Special Session 29: Dynamics of forced oscillators

Rafael Ortega, Universidad de Granada, Spain

On some forced oscillators at resonance

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We discuss the behaviour at resonance of a class of forced oscillators that includes for example oscillators driven by an asymmetric or a singular restoring force. We consider in particular the existence of periodic and unbounded solutions.

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Fucik Spectrum for nonautonomous periodic equations

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The main of the talk is to describe some new properties of the set

$$\left\{ (a,b) \in \mathbb{R}^2 \Big/ \begin{array}{c} u'' + p(t)(au^+ - bu^-) = 0 \text{ has a} \\ \text{nontrivial 1-periodic solution} \end{array} \right\}$$

where $p : \mathbb{R} \to \mathbb{R}$ is a continuous and 1-periodic function. We will show also some connections between the corresponding Dirichlet and Neumann problems.

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Silnikov Chaos in the Semiconductor Laser Equations

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A free-running semiconductor laser is an electronic device the behavior of which is described by a system of two first-order ordinary differential equations. The light from such a laser can be modulated to transmit data over large telecommunication networks. The high bit rate involved in such applications requires the laser to be modulated at very high speeds, a process which introduces noise in the generated signal. Some techniques for suppressing the unwanted noise involve an injection process, whereby light from another laser with comparable frequency is injected into the cavity of the modulated laser. However, a laser subject to injection is governed by a system of three first order ordinary differential equations, no longer two. The extra dimension accounts for a range of complex phenomena observed in injected semiconductor laser devices. These phenomena include Silnikov chaos, which makes its presence felt as the device is made to lase in a neighborhood of a Silnikov trajectory in the phase space of the rate equations. I will present an analytical method for detecting the presence of these Silnikov homoclinic orbits. This is joint work with C. K. R. T. Jones.

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Existence of periodic solutions for enzyme-catalyzed reactions with periodic substrate input

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We consider a basic enzyme-catalyzed reaction in which the rate of input of the substrate varies periodically in time. Such a situation often arises in biological systems. We prove a necessary and sufficient condition for the existence of a periodic solution of the reaction equations. The proof uses Leray-Schauder degree theory.

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Subharmonic bifurcations from infinity

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We consider periodic trajectories with norms tending to infinity for discrete time systems $x_{k+1} = U(x_k; \lambda)$, $x \in \mathbb{R}^N$, with a complex parameter λ. The expansion of the map $U(\cdot;\lambda)$ at infinity is assumed to contain a principal linear term, a bounded positively homogeneous nonlinear term, and a smaller part. We describe the sets of parameter values for which the large-amplitude n-periodic trajectories exist. In the related problems on small-amplitude periodic orbits near an equilibrium, similarly defined parameter sets are known as the Arnold tongues. Our main finding is that the Arnold tongues in the problem at infinity are thick triangles. This contrasts to the standard picture associated with the Neimark-Sacker bifurcation of smooth discrete time systems at an equilibrium, where the Arnold tongues have infinitely small angular width except for the strong resonance points. The other shape of the tongues in the problem at infinity is due to the non-polynomial form of the nonlinear term of the map $U(\cdot;\lambda)$, which implies non-degeneracy of the nonlinear terms in the expansion of the map iterations and non-degeneracy of the corresponding resonance functions. Asymptotic estimates for the length of the Arnold tongues are presented. Results obtained are applicable to bifurcation analysis of large-amplitude periodic solutions of the forced oscillator with a linear part near resonance and a nonlinearity with saturation.

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The Stability of equilibrium of quasi-periodic planar Hamiltonian and Reversible Systems

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In this talk, we deal with the stability of zero solutions of planar Hamiltonian and reversible systems which are quasi-periodic in the time variable. Under some reasonable assumptions, the existence of quasi-periodic solutions in a small neighborhood of zero solutions and the stability of zero solutions are proved.

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Invariant curves via the differentiability of the flow of a control system

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R. Ortega and C. Rebelo

We show that the differentiability of the flow of a control system with respect to controls is sufficient to investigate the C^4 proximity between the corresponding Poincaré operator and a twist map. If one is interested only in qualitative results, this permits a direct application of Moser's Small Twist Theorem to get the existence of families of invariant curves. We use this perspective to investigate a stability property of a T-periodic Hamiltonian system in a disk which models a fluid stirring in a cilindrical tank.

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Strange Non-chaotic Attractors in the oscillators dynamics

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Conditions establishing the occurrence of almost automorphic almost periodic dynamics for non-autonomous almost periodically forced oscillators are established. These conditions combine elements coming from the classical method of upper and lower solutions with topological concepts, as exponential dichotomy and disconjugation. The relation between almost-automorphic and not almost periodic dynamics and Strange Non-chaotic Attractors is analyzed. A rigorous proof of the existence of these objects in some cases of non-forced oscillators is presented.

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The Dynamics of Impact Oscillators

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Impact oscillator is the typical model with nonsmoothness duo to the impacts. In this talk, we concern the investigation of the periodic and Aubry-Mather tpye solutions for asymptotically linear impact oscillators. A new coordinate transformation for impact oscillators is performed and the relationship between the rotation numbers and the characteristic values forHill's type impact oscillators is showed.

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Multiplicity of solutions of Dirichlet problems associated to second order equations in $\mathbb{R}^2$

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We are interested on the existence of multiple solutions to the Dirichlet problem

$$\begin{cases} x'' + A(t,x)x = 0,\ x \in \mathbb{R}^2, t \in [0,\pi] \\ x(0) = x(\pi) = 0, \end{cases} \tag{1}$$

where $A : [0,\pi] \times \mathbb{R}^2 \to GL_s(\mathbb{R}^2)$,

$$A(t,x) = \begin{bmatrix} a_{11}(t,x) & a_{12}(t,x) \\ a_{12}(t,x) & a_{22}(t,x) \end{bmatrix},$$

is a continuous function such that uniqueness of solutions of Cauchy problems associated to system (1) is guaranteed. We assume that

$$\lim_{|x|\to 0} A(t,x) = A_0(t) \text{ uniformly in } t \in [0,\pi],$$
$$\lim_{|x|\to \infty} A(t,x) = A_\infty(t) \text{ uniformly in } t \in [0,\pi],$$

and prove that if $a_{11}(t,x) < 0$, $a_{22}(t,x) < 0$ and $a_{12}(t,x) \neq 0\ \forall (t,x) \in [0,\pi] \times \mathbb{R}^2$ then (1) admits at least $2|i(A_0) - i(A_\infty)|$ nontrivial solutions where we denote by $i(A)$ the index of a symmetric matrix A.

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Interaction of normal modes and local bifurcation

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For a class of autonomous systems, the existence of small amplitude oscillations near elliptic equilibria, which mix different normal modes, is considered. The reference problem is the so called cubic Fermi-Pasta-Ulam model. More precisely, an interaction mechanism between modes is described, which explains the arising of secondary bifurcations from the primary unimodal branches of periodic orbits. This mechanism depends on two conditions: one involves only the arithmetic properties of the eigenvalues of the linearized system at the rest, while the other takes into account the nonlinear character of the interaction between normal modes. Both conditions may be checked for the reference problem.

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Homoclinic solutions in a differential equation arising in Nonlinear Optics

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Motivated by the study of the propagation of electromagnetic waves through a multilayered fiber, we prove the existence of two different homoclinic solutions to the origin in the scalar nonlinear differential equation

$$-\ddot{u}(x) + a(x)u(x) = b(x)f(u(x)),$$

where $a, b \in L^\infty(\mathbb{R})$ are non-negative almost everywhere and f is a superlinear nonlinearity. The main assumption is the compactness of the support of b. We use a Krasnoselskii fixed point Theorem on compressing cones together with a compactness criterion due to K. Zima. The main result is illustrated with concrete examples of practical interest in Nonlinear Optics like self-focusing nonlinearities of Kerr and non-Kerr type.

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Invariant manifolds near a minimizer

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A classical result, studied, among others, by Carathéodory [1], states that, for second-order, scalar equations, nondegenerate periodic minimizers are hyperbolic. Consequently, the stable/unstable manifold theorem applies, and implies that, at least locally, the stable and unstable sets are regular curves intersecting transversally at the nondegenerate minimizer.

For analytic equations, there is a version of this fact which holds for isolated, but not necessarily nondegenerate, minimizers.

[1] Carathéodory, C. *Calculus of variations and partial differential equations of the first order.* New York : Chelsea, 1989.

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