Special Session 41: The Navier-Stokes equations and related problems

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The programme of the Session focuses on the recent development in mathematical fluid mechanics. Both compressible and incompressible fluids will be discussed. The emphasis is put on new theoretical results on the qualitative properties of solutions to the above mentioned problems.

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Regularity for the Navier–Stokes Equations with Slip Boundary Condition

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For the Navier-Stokes equations with slip boundary conditions, we obtain the pressure in terms of the velocity. Based on the representation, we consider the relationship in the sense of regularity between the Navier-Stokes equations in the whole space and those in the half space with slip boundary data.

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Stability problems for a spherical model of gaseous star

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We present new results concerning the stabilization of 1D spherical flows for a barotropic model of selfgravitating star.

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Navier's Slip and Evolutionary N-S equations with temperature dependent viscosity

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There are plenty of experimental studies confirming the dependence of the viscosity of a fluid on the temperature. We investigate mathematical properties of internal unsteady three-dimensional flows of such incompressible fluids subject to Navier's slip at the boundary.

Instead of equation for internal energy we consider the equation for global energy. Because the pressure is in the weak formulation of the global energy equation we have to construct the pressure from the beginning. The key-role in this construction will play the Navier's slip at the boundary. We establish the large-data and long-time existence of weak solution provided that the viscosity and heat conductivity are bounded positive functions of the temperature.

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Numerical investigation of cavitation in multi-D compressible flow

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We consider the compressible Navier-Stokes equations in several space dimensions. The flow is assumed to be symmetric about the origin with a purely radial velocity field. We perform a numerical study of the solutions generated by data with an everywhere positive density and a rapidly expanding (discontinuous) initial velocity field. The results indicate that there are weak solutions to the Navier-Stokes system in two and three space dimensions which exhibit cavitation. We compare the results with computations for one-dimensional flow, where it is known that, under the same conditions on the data, there exist a solution without vacuums.

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L^p estimates of stokes and Oseen type problem arising from flow around a rotating body

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Considering time-periodic linear flow around a rotating body in R^3 we prove weighted a priori estimates in L^p spaces for the whole space problem. After a

time-dependent change of coordinates the problem is reduced to a stationary problem with the additional term $(\omega \times x) \cdot \nabla u$ in teh equation of momentum where ω denotes the angular velocity.

In cylindrical coordinates attached to the rotating body we allow for Muckenhoupt weights which may be anisotropic or even depend on the angular variable and we prove weighted L^p estimates using a weighted theory of Littlewood- Paley decomposition and of maximal operators.

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Stability of a steady solution of a quasilinear parabolic system

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The question of stability of a steady solution of a quasilinear parabolic system (particularly also of the Navier–Stokes equation) can be transformed to the question of stability of the zero solution of the equation

$$\frac{du}{dt} = Lu + Nu$$

where L is a linear operator and N is a nonlinear operator in an appropriate function space. We do not assume that operator L is dissipative and in spite of presence of the eseential spectrum which touches the imaginary axis, we formulate sufficient conditions for stability based on the spectral analysis of operator L.

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Singular limits in the full Navier-Stokes-Fourier system

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We investigate low Mach number limits for weak variational solutions to the full Navier-Stokes-Fourier system

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On the long time behaviour of solutions to the Navier-Stokes-Fourier system with a time dependent driving force

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We will study the full system of equations taking into account the second law of thermodynamics and, in particular, the entropy production corresponding to the irreversible transfer of the mechanical energy into heat. We will show that the long time dynamics of energetically isolated (fluid) system is quite simple: either the total energy of the system goes up to infinity with growing time or all solutions tend to a uniquely determined static state.

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Regular solutions to steady compressible Navier–Stokes equations

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We consider the steady compressible Navier–Stokes equations in a bounded regular domain in three space dimensions. We show that for the pressure $p(\rho) \sim \rho^\gamma$, $\gamma > 3$ and the velocity satisfying the slip boundary condition, there exists a solution such that the density $\rho \in L^\infty(\Omega)$ and the velocity $\mathbf{u} \in W^{1,p}(\Omega)$, $p < \infty$. Next we show a similar result also for the full steady compressible Navier–Stokes–Fourier system (i.e. the heat conducting compressible fluid), with the third-type boundary condition for the temperature. The technique is based on the special approximation scheme introduced in: Mucha, P.B., Pokorný, M.: *On new approach to the issue of existence and regularity for the steady compressible Navier–Stokes equations.*

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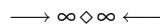
On Lyapunov functionals to the Navier-Stokes equations for compressible flow

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Some Lyapunov functionals for the Navier-Stokes equations of compressible flow will be presented and consequences for the global behavior of solutions will be dis-

cussed.

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Two-equation model of mean flow resonances in sub-critical flow systems

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Amplitude equations of the Landau type, which describe the dynamics of the most amplified periodic disturbance waves in slightly supercritical flow systems, have been known to form reliable and sufficiently accurate low-dimensional models capable of predicting the asymptotic magnitude of saturated perturbations in weakly nonlinear regimes beyond the bifurcation point. However the derivation of similar models capable of predicting the threshold disturbance amplitude (below which the solution returns to its undisturbed state and above which it undergoes the transition to a different nonlinear state) in systems bifurcating subcritically faces multiple resonances which lead to a singularity of model coefficients. The observed resonances are traced back to the interaction between the mean flow distortion induced by the decaying fundamental disturbance harmonic and other decaying disturbance modes. Here we illuminate the reasons why such difficulties are not encountered in supercritical systems and suggest a methodology of deriving a two-equation dynamical system of coupled cubic amplitude equations with non-singular coefficients which resolve the resonances and are capable of predicting the threshold amplitude for weakly nonlinear subcritical regimes and the qualitative features of such flows. The suggested reduction procedure is based on the consistent use of the appropriate orthogonality condition which is different from a conventional solvability condition (i.e. the orthogonality of the forcing term with the adjoint eigenvector). As an example, the developed procedure is applied to a system of the Navier-Stokes equations describing a subcritical plane Poiseuille flow. The developed model predictions are found to be in reasonable agreement with experimentally detected threshhold amplitudes reported in literature.

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On the Long-time Stability of the Implicit Euler Scheme for the 2d Navier-Stokes Equations

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In this talk I will discuss the stability for all positive time of the fully implicit Euler scheme for the 2D Navier-Stokes equations. More precisely, I will discretize the Navier-Stokes equations in time using the implicit Euler scheme and with the aid of a discrete Gronwall lemma and of a discrete uniform Gronwall lemma, I will prove that the numerical scheme is unconditionally stable (uniformly in time).

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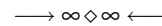
On Phase Transition Dynamics

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A multidimensional model is introduced for the phase transition dynamics of a mixture of compressible fluids. The model presented here can accommodate various physical contexts, namely "liquid - liquid", "gas - liquid" phase equilibria, as well as the phase transition of a mixture of two distinct gases, to product species due to combustion, the evolution of gaseous stars in astrophysics, the phase transition dynamics associated with semiconductors and others. The model is formulated by the Navier-Stokes equations in Euler coordinates, which is now expressed by the conservation of mass, the balance of momentum and entropy and the species conservation equation. These equations take now a new form due to the choice of rather complex constitutive relations which are able to accommodate the physical character of the mixture. The existence of global weak solutions is established by using weak convergence methods, compactness and interpolation arguments in the spirit of Feireisl and P.L. Lions.

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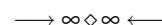
Some recent results on the analysis of fluid-structure interactions

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The aim of this presentation is to highlight some recent advances on the numerical analysis and the control of fluid-rigid body interactions. The motion of the fluid is governed by the incompressible Navier-Stokes equations and the standard conservation's laws of linear and angular momentum rule the dynamics of the structure. The time variation of the fluid domain (due to the motion of the structure) is not known a priori, so we deal with a free boundary value problem.

We first tackle wellposedness and approximation issues. This situation is by now well understood. We describe some recent global existence results (see, for instance, [1]. The global character of the solutions we obtain is due to the fact that we don't need any assumption concerning the lack of collisions between several rigid bodies or between a rigid body and the boundary. We give estimates of the velocity of the bodies when their mutual distance or the distance to the boundary tends to zero. We next describe and discuss the convergence of a numerical method introduced in [2]. One of the main features of this method is that it is based on a fixed mesh.

The second part of this presentation is devoted to a control problem.

References:

[1] J. A. San Martín, V. Starovoitov, M. Tucsnak,Global weak solutions for the two-dimensional motion of several rigid bodies in an incompressible viscous fluid, *Arch. Ration. Mech. Anal.*, **161** (2002), p. 113–147.

[2] J. S. Martin, J.-F. Scheid, T. Takahashi, M. Tucsnak, Convergence of the Lagrange-Galerkin method for the Equations Modelling the Motion of a Fluid-Rigid System, *SIAM J. on Numerical Analysis*, to appear.

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