separately in literature. To study complex flows, a single equation of motion is used to describe all three regimes and unify the mathematical treaments. Several scenarios and models are then considered for slightly compressible fluids. A nonlinear parabolic equation for the pressure is derived, which is degenerate when the pressure gradient is either small or large. We estimate the pressure and its gradient in terms of initial and boundary data. Moreover, we establish the continuous dependence of the solutions on the initial, boundary data, and other physical parameters. This is a joint work with Luan Hoang, Akif Ibragimov and Thinh Kieu.

Symmetry Classification and Peakon Solutions for a Homogeneous Family of Partial Differential Equations

Priscila da Silva
Universidade Federal do ABC, Brazil
Igor Leite Freire

In this talk we discuss a 4-parameter homogeneous family of equations in terms of weak solutions and conservation laws. A Lie point symmetry classification is performed. We also show that the L^2 norm is conserved in the solutions of the equation. Furthermore, we find relations between the parameters so the equation admits a special kind of solution with peaks.

Some Conservation Laws and Exact Solutions for a New Class of Nonlinear Dispersive Equations

Erica de Mello Silva
Federal University of Mato Grosso, Brazil
Wescley Luiz de Souza

In this talk we consider a class of highly nonlinear dispersive equations with variable coefficients, $vcK(m, n)$, that was recently derived from the well known Rosenau-Hyman compacton equation by means of the Lie symmetry approach. We discuss about its nonlinear self-adjointness and present some conservation laws and exact solutions for the particular $vcK(1, 1)$, $vcK(2, 2)$, and $vcK(3, 3)$ equations.

Global Stability of a Multistrain SIS Model with Superinfection

Attila Denes
Bolyai Institute, University of Szeged, Hungary
Yoshiaki Muroya, Gergely Röst

We study the global stability of a multistrain SIS model with superinfection. We present an iterative procedure to calculate a sequence of reproduction numbers, and we prove that it completely determines the global dynamics of the system. We show that for any number of strains with different infectivities, the stable coexistence of any subset of the strains is possible, and we completely characterize all scenarios. As an example, we apply our method to a three strains model. We also show a generalization to a group multistrain model with patch structure.

Discovery of Hysteresis Models Through the Symmetry Principle

Bassirou Diatta
Jackson State University, USA
Celestin Wafo Soh

Our goal in this talk is to employ the symmetry principle to obtain rate-independent hysteresis operators. The model we consider is a diffusion equation with hysteresis. Since the combination of the diffusion equation and the hysteresis constraint is a differential-algebraic equation, we assume that the hysteresis operator is smooth enough so that we may replace the hysteresis constraint by the vanishing of the differential of the hysteresis operator. We analyze this new form viz. the coupling of the diffusion model and the vanishing of the differential of the hysteresis operator, using Lie symmetry algorithm. Specifically, we seek maximally symmetric hysteresis models. The direct Lie approach to this problem leads to determining equations for the symmetries that are extremely difficult to solve. In order to mitigate this problem, we employ the classification of low-dimensional Lie algebra following the suggestion of Lahno et al. (Journal of Physics A: Mathematical and General 32.42 (1999): 7405).

Lie Symmetry Treatment for Pricing Options with Transactions Costs Under the Fractional Black-Scholes Model

Bienvenue Feugang Nteumagne
University of Pretoria, So Africa
B F Nteumagne, E Mare, E Pindza

We apply Lie symmetries analysis to price and hedge options in the fractional Brownian framework. The reputation of Lie groups is well spread in the area of Mathematical sciences and lately, in Finance. In the presence of transactions costs and under fractional Brownian motions, analytical solutions become difficult to find. Lie symmetries analysis allows us to simplify the problem and obtain new analytical solution. In this paper, we investigate the use of symmetries to reduce the partial differential equation obtained and find the analytical solution. We then proposed a hedging procedure and calibration technique for these types of options, and test the model on real market data. We show the robustness of our methodology by its application to the pricing of digital options.

A Camassa-Holm Type System Admitting Peakon and Kink Solutions

Igor Freire
UFABC, Brazil
Priscila Leal da Silva

In this talk we discuss a two component Camassa-Holm type system depending on a continuous parameter, controlling the nonlinearities. A Lie point symmetry classification is carried out and the existence of solutions, particularly peakon, multi-peakon, kink and multi-kink, is investigated.

A Continuous-In-Time Financial Model.

Emmanuel Frenod
Universite Bretagne Sud, France
Tarik Chakkour

I will present a continuous-in-time model which is designed to be used for the finances of public institutions. This model is based on using measures over time interval to describe loan scheme, reimbursement scheme and interest payment scheme; and, on using mathematical operators to describe links existing between those quantities. The consistency of the model with respect to the real world will be illustrated using simple examples. Then the model will be used on simplified examples in order to show its capability to be used to forecast consequences of a decision or to set out a financial strategy

Group Analysis Applied to Chemotaxis

Kesh Govinder
UKZN, So Africa
P M Tchepmo Djomegni

Chemotaxis is the phenomenon of cell movement in response to stimuli. This broad concept covers a variety of processes in the cell cycle and, in particular, is crucial to the understanding of the evolution of tumours.
We undertake a group theoretic approach to find travelling wave solutions for a hyperbolic chemotaxis model. We forsake the usual ansatz approach and, using the invariance properties of the equations, show that more general solutions can be obtained. This systematic approach enables us to maintain the diffusivity terms in the model, thereby obtaining truly diffusing solutions.
We expand our model by considering diffusion of substrates, degradation of chemoattractants and cell growth (constant and linear growth rate). In all cases we show that the system is still tractable. By applying realistic boundary conditions we isolate biologically applicable solutions.
Chemotaxis provides another example of the utility of Lie's method of extended groups applied to differential equations.

Parabolic Problems with Dynamic Or Wentzell Boundary Conditions in Spaces of Hölder Continuous Functions

Davide Guidetti
University of Bologna, Italy

It is aim of this talk to illustrate some maximal regularity results in spaces of Hölder continuous functions for parabolic systems with dynamic or Wentzell boundary conditions, recently obtained by the author. We shall consider, in particular, the case that in the boundary condition there appears a diffusion term consisting of a second order strongly elliptic tangential operator.

Estimation of Jump Times in Daily Share Prices of Nikkei 225 Stock Index Using a Jump Diffusion Model

Shuya Kanagawa
Tokyo City University, Japan

We investigate the daily share prices of the Nikkei 225 stock index to identify jump times of the stock index using a jump diffusion model, which consists of the Black-Scholes model with stochastic volatility and a compound Poisson process. Since the data of daily share prices of the Nikkei 225 stock index are observed at discrete times, it is difficult to find real jump times from the data. In this paper, we consider how to separate jump times from the observed times. The volatility of the stock index is estimated by the historical volatility from the observation of daily share prices. We also refer to the number of daily share prices for historical volatility and show that the number is essential for the accuracy of identifying of jump times.

Optimal Dosing Strategies in the Presence of Drug Resistant Bacteria

Adnan Khan
Lahore University of Management Sciences, Pakistan
Mudassar Imran

In this talk we discuss an ODE based model for the appearance of drug resistant bacteria and efficient antibiotic dosing strategies. We examine both periodic and optimal discrete antibiotic dosing. Drug resistant bacteria are affected to a lesser degree by antibiotic treatments; we are interested in finding efficient and successful antibiotic dosing strategies in the presence of both susceptible and resistant bacteria. We propose and analyze a model for the appearance of drug resistant bacteria, in the presence of antibiotic treatment. Using optimal control theory, we discuss antibiotic treatment strategies that minimize the total bacterial population while minimizing the antibiotic costs at the same time.

Stagnation Point Flow Over a Stretching Cylinder with Variable Viscosity and Heat Generation/absorption

Farzana Khan
Quaid-i-azam University, Pakistan
M.Y Malik

This article is focused to discuss the behavior of stretching cylinder of viscous fluid near stagnation point with variable viscosity. The effects of heat generation/absorption are also encountered. Comparison between solutions obtained through HAM and shooting method is presented. The effect of variation in parameters on velocity and temperature fields are shown graphically.

Solitons and Lumps by Bilinear Techniques

Wen-Xiu Ma
University of South Florida, USA

We will talk about bilinear techniques to solve nonlinear partial differential equations. Resonant solitons and lumps are generated through computational algorithms implemented in Maple. The key step is to realize applicability of the linear superposition principle and positivity of real multivariate polynomials.

Properties of First Integrals for Scalar Linear Third-Order ODEs of Maximal Symmetry

Komal Mahomed
Wits University, So Africa

The simplest scalar linear third-order ordinary differential equation has 7 Lie point symmetries. We find the classifying relation between the symmetries and the first integrals for the simplest equation. It is shown that the maximal Lie algebra of an integral for the simplest equation is unique and 4-dimensional. Further, we show that the Lie algebra of the simplest linear third-order equation is generated by the symmetries of the 2 basic integrals. We also present counting theorems of the symmetry properties of the integrals for linear third-order ODEs. We provide insights as to how one can generate the full Lie algebra of higher-order ODEs of maximal symmetry from two of their basic integrals.

Numerical Aspects of Carreau Fluid Over a Stretching Sheet with Slip Conditions

Muhammad Yousaf Malik
Quaid-i-Azam University, Pakistan
Taimoor Salahuddin

In this paper Carreau fluid model over a stretching sheet with slip conditions is discussed. The non-linear partial differential equation is converted into ordinary differential equation by using similarity transformations. The solution of the required equation subject to required boundary conditions is obtained with the help of implicit finite difference scheme known as Keller box method. The impact of different pertinent physical parameters on velocity profile are plotted through graphs. Skin friction coefficient is also tabulated. Moreover, comparison has been made with the previous literature in order to check the accuracy of the method.

Dynamics Analysis and Optimal Control in a Coupled Environment-Growth Model

Tufail Malik
Khalifa University, United Arab Emirates
Davide La Torre, Danilo Liuzzi, Oluwaseun Sharomi, Rachad Zaki

We analyse the interaction between economic growth and pollution accumulation, where pollution is a by-product of production. The stock of pollution negatively affects production. A share of the investments is devoted to an environmental tax that is used to dampen the accumulation of pollution. This economic-environmental model is described by a pair of ordinary differential equations whose dynamics and steady state characteristics are studied. Then we look at this ambient environment from the point of view of a social planner who can act on consumption and taxation, taking the dynamics of capital and pollution as constraints.

Entropy Generation Analysis for the Peristaltic Transport of a Nano Fluid in a Curved Channel Having Compliant Walls

Ehnber Maraj
Heavy Industries Taxila Education City University, Pakistan

In this talk I will inspects the peristaltic flow of a nano fluid in a curved channel. The governing equations of nano fluid model for curved channel with curvature effects will be presented. The coupled differential equations in simplified by using long wave length and low Reynolds number assumptions will be discussed. Homotopy perturbation solution for the governing differential equations will be discussed. Entropy generation analysis with the physical features of pertinent parameters have been presented for velocity, temperature, concentration, entropy number and stream functions at the end of the talks.

Conservation Laws and Structure-Preserving Integration Methods for a Special Class of PDEs

Brian Moore
University of Central Florida, USA

Conformal Hamiltonian systems, which are characterized by contraction of a symplectic 2-form and dissipation of energy, are generalized to nonlinear PDEs with convection, damping, diffusion, and/or dispersive terms. The structure of these PDEs is very special, but there are several PDE models with this structure that arise in applications, and their conservation laws have a straightforward interpretation in a discrete setting, which paves the way for construction of discretizations that preserve them. Presentation of numerical methods of this type will be followed by a brief exploration of the benefits of using such discretizations.

Practical Implementation of Algorithmic Trading

Evgeny Mozgunov
Caltech, USA

Creating a winning trading algorithm is a multistep process. The steps are: (i) get familiar with the trading platform and its limitations (ii) set up optimization and crossvalidation routines (iii) come up with several strategies and backtest them using (ii). Every step has its own tricks of trade that may be counterintuitive for the beginner. Some economists even think that stock prices fundamentally cannot be predicted. To be confident in either point of view, one should take a machine learning approach to the problem. The problem of predicting stock prices can be compared to image recognition and other machine learning classics. Difference is that it's not enough to predict well, one should find out how to make money using a prediction.

Nonstandard Finite Difference Methods for Weakly and Strongly Coupled Systems of Convection-Diffusion Equations

Justin B. Munyakazi
University of the Western Cape, So Africa

There have been relatively few publications on coupled systems of singularly perturbed convection-diffusion equations in comparison with their scalar counterparts. In this talk we consider both weakly and strongly coupled systems. While the former satisfy a maximum principle, the latter do not. We propose two nonstandard finite difference methods to integrate these systems (one for either case) and prove their uniform convergence. Numerical investigations are presented to confirm our theoretical findings. Finally, comparison with existing methods is given.

A New Method to Construct Approximate First Integrals of Generalized Hamiltonian Systems

Imran Naeem
Lahore University of Management Sciences, Pakistan
R. Naz, I. Naeem

The notion of approximate generalized Hamiltonian system is presented. The formulas for approximate Hamiltonian operators determining equations and approximate first integrals are provided explicitly. In order to show effectiveness of approach developed here, it is applied to establish the approximate first integrals of two perturbed coupled Duffing-Van der Pol oscillators. Both resonant and non-resonant cases are investigated in detail. Under a certain parameter restriction, for resonant case we obtain two stable approximate first integrals whereas only one stable first integral is attained for the non-resonant case.

A Current Value Hamiltonian Approach for Discrete Time Optimal Control Problems Arising in Economic Growth Theory

Rehana Naz
Lahore School of Economics, Pakistan

The notion of current value Hamiltonian is presented for the discrete time optimal control problems for economic growth models. Pontrygin-type maximum principle is developed for the current value Hamiltonian systems of nonlinear difference equations. A new method termed as a discrete time current value Hamiltonian method is established for the construction of first integrals for current value Hamiltonian systems of ordinary difference equations arising in Economic growth theory. The newly developed technique is explained with the help of a simple illustrative example. In order to show effectiveness of approach it is applied to the discrete time fundamental models of economic growth theory: the Ramsey model and the Lucas-Uzawa model.

Dynamic Optimization Problems in Economic Growth Theory: Current Value Hamiltonian and Current Value Lagrangian Techniques

Rehana Naz
Lahore School of Economics, Pakistan
F. M. Mahomed, Azam Chaudhry

The dynamic optimization problems can be solved by calculus of variation and by optimal control techniques. We have developed a new approach termed as a discount free or current value Lagrangian method

for construction of first integrals for dynamical systems of ordinary differential equations arising in Economic growth theory. It is shown how one can utilize the Legendre transformation in a more general setting to provide the equivalence between a current value Hamiltonian and a current or discount free Lagrangian when it exists. The approach is algorithmic and applies to many state variables of the Lagrangian. The discount free Lagrangian naturally arises in economic growth theory and many other economic models when the control variables can be eliminated at the outset which is not always possible in optimal control theory applications of economics. We explain our method with the help of few widely used economic growth models. We point out the difference of this approach and that of the more general current value Hamiltonian method proposed earlier for a current value Hamiltonian which is applicable in a general setting involving time, state, costate and control variables. It is worthy to mention here that a discount free or current value Lagrangian method deals with calculus of variation problems whereas the current value Hamiltonian method is applied to the optimal control problems.

Criteria of Existence of Lorenz Attractors

Ivan Ovsyannikov
University of Bremen, Germany

The Lorenz attractor is the well-known example of a robust chaotic behavior. Lorenz attractors were observed in various models, both theoretical an applied. However, a fully analytic proof of the birth of such attractors has been not generally given so far. I will demonstrate how the criteria developed by Shilnikov and his group can help in this.
The second part of the talk will be devoted to the proof of the birth of discrete Lorenz attractors in diffeomorphisms. Such attractors have a much more complicated structure than the classical ones. It is expected that they may allow homoclinic and heteroclinic tangencies, i.e. demonstrate the so-called “wild“ behavior.

Intrinsic Nonlinear Dynamical Systems in Isotropic Turbulence

Zheng Ran
Shanghai University, Peoples Rep of China

The central problem of fully developed turbulence is understanding the energy cascading process and multiscale interaction. Update, there is no deductive theory which leads to a full physical understanding or mathematical formulation. Based on the Karman-Howarth equation in 3D incompressible fluid, a new isotropic turbulence scale evolution equation and its related theory progress, the existence of nonlinear dynamic system mearsured by turbulence Taylor microscale was proven. The present results indicate that the enery cascading process has remarkable similarities with the deterministic construction rules of the logistic map. The cascade appears as an infinite sequence of period-doubling vortex bifurcations. Based on this new approach, this project treats isotropic turbulence from the point of view of dynamical systems. The exposition centres around a number of important issues for turbulence behavior in the new nonlinear dynamical system we found. Thus, the modern theory of fractal and multi-fractal now plays a major role in understanding the scaling behavior in isotropic turblence.

Optimal Control of the Dendrite Structure Using Magnetic Field

Amer Rasheed
Lahore University of Management Sciences (LUMS)
Pakistan, Pakistan

In this paper, we present the optimal control of a phase field model, recently developed by A. Rasheed and A. Belmiloudi [1], which represents the effect of magnetic field on the evolution of dendrite during the solidification process of a binary alloy in an isothermal environment. The aim of this study is to control the desired dynamics of the dendrite by using magnetic field as a control variable. In the control problem, the cost functional measures the distance between the calculated and desired dynamics. We have established the existence results and optimality conditions along with the adjoint problem.

References

[1] A. Rasheed, A. Belmiloudi, Mathematical modelling of numerical simulations of dendrite growth using phase-field method with a magnetic field effect, Communications in Computational Physics, 14(2), pp. 477-508, 2013.

Well-Posedness and Global Attractors for a Non-Isothermal Viscous Relaxation of Nonlocal Cahn-Hilliard Equations

Joseph Shomberg
Providence College, USA

In this preliminary report we will discuss a non-isothermal viscous relaxation of some nonlocal Cahn-Hilliard equations. This perturbation problem generates a family of solution operators, exhibiting dissipation and conservation. The solution operators admit a family of compact global attractors that are bounded in a more regular phase-space. Our aim is to prove that the family of global attractors satisfies an upper-semicontinuity type estimate, whereby the difference between trajectories of the relaxation problem and the limit isothermal non-viscous problem is explicitly controlled, in the topology of the relaxation problem, in terms of the relaxation parameters as well as other structural parameters.

Dynamics of Price and Wealth in a Multi-Group Asset Flow Model

David Swigon
University of Pittsburgh, USA
M. DeSantis

Recently developed model of asset flow dynamics allows one to use the tools of nonlinear dynamics to study various market conditions and trading scenarios. I will present the results of a study in which we analyzed the influence of investor strategies on their wealth. We find that the constant rebalanced portfolio (CRP) strategy, in which investor divides his wealth equally between different types of assets and maintains those proportions constant as the price changes, is optimal in that it minimizes the potential losses incurred during price fluctuations in the market. We also show that any other trading strategy can be taken advantage by other investors in the market and lead to a loss of wealth, via predatory trading behavior.

Approximation of Solutions of Multi-Dimensional Linear Stochastic Differential Equations

Hiroshi Takahashi
Tokyo Gakugei University, Japan

Yoshihara et al. showed that solutions of some linear stochastic differential equations may be approximated by the solutions of the corresponding difference equations defined by strong mixing sequences, In this talk, we generalized the result obtained by Yoshihara et al. to the multi-dimensional linear stochastic differential equations. We also give some examples concerning some finance models.

Symmetry Reductions for a Quantum Hydrodynamical Model

Rita Tracina
University of Catania, Italy

For the description of charge carrier transport in semiconductors, continuum models have interested in the last years applied mathematicians and engineers on account of their applications in the design of electron devices. With shrinking dimensions of submicron semiconductor devices, the quantum effects are no longer negligible. New symmetry reductions and exact solutions are presented for a quantum hydrodynamical model.

Applications of PDE Methods on Networks to Data and Image Analysis

Yves van Gennip
University of Nottingham, England

In this talk we will see some applications in data analysis and image analysis, that use partial differential equations techniques to solve problems on networks. An example is graph based image segmentation.

Hypecomplexification of Systems of Ordinary Differential Equations and Lagrangian

Celestin Wafo Soh
Jackson State University, USA
F. M. Mahomed

Consider a system of even-order ordinary differential equations that can be hypercomplexified. Can we always construct a Lagrangian of the system from that of the base equation? We provide a positive answer to this question under mild nondegeneracy assumptions. Specifically, we establish that under these restrictions, the components of the Lagrangian of the base equation are Lagrangians of the hypercomplexified system.

Power Spectra Methods Linking Stochastic Reaction-Diffusion Systems and Their Deterministic Limits

Thomas Woolley
University of Oxford, England
Ruth E. Baker, Eamonn A. Gaffney, Philip K. Maini

Being able to create and sustain robust, spatial-temporal inhomogeneity is an important concept in many areas, including developmental biology. Generally, the mathematical treatments of such systems have used continuum hypotheses of the interacting populations to produce reaction-diffusion partial differential equations (PDEs). However, these ignore any sources of intrinsic stochastic effects. We address this concern by developing analytical Fourier methods which allow us to rigorously link the deterministic PDEs with their stochastic analogues.
Further, we include domain growth into our framework as it has been recently shown that growth allows a deterministic reaction-diffusion patterning system to robustly double its pattern mode. However, this robustness feature is lost when stochasticity is included and, thus, we seek to use the presented Fourier methods to understand this patterning doubling breakdown. Further, we use our insights to suggest how to regain the robustness property, even in this stochastic setting.

Statistical Analysis on Modified Black-Scholes Models with Heavy-Tailed Distributions

Xing Yang
Jackson State University, USA

In this paper the Black-Scholes equation is modified with several heavy-tailed distributions. A statistical analysis is conducted on the new modified models and simulations are implemented. Results from simulation are compared and discussed to choose the more realistic model for option pricing.

The Exact and Approximate Solutions of the Fractional Delay Partial Differential Equations Using the Reproducing Kenel Method

Lihong Yang
Harbin Engineering University, Peoples Rep of China
Zhong Chen, Zhisheng Shuai

Fractional delay partial differential equations have recently been applied in various areas of engineering, finance,bio-engineering and others. In this talk, we use the reproducing kernel method to solve the fractional delay partial differential equations in the Caputo derivative sense. Numerical simulation with the exact solutions is presented.

Numerical Investigation of Couple Stress Fluid Over an Elasting Stretching Sheet with Pressure Dependent Viscosity

Iffat Zehra
Air University Islamabad Pakistan, Pakistan
Malik Muhammad Yousaf, Sohail Nadeem

In this paper, we have examined the steady flow of couple stress fluid with pressure dependent viscosity between two parallel plates in which the lower plate is stretched with while the upper plate is moving with constant velocity. The governing two dimensional nonlinear partial differential equations are simplified by using suitable form of velocities. The simplified nonlinear ordinary differential equations are solved numerically. The physical features of pertinent parameters have been investigated through graphs and numerical data.

Special Session 98: Inverse Problems and Imaging and Their Applications

Kai Huang, Florida International University, USA
Yuanchang Sun, Florida International University, USA

Inverse problems and imaging have found numerous applications in many areas from science to engineering. The mathematical studies of these problems pose significant analytical and computational challenges. This special session of AMIS conference aims to bring together researchers to promote exchange of ideas, and present recent developments on the mathematical analysis and computational methods in this area.

A Rigorous Mathematical Theory for Electromagnetic Field Enhancement in Metallic Nanogaps

Junshan Lin
Auburn University, USA

Subwavelength apertures and gaps on surfaces of noble metals (e.g., gold or silver) induce strong electric field and extraordinary optical transmission. This remarkable phenomenon can lead to novel applications in biological and chemical sensing, spectroscopy, and THz semiconductor devices. In this talk, I will present a quantitative analysis for the field enhancement when an electromagnetic wave passes through tiny metallic gaps. Based upon a rigorous study of the perfect electrical conductor model, we show that enormous electric field enhancement occurs inside the nanogap when there is extreme scale difference between the wavelength of radiation and the thickness of metal films. The analysis also leads to efficient asymptotic numerical method for calculating the wave field in the nanostructure. The ongoing work along this research direction will also be highlighted.

The Difference of L1 and L2 for Compressive Sensing and Image Processing

Yifei Lou
University of Texas Dallas, USA
Penghang Yin, Jack Xin

A fundamental problem in compressed sensing (CS) is to reconstruct a sparse signal under a few linear measurements far less than the physical dimension of the signal. Currently, CS favors incoherent systems, in which any two measurements are as little correlated as possible. In this talk, I will present a novel non-convex approach, which is to minimize the difference of L1 and L2 norms (L1-L2) in order to promote sparsity. Some theoretical aspects of L1-L2 minimization are discussed and efficient minimization algorithms are constructed and analyzed based on the difference of convex (DC) function methodology. Experiments demonstrate that L1-L2 improves L1 consistently and it outperforms Lp for highly coherent matrices. Finally, a recovery problem of point sources from a set of low-frequency measurements will be present, showing advantages of L1-L2 over L1 when a necessary condition for perfect reconstruction is not satisfied.

The Degrees of Freedom of Partly Smooth Regularizers

Samuel Vaiter
CNRS & Univ. Bourgogne, France
Charles-Alban Deledalle, Jalal M. Fadili, Gabriel Peyré, Charles Dossal

In this talk, we are concerned with regularized regression problems where the prior regularizer is a proper lower semicontinuous and convex function which is also partly smooth relative to a Riemannian submanifold. This encompasses as special cases several known penalties such as the Lasso (ℓ^1-norm), the group Lasso ($\ell^1 - \ell^2$-norm), the ℓ^∞-norm, and the nuclear norm. This also includes so-called analysis-type priors, i.e. composition of the previously mentioned penalties with linear operators, typical examples being the total variation or fused Lasso penalties. We study the sensitivity of any regularized minimizer to perturbations of the observations and provide its precise local parameterization. Our main sensitivity analysis result shows that the predictor moves locally stably along the same active submanifold as the observations undergo small perturbations. This local stability is a consequence of the smoothness of the regularizer when restricted to the active submanifold, which in turn plays a pivotal role to get a closed form expression for the variations of the predictor w.r.t. observations. We also show that, for a variety of regularizers, including polyhedral ones or the group Lasso and its analysis counterpart, this divergence formula holds Lebesgue almost everywhere. When the perturbation is random (with an appropriate continuous distribution), this allows us to derive an unbiased estimator of the degrees of freedom and of the risk of the estimator prediction. Our results hold true without requiring the design matrix to be full column rank. They generalize those already known in the literature such as the Lasso problem, the general Lasso problem (analysis ℓ^1-penalty), or the group Lasso where existing results for the latter assume that the design is full column rank.

A Gesture-Based Instruction and Input Device Using Acoustic Waves

Yuliang Wang
Hong Kong Baptist University, Peoples Rep of China
Hongyu Liu, Can Yang

A novel method is proposed for the recognition of gestures using acoustic waves. The gestures are modeled by acoustic scatterers whose shapes are drawn from a prescribed dictionary and the recoginition is modeled as an inverse acoustic scattering problem. The incident wave is generated from a fixed point exterior of the scatterer and the scattered field is measured at a bounded surface containing the source point. The recognition algorithm consists of two steps and requires two incident wave of different wavenumber. The approximate location of the scatterer is firstly determined by using the measured data at small wavenumber and the shape of the scatterer is then identified using the computed location of the scatterer and the measured data at a regular wavenumber. Numerical experiments show the proposed method is computationally efficient and works with full or phaseless backscattering data of small aperture.

Asynchronous Decentralized Consensus with Delayed Stochastic Gradient

Xiaojing Ye
Georgia State University, USA

Decentralized consensus optimization plays a critical role in the emerging technology of decentralized network computing. In contrast to traditional centralized computing, the nodes in a decentralized network hold partial objective/data privately and the goal is to let them collaboratively solve for the global optimum. In this talk, we provide several efficient decentralized consensus algorithms. In particular, we focus on the accelerated consensus algorithm and the cases where only delayed stochastic gradient information are available during computing process. Numerical results including application in seismic tomography using wireless sensor networks will be presented.

Constrained and Regularized Non-Negative Deconvolution: an Application to Speech Signal Dereverberation

Meng Yu
Knowles Corp., USA

A multi-channel spectral decomposition method is proposed based on a constrained and regularized non-negative deconvolution framework. The reverberation of speech signal is modeled as a convolution operation in the spectral domain. Using the generalized Kullback-Leibler (KL) divergence, we decompose the reverberant magnitude spectrum into clean magnitude spectrum convolved with a deconvolution filter. The multi-channel deconvolutions are jointly estimated by enforcing the cross-channel cancellation constraint and iteratively solved by a multiplicative algorithm to achieve multi-channel speech dereverberation.

Special Session 99: Theory and Applications of Boundary-Domain Integral and Pseudodifferential Operators

Sergey Mikhailov, Brunel University London, England
David Natroshvili, Georgian Technical University, Rep of Georgia

Integral equation technique is a very efficient tool in the study of boundary and initial-boundary value problems for linear and nonlinear partial differential equations arising in many static and dynamic mathematical models of physical, biological, and engineering processes. The goal of this special session is to discuss recent progress in the theory of boundary-domain integral and pseudodifferential operators and their applications in Mathematical Physics, Solid and Fluid Mechanics, Wave Scattering Problems, Engineering Mathematics, etc.

Asymptotic Analysis of Dynamical Interface Crack Problems for Metallic and Electro-Magneto Elastic Composite Structures

Otar Chkadua
Razmadze Mathematical Institute, Rep of Georgia
T.Buchukuri, D. Natroshvili

We investigate the solvability and asymptotic properties of solutions to 3-dimensional dynamical interface crack problems for metallic and electro-magneto-elastic composite bodies. We give a mathematical formulation of the physical problems when the metallic and electro-magneto-elastic bodies are bonded along some proper parts of their boundaries where interface cracks occur.
Using the Laplace transform, potential theory and theory of pseudodifferential equations on a manifold with boundary, the existence and uniqueness theorems are proved. We analyse the regularity and asymptotic properties of the mechanical and electromagnetic fields near the crack edges and near the curves where the different boundary conditions collide. In particular, we characterize the stress singularity exponents and show that they can be explicitly calculated with the help of the principal homogeneous symbol matrices of the corresponding pseudodifferential operators. For some important classes of anisotropic media we derive explicit expressions for the corresponding stress singularity exponents and show that they essentially depend on the material parameters. The questions related to the so called oscillating singularities are treated in detail as well.
Acknowledgements:This research was supported by Rustaveli Foundation grant No. FR/286/5-101/13: Investigation of dynamical mathematical models of elastic multicomponent structures with regard to fully coupled thermo-mechanical and electromagnetic fields.

A Time-Dependent Boundary-Field Equation Approach to Fluid-Thermoelastic Solid Interaction

George Hsiao
University of Delaware, USA

This paper presents a combined field and boundary integral equation method for solving time-dependent scattering problem of a thermoelastic body immersed in a compressible, inviscid and homogeneous fluid. The approach here is a generation of the coupling procedure employed in [G.C. Hsiao, J.F. Sayas and R.J. Weinacht. A Time-Dependent Fluid-Structure Interaction, *Math. Meth. Appl. Sci.*, DOI: 10.1002/sim.0000] for treating the time-dependent fluid-structure interaction problem. By using integral representation for the solution in the infinite exterior domain occupied by the fluid, the problem is reduced to one defined only over the finite region occupied by the solid, with nonlocal boundary conditions. We analyze this nonlocal boundary problem as the Lubich approach for time-dependent boundary integral equations via the Laplace transform with an essential feature in terms of data in the time domain directly. Existence and uniqueness results are established. Galerkin semi-discretization approximations are derived and error estimates are obtained. A full discretization based on the convolution quadrature method is also outlined. Finally, to demonstrate the validity of the method, a numerical experiment is included for the special case of the time-dependent fluid-structure interaction.

The Haseman Boundary Value Problem with Slowly Oscillating Data

Yuri Karlovich
Universidad Autónoma del Estado de Morelos, Mexico

The talk is devoted to studying the Haseman boundary value problem$\Phi^{\,+} \circ \alpha = G\Phi^{-} + g$ on a star-like Carleson curve Γ composed by logarithmic spirals in the setting of Lebesgue spaces, where$\Phi^{\,\pm}$ are angular boundary values of an unknown analytic function Φ on Γ ,G and g are given functions, and α is an orientation-preserving homeomorphism of Γ onto itself. This problem is reduced to the equivalent singular integral operator with a shift $T = V_\alpha P_+ + GP_-$

on a Lebesgue space $L^p(\Gamma)$, where the operators $P_\pm = 2^{-1}(I \pm S_\Gamma)$ are related to the Cauchy singular integral operator S_Γ, and the shift operator V_α is given by $V_\alpha f = f \circ \alpha$. Applying the theory of Mellin pseudodifferential operators with non-regular symbols of limited smoothness, we establish a Fredholm criterion and an index formula for the operator T provided that the shift derivative α‘ and the coefficient G are slowly oscillating functions on Γ.

Boundary Value Problems for Nonlinear Brinkman and Navier-Stokes Equations with Variable Coefficients in Lipschitz Domains

Mirela Kohr
Babes-Bolyai University, Cluj-Napoca, Romania
Massimo Lanza de Cristoforis, Sergey E. Mikhailov, Wolfgang L. Wendland

In this talk we present recent existence and uniqueness results in Sobolev and Besov spaces for boundary value problems for nonlinear Brinkman and Navier-Stokes equations with variable coefficients in Lipschitz domains in $\mathbb{R}^3$ and in compact Riemannian manifolds. First, we analyze boundary value problems for the linear Stokes and Brinkman systems with variable coefficients, and show the well-posedness of such a linear problem. A key role in this analysis is played by the mapping properties of Newtonian and layer potential operators, which describe these equations in appropriate Sobolev and Besov spaces. Next the well-posedness results in the linear case are combined with a fixed point theorem to show the existence of a solution in L^p-based Sobolev spaces for a boundary value problem for variable-coefficient nonlinear Brinkman or Navier-Stokes equations.

A Functional Analytic Approach to the Analysis of a Two-Parameter Homogenization for a Nonlinear Robin Problem

Massimo Lanza de Cristoforis
University of Padua, Italy
Paolo Musolino

We consider a nonlinear Robin problem for the Poisson equation in an unbounded periodically perforated domain. The domain has a periodic structure, and the size of each cell is determined by a positive parameter δ. The relative size of each periodic perforation is instead determined by a positive parameter ϵ. We prove the existence of a family of solutions which depends on ϵ and δ and we analyze the behavior of such a family as $(\epsilon, \delta,)$ tends to $(0, 0)$ by an approach which is alternative to that of asymptotic expansions and of classical homogenization theory.

Brinkman-Oseen Transmission Problem

Dagmar Medkova
Mathematical Institute, Czech Rep
M. Kohr, W .L. Wendland

We develop a layer potential analysis in order to show the well-posedness result of a transmission problem for the Oseen and Brinkman systems in open sets in Euclidean spaces with compact Lipschitz boundaries and around a lower dimensional solid obstacle, when the boundary data are q-integrable.

Analysis of Segregated Boundary-Domain Integral Equations for Variable-Coefficient Scalar BVPs with General Data

Sergey Mikhailov
Brunel University London, England

Segregated direct boundary-domain integral equations (BDIEs) based on a parametrix and associated with the Dirichlet and Neumann boundary value problems for the linear stationary diffusion partial differential equation with a variable coefficient are formulated. The PDE right hand sides belong to the Sobolev space $H^{-1}(\Omega)$ or $\tilde{H}^{-1}(\Omega)$, when neither classical nor canonical co-normal derivatives are well defined. Equivalence of the BDIEs to the original BVP, BDIE solvability, solution uniqueness/non-uniqueness, and as well as Fredholm property and invertibility of the BDIE operators are analysed in Sobolev (Bessel potential) spaces. It is shown that the BDIE operators for the Neumann BVP are not invertible, and appropriate finite-dimensional perturbations are constructed leading to invertibility of the perturbed operators. The contribution is based on and develops some results of [1-3].

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The BMO-Dirichlet Problem for Elliptic Systems in the Upper-Half Space and Quantitative Characterizations of VMO

Dorina Mitrea
University of Missouri, USA
J.M. Martell, I. Mitrea, M. Mitrea

Let L be a homogeneous, second order, constant complex coefficient elliptic system L in $\mathbb{R}^n$. In this talk I will discuss the well-posedness of the Dirichlet problem in $\mathbb{R}^n_+$ for L with boundary data in $\mathrm{BMO}(\mathbb{R}^{n-1})$ in the class of functions u for which the Littlewood-Paley measure associated with u, namely $d\mu_u(x', t) := |\nabla u(x', t)|^2\, t\, dx' dt$, is a Carleson measure in $\mathbb{R}^n_+$. The regularity result for this problem corresponding to the boundary datum being in Sarason's space $\mathrm{VMO}(\mathbb{R}^{n-1})$ will be discussed as well. Moreover, a new characterizations of the space VMO as the closure in BMO of classes of smooth functions contained in BMO within which uniform continuity may be suitably quantified (such as the class of smooth functions satisfying a Hölder or Lipschitz condition) will be presented. This improves on Sarason's classical result describing VMO as the closure in BMO of the space of uniformly continuous functions with bounded mean oscillations.

How to Custom-Tailor Boundary Layer Potentials for a Given Differential Operator on a Manifold

Marius Mitrea
University of Missouri, USA

Given an elliptic, second-order differential operator L acting between two vector bundles over a Riemannian manifold M, I will discuss a procedure that associates to L a double-layer operator with respect to some uniformly rectifiable subdomain of M. This is a natural adaptation of the blue-print according to which the familiar harmonic double-layer is associated with the Laplacian. Indeed, this generalized double-layer turns out to enjoy all trademark properties that the classical harmonic double-layer possesses such as nontangential maximal function estimates, jump relations, and square-function estimates.

Potential Operators Method for Domains with Smooth Unbounded Boundaries for Anisotropic Helmholtz Operators

Vladimir Rabinovich
National Polytechnic Institute of Mexico, Mexico

The paper is devoted to the potential operators method for the boundary and transmission problems in domains in $\mathbb{R}^n$ with smooth unbounded boundaries for the anisotropic Helmholtz operators

$$\mathcal{H}u(x) = \nabla \cdot a(x)\nabla u(x) + b(x)u(x), x\in \mathbb{R}^n$$

with variable coefficients, where $a = \left(a^{k,l}\right)_{k,l=1}^{n}$ are real-valued symmetric matrix on $\mathbb{R}^n$ with entries $a^{k,l} \in C_b^\infty(\mathbb{R}^n), k, l = 1, ..., n$. We suppose that the operator $\mathcal{H}$ is strongly elliptic, $b(x) = \omega^2 b_0(x)$, $\omega > 0$ is the frequency of the harmonic vibrations, $b_0(x)$ is the refractive index which satisfies the following conditions: $b_0 \in C_b^\infty(\mathbb{R}^n)$,

$$\Re b_0(x) > 0, \Im b_0(x) \geq 0, x \in \mathbb{R}^n, \liminf_{\mathbb{R}^n \ni x \to \infty} \Im b_0(x) > 0.$$

We introduce the single and double layer potentials associated with the operator $\mathcal{H}$, and reduce by means of these potentials the Dirichlet, Neumann, Robin, and transmission problems for domains with unbounded smooth boundary ∂D to pseudodifferential equations on ∂D. Applying the limit operators method we study the Fredholm properties and the invertibility of the boundary pseudodifferential operators in the Sobolev spaces $H^s(\partial D), s \in \mathbb{R}$.

Special Session 100: Nonstandard Analysis, Quantizations and Singular Perturbations

Kiyoyuki Tchizawa, Tokyo City University, Japan
Siavash H. Sohrab, Northwestern University, USA

Nonstandard Analysis is effective in the context of discretizations and changes of scale, for example blow-ups. Notably in the case of singular perturbations and quantum-mechanics phenomena are studied which involve simultanuously the small and the observable. The different orders of magnitude of the parameters may then be related by general principles of nonstandard analysis, like the Transfer Principle. In some cases blowing up, the Idealization and the Standardization Principle may also be needed.

Extended Wiener Measure by Nonstandard Analysis for Financial Time Series

Shuya Kanagawa
Tokyo City University, Japan
Kiyoyuki Tchizawa

We propose a new approach to construct an extended Wiener measure using nonstandard analysis by E. Nelson. For the new definition we construct nonstandardized convolution of probability measure for independent random variables. As an application we consider a simple calculation of financial time series.

No Waste Assumption, Construction of Optimal Solutions, and Transversality Condition for Infinite Horizon Problems

Takashi Nitta
Mie University, Japan
Cai Dapeng

By imposing a no waste assumption, which assumes that the entire stock is depleted before the planning horizon terminates, we modify the usual optimality criterion that is based on the standard overtaking criterion.

Some Implications of Invariant Model of Statistical Mechanics to Set Theory, Nonstandard Analysis, and Noncommutative Geometry

Siavash Sohrab
Northwestern University, USA

A scale-invariant model of statistical mechanics is applied to describe the foundation of multicomponent (mixture) set theory and some of its implications to ZFC axioms. Normalized spacing between nontrivial zeros of Riemann zeta function are shown to follow normalized Maxwell-Boltzmann distribution function. The zeros of zeta function are related to the zeros of particle velocities hence their stationary states. An invariant theory of hyperreal ordered sets on a line composed of nonstandard infinitesimals, reals, and nonstandard transfinite is introduced. Physical space or Casimir vacuum is identified as a quantum tachyon fluid, Dirac stochastic ether, that is governed by Heisenberg matrix mechanics and hence described by noncommutative geometry. The implication of modified theory of nonstandard analysis to microstructure of normal shock waves is described.

Canards in R^3 and R^4 with Co-Dimension 2

Kiyoyuki Tchizawa
Tokyo City University, Japan

This paper gives a survey about the existence of the canards in a slow-fast system in R^{1+2} and R^{2+2} with an invariant manifold. It has a 4-dimensional canard having a relatively stable region when there exists the invariant manifold near the pseudo singular node point.

Nonstandard Methods in Combinatorics

Keita Yokoyama
Japan Advanced Institute of Science and Technology, Japan

Nonstandard methods are available in various fields of mathematics, as well as analysis. In this talk, I will give a brief introduction of an application of nonstandard methods to combinatorics and graph theory. With nonstandard methods, the relation between finite combinatorial statements and infinite combinatorial statements can be understood clearly. Moreover, proof-theoretic analysis of nonstandard methods give more information to combinatorics, which provides some useful information to computer science.

Special Session 101: Randomness meets Life

Peter Hinow, University of Wisconsin - Milwaukee, USA
Blerta Shtylla, Pomona College, USA

We will bring together researchers that employ stochastic methods to model a rich set of biological phenomena. Stochastic effects are important when modeling small scale phenomena such as biochemical reactions, intra-cellular interactions, cellular pattern formation and transport due to the small scale of the environment. In these cases stochasticity is not a mere afterthought but the engine that mediates exquisitely complex phenomena. Stochastic effects are also important at larger scales, as in animal dispersion, spread of epidemics, mathematical immunology and cancer. Despite the large difference in scales the techniques used for model development and analysis are often similar. We intend to bring researchers together that have worked at both large and small scales in order to stimulate the exchange of ideas. An important aspect of the session is that we will encourage speakers to give large picture ideas of their respective fields which is important for cross pollination and also for young researchers interested in work on mathematical modeling of new biological applications.

An Stochastic Model of Cancer Evolution and Treatment Resistance in the Bone

David Basanta
H. Lee Moffitt Cancer Center, USA
Arturo Araujo, Leah Cook, Conor Lynch

The American Cancer Society predicts that approximately 6 Floridian men will die from prostate cancer each day in 2015. These deaths are due to the cancer spreading to secondary sites. Prostate cancer frequently metastasizes to the skeleton where it promotes extensive bone destruction and formation causing great pain to the patient. The bone metastases are incurable and thus we need to identify new therapies and rapid approaches in regards to how therapies are applied to our patients. Experimental animal models provide insights but are limited in terms of their ability to tease apart the drivers of cancer growth and evolution. Computational models on the other hand produce rich and high-resolution data, can yield new clinically interesting hypotheses and can accelerate biological research. We propose that integrating computational modeling with biological experimentation will allow for the rapid optimization of existing and new therapies for the treatment and cure of bone metastatic prostate cancer. Here, we illustrate how a biology-driven agent-based computational model can dissect the complex effects of inhibiting new therapeutic targets such as, transforming growth factor Beta (TGF-Beta). TGF-Beta is a powerful growth factor in the bone microenvironment that can have differential effects on bone building, bone destroying and cancer cells. Because of its important and diverse roles, we thought it to be the ideal candidate with which to begin to test our integrated computational and biological approaches. Computational results predict that TGF-Beta inhibition, applied in a specific therapeutic window, impacts prostate cancer cell growth directly but also, by limiting bone destruction and unexpectedly, promoting bone formation; results that in turn were validated in biologically relevant mouse models of the disease. We also demonstrate how the computational model can be modified on an individual patient basis to address cancer cell heterogeneity and their response to applied therapies. Collectively, our results demonstrate the power of a combined computational/experimental approach in optimizing the efficacy of applied therapies and measuring their impact on the tumor-bone microenvironment. We believe that the development, refinement and validation of our computational models will have a profound impact on the health of Floridian men suffering with bone metastatic prostate cancer.

Periodically Driven Noisy Neuronal Models: a Spectral Approach

Alla Borisyuk
University of Utah, USA
Firas Rassoul-Agha

Neurons are often driven by (noisy) periodic or periodically modulated inputs. In many such cases neuronal firing can be characterized by a stochastic phase response map (SPRM) that maps phase of the current spike into the phase of the subsequent spike. More generally, SPRMs represent Markov chains on a circle. In our spectral approach to studying such maps, we analyze path-wise dynamic properties of the Markov chain, such as stochastic periodicity (or phaselocking) and stochastic quasiperiodicity, and show how these properties are read off of the geometry of the spectrum of the transition operator. I will also discuss how SPRMs can be computed for some neuronal models, their relationship with phase response curves, and how they are affected by changes in the ionic channels.

Computer Simulations of Yeast Mating Reveal Robustness Strategies for Cell-Cell Interactions

Ching-Shan Chou
Ohio State University, USA
Weitao Chen, Tau-Mu Yi, Qing Nie

Cell-cell communication is important to cell functionality. Successful cell communication requires coordination of intricate intracellular and extracellular pathways with the presence of noises. In this work, we build a computational framework that accounts for the molecular dynamics inside and outside the cell, as well as cell morphogenesis. Through computer simulations, we found strategies that budding yeast cells use for efficient and successful mating.

Ergodicity and Loss of Capacity for a Family of Concave Random Maps

Peter Hinow
University of Wisconsin - Milwaukee, USA
Ami Radunskaya

Random fluctuations of an environment are common in ecological and economical settings. We consider a family of concave maps on the unit interval, $f_\lambda(x) = x(1+\lambda-x)$, that model a self-limiting growth behavior. The maps are parametrized by an independent, identically distributed random variable λ with values in the unit interval. We show the existence of a unique invariant ergodic measure of the resulting random dynamical system for arbitrary parameter distributions supported on certain subintervals of $[0, 1]$. Moreover, there is an attenuation of the mean of the state variable compared to the constant environment with the averaged parameter. We also provide an example of a family of just two maps such that the invariant probability measure is supported on a Cantor set.

This work has been supported by the grant "Collaborative Research: Predicting the Release Kinetics of Matrix Tablets" (DMS 1016214 and DMS 1016136) of the National Science Foundation of the United States of America.

Cross Scale Dynamics and The Evolutionary Emergence of Infectious Diseases

Ruian Ke
North Carolina State University, USA
Sebastian J. Schreiber, Claude Loverdo, Miran Park, Prianna Ahsan, James O. Lloyd-Smith

Emerging infectious diseases typically involve pathogens that are exposed to novel environments, such as pathogens from animal populations entering human populations or human pathogens exposed to new drug regimens. When pathogens are poorly adapted to these novel environments, emergence requires the pathogen to evolve sufficiently fast to avoid extinction. This evolution involves pathogen traits at multiple scales, such as within-host viral replication rates and between-host transmissibility. We introduce and analyze a stochastic cross-scale model , and determine how the likelihood of pathogen emergence is governed by selective pressures at these two scales. Our cross-scale analysis opens the door for a new generation of integrative risk assessment models which will link growing streams of data collected in laboratories and field surveillance programs.

Modelling Evolution of Post-Menopausal Human Longevity: the Grandmother Hypothesis

Peter Kim
University of Sydney, Australia
John McQueen, James Coxworth, Kristen Hawkes

Human post-menopausal longevity makes us unique among primates, but how did it evolve? One explanation, the Grandmother Hypothesis, proposes that as grasslands spread in ancient Africa displacing foods ancestral youngsters could effectively exploit, older females whose fertility was declining left more descendants by subsidizing grandchildren and allowing mothers to have new babies sooner. As more robust elders could help more descendants, selection favoured increased longevity while maintaining the ancestral end of female fertility.
We develop a probabilistic agent-based model that incorporates two sexes and mating, fertility-longevity tradeoffs, and the possibility of grandmother help. Using this model, we show how the grandmother effect could have driven the evolution of human longevity. Simulations reveal two stable life-histories, one human-like and the other like our nearest cousins, the great apes. The probabilistic formulation shows how stochastic effects can slow down and prevent escape from the ancestral condition, and it allows us to investigate the effect of mutation rates on the trajectory of evolution.

Fluctuation Models for Suspensions of Swimming Microorganisms

Peter Kramer
Rensselaer Polytechnic Institute, USA
Yuzhou Qian, Patrick Underhill

The collective dynamics of swimming microorganisms ("microswimmers") such as bacteria and algal cells have been of considerable recent interest, both as paradigms of collective patterns arising from individual autonomous agents and for their relevance to technological issues such as biofilm formation and power sources for microdevices. We will discuss some recent efforts to characterize stochastic fluctuations in a continuum "mean field" partial differential equation framework for the effective microswimmer dynamics in a suspension.

Evolutionary Games on the Lattice: Best-Response Dynamics

Nicolas Lanchier
Arizona State University, USA
Stephen Evilsizor

The best-response dynamics is an example of evolutionary game where players are located on the infinite square lattice and update their strategy in order to maximize their payoff. In the presence of two strategies, and calling a strategy selfish or altruistic depending on a certain ordering of the coefficients of the underlying payoff matrix, a simple analysis of the non-spatial mean-field approximation of this process shows that a strategy is evolutionary stable if and only if it is selfish, making in particular the system bistable when both strategies are selfish. The main objective of this talk is to show that, in contrast with the mean-field approximation, only the most selfish strategy is evolutionary stable for the stochastic process. The main ingredients of the proof are monotonicity results and a coupling between the process properly rescaled in space with bootstrap percolation.

Diffusion in a Randomly Switching Environment

Sean Lawley
University of Utah, USA

Driven by diverse applications to biochemistry and physiology, several recent models impose randomly switching boundary conditions on either a PDE or SDE. The PDE models arise from considering a density of particles diffusing in a random environment, whereas the SDE models arise from considering only finitely many particles diffusing in a random environment. In this talk, I will describe the mathematical tools for analyzing these systems and highlight the interesting behavior that they can exhibit. Special attention will be given to establishing mathematical connections between these classes of stochastic processes.

Mesoscopic Modeling of DNA Transport in an Array of Entropic Barriers

Anastasios Matzavinos
Brown University, USA

In this talk, we discuss dissipative particle dynamics (DPD) simulations of the dispersion of DNA molecules conveyed by a pressure-driven fluid flow across a periodic array of entropic barriers. We compare our simulations with nanofluidic experiments, which show the DNA to transition between various types of behaviors as the pressure is increased, and discuss physical insights afforded by the ability of the DPD method to explicitly model flows in the system. Finally, we present anomalous diffusion phenomena that emerge in both experiment and simulation, and we illustrate similarities between this system and Brownian motion in a tilted periodic potential. This is a joint work with Clark Bowman, Daniel Kim, and Derek Stein.

Anomalous Diffusion and Random Encounters in Living Systems

Scott Mckinley
Tulane University, USA
Rebecca Borchering, Jason Flynn, Steven Bellan, Juliet Pulliam

Due to the rapid growth of animal movement data obtained by GPS, radio tracking collars and other means, there is a growing recognition that classical models of encounter rates among animal populations should be revisited. Recent theoretical investigations have demonstrated that biologically relevant modifications to classical assumptions about individual behavior can bring about non-trivial changes in the formulation of population-scale dynamical systems. In particular, the combination of tracking data with habitat information has revealed the substantial impact that environmental factors have on animal movement and sociality. In this talk, I will review some of the existing conventional wisdom that supports the use of so-called “Levy flight“ models that seek to describe animal movement in the absence of environmental cues. However, through a few examples, I will make the case that animal movement patterns should not be separated from the spatial environmental features that shape them. In fact, animal sensing and decision-making are “leading-order“ effects, and their study gives rise to new ecological observations and novel mathematical challenges.

Sensory Feedback in a Bump Attractor Model of Path Integration

Daniel Poll
University of Houston, USA
Khanh Nguyen, Zachary Kilpatrick

Mammalian spatial navigation systems utilize several different sensory information channels.
This information is converted into a neural code that represents the animal's current position in space by engaging place cell, grid cell, and head direction cell networks. In particular, sensory landmark (allothetic) cues can be utilized in concert with an animal's knowledge of its own velocity (idiothetic) cues to generate a more accurate representation of position than path integration provides on its own (Battaglia et al, 2004). We develop a computational model that merges path integration with feedback from external sensory cues that provide a reliable representation of spatial position along an annular track. Starting with a continuous bump attractor model, we explore the impact of synaptic heterogeneity and noise fluctuations, which disrupt the position code of the path integration process. We use asymptotic analysis to reduce the bump attractor model to a single scalar equation. Such imperfections cause errors to build up when the network performs path

integration, but these errors can be corrected by an external control signal representing the effects of sensory cues. We demonstrate that there is an optimal strength and decay rate of the control signal when cues appear either periodically or randomly.

Dimension Reduction for Stochastic Conductance Based Neural Models with Time Scale Separation

Deena Schmidt
University of Nevada, Reno, USA
Peter Thomas, Roberto Galan

The Stochastic Shielding Approximation (SSA) is a fast, accurate simplification of randomly gated ion channel models (Schmandt and Galan 2012, Schmidt and Thomas 2014). Viewing the channel as a discrete process on a directed graph, driven by an independent noise source for each edge, the SSA accurately represents the process using independent noise sources for only a small subset of the edges. This approximation preserves the mean field behavior while selectively incorporating only the underlying noise sources that contribute the most significantly to observable system behavior. Thus the stochastic shielding heuristic provides an analytically tractable example of incorporating noise in a manner relevant to the network's physiological function. Here we investigate the limits of the SSA by studying its accuracy for systems exhibiting large separation of time scales, which is often the case in neural systems.

Stochastic Models of Force Generation and Measurement in Dividing Cells

Blerta Shtylla
Pomona College, USA

In this talk, we discuss mathematical models that track bias generation by nano-machines operating during cell division. Several nano-machines are involved in cell division and their efficient operation requires specific interactions with dynamic bio polymers. We provide two examples of such nano-machines: one working in eukaryotic cells and one in prokaryotic cells. We use first passage techniques to derive mesoscale properties for these constructs using microscale rates and reactions. In each case, we show how these nano-machines could be employed by cells to make the precise measurements needed for equipartition of DNA.

Special Session 102: Recent Developments of High-Order Numerical Methods

Ying Wang, University of Oklahoma, USA
Yang Yang, Michigan Technological University, USA

This mini-symposium is to bring researchers together to discuss the recent advances and exchange ideas in the algorithm design of high-order numerical methods for diffusion-dominated and other high-order partial different equations, including the implementation, numerical analysis, and applications. In the mini-symposium, the speakers will apply those high-order numerical methods to computational fluid, biology and physics, etc. This mini-symposium provides a good opportunity for discussions among researchers from different areas, and explore more applications and future research collaborations.

Maximum-Principle-Satisfying Third Order Direct Discontinuous Galerkin Methods for Time Dependent Convection Diffusion Equations on Unstructured Triangular Mesh

Zheng Chen
Oak Ridge National Laboratory, USA
Hongying Huang, Jue Yan

In this talk, we show Direct discontinuous Galerkin method (DDG) and its variations satisfy the strict maximum principle with at least third order accuracy for convection diffusion equations on unstructured triangular meshes. The key contribution of DDG is the introduction of numerical flux to approximate the solution derivative at the discontinuous element boundaries. We carefully calculate the normal derivative numerical flux across the element edges and prove that with proper choice of parameters in the numerical flux and suitable CFL condition, the piecewise quadratic polynomial solution satisfies strict maximum principle and maintains the third order accuracy at the same time. The implementation is very efficient, with a simple Maximum-Principle-Satisfying (M-P-S) limiter applied after each time stepping. These methods have potential applications in physical models which require the solutions satisfying maximum principle or preserving positivity during time evolution. Numerical examples including incompressible flows are carried out to show the optimal 3rd order of accuracy is maintained with the M-P-S limiter applied.

Reduced Basis Methods for Linear and Nonlinear Equations and Their Applications in Data Science

Yanlai Chen
University of Massachusetts Dartmouth, USA

Models of reduced computational complexity is indispensable in scenarios where a large number of numerical solutions to a parametrized problem are desired in a fast/real-time fashion. These include simulation-based design, parameter optimization, optimal control, multi-model/scale analysis, uncertainty quantification. Thanks to an offline-online procedure and the recognition that the parameter-induced solution manifolds can be well approximated by finite-dimensional spaces, reduced basis method (RBM) and reduced collocation method (RCM) can improve efficiency by several orders of magnitudes. The accuracy of these spectrally convergent methods is maintained through a rigorous a posteriori error estimator whose efficient development is critical.
In this talk, I will give a brief introduction of the RBM, discuss recent and ongoing efforts to develop RCM for linear and nonlinear equations, explain how the newly-designed Reduced Basis Decomposition can be used for data compression and face recognition.

Semi-Implicit Integration Factor Methods on Sparse Grids for High-Dimensional Systems

Weitao Chen
University of California, Irvine, USA
Dongyong Wang, Qing Nie

Numerical methods for partial differential equations in high-dimensional spaces are often limited by the curse of dimensionality. Though the sparse grid technique is popular for handling challenges such as those associated with spatial discretization, the stability conditions on time step size due to temporal discretization, such as those associated with high-order derivatives in space and stiff reactions, remain. Here, we incorporate the sparse grids with the implicit integration factor method (IIF) that is advantageous in terms of stability conditions for systems containing stiff reactions and diffusions. We combine IIF, in which the reaction is treated implicitly and the diffusion is treated explicitly and exactly, with various sparse grid techniques based on the finite element and finite difference methods and a multi-level combination approach. The overall method is found to be efficient in terms of both storage and computational time for solving a wide range of PDEs in high dimensions. In particular, the IIF with the sparse grid combination technique is flexible and effective in solving systems that may include cross-derivatives and non-constant diffusion coefficients.

A New Discontinuous Galerkin Method, Conserving the Discrete H^2-Norm, for Third-Order Linear Equations in One Space Dimension

Bo Dong
University of Massachusetts Dartmouth, USA
Yanlai Chen, Bernardo Cockburn

We introduce a Bassi-Rebay type discontinuous Galerkin (DG) method for both stationary and time-dependent third-order linear equations. This method is the first DG method which conserves the mass and the L^2-norm of the approximations of the solution and that of its first and second derivatives. For the stationary case, L^2-projections of the errors (in the approximation of the solution, its first and second derivatives) are proven to have optimal convergence rates when the polynomial degree k is even and the mesh is uniform, and to converge sub-optimally, but sharply, with order k when k is odd or the mesh is non-uniform. We show that suitably defined projections of the errors superconverge with order $k+1+\min\{k, \frac{1}{2}\}$ on uniform meshes and converge optimally on non-uniform meshes. The numerical traces are proven to superconverge with order $2k$ if k is odd or the mesh is non-uniform. For even k and uniform meshes, we show that the numerical traces superconverge with order $2k+\frac{3}{2}$. If in addition, the number of intervals is odd, the convergence order is improved to $2k+\frac{3}{2}+\min\{k, \frac{1}{2}\}$. This allows us to use an element-by-element postprocessing to construct new approximations that superconverge with the same orders as the numerical traces. For the time-dependent case, the errors are proven to be of order $k+1$ for even k on uniform meshes, and of order k when k is odd or the mesh is nonuniform. Numerical results are displayed which verify all of the above-mentioned theoretical orders of convergence as well as the conservation properties of the method. We also show that the orders of convergence of the the stationary case also hold for the time-dependent case.

Computational Methods for Extremal Steklov Problems

Chiu-Yen Kao
Claremont McKenna College, USA
Eldar Akhmetgaliyev, Braxton Osting

We develop a computational method for extremal Steklov eigenvalue problems and apply it to study the problem of maximizing the p-th Steklov eigenvalue as a function of the domain with a volume constraint. In contrast to the optimal domains for several other extremal Dirichlet- and Neumann-Laplacian eigenvalue problems, computational results suggest that the optimal domains for this problem are very structured. We reach the conjecture that the domain maximizing the p-th Steklov eigenvalue is unique (up to dilations and rigid transformations), has p-fold symmetry, and an axis of symmetry. The p-th Steklov eigenvalue has multiplicity 2 if p is even and multiplicity 3 if $p \geq 3$ is odd.

Ultraconvergence of Finite Element Method by Richardson Extrapolation

Runchang Lin
Texas A&M International University, USA
Wen-ming He, Zhimin Zhang

In this talk, two novel Richardson extrapolation operators are proposed to investigate local $2k$th order ultraconvergence properties of the kth order Lagrange finite element method for the second order elliptic problems. We show that, for both tensor product Q_k element and simplicial P_k element, the post-processed displacement and gradient both have $2k$th order ultraconvergence at interior mesh nodes away from the boundary. Numerical results are provided to demonstrate the theoretical findings.

A Partitioned Approach for Computing Fluid-Structure Interaction

Jin Wang
University of Tennessee at Chattanooga, USA

The interactions between fluid flows and immersed solid structures are nonlinear multi-physics phenomena that have applications to a wide range of scientific and engineering disciplines. In this talk, we present a new partitioned approach to compute fluid-structure interaction (FSI) by extending the original direct-forcing technique and integrating it with the immersed boundary method. The fluid and structural equations are calculated separately via their respective disciplinary algorithms, and their solutions only communicate at the fluid-structure interface. The computational framework is capable of handling FSI problems with sophisticated structures described by detailed constitutive laws.

Two-Competing-Species Chemotaxis Models

Qi Wang
Southwestern University of Finance and Economics, Peoples Rep of China
J. Hu, J. Yang, L. Zhang

In this talk, we consider a two-species chemotaxis model with Lotka-Volterra dynamics. We show the existence of both stationary and time-periodic patterns to this system. The effects of chemotaxis rate and cellular growth are examed. It is shown that this system has very rich spatial-temporal dynamics.

The Modified Buckley-Leverett Equation

Ying Wang
University of Oklahoma, USA
Yulong Xing

In this talk, I will present the modified Buckley-Leverett (MBL) equation, which models the underground oil recovery process. I will show some preliminary results for solving this equation using the DG methods. This is a joint work with Yulong Xing.

A Posteriori Error Estimates of Local Discontinuous Galerkin Methods for the Generalized Korteweg-De Vries Equations

Yulong Xing
University of California Riverside, USA

The Korteweg-de Vries (KdV) equation is a nonlinear mathematical model for the unidirectional propagation of waves in a variety of nonlinear, dispersive media. Recently it has attracted increasing attention as test-bed for the competition between nonlinear and dispersive effects leading to a host of analytical issues such global existence and finite time blowup, etc.
In this presentation, we construct, analyze, and numerically validate a class of discontinuous Galerkin schemes for the generalized KdV equation. We will provide a posteriori error estimate through the concept of dispersive reconstruction, i.e. a piecewise polynomial function which satisfies the GKdV equation in the strong sense but with a computable forcing term enabling the use of a priori error estimation techniques to obtain computable upper bounds for the error. Both semi-discrete and fully discrete approximations are studied.

Direct Discontinuous Galerkin Method and Its Variations for Second Order Elliptic Equations

Jue Yan
Iowa State University, USA
Hongying Huang, Zheng Chen

In this paper, we study direct discontinuous Galerkin method and its variations for 2nd order elliptic problems. A priori error estimate under energy norm is established for all four methods. Optimal error estimate under L^2 norm is obtained for DDG method with interface correction and symmetric DDG method. A series of numerical examples are carried out to illustrate the accuracy and capability of the schemes. Numerically we obtain optimal $(k+1)$th order convergence for DDG method with interface correction and symmetric DDG method on none uniform and unstructured triangular meshes. An interface problem with discontinuous diffusion coefficients is investigated and optimal $(k+1)$th order accuracy is obtained. Peak solutions with sharp transitions are captured well. Highly oscillatory wave solutions of Helmholz equation are well resolved.

Discontinuous Galerkin Methods for Chemotaxis Models

Yang Yang
Michigan Technological University, USA
Xingjie Li, Chi-Wang Shu

In this talk, we will focus on local discontinuous Galerkin methods for Chemotaxis model, which might yield blow-up solutions. We first give the error estimates based on two different finite element spaces, and then proceed to the positivity-preserving technique to obtain positive numerical approximations. Finally, we will numerically demonstrate how to find the blow-up time.

Recent Results on the Optimal Error Estimates of the Local Discontinuous Galerkin Method When Solving the Convection Diffusion Equations

Qiang Zhang
Nanjing University, Peoples Rep of China

In this talk we will present two results about the optimal error estimates of local discontinuous Galerkin methods when solving the time-dependent convection diffusion equations. The first one is the optimal error estimates in the L^2-norm when the generalized alternating numerical flux is used, where the generalized Gauss-Radau projection plays the important role. The second one is the optimal error estimates in the L^2-norm on the fully-discrete local discontinuous Galerkin methods with the implicit-explicit Runge-Kutta time-marching. Some numerical experiments are also given.

Special Session 103: Mixing in Dynamical Systems: Theory, Modelling, and Applications, from Micro- to Geophysical Scales

Sanjeeva Balasuriya, University of Adelaide, Australia
Gary Froyland, Univesity of New South Wales, Australia

The notion of mixing is central to the study of dynamical systems. This special session is devoted to various aspects of mixing, including the optimisation or control of mixing, the quantification of mixing by statistical laws, and the mathematical modelling of mixing processes in physical and geophysical systems. The session brings together theoreticians, modellers, and practitioners from small scale microfluidic devices to planetary scale geophysical processes.

Eigenmode Analysis of Advective-Diffusive Transport by the Compact Mapping Method

Patrick Anderson
Eindhoven University of Technology, Netherlands
MFM Speetjens, M. Giona

The present study concerns an efficient spectral analysis of advective-diffusive transport in periodic flows by way of a compact version of the diffusive mapping method. Key to the compact approach is the representation of the scalar evolution by only a small subset of the eigenmodes of the mapping matrix, and capturing the relevant features of the transient towards the homogeneous state. This has been demonstrated for purely advective transport in an earlier study by Gorodetskyi et al. Phys. Fluids 24, (2012). Here this ansatz is extended to advective-diffusive transport and more complex 3D flow fields, motivated primarily by the importance of molecular diffusion in many mixing processes. The study exposed an even greater potential for such transport problems due to the progressive widening of the spectral gaps in the eigenvalue spectrum of the mapping matrix with increasing diffusion. This facilitates substantially larger reductions of the eigenmode basis compared to the purely advective limit for a given approximation tolerance. The compact diffusive mapping method is demonstrated for a representative three-dimensional prototype micro-mixer. This revealed a reliable prediction of (transient) scalar evolutions and mixing patterns with reductions of the eigenmode basis by up to a factor 2000. The accurate estimation of the truncation error from the eigenvalue spectrum enables systematic determination of the spectral cut-off for a desired degree of approximation. The validity and universality of the presumed correlation between spectral cut-off and truncation error has been established. This has the important practical consequence that the cut-off can a priori be chosen such that the truncation error remains within a preset tolerance. This offers a way to systematically (and reliably) employ the compact mapping method for in-depth analysis of advective-diffusive transport.

Detecting Coherent Sets with Spacetime Diffusion Maps

Ralf Banisch
University of Edinburgh, Scotland
Peter Koltai

Intuitively, coherent sets are subsets of the configuration space that stay together under the (possibly chaotic) dynamics. Many different approaches for making this notion precise exist in the literature. For example, one approach defines coherent sets via spectral properties of the transfer operator, and another defines coherent sets as tight bundles of trajectories by specifying a euclidean distance metric in spacetime. We show that these two approaches can be reconciled: By replacing the Euclidean distance in spacetime with an augmented version of the distance used in diffusion maps, one can make contact with the transfer operator notion of coherence in the infinite data limit. The resulting numerical method, which can be used to extract coherent sets directly from trajectory data, is related to similar methods that have been discussed in the past. We demonstrate its performance on several examples.

Mixing Asymptotics and Limit Laws for Random Intermittent Maps

Chris Bose
University of Victoria, Canada
Wael Bahsoun

Non-uniformly expanding maps of the interval have been intensively studied, starting at least 20 years ago, primarily since they are amongst the simplest systems that model intermittency: for example polynomial mixing rates and non-CLT-type limit laws. Here we study random maps constructed from a parameterized family of intermittent maps, drawing two conclusions: 1. The speed of correlation decay (for Holder data) is completely determined by the map with the fastest mixing rate, independent of the randomizing process and 2. in cases where correlation decay fails to be square summable, establishing a CLT or stable law (as appropriate) is dependent on both the maps and the randomizing process. Specific examples will be discussed.

Optimal Mixing Enhancement of Flows

Gary Froyland
University of New South Wales, Australia
Naratip Santitissadeekorn

We introduce a general purpose method for optimising the mixing rate of steady or periodically-driven flows. A nearby vector field is selected from a prespecified neighborhood of the original vector field so as to maximise the mixing rate for flows generated by vector fields in that neighborhood. A linear optimization problem is solved to identify the optimal vector field. The perturbed flow may be easily constrained to preserve the same invariant density (e.g. preserve volume) as the original flow, and various other natural geometric constraints can be simply applied.

Transport and Mixing in Dynamical Systems Via Transfer Operators

Cecilia Gonzalez Tokman
University of Queensland, Australia
Gary Froyland, Anthony Quas, Thomas Watson

Transport and mixing properties of dynamical systems are of interest to geophysical (among other) scientists, and they pose interesting challenges for mathematicians. For example, large scale structures such as oceanic eddies and atmospheric vortices are connected with important features of the global climate, and their detection and tracking in complex models of the real world has been an active topic of mathematical research in recent years. In this talk we will discuss recent advances and challenges, arising from the use of transfer operators to investigate analytical and computational problems in the area, ranging from mixing optimization to the detection and approximation of coherent structures in non-autonomous dynamical systems.

On Fast Computation of Finite-Time Coherent Sets Using Radial Basis Functions

Oliver Junge
Technical University Munich, Germany
Gary Froyland

Finite-time coherent sets inhibit mixing over finite times. The most expensive part of the transfer operator approach to detecting coherent sets is the construction of the operator itself. We present a numerical method based on radial basis function collocation and apply it to a recent transfer operator construction that has been designed specifically for purely advective dynamics. The construction is based on a dynamic Laplacian operator and minimises the boundary size of the coherent sets relative to their volume. The main advantage of our new approach is a substantial reduction in the number of Lagrangian trajectories that need to be computed, leading to large speedups in the transfer operator analysis when this computation is costly.

Hyperbolic Mixing Sets in Control-Affine Systems

Christoph Kawan
University of Passau, Germany
Adriano Da Silva

A control-affine system is a control system of the form

$$\Sigma: \ \dot{x}(t) = f_0(x(t)) + \sum_{i=1}^{m} u_i(t) f_i(x(t)), \quad u \in \mathcal{U},$$

where $f_0, f_1, \ldots, f_m$ are vector fields on a smooth manifold M and $\mathcal{U}$ denotes the set of admissible control functions $u : \mathbb{R} \to \mathbb{R}^m$. Assuming $\mathcal{U} = L^\infty(\mathbb{R}, U)$ for a compact and convex set $U \subset \mathbb{R}^m$, the set $\mathcal{U}$ can be equipped with the weak*-topology of $L^\infty(\mathbb{R}, \mathbb{R}^m) = L^1(\mathbb{R}, \mathbb{R}^m)^*$ and becomes a compact metrizable space. A natural dynamical system on $\mathcal{U}$ is the shift flow $\theta_t u = u(\cdot + t)$, $t \in \mathbb{R}$. Assuming that for every $u \in \mathcal{U}$ and every initial value x (at time 0) the solution $\varphi(t, x, u)$ of the above ODE is unique and defined on $\mathbb{R}$, the map $\varphi : \mathbb{R} \times M \times \mathcal{U} \to M$ turns out to be a cocycle over θ and

$$\Phi : \mathbb{R} \times (\mathcal{U} \times M) \to \mathcal{U} \times M, \quad \Phi_t(u, x) = (\theta_t u, \varphi(t, x, u)),$$

is a continuous skew-product flow, the so-called control flow of Σ. Mixing properties of Φ are strongly related to controllability properties of Σ. Under the assumption of local accessibility, there is one-to-one correspondence between the maximal subsets of M of approximate controllability (called control sets) and the maximal topologically mixing subsets of $\mathcal{U} \times M$. In this talk, we present several results about the structure and the properties of control sets which admit a uniformly hyperbolic splitting, revealing remarkable analogies to the classical theory of uniformly hyperbolic dynamical systems.

Coherent Families: Spectral Theory for Transfer Operators in Continuous Time

Peter Koltai
Freie Universität Berlin, Germany
Gary Froyland

The decomposition of the state space of a dynamical system into metastable or almost-invariant sets is important for understanding macroscopic behavior. This concept is well understood for autonomous dynamical systems, and has recently been generalized to non-autonomous systems via the notion of coherent sets. We elaborate here on the theory of coherent sets in continuous time for periodically-driven flows and describe a numerical method to find families of coherent sets without trajectory integration.

Response Operators for Markov Processes in a Finite State Space: Radius of Convergence and Link to the Response Theory for Axiom a Systems

Valerio Lucarini
University of Hamburg, Germany

Using linear algebra we derive response operators describing the impact of small perturbations to finite state Markov processes. The results can be used for studying empirically constructed finite state approximation of statistical mechanical systems. Recent results on the convergence of the statistical properties of finite state Markov approximation of the dynamics of a system in the limit of finer and finer partitions of its phase space suggest that our results are robust in the Axiom A case. Our findings give closed formulas for the response theory at all orders of perturbation and provide matrix expressions that can be directly implemented in any coding language, plus providing bounds on the radius of convergence of the perturbative theory. We relate the convergence to the rate of mixing of the unperturbed system. Our formulas can be used to recover previous findings obtained on the response of continuous time Axiom A systems, by considering the generator of time evolution for the measure and for the observables. A very basic, low-tech, and computationally cheap analysis of the response of the Lorenz '63 model provides encouraging results regarding the possibility of using the approximate representation given by finite state Markov processes to compute a system's response.

Vortices in a Street Canyon

Rua Murray
University of Canterbury, New Zealand
Gary Froyland, Jamie de Jong, Miguel Moyers-Gonzalez, Benjamin Roberts, Phil Wilson

Among the many factors affecting urban air quality is the pattern of air circulation around clusters of tall buildings. Under steady wind conditions, each "street canyon" can support vortices that recirculate contaminated air and delay the dispersal of pollutants. The number of vortices can depend on the relative heights and spacing of the structures. This talk will report on the use of transfer operator methods for open systems to identify vortices and study their dependence on the geometry of the street canyon.

Memory Loss for Nonstationary Open Dynamical Systems

William Ott
University of Houston, USA
Brett Geiger, Andrew Torok

This talk is about an analog of decay of correlations for nonstationary open dynamical systems. By *nonstationary*, we mean that the dynamical model itself varies in time. Examples include dynamical processes evolving in slowly varying environments and dynamical systems with time-dependent parameters. By *open*, we mean that the phase space contains (possibly moving) holes through which trajectories escape. We formulate a notion of memory loss appropriate for nonstationary open systems and we provide a theoretical framework that allows one to prove that memory loss occurs at an exponential rate. We then apply this theory to a sample setting - a class of maps studied by Cowieson and Saussol. We emphasize that unlike the random dynamical systems setup, the stationarity of the process is entirely irrelevant for our purposes.

Cluster-Based Extraction of Finite-Time Coherent Sets from Trajectory Data

Kathrin Padberg-Gehle
TU Dresden, Germany
Gary Froyland

Coherent features in time-dependent dynamical systems are difficult to identify. Most identification algorithms require knowledge of the dynamical system or high-resolution trajectory information, which in applications may not be available. We present a fast and simple method that is based on spatio-temporal clustering of trajectory data. It provides a rough and rapid coherent structure analysis and is particularly aimed at situations where the available information is poor: there are few trajectories, the available trajectories do not span the full time duration under consideration, and there are missing observations within trajectories.

Multiobjective Optimal Control of Coherent Structures Using Reduced Order Modeling

Sebastian Peitz
Paderborn University, Germany
Sina Ober-Bløbaum, Michael Dellnitz

In a wide range of applications it is desirable to optimally control a dynamical system with respect to concurrent, potentially competing goals. This gives rise to a multiobjective optimal control problem where, instead of computing a single optimal solution, the set of optimal compromises has to be approximated. When the problem under consideration is described by a partial differential equation (PDE), as is the case for fluid flow, the computa-

tional cost rapidly increases and renders a direct treatment infeasible. Reduced order modeling is a popular method to reduce the computational cost, in particular in a multiquery context such as optimization. In this presentation, we show how to combine reduced order modeling and multiobjective optimal control techniques in order to efficiently solve multiobjective optimal control problems constrained by PDEs. We consider a global, derivative free optimization method as well as a local, gradient based approach for which we derive the optimality system. The methods are compared with regard to the solution quality as well as the computational effort, and they are illustrated using the example of the two-dimensional incompressible flow around a cylinder where we want to minimize the occurence of coherent structures (i.e. vortex shedding) and the control cost.

Global B-Pullback Attractors for Cocycles Generated by Discrete-Time Cardiac Conduction Models

Volker Reitmann
St. Petersburg State University, Russia
Anastasia Maltseva

We investigate the existence and the structure of global B-pullback attractors for cocycles generated by nonautonomous difference equations. As an example of such equations some discrete-time models of cardiac conduction systems are considered. We use the transfer function of the linear part of the system and some properties of the nonlinear part in order to prove the dissipativity of the given system with the help of the Yakubovich-Kalman frequency theorem. Employing the dissipativity the existence of the global B-pullback attractor for the discrete system under a perturbation, which is considered as a discrete-time cocycle, is shown. We state conditions under which the following is true: if the class of perturbations is almost periodic then the global B-pullback attractor is also almost periodic, in the case of perturbations given by a stationary Gaussian process, which is mixing, the resulting global B-pullback attractor of the cocycle is also represented by a mixing stationary Gaussian process.

Lagrangian Coherent Structures and The Non-Advective Transport of Potential Vorticity in Tropical Cyclones

Blake Rutherford
Northwest Research Associates, USA

During tropical cyclogenesis, a flow topology emerges in a wave-relative reference frame that indicates a quasi-closed circulation in the lower troposphere. Considering time-dependent flows introduces topological changes in the flow field. While a layer-wise 2D approximation is reasonable under tropical approximations, various 3D flow components are important to the evolution of the vorticity field. These 3D mechanisms may arise from small-scale convection or from large-scale environmental flow, but they have important consequences for development of the cyclone. Specifically, they can further alter the topology of the flow field and allow for the combination of regions with very different thermodynamic properties. By viewing Lagrangian coherent structures that arise purely from the 2D approximation, we can see all of the layer-wise topological changes in the flow. For any monotonic vertical coordinate, the impermeability principle states that there is no net vertical transport of the potential vorticity (pv). Variations in pv follow from a pv tendency equation and represent changes due to vortex tilting, diabatic heating and friction. An evaluation of the tendency equations along time evolving Lagrangian coherent structures gives a complete picture of the mixing of pv as a layer-wise 2D component tied to topological rearrangement plus a 3D flux of pv that can be viewed as acting only at the boundary due to Stokes‘ theorem.

Linear Response in the Intermittent Family

Benoit Saussol
University of Brest, France
Wael Bahsoun

A standing question in dynamical systems is to understand how statistical properties of a perturbed system are related to the original system. In particular the physical measure (or SRB measure) of the system may depend in a differentiable way on the parameters of the system. In the physics literature this is called linear response. We prove a linear response formula for the intermittent family $x \mapsto x + 2^\alpha x^{1+\alpha}$, that is the differentiability of the density h_α with respect to the parameter α.

Bounds for Generalised Lyapunov Exponents for Random Products of Shears

Rob Sturman
University of Leeds, England
Jean-Luc Thiffeault

We give lower and upper bounds on both the Lyapunov exponent and generalised Lyapunov exponents for the random product of positive and negative shear matrices. These types of random products arise in applications involving randomized stirring devices. The bounds, obtained by considering invariant cones in tangent space, give excellent accuracy compared to standard and general bounds, and are increasingly accurate with increasing shear. Bounds on generalised exponents are useful for testing numerical methods, since these exponents are difficult to compute.

Improving Mixing in Micro Fluidic Systems by Introducing Partial Slip to Break Symmetry

Pushpavanam Subramaniam
I.I.T. Madras, India
Gowtham Sankaran, Piyush Garg, Jason Picardo

Mixing in flow systems in micro-fluidic systems is a challenge in view of the small length scales which prevail in these devices. In the literature several mechanisms for mixing have been proposed which also end up increasing the pressure drop. In this work we discuss how a serpentine geometry can be used to improve mixing. Here the naturally occurring Deans vortices cause mixing in the direction normal to the flow. The symmetry in these vortices is broken by introducing a a periodic slip on the walls. The periodicity in the two walls is phase shifted. This results in creating a situation which is similar to the classical blinking vortex. The blinking vortex now evolves spatially. as the liquid moves through the channel. The flow is analysed using Poincare maps and other scalar indicators. In addition to single phase flows we discuss how the analysis can be extended to a slug flow regime in two phase flows.. The partial slip results in a decrease in the pressure drop across the channel. The flow field is computed using a semi- analytical approach which makes the algorithms computationally efficient.

Scalar Density Evolution with Lagrangian Measures

Wenbo Tang
Arizona State University, USA
Phillip Walker

In integrable shear flows, the second order moments of a passively advected and diffusive scalar can be computed analytically to obtain the spatial- and temporal-dependent effective diffusivity. In non-integrable flows, such analytical results are not accessible. In this talk, we discuss a semi-analytic framework, which reconstructs scalar density evolution from finite-time Lagrangian measures. We pay close attention to the stretching and shearing components of the deformation tensor, both playing important roles in shaping scalar patches. With proper choice of time-scale for mapping the scalar density forward, we show that the scalar density field can be resolved without having to solve the transport equation.

Reduced Transfer Operator Approach to Mixing and Stability in Chaotic and Stochastic Systems

Alexis Tantet
IMAU, Utrecht University, Netherlands
Mickael D. Chekroun, Valerio Lucarini, J. David Neelin, Henk A. Dijkstra, Frank Lunkeit

Much can be learned about systems exhibiting complex dynamics by studying the evolution of probability densities rather than single trajectories. This evolution is governed by the semigroup of transfer operators which allows to connect the correlation function to the Liouville or the Fokker-Planck equation. Yet, their approximation quickly becomes intractable when the dimension of the phase space is large.
We propose to approximate the transfer operators by Markov operators on a reduced space. While these Markov operators do not in general constitute a semigroup, rigorous results can be obtained regarding their spectral properties, in particular allowing to reconstruct correlation functions and quantify mixing in the reduced space.
The approach is applied to the study of the variability and the stability of chaotic and stochastic systems relevant for climate. New analytical and numerical results are shown for the Hopf bifurcation with additive noise, bringing new insights on the phenomena of noise-induced oscillation and phase diffusion. Finally, it is found for a chaotic attractor crisis that the slowing down of the decay of correlations is associated with the shrinkage of the reduced spectral gap, improving our understanding of early-warning signals for high-dimensional systems.

Can We Define Finite-Time Generalized Lyapunov Exponents?

Jean-Luc Thiffeault
University of Wisconsin – Madison, USA
Marko Budisic

One way to define Lyapunov exponents is in terms of the stretching of small vectors. If the relative growth of vectors is a random variable $\ell(t)$, then the exponent is $\lambda = t^{-1}\langle\log\ell\rangle$, where the brackets are an ensemble average over trajectories. Under suitable assumptions, this quantity converges as $t \to \infty$ or as the number of samples increases. This double convergence allows the definition of finite-time Lyapunov exponents, which have proved useful in analyzing the detailed structure of mixing flows. Generalized Lyapunov exponents are defined in terms of the expectation $\langle\ell^p\rangle$, for some constant p. These have very different convergence properties, which we discuss. We ask if anything can be done to define these in a finite-time sense, and what could be learned from such a definition.

Central Limit Theorems for Sequential and Random Intermittent Dynamical Systems

Andrew Torok
University of Houston, USA
M. Nicol, S. Vaienti

We establish self-norming central limit theorems for non-stationary time series arising as observations on sequential maps possessing an indifferent fixed point. These transformations are obtained by perturbing the slope in the Pomeau-Manneville map. We also obtain quenched central limit theorems for random compositions of these maps.

Special Session 104: Nonlinear Elliptic Equations and Fractional Laplacian

Qianqiao Guo, Northwestern Polytechnical University, Peoples Rep of China
Wenxiong Chen, Yeshiva University, USA

We will exchange new developments, ideas, and methods in the area of nonlinear elliptic partial differential equations and systems including those involving fractional Laplacians and other nonlocal operators. We will study qualitative properties, such as existence, symmetry, classification, monotonicity, asymptotic behavior, and nonexistence of solutions for the above mentioned equations.

Global Dynamics of Competition Models with Fractional Laplacian/ Nonlocal Dispersal

Xueli Bai
Northwestern Polytechnical University, Peoples Rep of China
Fang Li

In this talk, we consider the global dynamics of sevral competition models with fractional Laplacian or nonlocal dispersal. Since all the systems we considered here are monotone, thus our aim is deriving the uniqueness of the positive steady state..

Uniform Approach to Sharp Hardy-Littlewood-Sobolev Type Inequalities

Qianqiao Guo
Northwestern Polytechnical University, Peoples Rep of China
Jingbo Dou, Meijun Zhu

In this talk we try to give a (new) uniform approach to prove the sharp Hardy-Littlewood-Sobolev type inequalities, including reversed Hardy-Littlewood-Sobolev type inequalities, which should be useful to some integral equations.

Ground State of Nonlinear Schrodinger Equation with Fractional Laplacian

Zhiqing Han
Dalian University of Technology, Peoples Rep of China
Zupei Shen, Chuanfang Zhang

In this talk, I will present some results for the equation involving a fractional Laplacian: $(-\Delta)^{\alpha} + u = f(u), x \in R^N$. By using Mountain Pass Theorem with a rearrangement argument, we prove the existence of symmetry mountain pass solutions without the Ambrosetti-Rabinowitz condition. We aslo get a nonnegative symmetry ground state in the fractional Sobolev space $H^{\alpha}(R^N)$. rearrangement

The Diffusive Competition Problem with a Free Boundary in Heterogeneous Time-Periodic Environment

Fengquan Li
Dalian University of Techology, Peoples Rep of China
Qiaoling Chen, Feng Wang

In this talk, we will discuss the diffusive competition problem with a free boundary and sign-changing intrinsic growth rate in heterogeneous time-periodic environment, consisting of an invasive species with density u and a native species with density v. We assume that v undergoes diffusion and growth in R^N, and u exists initially in a ball $B_{h_0}(0)$, but invades into the environment with spreading front $\{r = h(t)\}$. The effect of the dispersal rate d_1, the initial occupying habitat h_0, the initial density u_0 of invasive species u, and the parameter μ (see (1.3)) on the dynamics of this free boundary problem are studied. A spreading-vanishing dichotomy is obtained and some sufficient conditions for the invasive species spreading and vanishing are provided. Moreover, when spreading of u happens, some rough estimates of the spreading speed are also given.

Symmetry Results for a System Involving the Fractional Laplacians

Yan Li
Yeshiva University, USA
Pei Ma

In this paper, we study non-negative solutions for a system involving the fractional Laplacians:

$$\begin{cases} (-\triangle)^{\alpha/2}u(x) = f(v(x)), \\ (-\triangle)^{\beta/2}v(x) = g(u(x)), \\ u, v \geq 0, \qquad x \in R^n, \end{cases} \tag{1}$$

where $\alpha,\beta \in (0, 2)$. Using a direct method of the moving planes, we obtain symmetry results of solutions.

Nonexistence of Positive Solutions for a System of Semilinear Indefinite Fractional Laplacian Problem

Ye Li
Central Michigan University, USA
Jingbo Dou

We consider a system of semilinear indefinite equations involving the fractional Laplacian in the Euclidean space $\mathbb{R}^n$:

$$\begin{cases}(-\Delta)^{\alpha/2}u(x) = f(x_n)v^p(x)\\(-\Delta)^{\alpha/2}v(x) = g(x_n)u^q(x)\end{cases}$$

in the subcritical case 1

Controllability for a Class of Semilinear Fractional Evolution Systems Via Resolvent Operators

Yansheng Liu
Shandong Normal University, Peoples Rep of China
Daliang Zhao

This talk is concerned with the exact control for a class of fractional evolution systems in a Banach space. First, we introduce a new concept of exact control and give the mild solutions of considered evolution system via re-solvent operators. Second, by utilizing the semi-group theory, fixed point strategy and measure of non-compactness, the exact control of the evolution system has been investigated without Lipschitz continuity and growth conditions imposed on nonlinear functions. The results are established under the hypothesis that re-solvent operator is differentiable and analytic respectively instead of supposing that the semi-group is compact. An example is provided to illustrate the abstract results.

The Pohozaev Identity of the Fractional Laplacian System

Pei Ma
Yeshiva University, USA
Fengquan Li, Yan Li

In this paper, we study the Pohozaev identity associated with a Hénon-Lane-Emden system involving the Fractional Laplacian:

$$\begin{cases}(-\Delta)^s u = |x|^a v^p, & x \in \Omega,\\(-\Delta)^s v = |x|^b u^q, & x \in \Omega,\\ u = v = 0, & x \in R^n \backslash \Omega,\end{cases}$$

in a star-shaped and bounded domain for $s \in (0,1)$. And then using the Pohozaev Identity result, we derive the nonexistence of positive solutions.

Time-Harmonic Solutions to Nonlinear Maxwell Equations on a Bounded Domain

Jaroslaw Mederski
Nicolaus Copernicus University, Poland
Thomas Bartsch

We find solutions $E : \Omega \to \mathbb{R}^3$ of the problem

$$\begin{cases}\nabla \times (\mu(x)^{-1}\nabla \times E) - \omega^2\varepsilon(x)E = \partial_E F(x,E) & \text{in } \Omega\\ \nu \times E = 0 & \text{on } \partial\Omega\end{cases}$$

on a bounded Lipschitz domain$\Omega \subset \mathbb{R}^3$ with exterior normal $\nu : \partial\Omega \to \mathbb{R}^3$. Here $\nabla\times$ denotes the curl operator in $\mathbb{R}^3$. The equation describes the propagation of the time-harmonic electric field $\Re\{E(x)e^{i\omega t}\}$ in an anisotropic material with a magnetic permeability tensor $\mu(x) \in \mathbb{R}^{3\times 3}$ and a permittivity tensor $\epsilon(x) \in \mathbb{R}^{3\times 3}$. The boundary conditions are those for Ω surrounded by a perfect conductor. It is required that $\mu(x)$ and $\epsilon(x)$ are symmetric and positive definite uniformly for $x \in \Omega$, and that $\mu,\epsilon \in L^\infty(\Omega, \mathbb{R}^{3\times 3})$. The nonlinearity $F : \Omega \times \mathbb{R}^3 \to \mathbb{R}$ is superquadratic and subcritical in E, the model nonlinearity being of Kerr-type: $F(x,E) = |\Gamma(x)E|^p$.

On Nonnegative Solutions to Elliptic Differential Inequalities on Riemannian Manifolds

Yuhua Sun
Nankai University, Peoples Rep of China
A.Grigor'yan

We provide optimal condition in terms of the volume growth of a Riemannian manifold that ensures that any non-negative solution to the inequality elliptic differential inequalities on this manifold is identically equal to 0.

Properties of Solutions of Sub-Elliptic Equations with Singular Nonlinearities on Heisenberg Group

Xinjing Wang
Northwestern Polytechnical University, Peoples Rep of China
Pengcheng Niu

In this work, we consider positive solutions for the zero Dirichlet boundary condition to the singular semilinear sub-elliptic equation on the Heisenberg group

$$-\Delta_{\mathbb{H}} u = \frac{1}{u^\gamma} + f(u),$$

in a bounded smooth domain in the Heisenberg group. We provide the monotonicity and the symmetry of cylindrical solutions to the problem. The main technique is the generalization of moving plane method to the Heisenberg group.

Unified Weighted Poincaré Inequalities in Metric Measure Space and Applications

Huiju Wang
Northwestern Polytechnical University, Peoples Rep of China
Pengcheng Niu

In this work we establish unified weighted Poincaré inequalities in metric measure spaces. A new class of higher order Poincaré inequalities in the Euclidean space is given. We obtain weighted higher order Poincaré inequalities in the Euclidean space and stratified Lie groups, respectively.

Some Stability Results of the Large-Amplitude Traveling Fronts for Traffic Flow Models

Lina Wang
Beijing Technology and Business University, Peoples Rep of China
Jingyu Li, Tong Li, Yaping Wu

In this talk, we will focus on the stability of the traveling fronts for traffic flow models with uniform road width and non-uniform road width. By applying geometric singular perturbation method, special Evans function estimates, detailed spectral analysis and C_0 semigroup theories, the linear exponential stability of the non-degenerate waves with large wave strength in some exponentially weighted spaces will be shown. At the same time, the convergence rate of solutions to traveling fronts with small wave strength will also be given.

Green Functions for Weighted Subelliptic p-Laplacian Operators Constructed by Hörmander's Vector Fields

Leyun Wu
Northwestern Polytechnical University, Peoples Rep of China
Pengcheng Niu

This work deals with the following weighted subelliptic p-Laplacian constructed by Hörmander's vector fields:

$$L_p u = div_X \left(\langle A(x)Xu(x), Xu(x) \rangle^{\frac{p-2}{2}} A(x)Xu(x) \right),$$

where $u \in W^{1,p}(U,w), 1 < p < Q, A(x)$ is a bounded measurable and $m \times m$ symmetric matrix satisfying

$$\lambda^{-1} w(x)^{2/p} |\xi|^2 \leqslant \langle A(x)\xi,\xi \rangle \leqslant \lambda w(x)^{2/p} |\xi|^2,$$
$$\xi \in \mathbb{R}^m, w(x) \in A_p.$$

We first prove existence of the modified Green function of L_p by virtue of Minty-Browder theorem and then existence of the Green function of L_p by proving the convergence of sequence of modified Green functions. Next, we derived upper bounds of the modified Green function of L_p by establishing the interpolation inequality in the weighted weak L^p spaces. Finally, the bounds of the Green function of L_p are also obtained by virtue of results for the modified Green function and the weighted compact embedding theorem.

On the Principal Eigenvalue of a Perturbed Fractional Laplace Operator

Guangyu Zhao
Central Michigan University, USA

This talk presents our recent study on the principal eigenvalue problem of a perturbed fractional Laplace operator:

$$\begin{cases} (-\Delta)^s u + c(x)u = \lambda u, & \text{in}\Omega\ , \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega. \end{cases}$$

Here Ω is a bounded domain of $\mathbb{R}^n$ ($n \geq 1$) with smooth boundary, $c \in L^\infty(\Omega)$, and $s \in (0,1)$. Our work extends a number of well-known properties of the principal eigenvalue of the Laplace operator to the aforementioned fractional Laplace operator. More specifically, the established results, among the other things, reveal the equivalence between the validity of a strong maximum principle and the positivity of the principal eigenvalue. The similar characterizations were also obtained for a fractional Laplace operator associated with weakly coupled cooperative systems. As an application, these results are utilized to investigate the spatio-temporal dynamics of a few mathematical models that arise from population biology and mathematical epidemiology.

Extension of Hardy-Littlewood-Sobolev Inequality on Compact Manifolds

Meijun Zhu
University of Oklahoma, USA

In this talk, I will survey our recent work on the extension of Hardy-Littlewood-Sobolev inequality on compact Riemannian manifolds with or without boundary. The study of the sharp forms and related integral equations will be addressed.

Special Session 105: Recent Advances in Computational PDEs and their Applications

Xinfeng Liu, University of South Carolina, USA
Hong Wang, University of South Carolina, USA
Huanzhen Chen, Shandong Normal University, Peoples Rep of China

This minisymposium focuses on recent advances on efficient and accurate numerical methods for PDEs and their various applications in physical or biological systems. The invited researchers from diverse background will discuss a wide range of computational methods ranging from efficient finite element and finite difference methods, adaptive methods, multiscale methods, to spectral methods and kinetic Monte Carlo simulations. Computational challenges will be discussed, and new computational techniques will be introduced for various applications.

Adjoint-Free Calculation Method for Conditional Nonlinear Optimal Perturbations

Ming Cui
Beijing University of Technology, Peoples Rep of China

Adjoint-free calculation method is proposed to compute conditional nonlinear optimal perturbations (CNOP) combined with initial perturbations and model parameter perturbations. The new approach avoids the use of adjoint technique in the optimization process. CNOPs respectively generated by ensemble-based and adjoint-based methods are compared based on a simple theoretical model.

Continuous Galerkin Method for Delay Differential Equations of Pantograph

Qiumei Huang
Beijing University of Technology, Peoples Rep of China
Hermann Brunner, Xiuxiu Xu

We analyze the optimal global convergence and local superconvergence properties of continuous Galerkin (CG) solutions on uniform meshes and quasi-geomteric meshes for delay differential equations with proportional delay. It is shown that the attainable order of nodal superconvergence of CG solutions under quasi-geomteric meshes is higher than of the one under uniform meshes. The theoretical results are illustrated by a broad range of numerical examples.

Numerical Simulation for Fractional-Order Diffusion Equations

Chen Huanzhen
Shandong Normal University, Peoples Rep of China

In this talk we adopt the saddle-point theoretical framework to analyze the conservative space-fractional diffusion equations. By introducing an intermediate Hilbert space and a fractional-order flux as auxiliary variable, we establish the well-posedness of the saddle-point variational formulation and better regularity of the solution. A locally-conservative mixed finite element procedure based on the formulation is proposed to approximate the unknown, its derivative and the fractional flux directly. Existence and uniqueness results are proven and the error estimates are derived. Numerical experiments are included that confirm our theoretical findings.

Petrov-Galerkin Methods for Fractional Convection Diffusion Problem

Bangti Jin
University College London, England
Raytcho Lazarov, Zhi Zhou

In this work, we develop variational formulations of Petrov-Galerkin type for boundary value problems involving either a Riemann-Liouville or Caputo derivative of order $\alpha \in (3/2, 2)$ in the leading term and a convection term. The well-posedness of the formulations and sharp regularity pickup of the weak solutions are established. A novel finite element method is developed, which employs continuous piecewise linear finite elements and “shifted” fractional powers for the trial and test space, respectively. It admits optimal error estimates in both L^2- and H^1-norms. Extensive numerical results are presented to verify the theoretical analysis and robustness of the numerical scheme.

Efficient and Tunably Accurate Spectral Methods for Fractional Differential Equations on the Half-Line

Anna Lischke
Brown University, USA
Mohsen Zayernouri, George Karniadakis

In this talk, we introduce new Laguerre Petrov-Galerkin spectral methods for fractional differential equations on the half-line. We demonstrate the tunable accuracy of these methods and the sensitivity of the accuracy due to the tuning parameter using numerical experiments. We also show that these approaches result in computationally efficient methods for solving multi-term FDEs on the half-line.

Integration Factor Method for a Class of Differential Equations

Xinfeng Liu
University of South Carolina, USA

In this talk, we will present an efficient high-order integration factor method for solving a family of high order differential equations, in which the linear high order derivatives are explicitly handled and the computational cost and storage remain the same as to the classic integration factor methods for second-order problems. In particular, the proposed method can deal with not only stiff nonlinear reaction terms but also various types of homogeneous or inhomogeneous boundary conditions. Also such method has recently been extended to solve a hydrodynamic phase field model for a binary fluid mixture of two immiscible viscous fluids.

New Central Schemes on Overlapping Cells for Solving Ideal Magnetohydrodynamic Equations

Yingjie Liu
Georgia Institute of Technology, USA
Zhiliang Xu

We develop a new central DG-type method on overlapping cells for solving MHD equations on triangular meshes. This method is fully conservative for the magnetic field. New features are introduced to reduce the complexity.

Polynomial Approximate Solutions for a Non-Darcy Groundwater Flow Equation

Aleksey Telyakovskiy
University of Nevada, Reno, USA
Jeffrey Olsen, Jeff Mortensen

Certain kinds of flows in groundwater aquifers are modeled by a nonlinear Forchheimer equation. We consider semi-infinite aquifer that is initially dry. At the inlet boundary conditions are specified. For certain types of the boundary conditions problem can be reduced using similarity variables to a boundary-value problem for a nonlinear ordinary differential equation. We derive polynomial approximate solutions to that nonlinear ordinary differential equation. Our approach shows good results when it is compared with the numerical solution obtained with a rescaling algorithm.

Local High Order Absorbing Boundary Conditions in Terms of Farfield Expansions

Vianey Villamizar
Brigham Young University, USA
Sebastian Acosta, Blake Dstrup

A new local high order absorbing boundary condition (ABC) for scattering of time-harmonic waves from obstacles of arbitrary shape is devised. First the infinite domain Ω is truncated by means of an artificial boundary B. This results in a division of the original infinite domain into a finite computational domainΩ^- and an exterior infinite domain Ω^+. Then, we define interface conditions at the artificial boundary B from truncated versions of Wilcox's farfield expansion in 3D and Karp's farfield expansion in 2D. As a result, we obtain a new local ABC for a bounded problem onΩ^-, which effectively accounts for the outgoing behavior of the scattered field. Contrary to what happens to other ABCs previously defined, the order of approximation of the farfield pattern can be increased to any order. We accomplish this by adding as many terms as needed to the truncated farfield expansions. We include numerical results which demonstrate the improved accuracy when compared to other absorbing boundary conditions.

A Fast Collocation Method for a Bond-Based Peridynamic Model

Hong Wang
University of South Carolina, USA
Xuhao Zhang

We develop a fast collocation method for a two-dimensional linear steady-state bond-based peridynamic model, which provides an appropriate description of the planar deformation of a continuous elastic body involving discontinuities or other singularities. The method reduces the computational cost of evaluating and assembling the stiffness matrix from O(N2) to O(N), where N is the number of unknowns in the discrete system. The method also reduces the computational work from O(N2) to O(N log N) per Krylov subspace iteration and the memory requirement from O(N2) to O(N). All of this is achieved by carefully exploring the structure of the stiffness matrix of the collocation scheme, without any lossy compression involved. Numerical results are presented to show the utility of the method.

Split-Step Orthogonal Spline Collocation Methods for Nonlinear Schrodinger Equations in One, Two, and Three Dimensions

Shanshan Wang
Nanjing University of Aeronautics and Astronautics, Peoples Rep of China
Luming Zhang

Split-step orthogonal spline collocation (OSC) methods are proposed for one-, two-, and three-dimensional nonlinear Schrodiner (NLS) equations with time-dependent potentials. The original NLS equations are separated into two nonlinear equations, and one or more one-dimensional equations by the split-step method. Discrete-time OSC Schemes are applied to solve the linear subproblems. Commonly, the nonlinear subproblems could be integrated directly and accurately, but it fails when the time-dependent potential cannot be integrated exactly. In this case , three approximations based on the quadrature formulae is used, and split order in not reduced. Extensively numerical tests are carried out to verify the reliability and efficiency of the present method. The three approximations are applied to the split-step finite difference methods and the time-splitting spectral methods, and they also work well.

A Phase Field Model and Energy Stable Scheme for Simulating Multi-Phase Viscoelastic Incompressible Flow

Zhiliang Xu
University of Notre Dame, USA
Shixin Xu, Mark Alber

Diffusion interface (phase field) method is one of the most important approaches for studying multi-phase fluids due to its treatment of the interface as a physically diffuse thin layer. In this talk, we will present a thermodynamically consistent model for the mixture of Newtonian and viscoelastic fluids derived by using the Energetic Variational Approach. Elastic property of the viscoelastic fluid is described by the deformation gradient tensor in an Eulerian framework. Different densities and viscosities of distinct fluid phases are taken into account. This new model is shown to satisfy the law of energy dissipation automatically. An energy stable numerical scheme is proposed to solve the coupled system of model equations. Numerical experiments using problem with large density ratio and problem with mixture of Newtonian and viscoelastic fluids are carried out to validate the model and the scheme. Specifically, the model is used to simulate deformation of a blood clot under shear flow.

Special Session 106: Nonlinear Waves: Coherent Structures and Complex Dynamics

S. Roy Choudhury, University of Central Florida, USA
Andrei Ludu, Embry-Riddle Aeronautical University, USA

This session will consider recent advances in the field of nonlinear wave propagation, including coherent structures, Bose-Einstein condensates, and a variety of dynamical behaviors. Application will primarily be drawn from, but will not limited to, water waves and nonlinear optics.

Quantitative Dynamics and Routes to Chaos in a Chemical System

Sudipto Choudhury
University of Central Florida, USA

A global study of an generic family of chemical reactions is carried out to analyze their primary complex dynamical behaviors. The study proceeds from a construction of periodic solutions of the system constructed via a harmonic balance method. Bifurcations of such periodic orbits are then systematically considered, with secondary Hopf (Neimark-Sacker) bifurcations leading to quasiperiodic solutions and one route to chaos, a first period doubling initiating an infinite sequence in another well-known path to chaos, and so on. The analytical results give a fairly comprehensive general picture of possible system behaviors in the parameter space, and compare well when tested against numerical simulations. The predictions also provide a good guide to numerically searching for different dynamical behaviors in various parts of the parameter space.

Fractional Partial Differential Equation for Rapid Growing Systems

Andrei Ludu
Embry-Riddle Aeronautical University, USA

A short review on fractional ordinary/partial differential equations (FODE, FPDE) and some of their novel applications developed in the last decade will be presented, followed by a presentation of the improved fractional sub-equation method and the spectral method to solve such equations, including nonlinear PDE inspired by solitons in fluid dynamics. A new type of super-dynamical FPDE will be presented together with its potential applications in very fast time growing systems like those related to Kryder's, Reed's, Nielsen's, and Carlson's laws, etc.

JONSWAP Rogue Waves Over Non-Constant Backgrounds

Constance Schober
University Central Florida, USA
Anna Calini

JONSWAP rogue waves are large amplitude waves on a non-constant background that are distinct from prior analytical models, including spatially periodic breathers and rational solutions of the Nonlinear Schrödinger (NLS) equation, which are all constructed over a constant background. In this talk we classify JONSWAP rogue waves using the inverse spectral theory of the NLS equation and construct NLS solutions that model JONSWAP rogue waves. The associated spectral configurations are more complex that those of prior models, but also in these cases proximity to instabilities is the main indicator of rogue wave occurrence. To support this claim, we correlate the maximum wave strength as well as the higher statistical moments with elements of the nonlinear spectrum. The result is a diagnostic tool widely applicable to both model and field data for predicting the likelihood of rogue waves.

Solitary Waves for Long Wave-Short Wave Interaction System

Sharad Silwal
Jefferson College of Health Sciences, USA
Santosh Bhattarai

In this talk, we prove the existence of solitary wave solutions of a system of long wave-short wave interaction equations. The system here describes the interaction between multiple short waves and a long wave. The existence of solitary waves is established via a variational method and using the concentration compactness argument. We also prove the stability of the set of minimizers.

Numerical Investigation of the Dynamics of High-Intensity Ultrashort Light Pulses

Michail Todorov
Technical University of Sofia, Bulgaria

The pulse propagation in nonlinear bulk medium governed by the (3+1)D nonlinear Schrödinger equation (NLSE) as well as by the (3+1)D nonlinear envelope equation (NEE) is investigated. Implicit finite-difference schemes for the both dynamical systems are developed. An operator splitting by physical factors of these types of equations is applied and its suitability is grounded. For the non-linear terms of the equations is used the so-called internal iteration. Numerical treatment in complex arithmetic is carried and the generalized Thomas algorithm for multidiagonal complex banded matrices with pivoting is used.

Significant and consistent physical results are obtained. The minimal number of processes providing a self-compression of the pulse is established. A new regime of propagation of the pulse is found and a controllable guidance concept as alternative to the soliton concept based on (3+1)D NLSE is proposed. For (3+1)D NEE realistic propagation regimes are considered and a self-compression of the pulse as well as a stable propagation of the compressed pulse are found out. The strong influence of ionization on the group-velocity dispersion can cause even its inversion from positive to negative one.

Special Session 107: Analysis of Nonlinear Dispersive Wave Equations and Integrable Systems

Stephen Anco, Brock University, Canada
Robin Ming Chen, University of Pittsburg, USA
Yue Liu, University of Texas at Arlington, USA
Changzheng Qu, Ningbo University, Peoples Rep of China

This session will bring together researchers at all career stages to share their recent results on various aspects in the analysis of nonlinear dispersive wave equations and integrable structures. It will focus on (but not be restricted to) developments connected with derivation of nonlinear equations from physical models and integrable structures, including well-posedness, stability analysis, blow-up, wave breaking, and geometric aspects. Of special interest are equations related to the theory of water waves, such as the Camassa-Holm equation, Whitham equation, Hunter-Saxton equation, Degasperis-Procesi equation, NLS equation, and their various recent generalizations.

On the Symmetry of Traveling Wave Solutions to the Whitham Equation

Gabriele Bruell
NTNU Trondheim, Norway
Mats Ehrnstrom, Anna Geyer, Long Pei

The Whitham equation is a nonlocal, nonlinear dispersive wave equation introduced by G. B. Whitham as an alternative wave model equation for the Korteweg-de Vries equation, describing the wave motion at the surface on shallow water. Knowing that traveling wave solutions to the Whitham equation exist, we prove that any solitary wave solution is symmetric and has exactly one crest. Moreover, the structure of the Whitham equation allows to conclude that conversely any classical symmetric solution constitutes a traveling wave. In fact, the latter result holds true for a large class of partial differential equations sharing a certain structure.

Asymptotic Behavior of Bound States for a Class of Schrodinger-Poisson System

Jianqing Chen
Fujian Normal University, Peoples Rep of China
Yongqing Li, Zhengping Wang, Xiaoju Zhang

For the following Schrödinger-Poisson equation
(P_μ)
$$\begin{cases} -\Delta u + V(x)u + \lambda\phi(x)u = \mu u + |u|^{p-1}u, \ x \in \mathbb{R}^3, \\ -\Delta\phi = u^2, \ \lim_{|x|\to+\infty}\phi(x) = 0, \end{cases}$$

we study the existence of ground state and bound state as well as their asymptotic behavior depending on μ. Here $p \in (3,5)$, $\lambda > 0$, $V \in C(\mathbb{R}^3, \mathbb{R}^+)$ and $\lim_{|x|\to+\infty} V(x) = \infty$. We prove that for any $\lambda > 0$, there exists $\delta_1(\lambda) > 0$ such that for $\mu_1 < \mu < \mu_1 + \delta_1(\lambda)$, problem (P) has a nonnegative ground state, which bifurcates from zero solution; problem (P) has a nonnegative bound state, which bifurcates from a solution of (P_{μ_1}). Here μ_1 is the first eigenvalue of $-\Delta + V$. We also analysis the stability of bound states. Similar results are also proven for a more general version of SP system.

Asymptotic Analysis of the Models Arising from the Shallow Water with the Coriolis Effect

Guilong Gui
Northwest University, Peoples Rep of China
Yue Liu, Ting Luo, Junwei Sun

This talk is concerned with asymptotic analysis of the models arising from the shallow water with the Coriolis effect. It is shown that the rotation-Camassa-Holm (R-CH) equation captures stronger nonlinear effects than the classical nonlinear dispersive rotation-Korteweg-de Vries (R-KdV) equation. It is also demonstrated that the classical KdV equation is the limit approximation of the R-KdV model as the Coriolis effect vanishes. Moreover, it is established that the rotation-free limit of the R-CH equation is the classical CH equation.

On a 4-Parameter Family of Equations with Peakon Traveling Waves

Alex Himonas
University of Notre Dame, USA
Dionyssios Mantzavinos

The Cauchy problem for a novel 4-parameter family of evolution equations, which are nonlinear and nonlocal and possess peakon traveling wave solutions, is studied on both the line and the circle. It is proved that this family of equations is well-posed in the sense of Hadamard when the data belong to the Sobolev spaces H^s, $s > 5/2$. Also, it is shown that the data-to-solution map is not uniformly continuous. However, it is Hölder continuous in a weaker Sobolev norm.

Liouville Correspondence Between the Short-Pulse Hierarchy and The Sine-Gordon Hierarchy

Jing Kang
Northwest University, Peoples Rep of China

This talk considers the whole hierarchy of bi-Hamiltonian integrable equations associated to each of the Short-Pulse (SP) equation and the Sine-Gordon (SG) equation. We prove that the transformation that relates the SP equation with the SG equation also serves to establish the correspondence between their flows and Hamiltonian conservation laws in respective hierarchy.

Stability of the Camassa-Holm Peakons and Train of Peakons in the Dynamics of a Shallow-Water-Type System

Xiaochuan Liu
Northwest University, Peoples Rep of China

The stability of the Camassa-Holm (periodic) peaks and train of peakons in the dynamics of an integrable shallow-water-type system is investigated. A variational approach with the use of the Lyapunov method is presented to prove the variational characterization and the orbital stability of the peaked wave patterns. Furthermore, the energy method is used to show the stability issue of the train of peakons.

Orbital Stability of Solitary Waves in the Two-Component Camassa-Holm Systems

Ting Luo
The University of Texas at Arlington, USA

In this talk, we consider the orbital stability of solitary waves in the two-component Camassa-Holm systems. Using the property of almost monotonicity and the local coercivity of the solitary-wave solution, it is shown that the train of N-smooth solitary waves of the generalized two-component Camassa-Holm system is dynamically stable to perturbations in energy space with a range of parameters. In addition, stability results on multi-peakons and multi-antipeakon-peakons of the two-component Camassa-Holm system on the ground state is obtained by taking advantage of the conservation laws.

Multiple Solutions for Kirchhoff Type Problems Involving Super-Linear and Sub-Linear Terms

Cao Xiaofei
Southeast University, Peoples Rep of China
Junxiang Xu

The paper concerns the multiplicity of solutions for a class of Kirchhoff type problems with concave and convex nonlinearities on an unbounded domain. Under suitable hypotheses, it is proved that the Kirchhoff problem has at least two positive solutions, one of which has negative energy and the other positive energy. The proof is based on Ekeland's variational principle, Jeanjean's monotone method and the Pohozaev identity.

Blow-Up Phenomena and Persistence Property for the Modified b-Family of Equation

Ying Wang
University of Electronic Science and Technology of china, Peoples Rep of China

We investigate the blow-up mechanism and persistence property of solutions to the modified b-family of Equation. The dynamics of the blow-up quantity along the characteristics is established by the Riccati-type differential inequality. Furthermore, the persistence results for the solution in weighted spaces are established in the case of data decaying slower than solitons.

The Existence and Mass Concentration of L^2-Normalized Solutions for Nonlinear Fractional Schrodinger Equations

Fubao Zhang
Southeast University, Peoples Rep of China
Miao Du, Lixin Tian, Jun Wang

In this talk we will prove the existence and mass concentration of L^2-normalized solutions for the following Schrodinger equations with fractional Laplacian: $(-\Delta)^s u + V(x)u = \mu u + af(u), \quad x \in \mathbb{R}^N$.

Blow-Up of Solutions to the Periodic Modified Camassa-Holm Equation with Varying Linear Dispersion

Min Zhu
Nanjing forestry University, Peoples Rep of China
Shuanghu Zhang

Considered herein is the blow-up mechanism to the periodic modified Camassa-Holm equation with varying linear dispersion. We first consider the case when linear dispersion is absent and derive a finite-time blow-up result. The key feature is the ratio between solution and its gradient. Using the continuity of the solutions and the right transformation, we then obtain this blow-up criterion to the case with negative linear dispersion and determine that the finite time blow-up can still occur if the initial momentum density is bounded below by the magnitude of the linear dispersion and the initial datum has a local mild-oscillation region. Finally, we demonstrate that when the linear dispersion is non-negative, formation of singularity can be induced by an initial datum with a sufficiently steep profile.

Special Session 108: New Developments in Porous Media

Akif Ibragimov, Texas Tech University, USA
Yuliya Gorb, University of Houston, USA
Luan Hoang, Texas Tech University, USA

Porous media problem attract attention of scientists and mathematician since classical work by Henry Darcy. Fundamental developments in the area of porous media led to big spectrum of application spanning problems from engineering, and Geo-sciences to biochemistry and environmental sciences. Many problems in the porous media are highly non-linear involving modeling of coupling physical and chemical processes at different scales. The goal of special session is to present a new development of the modeling of the processes in the porous media from mathematical analysis point of view and discuss most challenging up-today problems.

Multiscale Numerical Methods for Solving Nonlinear Forchheimer Equation in Highly Heterogeneous Porous Media

Manal Alotibi
Texas A&M University, USA
Yalchin Efendiev, Eric Chung

Abstract. In this talk, I will present a local multiscale model reduction for nonlinear flows in heterogeneous porous media. I will consider generalized Forchheimer equations. The generalized Forchheimer equation describes flows at Darcy scales and arises when the pore-scale velocity is large. We consider the two term law form of Forchheimer equation and write the resulting system in terms of a degenerate nonlinear flow equation for the pressure. Our multiscale model reduction can be considered a generalization of recently introduced upscaling and numerical homogenization techniques, where the authors consider problems with scale separation. In the proposed approach, we construct local reduced-order model by constructing appropriate snapshot spaces and local spectral problems within the framework of Generalized Multiscale Finite Element Method (GMsFEM). To save the computational time, we use empirical interpolation techniques in estimating the nonlinear terms. I will discuss the use of adaptive procedures both in offline and online stages of the computation. We present numerical and theoretical results for the proposed method.

Some New Approaches to Simulating Two-Phase Flow in Porous Media on Hexahedral Meshes

Todd Arbogast
University of Texas at Austin, USA
Maicon Correa, Chieh-Sen Huang, Zhen Tao, and Xikai Zhao

Subsurface geology often dictates that relatively general hexahedral computational meshes be used when simulating flow in porous media. The equations of two-phase flow divides into a parabolic pressure equation for the flow and a degenerate parabolic (convection-diffusion) saturation equation for the transport. We present three new approaches. (1) The elliptic part of both equations is often approximated using mixed finite elements, which are defined by mapping from a reference cube using the Piola transformation. This destroys the approximation properties of the method. We describe a new family of finite elements (AC elements) that overcomes the problem. (2) The saturation equation exhibits degeneracy in its elliptic diffusion term due to loss of capillarity when a phase is lost. We present a new mixed formulation that is stable when approximating degeneracies. It is suitable for approximation with the new AC elements. (3) The convection part of the saturation equation is often approximated, for example, using discontinuous Galerkin (DG) methods, since DG can handle general meshes. Traditional WENO methods are very accurate but restricted to rectangular meshes. We present a new approach using high order WENO reconstructions on logically rectangular meshes.

Productivity Index for Darcy and Pre-/post-Darcy Flow

Lidia Bloshanskaya
SUNY New Paltz, USA
Akif Ibragimov, Fahd Siddiqui, Mohamed Y. Soliman

We investigate the impact of nonlinearity of high and low velocity flows on the well productivity index (PI). Experimental data shows the departure from the linear Darcy relation for high and low velocities. High-velocity (post-Darcy) flow occurring near wells and fractures is described by Forchheimer equations and is relatively well-studied. While low velocity flow receives much less attention, there is multiple evidence suggesting the existence of pre-Darcy effects for slow flows far away from the well. This flow is modeled via pre-Darcy equation. We combine all three flow regimes, pre-Darcy, Darcy and post-Darcy, under one mathematical formulation subjected to certain critical transitional velocities. This allows to use our previously developed framework to obtain the analytical formulas for the PI for the cylindrical reservoir. We study the impact of non-Darcy effect on the PI depending on the well-flux and the parameters of the equations.

Studying Generalized Forchheimer Flows in Heterogeneous Porous Media

Emine Celik
Texas Tech University, USA
Luan Hoang

We study the generalized Forchheimer flows of slightly compressible fluids in heterogeneous porous media where the derived nonlinear partial differential equation for the pressure can be singular and degenerate in the spatial variables, in addition to being degenerate for large pressure gradient. Suitable weighted Lebesgue norms for the pressure, its gradient and time derivative are estimated. The continuous dependence on the initial and boundary data is established for the pressure and its gradient with respect to those corresponding norms. Asymptotic estimates are derived even for unbounded boundary data as time tends to infinity. We also obtain the estimates for the L^∞-norms of the pressure and its time derivative by implementing De Giorgi's iteration in the context of the above weighted norms. This is a joint work with Luan Hoang.

Self-Similar Viscous Gravity Currents in Heterogeneous Porous Media: Second-Kind Solutions

Ivan Christov
Purdue University, USA
Z. Zheng, H. A. Stone

We summarize our recent combined experimental-theoretical-computational study of the effects of horizontal heterogeneities on the propagation of viscous gravity currents with applications to porous media flows. Our model geometry is a horizontal channel (specifically, a Hele-Shaw cell) with variable gap thickness in the streamwise direction in the form of a power law. We demonstrate that two types of self-similar behaviors emerge as a result of such horizontal heterogeneity: (a) a "first-kind" solution is found using dimensional analysis for currents that propagate away from the origin (a point of zero permeability); (b) a "second-kind" solution is found using a phase-plane analysis for viscous gravity currents that propagate toward the origin. Using the phase-plane formalism, we are able to construct the universal second-kind self-similar current shape. Additionally, still employing self-similar intermediate asymptotics and the phase-plane formalism, we identify self-similar behaviors in the post-closure regime, i.e., once the current reaches the geometric origin and begins to fill the model porous medium. The theoretical predictions show good agreement with lab-scale experiments using Hele-Shaw cells and also numerical solutions of the governing partial differential equation developed under the lubrication approximation. Z. Zheng, I.C. Christov, H.A. Stone, *J. Fluid Mech.* **747** (2014) 218-246, doi:10.1017/jfm.2014.148.

Analysis of a Turbulence K-Epsilon Model with Applications in Porous Media

Hermenegildo de Oliveira
Universidade do Algarve, Portugal
Ana Paiva

In this talk we will analyse a one-equation turbulence model of the k-epsilon type that is being used to describe turbulent flows through porous media. The considered equations are in the steady-state and we supplement them with homogeneous Dirichlet boundary conditions. The novelty of the problem relies on the consideration of the classical Navier-Stokes equations with feedback's forces field, whose presence in the momentum equation will affect the equation for the turbulent kinetic energy (TKE) with a new term that is known as the production and represents the rate at which TKE is transferred from the mean flow to the turbulence. For the considered problem, we prove the existence and uniqueness of weak solutions by assuming suitable growth conditions together with monotone conditions on the feedback terms. In this talk we will also address the issue of existence by considering strongly nonlinear feedback terms and the question of partial regularity of the solutions will be analyzed as well.

An Expanded Mixed FEM for Generalized Forchheimer Flows of Slightly Incompressible Fluids in Porous Media

Thinh Kieu
University of North Georgia, USA
Akif Ibragimov

We study the expanded mixed finite element method applied to the generalized Forchheimer equation with the Dirichlet boundary condition. The bounds for the solutions are established. In both continuous and discrete time procedures, utilizing the monotonicity properties of Forchheimer equation and boundedness of solutions, we establish the error estimates for the solutions in several Lebesgue norms. A numerical example using the lowest order Raviart-Thomas (RT_0) mixed element agrees our theoretical result regarding convergence rate.

Interior $W^{1,q}$ Estimates for Solutions of Nonlinear Degenerate Parabolic Systems

Truyen Nguyen
University of Akron, USA

We consider nonlinear parabolic systems of the form $u_t = \operatorname{div} \mathbf{A}(x,t,u,\nabla u) + \mathbf{B}(x,t,u,\nabla u)$ which include those of p-Laplacian type. In this talk, we will discuss some results concerning local integrability of gradients of weak solutions to the system. In particular, we derive interior L^q estimates for the gradient when

A is possibly dicontinuous in the x variable. The dependence of the principal part on the u variable made it difficult to perform any scaling analysis and we handle it by using the intrinsic geometry method of DiBenedetto together with our two-parameter scaling technique.

Reconstruction of Dynamic Tortuosity from Data at Distinct Frequencies

Yvonne Ou
University of Delaware, USA

Dynamic tortuosity plays an important role in the energy dissipation of wave propagation in poroelastic materials by being the kernel function of the memory terms in the time-domain Biot-Johnson-Koplik-Dashen (JKD) wave equations. In this talk, the integral representation formula (IRF) of dynamic tortuosity will be presented and the mathematical strategy for constructing the dynamic tortuosity function from dynamic permeability at distinct frequencies, utilizing the IRF, will be explained. Numerical results for JKD tortuosity will be demonstrated in this talk.

Reduced Order Hybrid Modeling from Pore-Scale to Core-Scale

Malgorzata Peszynska
Oregon State University, USA
Tim Costa, Anna Trykozko

We propose a new paradigm for modeling flow and transport when the pore-scale geometry is changing due to, e.g., reactive transport, phase transitions, bioclogging, proppant, and/or matrix swelling; (denoted by the proxy u). Such changes are sometimes accounted for with ad-hoc algebraic relationships such as Carman-Kozeny for Darcy conductivities $K(\phi)$. Based on our experience with real pore-scale imaging data, we propose a new reduced order hybrid dynamic methodology for $K(u)$. We do not require transient simulations at pore-scale, but rather we rely on a set of values computed offline based on a) a stochastic parametrization of the modified pore-geometries, b) pore-scale flow solver with an Immersed Boundary, c) and a reduced order model which approximates $K(u)$.
We (i) use a probability distribution $K(\phi,\omega)$ instead of $K(\phi)$ to accurately account for the evolving pore-scale. Next, (ii) given the character of u (e.g., pore-filling, or pore-coating), we sample efficiently from the corresponding distribution of $K(\phi,\omega)$. In addition, (iii) we account for the local in time and space changes in $K(u)$ by introducing the intermediate third scale of pore-network. The latter step prevents the prohibitive complexity of local pore-scale transport computations.

Phase-Field Modeling of Proppant-Filled Fractures in a Poroelastic Medium

Mary Wheeler
The University of Texas At Austin, USA
S. Lee, A. Mikelic, T. Wick

This work presents proppant and fluid-filled fracture with quasi-Newtonain fluid in a poroelastic medium. Lower-dimensional fracture surface is approximated by using the phase field function. The two-field displacement phase-field system solves fully-coupled constrained minimization problem due to the crack irreversibility. This constrained optimization problem is handled by using active set strategy. The pressure is obtained by using a diffraction equation where the phase-field variable serves as an indicator function that distinguishes between the fracture and the reservoir. Then the above system is coupled via a fixed-stress iteration. The transport of the proppant in the fracture is modeled by using a power-law fluid system. The numerical discretization in space is based on Galerkin finite elements for displacements and phase-field, and an Enriched Galerkin method is applied for the pressure equation in order to obtain local mass conservation. The concentration is solved with cell-centered finite elements. Nonlinear equations are treated with Newton's method. Predictor-corrector dynamic mesh refinement allows to capture more accurate interface of the fractures with reasonable number for degree of freedoms.

References

[1] Lee, S., Wheeler, M., and Wick, T.; Pressure and fluid-driven fracture propagation in porous media using an adaptive finite element phase field model

[2] Lee, S., Mikelic, A., Wheeler, M., and Wick, T.; Phase-field modeling of proppant-filled fractures in a poroelastic medium

Effect of Pore Size Distribution on the State of Hydrocarbon Phases During Pressure Depletion

Xiaolong Yin
Colorado School of Mines, USA

In shale gas and shale oil reservoirs, hydrocarbons are generally stored in pores that are nanometers to tens of nanometers in size. In these pores, capillary pressure affects the phase behavior of hydrocarbon mixtures. In this talk, I will present modeling of the effect of pore size distribution on vapor-liquid equilibrium in porous media with narrow pores and strong capillary pressures. Such models are important for predicting the saturations and properties of gas and oil phases during primary production. Take a porous medium that is initially saturated with oil as an example. Gas saturation will appear first in large pores; the initially formed gas changes the composition of the remaining oil, and the compositional change in turn alters the equilibrium condition at which oil is vaporized in smaller pores. To properly predict the state of phases confined in this porous medium, one therefore must trace the entire pressure-saturation history and cannot just rely on a single phase diagram. Our calculations show that the capillary force increases with increasing gas saturation, and it is likely that the smallest pores are always filled with liquid during production, no matter whether the reservoir is initially filled with oil or gas.

Diffusion in Random Networks

Duan Zhang
Los Alamos National Laboratory, USA
Juan C. Padrino

We study mass diffusion in an ensemble of random networks consisting of junction pockets connected by tortuous microchannels. Inside the channels, the mass diffusion is governed by the one-dimensional diffusion equation. Using the ensemble averaging technique to derive an averaged equation for these processes, we find that the average concentration evolution is governed by an integro-differential equation. In the case of diffusion in a semi-infinite domain, this equation predicts that for an early time compared to the characteristic time of channel diffusion, there is a similarity variable $xt^{-1/4}$ for the average concentration in these inhomogeneous media, instead of the traditional $xt^{-1/2}$ in a homogeneous medium, where x is the distance from the boundary, and t is the time. This early time similarity is a result of the time required to establish the linear concentration profile inside the channels and can be explained by the random walk theory.
Work sponsored by LANL LDRD project 20140002DR.

Flow Regimes for Fluid Injection Into a Confined Porous Medium

Zhong Zheng
Princeton University, USA
Bo Guo, Ivan C. Christov, Michael A. Celia, Howard A. Stone

We report theoretical and numerical studies of the flow behaviour when a fluid is injected into a confined porous medium saturated with another fluid of different density and viscosity. For a two-dimensional configuration with point source injection, a nonlinear convection-diffusion equation is derived to describe the time evolution of the fluid-fluid interface. In the early time period, the fluid motion is mainly driven by the buoyancy force and the governing equation is reduced to a nonlinear diffusion equation with a well-known self-similar solution. In the late time period, the fluid flow is mainly driven by the injection, and the governing equation is approximated by a nonlinear hyperbolic equation that determines the global spreading rate; a shock solution is obtained when the injected fluid is more viscous than the displaced fluid, whereas a rarefaction wave solution is found when the injected fluid is less viscous. In the late time period, we also obtain analytical solutions including the diffusive term associated with the buoyancy effects (for an injected fluid with a viscosity higher than or equal to that of the displaced fluid), which provide the structure of the moving front. Numerical simulations of the convection-diffusion equation are performed; the various analytical solutions are verified as appropriate asymptotic limits, and the transition processes between the individual limits are demonstrated. The flow behaviour is summarized in a diagram with five distinct dynamical regimes: a nonlinear diffusion regime, a transition regime, a traveling wave regime, an equal-viscosity regime, and a rarefaction regime.

Special Session 109: Applied and Integrable Nonlinear PDEs

Stephen Anco, Brock University, Canada

This session will focus on a number of recent developments in the very active area of nonlinear PDEs: integrable systems; Hamiltonian structures; symmetry reductions and constraints;Hamiltonian splitting; special solutions e.g. solitons on non-zero backgrounds, rogue waves, peakons, compactons, lump solutions and complexiton solutions; exact solution methods. Directions of work related to new methods and their applications to integrable systems and nonlinear PDEs will be emphasized, with the aim of bringing together a number of leading researchers and young scientists working in on these topics.

Equivalence Group of a Generalized Kuramoto-Sivashinski Equation and Conservation Laws

Maria Bruzon
University of Cadiz, Spain
Rafael de la Rosa

The aim of this paper is to carry out an exhaustive analysis of the symmetries of a Generalized Kuramoto-Sivashinsky equation with dispersive effects. To achieve this objective, we obtain the continuous equivalence transformations of this class. The generators of the equivalence group allow us to determine for which types of arbitrary functions the equation admits additional symmetries. In addition, we get the corresponding group invariant solutions. Furthermore, we derive some conservation laws by applying the direct construction method of Anco and Bluman.

Symmetry Analysis and Conservations Laws for Some Equations with Compacton Solutions

Maria Luz Gandarias
University of Cadiz, Spain
Maria Rosa

In this talk we consider some equations that admit compacton solutions induced by a non-convex convection. We derive symmetry reductions and we find some of the reduced ordinary differential equations suitable for a qualitative analysis. By using the multipliers method we find a classification of the low-order conservation laws for these equations.

An Integrable Hamiltonian Hierarchy in sl$(2,\mathbb{R})$ and Its Counterpart in so$(3,\mathbb{R})$ with Three Potentials

Xiang Gu
University of South Florida, USA
Wen-Xiu Ma, Wen-Ying Zhang

Starting from two specific matrix spectral problems associated respectively with sl$(2,\mathbb{R})$ and so$(2,\mathbb{R})$ matrix Lie algebras, we engender two integrable Hamiltonian hierarchies with three potentials. The computation and analysis on their Hamiltonian structures by means of the trace identity reveal the Liouville integrability of both hierarchies; namely, that they both consist of infinitely many independent commuting conserved functionals and symmetries.

Residual Symmetries of the MKdV Equation and The AKNS Equations

Ping Liu
University of Electronic Science and Technology of China Zhongshan Institute, Peoples Rep of China

The residual symmetries of the mKdV equation and the AKNS equations are obtained by the truncated Painleve analysis. The residual symmetries for the mKdV equation are proved to be nonlocal and the nonlocal residual symmetries are extended to the local Lie point symmetries by means of enlarging the mKdV equations. It is noted that we researched the twofold residual symmetries by means of taking the mKdV equation as an example. Similarity solutions and the reduction equations are demonstrated for the extended mKdV equations related to the twofold residual symmetries. The residual symmetries for the AKNS equations are proved to be nonlocal and the nonlocal residual symmetries are extended to the local Lie point symmetries of a prolonged AKNS system. The local Lie point symmetries of the prolonged AKNS equations are composed of the residual symmetries and the standard Lie point symmetries, which suggests that the residual symmetry method is a useful complement to the classical Lie group theory. Several types of exact solutions for the AKNS equations are obtained with the help of the symmetry method and the Backlund transformations between the AKNS equations and the Schwarzian AKNS equation.

The Darboux Transformation for a Generalized Associated Camassa Holm Equation

Lin Luo
Shanghai Second Polytechnic University, Peoples Rep of China

In this talk, we discuss an integrable generalization of the associated Camassa Holm equation. The generalized system is shown to be integrable in the sense of Lax pair. Meanwhile, the Darboux transformation for the system is derived with the help of the gauge transformation between two Lax pairs. As an application, soliton and periodic wave solutions are given through the Darboux transformation.

Quasideterminant Solutions of NC Painlevé II Equation with the Toda Solution at $n = 1$ As a Seed Solution in Its Darboux Transformation

Irfan Mahmood
University of the Punjab, Pakistan

In this paper, the Darboux-quasideterminant solutions for the noncommuting elements ϕ and ψ of noncommutative (NC) Toda system at $n = 1$ are presented. Their Darboux transformations have constructed with the zero curvature representations of the associated systems of non-linear differential equations. I have also derived the quasideterminant solutions to the NC Painlevè II equation by taking the Toda solutions at $n = 1$ as a seed solution in its Darboux transformations. Further by iteration, I generalize the Darboux transformations of these solutions to the N-th form.

A Two-Component Short Pulse System Produced Through a Negative Integrable Flow

Zhijun (George) Qiao
University of Texas - Rio Grande Valley, USA
Qilao Zha, Qiaoyi Hu

In this paper, we study a two-component short pulse system, which was produced through a negative integrable flow associated with the WKI hierarchy. The Cauchy problem and the multi-soliton solutions for the two short pulse system investigated, in particular, one-, two-, three-loop soliton, and breather soliton solutions are discussed in details with interesting dynamical interactions and shown through figures.

A Nonlinear Generalization of Camassa-Holm and Modified Camassa-Holm Equations with Multi-Peakon Solutions

Elena Recio
Brock University, Canada
Stephen Anco

In this work, a 2-parameters family of equations generalizing the Camassa-Holm equation and the modified Camassa-Holm equation is considered. This equation reduces to the Camassa-Holm equation when $p = 2$ and the modified Camassa-Holm equation when $p = 3$ and shares one of the Hamiltonian structures of both equations. In addition, this equation admits multi-peakon solutions.

Applications of Wu Method for PDE Symmetry Calculation, Classification, Decision and Extension

Chaolu Termuer
Shanghai Maritime University, Peoples Rep of China

In this talk, we give a review on characteristic set algorithm for PDE symmetry calculation, classification, decision and extension done in recent years by author. A key step of solving these problems is to analysis and solves so called determining system. The essential idea of our research for the purposes is in the following. First we turn the problems of PDE symmetry calculation, classification, decision and extension equivalently into ones of dealing with the zero point set of differential polynomial system (d.p.s). The target of the idea is that we focus our attention on the analysis of the zero point set and algebra properties of the d.p.s corresponding to determining equations of the symmetry. Then we use the characteristic set theory and algorithm fundamental tool in differential algebra on the transferred problems and solve original problems. Doing so, we can get an alternative ways to discuss these problems from differential algebra point and get automatic algorithm to obtain concrete symmetry and decision of (non classical) symmetry existence for a PDE. Meantime, the algorithm can give symmetry classification and symmetry extension of a PDE with arbitrary parameters.

Complexiton Solutions and Linear Superposition Principle to Bilinear PDEs

Yuan Zhou
University of South Florida, USA
Wen-Xiu Ma

Based on previous work of Ma and his collaborators, we analyzed the linear superposition principle of complexitons (exponential and trigonometric traveling waves), we proposed a method to construct a sub-class of exact solutions to some bilinear PDEs by linear combinations of complexitons and solitons under weak conditions. We obtained multi-complexiton solutions for some integrable systems such as KdV, KP equations via applying Hirota's direct method. We also considered the counterpart of generalized bilinear derivative case. Finally, We presented some examples in different situations.

Special Session 110: Computational and Mathematical Methods for Complex Biological Systems

Andrew Nevai, University of Central Florida, USA
Libin Rong, Oakland University, USA
Zhisheng Shuai, University of Central Florida, USA

The characterization in space and time scales often leads to extraordinary complexity in biology systems. Mathematics and computation now play a much greater role in the study of complex biological systems. This special session will gather researchers who are actively in development and application of computational and mathematical methods in mathematical biology. The purpose is to present the most cutting-edge studies in the field and to promote further method development and practical application.

The Importation, Establishment and Transmission Dynamics of the Mosquito-Borne Disease

Jing Chen
University of Miami, USA
Shigui Ruan

From 2006-2013, all the cases of chikungunya virus infection in the United States were travelers visiting or returning to the United States from affected areas. From the beginning of 2014, 12 locally-transmission cases were reported from Florida in 2014. As the development of transportation systems, it is more convenient and frequent to have long-distance travel today. Therefore, they are not the mosquito, but the humans who carry the viruses, move to another country and spread the disease. Our hypothesis is the imported cases first spread the mosquito-borne disease to local mosquitoes, which later cause local infections on humans. Based on this, we propose a mathematical model to study how these movements of humans affect the establishment and transmission dynamics of the mosquito-borne disease.

On the Turnover Rate of the HIV Latent Reservoir in Untreated Patients

Ruian Ke
North Carolina State University, USA
Kai Deng

The stability of the HIV latent reservoir, a population of cells latently infected by replication competent HIV but do not actively produce viruses, represents a major barrier to cure HIV infection. The mechanism underlying this stability is not clear. It may be due to a low death rate of latently infected cells; alternatively, it may be due to a dynamical balance between high production and high death rate. Recent data shows that the frequency of cytotoxic T lymphocyte (CTL) escape mutant viruses in the latent reservoir rises to 98% in a majority of untreated patients between 6 months to 2 years after HIV infection. Here, we construct mathematical model to describe the dynamics of transmitted founder virus and CTL escape mutants in the plasma and the reservoir, and use the model to estimate the rate of turnover of the HIV latent reservoir from the recent data. The results suggest that the half-life of the reservoir is at least 10 times shorter in untreated patients than the half-life estimated from treated patients. We further explore possible mechanisms that drive a fast turnover rate in untreated patients. This work sheds light on the dynamic nature of the reservoir and has implications for HIV eradication strategies.

Systematic Measures of ODE-Modeled Complex Networks

Yao Li
University of Massachusetts Amherst, USA
Yingfei Yi

Proposed in systems biology, systematic measures of complex biological systems, including degeneracy, complexity, and robustness, have been frequently used by biologists. Measuring the ability of structurally different components to perform the same function, degeneracy is known to have close ties with both structural complexity and robustness of complex systems. In this talk I will report our efforts of quantifying these systematic measures. In addition, we will also discuss our results about connections among degeneracy,complexity, and robustness.

On the Principle for Host Evolution in Host-Pathogen Interactions

Maia Martcheva
University of Florida, USA
Necibe Tuncer, Yena Kim

We use a two-host one pathogen immuno-epidemiological model to argue that the principle for host evolution, when the host is subjected to a fatal disease, is minimization of the case fatality proportion $\mathcal{F}$. This principle is valid whether the disease is chronic or leads to recovery. In the case of continuum of hosts, stratified by their immune response stimulation rate a, we suggest that $\mathcal{F}(a)$ has a minimum because a trade-off exists between virulence to the host induced by the pathogen and virulence induced by the immune response. We find that the minimization of the case fatality proportion is an evolutionary stable strategy (ESS) for the host.

Effects of Macroalgal Toxicity and Overfishing on the Resilience of Coral Reef

Samares Pal
University of Kalyani, India
Joydeb Bhattacharyya

Competition between macroalgae and corals for occupying the available space in sea bed is an important ecological process underlying coral-reef dynamics. While herbivorous reef-fish play a beneficial role in decreasing the growth of macroalgae, macroalgal toxicity and overfishing of herbivores leads to the proliferation of macroalgae in coral reef ecosystem, which eventually changes the community structure towards macroalgae-dominated reef ecosystem. We analyze a mathematical model of interactions between coral, macroalgae and herbivores to investigate coral-macroalgal phase shifts by assuming that the growth of herbivorous Parrotfish is limited by coral cover. It is observed that in presence of macroalgal toxicity and overfishing of Parrotfish the system exhibits hysteresis through saddle-node bifurcation and transcritical bifurcation. We examine the effects of macroalgal toxicity and herbivore-harvesting in the resilience of coral-macroalgae coexistence steady state. Further, we study the non-autonomous version of the model by incorporating synchronous or asynchronous seasonal variations in different parameters. By using of Mawhin's continuous theorem of coincidence degree theory, a sufficient condition is obtained for the existence of a positive periodic solution. Computer simulations have been carried out to illustrate different analytical results.

Stochastic Models in a Heterogeneous Host Population

Donald Porchia
University of Central Florida, USA
R. N. Mohapatra, Zhisheng Shuai

Heterogeneity and stochasticity are two important characteristics in modeling infectious diseases. We propose and investigate basic stochastic models in a heterogeneous host population. Simulations are carried out to demonstrate the impact of heterogeneity and stochasticity.

Virus and T Cell Dynamics in HIV-Infected Individuals

Libin Rong
Oakland University, USA

In this talk, I will present some recent work on modeling HIV infection and treatment, such as HIV latency and persistence, viral blips, virus dynamics under different classes of drugs, treatment intensification with additional drugs, and the slow time scale of CD4+ T cell depletion in untreated patients. Model formulation, mathematical analysis, numerical simulation and comparison with data will be presented. Implications of modeling results for viral control strategies will also be discussed.

Modeling Zika As a Mosquito-Borne and Sexually Transmitted Disease

Shigui Ruan
University of Miami, USA
Daozhou Gao, Yijun Lou, Daihai He, Travis C. Porco, Yang Kuang, Gerardo Chowell

The ongoing Zika virus (ZIKV) epidemic poses a major global public health emergency. It is well-known that ZIKV is spread by Aedes mosquitoes, recent studies show that ZIKV can also be transmitted via sexual contact and several cases of sexually transmitted ZIKV of the current outbreak have been confirmed in the U.S. and France. We presented a mathematical model to investigate the impact of mosquito-borne transmission and sexual transmission on prevention and control of ZIKV and used the model to fit the ZIKV data up to February 2016 in Brazil, Colombia, and El Salvador. Our study indicates that sexual transmission increases the risk of infection and epidemic size and prolongs the outbreak. In order to prevent and control the transmission of ZIKV, it must be treated as not only a mosquito-borne disease but also a sexually transmitted disease.

A Mathematical Model for Feline Leukemia

Jeff Sharpe
University of Central Florida, USA
Andrew Nevai

A model of feline leukemia in a population of feral cats is studied. Building upon a multi-compartment model involving kittens, adult females, and adult males, the model accounts for multiple methods of virus transmission. A basic reproduction R_0 can be defined which distinguishes between disease persistence and disease eradication. Vaccination strategies are also considered.

Mathematical Models for the Trojan Y-Chromosome Eradication Strategy of an Invasive Species

Xueying Wang
Washington State University, USA
Jay R Walton, Rana D Parshad

The Trojan Y-Chromosome (TYC) strategy, a genetic biocontrol method, has been proposed to eliminate invasive species by introducing sex-reversed trojan females. Because constant introduction of the trojans for all time is not possible in practice, there arises the question: What happens if this injection is stopped after some time? Can the invasive species recover? To answer that question, we study this strategy through deterministic and stochastic models. Our results show that: (1) with the inclusion of an Allee effect, the number of the invasive females is not required to be very low when this injection is stopped, and the remaining trojan population is sufficient to induce extinction of the invasive females; (2) incorporating diffusive spatial spread does not produce a Turing instability, which would have suggested that the TYC eradication strategy might be only partially effective; (3) the probability distribution and expectation of the extinction time of invasive females are heavily shaped by the initial conditions and the model parameters; (4) elevating the constant number of the trojan females being introduced into the population will lead to a decrease in the expected extinction time for wild-type females, as opposed to an increase in the extinction probability within an application time.

Evaluation on Outcomes of Vaccine Strategies for Areas with Different Demographic Structures

Yanyu Xiao
University of Cincinnati, USA
M. Laskowski, N. Charland S. M. Moghadas

Ongoing research and technology developments hold the promise of rapid production and large-scale deployment of strain-specific or cross-protective vaccines for novel influenza viruses. We sought to investigate the impact of early vaccination on age-specific attack rates and evaluate the outcomes of different vaccination strategies that are influenced by the level of single or two-dose vaccine-induced protections. We developed and parameterized an individual-based model for two population demographics of urban and remote areas in Canada. Our results demonstrate that there is a time period before and after the onset of epidemic, during which the outcomes of vaccination strategies may differ significantly and are highly influenced by demographic characteristics.

Special Session 111: Geometric Methods in Mechanics and Differential Equations

Dmitry Zenkov, North Carolina State University, USA
Irina Kogan, North Carolina State University, USA

This session will concentrate on the contemporary geometric techniques in ordinary and partial differential equations, finite and infinite dimensional mechanics, and control. The geometric viewpoint provides a unified and systematic framework for addressing a broad range of questions arising in variational calculus, variational geometry, fluid dynamics, mechanics of coupled systems, and mathematical physics. Geometric method include structure-preserving integration, symmetry reduction, moving frame calculus, and integrability. This session will become a venue for mathematicians, applied mathematicians, and engineers to explore their common interest in the geometric approach to mechanics, and more generally to non-linear differential equations.

Conservation Laws with Coinciding Shock and Rarefaction Curves

Michael Benfield
NC State University, USA
Kris Jenssen, Irina Kogan

Hyperbolic conservation laws are used to model waves propagating at finite speed. Solving conservation laws involves analyzing wave interactions: what happens when two waves collide? A particularly simple case is when the shock and rarefaction curves of the conservation law coincide. Here we prove several results related to such systems and introduce the class of quadratically interacting systems.

Control and Sensitivity Analysis in Fluid-Elasticity Interactions

Lorena Bociu
NC State University, USA

Free and moving boundary fluid-structure interactions are ubiquitous in nature, with most known examples coming from industrial processes, aero-elasticity, and biomechanics. We consider optimal control problems subject to interactions between viscous, incompressible fluid and nonlinear elasticity. We discuss existence of optimal controls, sensitivity equations, and optimality conditions. One of the main challenges of applying optimization tools to free and moving boundary interactions is the proper derivation of the adjoint sensitivity information with correct adjoint balancing conditions on the common interface. As the coupled fluid-structure state is the solution of a system of PDEs that are coupled through continuity relations defined on the free interface, sensitivity of the fluid state, which is an Eulerian quantity, with respect to the motion of the solid, which is a Lagrangian quantity, falls into moving shape analysis framework.

Lipschitz Metric for Variational Wave Equation

Geng Chen
Georgia Institute of Technology, USA
Alberto Bressan

The nonlinear wave equation: $u_{tt}-c(u)[c(u)u_x]_x = 0$ is a natural generalization of the linear wave equation. In this talk, we will discuss a recent breakthrough addressing the Lipschitz continuous dependence of solutions on initial data for this quasi-linear wave equation. Our earlier results showed that this equation determines a unique flow of conservative solution within the natural energy space $H^1(R)$. However, this flow is not Lipschitz continuous with respect to the H^1 distance, due to the formation of singularity. To prove the desired Lipschitz continuous property, we constructed a new Finsler type metric, where the norm of tangent vectors is defined in terms of an optimal transportation problem. For paths of piecewise smooth solutions, we carefully estimated how the distance grows in time. To complete the construction, we proved that the family of piecewise smooth solutions is dense, following by an application of Thom's transversality theorem. This is a collaboration work with Alberto Bressan.

Nonholonomic Dynamics As Limit of Infinite Friction

Jaap Eldering
Universidade de Sao Paulo - ICMC, Brazil

Nonholonomic dynamics is Lagrangian mechanics extended with d'Alembert's a principle to incorporate constraints on velocities. Such no slipping constraints often show up in mechanical systems and it is natural to think of them as enforced by strong contact friction forces.
I will show how this idea can be made rigorous by considering the limit dynamics with infinite (linear) friction forces added, using singular perturbation theory. We indeed recover nonholonomic dynamics (thus providing a justification for d'Alembert's principle), but moreover, obtain a simplified method to study systems with non-ideal, i.e. large but finite friction. This improves previous results by Karapetian, Brendelev and others.

I will illustrate the results applied to the Chaplygin sleigh toy model and discuss its possible use in studying the tippe top.

Geometric Integration of Surface Isometric Embeddings

Thomas Ivey
College of Charleston, USA
Jeanne Clelland, Ben McKay, Peter Vassiliou

We formulate the system for isometrically embedding a surface into Euclidean 3-space as an exterior differential system on the product of frame bundles. We identify surface metrics for which this system is integrable as those realized by certain surfaces of revolution. We develop two complementary methods for integrating these equations, the first using Weierstrass-type representation, and the second using superposition formulas and the action of the Vessiot group.

Multiscale Dynamics and Synchronization in Vibrationally Driven Nonholonomic Systems

Scott Kelly
UNC Charlotte, USA

A mechanical system subject to periodic forcing in the presence of nonholonomic constraints may exhibit aperiodic behavior. This principle underpins the self-propulsion of a class of planar robotic vehicles that includes the snakeboard and the Chaplygin sleigh surmounted by a balanced rotor. Internal actuation enables each of these vehicles to propel itself atop a stationary platform, but endowing the platform with dynamics of its own can induce a dynamically rich response from a similar vehicle without internal actuation. This talk will explore the dynamics of simple wheeled vehicles atop non-stationary platforms that are subject to exogenous forcing or that passively permit energy transfer between vehicles.

Hyperbolic Conservation Laws with Prescribed Eigenfields.

Irina Kogan
North Carolina State University, USA
Michael Benfield, Kris Jenssen

Eigenvectors of the Jacobian of a flux play an important role in determining wave curves of a hyperbolic system of conservation laws, and hence in constructing solutions of the system. Since eigenvectors depend on a point in the state space, they are called eigenfields. In this talk, we concentrate on systems of three equations in one spatial dimension and discuss the following questions. Given three independent vector fields on an open subset of R^3 (a local frame), does there exist a strictly hyperbolic system, such that the given three vector fields are eigenfields of its flux? If yes, what is the degree of freedom for finding such systems? What happens if we are only given two independent vector fields (an incomplete local frame) on an open subset of R^3? Is there a strictly hyperbolic flux, such that the given two vector fields are eigenfields of the flux? If yes, how many are there? These questions are part of a larger project of determining the effects of geometric properties of the wave curves on the behavior of the solutions of the conservative systems.

Global Asymptotic Controllability for Control Systems with Unbounded Inputs

Anna Chiara Lai
Sapienza University of Roma, Italy
Monica Motta, Franco Rampazzo

We present some results on control systems characterized by a Lagrangean with non-negative values and by the unboundedness of controls. In the first part of the talk we show sufficient conditions for global asymptotic controllability and a state-dependent upper bound for the infima. The result holds under a quite mild assumption concerning the dependence of the data on inputs: such condition is met, for instance, by control vector fields that are compositions of Lipschitz maps with polynomials and exponentials of the control variable. We then discuss the particular case of systems with a polynomial dependence on controls. This class of systems includes control-quadratic systems, that are suitable to model mechanical systems controlled via time-varying,frictionless, holonomic constrains. We show that algebraic and convexity properties of control-polynomial systems provide simplified versions of the main result.

Local Study of Causal Geometries and Related Structures

Omid Makhmali
McGill University, Canada

A causal structure on M^{n+1} is given by a codimension one sub-bundle $\mathcal{C} \subset \mathbb{P}TM$ with tangentially nondegenerate fibers. There structures are special classes of cone structures whose study is motivated partly by the Hwang-Mok program of differential geometric characterization of uniruled varieties and, by the geometrization program of differential equations in the sense of Cartan. In this work, the equivalence problem for causal structures is solved using Cartan's method of equivalence, and the local invariants are obtained. The result shows that the relation between these structures and Finsler geometry are analogous to that of conformal structures and Riemannian geometry. Moreover, causal geometries are the structures associated to certain distributions with growth vector $(n, 2n-1, 2n)$ on an $2n$-dimensional manifold. Several properties and related structures obtained via twistor correspondences will be discussed.

On the Construction and Properties of Weak Solutions Describing Dynamic Cavitation

Alexey Miroshnikov
University of Massachusetts Amherst, USA
Athanasios Tzavaras

In this work we study the problem of dynamic cavity formation in isotropic compressible nonlinear elastic media. Cavitating solutions were introduced by J.M. Ball (1982) in elastostatics and by K.A. Pericak-Spector and S. Spector (1988) in elastodynamics. They turn out to decrease the total mechanical energy and provide a striking example of non-uniqueness of entropy weak solutions (in the sense of hyperbolic conservation laws) for polyconvex energies. In our work we established various further properties of cavitating solutions. For the equations of radial elasticity we construct self-similar weak solutions that describe a cavity emanating from a state of uniform deformation. For dimensions d = 2,3 we show that cavity formation is necessarily associated with a unique precursor shock. We also study the bifurcation diagram and do a detailed analysis of the singular asymptotics associated to cavity initiation as a function of the cavity speed of the self-similar profiles. We show that for stress-free cavities the critical stretching associated with dynamically cavitating solutions coincides with the critical stretching in the bifurcation diagram of equilibrium elasticity.

Controlled Lagrangians and Stabilization of Discrete Spacecraft with Rotor

Yuanyuan Peng
Claflin University, USA
Syrena Huynh, Dmitry V. Zenkov, Anthony M. Bloch

The method of controlled Lagrangians for discrete mechanical systems is extended to the problem of stabilization of the rotations of a spacecraft with a symmetric rotor. In particular, stabilization about its intermediate axis of inertia is considered. The Moser-Veselov discretization is used to obtain the discrete dynamics of the system. Stabilization conditions for the continuous model and its discretization are compared. It is shown that stability of the discrete system is sufficient for stability of its continuous counterpart but not vice versa.

Optimal Control of Nonholonomic Mechanical Systems

Stuart Rogers
University of Alberta, Canada
Vakhtang Putkaradze, Stuart Rogers

We investigate the optimal control of mechanical systems with nonholonomic constraints. For algebraic simplicity, we focus mainly on Suslov's problem, which is a classical example of a nonholonomic mechanical system. This system considers the motion of a rigid body rotating about a fixed point subject to the constraint$\Omega \cdot \xi = 0$, where Ω denotes the angular velocity and ξ is a prescribed vector, both expressed in the body frame. Permitting ξ to vary with time, we derive the optimal control equations given a cost function $C(\Omega, \xi, \dot{\xi})$. In addition to numerical solutions, we show that in several particular cases, we can either find analytical solutions or asymptotic solutions using singular perturbation methods. We shall also touch briefly on the optimal control of a substantially more complicated nonholonomic system, namely, a ball with moving internal masses rolling without slipping on a horizontal surface. Time permitting, derivations of the uncontrolled and controlled equations of motion, as well as numerical simulations, for this system will be presented.

Lie-Dirac Reduction on Semidirect Products and Nonholonomic Mechanics

Hiroaki Yoshimura
Waseda University, Japan
Francois Gay-Balmaz

For the sake of modeling nonholonomic systems, the theory of Lagrange-Dirac systems is quite a useful tool since Dirac structures systematically show how the system components are interconnected through energy flow, even for the case of degenerate Lagrangians. In particular, the case in which a configuration space is given by a Lie group can be understood in the context of the so-called Lie-Dirac reduction theory. In this talk, we consider Lagrange-Dirac systems for nonholonomic mechanical systems on Lie groups with broken symmetry. We show reduction of Lagrange-Dirac systems as well as Hamilton-Dirac systems in the context of the Lie-Dirac reduction with advected parameters. We illustrate our theory with some examples of Chaplygin's ball and the second-order Rivlin-Ericksen Navier-Stokes fluids.

Weak* Solutions of Conservation Laws

Robin Young
University of Massachusetts, USA
Alexey Miroshnikov

We introduce a new concept of solution for systems of conservation laws, which allows us to rigorously handle certain measure-valued solutions such as the vacuum in a Lagrangian frame. In particular, the PDEs can be regarded as (weak*) solutions of an ODE in Banach space, and inherit certain regularity. We show that BV weak solutions are weak* solutions, and describe solutions with vacuums as an example. We further discuss approximations to such solutions.

Hamel's Formalism for Infinite-Dimensional Mechanics

Dmitry Zenkov
North Carolina State University, USA

Separation of the position and velocity measurements in mechanics originated in Euler's work on the dynamics of rigid body and fluid. Hamel extended this formalism from the rigid body setting to arbitrary finite-dimensional Lagrangian mechanical systems. This talk will introduce Hamel's formalism for infinite-dimensional mechanics, with an emphasis on the dynamics of constrained continuum-mechanical systems.

Special Session 113: Inverse Problems, Variational Inequalities, and Applications

Akhtar A. Khan, Rochester Institute of Technology, USA
Patrizia Daniele, University of Catania, Italy
Baasansuren Jadamba, Rochester Institute of Technology, USA

The special session "Inverse Problems, Variational Inequalities, and Applications" will focus on recent developments in variational methods for inverse problems, variational inequalities, the cross fertilization of ideas in these two disciplines, and their applications focusing on projected dynamical systems, equilibrium problems, imaging and signal processing, elasticity imaging, regularization, among others.

A New Topological Degree Theory and Applications to Nonlinear Hyperbolic Problems with Nonmonotone Nonlinearities

Teffera Asfaw
Virginia Tech, USA

Let H be a real Hilbert space. Let $T : H \supseteq D(H) \to 2^H$ be maximal monotone, $C : H \to H$ be demicontinuous such that there exist nonegative constants τ and ν satisfying $\|Cx\| \le \tau\|x\| + \nu$ for all $x \in H$, $S : H \to 2^H$ be bounded of type (S_+) and $L : H \supseteq D(L) \to H$ be linear, closed, range closed and $L^{-1} : R(L) \to H$ be compact. A new degree theory is developed for operators of the type $L(T+S)+C$. The generalization of this theory also holds if S is bounded pseudomonotone and $T+S$ is of type (S). The theory developed herein is applied to prove existence of weak solution (s) for nonlinear hyperbolic equations of the type

$$\begin{cases} u_{tt} - \Delta u - \dfrac{\partial}{\partial t}\Delta u + u_t + \alpha(\Delta u - \Delta^2 u) + g(x,t,u) \\ \quad = f(x,t) \quad (x,t) \in Q_T \\ u(x,t) = 0 \quad (x,t) \in \partial Q_T \\ u(x,0) = u(x,T) \quad\ x \in \Omega, \end{cases}$$

where $\alpha \ge 0$, $Q_T = \Omega \times (0,T)$, $\partial Q_T = \partial\Omega \times (0,T)$ and $g : \overline{\Omega} \times [0,T] \times \mathbb{R} \to \mathbb{R}$ satisfies suitable sublinearity condition. Analogous existence theorems are applied to show solvability of hyperbolic variational inequality problems in appropriate Hilbert spaces.

Inverse Problems of Identifying Nonlinear Parameters in Variational Problems

Manki Cho
Rochester Institute of Technology, USA
Baasansuren Jadamba, Akhtar A. Khan

This work is on a reliable computational framework for an inverse problem of identifying nonlinear parameters using an output least-squares(OLS) approach. The proposed framework is based on a second-order adjoint method for the computation of the second-order derivative of the regularized OLS functional. The feasibility of the proposed framework is supported by numerical experiments. This is joint work with Baasansuren Jadamba and Akhtar A. Khan.

Generalized Nash Games, Vaccination Policies and Environmental Accords

Monica Cojocaru
University of Guelph, Guelph ON, Canada
E. Wild, A. Small

We present two numerical methods to compute the entire solution sets of Nash points in a generalized Nash game. We then show how two very different models of policy games can be analyzed by being formulated as generalized Nash games with shared constraints. This is joint work with Erin Wild and Allison Small.

Nonlinear Parameter Identification in Variational Problems

Baasansuren Jadamba
Rochester Institute of Technology, USA

Inverse problems of identifying parameters in partial differential equations (PDEs) constitute an important class of problems emerging from diverse applications. These inverse problems are most conveniently studied using an optimization framework and there are various optimization formulations. Although a non-convex output least-squares (OLS) objective is commonly used, a convex modified output least-squares (MOLS) gained considerable interest in recent years. However, the convexity of the MOLS has only been established for parameters appearing linearly in the PDEs. The primary objective of this work is to introduce and analyze a variant of the MOLS for the inverse problem of identifying parameters that appears nonlinearly in general variational problems. We are interested in understanding what geometric properties of the original MOLS can be retained for the nonlinear case. Besides giving an existence result for the optimization formulation of the inverse problem, we give a thorough derivation of the first-order and second-order derivative formulas for the new objective functional. The derivative formulae suggest that the convexity of the MOLS cannot be retained for the parameters appearing nonlinearly without imposing additional assumptions on the data involved.

Parameter Identification in Variational and Quasi-Variational Inequalities

Akhtar Khan
Rochester Institute of Technology, USA
Joachim Gwinner, Baasansuren Jadamba, Miguel Sama

In this talk, we will focus on the inverse problem of identifying variable parameters in certain variational and quasi variational inequalities. We develop a trilinear form based optimization framework that has been used quite effectively for parameter identification in variational equations emerging from partial differential equations. An abstract nonsmooth regularization approach is developed that encompasses the total variation regularization and permits the identification of discontinuous parameters. We investigate the inverse problem in an optimization setting using the output-least squares formulation. We give existence and convergence results for the optimization problem. We also penalize the variational inequality and arrive at optimization problem for which the constraint variational inequality is replaced by the penalized equation. For this case, the smoothness of the parameter-to-solution map is discussed and convergence analysis and optimality conditions will be discussed.

Coupling of Regularization and H-(hp-Adaptive) BEM for Hemivariational Inequalities Modelling a Delamination Problem

Nina Ovcharova
University of the Army Munich, Germany
Lothar Banz

We couple the regularization techniques of nondifferentiable optimization with the h resp. hp-adaptive versions of the boundary element method (BEM) to solve nonsmooth variational problems arising in contact mechanics. As a model example we consider the delamination problem. The variational formulation of this problem leads to hemivariational inequality with a nonsmooth functional deffned on the contact boundary. This problem is first regularized and then, discretized by a h- resp. hp-BEM. We give conditions for the uniqueness of the solution, prove convergence of the BEM Galerkin solution of the regularized problem in the energy norm, and obtain an a-priori error estimate for the regularized problem based on a novel Cea-Falk approximation lemma. Furthermore, we derive a reliable a-posteriori error estimate based on an equivalent regularized mixed formulation, thus enabling hp- adaptivity. Numerical experiments illustrate the behavior, strengths and weaknesses of the proposed approximation scheme.

Some Properties of Affine Variational Inequalities

Stephen Robinson
University of Wisconsin-Madison, USA

Affine variational inequalities (AVI) are basic to modeling in economics and in several areas of engineering. For some problems, the AVI provide suitable models; for other problems the models require variational inequalities with nonlinear functions but AVI can provide suitable approximations to support Newton-type methods for solution.
Although AVI look quite simple, proofs of some of their useful properties require considerably more work than proofs of analogous properties of ordinary linear equations (which are the simplest cases of AVI, in which the underlying set is the whole space). In particular, the property of injectivity—which in linear equations is equivalent to nonsingularity of a square matrix—has been characterized by a variety of different arguments, some accessible to students in the fields noted above and some not. Even now, after years of work, the characterization can be expressed in either of two different ways, which must be equivalent but of which the equivalence is not evident.
In this lecture we will look at some basic properties of AVI, including injectivity, and will ask how the proofs supporting those properties might be made as transparent as possible so that students with a basic mathematical background could follow the proofs.

Special Session 114: Uncertainty Quantification

Kody Law, Oak Ridge National Laboratory, USA
Clayton Webster, Oak Ridge National Laboratory, USA

Uncertainties in the parameters which define differential equations give rise to distributions of solutions, hence distributions of functionals of the solutions. Quantities of interest can be represented as expectations of such functionals. The resulting high-dimensional integrals, involving expensive function evaluations, has lead to a wealth of new Mathematics. Further complexity is introduced if data is available. The Bayesian framework gives rise to a probabilistic interpretation of inverse problems. This special session aims to bring together researchers in high-dimensional approximation theory, numerical and computational methods for stochastic partial differential equations, and Bayesian inverse problems to share ideas and discuss these interesting problems and the interplay between them.

Sparse Sampling Methods for Neutron Data

Rick Archibald
ORNL, USA

This talk will focus on mathematics developed to help with the mathematical challenges face by the DOE at the experimental facilities. This talk will discuss sparse sampling methods and fast optimization developed specifically for neutron tomography. Sparse sampling has the ability to provide accurate reconstructions of data and images when only partial information is available for measurement. Sparse sampling methods have demonstrated to be robust to measurement error. These methods have the potential to scale to large computational machines and analysis large volumes of data.

A Partial Domain Inversion Approach for Large-Scale Baysian Inverse Problems in High Dimensional Parameter Spaces

Tan Bui-Thanh
The University of Texas at Austin, USA
Vishwas Hebbur Venkata Subba Rao

While Bayesian inference is a systematic approach to account for uncertainties, it is often prohibitively expensive for inverse problems with large-scale forward equation in high dimensional parameter space. We shall develop a *partial domain inversion strategy* to only invert for distributed parameters that are well-informed by the data. This is an efficient data-driven reduction method for both forward equation and parameter space. This approach induces several advantages over existing methods. First, depending on the size of truncated domain, solving the truncated forward equation could be much less computationally demanding. Consequently, the adjoint (and possibly incremental forward and adjoint equations if Newton-like method is used is much less computationally intensive. Second, since the parameter to be inverted for is now restricted, the curse of dimensionality encountered when exploring the parameter spaces (to compute statistics of the posterior density, for example) is mitigated. Third, this approach is well-suited for current and future extreme-scale computing systems (with decreasing memory per node) since it is naturally a low memory demanding algorithm. Fourth, it supports methods that are efficient for small-to-medium scale problems, such as (parallel) direct solvers.

Hierarchical Bayesian Level Set Inversion

Matthew Dunlop
University of Warwick, England
Marco Iglesias, Andrew Stuart

The level set approach has proven widely successful in the study of inverse problems, since its systematic development in the 1990s. Recently it has been employed in the context of Bayesian inversion, allowing for the quantification of uncertainty within reconstruction methods. However the Bayesian approach is very sensitive to the length and amplitude scales encoded in the prior probabilistic model. We demonstrate how the scale-sensitivity can be circumvented by means of a hierarchical approach, using a single scalar parameter. Together with careful consideration of the development of algorithms which encode probability measure equivalences as the hierarchical parameter is varied, this leads to well-defined Gibbs based MCMC methods found by alternating Metropolis-Hastings updates of the level set function and the hierarchical parameter. These methods demonstrably outperform non-hierarchical Bayesian level set methods.

Multilevel Markov Chain Monte Carlo Method for Bayesian Inverse Problems

Viet Ha Hoang
Nanyang Technological University, Singapore
Christoph Schwab, Andrew M. Stuart

For Bayesian inverse problems of partial differential equations with unknown random coefficients, the plain Markov Chain Monte Carlo (MCMC) method that straightforwardly combines Finite Element approximation with MCMC sampling is prohibitively costly. We present the Multilevel MCMC method that achieves a prescribed level of accuracy for approximating the posterior expectation of a quantity of interest, with essentially optimal complexity. Numerical results confirm our rigorous theory.

Multilevel Sequential Monte Carlo Samplers

Kody Law
ORNL, USA
Ajay Jasra, Yan Zhou, Alex Beskos, Raul Tempone

This talk will review the probabilistic formulation of the inverse problem, the sequential Monte Carlo (SMC) sampling framework, and the standard multilevel Monte Carlo (MLMC) framework. These ideas will coalesce into the MLSMC sampling algorithm for Bayesian inverse problems. A numerical example of permeability inversion through an elliptic PDE given observations of pressure will illustrate the theoretical results.

Special Session 117: Partial Differential Equations from Fluid Dynamics

Hongjie Dong, Brown University, USA
Robin Ming Chen, University of Pittsburg, USA
Dong Li, University of British Columbia, Canada

This session will serve to promote and disseminate recent developments on evolutionary PDEs governing the motion of fluids. The fundamental prototypes are the Navier-Stokes (NS) and Euler equations. These equations appear, with fixed or moving interfaces, either alone or coupled with other equations, in the study of many phenomena in aerodynamics, geophysics, meteorology, plasma physics, etc. This session will focus on (but not restricted to) issues regarding modeling, local well/ill-posedness, gobal well-poseness/finite-time blowup, and stability.

Review of Some Properties of Leray-Hopf Weak Solutions to the Navier Stokes Equations

Hao Jia
University of Chicago, USA

In the talk, we will review the properties of Leray Hopf weak solutions to the Navier Stokes equations and some related open problems. In particular, we will discuss a recent application in the construction of scale invariant solutions, and explain the connection of linear spectral property of scale invariant solutions with uniqueness problem of the weak solutions.

L_p-Estimates for Stationary Stokes Systems with Coefficients Measurable in One Direction

Doyoon Kim
Korea University, Korea
Hongjie Dong

We discuss L_p-estimates of solutions to stationary Stokes systems when the coefficients are measurable (i.e., no regularity assumptions) in one direction. We prove a priori L_p-estimates of solutions when the systems are defined on the whole Euclidean space or on a half space. We also prove the unique solvability in L_p-spaces of stationary Stokes systems on a bounded domain.

Spatial Asymptotics in the Euler Equation

Robert McOwen
Northeastern University, USA
Peter Topalov

We prove that the Euler equation describing the motion of an incompressible fluid in R^n is well-posed in a class of functions allowing partial asymptotic expansions of any order as $|x| \to \infty$.

Global Well-Posedness of 2D Nonlinear Schrödinger Equations of Indefinite Signature

Nathan D. Totz
University of Massachusetts Amherst, USA

We describe a new method to obtain global a priori bounds in time for solutions to nonlinear Schrodinger equations (NLS) on $\mathbb{R}^2$ having power nonlinearities of arbitrary odd degree, and with large initial data in Sobolev space. The method presented here applies to both the usual NLS equations associated to the Laplacian and with a nonlinearity of defocusing sign, as well as to the more difficult so-called "hyperbolic" NLS which is associated to an indefinite signature. The latter is particularly interesting since its long time behavior is to date unknown for large (and even very smooth) initial data. We show, by rigorously justifying that these equations govern the modulation limit of an artificially constructed equation with an advantageous structure, that every subcritical Sobolev norm of the solution increases a priori at most polynomially in time. Global existence in all subcritical Sobolev spaces then follows by standard local well-posedness results for NLS.

Existence and Qualitative Theory of Stratified Solitary Water Waves

Samuel Walsh
University of Missouri, USA
Robin Ming Chen, Miles H. Wheeler

In this talk, we will report some recent results concerning two-dimensional gravity solitary water waves with hereogeneous density. The fluid domain is assumed be bounded below by an impenetrable flat ocean bed, while the interface between the water and vacuum above is a free boundary. Our main existence result states that, for any smooth choice of upstream velocity and streamline density function, there exists a path connected set of such solutions that includes large-amplitude surface waves. Indeed, this solution set can be continued up to (but does not include) an "extreme wave" that possess a stagnation point.
We will also discuss a number of results characterizing the qualitative features of solitary stratified waves. In part, these include bounds on the Froude number from above and below that are new even for constant density flow; an a priori bound on the ve-

locity field and lower bound on the pressure; a proof of the nonexistence of monotone bores for stratified surface waves; and a theorem ensuring that all supercritical solitary waves of elevation have an axis of even symmetry.

Blow-Up of Critical Norms for the 3-D Navier-Stokes Equations

Wendong Wang
Dalian University of Technology, Peoples Rep of China
Zhifei Zhang

In this talk, we‘ll talk about the regularity of weak solutions for the 3-D Navier-Stokes equations in the critical norm.This is a joint work with Z.-F. Zhang etc.

Asymptotic and Integral Properties of Solitary Waves in Deep Water

Miles Wheeler
NYU Courant Institute, USA

We consider solitary waves on the surface of an infinitely deep two- or three-dimensional fluid, both with and without surface tension. Under a mild algebraic decay assumption, we prove precise asymptotics at infinity, and relate these asymptotics to several integral properties of the wave. This complements previous nonexistence results for waves in deep water without surface tension.

KdV Dynamics and Traveling Waves in Polyatomic FPU

J. Douglas Wright
Drexel University, USA

Using homogenization theory, we can derive and justify a Korteweg-deVries limit for a polyatomic Fermi-Pasta-Ulam lattice problem under the assumption that the material parameters vary periodically. While the KdV approximation predicts the existence of solutions which look like solitary waves for long times, it does not guarantee that such solutions remain coherent forever. We discuss recent results on the global in time existence of generalized solitary waves in diatomic FPU lattices.

Global Martingale Solution for the Stochastic Boussinesq System with Zero Dissipation

Kazuo Yamazaki
Washington State University, USA

In this talk, we discuss some new results on the stochastic analysis of PDE in fluid. In particular, we discuss the global existence of a martingale solution to the 2D stochastic Boussinesq system with zero dissipation. This result is new in contrast to previous work due to the lack of dissipation. If time permits, we also discuss the global existence of a martingale solution to the 3D stochastic nonhomogeneous (variable-density) magnetohydrodynamics system, ergodicity and large deviation principle results for the 2D stochastic micropolar and magneto-micropolar fluid systems.

The Weak Solutions to 3D Degenerate Compressible Navier-Stokes Equations

Cheng Yu
The University of Texas at Austin, USA
Alexis Vasseur

In this talk, we discuss the existence of global weak solutions for 3D compressible Navier-Stokes equations with degenerate viscosity. The method is based on the Bresch and Desjardins entropy conservation. The main contribution of this paper is to derive the Mellet-Vasseur type inequality for the weak solutions, even if it is not verified by the first level of approximation. This provides existence of global solutions in time, for the compressible Navier-Stokes equations, in three dimensional space, with large initial data possibly vanishing on the vacuum. This solves an open problem proposed by Lions.

Regularity Criteria of 3D Navier-Stokes Equations Involving the Pressure Term

Xinwei Yu
University of Alberta, Canada
Chuong V. Tran

In this talk we will present some new regularity criteria for the 3D Navier-Stokes equations. This new criteria involve combinations of the pressure and the velocity. They improve the classical Prodi-Serrin conditions. This is joint work with Prof. Chuong V. Tran of University of St. Andrews, Scotland.

Special Session 118: Mean Field Games and Applications

Daniela Tonon, CEREMADE Université Paris Dauphine, France
Adriano Festa, RICAM, Austria

Mean Field Games theory is a new and challenging mathematical topic which analyses the dynamics of a very large number of interacting rational agents. Introduced ten years ago, the MFG models are used in many areas such as economics, finance, social sciences and engineering. In this section we will present some recent developments of the topic with elements coming from mean field theories, optimal control and stochastic analysis, calculus of variations and partial differential equations. Modeling and numerical aspects of the matter will also be presented.

An Efficient Numerical Method for Stationary Mean Field Games

Simone Cacace
Sapienza University of Rome, Italy
Fabio Camilli

We propose a new approach to the numerical solution of Stationary Mean Field Games. It is based on a Newton-like method for solving inconsistent systems of nonlinear equations, arising in the discretization of the corresponding Ergodic Hamilton-Jacobi and Fokker-Planck equations. We show that our method is able to solve efficiently Mean Field Games on Euclidean spaces and Networks, also in the case of more competing populations. We present several numerical experiments in dimension one and two, showing the performance of the proposed method in terms of accuracy, convergence and computational time.

Bifurcation and Segregation in Quadratic Two-Populations Mean Field Games Systems

Marco Cirant
Universita di Milano, Italy
Gianmaria Verzini

In this talk we will consider stationary (ergodic) Mean Field Games systems of two competing populations. We will focus on the corresponding elliptic system of coupled semilinear equations obtained via the Hopf-Cole transformation, and in particular study the behavior of the two populations as the viscosity parameter goes to zero. We will discuss the existence of non-trivial solutions using variational and bifurcation methods; then, for selected families of non-trivial solutions, we will address the appearing of segregation in the vanishing viscosity limit by means of blow-up techniques.

A Semi-Lagrangian Scheme for the Hughes Model for Pedestrian Flow

Adriano Festa
RICAM - Austrian Academy of Science, Austria
Carlini, Silva, Wolfram

In this talk we present a Semi-Lagrangian scheme for a regularized version of the Hughes model for pedestrian flow. Hughes originally proposed a coupled nonlinear PDE system describing the evolution of a large pedestrian group trying to exit a domain as fast as possible. The original model corresponds to a system of a conservation law for the pedestrian density and an Eikonal equation to determine the weighted distance to the exit. We consider this model in presence of small diffusion and discuss the numerical analysis of the proposed Semi-Lagrangian scheme. Furthermore we illustrate the effect of small diffusion on the exit time with various numerical experiments.

Existence of Weak Solutions of Mean-Field Games by Monotonicity Methods

Diogo Gomes
King Abdullah University of Science and Technology, Saudi Arabia
Rita Ferreira

Here, we consider monotone mean-field games (MFGs) and study the existence of weak solutions. First, we introduce a regularized problem that preserves the monotonicity. Next, using variational inequality techniques, we prove the existence of solutions to the regularized problem. Then, using Minty's method, we establish the existence of solutions for the original MFG. Finally, we examine the properties of these weak solutions in several examples. Our methods provide a general framework to construct weak solutions to MFGs with local, nonlocal, or congestion terms.

Well-Posedness of Cournot Competition MFG Model

Philip Graber
University of Dexas at Dallas, USA
Alain Bensoussan

In their chapter on Mean Field Games and Applications in the Paris-Princeton Lectures on Mathematical Finance, Gueant, Lasry, and Lions introduced an MFG model of production of an exhaustible resource. Although such models have been studied numerically, there seem to be very few results in the literature giving well-posedness. Recently, Chan and Sircar introduced a similar model with a linear demand schedule, which corresponds to Cournot (or Bertrand) type oligopolistic competition with a large number of producers. In this talk we present a theorem on the existence and uniqueness of smooth, classical solutions to the model of Chan and Sircar. The proof depends on new a priori estimates, particularly for the “nonlocal coupling term.

Aggregative Control of Large-Scale Multi-Agent Systems

Sergio Grammatico
Eindhoven University of Technology, Netherlands

Many large-scale systems involve the interaction of a number of agents with loosely coupled dynamics and decisions, for instance in demand response management in electricity grids, where the agents locally optimize their decisions, but their eventual well being also depends on the aggregate of the decisions of all other agents. For such systems, it is typically impractical to impose a centralized control structure for a number of reasons (e.g. privacy concerns, computational and communication limitations). Instead one can provide suitable information to the agents and impose an appropriate penalty/reward feedback scheme to control the overall population using macroscopic commands only, so that the population exhibits a desirable macroscopic behaviour. In this talk, we discuss an aggregative control structure via a game theoretical approach, and present technical results based on fixed point operator theory. Finally, some open research opportunities are presented.

Mean Field Type Control with Congestion

Mathieu Lauriere
University Paris 7, France
Yves Achdou

The theory of mean field type control, developed by Bensoussan, Frehse and Yam, aims at describing the behaviour of a large number of interacting agents using a common feedback. A type of problems that have raised a lot of interest recently concerns congestion effects. They model situations in which the cost of displacement of the agents increases in the regions where the density is large (as, for instance, in crowd motion). I will present a system of partial differential equations arising in this setting. The main result is the existence and uniqueness of suitably defined weak solutions, which are characterized as the optima of two optimal control problems in duality. If time permits, I will also discuss a numerical method to solve this problem and present some numerical results.

A Dynamic Game Model of Collective Choice in Multi-Agent Systems

Jerome Le Ny
Polytechnique Montreal, Canada
Rabih Salhab, Roland Malhamè

We consider a mean field games-like scenario where a large number of agents have to make a choice among a set of different potential target destinations. Each individual both influences and is influenced by the group's decision, as well as the mean trajectory of all the agents. The model can be interpreted as a stylized version of opinion crystallization in an election or of collective decision making in animal societies for example. For our formulation, we show how to reduce the computation of the approximate Nash equilibria to a fixed point computation in a finite dimensional space, where we essentially look for consistent proportions of players choosing each destination. If time permits, we will also discuss a cooperative version of the model as well as the role that advertisers can play to influence the outcome.

Some Aspects of Finite-State Mean-Field Games

Roberto Machado Velho
KAUST, Saudi Arabia
Diogo A. Gomes, Marie-Therese Wolfram

In this talk, we consider finite-state mean-field game problems. First, we introduce the finite-state MFG problem and its analogy to the continuous case. Next, we examine for two-state problems and address their dual and potential formulation. Then, we propose a numerical method for the limit hyperbolic equation that is based upon the N-agent problem. Finally, we illustrate the behavior of two-state games via some examples and their simulations.

First Order Mean Field Games with Density Constraints: Pressure Equals Price

Alpar Richard Meszaros
UCLA, USA
Pierre Cardaliaguet, Filippo Santambrogio

We study first order Mean Field Game systems under density constraints as optimality conditions of two optimization problems in duality. A weak solution of the system contains an extra term, an additional price imposed on the saturated zones. We show that this price corresponds to the pressure field from the models of incompressible Euler's equations à la Brenier. By this observation we manage to obtain a minimal regularity, which allows to write optimality conditions at the level of single agent trajectories and to define a weak notion of Nash equilibrium for our model.

Numerical Method for Mean-Field Type Control Problems

Laurent Pfeiffer
Graz University, Austria

In this talk, we present two gradient-type numerical methods to solve mean-field type control problems. These problems consist in optimal control problem of stochastic differential equations, where the cost function is a function of the probability distribution of the state variable.
The first described method is based on a convexity property of the reachable set of probability distributions. We provide a convergence result. The second described method generates controls which are feedback controls.

Stationary Mean-Field Games in the Presence of Mild Singularities

Edgard Pimentel
PPGM-UFSCar, Brazil

In this talk, we address stationary mean-field game systems in the presence of mild singularities and prove well-posedness in the class of smooth solutions. Mild singularities have played a major role in regularity theory for (nonlinear) elliptic equations and free boundary problems, as it accounts - among other things - for cavitation and obstacle problems. In the realm of mean-field games, this class of couplings encodes a particular setting of preferences. More importantly, it introduces a new set of difficulties, as the integrability of mild singularities is weaker than the borderline case, i.e., logarithmic nonlinearities.

A Mean-Field Game of Evacuation in a Multi-Level Building

Hamidou Tembine
New York University, USA
Boualem Djehiche, Alain Tcheukam

This work puts forward a simple mean-field network game that captures some of the key dynamic features of crowd and pedestrian flows in multi-level building evacuations. It considers both microscopic and macroscopic route choice by strategic agents. To achieve this, we use mean-field differential game with local congestion measure based on the location of the agent in the building. Including the local mean-field term and its evolution along the path causes a sort of dispersion of the flow: the agents will try to avoid high density areas in order to reduce their overall walking costs and queuing cost at the exits. Each agent state is represented by a simple dynamical system. Each agent will move to one the closest exits that is safer and with less congested path. We first formulate the problem and derive optimality equations using maximum principle and dynamic programming with boundary conditions. Then, well-posedness and existence results are provided. Numerics and simulations are carried out to illustrate mean-field equilibria of a safer evacuation process.

Sobolev Regularity for Weak Solutions of First Order MFG

Daniela Tonon
CEREMADE Universite Paris Dauphine, France
Pierre Cardaliaguet, Alessio Porretta

The theory of mean field games in the deterministic case with a local coupling motivates the analysis for Hamilton-Jacobi equation with possibly unbounded right-hand side. We show Sobolev estimates and almost everywhere differentiability for solutions of first order Hamilton-Jacobi equations with Hamiltonians which are superlinear in the gradient variable and right-hand sides which are only bounded in Lebesgue spaces. The proof relies on an inverse Holder inequality. Applications to mean field games are discussed.

Vaccination and Markov Mean Field Games

Gabriel Turinici
Universitè Paris Dauphine, Paris, France

The vaccination is a public health policy often invoked to control an epidemic. However it is not exempt of debates which can, in severe cases, lead to vaccination campaign failures.
Several approaches have been proposed in the literature to model this collective behaviors as equilibrium between the individual (well-being optimizing) vision and the global, aggregate, dynamics; in this context we present in this talk our works which use the Mean Field Games framework (à la Lasry - Lions and Caines - Huang - Malhamè). We show that this perspective is very versatile and allows to treat rigorously a diversity of situations. From the technical point of view the individual undergoes a Markov chain dynamics and the society the resulting, deterministic, average master equation.
We discuss then the numerical approaches to find the equilibriums and apply to some practical situations.

Mean Field Stackelberg Games: Aggregation of Delayed Instructions

Phillip Yam
Chinese University of Hong Kong, Hong Kong
Alain Bensoussan, Michael Chau

In this talk, we propose to consider an N-player interacting strategic game in the presence of a (endogenous) dominating player, who gives direct influence on individual agents, through its impact on their control in the sense of Stackelberg game, and then on the whole community. Each individual agent is subject to a delay effect on collecting information, specifically at a delay time, from the dominating player. The size of his delay is completely known by the agent; while to others, including the dominating player, his delay plays as a hidden random variable coming from a common fixed distribution. By invoking a non-canonical fixed point property, we show that, for a general class of finite N-player games, each of them converges to the mean field counterpart which may possess an optimal solution that can serve as an epsilon-Nash equilibrium for the corresponding finite N-player game. Secondly, we provide, with explicit solutions, a comprehensive study on the corresponding linear quadratic mean field games of small agents with delay from a dominating player. A simple sufficient condition for the unique existence of mean field equilibrium is provided by tackling a class of non-symmetric Riccati equations. Finally, via a study of a class of forward backward stochastic functional differential equations, the optimal control of the dominating player is granted given the unique existence of the mentioned mean field equilibrium for small players.

Special Session 119: Geometric Functional Inequalities and Application to PDEs

Hidemitsu Wadade, Kanazawa University, Japan
Yohei Tsutsui, Shinshu University, Japan
Futoshi Takahashi, Osaka City University, Japan
Michinori Ishiwata, Osaka University, Japan

This special session mainly focuses on the functional inequalities of Sobolev type embeddings and related partial differential equations. Participants of this session will collect recent their studies of functional inequalities established by using the methods of the real analysis, Fourier analysis and variational analysis as basic mathematical tools. Our main purpose of this session is to requesting the possibility of combining those methods together for applying evolutionary partial differential equations and so on.

Diffeomorphisms with Prescribed Jacobian and Boundary Data on Sobolev-Class Domains

Ching-Hsiao Cheng
National Central University, Taiwan
Steve Shkoller

In this talk we consider the problem of finding diffeomorphisms whose Jacobian and boundary data are of Sobolev class and the domain under consideration is also of Sobolev class; that is, finding a diffeomorphism ψ between two Sobolev class domainsΩ_1 and Ω_2 satisfying

$$\begin{aligned} \det(\nabla\psi) = f \qquad & \text{in}\Omega_1 , \\ \psi = g \qquad & \text{on} \quad \partial\Omega_1 , \end{aligned}$$

where f and g are given functions possessing Sobolev class regularity. The result for the case that f, g and Ω_1 possess classical regularity is well-known; however, such kind of results appear to undergo loss of regularity in the sense that whenΩ_1 is of class $C^{k+3,\alpha}$ and the forcing are of class $C^{k,\alpha}$, the solution is of class $C^{k+1,\alpha}$. Our result show that when studying this problem on Sobolev class domain with Sobolev class forcing, there exists a solution to the equation above satisfying

$$\|\nabla\psi\|_{H^{k+1}(\Omega_1)} \le C\Big[\|f\|_{H^k(\Omega_1)} + \|g\|_{H^{k+0.5}(\partial\Omega_1)}\Big]$$

for some generic constant C depending on the H^{k+1}-regularity ofΩ_1. Therefore, no loss of regularity is encountered.

On the Elliptic Equations of Hardy-Sobolev Type with Multiple Boundary Singularities

Jann-Long Chern
National Central University, Taiwan
Xiang Fang, Chun-Hsiung Hsia

In this talk, we are interested in how the geometry of boundary singularities can affect the attainability of the respective best Caffarelli-Kohn-Nirenberg and Hardy-Sobolev constant.

Averaged Decay Estimates for Fourier Transforms of Measures Over Curves with Nonvanishing Torsion

Seheon Ham
Korea Institute for Advanced Study, Korea
Yutae Choi, Sanghyuk Lee

For a positive Borel measure with compact support, we consider L^2-averaged decay estimates of its Fourier transform. When the average is taken over the unit sphere, the decay estimates were studied extensively, in connection with the Falconer distance set problem, by Mattila, Sjölin, Bourgain, Wolff, Erdougan. In this talk, we study the case of space curves with non-vanishing torsion. We extend the previous known results for the unit circle to higher dimensions. Also we discuss sharpness of the estimates. This is a joint work with Yutae Choi (Pohang University of Science and Technology) and Sanghyuk Lee (Seoul National University).

Minimization Problem on the Hardy-Sobolev Inequality

Masato Hashizume
Osaka City University, Japan

We consider the attainability of the best constant for the Hardy-Sobolev inequality on bounded domains. It is well known that the attainability of this constant is affected by the position of the origin. Moreover, in the boundary singularity case, the mean curvature of the boundary at the singularity plays a crucial role in investigating the existence of minimizers. In this talk, we consider the attainability of the best constant on the continuous embedding $H^1(\Omega) \subset L^{2^*(s)}(\Omega, |x|^{-s}dx)$ in the interior singularity case. Different from the case of the Sobolev inequality, the attainability of the best constant changes according to the scale of the domain.

On the Long Time Stability of a Temporal Discretization Scheme for the Three Dimensional Primitive Equations.

Chun-Hsiung Hsia
National Taiwan University, Taiwan
Ming-Cheng Shiue

Abstract In this joint work with Ming-Cheng Shiue, a semi-discretized Euler scheme to solve three dimensional primitive equations is studied. With suitable assumptions on the initial data, the long time stability of the proposed scheme is shown by proving that the H1 norm (in space variables) is bounded.

Existence and Nonexistence of Solutions for a Heat Equation with a Superlinear Source Term

Norisuke Ioku
Ehime University, Japan
Yohei Fujishima

We consider a heat equation with an unbounded initial data and investigate local in time existence, nonexistence of solutions for the Cauchy problem with a superlinear source term. In particular, we reveal the threshold integrability of the initial data to classify existence and nonexistence of solutions for the Cauchy problem without any assumption on the growth rate of the nonlinear term.

Gradient Stability of the Sobolev Inequality: the Case $p > 2$

Robin Neumayer
University of Texas at Austin, USA
Alessio Figalli

The sharp Sobolev inequality in $\mathbb{R}^n$ gives control of the L^{p^*} norm of a function, $p^* = np/(n-p)$, in terms of the L^p norm of the gradient. Equality is achieved by the $(n+2)$-dimensional family of Talenti functions. In this talk, we show that, in the case $p \geq 2$, functions which almost attain equality in the Sobolev inequality are quantitatively close to Talenti functions at the level of gradients. To the furthest degree possible, we extend the Hilbert space methods employed in Bianchi and Egnell's proof of the analogous result for $p = 2$ (despite the fact that L^p is not a Hilbert space for $p > 2$), and then use an interpolation argument to reduce to a weaker stability result already shown by Cianchi, Fusco, Maggi, and Pratelli. This talk is based on joint work with Alessio Figalli.

Some Endpoint Estimates for Bilinear Paraproducts and Applications

Salvador Rodriguez-Lopez
Stockholm University, Sweden
W. Staubach

In this talk we will present some endpoint estimates for bilinear paraproducts of Bony's type. That is, operators of the form

$$\Pi(f,g)(x) = \int_0^\infty Q_t f(x)\, P_t g(x)\, m(t) \frac{\mathrm{d}t}{t},$$

where P_t and Q_t represent frequency localisation operators near the ball $|\xi| \leq C/t$ and the annulus $|\xi| \approx 1/t$, respectively, and m is a bounded function. More precisely, we will present some new boundedness estimates for bilinear paraproducts operators on local bmo spaces.
We will motivate this study by giving some applications to the investigations on the boundedness of bilinear Fourier integral operators and bilinear Coifman-Meyer multipliers.

Generalized Critical Hardy Inequalities with the Optimal Constant

Megumi Sano
Osaka City University, Japan

Let Ω be a bounded domain in $\mathbb{R}^N$ with $0 \in \Omega$, $R = \sup_{x\in\Omega} |x|, N \geq 2, a = 1, e$ and $q, \beta > 1$. In this talk, we consider the minimization problem associated with the optimal constant of the generalized critical Hardy inequality as follows:

$$F(\Omega; a) := \inf_{0 \neq u \in W_0^{1,N}(\Omega)} E(u)$$
$$:= \inf_{0 \neq u \in W_0^{1,N}(\Omega)} \frac{\int_\Omega |\nabla u|^N dx}{\left(\int_\Omega \frac{|u|^q}{|x|^N (\log \frac{aR}{|x|})^\beta} dx \right)^{\frac{N}{q}}}.$$

When $q = \beta = N$, it is known that the optimal constants $F(\Omega; a)$ are $\left(\frac{N-1}{N}\right)^N$ which are independent of Ω and are not attained both $a = 1$ and $a = e$.
Note that, in general, we can not apply the rearrangement technique for $F(\Omega; a)$ due to the non-monotonicity of the potential function $|x|^{-N} \left(\log \frac{aR}{|x|}\right)^{-\beta}$. Furthermore the quotient $E(u)$ does not have the scale invariance under the scaling $u_\lambda(x) = \lambda^{-\frac{N-1}{N}} u \left(\left(\frac{|x|}{aR}\right)^{\lambda-1} x \right)$ for $\lambda > 0$, because of the term $||\nabla u||_N$.
We prove the positivity and the attainability of $F(\Omega; a)$ under some conditions of (q, β, Ω).

Dispersive Estimates for the Stably Stratified Boussinesq Equations

Ryo Takada
Tohoku University, Japan
Sanghyuk Lee

We consider the initial value problem for the 3D Boussinesq equations for stably stratified fluids without the rotational effect. We establish the sharp dispersive estimate for the linear propagator related to the stable stratification. As an application, we give the explicit relation between the size of initial data and the buoyancy frequency which ensures the unique existence of global solutions to our system. In particular, it is shown that the size of the initial thermal disturbance can be taken large in proportion to the strength of stratification. This talk is based on the joint work with Sanghyuk Lee (Seoul).

Some Improvements for a Class of the Caffarelli-Kohn-Nirenberg Inequalities

Futoshi Takahashi
Osaka City University, Japan
Megumi Sano

In this talk, we concern a weighted version of the Hardy inequality:

$$\int_\Omega |\nabla u|^p |x|^{-pa} dx \geq \left(\frac{N-p-pa}{p}\right)^p \int_\Omega \frac{|u|^p}{|x|^{p(a+1)}} dx$$

for all $u \in C_0^\infty(\Omega)$, where Ω is a smooth bounded domain in $\mathbb{R}^N$ $(N \geq 3)$ with $0 \in \Omega$, or $\Omega = \mathbb{R}^N$, $1 < p < N$ and $-\infty < a < \frac{N-p}{p}$. This is a special case of the more general Caffarelli-Kohn-Nirenberg inequalities. On the whole space, we improve the inequality by adding a remainder term of the form of ratio of two weighted integrals. Also we derive a remainder term involving a distance from the manifold of the "virtual extremals". Finally on bounded domains, we prove the existence of remainder terms involving the gradient of functions. This talk is based on a joint work with Megumi Sano (Osaka City Univ.).

Div - Curl Estimate with Critical Power Weight

Yohei Tsutsui
Shinshu University, Japan

We treat with div - curl estimates due to Coifman-Lions-Meyer-Semmes in Hardy spaces with power weights. This inequality was applied to control the convection term in the incompressible Navier-Stokes equations. There is a critical degree of power weight concerning the optimal L^2-energy decay. Div - curl estimate with sub-critical power weights was already known. In this talk, we give an inequality with critical power weight, using a real interpolation spaces of weighted Hardy spaces, but our estimate involves quasi-Banach spaces.

On the Maximizing Problem Associated with Trudinger-Moser Type Inequalities

Hidemitsu Wadade
Kanazawa University / Institute of Science and Engineering, Japan
Michinori Ishiwata

In this talk, we consider the existence and non-existence of maximizers associated with Trudinger-Moser type inequalities. Recently, A Trudinger-Moser inequality was derived by B. Ruf, JFA, 219 (2005) for the two spacial dimension and extended to the higher spacial dimensions by Y. Li-B. Ruf, Indiana UMJ, 57 (2008), which are in-homogeneous type inequalities in the whole space. In the papers Li-B. Ruf, Indiana UMJ, 57 (2008) and M. Ishiwata, Math. Ann. 351 (2011), the authors considered the variational problems associated with these Trudinger-Moser type inequalities and proved that the existence and non-existence results depending on the exponents appearing in the exponential type integrals. We revisit the existence and non-existence problems for the above inequalities and clarify the effects of the norm-normalization to the structure of these variational problems.

L^p Estimates for Some Pseudo-Differential Operators

Lu Zhang
Wayne State University, USA
Guozhen Lu

We study the L^p estimates for a class of trilinear pseudo-differential operators with flag symbols and a bi-parameter bilinear Calderon-Vaillancourt theorem. For the trilinear pseudo-differential operators, the symbols are in the form of the product of two standard symbols from the Hormander class $BS^0_{1,0}$. Such operators are extensions from the trilinear operators with flag singularities, with the multipliers in the form of product of two Marcinkiewcz-Mikhlin-Hormander symbols. For the bi-parameter Calderon-Vaillancourt theorem, we take the symbols from the bi-parameter Hormander class $BBS^m_{0,0}$ and and study the Holder's type estimate.

Special Session 120: Global Bifurcations and Complex Dynamics

Ivan Ovsyannikov, University of Bremen, Germany

The session is devoted to the topics related to global bifurcations, which emerge in the theoretical context or various applications. It is well-known that the stable and unstable manifolds of fixed points (both in continuous and discrete-time systems) are usually embedded into the phase space in a very complicated way, forming, in particular, homoclinic or heteroclinic cycles. Moreover, small perturbations of such cycles may lead to the appearance of complex structures, such as finite or infinite sets of periodic orbits, invariant tori, formation of Smale horseshoes, strange attractors etc. At this session research talks on homoclinic and heteroclinic bifurcations are welcome, as well as the talks covering the study of the structures of stable and unstable invariant manifolds.

Bifurcations of Invariant Manifolds Near a Non-Central Saddle-Node Homoclinic Orbit

Pablo Aguirre
Universidad Tecnica Federico Santa Maria, Chile

We investigate the role of the two-dimensional global invariant manifolds near a codimension-two non-central saddle-node homoclinic point in a three-dimensional vector field. The main question is to determine how the arrangement of global two-dimensional manifolds changes through the unfolding and how this affects the topological organisation of basins of attraction. To this end, we compute the respective global invariant manifolds — rendered as surfaces in the three-dimensional phase space—, and their intersection curves with a suitable sphere, as families of orbit segments with a two-point boundary-value-problem setup. As a specific example to work on, we consider a laser model with optical injection which undergoes this codimension-two bifurcation. For this model vector field we present two-parameter bifurcation diagrams (with representative images) of the invariant manifolds and the relevant basins of attraction near the codimension-two singularity. In particular, this combination of bifurcation analysis and identification of basin boundaries in phase space allow us to find conditions for multipulse behaviour in the laser model depending on the global manifolds in open regions of parameter space and at the bifurcations involved.

Pulses with Oscillatory Tails and A Homoclinic Banana in the FitzHugh-Nagumo System

Paul Carter
Brown University, USA
Bjorn Sandstede

It is well known that the FitzHugh-Nagumo system exhibits stable, spatially monotone traveling pulses. Recently, it has been shown that this system also admits traveling pulse solutions with oscillatory tails. We discuss analytical results regarding the existence and stability of such pulses using geometric blow-up techniques and singular perturbation theory, and we outline a mechanism that explains the transition from single to double pulses along a so-called homoclinic banana that was observed in earlier numerical studies.

Deconstructing the Stunning Complexity of Global Bifurcations in a Far-Infrared Raman Laser Model

Krishna Pusuluri
Georgia State University, USA
Andrey Shilnikov

A new computational technique based on the symbolic description of complex homoclinic and heteroclinic bifurcations underlying the occurrence and dynamics of the Lorenz-like attractor has been employed to an example system from nonlinear optics - 6D model of the optically pumped, far-infrared three-level molecular laser. The bifurcation feature of this laser model is the coexistence of heteroclinic T-points of various kinds with accompanying self-similar fractal structures that they generate in the parameter plane - now the de-facto key signatures of most systems with the Lorenz attractor. Of special interest are quite unordinary codimension-two homoclinic bifurcations called inclination-switch, which has never been reported previously and can only occur in multi-dimensional systems. By employing the latest advancements in parallel computing techniques - OpenMP, CUDA, OpenACC and OpenMPI- we achieve significant performance improvements which allow us to study the complex laser system at stunning resolutions.

Melnikov Processes and Chaos in Randomly Perturbed Dynamical Systems

Kazuyuki Yagasaki
Kyoto University, Japan

We consider a wide class of randomly perturbed systems subjected to stationary Gaussian processes and show that chaotic orbits exist almost surely under some degenerate condition, no matter how small the random forcing terms are. This result is very contrast to the deterministic forcing case, in which chaotic orbits exist only if the influence of the forcing terms overcomes that of the other terms in the perturbations. To obtain the result, we extend Melnikov's method and prove that the corresponding Melnikov functions, which we call the Melnikov processes, have infinitely many zeros, so that infinitely many transverse homoclinic orbits exist. We illustrate our theory for the randomly perturbed Duffing oscillator subjected to the Ornstein-Uhlenbeck process.

Special Session 121: Recent Advancements in Computational Methods Involving Implicit or Non-parametric Interfaces

Catherine Kublik, University of Dayton, USA
Richard Tsai, KTH Royal Institute of Technology, Sweden and The University of Texas at Austin, USA

We are interested in gathering active researchers who are working on numerical methods for solving differential or integral equations on manifolds with no explicit parameterizations (e.g. using level set or closest point methods). This session targets applications where the interface is evolving in time, where the solution of a PDE is needed on some manifold, and is motivated by the recent need to work with unstructured point clouds sampled from an underlying manifold.

The Dirichlet-To-Neumann Operator with a Level-Set Function

Julien Dambrine
University of Poitiers, France
Nicolas Meunier

The motion of surfaces with a velocity depending on the Dirichlet-to-Neumann operator for a given elliptic problem appear in various practical applications ranging from the motion of cells to the geometrical optimisation of mechanical structures. The level-set framework is particularly interesting in this context of moving surfaces. In this work we focus on the computation of the Dirichlet-to-Neumann operator calculation for the Laplace equation, following the ideas developed in [1] intended for the computation of the bulk solution.

References

[1] Catherine Kublik, Nicolay M. Tanushev, Richard Tsai, An implicit interface boundary integral method for Poisson's equation on arbitrary domains, JCP, 2013.

On a Vector Field Embedding of Multiphase Geometries

Elliott Ginder
Hokkaido University, Japan

Tracking the evolution of multiphase geometries represents a fundamental problem that arises in a variety of scientific simulations. The phenomena under consideration can exhibit additional challenges to its numerical simulation, including the occurrence of topological changes, volume constraints, and inertial effects. We will introduce a method for treating these issues and investigate its application in the simulation of 2D and 3D multiphase parabolic and hyperbolic curvature flows. Volume constrained motions will also be investigated through the use of minimizing movements, and we will remark about aspects related to our method's numerical realization.

Implicit Interface Boundary Integral Methods and Their Application to the Mullins-Sekerka Problem

Catherine Kublik
University of Dayton, USA
Richard Tsai, Chieh Chen

We describe a simple formulation for integrating over smooth curves or surfaces that are described implicitly through a level set function or directly by their closest point mapping. Contrary to common practice with level sets, the volume integrals derived from our formulation coincide exactly with the boundary integrals. We use this formulation to simulate the Mullins and Sekerka dynamics on unbounded domains via boundary integral methods.

Solve Geometric PDEs on Manifolds Represented As Point Clouds and Applications

Rongjie Lai
Rensselaer Polytechnic Institute, USA

In this talk, I will discuss our recent work of solving geometric PDEs on manifolds sampled as point clouds. These methods can achieve high order accuracy and enjoy flexibility of solving different types of equations on manifolds with possible high codimesion. We use the proposed methods to consider special designed geometric PDEs on point clouds, which provides us a bridge to link local and global information. Based on this method, I will discuss a few applications to geometric understanding for point clouds, including computation of LB eigen-systems for point clouds, extraction of global skeletons structure from point clouds, extraction of conformal structures from point clouds, and intrinsic comparisons among point clouds etc. In addition, our methods can also be extended to solve PDEs on manifolds only represented as incomplete distance information. I will also discuss our results of this method for reconstructing and understanding distance data based on solutions of Laplace-Beltrame equations.

A New Level Set Method for Spirals Evolving by Eikonal-Curvature Equation and Its Application to Spiral Crystal Growth

Takeshi Ohtsuka
Gunma University, Japan

Theory crystal growth by rotating spiral steps was introduced by Burton, Cabrera and Frank in 1951. One can find evolving spiral patterns on the crystal surface including singularities caused by collision of several spirals or singular anisotropy of the motion. To describe such a motion of spirals we introduce a simple level set method describing spirals with a zero-level set of an auxiliary function minus pre-determined multi-valued function. Conventional level set method does not work well for spirals since the usual level set divide the domain into two disjoint sets although spirals do not make such a division. Our method can be applied not only isotropic but also anisotropic evolution. For anisotropic curvature with strong singularities, which is called as crystalline curvature, we shall introduce a new approach. As an application of our method, we present some numerical results on vertical growth rates of the surface by co-rotating spiral steps. In the theory of crystal growth, it is well known that if the centers of co-rotating pair are close together, then the vertical growth rate increases from the single one. We examine the growth rate and improve the estimate by Burton et al. numerically.

Using Evolving Interface Techniques to Solve Network Problems

Yves van Gennip
University of Nottingham, England

In recent years, ideas from the world of evolving interface PDEs have found their way into the arena of graph and network problems. In this talk I will discuss how techniques based on the Allen-Cahn equation and the Merriman-Bence-Osher threshold dynamics scheme can be used to detect particular structures in graphs, such as densely connected subgraphs (clustering and classification) and bipartite subgraphs.

Special Session 122: Variational Convergence and Degeneracies in PDEs: Fractal Domains, Composite Media, Dynamical Boundary Conditions

Maria Rosaria Lancia, Sapienza University of Rome, Italy
Maria Agostina Vivaldi,Sapienza University of Rome, Italy
Raffaela Capitanelli, Sapienza University of Rome, Italy

In the last decades there has been an increasing interest in studying degeneracies in BVPs such as those due to highly irregular domains as in the case of fractal boundaries or interfaces, or due to the presence of composite media . Other singularities arise when studying evolution problems with dynamical boundary conditions in fractal domains. In all these cases it is important, also in view of numerical approximations, to approximate these wild geometries by smoother ones and to study the convergence of approximating functions to the limit fractal one. This is obtained by variational convergence techniques. This session is devoted to new results on this type of degeneracies.

On the Effective Boundary Conditions Throughquasi-Filling Fractal Layers

Raffaela Capitanelli
Sapienza Univ. Roma, Italy
Cristina Pocci

We study the periodic homogenization of the stationary heat equation in domain with two connected components, separated by an oscillating interface defined on prefractal Koch type curves. The problem depends both on the parameter n, which is the index of the prefractal iteration, and ϵ, that defines the periodic structure of the composite material. We discuss about the commutative nature of the limits in ϵ and n.

Numerical Approximation of Non Local Venttsel Problems in (pre)fractal Domains

Simone Creo
University of Rome La Sapienza, Italy
M. Cefalo, M. R. Lancia, A. I. Nazarov, P. Vernole

We consider a diffusion problem of parabolic type in a polygonal domain$\Omega \subseteq \mathbb{R}^2$ having Koch type boundary K_n with non local Venttsel boundary conditions. We prove the existence and uniqueness of the weak solution, as well as regularity results in weighted Sobolev spaces. We consider the numerical approximation by finite element methods in space and finite differences in time. Crucial tools in proving a priori error estimates of the numerical approximation are the regularity properties of the weak solution.

Elliptic Problems for the Fractional Laplacian with a Singular Nonlinearity

Ida de Bonis
University Giustino Fortunato, Italy

We study the existence, regularity and uniqueness of solution of the following nonlocal problem

$$(P) \quad \begin{cases} (-\Delta)^s u &= F(x,u) \quad \text{in} \quad ,\Omega \\ u &= 0 \quad \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where Ω is a bounded smooth domain of $\mathbb{R}^N$, $N > 2s$, 0

Trigonometrical Inequalities on Regular Lattices.

Anna Chiara Lai
Sapienza University of Roma, Italy
Vilmos Komornik, Paola Loreti

Starting from Parseval's formula for multiple Fourier series, we consider non-harmonic Fourier expansions with exponents belonging to sets that can be decomposed in the finite union of translations of a fixed lattice. In the two dimensional case, this class of discrete sets includes well-known tilings of the plane, such as the honeycomb lattice and all the regular lattices. In this framework, we show Ingham type trigonometrical inequalities. The talk is based on a joint work with V. Komornik and P. Loreti.

Venttsel Boundary Value Problems in Fractal Domains

Maria Rosaria Lancia
S.B.A.I. Sapienza universita' di Roma, Italy
P. Vernole

Venttsel problems belong to the class of the so-called boundary value problems with dynamical boundary conditions. Problems with Venttsel type conditions have applications to various fields of science and technology e.g. water wave theory, models of heat transfer as well as engineering problems of hydraulic fracturing. It is to be pointed out that these conditions appear also under the name of effective boundary conditions (EBCs) for thin ferromagnetic layers or in laminar flows. Several natural and industrial processes lead to the formation of rough surfaces or occur across them. Fractal boundaries and fractal layers may be of great interest for those applications in which the surface effects are enhanced with respect to the surrounding volume. The use of fractal geometries is by now well diffused in some industrial applications; for instance with the increasing miniaturization of electronic chips and increasingly larger heat dissipation rates better design of cooling systems are necessary. Fractal boundaries could also be used in catalytic converters as well as, in all those applications in which diffusion phenomena take place in small volumes with large surfaces. Recently, we focused on the study of linear, semilinear, quasilinear and non local evolution Venttsel problems in domain with fractal boundaries and on their approximations. In this talk I present a survey of the results obtained in collaboration with P.Vernole and A.Velez-Santiago as well as some open problems.

Degenerate Elliptic Operators, Heat Kernel Estimates and Harnack Inequalities.

Luisa Moschini
Sapienza, University of Rome, Italy

We will deal with parabolic harnack inequalities, Liouville type theorems as well as heat kernel sharp two sided estimates in various context of degeneracies. Degeneracies will be given in some cases by distances to smooth k dimensional surfaces, in these cases they are also related to Scroedinger operators with critical potentials, involving inverse square distances to the same k dimensional surfaces. A brief approach to Schroedinger operators with fractional diffusion is also given as well as to the case of anisotropic degeneracies, that is to the case in which different powers of the same distance function are involved in the various derivatives.

On the Dynamics of Space-Filling Attractors

Umberto Mosco
Worcester Polytechnic Institute, USA

We present a family of impulsive Cauchy initial value problems in the plane driven by irregular acceleration fields on discrete synchronized space time grids with attractors of infinite perimeter that asymptotically are space filling. The 1D dynamics of these attractors spectrally interpolate the one dimensional and the two dimensional Laplace operators on planar domain. We also describe a fully discrete nonlinear self organized criticality model with finite time extinction dynamics, in which grid synchronization also plays an important role.

Short Time Heat Diffusion in Bounded Domains with Discontinuous Transmission Boundary Conditions

Anna Rozanova-Pierrat
CentraleSupelec, France
Claude Bardos, Denis Grebenkov

We consider a heat problem with discontinuous diffusion coefficients and discontinuous transmission boundary conditions with a resistance coefficient. For all bounded (ϵ,δ)-domains$\Omega \subset \mathbb{R}^n$ with a d-set boundary (for instance, a self-similar fractal), we find the first term of the small-time asymptotic expansion of the heat content in the complement of Ω, and also the second-order term in the case of a regular boundary. The asymptotic expansion is different for the cases of finite and infinite resistance of the boundary. The derived formulas relate the heat content to the volume of the interior Minkowski sausage and present a mathematical justification to the de Gennes' approach. The accuracy of the analytical results is illustrated by solving the heat problem on prefractal domains by a finite elements method.

Dirichlet Boundary Conditions for Degenerate and Singular Nonlinear Parabolic Equations

Marta Strani
Université Paris Diderot, IMJ-PRG, France
Fabio Punzo

We study existence and uniqueness of solutions to a class of quasilinear degenerate parabolic equations, in bounded domains$\Omega \subset \mathbb{R}^N$. We show that there exists a unique solution which satisfies possibly inhomogeneous Dirichlet boundary conditions. To this purpose some barrier functions are properly introduced and used.

Fractal Snowflake Domain Diffusion with Boundary and Interior Drifts

Alexander Teplyaev
University of Connecticut, USA
Michael Hinz, Maria Rosaria Lancia, Paola Vernole

We study evolution equations for a (elliptic measurable coefficients) diffusion in the classical snowflake domain in situations when there are diffusion and drift terms not only in the interior but also on the fractal boundary, which is a union of three copies of the classical Koch curve. We prove that under standard conditions related Cauchy problems possess unique strict solutions and explain in which sense they solve a rigorous formulation of our problem. As a second result we prove that functions that are intrinsically Lipschitz on the snowflake boundary ad-

mit Euclidean Lipschitz extensions to the closure of the entire domain. Our methods combine the fractal membrane analysis, the vector analysis for local Dirichlet forms and PDE on fractals, coercive closed forms, and the analysis of Lipschitz functions.

Magnetorheological Composites

Bogdan Vernescu
WPI, USA
G. Nika

We consider a magnetorheological composite material: i.e a suspension of solid magnetizable particles in a non-conducting viscous fluid in the presence of an applied external magnetic field. The Maxwell equations are coupled with the Stokes equations and the rigid particle motion to capture the magnetorheological effect. We obtain the effective equations consisting of a coupled nonlinear system in a connected phase domain as well as the new constitutive laws. Qualitative properties of the solution of this nonlinear system are studied.

Asymptotics for Venttsel Problems in Non Divergence Form in Irregular Domains

Paola Vernole
Roma Sapienza, Italy
M.R. Lancia, V. Regis Durante

In this talk we study a boundary value problem for a second order operator in non divergence form L with Venttsel's boundary conditions

$$(P)\begin{cases} u_t(t,P) - Lu(t,P) = f(t,P) & \text{in } [0,T]\times Q \\ u_t(t,P) - \Delta_S u(t,P) + b(P)u(t,P) = -\frac{\partial u}{\partial n_{\mathcal{A}}} + f(t,P) & \text{on } [0,T]\times S \\ \frac{\partial u}{\partial n_{\mathcal{A}}} = 0 & \text{on } [0,T]\times \partial Q\setminus S \\ u(0,P) = 0 & \text{in } Q, \end{cases}$$

where $Lu = \sum_{i,j=1}^{3} a_{ij}\partial_{ij}u + a_0 u$, a_{ij} are symmetric, uniformly Lipschitz functions in Q satisfying suitable ellipticity conditions and a_0 is a positive $L^\infty(Q)$ function, Q is the three-dimensional domain with lateral boundary $S = F \times [0,1]$, where F is the Koch snowflake;$\Delta\ _S$ is the fractal Laplacian on S , b is a continuous strictly positive function on S, $\frac{\partial u}{\partial n_{\mathcal{A}}}$ is the co-normal derivative across ∂Q to be defined in a suitable sense, $f(t,P)$ is a given function in $C^\theta([0,T];L^2(\overline{Q},m))$, $\theta \in (0,1)$ and m is the sum of the three-dimensional Lebesgue measure and of a suitable measure g supported on S .

Existence and uniqueness results are proved via a semigroup approach. From the point of view of numerical analysis it is also crucial to study the corresponding approximating (prefractal) problems in the domains Q_h, where $\{Q_h\}_{h\in N}$ is a sequence of increasing (invading) domains approximating Q, with lateral surface $S_h = F_h \times [0,1]$, the corresponding approximating polyhedral surfaces, where F_h is a prefractal curve approximating F. To this aim the asymptotic behavior, as $h \to \infty$, of the approximating solutions is studied. As to the asymptotic behavior of the solutions, the presence of the time derivative in the boundary conditions requires, as a natural functional setting for these problems, suitable varying Hilbert spaces. Moreover since the energy forms are not symmetric we use the results of Ma-Rockner, Rockner-Zhang and Toelle in order to prove the convergence of the forms, of the associated semigroups and hence of the solutions. Crucial tools to prove the convergence of the energy forms are density results for the energy spaces $V(Q,S)$.

A Second Order Elliptic Problem with Unnatural Boundary Condition Arising in Tensile Structures Design

Giuseppe Viglialoro
University of Cagliari, Italy

This talk is concerned with a mathematical problem modeling the equilibrium of a thin membrane structure (tensile structure), with rigid and cable boundaries. The general formulation is expressed by means of a second order elliptic PDE in terms of the membrane shape and its stress tensor; contrarily, the boundary constrains take into account a singular condition which makes the general analysis more complex. We discuss some partial results (both by the theoretical and numerical point of view) as well as present open questions.

P-Laplaceans and Mass Transport Problems

Maria Agostina Vivaldi
SBAI, Sapienza Universita di Roma, Italy

This talk deals with p-Laplaceans type problems on both Euclidean domains with fractal boundaries and fractal structures such as the Sierpinski Gasket. We use the approach of the variational convergences to investigate the asymptotic behavior of the solutions and the connection with the Kantorovich potential for the transport problem. Some numerical aspects are also discussed.

Contributed Session 1: ODEs and Applications

Tikhonov Regularization of a Singularly Perturbed Two-Point Boundary Value Problem

Sukla Adak
Indian Institute of Technology Madras, India
Arindama Singh

In this paper, we consider a linear second order two-point boundary value problem where a small parameter is multiplied with the highest order derivative. Since solutions of such a problem do not converge uniformly in the interval of integration, it may be considered as an ill-posed problem. We would like to explore the possibility whether a regularization method such as Tikhonov's can be used for solving this problem. It is found that the solution of the regularized problem converges to the original solution. When the noisy data is available and it is known, then a priori parameter choice rule can be given. Shishkin mesh gives an approximate length of a boundary layer once the location of layer is roughly known. The numerical experiments confirm that the computational solution is improved on a Shishkin mesh by Tikhonov's regularization technique.

Robustness of Isolated Non-Saddle Sets

Hector Barge
Universidad Complutense de Madrid, Spain
J.M.R. Sanjurjo

Isolated non-saddle sets arise in a natural way when dealing with the qualitative study of ODEs. Locally, these sets exhibit much of the properties of attractors and repellers. However the global picture of the flow may be dramatically different. In this talk we will introduce this kind of objects and we will focus on their continuation properties. It turns out that, in spite of the fact that attractors and repellers are robust under small pertubations, the property of being non-saddle is not. We will see some examples illustrating this fact and we will see that the continuation of this property has much to do with the continuation of some topological properties of the original non-saddle set. Some of the results presented here were obtained in collaboration with J.M.R. Sanjurjo.

Matrix- Valued Differential Equations with Spectral Singularities

Serifenur Cebesoy
Ankara University, Turkey
Elgiz Bairamov

The main aim of this paper is to obtain the Jost solutions and some spectral properties of a second order matrix nonself-adjoint differential equation on the whole axis. In this study, we investigate the analytical properties and asymptotic behaviors of these Jost solutions. Then, we find continuous spectrum of the operator L generated by matrix-valued differential equation and we get that the operator L has a finite number of spectral singularities and real eigenvalues.

Existence Results of Solutions for Nonlinear Boundary Value Problems on an Infinite Interval

Fulya Yoruk Deren
Ege University, Turkey
Nuket Aykut Hamal

This study is concerned with a boundary value problem of a second order differential equation on an infinite interval. Here, under some assumptions on the nonlinearities, the existence results of solutions are established by using some fixed point theorems.

On the Connectedness of the Attainabvle Set of Discountinuous Quantum Stochastic Differential Inclusions

Dauda Dikko
University of Ibadan, Nigeria
E. O. Ayoola

Using the notion of quantum stochastic differential inclusion, we establish a non-commutative generalisation which shows that the attainable sets of discountinuous quantum stochastic differential inclusions are connected. This result remain valid within the framework of the Hudson-Parthasarathy formulation of quantum stochastic differential inclusions.

Bifurcational and Topological Methods for the Global Qualitative Analysis of Low-Dimensional Dynamical Systems

Valery Gaiko
NAS Belarus, Belarus

We carry out the global qualitative analysis of low-dimensional polynomial dynamical systems. First, using new bifurcational and topological methods, we solve Hilbert's Sixteenth Problem on the maximum number of limit cycles and their distribution for the 2D general Liènard polynomial system and Holling-type quartic dynamical system. Then, applying a similar approach, we study 3D polynomial systems and complete the strange attractor bifurcation scenario for the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles which is related to Smale's Fourteenth Problem.

Improvements to the Averaging Theory of Periodic Differential Equations

Isaac Garcia
University of Lleida, Spain

We consider a family of T-periodic analytic differential equation in$\Omega \subset \mathbb{R}^n$ of the form

$$\dot{x} = F(t,x;\lambda,\varepsilon) = \sum_{i\geq 1} F_i(t,x;\lambda)\,\varepsilon^i,$$

where $\lambda \in \mathbb{R}^p$ are the parameters of the family and, for all i, the functions F_i are analytic and T-periodic in the t variable. Here, ε is the small real perturbation parameter.
We consider the Poincare displacement map at time T, that is, $d(z,\lambda,\varepsilon) = x(T;z,\lambda,\varepsilon) - z$ where, for each $z \in \Omega$, we denote $x(t;z,\lambda,\varepsilon)$ the solution of ODE with initial condition $x(0;z,\lambda,\varepsilon) = z$. . Clearly, the zeros of d are initial conditions for the T-periodic solutions of the differential equation. Expand $d(z,\lambda,\varepsilon) = \sum_{i\geq 1} f_i(z;\lambda)\,\varepsilon^i$, and denote the α-th component of the averaged function f_i by $f_i^{[\alpha]}$ with $\alpha \in \{1,\ldots,n\}$. Consider now the ideals $\mathcal{I}^{[\alpha]} = \langle f_i^{[\alpha]}(z;\lambda) : i \in \mathbb{N}\rangle$ in the Noetherian ring $\mathbb{R}\{z,\lambda\}_{(z_0,\lambda^*)}$ formed by all the real analytic functions at (z_0,λ^*).
We show the relationship between the cardinality of the minimal basis of the nontrivial ideals $\mathcal{I}^{[\alpha]}$ and the maximum number of isolated T-periodic solutions that can bifurcating from a fixed $z_0 \in \Omega$ for $\lambda = \lambda^*$ and $|\varepsilon| \ll 1$.

First Integrals, Inverse Integrating Factors and Lie Groups Admitted by N-Th Order Autonomous Systems

Yanxia Hu
North China Electric Power University, Peoples Rep of China

The methods of obtaining first integrals and inverse integrating factors of n-th autonomous systems using one-parameter Lie groups admitted by the systems are discussed and given. Some sufficient conditions on the existence of the inverse integrating factors of certain classes of n-th order autonomous systems are presented, and the explicit forms of the inverse integrating factors can be shown. Simultaneously, several related examples are given to illustrate the feasibility and the effectiveness of the proposed methods.

Spread of CTV Using Meta Population ODEs

Stephen Ippolito
ASTA, USA

The spread of the Cirtus tristeza virus(CTV) has been studied in Eastern Spain with the goal of determining the dynamics of the spread. Results do not support patterns in which the spread necessarily followed from adjacent trees which were already infected. The suggested mechanism for the results was inter- and intra-plot spread. Following a series of attempts to model this problem we consider a Bayesian ODE meta-population model where each meta population indicates a probability of infection at a particular time.

Oscillation of a Class of Delay Differential Equations with Impulses

Fatma Karakoc
Ankara University, Turkey

In this talk we present a class of linear delay differential equations with impulses. The equation which we introduce includes continuous argument as well as piecewise constant argument. Sufficient conditions for the oscillation of the solutions are obtained.

Cordial Volterra Integral Equations

Melusi Khumalo
University of Johannesburg, So Africa
Z. W. Yang

nt Cordial Volterra integral equations (CVIEs), associated with a noncompact cordial Volterra integral operator, have been prevalent in recent years since a number of real problems incorporate delayed history information. In this paper, we investigate some properties of cordial Volterra integral operators influenced by a vanishing delay. It is shown that to replicate all eigenfunctions λ, $\lambda = 0$ or R(λ)> 0, the vanishing delay must be a proportional delay. For such a linear delay, the spectrum, eigenvalues and eigenfunctions of the operators and the existence, uniqueness and solution spaces of solutions are presented. For a nonlinear vanishing delay, we show a necessary and sufficient condition such that the operator is compact, which also yields the existence and uniqueness of solutions to CVIEs with the vanishing delay

Stability Analysis of Impulsive Switched Singular Systems with Time-Delay

Humeyra Kiyak
University of Waterloo, Canada
Mohamad S. Alwan, Xinzhi Liu

In this work, we consider the impulsive switched singular systems with time-delay. The focus here is to address the problem of exponential stability of the system which consists of both stable and unstable subsystems. The novelty of this work is to establish some new sufficient conditions that guarantee the exponential stability of the system. The methodology of multiple Lyapunov function and the average dwell-time switching signal are used to achieve this goal. It has been observed that exponential stability of the system subject to perturbing impulses is guaranteed if the total activation time of stable subsystems is larger than that of the unstable ones. Numerical examples with simulations are presented to clarify the proposed result.

Some Results on a Third Order Differential Equation with a Piecewise Constant Argument

Mehtap Lafci
Ankara University, Turkey
Huseyin Bereketoglu, Gizem S. Oztepe

In this work, we consider a third order differential equation with a piecewise constant argument . We study existence and uniqueness of solutions of this equation and obtain some conditions that guarantee qualitative properties such as oscillation, nonoscillation and periodicity.

Periodic Solutions for Delay Differential Equations with Nonnegativity Constraints

David Lipshutz
Brown University, USA
Ruth Williams

Dynamical system models with delayed feedback and nonnnegativity constraints arise in a variety of applications in science and engineering. In some applications, oscillatory behavior is critical to the well functioning of the system. As a prototype for such models, we consider a one-dimensional delay differential equation with a nonnegativity constraint. In particular, the equation has discontinuous dynamics at zero. We study the existence, uniqueness and stability of slowly oscillating periodic solutions and illustrate our findings using a model for a simple genetic circuit. This is joint work with Ruth Williams.

Center Cyclicity of a Family of Quintic Polynomial Vector Fields

Susanna Maza
Universitat de Lleida, Spain
I. A. García, J. Llibre

We present a method for studying the Hopf cyclicity problem for the non-degenerate centers without the necessity of solving previously the Dulac complex center problem associated to the larger complexified family. As application we analyze the Hopf cyclicity of the centers of the quintic polynomial family written in complex notation as

$$\dot{z} = iz + z\bar{z}(Az^3 + Bz^2\bar{z} + Cz\bar{z}^2 + D\bar{z}^3).$$

The center problem for this family has been solved by J. Llibre and C. Valls, but the Hopf cyclicity is only stated for the easier case of having a focus at $z = 0$. Hence we will restrict our attention on the cyclicity problem of the center at $z = 0$ and our results are stated below.
Theorem. The following statements hold.
(a) Any nonlinear center at the origin of family has Hopf cyclicity at most 6 when we perturb it inside this family.
(b) There are perturbations of the linear center $\dot{z} = iz$ inside family producing 6 limit cycles bifurcating from the origin.
This is a work already published in J. Differential Equations, 258, (2015), no.6, 1990-2009.

Dynamics and Integrability of Quadratic Three-Dimensional Polynomial Vector Fields Having an Invariant Paraboloid

Marcelo Messias
Sao Paulo State University - UNESP, Brazil
Alisson de Carvalho Reinol

Invariant algebraic surfaces are frequently observed in vector fields or systems of ordinary differential equations arising in mathematical modeling of natural phenomena. In this work we concider quadratic polynomial vector fields defined in $\mathbb{R}^3$ having an elliptic paraboloid as an invariant algebraic surface. We give the normal form for these type of vector fields and state some results related to their integrability and to the existence of first integrals, Darboux invariants and exponential factors. For certain parameter values, we prove the existence of a limit cycle contained in the invariant paraboloid and show the ocurrence of centers and homoclinic orbits. We also investigate their dynamical behavior restricted to the invariant paraboloid, showing the occurrence of an interesting type of bifurcation, leading to the existence of infinitely many orbits homoclinic to a point at infinity, after compactification. To illustrate the possible applications of the obtained results, we

study the well-known Rabinovich system in the case it has invariant paraboloids and perform a detailed analysis of the dynamics and bifurcations of this system restricted to these invariant algebraic surfaces.

Volterra Integral Equations Related to Ordinary Differential Equations with Nondecreasing Nonlinearities

Wojciech Mydlarczyk
Wroclaw University of Science and Technology, Poland

We give a survey of the results on the solutions to the Volterra type integral equations in the convolution form $u(t) = \int_a^t k(t-s)g(u(s))ds$, where $a = -\infty$ or 0, the function $k \geq 0$ is locally integrable and the function $g \geq 0$ is continuous, nondecreasing with $g(0) = 0$. It is easily seen that one of the solutions is $u(t) \equiv 0$, called a trivial solution. However, from the point of view of applications only nontrivial solutions are interested. We focus on the criteria for the existence of the positive solutions and discuss conditions under which the solutions experience the blowing up behavior. The class of the equations under consideration include the important class of the fractional integral equations $u(t) = \int_a^t (t-s)^{\alpha-1} g(u(s))ds$, $\alpha > 0$. The considered problems originate in discussing the classical initial value problem for the ordinary differential equation $u^{(n)}(t) = g(u(t))$, $u(a) = u'(a) = \cdots = u^{(n-1)}(a) = 0$ and the famous Osgood condition.

Uniform Asymptotic Stability and Exponential Stability for a High-Dimensional Half-Linear Differential System

Masakazu Onitsuka
Okayama University of Science, Japan

In this talk, we deal with a high-dimensional half-linear differential system. In the special case, it becomes the n dimensional linear system $\mathbf{x} = A(t)\mathbf{x}$ where $A(t)$ is an $n \times n$ continuous matrix and $\mathbf{x}$ is an n dimensional vector. It is known that the zero solution of the linear system is uniformly asymptotically stable if and only if it is exponentially stable. However, in the case of nonlinear systems, uniform asymptotic stability does not imply exponential stability. Despite the fact that the high-dimensional half-linear system is nonlinear, will uniform asymptotic stability guarantee exponential stability? In this talk, we give the answer to this question.

Boundedness for the General Semilinear Duffing Equations Via the Twist Theorem

Daxiong Piao
Ocean University of China, Peoples Rep of China

We consider the boundedness of all solutions for the periodic semilinear equation $x'' + \omega^2 x + \psi(x,t) = 0$, where $\psi(x,t)$ does not necessarily satisfy the so called polynomial-like growth condition $\lim_{|x|\to+\infty} x^m \psi^{(m)}(x,t) = 0$ for some finite m. Usually this condition is needed in the references about boundedness problems of semilinear Duffing equations. Two cases of resonance and non-resonance are considered respectively.
* Joint work with Yiqian Wang, Zhiguo Wang, Lei Jiao and Xiao Ma

Collocation Method for Solving Singular ODEs and Higher Index DAEs

Ewa Weinmueller
Vienna University of Technology, Austria

We deal with a numerical solution of BVPs in ODEs which can exhibit singularities. Such problems have often the following form:

$$y'(t) = \frac{1}{t^\alpha} M(t)y(t) + f(t, y(t)),$$

$$t \in (0,1], \quad \alpha \geq 1, \quad b(y(0), y(1)) = 0.$$

The search for efficient numerical methods to solve the above BVP is strongly motivated by numerous applications from physics, chemistry, mechanics, ecology, or economy. In particular, problems posed on infinite intervals are frequently transformed to a finite domain taking above form with $\alpha > 1$. Also, research activities in related fields, like differential algebraic equations (DAEs) or singular Sturm-Liouville eigenvalue problems benefit from techniques developed for singular BVPs.
Polynomial collocation stays robust and shows advantageous convergence properties in context of singular problems. We illustrate how a collocation based open domain Matlab code bvpsuite can be used to solve BVPs from applications, complex Ginzburg-Landau equation, density profile in hydrodynamics, and generalized Korteweg-de Vries equation.
Finally, we consider higher index DAEs. Higher index DAEs constitute a really challenging class of problems due to the involved differentiation which is a critical operation to carry out numerically. We present a least-squares variant of the collocation method to deal with DAEs and discuss its performance by means of numerical experiments.

When Will a Formal Finite-Difference Expansion Becomes Real?

Chiang Yik Man
Hong Kong University of Science and Technology, Hong Kong
Shaoji Feng

It is a classical formula that one can expand a higher-order finite difference as an infinite sum of derivatives in a formal sense. We shall show that such a formal expansion becomes an equality when we restrict ourselves to meromorphic functions of small order (in the sense of Nevanlinna theory). The result is related to recent progress in complex function theory related to difference operators.

Contributed Session 2: PDEs and Applications

Nonuniform Dependence on Initial Data for the Whitham Equation

Mathias Arnesen
Norwegian University of Science and Technology, Norway

We consider the Cauchy problem for the Whitham equation on the torus and on the real line and prove that the solution map for the Whitham equation is not uniform in $H^s(\mathbb{T})$ or $H^s(\mathbb{R})$ for $s > \frac{3}{2}$. This is done by constructing two sequences of solutions in $H^s(\mathbb{T})$ ($H^s(\mathbb{R})$), converging to the same limit at the initial time while the distance at any later time is bounded below by a positive constant. The result is also extended to a wide class of Whitham-type equations by considering more general dispersive terms, covering, amongst other equations, fractional Korteweg-de Vries equations and the capillary Whitham equation.

Existence of Positive Solution for Boundary Value Problem of Non Linear Fraction Differential Equation

Muhammad Ibrahim Badsha Zada
Chinese Academy of Sciences, Peoples Rep of China
Anwarud Din

This paper is mainly concerned with the existence of solution for some non-linear fractional differential equation with boundary conditions. The uniqueness, existence and multiplicity of the solution have been tried here, using Banach contraction principle and fixed point theorems. The process is based upon the reduction of the given problem to the Equivalent fredholim integral equation and Greens function.

Quasicrystal Lattice Solitons in NLSM Systems

Mahmut Bagci
Istanbul Kavram Vocational College, Turkey
Ilkay Bakirtas, Nalan Antar

The properties of crystal and quasicrystal materials‘ surface are not necessarily the same as those of the bulk material. In the other words, although a material is centro-symmetric and the dynamics in this material can be described by the cubic nonlinear Schrödinger equation (NLSE), there are quadratic polarization effects at surface of the material. The nonlinear wave dynamics in quadratic ($\chi^{(2)}$) materials can be described by generalized NLSE with coupling to a mean term (NLSM system). The NLSM system with an external potential is given by

$$iu_z + \frac{1}{2}\Delta u + |u|^2 u - \rho\phi u - V(x,y)u = 0,$$

$$\phi_{xx} + \nu\phi_{yy} = (|u|^2)_{xx}$$

where $u(x,y,z)$ is the normalized amplitude of the electric field, $\phi(x,y,z)$ is the normalized static field (mean term), ρ is a coupling constant and ν is the anisotropy coefficient of the material. $V(x,y)$ is an external optical lattice.
In this study, we have investigated the quadratic nonlinearity effects on the quasicrystal lattice (Penrose tiling) solitons by the use of NLSM system as model. Using a fixed numerical scheme, we numerically obtained the soliton solutions of the NLSM systems with Penrose type lattices. The linear and nonlinear (in)stabilities of these solitons have been examined by direct computations of the NLSM systems.

Analysis of the Parallel Schwarz Method for the Solution of Chains of Particles in the Solvation Model

Gabriele Ciaramella
University of Geneva, Switzerland
Martin J. Gander

A new class of Schwarz methods was recently presented in the literature for the solution of solvation models, where the electrostatic energy contribution to the solvation energy can be computed by solving a system of elliptic partial differential equations; see [1] and related references. Numerical simulations have shown an unusual convergence behaviour of Schwarz methods for the solution of this problem, where each particle corresponds to a subdomain: the convergence of the Schwarz methods is independent of the number of particles [1], even though there is no coarse grid correction. Despite the successful implementation of Schwarz methods for this solvation model, a rigorous analysis for this unusual convergence behaviour is required, since no theoretical results are given in the corresponding literature.

In this talk, we analyze the behaviour of the Schwarz method for the solution of a chain of particles and show that its convergence does not depend on the number of particles. We use three different techniques to prove this result: the first technique is based on a Fourier expansion of the error and on the analysis of transfer matrices, the second consists in an application of the maximum-principle and the third is a variational approach that allows us to regard the Schwarz method as an alternating projection method.

References

[1] Cancés et al., Domain decomposition for implicit solvation models, J. of Chem. P. (2013).

Existence of a Unique Solution to the Nonlinear Poisson Equation

Diane Denny
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We consider the nonlinear Poisson equationΔ $u = f(u) - \frac{1}{|\Omega|}\int_\Omega f(u)d\mathbf{x}$, where $u(\mathbf{x}_0) = u_0$ at a given point $\mathbf{x}_0 \in \Omega$. We prove that if f and its first derivative f' are sufficiently small, then a unique classical solution u exists under periodic boundary conditions.

Nonlinear Picone's Identity for P-Biharmonic Operator and Its Applications

Gaurav Dwivedi
IIT Gandhinagar, India

Classical Picone's identity plays an important role in proving several qualitative result in the theory of second order elliptic PDE. Recently the classical Picone's identity has been extended to deal with problems concerning operators such as p-Laplace operator, polyharmonic operator etc. In this paper, we shall establish a nonlinear analogue of Picone's identity to deal with problems concerning p-biharmonic operator. As an application of the Picone's identity, we establish Hardy type inequality, Sturmian comparison principle, Caccioppoli inequality and some other qualitative results.

Envelope Solutions to PDEs Depending of Two Disjoint Sets of Variables

Maria Lewtchuk Espindola
Federal University of Paraiba, Brazil

There are a lot of applications for the envelope solutions to PDEs, the hypersurfaces that enclose one of the families of the hypersurfaces given by the complete solutions. The development and discussion of the existence of envelope solutions to PDEs that depends of two disjoint sets of variables are the main purpose of this research. As an example it is considered the canonical variables describing a mechanical system at the phase space in Hamiltonian Analytical Mechanics. As one of the possible extensions it can be discussed the development and the analyses of the existence of envelope solutions to the variational PDEs that involves functional depending of two disjoint sets of variables. As it occurs in Hamiltonian Analytical Mechanics applied to field theories where the dependence is of the field functions and the canonical variables represented by the density momenta.

Generalized Solution to the P-Laplace PDE: $u_x^2u_{xx} + 2u_xu_yu_{xy} + u_y^2u_{yy} = 0$

Maria Lewtchuk Espindola
Federal University of Paraiba, Brazil

The solutions to the p-Laplace partial differential equation $u_x^2u_{xx}+2u_xu_yu_{xy}+u_y^2u_{yy} = 0$ are enlarged in this article. For this purpose the Monge method for uniforme partial differential equations reduces the above equation to a Monge system. This system results in a PDE of first order of the type $f(p, q) = 0$. A method developed in previous papers by Espindola to the PDE $f(p, q) = 0$, furnishes a general solution which is a generalized solution of the p-Laplace as it contains one an arbitrary function.

Logarithmic Approximation of Non-Linear Ill-Posed Problems with Numerical Experiments

Matthew Fury
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Various approximation methods, including the quasi-reversibility method, have been applied in order to regularize the ill-posed Cauchy problem $du/dt = Au$, $t \in (0, T)$, $u(0) = x$ where A is an unbounded linear operator in a Banach space X and $x \in X$. In the case that A is positive, self-adjoint in Hilbert space, Boussetila and Rebbani apply a logarithmic approximation $A_\beta = -\frac{1}{pT}\ln(\beta + e^{-pTA})$, $\beta > 0$, $p \geq 1$ which induces a mild error order as compared with that of previous methods. In this presentation, we will outline the regularization of certain non-linear problems $du/dt = A(t)u + h(t, u(t))$, $t \in (0, T)$, $u(0) = x$ using Boussetila and Rebbani's perturbation in both Hilbert space and Banach space, where the operators $A(t)$ may be non-constant. In this case, the solution $v_\beta(t)$ of the corresponding well-posed problem converges in an appropriate norm to a supposed solution $u(t)$ of the original problem for each $t \in [0, T]$. Finally, we demonstrate applications of the theory to the backward heat equation and other partial differential equations through numerical experiments including a finite-difference method.

Double-Diffusive Convection for Nanofluids in MHD

Urvashi Gupta
Panjab University, India
Jyoti Sharma, Veena Sharma

The effect of vertical magnetic field on a horizontal layer of electrically conducting nanofluid, heated and soluted from below is studied analytically and numerically. For the analytical study, valid approximations are made in the complex expression for Rayleigh number to get useful and interesting results. The critical wave number increases whereas the frequency of oscillation (for the bottom heavy configuration) decreases when the Chandrasekhar number increases. For top heavy nanofluids, oscillatory motions are not possible and the instability through stationary convection increases with an increase in the nanoparticle concentration at the upper boundary of the fluid layer. The destabilizing effect of the higher concentration of nanoparticles at the top is so high that the magnitude of applied magnetic field must be increased so as to neutralize the effects of nanoparticles or the temperature at the lower boundary must be decreased. The numerical results for alumina-water nanofluid are studied using MATHEMATICA software. The heat transfer mode is oscillatory for bottom heavy nanofluids. The Soret parameter has largely destabilizing effect except around a small portion of the critical wave number whereas the effect of Dufour parameter is destabilizing.

Differential Equations in the Lovelock Modified Gravity Theory

Sudan Hansraj
University of KwaZulu Natal, So Africa
NK Dadhich, B Chilambwe

The recent discovery by LIGO of gravitational waves and the implied existence of black holes has served as yet another verification of the success of Einstein's general theory of relativity (GR). The result was conjectured by Einstein on studying the equations governing the coalescence of a binary black-hole system. This discovery further emphasises the need for theoretical explorations to accompany work on the observational front. Despite its major advantages, GR has failed to explain certain anomalous cosmological phenomena such as the late time accelerated expansion of the universe. Modified theories hold promise in these areas. In particular, we investigate the pure Lovelock theory of gravity which has the advantage of reducing to GR in the special first order case. The Lovelock Lagrangian consists of terms quadratic in the Riemann tensor, Ricci tensor as well as the Ricci scalar though remarkably the equations of motion turn out to be of second order only. This is in contrast to other gravity theories such as $f(R)$ theory where fourth order derivatives appear. The advantage of Lovelock theory is that the gains of GR are retained. Ordinary GR is well known for producing a highly coupled system of nonlinear partial differential equations that are ten in number in the worst case and these have been studied thoroughly over the last century. The Lovelock equations are far more complicated involving both higher curvature terms involving the spacetime dimension d as well as the order of the Lovelock polynomial N. We have succeeded for the first time in writing the full Lovelock equations for the gravitational behaviour of a spherically symmetric perfect fluid. Previously, only black-hole solutions were investigated. We are now in a position to construct models of the interiors of massive objects and to analyse their evolution in a different gravity regime. To date we have established that the famous Schwarzschild interior solution is universal in that the field equations are independent of d and N. Additionally, we have studied the case of an isothermal fluid sphere (PRD Vol 93 to appear) analogous to the Einstein case where a density and pressure fall-off is according to an inverse square law. A linear barotropic equation of state is automatically built in. We indicate further cases that are being studied and their potential to serve as models of compact objects in the universe. The major difficulty in this study is the complexity of the systems of partial differential equations. To work around this, assumptions such as the existence of symmetries, are made. These prescriptions on mathematical grounds then compromise the physical behaviour that is already observed. Resolving this tension between mathematical efficacy and observations is a cornerstone of research in gravity theories.

The Inviscid Limit for the Landau-Lifshitz Equation

Chunyan Huang
Central University of Finance and Economics, Peoples Rep of China
Zihua Guo

We prove that in dimension three and higher the Landau-Lifshitz equaiton is globally well-posed for small initial data in the critical Besov space uniformly with respect to the Gilbert damping parameters. Then we show the global solution converges to that of the Schroedinger maps as the Gilbert damping term vanishes.

Investigating Bi-Stability of Combustion Waves

Zhejun Huang
UNSW Canberra, Australia
Harvinder Sidhu, Isaac Towers, Zlatko Jovanoski, Vladimir Gubernov

When high temperature is applied to a material at one end by an ignition source, there is a possibility that a flame front is formed and this front propagates down the material. The propagating front is known as a combustion wave. We investigate properties of combustion waves for a competing two-stage exothermic reaction scheme.
Previous studies for the adiabatic case have shown the existence of bi-stability – fast (combustion waves

with high speeds) and slow (low speeds) solution branches co-exist for the same parameter values. We report the influence of heat loss on the existence of bi-stability. It is demonstrated that bi-stability can still exist for the non-adiabatic case if the heat loss parameter is below a certain threshold value. The transition between the fast and slow branches can be observed by perturbing the temperature profile for both adiabatic and non-adiabatic cases. Including heat loss into the system has a 'stabilizing' effect on the bi-stability behavior. In other words, for the non-adiabatic case the solutions do not 'jump' from one solution branch to the other unless a strong perturbation is applied to the system. Our results have clear implications for fire safety and industrial processes.

Conditional Lie-Backlund Symmetries and Functionally Generalized Separable Solutions of (n+1)-Dimensional Evolutions Equations

Muhammad Khan
Institute of Business Management, Pakistan
Amjad Faizan

We propose a generalization of the notion of invariance of $(n+1)-$dimensional partial differential equations with respect to Lie-Backlund vector field. Such generalization , known as conditional Lie-Backlund symmetries plays a vital role in constructing new ansatz that enables us to construct functionally generalized separable solutions of evolution equations which are defined on the invariant subspaces generated by the functionals. Using invariant subspaces, we reduce the PDE into a system of time dependent ODEs. A complete list of invariant subspaces of $(2+1)-$dimensional nonlinear diffusion equation with convection and source term is provided. Such generalization results in a number of new solutions which cannot be constructed by means of the symmetry reduction procedure.

Inertial Manifolds for Semilinear Parabolic Equations Which Do Not Satisfy the Spectral Gap Condition

Anna Kostianko
University of Surrey, England

We report on resent results concerning the existence of inertial manifolds for semilinear parabolic equations which do not satisfy the spectral gap conditions. The list of problems includes the 3D Cahn-Hilliard problem with periodic boundary conditions, the so-called modified Leray-α model which regularizes the 3D Navier-Stokes equations with periodic boundary conditions and 1D reaction-diffusion advection problems.

Asymptotic Behavior for Rayleigh's Problem Based on Kinetic Theory

Hung-Wen Kuo
National Cheng Kung University, Taiwan

We investigate the dynamics of the gas bounded by an infinite flat plate which is initially in equilibrium and set at some instant impulsively into uniform motion in its own plane. We use the BGK model equation to describe intermolecular collisions and assume the diffuse reflection to describe the interaction of the gas with the boundary. The Mach number of the plate is assumed to be small so that we can linearize the equation as well as the boundary condition. We establish the asymptotic expansions of the microscopic density function and the macroscopic velocity for small Knudsen numbers. The exact first two order terms of our asymptotic expansion of solutions provide a point of view to see the long time behavior for Rayleigh's problem.

A Nekhoroshev Type Theorem for the Beam Equation with Derivative Nonlinear Perturbation on the Torus

Chunyong Liu
Dalian University of Technology, Peoples Rep of China

In this paper, we prove a Nekhoroshev type theorem for the nonlinear beam equation with convolution potential under periodic boundary conditions. More precisely, we prove that if the initial datum is analytic in a strip of width $2\rho > 0$ whose norm on this strip is equal to ϵ, then if ϵ is small enough, the solution of the nonlinear beam equation above remains analytic in a strip of width $\rho/2$, with norm bounded on this strip by $C\epsilon$ over a very long time interval of order $\epsilon^{-\sigma|\ln\epsilon|^{\beta}}$, where 00 are positive constants depending on β and ρ.

Approximation of Non-Classical Shocks by Shadow Waves, Entropies and Wave Interactions

Marko Nedeljkov
University of Novi Sad, Yugoslavia

Our aim is to present a tool for dealing with non-classical solutions to conservation law systems containing delta functions or some other irregular objects. We call it Shadow Waves (SDWs, for sort), with the first reference [M. Nedeljkov, Shadow Waves: Entropies and Interactions for Delta and Singular Shocks, ARMA, 2010]. They are inspired by the well known Wave Front Tracking algorithm. SDWs are possibly unbounded nets of piecewise constant functions for fixed time depending on a small parameter ε..They have distributional limits as $\varepsilon \to 0$ and the typical examples are delta and singular shocks. The main advantage of SDWs is (a) possibil-

ity to use the Lax entropy pairs as an admissibility criteria and (b) easy dealing with wave interactions. We will give basic properties of SDWs (1-D case first) with several examples of their usefulness concerning the above points (a) and (b), and some of their applications in n-D conservation law systems.

Wave Propagation in Heterogeneous Geological Media with Cracks

Viktoria Savatorova
UNLV, USA
Alexey Talonov

Wave propagation through heterogeneous materials is of the utmost importance in various fields including structural dynamics, geophysical studies, earthquake engineering, oil exploration. Focusing on geological media, it is important to take into account such peculiarities of structure as soil layering, material heterogeneity, and cracks. The mathematical description of wave motion in heterogeneous media involves the solution of partial differential equations (PDEs) with variable coefficients. The aim of this talk is to present our results for multiscale modeling of wave propagation in geological materials with cracks. We consider multiscale model of heterogeneous medium consisting of randomly distributed micro-cracks (micro level) and periodic (layered) structure of heterogeneities at a higher scale (meso level). We justify using of averaging technique to perform the upscaling from micro scale (cracks) to mesoscale (layers), and then from mesoscale to macro (wave length) scale. As a result of upscaling we derive homogenized wave equation with effective coefficients, and investigate the influence of heterogeneities on micro and meso levels on the speed of propagation of waves, and their attenuation. Effective characteristics of the medium are determined from the solution of the problem defined on the periodic cell with taking into account the distribution of cracks.

Exact Solutions of a Non-Linear PDE Arising in Biological Processes

Muhammad Shabeer
Qatar University, Qatar
Muhammad Tahir Mustafa

The existence of travelling wave solutions of a class of non-linear PDEs, arising in biological processes, was shown in [J. D. Murray, Mathematical Biology I: An Introduction, Springer-Verlag, NewYork, 2002]. Applying the modified tanh method, we provide examples of travelling wave solutions of these non-linear PDEs.

A Numerical Study of Free Convection Heat and Mass Transfer in a Rivlin-Ericksenian Viscoelastic Flow Past an Impulsively Started Vertical Plate with Variable Temperature and Concentration

Veena Sharma
Himachal Pradesh University, India
Rajneesh Kumar, Ibrahim A. Abbas, Radhe Shyam

In this paper, an analysis of the effect of viscoelasticity on the natural free convective unsteady laminar heat transfer fluid flowing past an impulsively started vertical plate with variable surface temperature and mass concentration is considered. The Rivlin-Ericksen model is employed to simulate the rheological liquids. The transformed two-point boundary value problem is solved numerically using the Galerkin method for solving the partial differential equations relevant to the physical problem for the weak formulation. Numerical results are obtained to study the influence of viscoelasticity parameter, surface temperature power law exponent and surface concentration power law on the velocity, temperature, concentration fields ,the local skin-friction, the Nusselt number and the Sherwood number; which are graphically presented and discussed. It is found that increase in viscoelasticity parameter accelerates the velocity profiles, but has a little influence on the temperature and concentration fields; enhances the shear stress for $X > 0.28$ and reduces local Nusselt number and local Sherwood number for $X > 0.51$ in the boundary layer. It is also depicted that increase in power law exponents decelerate the velocity profiles, reduces temperature and the concentration fields of the diffusing species; but increases the surface shear stress for $X > 0.6$.

Reaction-Diffusion Systems with $(p(x,t), q(x,t))$-Growth and L^1 Data for Image Processing

Kehan Shi
Harbin Institute of Technology, Peoples Rep of China
Dazhi Zhang, Zhichang Guo, Jiebao Sun, Boying Wu

In this paper we investigate a class of reaction-diffusion systems coupled with $p(x,t)$-Laplacian equation and $q(x,t)$-Laplacian equation that arise from the study of image denoising problem and image decomposition problem. We extend the notion of entropy solution for reaction-diffusion systems with time-dependent variable exponents. The existence and uniqueness of entropy solutions for this problem with L^1 initial data is established.

Higher Order Boundedness of the Harmonic Projection Operator on Solutions to the Dirac-Harmonic System

Guannan Shi
Harbin Institute of Technology, Peoples Rep of China
Shusen Ding, Donna Sylvester, Yuming Xing

In this paper, higher order estimates for the harmonic projection operator applied to solutions of the Dirac-harmonic equations are established and the higher integrability of the projection operator is proven. As applications, new highter Sobolev imbedding inequalities of the projection operator and differential forms are constructed.

Potential Method in the Theory of Double Porosity Thermoelastic Materials

Merab Svanadze
Ilia State University, Rep of Georgia

The theory of thermoelasticity of double porosity materials, as originally developed for the mechanics of naturally fractured reservoirs, has found applications in many branches of civil engineering, geotechnical engineering, technology and biomechanics.
We shall consider the coupled linear theory of thermoelasticity for materials with double porosity structure. The boundary value problems (BVPs) of the steady vibrations are investigated. The fundamental solution of system of equations of steady vibrations is constructed. The Green's formulas and the representations of general solution of equations of steady vibrations are obtained. The basic properties of plane waves and the radiation conditions for regular vector are established. The uniqueness theorems of the internal and external BVPs of steady vibrations are proved. The basic properties of surface (single-layer and double-layer) and volume potentials are established. The existence of regular solution of the BVPs by means of the potential method (boundary integral equation method) and the theory of singular integral equations are proved.

A Range of Reynolds Number for Stationary Solution of the Viscous Incompressible Fluid Flow Down an Inclined Plane

Kyoko Tomoeda
Institute for Fundamental Sciences, Setsunan University, Japan
Yoshiaki Teramoto

We consider the two dimensional motion of a viscous incompressible fluid flowing down an inclined plane with an angle of inclination α, under the effect of gravity. The fluid motion is governed by the Navier-Stokes equations with the free boundary conditions. This problem contains two dimensionless quantities: Reynolds number and Weber number. When the Reynolds number and the angle α is sufficiently small, Nishida-Teramoto-Win (1993) proved the global existence of periodic solutions with an exponential decay rate for sufficiently small initial data. To obtain a specific range of this "sufficiently small Reynolds number", we examine the spectra of the compact operator arising the linearized problem. In this talk we discuss about a Range of the Reynolds number and Weber number, when the linear operator has a non-zero spectral value.

Existence of Viscosity Solutions of Singularly Perturbed Problem

Ram Baran Verma
IIT Gandhinagar, India
Jagmohan Tyagi

In this presentation, we will present the existence of two viscosity solutions of the following singularly perturbed problem

$$\begin{cases} -\epsilon^2\mathcal{M}^+_{\lambda,\Lambda}(D^2u) = f(x,u) \text{ in } \Omega \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1}$$

where f is a non-Lipschitz function and Ω is a smooth bounded domain in $\mathbb{R}^n$, $n > 2$. Here we will use inf regularization process to make the nonlinearity locally lipschitz continuous. That will allow us to apply the result that are available for locally Lipschitz nonlinearity.

A Stationary Core-Shell Assembly in a Ternary Inhibitory System

Chong Wang
George Washington University, USA
Xiaofeng Ren

A ternary inhibitory system motivated by the triblock copolymer theory is studied as a nonlocal geometric variational problem. The free energy of the system is the sum of two terms: the total size of the interfaces separating the three constituents, and a longer ranging interaction energy that inhibits microdomains from unlimited growth. In a particular parameter range there is an assembly of many core-shells that exists as a stationary set of the free energy functional. The cores form regions occupied by the first constituent of the ternary system, the shells form regions occupied by the second constituent, and the background is taken by the third constituent. The constructive proof of the existence theorem reveals much information about the core-shell stationary assembly: asymptotically one can determine the sizes and locations of all the core-shells in the assembly. The proof also implies a kind of stability for the stationary assembly.

Mathematical Analysis of a Class of Integro-Differential Equations

Weiqing Xie
Cal Poly Pomona, USA

A nonlinear integro-differential equations related to thermoelasticity will be studied. Mathematical analysis to this special nonlinear differential equations will be discussed and analyzed.

Permanence of Diffusive Models for Three Competing Species in Heterogeneous Environments

Benlong Xu
Shanghai Normal University, Peoples Rep of China
Zhenzhang Ni

In this talk, we address the question of the long term coexistence of three competing species whose dynamics are governed by the partial differential equations. We obtain criteria for permanent coexistence in a Lotka-Volterra system modeling the interaction of three competing species in a bounded habitat whose exterior is lethal to each species. It is also proved that if the inter-competing strength is very weak, the system is always permanent, provided that each single one of the three species can survive in the absence of the two other species.

The Existence and The Non-Existence of Global Solutions of a Free Boundary Problem

Rong Yin
Nantong University, Peoples Rep of China
Wanghui Yu

We study a free boundary problem of parabolic equations with a positive parameter τ included in the coefficient of the derivative with respect to the time variable t. This problem arises from some reaction-diffusion system. We prove that, if τ is large enough, the global solution exists while, if τ is small enough, the solution exists only in finite time.

A Bessel Type Pantograph Equation Arising in a Cell Growth Model

Ali Ashher Zaidi
Lahore University of Management Sciences (LUMS), Pakistan
Bruce van Brunt, Graeme Wake

A simple model for cell growth and division into $\alpha > 1$ daughter cells is given by the functional pde

$$-\frac{\partial^2}{\partial x^2}\left(D(x)n(x,t)\right) + \frac{\partial}{\partial x}\left(g(x)n(x,t)\right) + \frac{\partial}{\partial t}n(x,t) + bn(x,t) = b\alpha^2 n(\alpha x, t).$$

Here, n denotes the number density of cells of size x at time t , D is dispersion, g is the growth rate, and b is the division rate. ("Size" is usually measured by mass or DNA content.) The differential equation is supplemented by the condition

$$n(x,0) = n_0(x),$$

where n_0 is the initial cell size distribution, and the boundary condition

$$-\frac{\partial}{\partial x}\left(D(x)n(x,t)\right) + g(x)n(x,t) = 0,$$

at $x = 0$. The problem is thus of the initial-boundary value type. There is a paucity of analytical solution techniques for these problems; however, it is possible to solve the problem for some simple cases of interest. Although the leading order long time asymptotic behaviour of solutions to these problems is known for fairly general cases, the higher order terms are relatively unexplored. The exact solutions yield the higher order long time asymptotic behaviour of solutions for the special cases and may provide some insight into more general cases.

A Doubly Degenerate Diffusion Model for Multiplicative Noise Removal

Zhenyu Zhou
Harbin Institute of Technology, Peoples Rep of China
Zhichang Guo, Jiebao Sun, Dazhi Zhang, Boying Wu

Multiplicative noise removal is a challenging task in image processing. In this talk, the problem is addressed by using a doubly degenerate diffusion model, which is analyzed with respect to the existence of the weak solution. Experimental results illustrate effectiveness and efficiency of the proposed model.

Contributed Session 3: Modeling, Math Biology and Math Finance

Global Stabilizing Feedback Law for a Problem of Biological Control of Mosquito-Borne Diseases

Maria Soledad Aronna
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Pierre-Alexandre Bliman, Flàvio C. Coelho, Moacyr A.H.B. da Silva

The use of bacteria *Wolbachia* is a promising method presently considered to block the transmission of dengue, zika and chikungunya viruses. Systematic procedures for introduction of mosquitos infected by the bacteria in a healthy population are still to be studied. This is a central question, with heavy impact on the cost and efficiency.
The aim of the present study is the synthesis of a method allowing the reduction of the number of introduced mosquitos, and consequently the cost, without putting at risk the success of the infestation (something that could happen e.g. if the initial size of the population has been underestimated). Using the fact that measurements are completed during the whole release process, techniques from the theory of control of dynamical systems are used to define the quantity to be introduced. The original system is shown to have two stable equilibria, corresponding to *Wolbachia*-free and complete infestation situations. A simple feedback law is proposed and shown to have the capacity to asymptotically settle the bacteria. Up to our knowledge, this is the first attempt to use feedback for introduction of *Wolbachia* within a population of arthropods.
The techniques are based on the theory of monotone systems, recently extended by D. Angeli, E. Sontag and E. Enciso to analyze the asymptotic behavior of input-output monotone systems closed by negative feedback. Due to bistability, the considered input-output system has multivalued static characteristics, but the existing results are unable to prove almost-global stabilization, so *ad hoc* analysis has to be used.

Stock Pricing with New Dynamic Fractal Analysis Approach

Alireza Bahiraie
Semnan University and University of Malaya, Iran

Fractal analyzing of continuous processes have recently emerged in literatures in various domains. Existence of long memory in many processes including financial time series has been evidenced via different methodologies in many literatures in the past decade. This has inspired many recent literatures on quantifying the fractional Brownian motion (fBm) characteristics of financial time series. This presentation questions the accuracy of commonly applied fractal analyzing methods on explaining persistent or anti-persistent behavior of time series understudy. Rescaled range (R/S) and power spectrum techniques produce fractal dimensions for daily returns of twelve hundred S&P100 stocks from the most well performed firms are estimated. Zipf's law generates linear and logarithmic power-law distribution plots to evaluate the validity of estimated fractal dimensions on prescribing persistent and anti-persistent characteristics with less ambiguity. Findings of this study recommend a more thoughtful approach on classifying persistent and anti-persistent behaviors of financial time series by utilizing existing fractal analyzing methods.

Roots of Some Trinomial Equations with Applications to Financial Problems

Vanessa Botta
UNESP, Brazil

The main purpose of this paper is to determine the behavior of the roots of a kind of trinomial equation that appears in certain financial mathematics problems. In addition, we present the regions of the complex plane where these roots are located.

Spot Dynamics in a Plant Hair Initiation Model

Victor F. Brena-Medina
Centro de Ciencias Matematicas, UNAM Campus Morelia, Mexico
Michael J. Ward

Patch location dynamics of an initiation process in a plant root hair cell at a sub-cellular is thoroughly analysed. An earlier model proposed by Payne and Grierson captures key features of an interacting small G-protein family so-called Rho of Plants (ROPs). These proteins are in charge of promoting certain protuberances on root hair cells, which are crucial for nutrients uptake from the soil and anchorage, for instance. Auxins are a class of hormones that are known to take part on the morphogenesis of plants. As experimental observations show that a fast auxin flow is heterogeneously distributed along the cell at the ROPs diffusive scale, auxin catalysis is modelled as a spatially dependent coefficient controlling dominant cubic terms. Such a model consists of a generalised two-component Schnakenberg reaction-diffusion system, which is set up in a non-homogeneous domain. Upon considering a more realistic cell geometry, a two-dimensional root hair cell gathers the essential ingredients that allows to rigorously analyse whether shape and form are relevant for patch location dynamics. Numerical bifurcation analysis, as well as time numerical simulations, and the theory of semi-strong interactions are performed in order to shed light on the understanding of dynamical root hair morphogenesis.

Estimation of Natural Mortality for Isochronal Fish : Application to Sandfish in the Eastern Coastal Waters of Korea

Giphil Cho
Pusan National Univerisy, Korea
Il Hyo Jung

Studies of Estimating natural mortality play an important role in analyzing fisheries stock assessment and management, but current methods of estimation do not allow size-dependent instantaneous rate of natural mortality rates based on effective fecundity, growth and catch simultaneously. We propose stage-structure model that represent size-dependent natural mortality rates for isochronal fish because larger fish have fewer predators than smaller ones and natural mortality of larvae and juveniles is known to decrease with increasing body size. Here we represent an example of application to the sandfish (*Arctoscopus japonicus*) in eastern coastal waters of Korea. Effective fecundity was estimated by considering von bertalanffy growth equation, logistic equation of maturity and an exponential equation of fecundity with total size, which were derived from otolith and gonad analyses of sandfish collected from eastern coastal waters of Korea from 2005 to 2008. We assume that CPUE (Catch Per Unit Effort) for each year is direct proportion to average biomass for each year. As assuming various initial value, we can estimate optimal size-dependent natural mortality corresponding to the largest coefficient of determination (R^2) to account for interaction formula of assumption. We compared the results of previous studies with our result.

About New Mathematical Approaches for Inferring and Forecasting the Dynamics of Regulatory Network Models-A Numerical Study

Ozlem Defterli
Cankaya University, Turkey
Armin Fugenschuh,Gerhard-Wilhelm Weber

The analysis and anticipation of real-world complex phenomena, appearing frequently in the areas of life and environmental sciences, engineering sciences and finance, based on experimental data and environmental measurements is a challenging research problem of mathematical modeling.
In this study, we focus on such complex systems that appears in system biology, namely genetic regulatory networks. Based on the given experimental data and environmental measurements, the interactions of each gene with the others in a metabolic and genetic structure have to be identified clearly and the influences need to be predicted. Furthermore, anticipating the future behaviour of such complex systems requires the study of their discrete dynamics based on various numerical schemes.
Here, our model is a gene-environment network whose dynamics are represented by a class of time-continuous systems of ordinary differential equations based on the information provided by gene-expression levels and certain levels of environmental factors. The model contains unknown parameters to be optimized by considering the data at the sample times. Accordingly, time-discrete versions of that model class is studied and improved by higher-order explicit numerical methods. These numerical schemes are used to generate corresponding gene-expressions values for further time levels. We perform illustrative examples with simulations both for an artificial data set and also for a real-world data set. Furthermore, new extensions of this study will be mentioned briefly by considering new mathematical tools for the concept of uncertainty in the data.

Modeling TCP, Recurrence, and Second Cancer Risks Induced by Proton Radiation

Andrew Dhawan
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V. S. K. Manem

In the past few years, proton therapy has taken center stage to treat various tumor types, with increasing success due to volume-conforming dosing. However, the price of success may be the appearance of a secondary malignancy post-irradiation. The primary contribution of this study is to investigate the tumor control probability (TCP), relapse time, and the corresponding secondary cancer risks induced by proton beam radiation therapy. We incorporate tumor relapse kinetics into the tumor control probability framework, and calculate the associated second cancer risks, using the well-known initiation-inactivation-proliferation (IIP) formalism. We use the available in-vitro data for the linear energy transfer (LET) dependence of cell killing and mutation induction parameters. TCP, relapse and radiation induced second cancer risks are evaluated for protons in the clinical range of LETs. We show that compared to photon therapy, proton therapy markedly reduces the risk of secondary malignancies, and for equivalent dosing regimens achieves better tumor control as well as a reduced primary recurrence outcome, especially within a hypo-fractionation regimen. This study may serve as a framework for further work in this field, and, elucidates proton induced TCP and the associated secondary cancer risks, not previously reported in the literature.

Derivation of Windage Jump to Compensate Barrel Angular Rates on Flightpath of a Projectile

Fatih Geridonmez
Tubitak Sage, Turkey

This study describes the algorithm used in order to compensate the effect of muzzle angular rates on trajectory of a projectile. Starting with aerodynamic jump, the derivation of the windage jump is explained. The Windage Jump model is used in ballistic model of a projectile to compensate the effect of the crosswind on its flightpath. In ballistic theory, the wind-shear effect needs to be computed and added to the velocity vector of the projectile. The starting point in the derivation is the generalized aerodynamic jump effect on the spin stabilized projectiles due to cross wind.

Introduction of an Intervention Term to the Universal Law for Tumor Growth

Jaffar Ali Shahul Hameed
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Samantha Burns, Menaka Navaratna

Recent work in computational biology has led to the development of a universal mathematical model of tumor growth based on metabolism. Generally, the growth of tumor in terms of mass is represented as $\frac{dm}{dt} = f(m,t)$. In this paper, we introduce an intervention term, c, to this nonlinear computational model in order to analyze tumor growth in adverse cellular conditions, i.e. $\frac{dm}{dt} = f(m,t) - c$. We assess the intervention term, c, as a constant and as a function of time. Our simulations suggest that tumor treatment does not, in general, cause a monotonic decrease in mass and this finding is reflected in experimental data from the cancer literature. Our findings have potential applications for the development of cancer treatment by demonstrating that the duration and quantity of treatment can be optimized to suppress tumor growth, prevent metastasis, and prepare for tumor excision.

Population Dynamics with Immigration

Dan Han
University of North Carolina at Charlotte, USA
Stanislav A. Molchanov, Joseph M. Whitmeyer

We present population dynamic models by constructing Galton-Watson processes with immigration on the lattice Z^d. We use not the forward but the Kolmogorov backward equation, which is simpler and at the same time gives more detailed information on the limiting process. Asympototic analysis is given for the number of population as time tends to infinity. And we use these to prove a local central limit theorem result. The goal of the present paper is to prove the existence of a stationary limiting distribution for a wide class of branching random walks or processes in Z^d. Joint work with Dr. Stanislav A. Molchanov and Dr. Joseph M. Whitmeyer.

Analytical Issues in Pulsatile Blood Flow: Arterial Stiffness and Pulse Wave Velocity

Simona Hodis
Texas A&M University-Kingsville, USA
Mair Zamir

Vascular compliance is a major determinant of wave propagation within the vascular system, hence the measurement of pulse wave velocity (PWV) is commonly used clinically as a method of detecting vascular stiffening. The accuracy of that assessment is important because vascular stiffening is a major risk factor for hypertension. PWV is usually measured by timing a pressure wave as it travels from the carotid artery to the femoral or radial artery and estimating the distance that it travelled in each case to obtain the required velocity. A major assumption on which this technique is based is that the vessel wall thickness h is negligibly small compared with the vessel radius a. The extent to which this assumption is satisfied in the cardiovascular system is not known because the ratio h/a varies widely across different regions of the vascular tree and under different pathological conditions.
In this study we show that an expansion for small h/a of the classical solution of this problem does not in fact lead to valid results for values of h/a that are not infinitely small. An alternative solution for large values of h/a is presented, together with a method of patching the two solutions.

Understanding Pollution with Wiener-Hopf Lattice Factorizations

Hermen Jan Hupkes
Leiden University, Netherlands
E. Augeraud-Veron

We study optimal control problems with time delays posed on lattices, which can be used to weigh the costs and benefits of utilizing polluting agents to enhance crop yields. The conditions defining optimal strategies turn out to be Hilbert-space valued functional differential equations of mixed type (MFDEs). We develop tools such as exponential dichotomies and Wiener-Hopf factorizations for such systems to determine whether optimal strategies can retain their optimality under small variations in their initial conditions. Complications are caused by the fact that the modelling state space is only half of the natural mathematical state space.

Stoneley Waves at an Interface Between Thermoelasticdiffusion Solid Half Spaces

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This paper is concerned with the study of propagation of Stoneley waves at the interface of two dissimilar isotropic thermoelastic diffusion medium in the context of generalized theories of thermoelasticity (Lord-Shulman,1967 and Green-Lindsay,1972). The frequency equation of Stoneley waves is derived in the form of a determinant by using the boundary conditions. The dispersion curves giving the phase velocity and attenuation coefficients with wave number are computed numerically. Numerically computed results are shown graphically to depict the diffusion effect alongwith the relaxation times in thermoelastic diffusion solid half spaces for thermally insulated and impermeable boundaries, respectively. The components of displacement, stress, and temperature change are presented graphically for two dissimilar thermoelastic diffusion half-spaces. Several cases of interest under different conditions are also deduced.

Modeling and Simulation of Carrier-Based Aircraft Approach and Landing Based on Touchdown Precision

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Ting Yue, Lixin Wang

To establish the relationship between control system design and landing precision, a dynamics model integrated with 6-DOF nonlinear model of F/A-18, Automatic Carrier Landing System (ACLS), ship airwake influence and typical carrier deck motion is presented. The ACLS parameters are firstly designed for the powered approach flight condition according to frequency domain requirements. Touchdown errors and its dispersions then can be obtained through closed loop simulations with random initial settings of carrier motion and turbulence characteristics, which will be a significant evaluation index of carrier-based aircraft landing performance. The simulation result indicates the validity of the established model, and demonstrates the different effects of turbulence, gust and deck motion on touchdown accuracy. This analytical method also proves its availability and convenience in ACLS design and landing safety research.

Modeling the Trade-Off Between Transmissibility and Contact in Infectious Disease Dynamics

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Kristen A. Deger, Joseph H. Tien

Symptom severity affects disease transmission both by impacting contact rates, as well as by influencing the probability of transmission given contact. This involves a trade-off between these two factors, as increased symptom severity will tend to decrease contact rates, but increase the probability of transmission given contact (as pathogen shedding rates increase with symptom severity). This paper explores this trade-off between contact and transmission given contact, using a simple compartmental susceptible-infected-recovered type model. Under mild assumptions on how contact and transmission probability vary with symptom severity, we give sufficient, biologically intuitive criteria for when the basic reproduction number varies non-monotonically with symptom severity. Multiple critical points are possible. We give a complete characterization of the region in parameter space where multiple critical points are located in the special case where contact decreases exponentially with symptom severity. We consider a multi-strain version of the model with complete cross-immunity and no super-infection. We prove that the strain with highest basic reproduction number drives the other strains to extinction. This has both evolutionary and epidemiological implications, including the possibility of an intervention paradoxically resulting in increased disease prevalence.

Adsorption of Flexible Polymer Chains on a Surface

Paulo Martins
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João Plascak, Michael Bachmann

Polymer chains undergo a continuous adsorption/desorption transition onto a flat surface in such a way that for $T < T_c$ the system is in the adsorbed phase and for $T > T_c$ the polymer is desorbed, T_c being the critical temperature. In three dimensions, the precise values of the corresponding critical exponents remain still an open question. It has been noted that their values are not only size dependent, but also depend on the precise location of T_c. As a result, the known estimates of the critical exponent cover a broad range of values. Additionally, most studies only consider good solvent conditions, in which the monomer-monomer interaction is negligible. In the present work we estimate critical quantities for different solvent conditions showing that, in analogy to magnetic phase transitions, finite-size-scaling methods furnish quite good results by taking into account corrections to scaling.

Delay Differential Equations with Dynamical Systems: Numeric and Analytic

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In this paper, we show the consistency of delay differential equations with biological systems with memory, in which we present a class of mathematical models with time-lags in immunology, physiology, epidemiology and cell growth. We also incorporate optimal control parameters into a delay model to describe the interactions of the tumour cells and immune response cells with external therapy. We then study parameter estimations and sensitivity analysis with delay differential equations. Sensitivity analysis is an important tool for understanding a particular model, which is considered as an issue of stability with respect to structural perturbations in the model. We introduce a variational method to evaluate sensitivity of the state variables to small perturbations in the initial conditions and parameters appear in the model. The presented numerical simulations show the consistency of delay differential equations with biological systems with memory. The displayed results may bridge the gap between the mathematics research and its applications in biology and medicine.

Sympathetic Inhibition: Understanding a Dynamic Paradox When Inhibitors Promote Future Expressions

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An inhibitor is traditionally conceived as a substance that has a negative affect on the rate of change in the concentration of a particular state variable and/or that variable‘s equilibrium level, and there are many mechanisms by which suppression can occur. With some types of inhibitors, however, there are secondary effects which may actually boost expression to levels not normally achieved in systems in which the inhibitor is entirely absent. By converting expressible factors from a free to stored state, this sympathetic inhibition creates a potential reserve for future expression. When the inhibitor is removed from the system, this stored concentration is free to revert back to its regular state, leading to a large bloom of expression/variable concentration.

Feedback Linearization and Its Dynamics in a Prey Predator Model

Anuraj Singh
Graphic Era University, India

The stabilization problem through feedback linearization of a prey-predator model with is discussed. Feedback linearization is a technique through which complexity can be controlled and system may behave in a desired fashion. By approximate linearization approach, a feedback control law is obtained which stabilizes the closed loop system. On the other hand, by suitable change of coordinates in the state space, a feedback control law is obtained. This feedback control renders the complex nonlinear system to be linear controllable system such that the positive equilibrium point of the closed-loop system is globally asymptotically stable. Numerical experiments substantiate the analytical findings.

Notes of Mathematical Model in Quorum Sensing

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Sarangam Majumdar

In this present work, we discuss a mathematical model of quorum sensing mechanism of bacteria.Quorum sensing is density dependent behaviour which is widespread in bacteria like Vibrio Fischeri.This biochemical process is regulated by the quorum sensing molecules.To understand this biological phenomenon, we use a nonlinear dynamical system and study the stability analysis based on experiment.

Global Attractivity of Positive Almost Periodic Solution for a Density Dependent Predator-Prey System with Mutual Interference and Crowley-Martin Response Function

Jai Tripathi
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A nonautonomous density dependent predator-prey model with mutual interference and Crowley-Martin response function is proposed and studied. The dynamics of the system is analyzed mainly from the point of view of permanence, extinction, stability, existence and uniqueness of a positive almost periodic solution. It is also shown that the the obtained permanence conditions are only sufficient but not necessary. The sufficient conditions are derived for globally attractive unique positive solution by constructing suitable Lyapunov functional. It is shown that sufficient conditions obtained for globally attractive unique positive solution depend on both the predator density dependent death rate and Crowley-Martin coefficient. The obtained analytical results are illustrated with the help of numerical examples.

Mathematical Modeling, Analysis and Monte Carlo Simulation of Ebola Epidemics

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Boping Tian

Ebola virus disease is a fatal disease which become the headache of the whole world by now. As its fatality rate is the highest, an urgent solution(measure) has to be taken to control the disease. In this article we built a new mathematical model to study the dynamics of Ebola. Besides, as the spread of Ebola virus is random process simulation is done using Monte Carlo method(algorithm). Key words: Mathematical modeling, Monte Carlo, Epidemic model, Ebola

Cellular Blebs: Modelling Cellular Movement Through Dynamic Membrane Protrusions

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Ruth E. Baker, Eamonn A. Gaffney, James M. Oliver, Sarah L. Waters, Alain Goriely

Human muscle undergoes an age-related loss in mass and function. Preservation of muscle mass depends, in part, on stem cells, which navigate along muscle fibres in order to repair damage. Critically, these stem cells have been observed to undergo a new type of motion that uses cell protrusions known as “blebs“, which protrude from the cell and permit it to squeeze in between surrounding material.
In order to understand this dynamic motion we have created a model which combines a stochastic protrusion model with a mechanical membrane model. The resulting multiscale framework allows us to link easily generated experimental data on trajectories to mechanical properties of the membrane, which are much more difficult to explore experimentally. Through investigating multiple extensions of this model we find numerous results concerning size, shape and limiting factors of blebs based movement.

Multibody Dynamic Characteristics of Oblique-Wing Aircraft in the Wing Morphing Process

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Ting Yue, Lixin Wang

The multibody dynamic modeling and simulation of an oblique-wing aircraft in the wing morphing process is studied. When the wing is oblique and no longer symmetric, wing area, moments of inertia, aerodynamics and center of gravity alter considerably, and these will change the dynamic characteristics of this aircraft. Six degree of freedom nonlinear dynamic equations of oblique-wing aircraft in the wing morphing process are derived. On condition that the angular velocity of oblique wing is small and the unsteady aerodynamic effect can be ignored, the responses in different angular velocities are numerically simulated by quasi-steady aerodynamic supposition. The simulation results show that the dynamic response exhibits large variations in wing morphing process. Meanwhile, the aircraft dives quickly and cannot achieve a new state of level flight without control. Furthermore the rules of how dynamic characteristics are affected by the shift of center gravity position and variation of aerodynamics are investigated. It can be concluded that the key factor which influences the dynamic response in the wing morphing process is the variation of aerodynamics resulting from asymmetric wing.

Contributed Session 4: Control and Optimization

Pontryagin Maximum Principle for Optimal Nonpermanent Control Problems on Time Scales

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Limoges University, France
Emmanuel Trelat

Pontryagin maximum principle (in short, PMP) is a fundamental result of the optimal control theory. In its historical statement, the control of the dynamical system is considered as permanent, that is, the control is authorized to be modified at any real time. In many problems, it follows that achieving the optimal trajectory requires a permanent modification of the control. However, such a request is not conceivable in practice for human beings, even for mechanical or numerical devices. Therefore, piecewise constant controls (also called sampled-data controls or digital controls), for which only a finite number of modifications is authorized, are usually considered in Automatic and Engineering. Sampled-data controls are one example of nonpermanent controls. Another example concerns dynamical systems whose trajectories go across noncontrolled areas (like a mobile phone or a GPS device going under a tunnel). In this talk, we will present a new version of the PMP that can be applied to optimal nonpermanent control problems. This result was recently obtained in [1] and is stated with the help of the time scale calculus theory. Numerous properties about optimal permanent controls are well-known in the literature (such as the continuity of the Hamiltonian, or the saturation of the control constraints set when the Hamiltonian is affine, etc.). In this talk, we will be interested in the preservation (or not) of these classical properties when we consider nonpermanent controls. Finally, in the linear-quadratic case, we will state that the optimal sampled-data controls converge to the optimal permanent control when the distances between consecutive sampling times uniformly tend to zero.

References

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Multiplicity and Stable Manifolds for Time-Delay Systems: Further Remarks on the Rightmost Root

Islam Boussaada
Paris Saclay University and IPSA, France
Islam Boussaada, Silviu Iulian Niculescu, Hakki Unal

Multiple spectral values in dynamical systems are often at the origin of complex behaviors as well as unstable solutions. However, the starting point of this talk is an example of delay system where the maximal multiplicity of an appropriate delay-dependant negative spectral value leads to a negative spectral abscissa and, as a consequence, the asymptotic stability of the corresponding steady state solution holds. In algebraic terms, the manifold corresponding to such a multiple root defines a stable manifold for the steady state. It will be shown that such a multiple spectral value is nothing but the rightmost root. Motivated by the potential implication of such a property in control systems applications, this study is devoted to better explore the connexion between those manifolds.

Sufficient Conditions for L^1-Minimization in Celestial Mechanics

Zheng Chen
University Paris-Sud, France
Jean-Baptiste Caillau, Yacine Chitour

In this talk, we consider the L^1-minimization problem for celestial mechanics. First, some basic properties of the extremal flow including the existence of bang, singular, and chattering solutions will be presented as consequences of Pontryagin maximum principle. Then, we will deal with the sufficient optimality conditions for bang-bang extremals with regular switching points. The ability to establish and verify the sufficient conditions is a fundamental issue in optimal control theory. The crucial idea for establishing such conditions is to construct a parameterized family of extremals such that a bang-bang extremal with regular switching points can be embedded into a field of broken extremals. Two no-fold conditions for the canonical projection of the parameterized family of extremals are devised. For the scenario that the endpoints are fixed, these no-fold conditions are sufficient to guarantee that the bang-bang extremal is locally minimizing. If the final point is not fixed but lies on a smooth submanifold, another sufficient condition involving the geometry of the target submanifold is established. Finally, two numerical examples in celestial mechanics will be shown to verify these sufficient conditions.

P-th Moment and Almost Sure Stability of Neutral Stochastic Switched Nonlinear Systems

Caixia Gao
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Haibo Gu

As one of the major issues in the study of control theory, the stability of stochastic system has stirred some initial research interest. Since ψ^γ stability contains exponential stability and polynomial stability, it has a wide applicability. However, there are few research results about p-th moment and almost sure ψ^γ stability for neutral stochastic switched nonlinear systems. In this paper, we attempt to investigate p-th moment and almost sure ψ^γ stability of neutral stochastic switched nonlinear systems. Firstly we introduce Razumikhin-type theorems and Lyapunov methods. By the aid of Lyapunov-Razumikhin approach, we obtain the p-th moment ψ^γ stability of neutral stochastic switched systems. In order to establish the criterion on almost surely ψ^γ stability of neutral stochastic switched systems, the Holder inequality, Burkholder-Davis-Gundy inequality and Borel-Cantelli's lemma are utilized in this paper. Finally one numerical example is provided to demonstrate the effectiveness of the proposed results.

Chains of Minimal Image Sets Can Attain Arbitrary Length

Byungik Kahng
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It is known that the maximal invariant set of a continuous iterative dynamical system in a compact Hausdorff space is equal to the intersection of its forward image sets, which we will call the *first minimal image set*. In this talk, we discuss the corresponding relation for a class discontinuous self maps that are on the verge of continuity, or *topologically almost continuous endomorphisms*. We prove that the iterative dynamics of a topologically almost continuous endomorphisms yields a chain of minimal image sets that attains a unique transfinite *length*, which we call the *maximal invariance order*, as it stabilizes itself at the maximal invariant set. We prove the converse, too. Given ordinal number ξ, there exists a topologically almost continuous endomorphism f on a compact Hausdorff space X with the maximal invariance order ξ.

Uncountable Set of Real Numbers Having No Uncountable Subset of the First Category in $\mathbb{R}$

Paula Kemp
Missouri State University, USA

In this paper, it is shown that the statement, Every Uncountable set of Real Numbers $\mathbb{R}$ has an uncountable subset which is of the first category in $\mathbb{R}$ is independent of the axioms of the ZFC Set Theory, (Zermelo-Fraenkel with the Axiom of Choice). Also, an example of a nonmeasurable set is given.

Chain Sequences of Some Finite Classes of Classical Orthogonal Polynomials

Pradeep Malik
University of Petroleum and Energy Studies, Dehradun, India

In this work, we find chain sequences for Laguerre polynomials and some finite classes of classical orthogonal polynomials (COP). Chain sequences are much useful in the classification of the birth and death process.

The Stochastic Linear Quadratic Control Problem with Singular Estimates

Hermann Mena
University of Innsbruck, Austria

We study an infinite dimensional finite horizon stochastic linear quadratic control (SLQ) problem in an abstract setting. We assume that the dynamics of the problem are generated by a strongly continuous semigroup, while the control operator is unbounded and the multiplicative noise operators for the state and the control are bounded. We prove an optimal feedback synthesis along with well-posedness of the Riccati equation for the finite horizon case. In addition, we investigate a numerical framework for the SLQ problem, in particular, the convergence of the Riccati operators.

A Computed Feedforward Compensation and Robust Dynamics Force Feedback Control for a 6DOF Stewart-Type Nanoscale Platform

Yung Ting
Chung Yuan Christian University, Taiwan

In this article, a new computed feedforward compensation for a 6DOF Stewart-type nanoscale platform is investigated to enhance the tracking performances of the system due to the nonlinear hysteresis, creep effect and drifting disturbance of the piezoelectric actuators, which are the main driving resource of the system. Exponentially Weighted Moving Average (EWMA) method has been widely used in process control and verified its capability of overcoming systematic change and drift disturbance. A double Exponentially Weighted Moving Average (dEWMA) instead is used in combination of a dynamic inverse hysteresis Preisach model for the feedforward computed controller. Besides using feedforward control, an improved robust dynamics force feedback controller, which is based on the idea of combining the classical impedance control and predictive force control, is also proposed to face with unacknowledged interacting forces in nano-cutting applications. The algorithm provides a way to increase the robustness of force control scheme with respect to a variation of the environment characteristics. The experiment is performed to evaluate the effectiveness and robustness of the proposed controller above for in-feed and cross-feed motion of the 6DOF Stewart-type nanoscale platform.

Internal Model Control with Run-To-Run EWMA for Speed Control of a Co-Planar Stage Driven by Piezoelectric Motors

Yung Ting
Chung Yuan Christian University, Taiwan
Mark Leorna

In this article, speed control for a co-planar stage driven by a bimodal piezoelectric motor is investigated. Such a motion control system is subject to disturbance such as friction, preload and operating temperature rise. Especially, operating temperature rise is an essential problem of using piezoelectric motor, but very few research works address this topic in depth. Exponentially Weighted Moving Average (EWMA) method has been widely used in process control and verified its capability of overcoming systematic change and drift disturbance. Besides using EWMA for speed control, it is attempted to map the EWMA method into a run-to-run (RtR) Internal Model Control (IMC) structure to achieve a RtR-IMC adaptive control scheme (RtR-IMC_EWMA). Based upon this control structure, a PI controller is added in the feedforward path and proved to be able to deal with the ramp disturbance determined in practical experiment at variant operating temperature. Friction causes dead-zone area, a dead-zone compensator is thus designed based on a predictive friction model in order to reduce the friction effect. Associated with the friction compensator, performance of several control methods including a general PID controller with optimal gains, RtR-IMC_EWMA, and RtR-IMC_EWMA with PI controller (RtR-IMC_EWMA+PI) are examined. From experiment, the proposed RtR-IMC_EWMA+PI control is superior to other methods. Such a new adaptive control scheme is easy to establish and provides flexibility of adding suitable controller to enhance system robustness.

Limit Theorems for One Class of Ergodic Markov Chains

Nezihe Turhan
University of North Carolina at Charlotte, USA
Stanislav Molchanov

In this talk, I will start with giving an intuitive background on the limit theorems for Markov chains. Since my work includes both discrete and continuous-time Markov chains, I provide some preliminary work on both cases. Later on, I will briefly explain two methods, namely Doeblin method and Martingale approximation, to prove the Central Limit Theorem for the Loop Markov chains. Subsequently, I introduce three models of Loop Markov chain, and I prove the Central Limit Theorem for a special class of functionals. As an example, I talk about Random Number Generators (RNGs) which are appropriate applications of Loop Markov chains. Lastly, I analyse convergence to the stable limiting distributions, where we consider not a special class functionals but arbitrary ones.

Optimal Harvesting Control of a Diffusive Population Model with Size Random Growth and Distributed Recruitment

Qiangjun Xie
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Ze-Rong He, Zhaosheng Feng

We investigate an optimal harvesting control problem for a spatial diffusion population system, which incorporates individual's random growth of size and distributed style of recruitment. The existence and uniqueness of nonnegative solutions to this practical model are shown by means of Banach's fixed point theorem, and the continuous dependence of the population density on the harvesting effort is analyzed. The optimal harvesting strategies are established via normal cone and adjoint techniques. Some conditions are presented to assure that there is only one optimal policy.

Contributed Session 5: Scientific Computation and Numerical Algorithms

Stability and Convergence of a Vector Penalty Projection Scheme for the Incompressible Navier-Stokes Equations with Moving Body.

Adrien Doradoux
Bordeaux University, France
Vincent Bruneau, Pierre Fabrie

This work is devoted to the study of a Vector Penalty Projection scheme to solve the incompressible viscous flow around a moving body in the case where the interaction of the fluid forces on the solid can be neglected. The velocity inside the solid region is enforced using a penalization technique. The stability of the system is shown using energy estimates on each equation of the splitting. This result additionally leads to a bound of the velocity in a Nikolskii space which is useful to prove the strong convergence of the velocity through Simon's compactness Lemma. We therefore show that there exist $u \in L^\infty(]0;T[,\mathbf{L}^2) \cap L^2(]0;T[,\mathbf{H}^1)$ and $p \in W^{-1,\infty}(]0;T[,\mathbf{L}_0^2)$ limit of the scheme when the time step and the corection parameter that ensures a small velocity divergence tend to 0. Moreover, the couple (u,v) is a weak solution of the penalized Navier-Stokes equations. Finally, studying the convergence when the penalty parameter that enforces the velocity inside the body tends to 0, we give a new proof of the existence of weak solutions to the Navier-Stokes problem with a no sleep boundary condition on the moving body boundary.

RBF Solution of Natural Convection of Nanofluids in a Cavity

Bengisen Pekmen Geridonmez
TED University, Turkey

In this study, natural convection in a unit square cavity filled with a nanofluid is solved numerically utilizing the multi quadric radial basis function pseudo spectral (MQ RBF-PS) in space domain and differential quadrature method (DQM) in time domain. The governing dimensionless equations are solved in terms of stream function, temperature and vorticity. In cavity, thermally insulated top and bottom walls are maintained while the left and right walls are at constant temperatures. Numerical solutions present the average Nusselt number variation as well as streamlines, isotherms and vorticity contours. The problem parameters, Rayleigh number Ra and solid volume fraction χ are varied as $10^3 \leq Ra \leq 10^6$ and $0 \leq \chi \leq 0.2$, respectively. It is found that the heat transfer is enhanced in presence of nanoparticles.

Magneto-Convection in Binary Nanofluids: a Revised Model

Urvashi Gupta
Panjab University, India
Jyoti Sharma, R. K. Wanchoo

The paper presents the effect of vertical magnetic field on double-diffusive nanofluid convection with the assumption that nanoparticle flux is zero along the boundaries of the layer. The nanofluid layer incorporates the effect of Brownian motion and thermophoresis due to the presence of nanoparticles and effect of Dufour and Soret parameters due to the presence of solute. Normal mode technique and single term Galerkin method are used to solve the conservation equations related to the system.For the analytical study, valid approximations are made in the complex expression for the Rayleigh number to get useful and interesting results. Due to the inclusion of magnetic field, Lorentz force term is added in the momentum equation, which results in strong stabilizing effects of the magnetic field parameter (the Chandrasekhar number) on the fluid layer. Oscillatory motions are not possible and hence mode of convection is invariably through stationary mode. Binary nanofluids are found to be much less stable than regular fluids. Numerical computations are carried out for water based nanofluids to analyze solutal effects on the stability of the system using the software Mathematica. Higher conductivity and density of metallic nanofluids make them less stable as compared to nonmetallic/semiconducting nanofluids.

Computable Error Estimates for Monte Carlo Finite Element Approximation of Elliptic PDE with Rough Lognormal Diffusion Coefficients

Eric Hall
University of Massachusetts Amherst, USA
Håkon Hoel, Mattias Sandberg, Anders Szepessy, Raul Tempone

The Monte Carlo (and Multi-level Monte Carlo) finite element method can be used to approximate observables of solutions to diffusion equations with lognormal distributed diffusion coefficients, e.g. modeling ground water flow. Typical models use lognormal diffusion coefficients with Hølder regularity of order up to 1/2 almost surely. This low regularity implies that the high frequency finite element approximation error (i.e. the error from frequencies larger than the mesh frequency) is not negligible and can be larger than the computable low frequency error. We address

how the total error can be estimated by the computable error and propose goal-oriented estimates for the pathwise Galerkin and expected quadrature errors that are derived using easily validated assumptions.

Discontinuous Galerkin Method with Weighted Numerical Flux for Time-Dependent Equation

Jia Li
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Boying Wu, Jiebao Sun, Shengzhu Shi

This article studies the numerical scheme by using weighted numerical flux for solving time-dependent equation. The numerical flux is a vital item for discontinuous Galerkin methods, that it is the key for the solvability and properties of scheme. The weighted flux (up-wind biased flux) this article studies is a special form of numerical fluxes. Unlike up-wind and central flux taking the value of one side or average of two side at the nodes, it takes the weighted value of two side, which made the scheme be more adaptive for complex equations and have better convergence. So weighted flux has better properties than others in some ways. This article develops a numerical scheme to solve heat equation by discontinuous Galerkin methods applies the weighted flux, and studies the stability analysis and error estimate. Meanwhile, numerical examples are consistent with the error estimate, which proves the validity of scheme.

Semi-Lagrangian Numerical Methods for Systems of Time-Dependent Partial Differential Equations

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Semi-Lagrangian methods are numerical methods designed to find approximate solutions to particular time-dependent partial differential equations (PDEs) that describe the advection process. We propose semi-Lagrangian one-step methods for numerically solving initial value problems for two general systems of partial differential equations:

$$\left\{\begin{array}{rcll} \frac{\partial y_1}{\partial t}+\omega\frac{\partial y_1}{\partial x} & = & f_1(t,x,y_1,\ldots,y_n), & y_{1_0}=y_1(0,x),\\ \vdots \qquad \vdots & & \vdots & \vdots\\ \frac{\partial y_n}{\partial t}+\omega\frac{\partial y_n}{\partial x} & = & f_n(t,x,y_1,\ldots,y_n), & y_{n_0}=y_n(0,x), \end{array}\right.$$

where $(t,x)\in[0,\infty)\times[a,b]$ and ω can take on one of three cases:

1. $\omega\in\mathbb{R}$,
2. $\omega=\omega(t,x)$, $|\omega|<\infty$,
3. $\omega=\omega(t,x,\boldsymbol{y})$, $|\omega|<\infty$,

and

$$\left\{\begin{array}{rcll} \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} & = & f(t,x,y,u,v), & u_0=u(0,x,y),\\ \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y} & = & g(t,x,y,u,v), & v_0=u(0,x,y), \end{array}\right.$$

where $(t,x,y)\in[0,\infty)\times[a,b]\times[c,d]$.
Along the characteristic lines of the PDEs, we will use ordinary differential equation (ODE) numerical methods to solve the PDEs. The main benefit of our methods is that we have managed to achieve high order local truncation error through the use of Runge-Kutta methods along the characteristics. In addition, we have established numerical analysis precedents for semi-Lagrangian methods applied to systems of PDEs: stability, convergence, and maximum error bounds.

Transformation Optics Based Finite-Difference Time-Domain Method for Solving the Maxwell Equations

Jinjie Liu
Delaware State University, USA
Moysey Brio, Jerome V. Moloney

Transformation optics (TO) is an elegant coordinate transformation technique to design the metamaterial invisibility cloak. An important feature of the TO technique is the invariance of the Maxwell equations after a coordinate mapping, while the transformed material becomes electrically and magnetically anisotropic.
Recently, we have developed a stable TO based mesh refinement method to numerically solve the Maxwell's equations. The TO technique is applied to the Maxwell equations to achieve local mesh refinement, by mapping a structured non-orthogonal grid to a rectangular mesh. Then the transformed anisotropic Maxwell equations are solved using a stable anisotropic Finite-Difference Time-Domain (FDTD) method. With TO based local mesh refinement, the computational efficiency is significantly improved. In comparison to other subgridding FDTD methods, one of the major advantages of our method is the stability property of the numerical methods applied to the anisotropic Maxwell equations, while other subgridding FDTD methods often suffer from the late-time instability problems. In this talk, we will discuss the recent progress in the TO based numerical algorithm for solving Maxwell equations and its applications.

Conservative Finite-Difference Scheme for Computer Simulation of the Field Optical Bistability in Semiconductor

Maria Loginova
Lomonosov Moscow State University, Russia
Vyacheslav A. Trofimov, Vladimir A. Egorenkov

We consider femtosecond laser pulse interaction with 2D semiconductor under the condition of field optical bistability occurrence. This is very promising phenomenon for the creation and developing of all-optical data processing. The optical bistability is based on semiconductor absorption coefficient dependence from strength of an electric field induced by a laser radiation. The laser pulse interaction with a semiconductor is described by the set of nonlinear differential equations concerning the electric field potential, and a free electron concentration, and ionized donor concentration. Laser pulse propagation in the semiconductor is described by either a nonlinear equation with respect to its intensity or the nonlinear Schrodinger equation. For a set of these equations we develop conservative finite-difference scheme. This finite-difference scheme is nonlinear one. To realize it we propose an original two-step iteration process, which is also conservative one on each iteration and allows to provide a calculation on long time interval with high accuracy. Therefore, our finite-difference scheme possesses an asymptotic stability property. This is very important feature because we have to do a calculation during very long time interval. An approach for the nonlinear finite-difference scheme realization can be easy generalized for multidimensional problem.

Numerical Study of Magneto-Thermal Convection of Ferromagnetic Fluids in the Presence of Hall Currents in a Porous Medium

Veena Sharma
Himachal Pradesh University, India
Anukampa Thakur, Urvashi Gupta

In this study, the effect of Hall currents on the onset of convection in a porous medium layer saturated by an electrically conducting ferromagnetic fluid heated from below using linear stability analysis is investigated. Darcy law for the ferromagnetic fluid is used to model the momentum equations for a porous medium. The employed model incorporates the effects of polarization force and body couple. Ferromagnetic fluid behaves as a homogenous continuum and exhibits a variety of interesting phenomenon. Hall currents are likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. The coupled partial differential equations governing the physical problem are reduced to a set of ordinary differential equations using normal mode technique. These equations are solved analytically for stress free boundaries and numerically using the software Mathematica for rigid-free boundaries by obtaining approximate solutions using Galerkin method. For the case of stationary convection, it is found that the magnetic field and magnetization have a stabilizing effect as such their effect is to postpone the onset of thermal instability; whereas Hall currents are found to hasten the same. The medium permeability hastens the onset of convection under certain conditions.

Split-Step Compact Finite Difference Method for Cubic-Quintic Complex Ginzburg-Landau Equations

Shanshan Wang
Nanjing University of Aeronautics and Astronautics, Peoples Rep of China
Luming Zhang

Split-step compact finite difference method is proposed for the cubic-quintic complex Ginzburg-Landau (CQ CGL) equations both in one dimension and in multi-dimensions. The orginal CQ CGL equations are separated into two nonlinear subproblems and one or several linear ones by the split-step method. The linear subproblems are solved by the compact finite difference schemes. As the nonlinear subproblems cannot be solved exactly as usual, the Runge-Kutta method is applied and the total accuracy order is not reduced. Extensive numerical experiments are carried out to examine the performance of this method for nonlinear Schrödinger equations, the cubic complex Ginzburg-Landau equation, and the CQ CGL equations.

Split Bregman Method for Minimization of Convex Image Segmentation Model Based on Local and Global Intensity Fitting Energy

Yunyun Yang
Harbin Institute of Technology Shenzhen Graduate School, Peoples Rep of China
Yi Zhao, Boying Wu

In this paper we apply the split Bregman method to minimize our proposed convex image segmentation model based on local and global intensity fitting energy, which can be used to segment more general images more accurately. We emphasize several important theoretical results including the convergence result of our proposed model and present proofs for them. We also show several experimental results to demonstrate the segmentation accuracy and efficiency of the proposed model.

The Numerical Method for Time-Fractional Convection-Diffusion Problems with High-Order Accuracy

Wenjuan Yao
Harbin Institute of Technology, Peoples Rep of China
Boying Wu, Jiebao Sun, Jia Li

In this paper, we consider the numerical method for solving the two-dimensional fractional convection-diffusion equation with a time fractional derivative of order α $(1 < \alpha < 2)$. By combining the compact difference approach for spatial discretization and the alternating direction implicit (ADI) method in the time stepping, a compact ADI scheme is proposed. The unconditional stability and H^1 norm convergence of the scheme are proved rigorously. The convergence order is $O(\tau^{3-\alpha} + h_1^4 + h_2^4)$, where τ is the temporal grid size and h_1, h_2 are spatial grid sizes in the x and y directions, respectively. It is proved that the method can attain $(1 + \alpha)$ order accuracy in temporal for some special cases. Numerical results are presented to demonstrate the effectiveness of theoretical analysis.

Contributed Session 6: Bifurcation and Chaotic Dynamics

Flip Bifurcation for P-Periodic Maps

Muna Abu Alhalawa
Birzeit University, Israel
Henrique Oliveira

In this talk we extend the theory of period doubling bifurcation in one dimensional autonomous maps to one dimensional non-autonomous periodic maps. We deduce the invariance of the bifurcation equations, including the degeneracy, non-degeneracy and transversality conditions under cyclic permutations of composition of maps.
Therefore, the results in this work establish that the definitions of bifurcation with eigenvalue -1, i.e., the flip D_1 and degenerate flip bifurcations with codimensions two and μ, D_2 and D_μ resp., with μ natural, in one dimensional p-periodic non-autonomous iteration of families of maps are well posed. We prove that the bifurcation equations of the double composition of the maps defining the p-periodic non-autonomous maps are invariant, thus proving the conditions on the flip bifurcations.
We also give a catalog and some examples of the occurrence of the degenerate type of bifurcations in periodic non-autonomous maps with codimension 2 .
The new results extend the invariance of the bifurcation conditions of the bifurcations with eigenvalue $+1$, i.e., the A_μ class in Arnold's classification, in non-autonomous p-periodic maps generated by parameter depending families with p maps. It is proved that the conditions of degeneracy, non-degeneracy and transversality are invariant relative to cyclic order of compositions for any natural number μ.

Mode-Matching Approach for Acoustic Scattering in Flexible Walled Bifurcated Waveguides

Muhammad Afzal
Capital University of Science and Technology, Islamabad, Pakistan

This paper deals with the propagation and scattering of acoustic wave through a bifurcated flexible channel and discontinuity. The orthogonal and non-orthogonal duct modes across the interface are matched via the continuity of pressure and normal velocities. It enables to determine the amplitudes of scattering duct modes. However, there appear some oscillations in normal velocities which can be removed by using the Lanczos filters to extract the required useful information. Further, the accuracy of algebra can be seen through the conserve power identity , and that is also useful to insights the problem physically. It is interesting to note that the fundamental duct mode incident, which contains the characteristics of rigidly bounded duct, is scattered through the bifurcation of flexible channel and the discontinuity of structure.

Probability Density Functions of Stochastic Dynamics and The Path Integration Methods

Linghua Chen
Norwegian University of Science and Technology, Norway
Espen Robstad Jakobsen, Arvid Naess

In this talk we present new results for Fokker-Planck equations with unbounded coefficients: solvability and the strong convergence of numerical schemes in L1 space. The solution of such equations corresponds to the evolution of probability density functions of stochastic differential equations (SDE). The latter has a large number of applications in various areas: including physics, economics, and finance. Existence and uniqueness of a mild solution is derived. On the numerical side, we study the so-called discrete path integration method which produces approximate probability density functions for the solutions of corresponding SDEs. We prove that this scheme strongly converges in L1 norm, uniformly in any finite time horizon. Specifically, we use the concept of dissipative operators, combined techniques from semigroup and PDE theory, as well as methods from stochastic analysis.
This is a joint work with Arvid Naess and Espen Robstad Jakobsen.

Chaotic Behaviours of the Shift Map on the Generalized M-Symbol Space and Their Topological Conjugacy

Tarini Dutta
Gauhati University, India

This paper deals with some illuminating results in connection with various chaotic behaviors of the forward shift map σ^+ on the generalized one-sided symbol space
$\sum_m^+, m(\geq 2) \in N$. We prove that

(i) the shift map σ^+ on $\sum_m^+$ is Devaney chaotic, exact Devaney Chaotic, mixing Devaney Chaotic, weak mixing Devaney Chaotic, Auslander-Yorke Chaotic and generically δ-Chaotic;
(ii) σ^+ on $\sum_m^+$ is topologically conjugate to the map $f_m(x) = mx(mod 1)$ on the circle S^1;
(iii) some characteristic features of the space $\sum_m^+$, viz, perfectness, connectedness, etc. are fruitfully investigated and
(iv) a few applications and open problems on period-doubling bifurcation as well as Hopf-bifurcation are demonstrated for further research work.

Bifurcation Analysis for a Billiard Problem in Nonlinear and Nonequilibrium Systems

Tomoyuki Miyaji
Meiji University, Japan

We study a four-dimensional dynamical system defined by a system of ordinary differential equations. It is a mathematical model of self-propelled motions of a camphor disk floating on a water surface. A camphor disk moves as if it is a billiard ball: it repeats a uniform motion and reflection. There are some differences from the mathematical billiards in which a particle exhibits a uniform motion and a completely elastic reflection. A camphor disk changes the direction of motion without collision at the boundary and the reflection is not completely elastic. Moreover, when the domain is square-shaped, orbits of the mathematical model eventually tends to a limit cycle as time passes. In this study, we consider behavior of the system in a rectangular domain. It is known by numerical study that there exists an attractor, which may be periodic, quasi-periodic, or chaotic depending on the aspect ratio of the rectangle. We apply methods of dynamical systems and bifurcation theory for understanding bifurcations of the system. We reveal that the Hopf-Hopf bifurcation of the rest state is the organizing center of a complicated bifurcation structure of the system.

Poster Session

Mathematical Modeling of HER2 Signaling Pathway: Implications for Breast Cancer Therapy

Sameed Ahmed
University of South Carolina, USA
Xinfeng Liu, Hexin Chen

The cancer stem cell hypothesis states that there is a small subset of tumor cells, called cancer stem cells (CSCs), that are responsible for the proliferation and resistance to therapy of tumors. CSCs have the ability to self renew and differentiate to form the nontumorigenic cells found in tumors. Overexpression of human epidermal growth factor receptor 2 (HER2) plays a role in regulation of CSC population in breast cancer. Current cancer therapy includes drugs that block HER2, however, patients can develop anti-HER2 drug resistance. Downstream of HER2 is nuclear factor κB (NFκB). The aberrant regulation of NFκB leads to cancer growth, which makes it a promising target for cancer therapy, especially for those who have developed resistance to anti-HER2 treatment. Our collaborator's lab has discovered that IL1, which is downstream of HER2, is responsible for NFκB activation, thus making it a potential target for cancer treatment. We have developed a mathematical model to represent this new signaling pathway, and simulations of the model match the some of the results from the lab. We will use the mathematical model to make predictions for different scenarios, and it will be updated and expanded based upon new experiments.

A Basis for Improving Numerical Weather Prediction in the Gulf Area by Assimilating Doppler Radar Radial Winds

Mohamed-Naim Anwar
United Arab Emirates University, United Arab Emirates
F.A. Rihan, C.G. Collier

Numerical Weather Prediction (NWP) is considered as an initial-boundary value problem: given an estimate of the present state of the atmosphere, the model simulates (forecasts) its evolution. Specification of proper initial conditions and boundary conditions for numerical dynamical models is essential in order to have a well-posed problem and subsequently a good forecast model (A well-posed initial/boundary problem has a unique solution that depends continuously on the initial/boundary conditions). The goal of data assimilation is to construct the best possible initial and boundary conditions, known as the analysis, from which to integrate the NWP model forward in time. In this paper, we describe an approach to assimilate Doppler radar radial winds into a high resolution NWP model using 3D-Var system. We discuss the types of errors that occur in radar radial winds. Some related problems such as nonlinearity and sensitivity of the forecast to possible small errors in initial conditions, random observation errors, and the background states are also considered. The technique can be used to improve the model forecasts, in the Gulf area, at the local scale and under high aerosol (dust/sand/pollution) conditions.

Continuous Simulation of the Dynamical Forces Acting on Cervical Spine

Murat Bakirci
Old Dominion University, USA
Ashley Lara

Cervical syndrome is a range of symptoms which are due to the degenerative changes of the cervical spine, and cervical muscles. It is quite necessary to consider the strength of the cervical muscles because some people with weak muscles in their neck region suffer from cervical syndrome. In particular, when riding in a car for extended periods of time, they suffer awful headache attacks, because their head vibrates (oscillates) with the vibrations of the car on the road, since their head is not attached stiffly enough to the shoulders. To a first approximation, a human body could be modeled as a translational mechanical system and so it can be considered as a combination of different sub-parts. When dealing with the relative positions of the human body parts or relationship between those parts such as anterior, proximal, or distal movement, a translational mechanical system approximation is sufficient enough to model the entire system. In this study, dynamical forces acting on cervical spine in response to the vertical movement of human body has been investigated. The extended model is obtained by applying Newton's law to each body parts and the obtained set of differential equations has been solved numerically.

Simulation of the Electroosmotic Flow Induced by AC Electric Field in a 2D Rectangular Micropore

Murat Bakirci
Old Dominion University, USA
George Pratt

Ions and molecules passing through a micropore must interact with the pore walls. Often the pore walls have a non-zero surface charge density which affects ionic concentrations within the pore. This exposes a mathematical difficulty that is usually treated with numerical techniques. A charged surface in contact with an ionic solution attracts ions of opposite charge (counterions) and repel ions of like charge (coions), and form electric double layer (EDL) in the vicinity of the charged surface. Within the EDL, the concentration of the counterions is much higher than that of coions. Hence, the EDL plays an important role in ionic mass transport in micropores such as a negatively charged conical micropore has been shown to transport positive ions with higher selectivity which means that most of the ionic current through the micropore is carried by positive ions. In this research, AC electroosmotic flow through a micropore under the effect of discrete surface potential has been investigated. An electrokinetic model that combines the Poisson-Boltzmann distribution for ions with Stokes equation for the fluid flow in the absence of a hydrostatic pressure gradient has been developed and numerically solved for the given computational domain.

Dynamical Analysis of Connected Neuronal Motifs with OpenAcc and OpenMPI

Sunitha Basodi
Georgia State University, USA
Krishna Pusuluri, Andrey Shilnikov

Large scale analysis of the dynamical behavior of Central Pattern Generators (CPGs) formed by neuronal networks of even small sizes is computationally intensive and grows exponentially with network size. We have developed a suite of tools to exhaustively study the behavior of such networks on modern GPGPU accelerators using the directive based approach of OpenAcc. We also achieve parallelization across clusters of such machines using OpenMPI. Directive based approaches simplify the task of porting serial code onto GPUs, without the necessity for expertise in lower level approaches to GPU programming, such as CUDA and OpenCL. 3-cell neuronal CPGs have been explored previously using various GPGPU tools [1]. As motifs form the building blocks of larger networks, we have employed our framework to study 4-cell CPGS and two connected 3-cell motifs. We discuss the performance improvements achieved using this framework and present some of our results.

The Impact of Resource Abundance on Pathogen Invasion Risk

Rebecca Borchering
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J.M. Flynn, S.E. Bellan, J.R.C. Pulliam, S.A. McKinley

Territorial animals share a variety of common resources, which can be a major driver of conspecific encounter rates. We investigate how changes in resource density influence the rate of encounters between individuals in a population. We develop a model of resource selection by consumers on a spatial resource landscape and estimate changes in encounter rates as a function of resource availability. Using simulations and asymptotic analysis we show that the relationship between resource availability and consumer encounter rate is nonmontonic. We also find that the maximum distance at which consumers are able to detect resources greatly influences the expected consumer encounter rate. We discuss these theoretical results in the context of a jackal population which has access to a seasonally varying number of carcasses and their subsequent vulnerability to rabies virus outbreaks.

Dynamics of Target Mediated Drug Disposition in PKPD

Jonghyuk Byun
Busan National Univerisity, Korea
Il Hyo Jung

Target Mediated drug disposition (TMDD) is a model which concerns the relation between a drug and its target (receptor) in vivo or vitro. TMDD predicts Pharmacokinetics & Pharmacodynamics (PKPD) behavior of various drugs such as monoclonal antibodies(mAbs). We introduce two compartment TMDD model and simplify the TMDD with quasi equilibrium, quasi steady state or Michaleis-Menton models. Furthermore, we also present strong and weak points of those models and how parameters affect the concentration of drug, receptor and drug-receptor complex in TMDD.

A Mathematical Analysis on the Transmission Dynamics of Neisseria Gonorrhoeae

Christine Craib
University of North Carolina at Wilmington, USA

In this project, we analyze an epidemiological model describing the transmission of gonorrhea, with a core sexual activity class and a noncore sexual activity class. We discuss the behavior of the model around the two equilibrium points, a disease-free equilibrium and a coexistence equilibrium. The focus of the project is to identify equilibrium points, analyze the stability of these points, and discuss the results in terms of the epidemiological model. Ultimately, the goal of the project is to find conditions of an endemic state, and the conditions that ensure the eradication of gonorrhea.

Interaction of Scroll Waves in an Excitable Medium

Nirmali Prabha Das
Indian Institute of Technology Guwahati, India
Sumana Dutta

Spiral waves occur in systems ranging from biology to astrophysics, fluids to superconductors. Scroll waves are the three dimensional counterparts of spiral waves that rotate around a one dimensional phase singularity, known as filament. Their presence in the cardiac tissues is many times the cause arrythmia that finally lead to heart failure. So the interaction of scroll waves may have far-reaching consequences on cardiac activity. In fluids and liquid crystals, there is evidence of vortex interaction leading to interesting phenomena like filament reconnection. If likewise, scroll rings interact and reconnect, then small rings may merge and form large ones that will have enhanced life-times. If this happens in heart tissues, it will ensure a long life of the filaments which in turn will have a detrimental effect on cardiac health. The work reported here is motivated by these concerns. Here, we report the first experimental evidence of scroll wave reconnection. Our results demonstrate that when two scroll rings are brought close enough, they can either attract each other, and reconnect to form a large scroll ring, or they can repel so that they rupture on touching the boundaries. We also carry out simple numerical simulations that helps explain the filament behavior in our experiments.

New a Priori Estimates for Mean-Field Games with Congestion

David Evangelista da Silveira Junior
KAUST, Saudi Arabia
Diogo Gomes

We present recent developments in crowd dynamics models (e.g. pedestrian flow problems). Our formulation is given by a mean-field game with congestion. We start by reviewing earlier models and results. Next, we develop our model. We establish new a priori estimates that give partial regularity of the solutions. Finally, we discuss numerical results.

Relativistic Compact Objects with a Linear Equation of State

Megandhren Govender
Durban University of Technology, So Africa

In this work we present a general framework for generating exact solutions to the Einstein field equations for static, anisotropic fluid spheres in comoving, isotropic coordinates obeying a linear equation of state of the form $p_r = \alpha\rho - \beta$. We show that all possible solutions can be obtained via a single generating function defined in terms of one of the gravitational potentials. The physical viability of our solution-generating method is illustrated by modeling a static fluid sphere describing a strange star.

Stability and Uniqueness of Slowly Oscillating Periodic Solutions to Wright's Equation

Jonathan Jaquette
Rutgers University, USA
Jean-Philippe Lessard, Konstantin Mischaikow

Jones' conjecture states that when $\alpha > \pi$, then there exists an unique slowly oscillating periodic solution (SOPS) to Wright's equation: $y'(t) = -\alpha y(t-1)[1+y(t)]$. While Jones proved the existence of SOPS in 1962, the conjecture has yet to be proven in whole. To prove uniqueness, it suffices to show that all SOPS to Wright's equation are asymptotically stable. By developing asymptotic bounds on the behavior of SOPS and then estimating their Floquet multipliers, in 1991 Xie was able to prove the conjecture for $\alpha > 5.67$. We sharpen Xie's result by using rigorous numerics to estimate the Floquet multipliers of SOPS to Wright's equation. In this manner we are able to make progress on a long standing conjecture in delay differential equations.

Interaction Energy Between Colloidal Particles

Wai-Ting Lam
Yeshiva University, USA

One of the central problems on colloids is their stability. That is, whether under known conditions (such as concentration) the system will coagulate or remain indefinitely stable.
Mathematically, the stability depends on the details of the pairwise energy as a function of separation of colloidal particles. One approach to obtain that energy is to solve Poisson-Boltzmann equation (PB), which gives the charge density and electrostatic potential of the solution surrounding the colloidal particles. PB is a differential equation that has an exact known solution only for one-dimensional geometries. In the case of general interest, the problem is three-dimensional and not amenable to analytical solutions. In this paper we solve the PB equation approximately via variational methods. We introduce the density and corresponding electrostatic potential parametrically and minimize the PB functional with respect to the parameters. We also find useful polynomial approximations for the parameters as functions of separation and boundary conditions. Finally, we produce energy-separation curves and conclude with the stability properties predicted by this approach.

Some Numerical Aspects on Crowd Motion - the Hughes Model

Roberto Machado Velho
KAUST, Saudi Arabia
Diogo A. Gomes

We study the Hughes model for crowd motion and we present a numerical approach to solve it.
The model comprises a Fokker-Planck equation coupled with an Eikonal with Dirichlet or Neumann data:

$$\begin{cases} \rho_t(x,t) - \operatorname{div}(\rho(1-\rho)^2 Du) = \Delta\rho, \\ |Du(x)|^2 = \frac{1}{(1-\rho)^2}. \end{cases}$$

The Fokker-Planck equation gives the evolution of the crowd density ρ. The Eikonal equation determines the optimal direction of movement.
We establish a priori estimates for the solution and identify a shock formation mechanism. We illustrate the existence of congestion, the breakdown of the model, and trend to equilibrium.

Our proposed numerical method explores the adjoint structure in this system and we illustrate through some examples. The method is valid in arbitrary dimension, conserves of mass and positivity.

On the Existence of Limit Cycles in a 3D Piecewise Linear Van Der Pol Like Memristor Oscillator

Marcelo Messias
Sao Paulo State University - UNESP, Brazil
Anderson Luiz Maciel

In this work we study a mathematical model for a Van der Pol like oscillator formed by three fundamental electronic elements: one memristor, one capacitor and one inductor. The model is given by a discontinuous piecewise linear system of ordinary differential equations, defined on three zones in $\mathbb{R}^3$, given by $|z| \leq 1$, the internal zone, and $|z| \geq 1$, the external zones. We show that the z–axis is filled with equilibrium points of the system and analyze the linear stability of the equilibria in each zone. We prove that, due to the existence of this line of equilibria, the phase space is foliated by parallel invariant planes transversal to the z–axis, so the dynamics is essentially two-dimensional and no chaotic behavior may occur. We also determine numerically the occurrence of nonlinear oscillations, given by the existence of limit cycles belonging to the invariant planes and passing by two of the three zones or passing by the three zones. These periodic orbits arise due to homoclinic and heteroclinic bifurcations, obtained varying one of the parameters of the studied model. The analytical and numerical results obtained extend the analysis presented in the literature

Dynamics Analysis of a Inflammatory Cytokines Model with State Dependent Impulsive Effect in Autoimmune Disease

Anna Park
Pusan National University, Korea
Il Hyo Jung

An autoimmune disease is a chronic inflammatory disease and triggered by abnormal immune response. It occurs when the immune system attacks its own tissue and consequently destroys and damages tissue. This is related to a break down in immune tolerance. An autoimmune disease is characterized by the breakdown of tolerance of the pro-inflammatory cytokine and anti-inflammatory cytokine. Our propose is to restore the immune tolerance by using control strategy with state dependent impulsive effect. We present a two-variable model for the interactions between pro-inflammatory cytokine and anti-inflammatory cytokine. Our result illustrate how feasibility condition protects pro-inflammatory cytokine increase. Numerical simulation imply that the presence of pulses makes the dynamic behavior more complex.

On the Computation of Expansion Entropy

Eric Roberts
University of California, Merced, USA

Despite the rich history in the field of nonlinear dynamics and chaos, an exact definition for the term chaos is still very much up for debate. In this poster, we investigate a recently proposed entropy-based definition of chaos which measures the difference between the exponential growth rate of volume expansion in an arbitrary region with the exponential growth rate at which volume escapes. Implementation of this method could have a large impact in wide array of applications, including parameter control for chaos detection. We advocate for this new, broadly applicable, more computationally feasible measure for complexity and explore certain numerical computational aspects of this expansion entropy for discrete mappings. Namely, by way of calculating the singular values of the iterated Jacobian mapping, we use expansion entropy to help extend the concepts of turnstile and transport into 3D volume preserving maps.

An Analytic Approach to the Design and Payback Time of PV Plants

Angela Sciammetta
Università degli Studi di Messina, Italy
Gabriele Bonanno, Salvatore De Caro, Tommaso Scimone, Antonio Testa

Grid connected photovoltaic plants for residential applications are normally designed through deterministic methods on the basis of system requirements as: PV array peak power, available budget, or available space. Unfortunately, these approaches give only suboptimal results. More effective design techniques are those oriented to fulfill given performance targets as minimum pay-back time, minimum life cycle cost, or minimum annual electrical energy cost. However, they require very complex optimization techniques, especially if the plant encompasses energy storage systems. An analytic approach is presented to optimally sizing the photovoltaic module array and the energy storage system in a grid-connected generator serving a set of residential loads, on the basis of annual average solar radiation data and load demand. The configuration with the shorter pay-back time is identified through a mathematical model, describing the variation of the annual energy cost as a function of the plant configuration. The proposed method can be easily modified to take into account different optimization targets, as maximum life-cycle profit.

Numerical Investigation of the Hierarchical Stability of the Caledonian Symmetric Four Body Problem

Anoop Sivasankaran
Khalifa University of Science Technology and Research, United Arab Emirates
Muhammad Shoaib

The Caledonian Symmetric Four Body Problem (CSFBP) is a restricted four body problem developed by Steves and Roy (1998) with a symmetrically reduced phase space which can be applied to study the stability and evolution of symmetric quadruple stellar clusters and exo-planetary systems. Sivasankaran, Steves and Sweatman (2010) developed a global regularization scheme for CSFBP that consists of adapted versions of several known regularisation transformations such as the Levi-Civita-type coordinate transformations; that together with a time transformation, removes all the singularities due to colliding pairs of masses. Using this newly developed numerical algorithm, we numerically investigate the relationship between the hierarchical stability of the system and the analytical stability parameter characterised by the Szebehely constant C_0, which is a function of the total energy and angular momentum of the system. We also show that for $C_0 \geq 0.046$ all the CSFBP systems will be hierarchically stable for all time.

Generalized Ordinary Differential Equations and Measure Functional Differential Equations

Patricia Tacuri
Sao Paulo State University, Brazil
M. Federson, J. Mesquita, M. Frasson

We introduce the equations called neutral measure functional differential equations and we prove that these equations can be also related with a class of abstract generalized ordinary differential equations (GODEs). Then, using these correspondence, we are able to prove existence and uniqueness of solutions and continuous dependence results for neutral measure functional differential equations.

Bayesian Logistic Modeling of Disease Dispersal with Partial Differential Equation

Kim Taekyoung
Chonnam National University, Korea
Taekyoung Kim and Ilsu Choi

Bayesian inference provide good parameter estimates, parsimonious descriptions of observed data, predictions for missing data and forecast of future data, a computational structure for model estimation selection and validation. Partial diffrential equation models give describing disease dispersal with population process and have been used extensively to evaluate the effects of spatial variation on populations. The transmisson is focused in disease prevalence and control with new kinds of virus and bacteria. We consider basic finite difference methods and spatial-temporal Bayesian logistic models for spatial-temporal binary data which are observed on visiting hospital and apply the methods with an example of outcomes of Noro-virus on Korea peninsula. We discuss dispersal, disease invasions, and diffusion spatial pattern as well as interaction between space and time.

Symbolic Computation and Abundant Traveling Wave Solutions to Modified Burgers' Equation

Muhammad Younis
University of the Punjab, Pakistan
M. Younis, S.T.R.Rizvi, A. Sardar

In this article, the novel G'/G expansion scheme is successfully applied to construct the abundant travelling wave solutions to the modified Burgers equation with the aid of symbolic computation. The applied scheme is reliable and useful which gives more and general exact travelling wave solutions than the other existing schemes. These obtained solutions are in the form of hyperbolic, trigonometric and rational functions including solitary, singular and periodic solutions as well, which have many other potential and useful applications in physical science and engineering. Most of these solutions are new and some have already been constructed using different techniques. Additionally, the constraint conditions, for the existence of the solutions are also listed.

Student Paper Competition Session

Picone's Identity for P-Biharmonic Operator and Its Applications

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In this article we prove the nonlinear analogue of Picone's identity for p-biharmonic operator. As an application of our result, we show that the Morse index of the zero solution to a p-biharmonic boundary value problem is 0. We also prove a Hardy type inequality and Sturmian comparison principle. We also show the strict monotonicity of the principle eigenvalue and linear relationship between the solutions of a system of singular p-biharmonic system. In the end, we establish a Caccioppoli type inequality for p-biharmonic operator.

A Mathematical Model of the Human Papillomavirus (HPV) with a Case Study in Japan

Arielle Gaudiello
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The human papillomavirus (HPV) is a sexually transmitted infection prominent among young adults in most countries. The disease typically clears in two years naturally, but causes increased risk of development of cancerous cells post-infection. We develop an ordinary differential equation model including vaccination of the female classes and investigate existence and stability of the disease-free equilibrium and endemic equilibrium. We also discuss applications to recent issues arising in Japan, where government-based vaccination programs have terminated and vaccination has drastically decreased.

Inertial Manifolds for Semilinear Parabolic Equations Which Do Not Satisfy the Spectral Gap Condition

Anna Kostianko
University of Surrey, England

We report on resent results concerning the existence of inertial manifolds for semilinear parabolic equations which do not satisfy the spectral gap conditions. The list of problems includes the 3D Cahn-Hilliard problem with periodic boundary conditions, the so-called modified Leray-α model which regularizes the 3D Navier-Stokes equations with periodic boundary conditions and 1D reaction-diffusion advection problems.

On a Homogeneous Evolution Equation: Lax Pair and Peakon Solutions

Priscila Leal da Silva
Universidade Federal do ABC, Brazil

In this talk we discuss integrability and peakon solutions of a family of homogeneous evolution equations. Recursion operators are obtained and two members related to KdV-type equations are shown to be completely integrable using a Lax representation and the existence of an infinite number of conserved quantities. Conditions for the existence of peakon solutions are then given.

Dark Soliton Linearization of the 1D Gross-Pitaevskii Equation: Long-Time Dynamics

Numann Malik
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Consider the nonlinear Schrödinger equation

$$i\partial_t u(t,x) + \partial_x^2 u(t,x) - 2u(t,x)(|u(t,x)|^2 - 1) = 0$$

in spatial dimension one, subject to the non-vanishing boundary conditions $|u(t,x)| \to 1$ as $|x| \to \infty$. Linearizing around the black soliton $q(x) = \tanh(x)$ yields an evolution equation $i\partial_t \vec{w} = \mathcal{L}\vec{w} + O(w^2)$ where $w = w(t,x)$ is a complex valued perturbation and

$$\mathcal{L} = \begin{bmatrix} -\partial_x^2 + 2 & 2 \\ -2 & \partial_x^2 - 2 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} \operatorname{sech}^2(x).$$

We describe the long-time dynamics by carrying out a spectral out a spectral and Fourier analysis of $\mathcal{L}$ to derive a formula for the propagator $e^{-it\mathcal{L}}$ using the resolvent kernel. This is performed via a natural distorted Fourier transform obtained from explicit squared Jost solutions f to $\mathcal{L}f = Ef$.

Global Existence and Asymptotic Stability of Solutions to a Two-Species Chemotaxis System

Masaaki Mizukami
Tokyo University of Science, Japan

Nowadays, there are a lot of lives in the high civilization developed by mathematics, physics, chemistry, biology, and so on. One of the things which play an important role in their lives is chemotaxis. Chemotaxis is the directed movement in response to a chemoattractant. This talk is concerned with a two-species chemotaxis system with logistic source. This system describes a situation in which multi populations react on single chemoattractant. The main result asserts boundedness of solutions and asymptotic

stability for the system with "any" chemical diffusion. This result improves the previous results for "slow" or "non"-diffusive chemoattractant by Negreanu and Tello in 2014, 2015. This work will be the first step to study a two-species chemotaxis system with Lotka-Volterra competitive interaction.

Slow Motion for the One-Dimensional Swift-Hohenberg Equation

Matteo Rinaldi
Carnegie Mellon University, USA

The behavior of solutions of the Swift–Hohenberg equation in a bounded interval $I \subset \mathbb{R}$ with periodic boundary conditions is studied. Combining results from Γ–convergence and ODE theory it is shown that solutions that start L^1–close to a jump function v, remain close to v. This can be achieved by regarding the equation as the L^2–gradient flow of a given energy functional and studying the asymptotic behavior of solutions of its Euler–Lagrange equation. The linearization of such equation provides almost sharp estimates on the tail of the associated energy.

Reaction-Diffusion Approximation for Understanding Pattern Formations Through Non-Local Interactions

Yoshitaro Tanaka
Meiji University, Japan

Motivated by the problem of studying the relationship between non-local interaction and local dynamics in pattern formation, we analyze the following mathematical model with the Mexican-hat interaction:

$$\begin{cases} u_t = d_u u_{xx} + (J * u)u + f(u), & \text{in } \mathbb{T} \times \{t > 0\}, \\ u(x,0) = u_0(x), & \text{on } \mathbb{T}, \end{cases}$$

where $u(x,t)$ is the theoretical concentration at the position x at time t, $\mathcal{T} = [-L, L]$ with periodic conditions, $J \in L^1(\mathbb{T})$ is a kernel satisfying $\int_{\mathcal{T}} J(x)\,dx = 0$, f is a C^1 function from $\mathbb{R}$ to $\mathbb{R}$, and $d_u > 0$ is a diffusion coefficient.

Unified Weighted Poincaré Inequalities in Metric Measure Space and Applications

Huiju Wang
Northwestern Polytechnical University, China

In this paper we establish unified weighted Poincaré inequalities in metric measure spaces. As applications, we obtain weighted higher order Poincaré inequalities in the Euclidean space and stratified Lie groups, respectively, in which a new class of higher order Poincaré inequalities in the Euclidean space is given.

Global Wellposedness of Cubic Camassa-Holm Equations

Qingtian Zhang
Penn State University, USA

Cubic Camassa-Holm equation is a model for shallow water dynamic. I will discuss the Cauchy problem of cubic Camassa-Holm equation. We prove the global existence of entropy weak solution to this problem in space H^1 with its derivative in BV. The stability and uniqueness of entropy weak solution are obtained in $W^{1,1}$.

List of Contributors